Nuclear Science Computing Center at CCNU

# Chiral properties of QCD in strong magnetic fields

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HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505

HTD, S.-T. Li, J.-H. Liu, X.-D. Wang, Phys.Rev.D 105 (2022) 3, 034514

Topology, Flux Tubes and Holographic Models

May 23-27, 2022, ECT*

# Outline

- QCD in strong magnetic fields at zero temperature

  HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 104 (2021) 014505

  - Gell-Mann-Oakes-Renner relation
  - Masses of neutral and charged pseudoscalar mesons

- QCD in strong magnetic fields at nonzero temperature

  HTD, S.-T. Li, J.-H. Liu, X.-D. Wang, PRD 105 (2022)034514

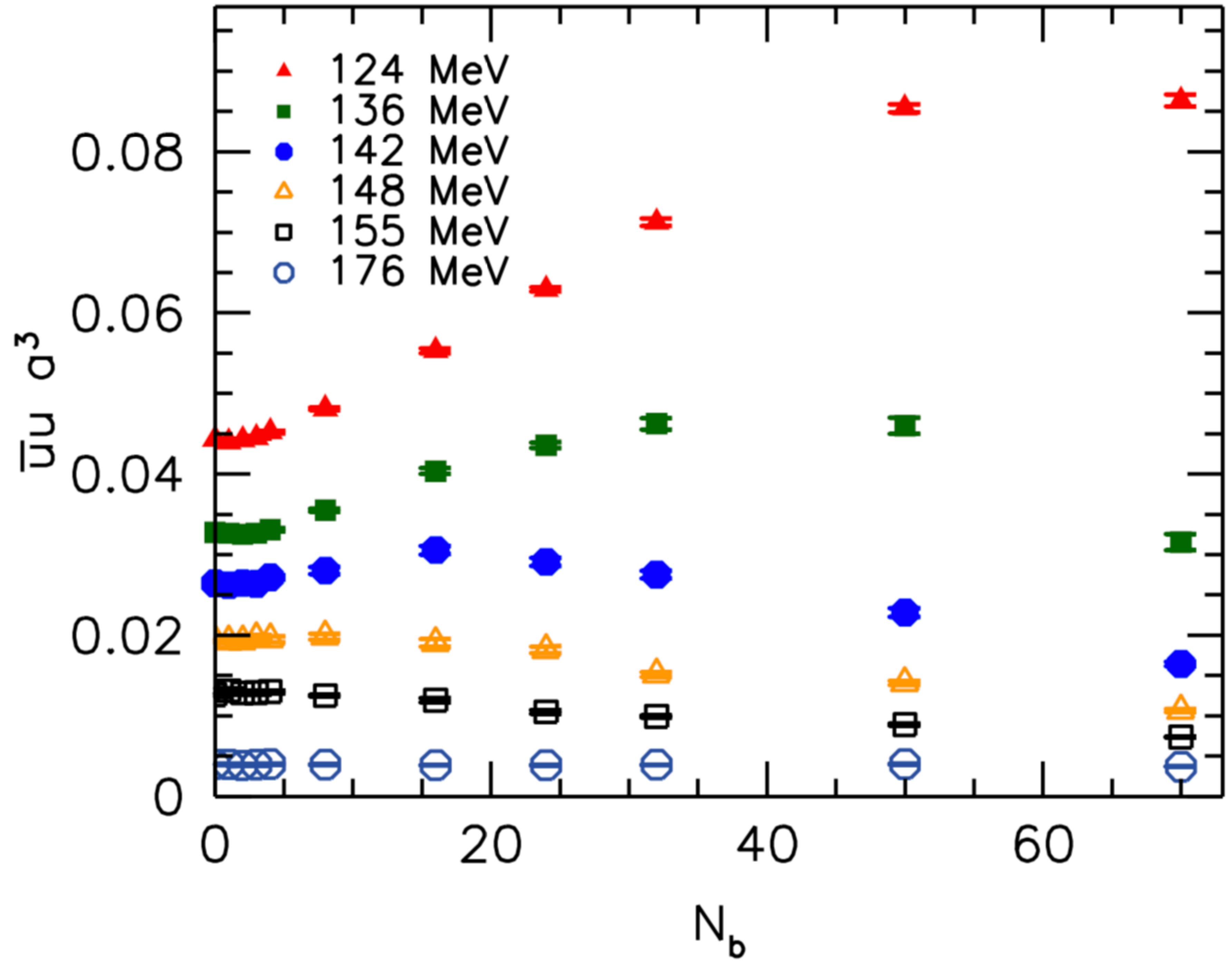
  - Intrinsic connection between (inverse) magnetic catalysis and screening masses of neutral pseudo scalar mesons via Ward-Takahashi identity

# Inverse magnetic catalysis and reduction of $T_{pc}$

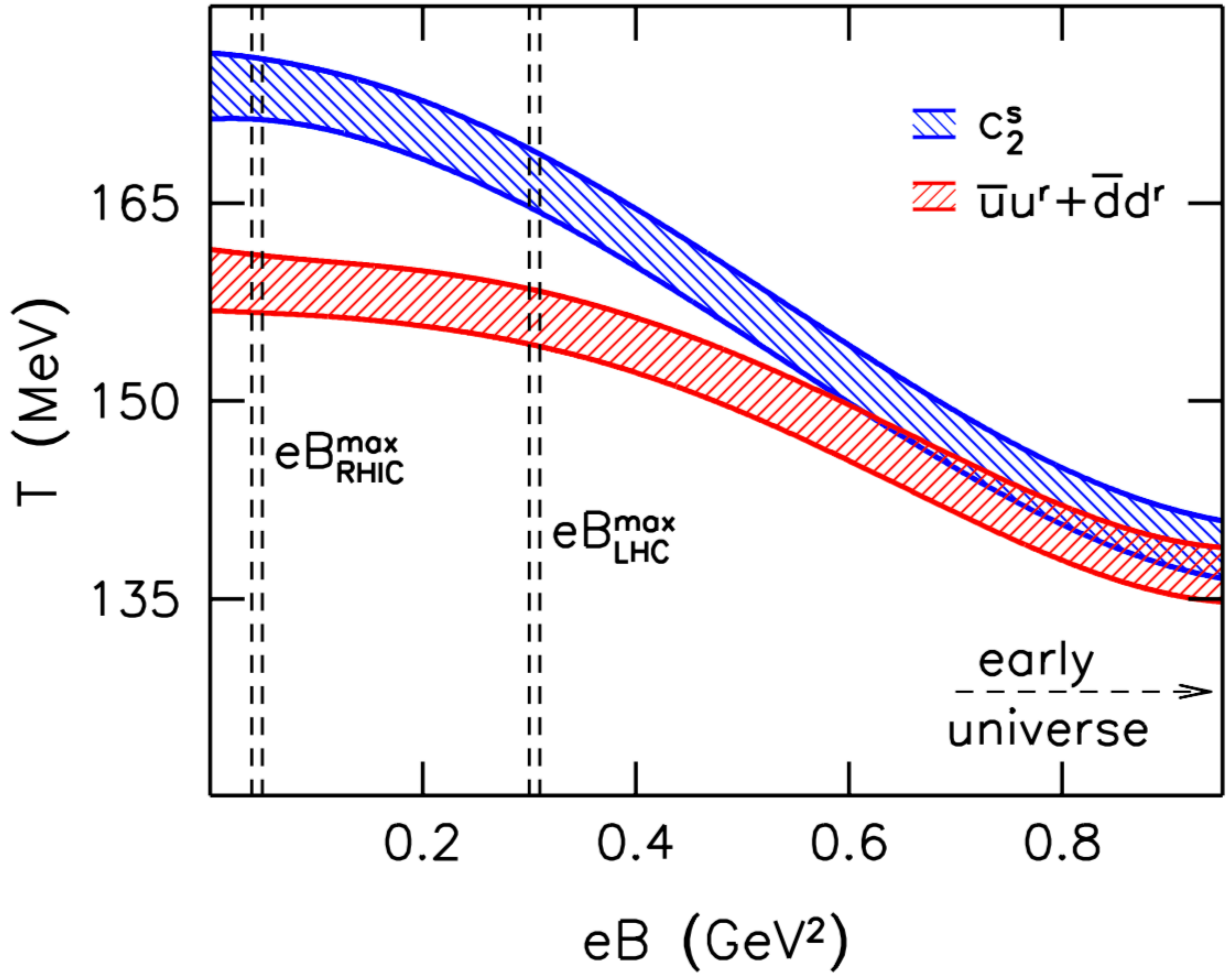
Continuum extrapolated lattice QCD results with physical pion mass

Bali et al., JHEP02(2012)044

## Inverse magnetic catalysis




## eB↑ $T_{pc}$↓

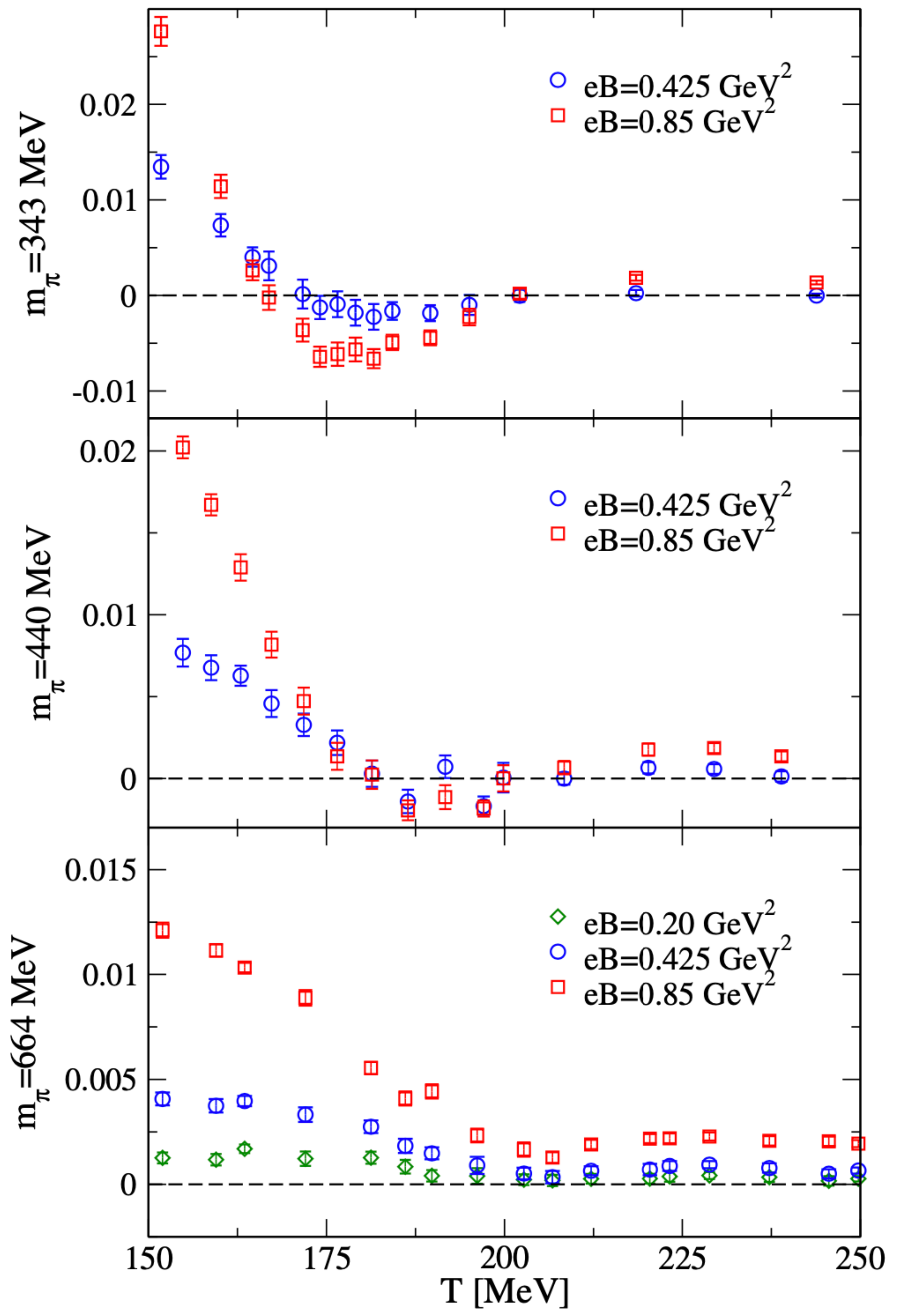


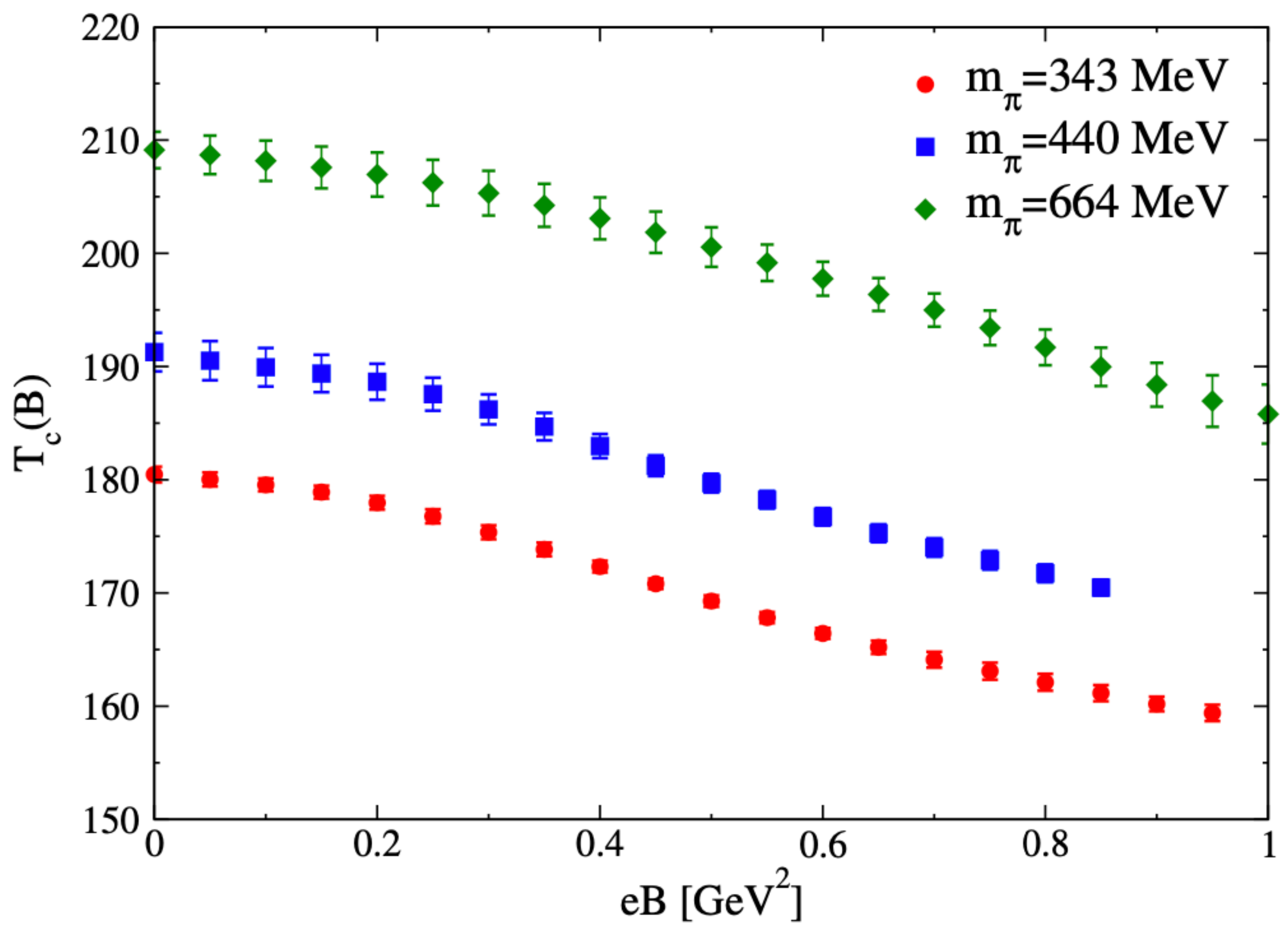

See recent reviews e.g.
G. Cao, arXiv:2103.00456
Andersen et al., Rev. Mod. Phys. 88(2016)02001

Chiral condensate always increases as eB at $T<<T_{pc}$

Reduction of $T_{pc}$ associated with inverse magnetic catalysis?

# IMC and reduction of $T_{pc}$: Not necessarily connected




M. D'Elia et al., Phys.Rev. D 98 (2018) 054509

Similar results from G. Endrodi et al., JHEP07(2019)007

# Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

Explicit chiral symmetry breaking

$$(m_u + m_d)\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2\,(1-\delta_\pi)$$

Spontaneous chiral symmetry breaking

At physical pion mass $\delta_\pi \sim 6\%$

- At T=0, in the weak magnetic field the 2-flavor GMOR relation holds true for chiral (& point-like) pions from LO ChPT Shushpanov and Smilga, PLB402(1997)351
- At eB=/=0, additional pion decay constants appear due to a nonzero pion-to-vacuum transition via the vector electroweak current Fayazbakhsh & Sadooghi, PRD 88(2013)065030

Bali et al., PRL121(2018)072001

Coppola et al., PRD.99 (2019)0540312

# Lattice setup

HTD et al., arXiv:2008.00493,2104.06843

- Symanzik-improved gauge action with HISQ fermions

- T=0: mostly $32^3$x96 lattices, with a=0.117 fm ($a^{-1}$=0.17 GeV), $m_l/m_s$ =1/10 ($M_\pi$ =220 MeV)

- In our setup $f_\pi$= 96.93(2) MeV, $f_K$=112.50(2) MeV, $f_K/f_\pi$=1.1606(3)

FLAG 2019: At physical mass point $f_\pi$= 92.1(6) MeV, $f_K$=110.1(5) MeV, $f_K/f_\pi$=1.1917(37)

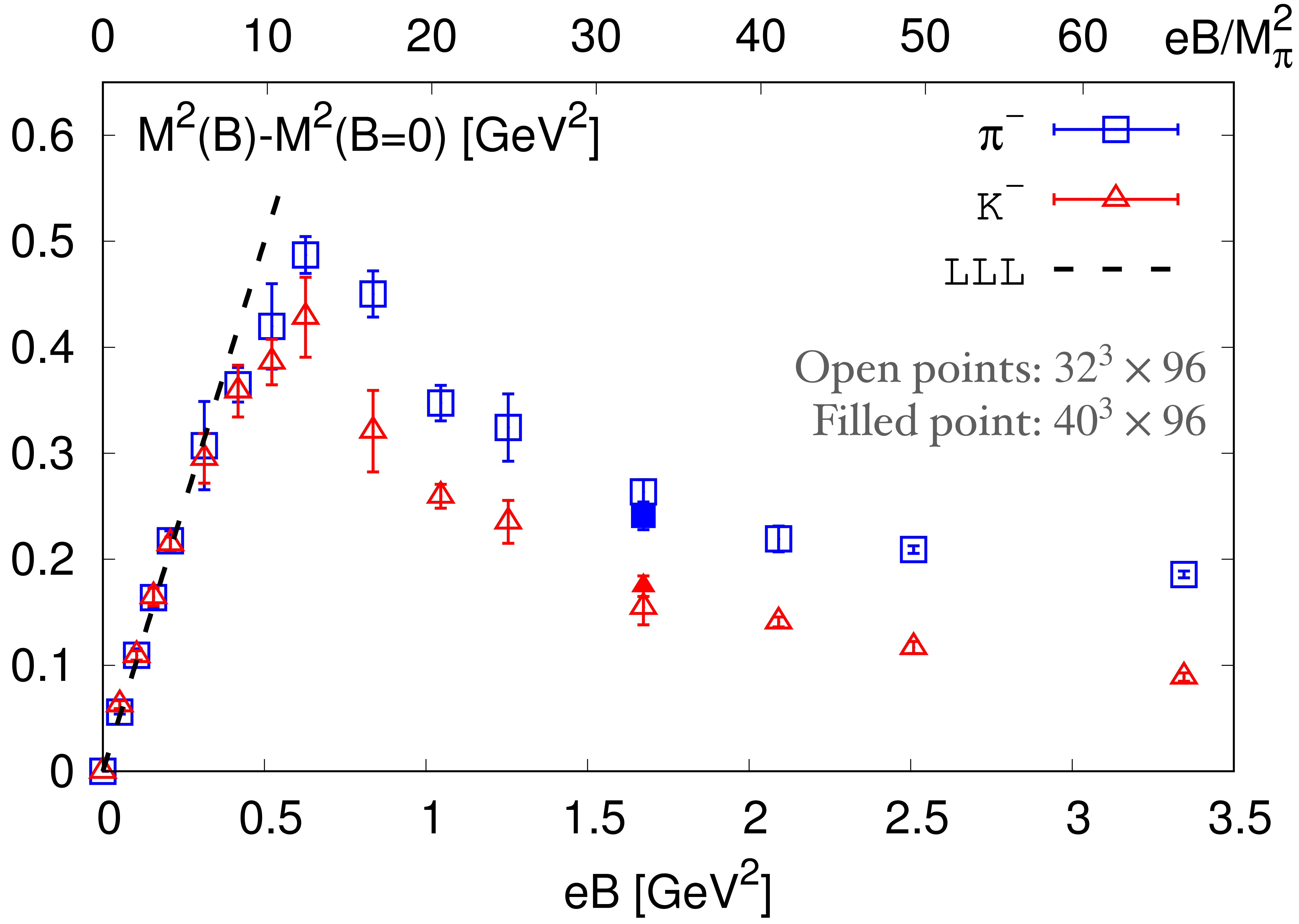
✦ Magnetic field along z direction is quantized as $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$

✦ Magnetic flux: $N_b$=0,1,2,3,4,6,8,10,12,16,20,24,32,48 & 64

✦ 0 ≤ eB ≤ 3.35 $GeV^2$ (~70 $M_\pi^2$)

✦ T=/=0: Fixed scale approach to nonzero T up to 281 MeV

# Masses of charged pseudo scalar mesons




Lowest Landau-Level (LLL):

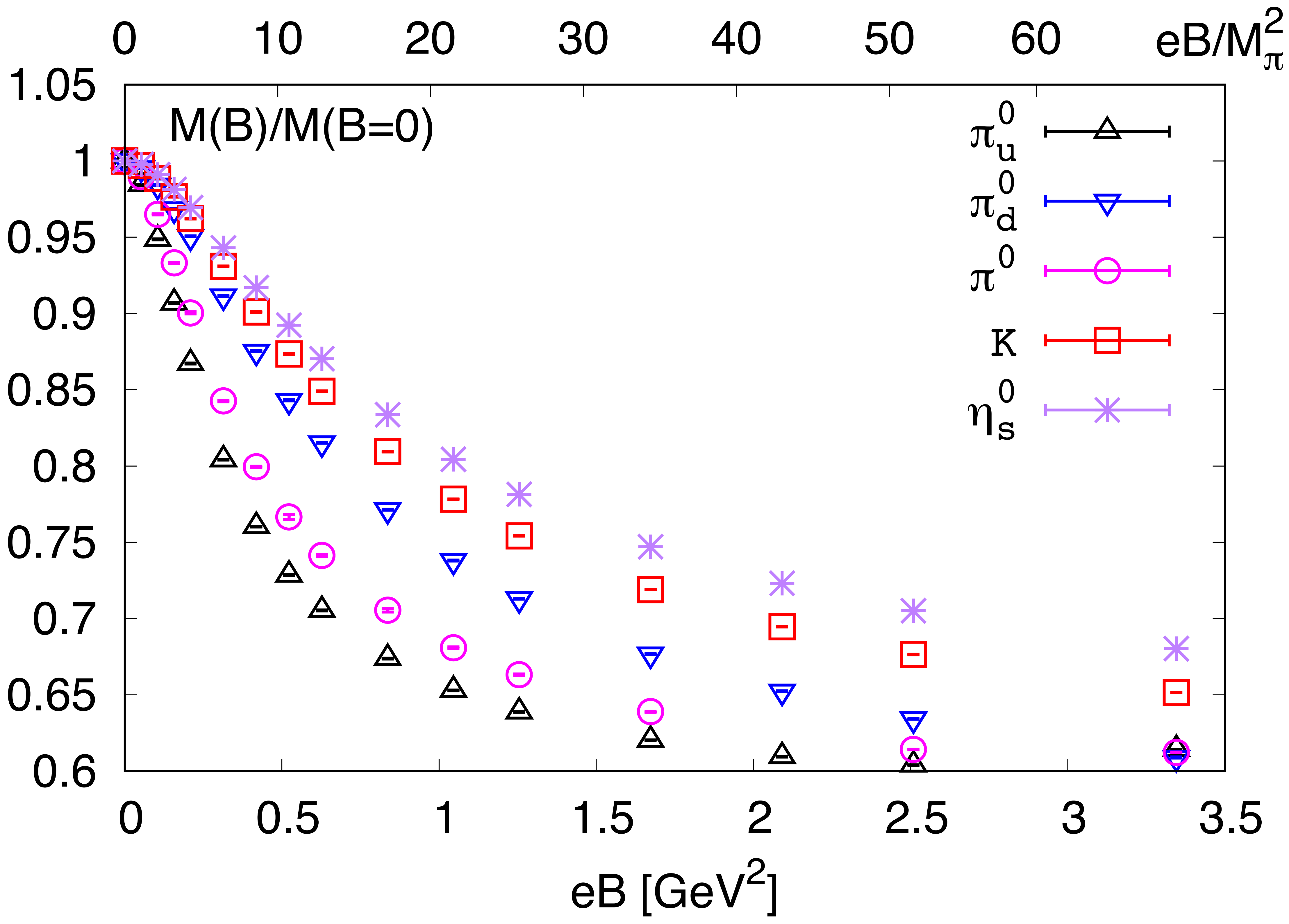
$$M^{\pm}_{\mathrm{ps}}(B) = \sqrt{\left(M^{\pm}_{\mathrm{ps}}(B=0)\right)^2 + |eB|}.$$

In contrast to Quenched QCD results where M increases monotonously with *eB*

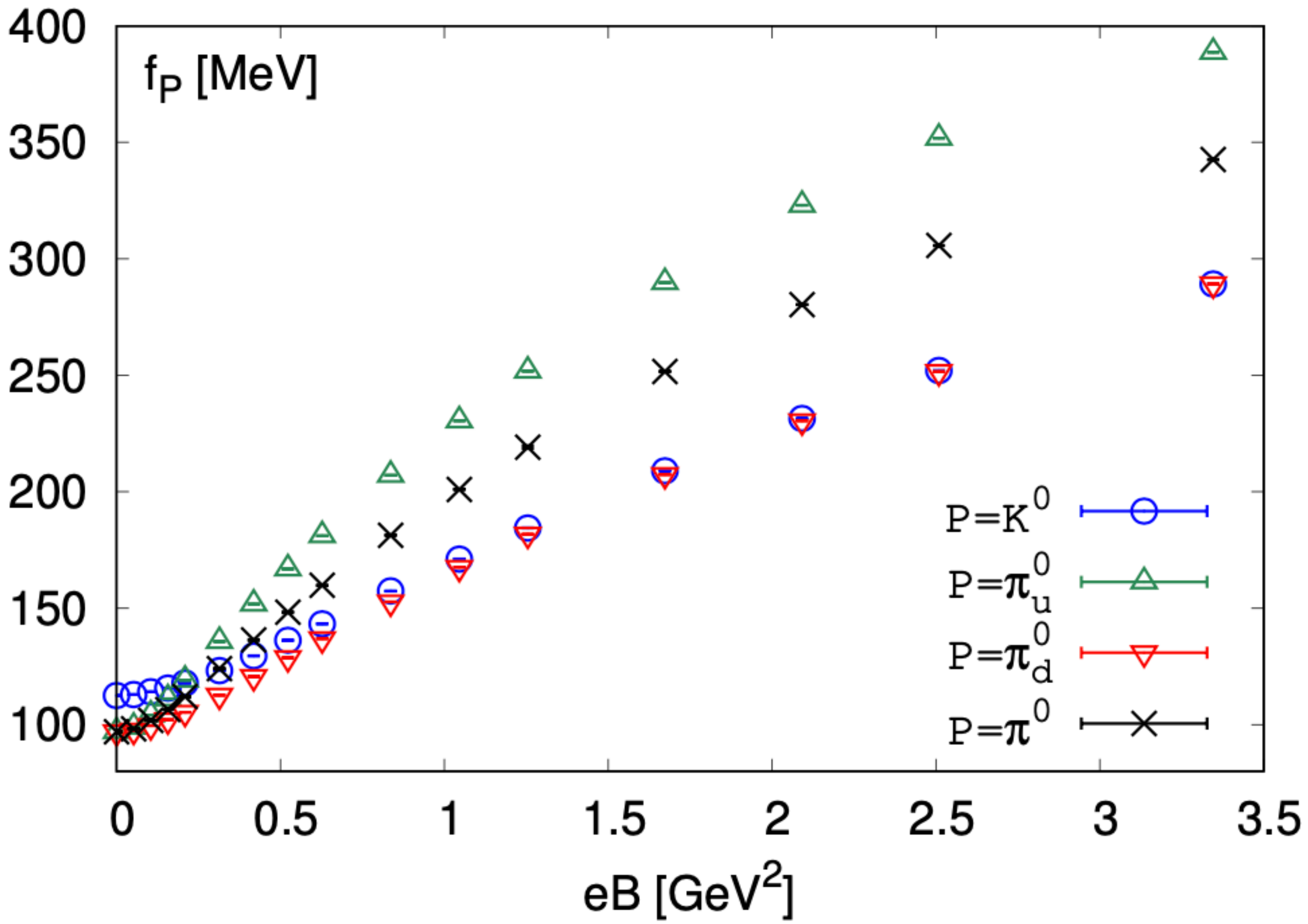
Bali et al., PRD 97, 034505 (2018) Luschevskaya et al, PLB 761 (2016) 393

Not point particles anymore at eB>0.3 GeV$^2$ ! Effects from dynamic quarks?

# Masses and decay constants of neutral pseudo scalar mesons




Mass of neutral pseudo scalar meson decreases with eB




Decay constants of neutral pseudo scalar meson increases with eB

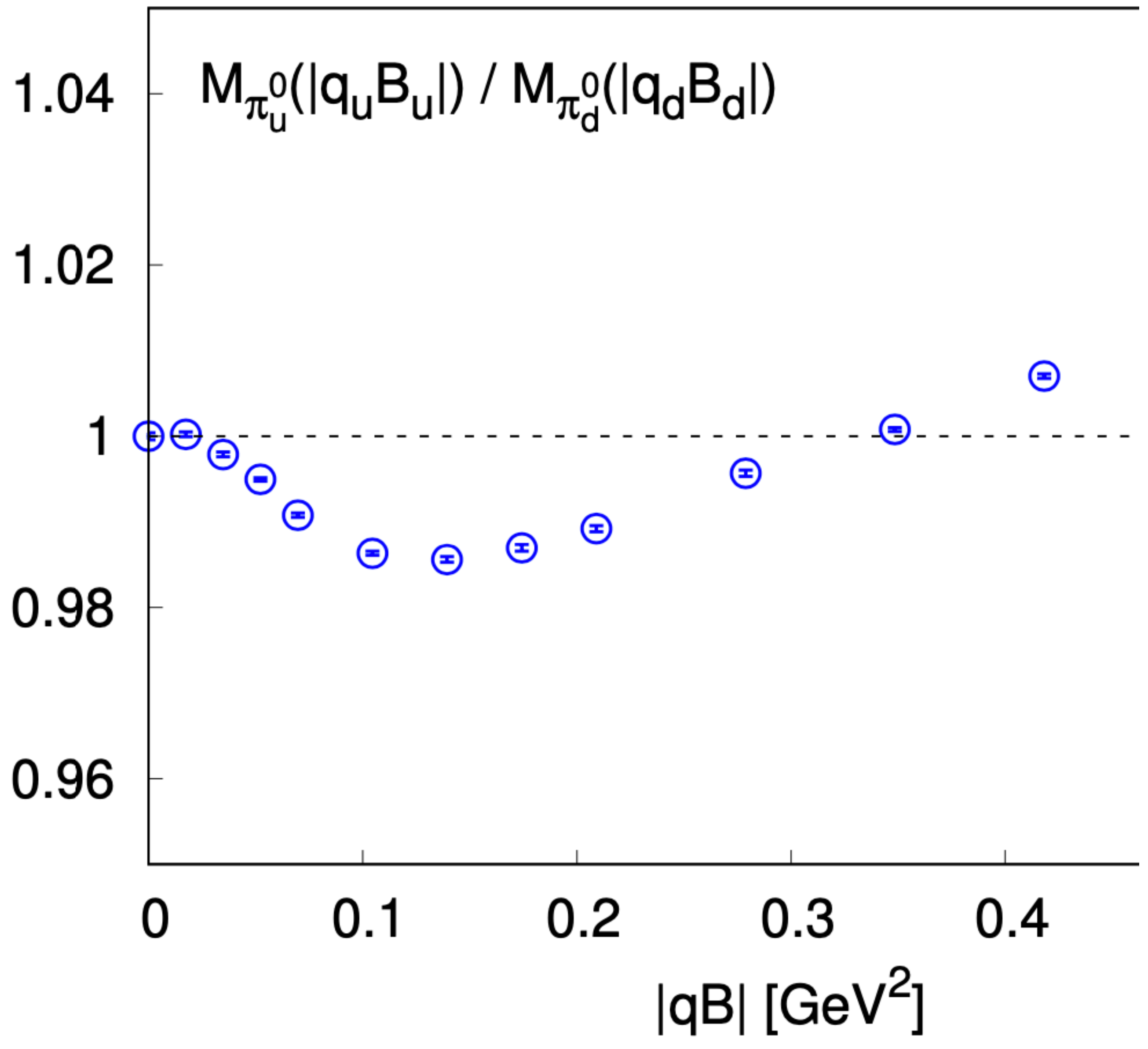
# qB scaling of up and down quark components of $M_{\pi^0}, f_{\pi^0}$ and chiral condensates

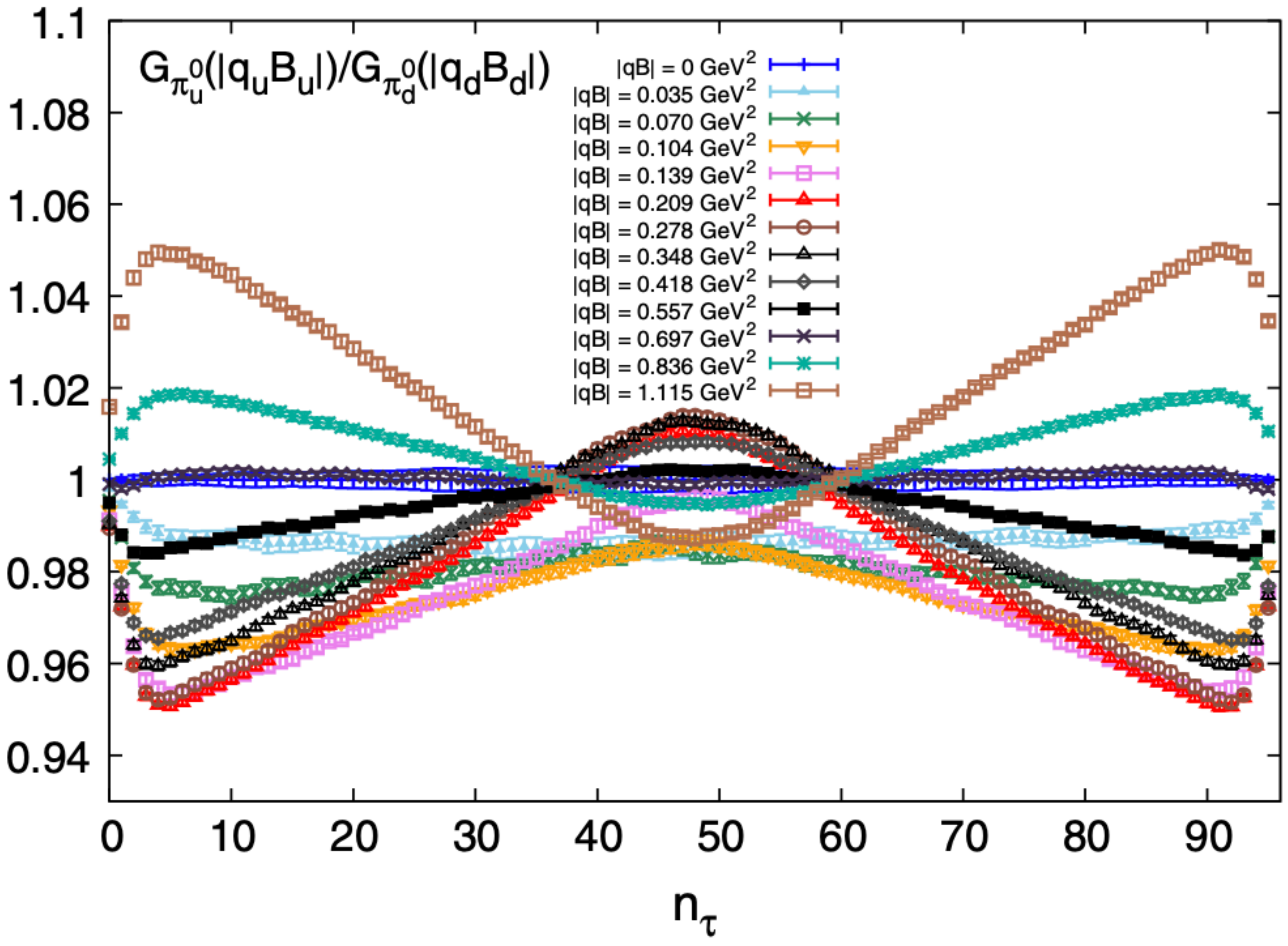
$\mathcal{O}_u$ and $\mathcal{O}_d$ agree at the same value of $|qB|=|q_u B_u|=|q_d B_d|$

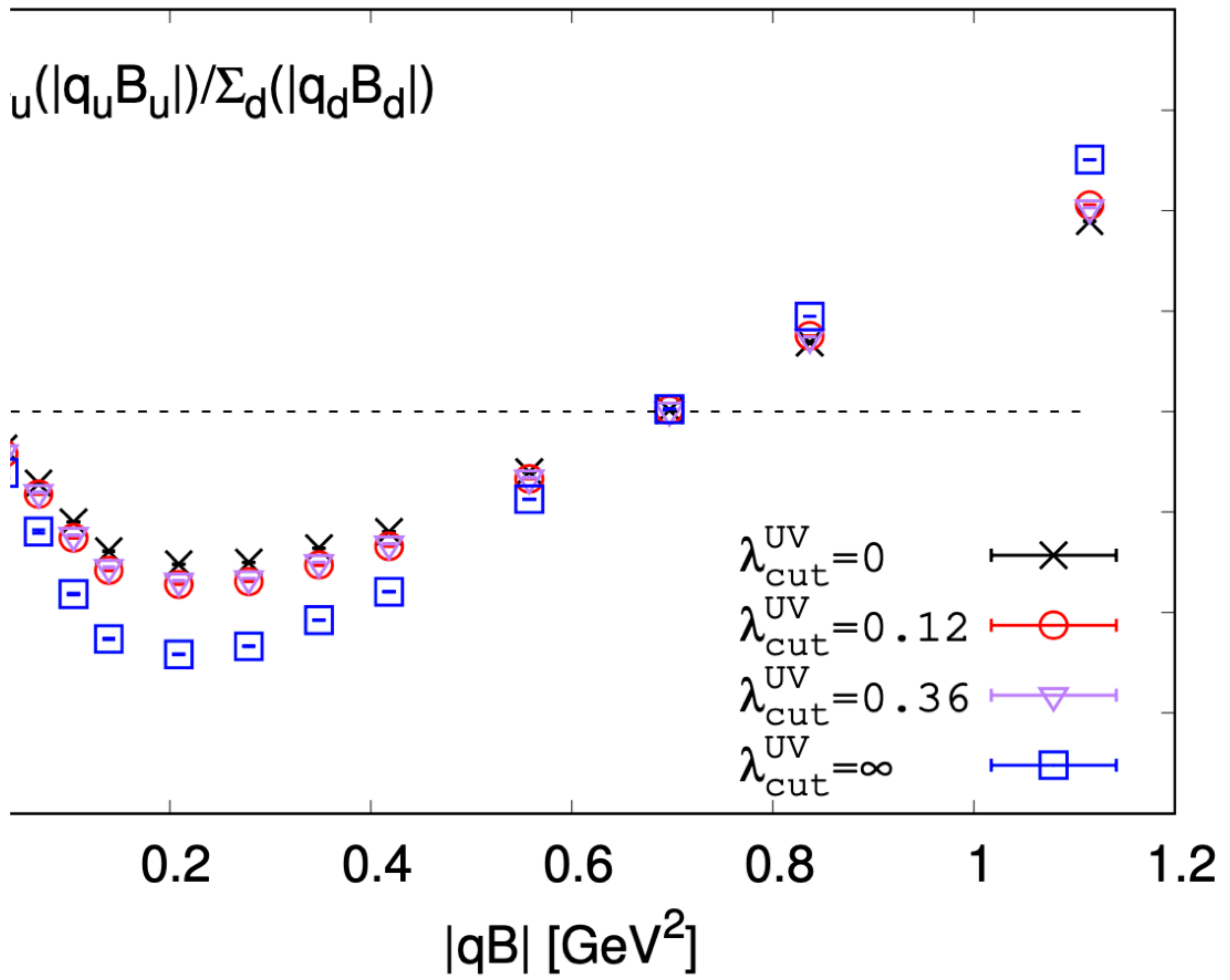

$$q_u = \frac{2}{3}e \qquad q_d = -\frac{1}{3}e$$

# qB scaling of up and down quark components of $M_{\pi^0}, f_{\pi^0}$ and chiral condensates

$\mathcal{O}_u$ and $\mathcal{O}_d$ agree at the same value of $|qB|=|q_uB_u|=|q_dB_d|$




The qB scaling of these quantities originates from pi0 correlation function

# UV divergence of chiral condensate

$$(m_u + m_d)\ \left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2 f_\pi^2 M_\pi^2 \left(1 - \delta_\pi\right)$$

- UV-divergence term dominates by the linear-in-quark-mass term

$$\langle\bar{\psi}\psi\rangle_{q,UV-div} = \frac{\nu_f}{2}\left(\frac{\pi}{a}\right)^2 \frac{1}{(2\pi)^2} m_q + \frac{\nu_f}{2} ln(\frac{a m_q}{2\pi}) \frac{1}{(2\pi)^2} m_q^3.$$

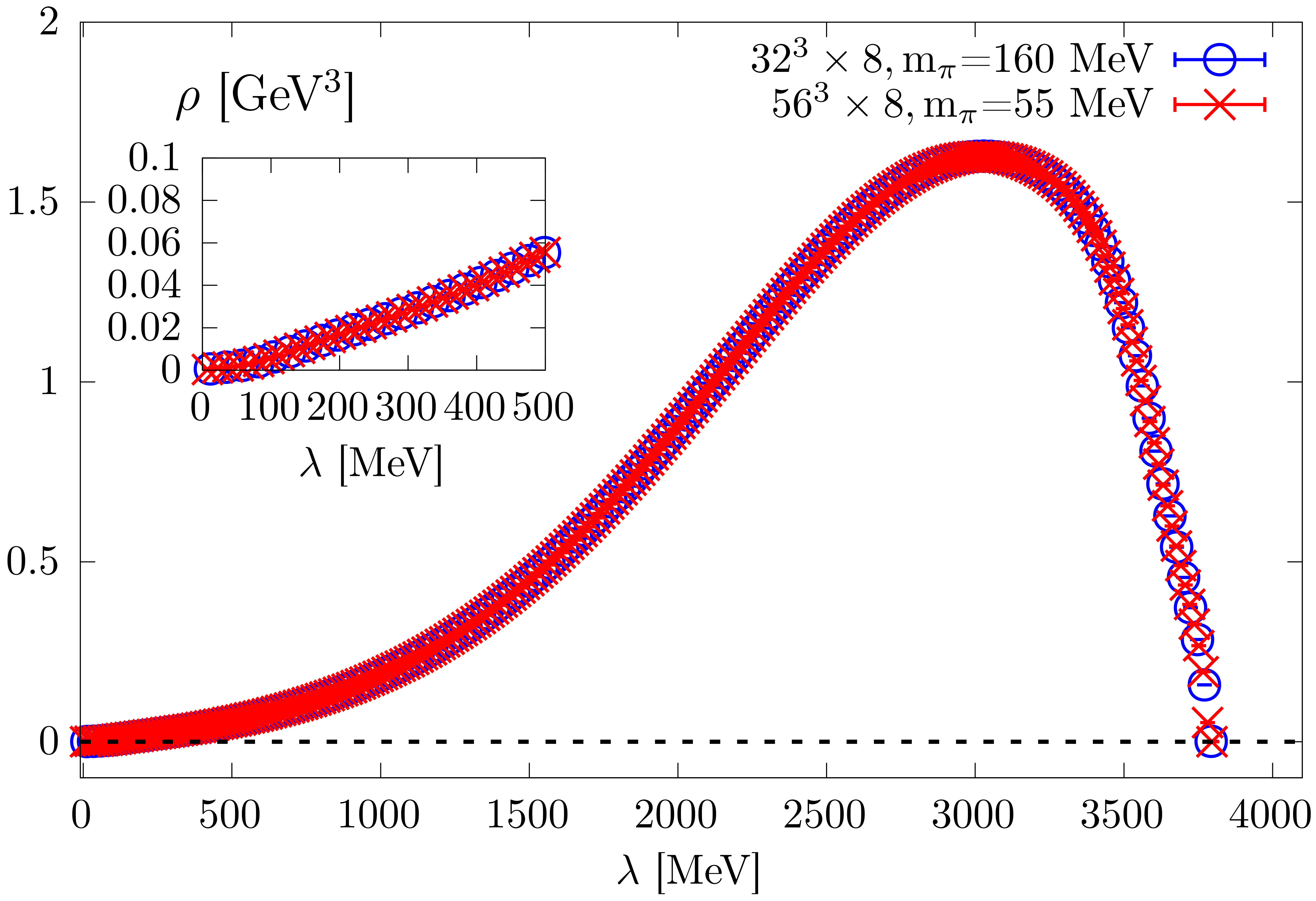
- Commonly used methods to get rid of the UV-divergence part

Subtracted chiral condensate: $\langle\bar{\psi}\psi\rangle_{sub} = \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s$ ✗

Zero T/eB subtraction: $\langle\bar{\psi}\psi\rangle_{UV free} = \langle\bar{\psi}\psi\rangle_l(eB \neq 0) - \langle\bar{\psi}\psi\rangle_l(eB = 0)$ ✗

# A complete Eigenvalue spectrum

$$\langle\bar{\psi}\psi\rangle = \int_0^\infty \frac{4m_l\,\rho}{\lambda^2+m_l^2}\,\mathrm{d}\lambda$$



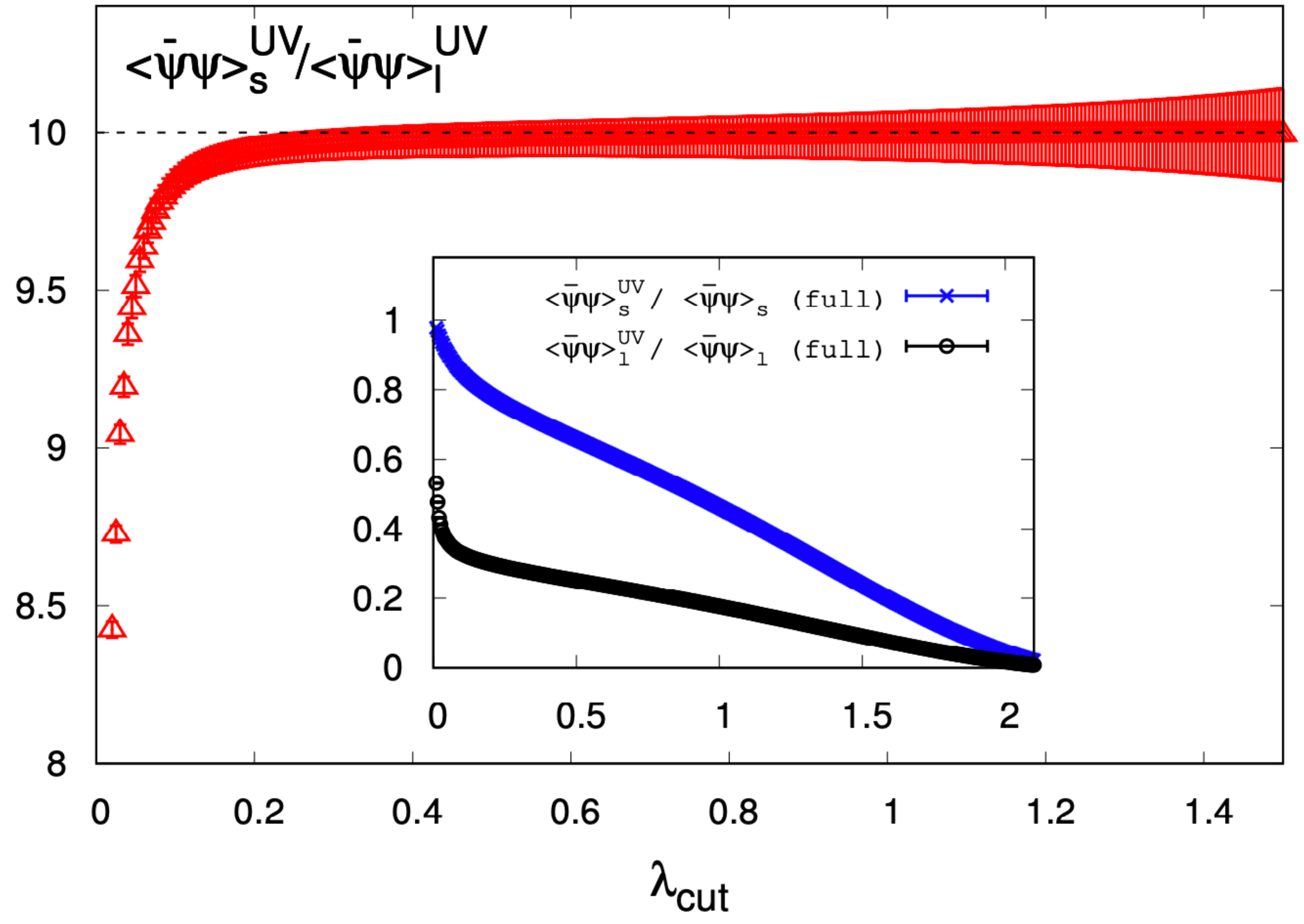

via Chebyshev Polynomial filtering technique

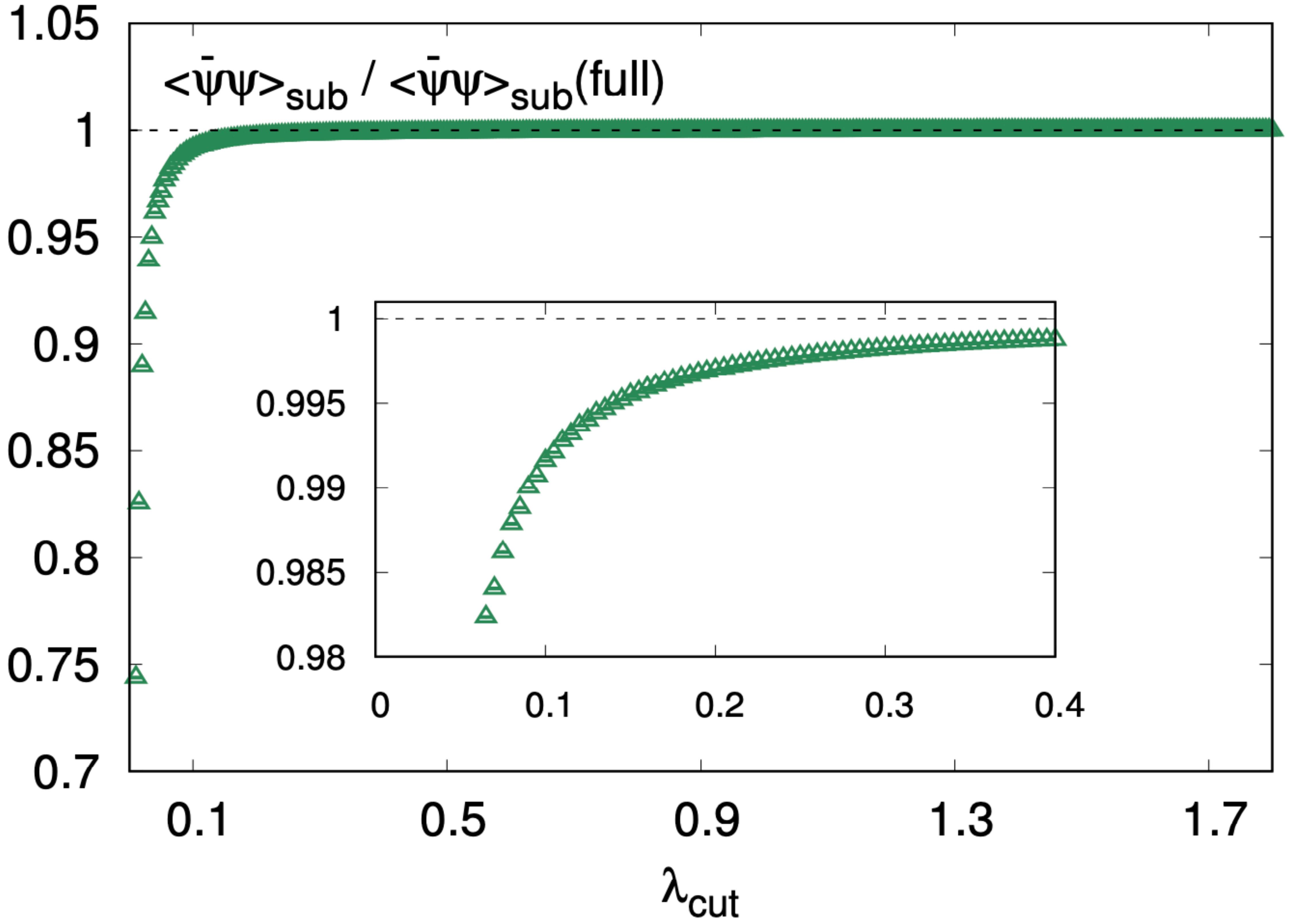
HTD, S.-T. Li, S. Mukherjee, A. Tomiya, X.-D. Wang, Y. Zhang, Phys. Rev. Lett. 126 (2021) 082001

# UV-free chiral condensate

$$\langle\bar\psi\psi\rangle^{\rm UV}_{l,s} = \int_{\lambda^{\rm UV}_{cut}}^{\infty} \frac{2\, m_{l,s}\, \rho(\lambda)}{\lambda^2 + m^2_{l,s}}\, d\lambda.$$

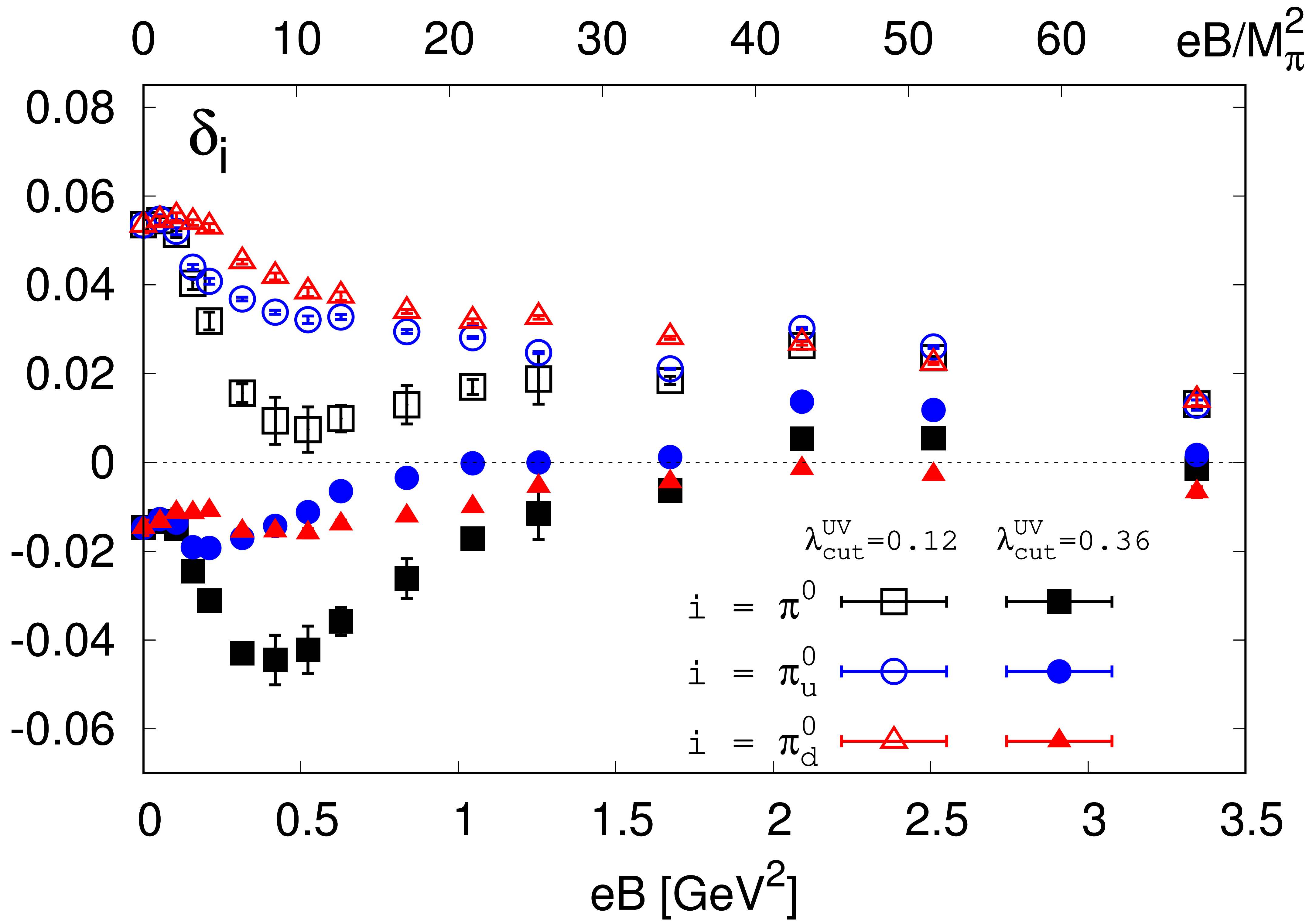
$$\langle\bar\psi\psi\rangle_{sub} \equiv \langle\bar\psi\psi\rangle_l - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_s = \int_0^\infty \frac{2m_l\,(m_s^2 - m_l^2)\rho(\lambda)}{(\lambda^2+m_l^2)(\lambda^2+m_s^2)}\, d\lambda.$$




$$\lambda^{\rm UV}_{cut} \in [0.12, 0.36]$$

# Corrections to Gell-Mann-Oakes-Renner relation




$$4m_u \langle\bar\psi\psi\rangle_u = 2f^2_{\pi^0_u} M^2_{\pi^0_u}\,(1-\delta_{\pi^0_u})$$
$$4m_d \langle\bar\psi\psi\rangle_d = 2f^2_{\pi^0_d} M^2_{\pi^0_d}\,(1-\delta_{\pi^0_d})\,.$$

$$(m_u+m_d)\ \big(\langle\bar\psi\psi\rangle_u + \langle\bar\psi\psi\rangle_d\big) = 2f_\pi^2 M_\pi^2\,(1-\delta_\pi)$$

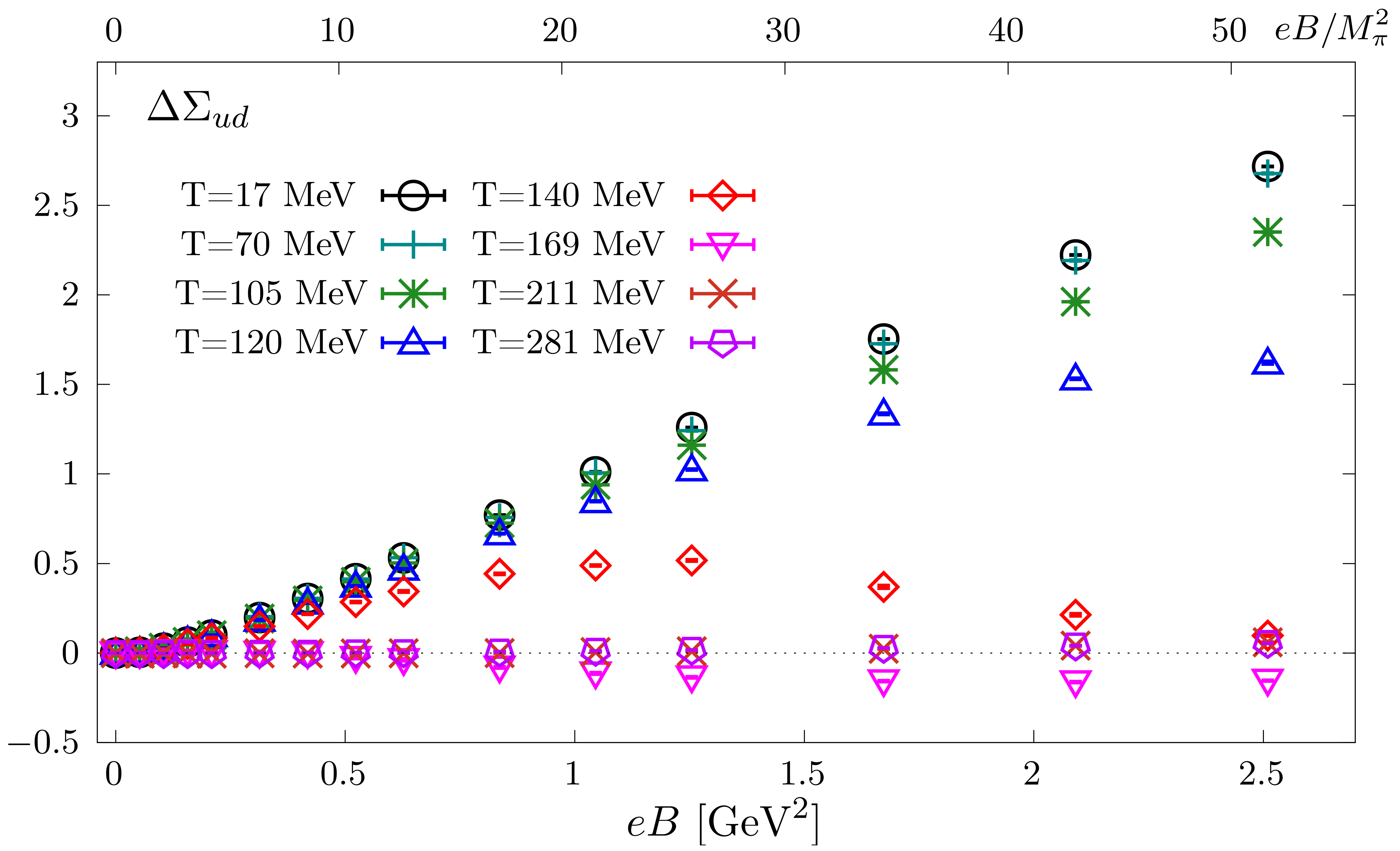
$|\delta_i|$ is less than 6%

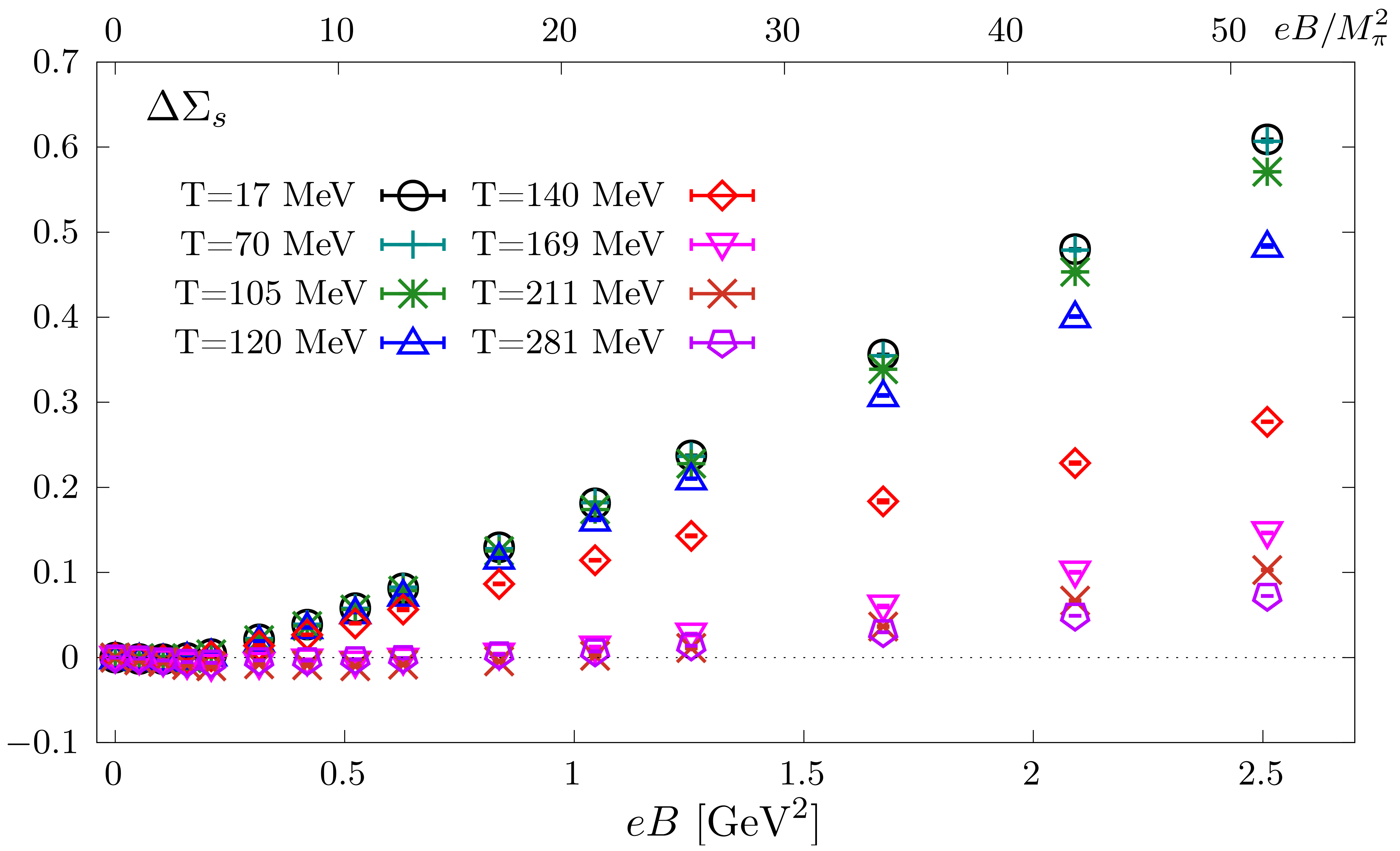
# Summary for T=0 results

# (Inverse) magnetic catalysis at T≠0

$$\Delta\Sigma_{ud}(B,T) = \frac{m_u + m_d}{2M_\pi^2 f_\pi^2} \sum_{f=u,d} \left(\langle\bar{\psi}\psi\rangle_f(B,T) - \langle\bar{\psi}\psi\rangle_f(0,T)\right)$$

$$\Delta\Sigma_s(B,T) = \frac{m_d + m_s}{2M_K^2 f_K^2} \left(\langle\bar{\psi}\psi\rangle_s(B,T) - \langle\bar{\psi}\psi\rangle_s(0,T)\right)$$




Both IMC and MC are observed in $\Delta\Sigma_{ud}$

See similar results of $\Delta\Sigma_{ud}$ in e.g. Bali et al., PRD86(2012)071502

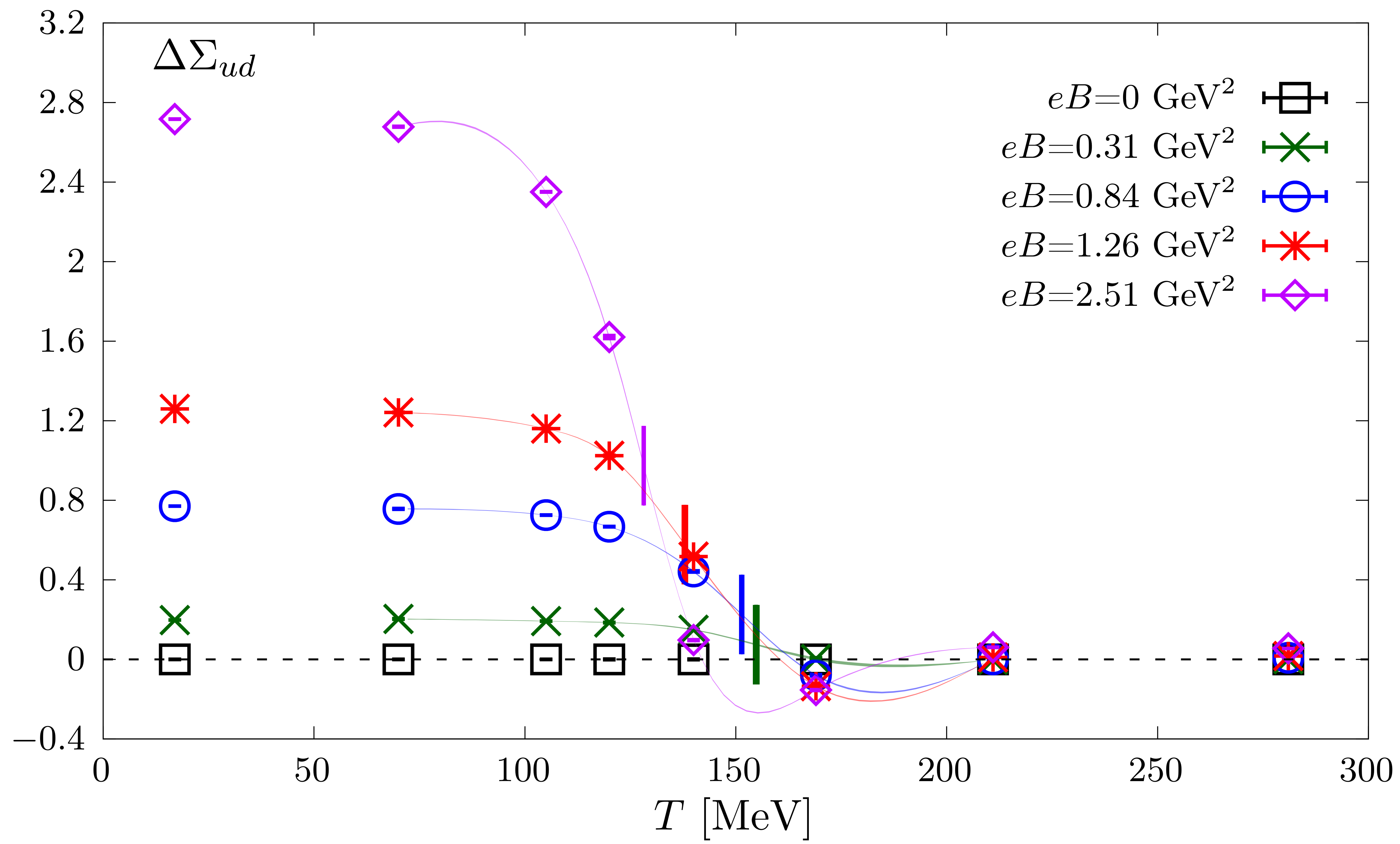
Only MC is observed in $\Delta\Sigma_s$

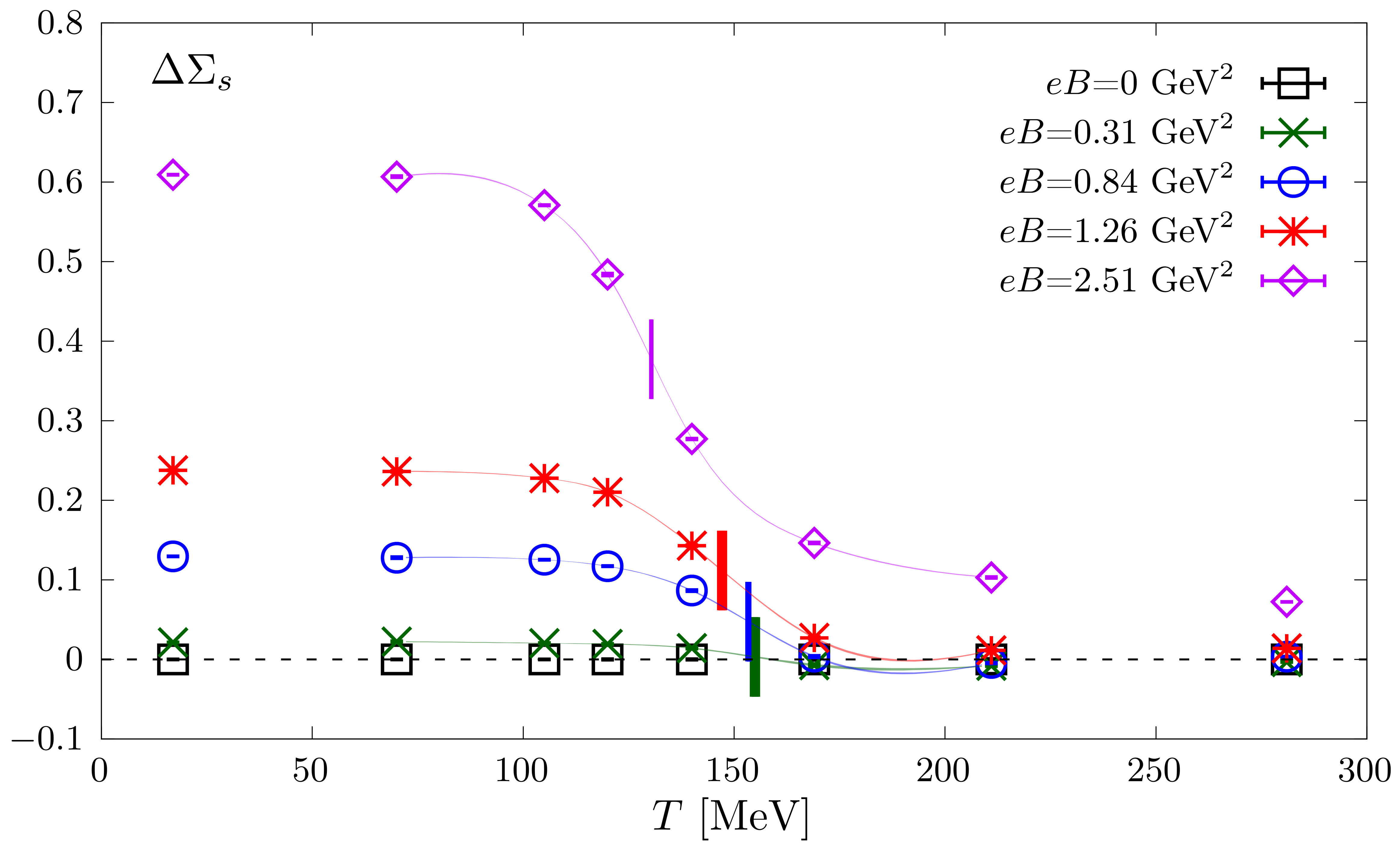
HTD, S.-T. Li, J.-H. Liu, X.-D. Wang, Phys.Rev.D 105 (2022) 3, 034514

# $T_{pc}$ determined from inflection points of $\Delta\Sigma_{ud}$ and $\Delta\Sigma_s$

$$\Delta\Sigma_{ud}(B,T) = \frac{m_u + m_d}{2M_\pi^2 f_\pi^2} \sum_{f=u,d} \left(\langle\bar{\psi}\psi\rangle_f(B,T) - \langle\bar{\psi}\psi\rangle_f(0,T)\right)$$

$$\Delta\Sigma_s(B,T) = \frac{m_d + m_s}{2M_K^2 f_K^2} \left(\langle\bar{\psi}\psi\rangle_s(B,T) - \langle\bar{\psi}\psi\rangle_s(0,T)\right)$$




The transition temperature $T_{pc}$ always decreases as eB increases

# Chiral condensates v.s. screening masses of neutral PS mesons

Ward Identity: $\left\langle O \frac{\delta S_{\rm QCD}}{\delta \alpha^j(x)} \right\rangle = \left\langle \frac{\delta O}{\delta \alpha^j(x)} \right\rangle$

At nonzero magnetic field: $D_\mu \to \tilde{D}_\mu = \partial_\mu - \mathrm{i}gG_\mu - \mathrm{i}eA_\mu Q^3, \quad Q^3 = \frac{1}{6}\sigma^0 + \frac{1}{2}\sigma^3 = \frac{1}{6}\mathbf{1} + t^3$

Consider $\mathcal{O}$ as a pseudoscalar operator, and integrate the WI over space-time:

$$\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d = (m_u + m_d)\,\chi_{\pi^0}$$
$$\langle\bar{\psi}\psi\rangle_d + \langle\bar{\psi}\psi\rangle_s = (m_d + m_s)\,\chi_{K^0}$$
$$\langle\bar{\psi}\psi\rangle_s = m_s \chi_{\eta^0_{s\bar{s}}}$$

HTD et al., PRD 2022,2021

See also cases at eB=0, Kilcup and Sharpe, NPB 1987

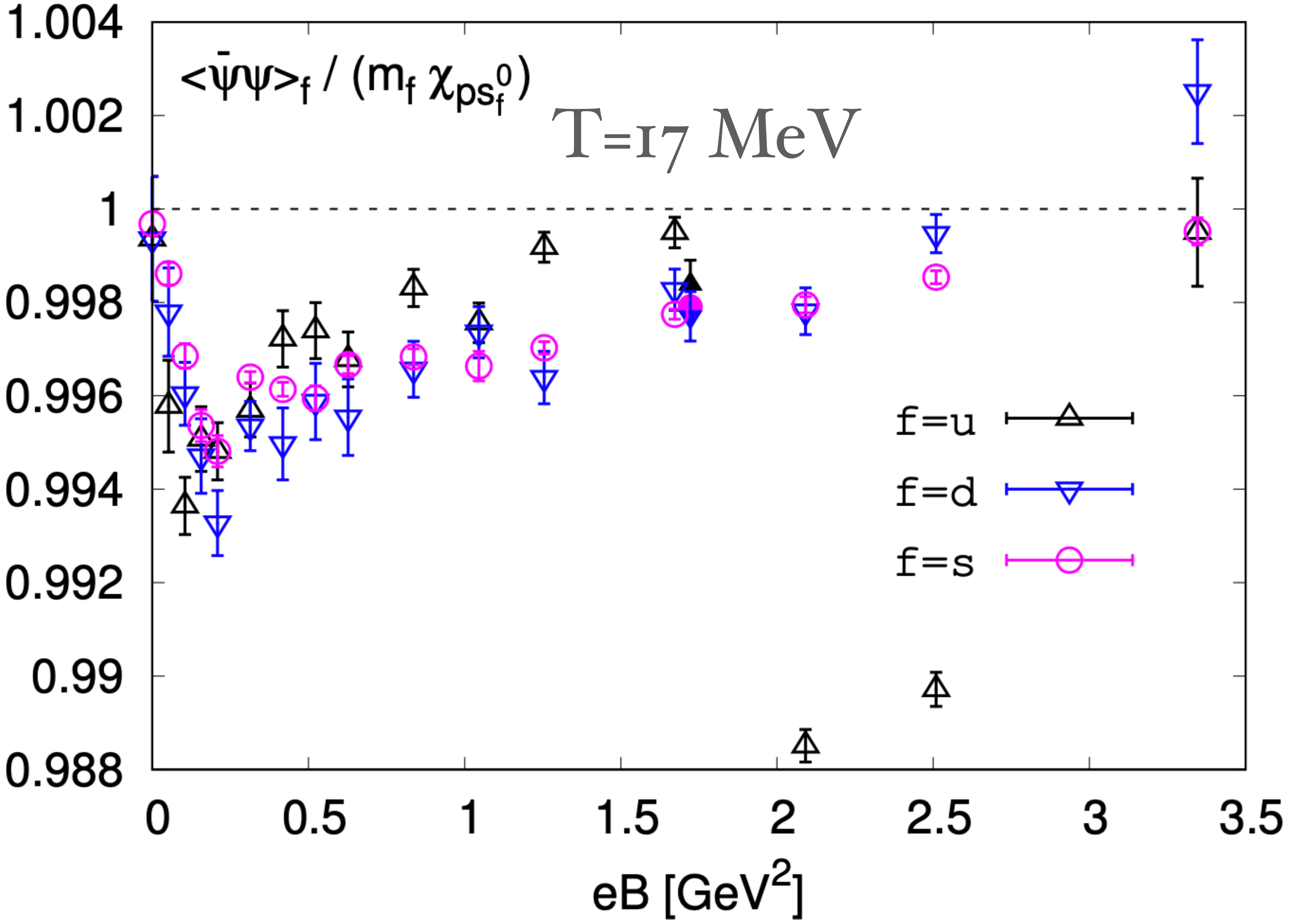
Hadron susceptibility: Integral of spatial correlation function

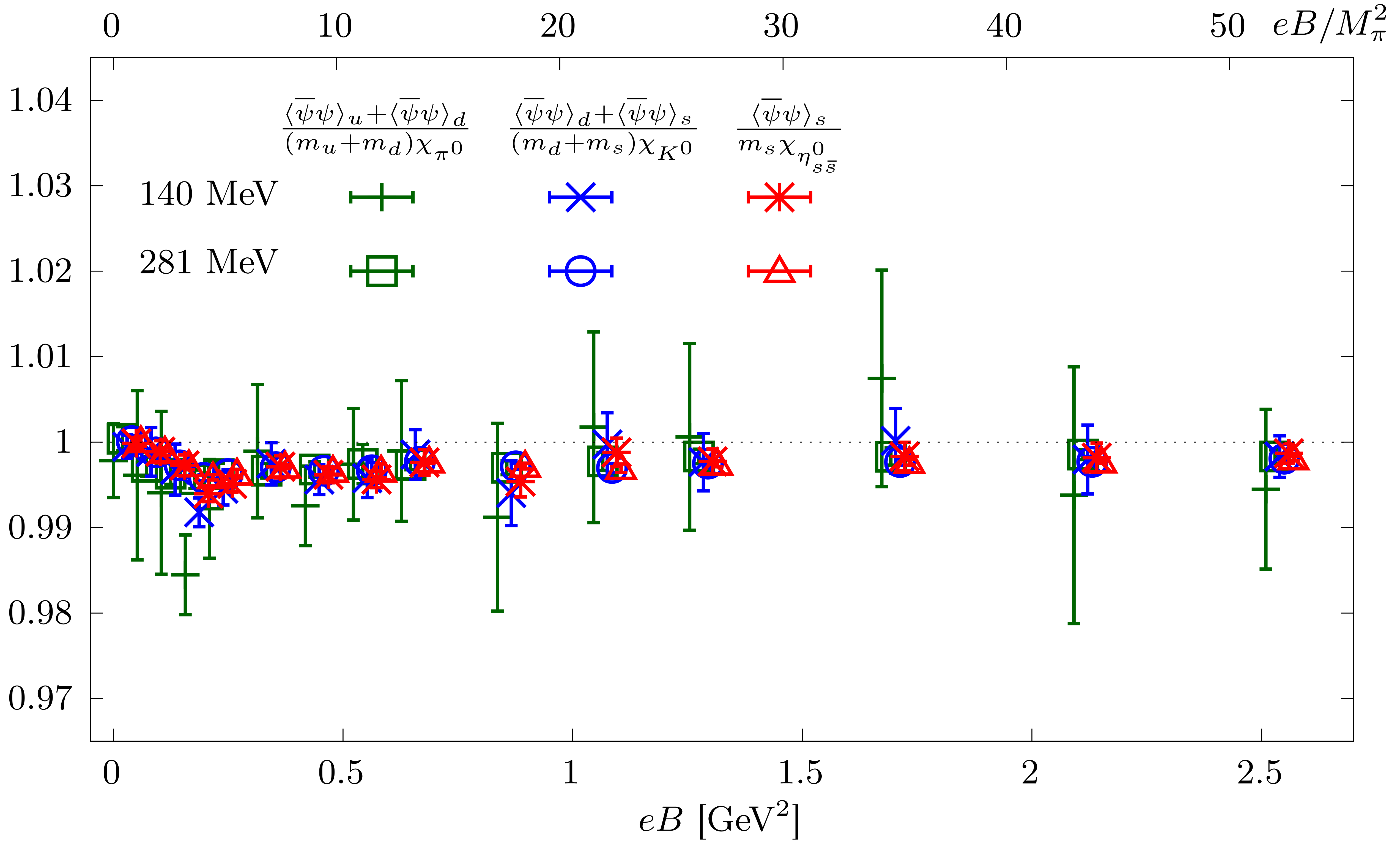
$$\chi_H = \int \mathrm{d}z\, G_H(z)$$

Screening mass $M_H$:

$$\lim_{z\to\infty} G_H(z) = A_H e^{-M_H z}$$

# Numerical demonstration of the integrated Ward Identities




HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505

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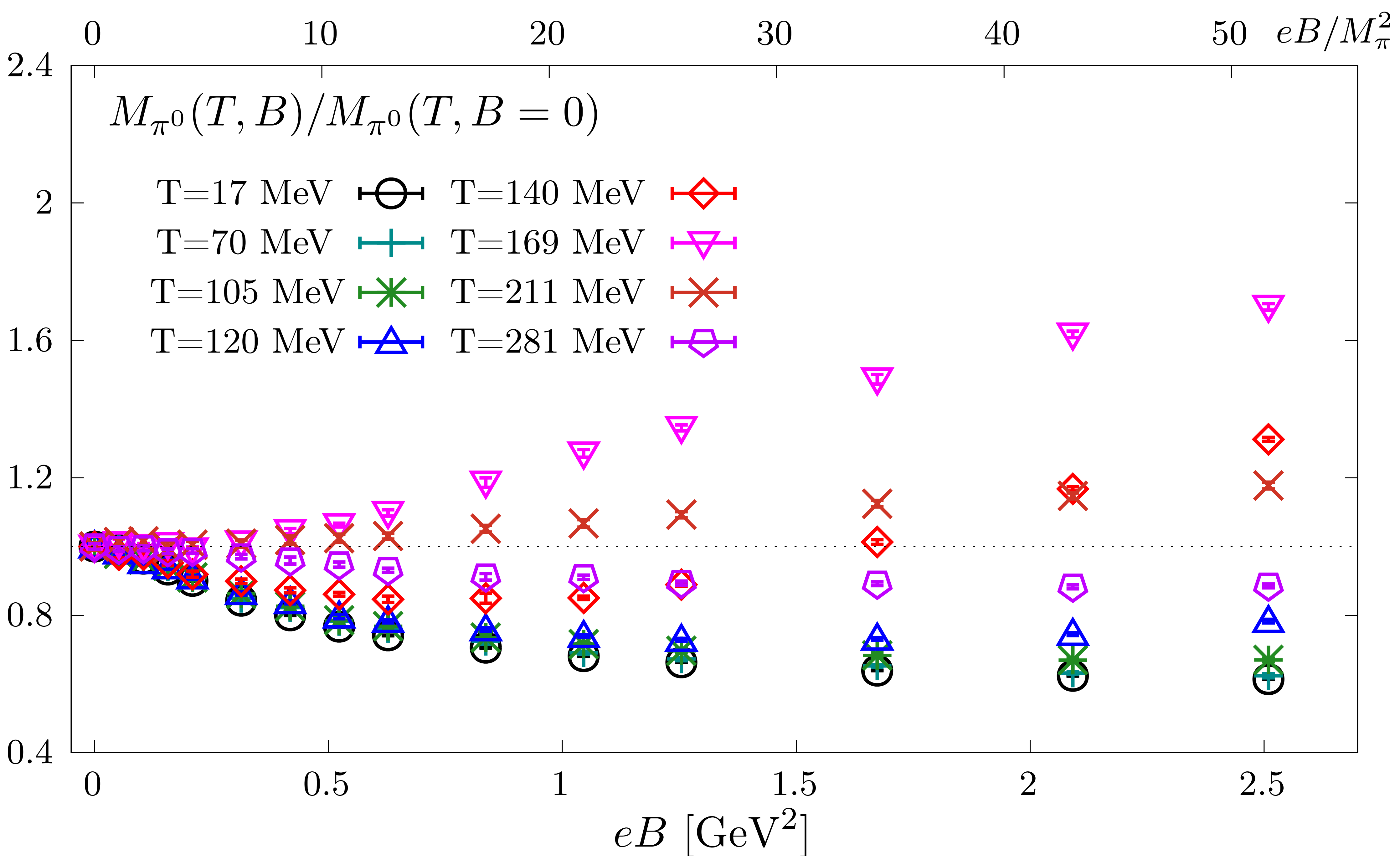
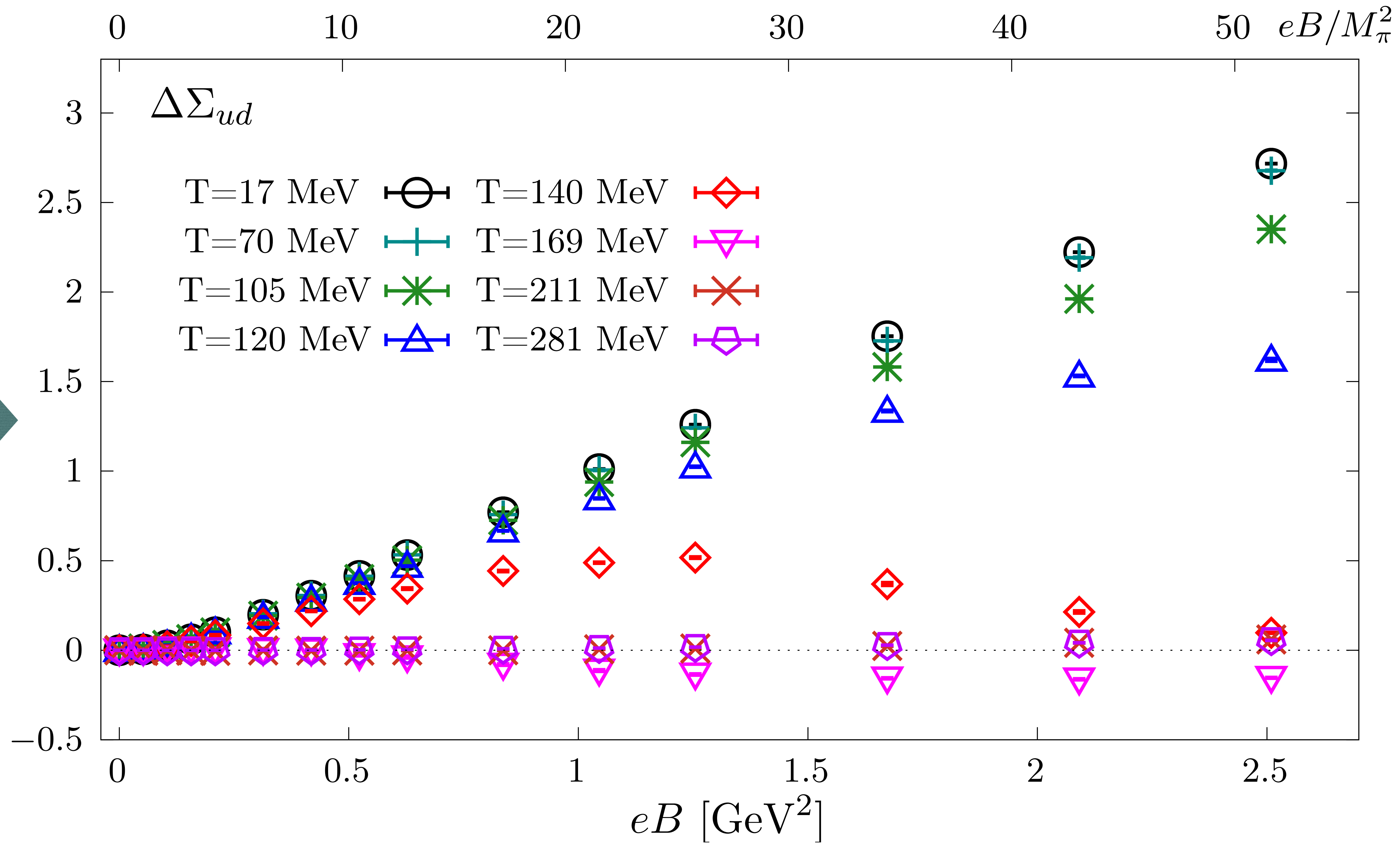
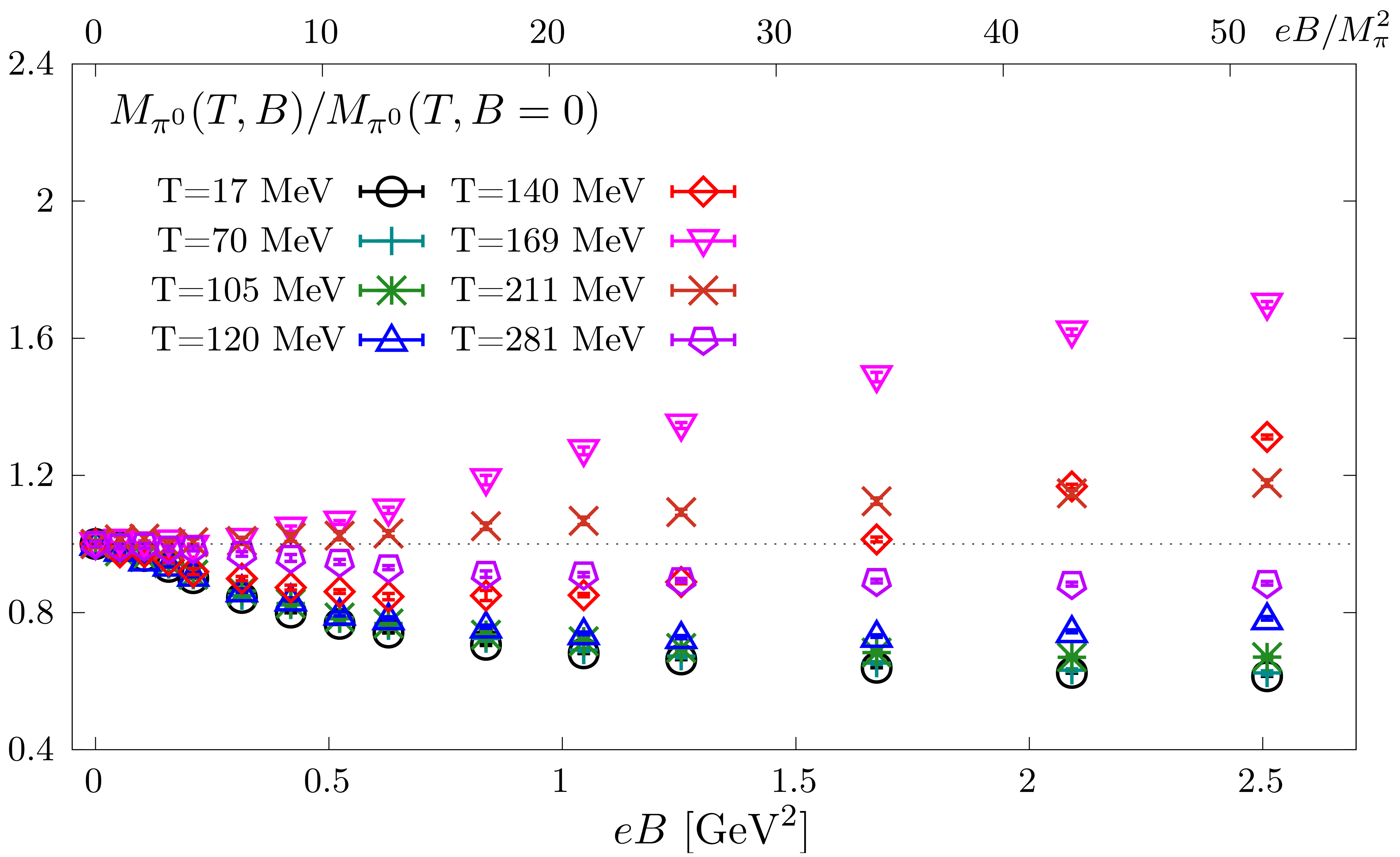
# Connection between screening masses and chiral condensates



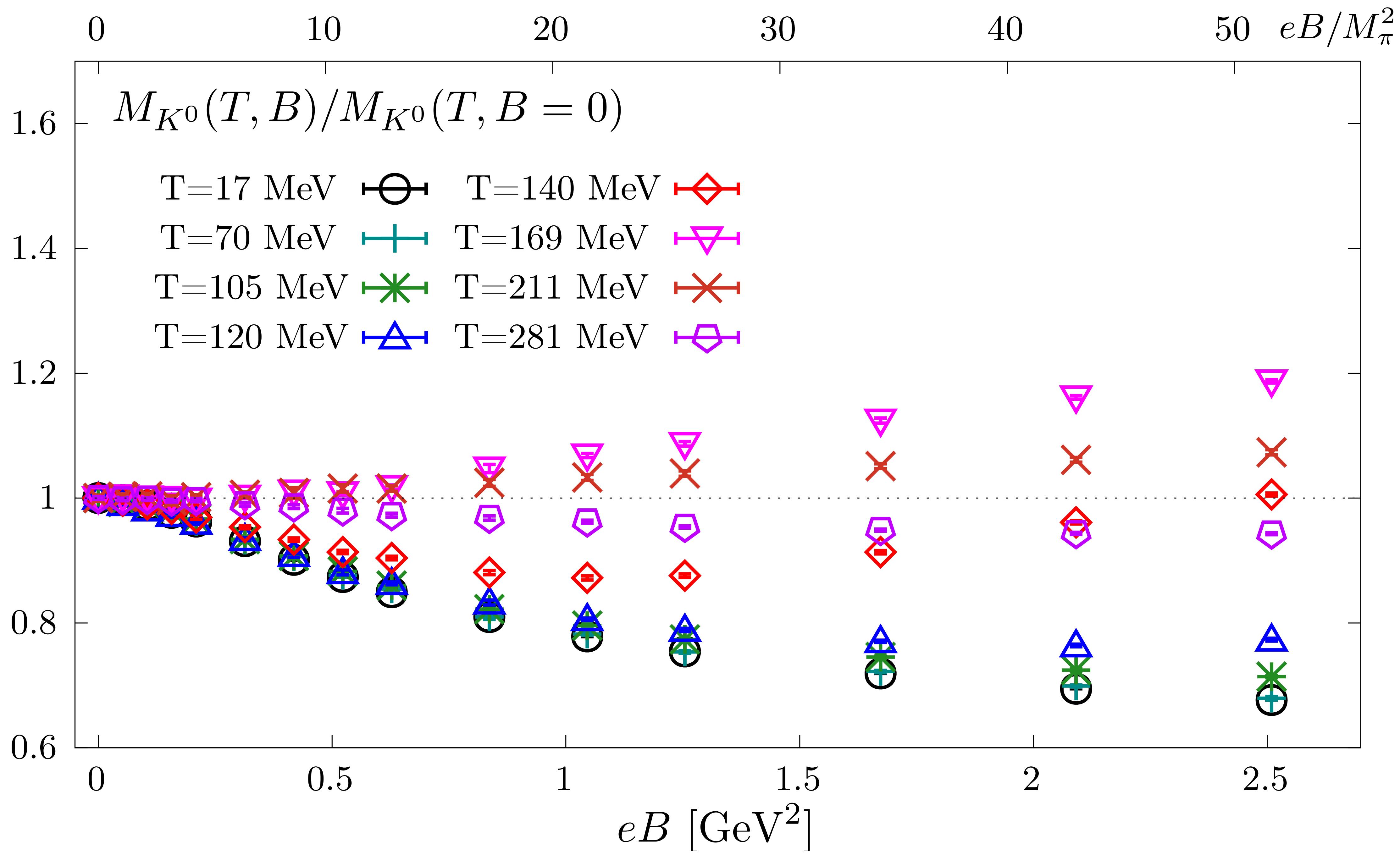



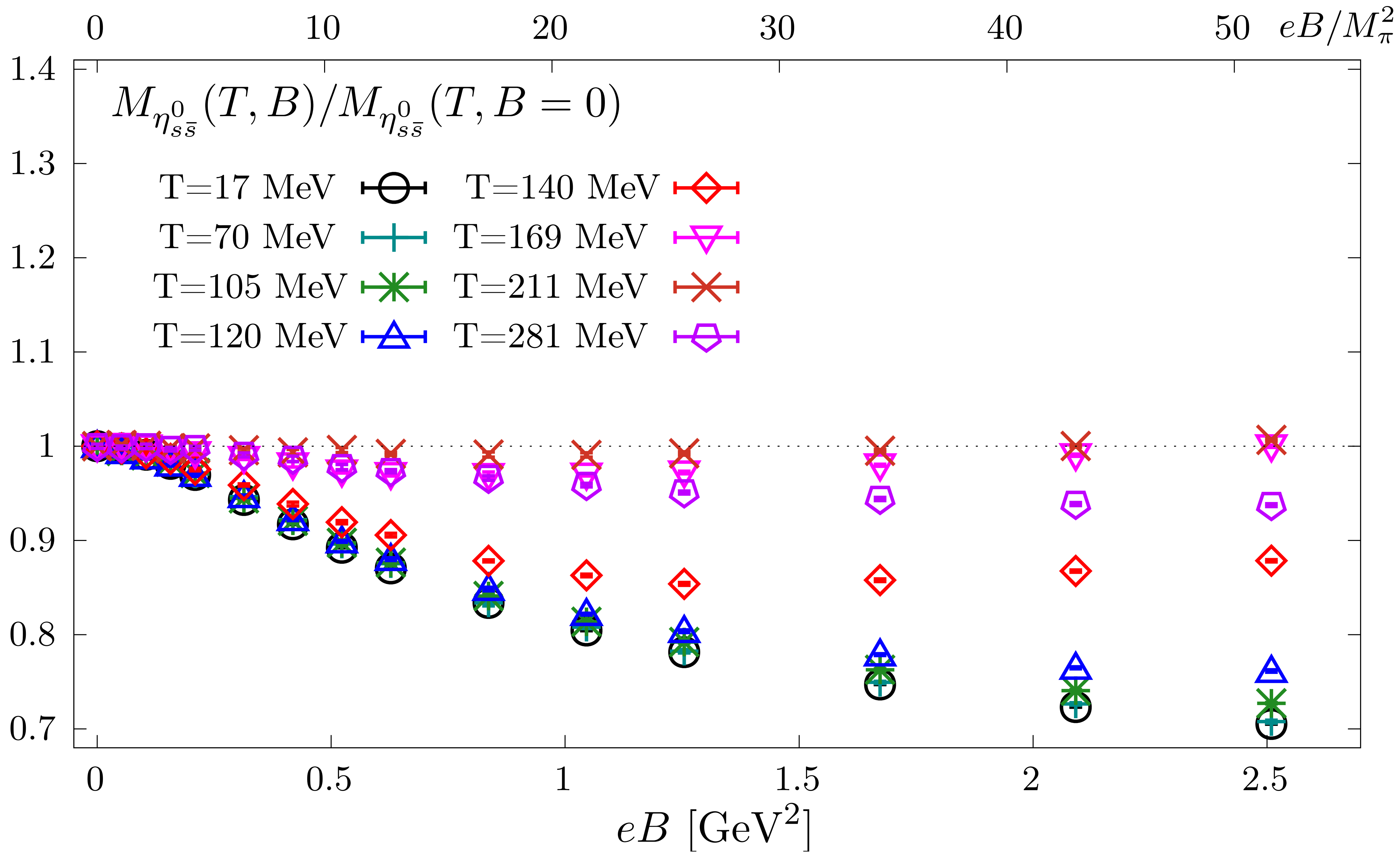

$$\lim_{z\to\infty} G_{\pi^0}(z) = A_{\pi^0} e^{-M_{\pi^0} z}, \qquad \langle\bar\psi\psi\rangle_u + \langle\bar\psi\psi\rangle_d = (m_u+m_d)\,\chi_{\pi^0}\,, \qquad \chi_{\pi^0} = \int \mathrm{d}z\, G_{\pi^0}(z)$$

- Complex non-monotonous behavior consistent with chiral condensates

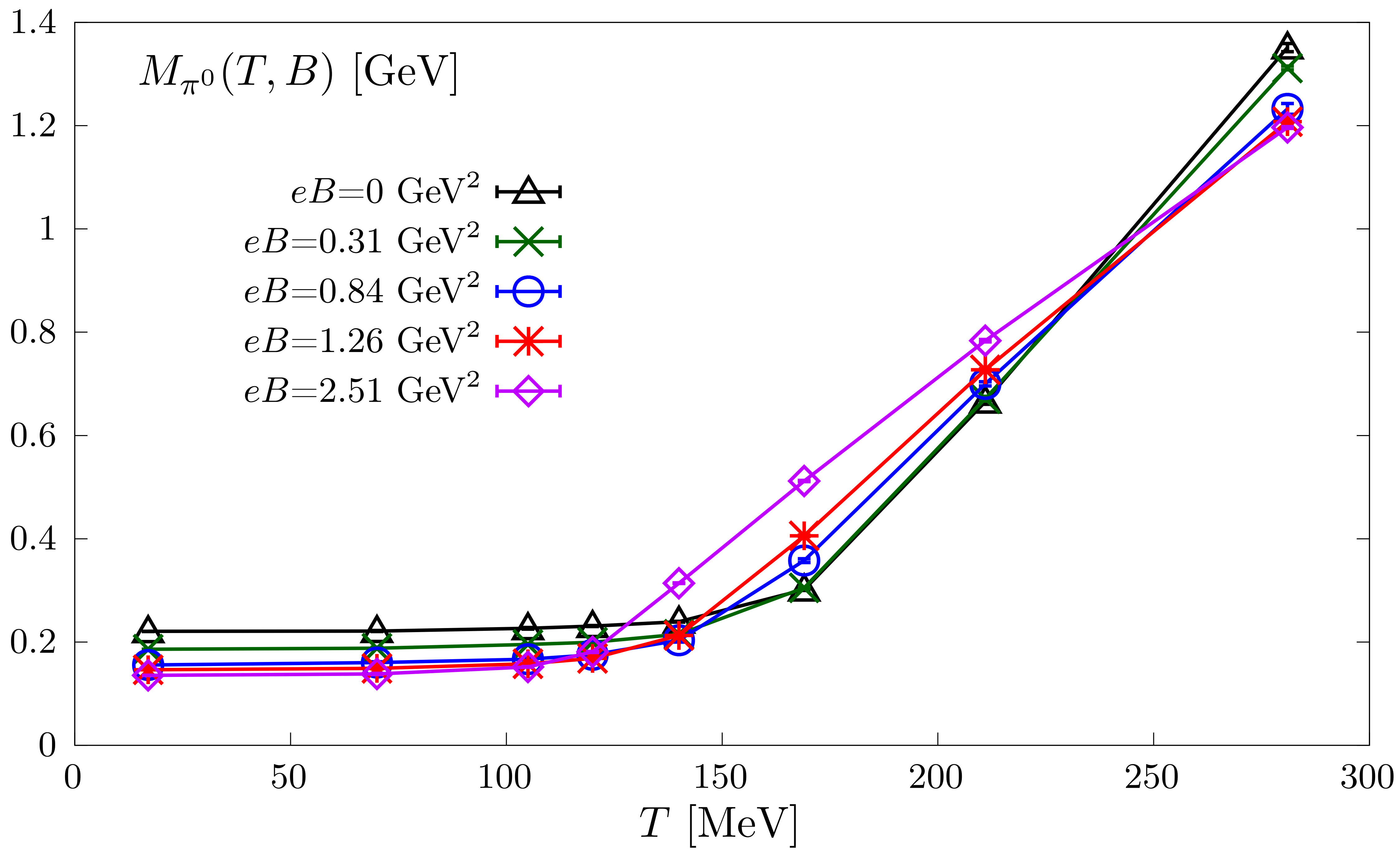


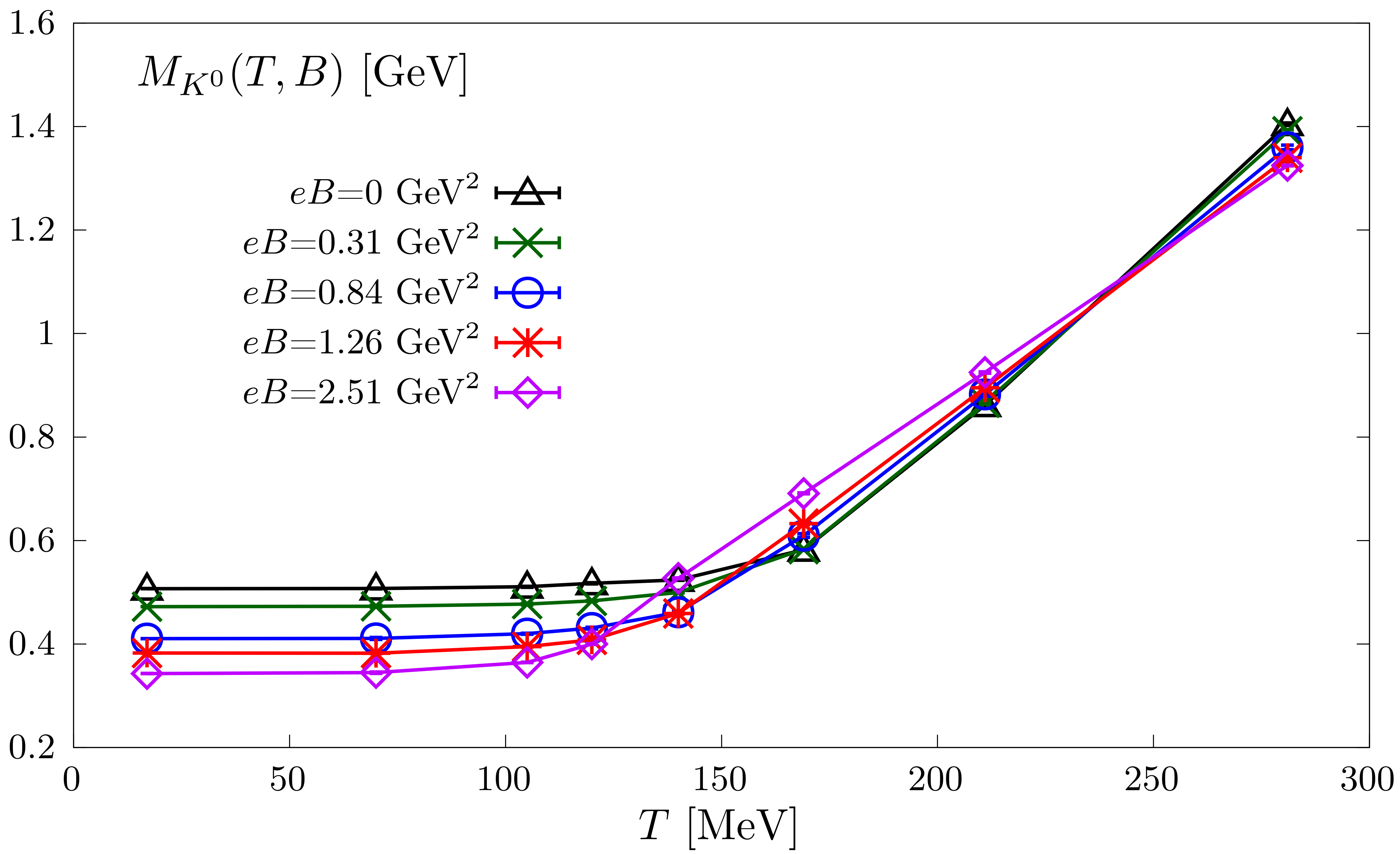

# Heavier mesons less affected by eB

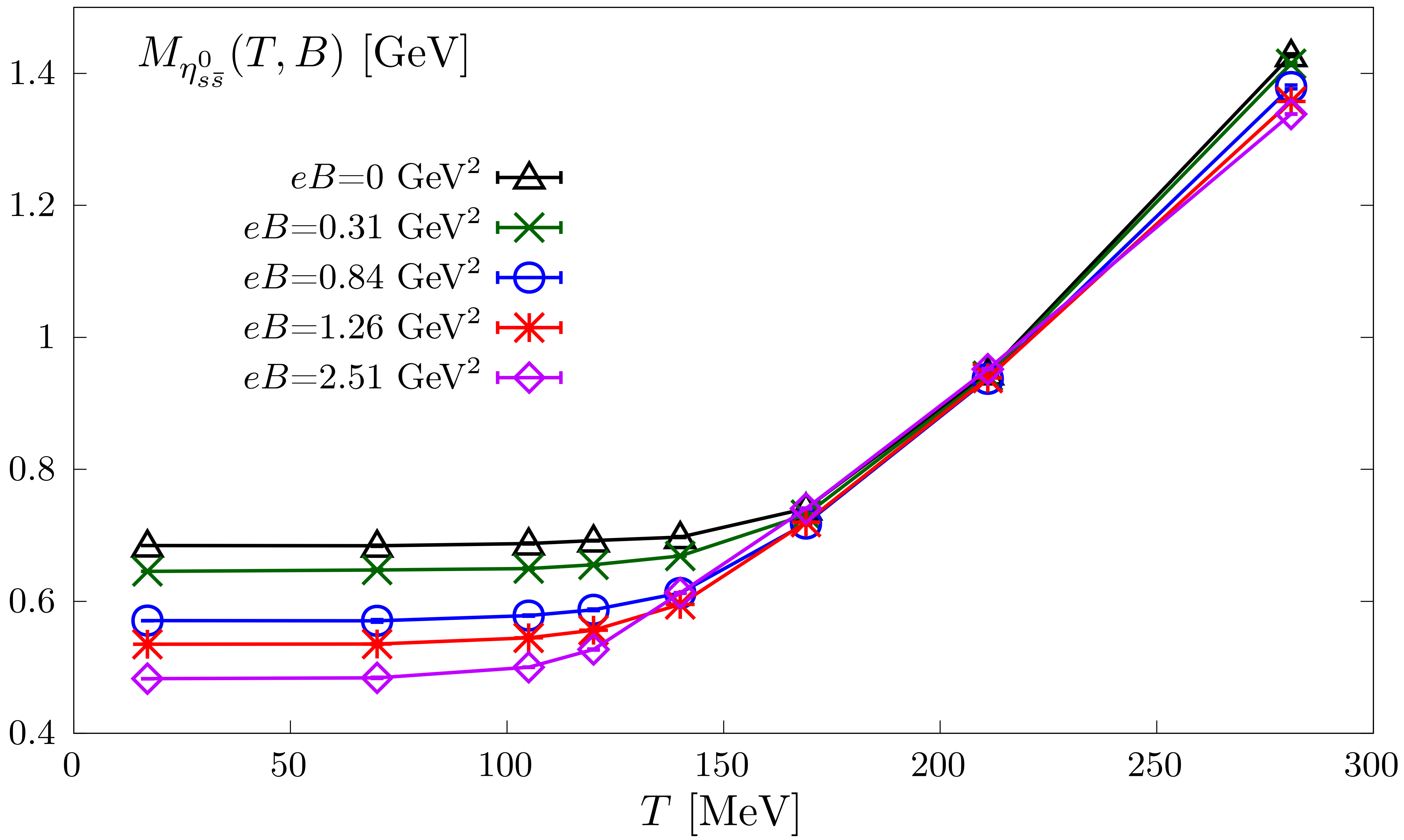


# Screening masses of neutral pseudoscalar mesons




Screening masses of $\pi^0$,$K^0$ and $\eta^0_{s\bar{s}}$ start to increase simnifically at lower T with stronger eB

# Sea and valence contributions to correlation functions

$$G_H(B,T,z) = \frac{\int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}y \int \mathrm{d}x}{Z(B,T)} \int \mathcal{D}U\, e^{-S_g} \times \prod_{f=u,d,s} \det M(U, q_f B, m_f)\; \mathcal{G}_{f_1 f_2}(B,\mathbf{x}),$$

- Valence effects to the correlation function:

$$G_H^{\mathrm{val}}(B,T,z) = \frac{\int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}y \int \mathrm{d}x}{Z(B=0,T)} \int \mathcal{D}U\, e^{-S_g} \times \prod_{f=u,d,s} \det M(U, q_f B = 0, m_f)\; \mathcal{G}_{f_1 f_2}(B,\mathbf{x})$$

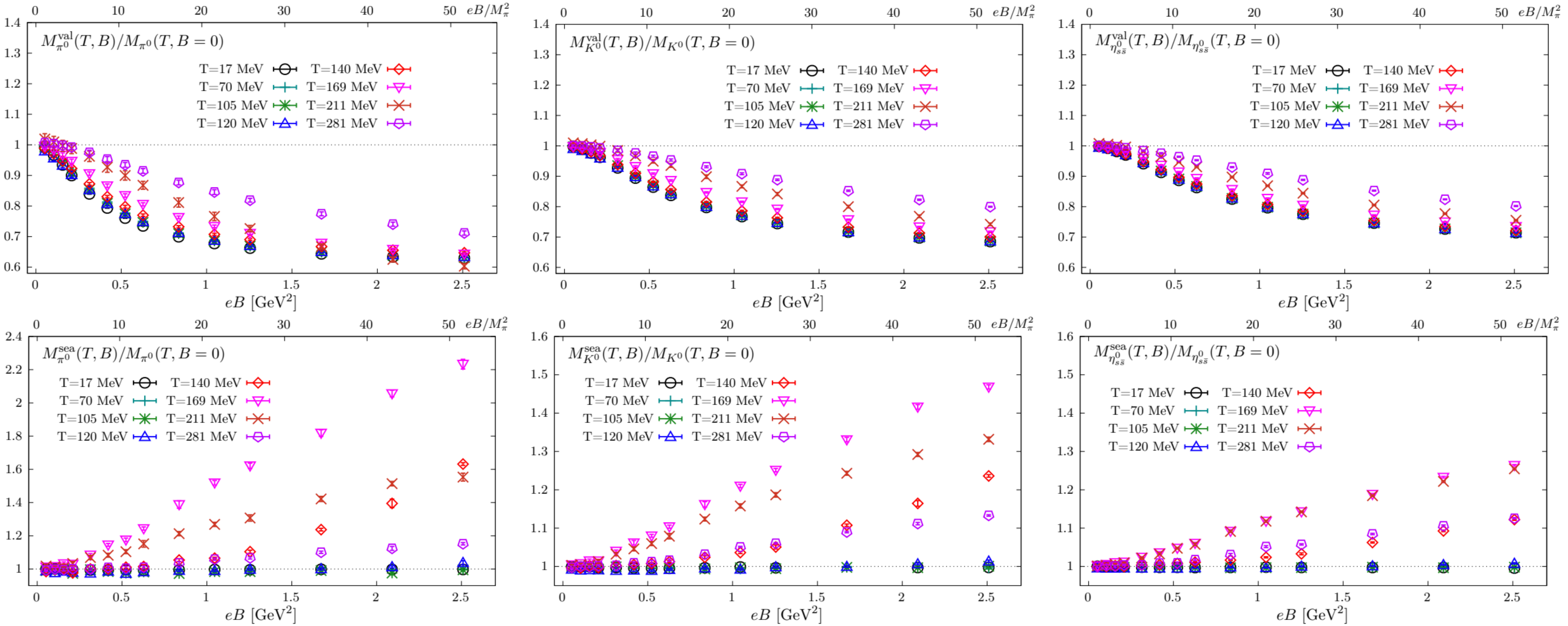
- Sea effects to the correlation function:

$$G_H^{\mathrm{sea}}(B,T,z) = \frac{\int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}y \int \mathrm{d}x}{Z(B,T)} \int \mathcal{D}U\, e^{-S_g} \times \prod_{f=u,d,s} \det M(U, q_f B, m_f)\; \mathcal{G}_{f_1 f_2}(B=0,\mathbf{x})$$

Inspired by the techniques applied to chiral condensates:
D'Elia PRD83(2011)114028, Bruckmann et al., JHEP 04(2013)112

# Valence and sea contributions to screening masses

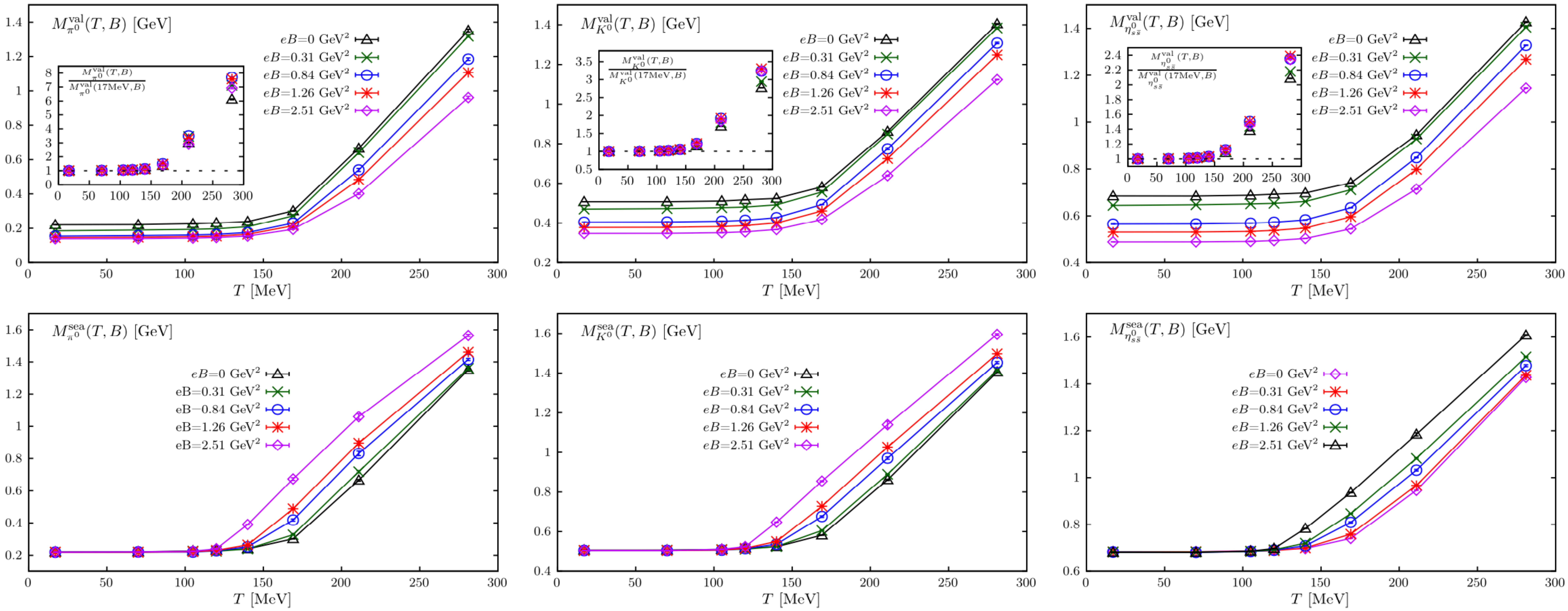
Valence effects: $M_{src} \downarrow$ MC;    Sea effects: $M_{src} \uparrow$ IMC




IMC or MC: competition between sea and valence effects

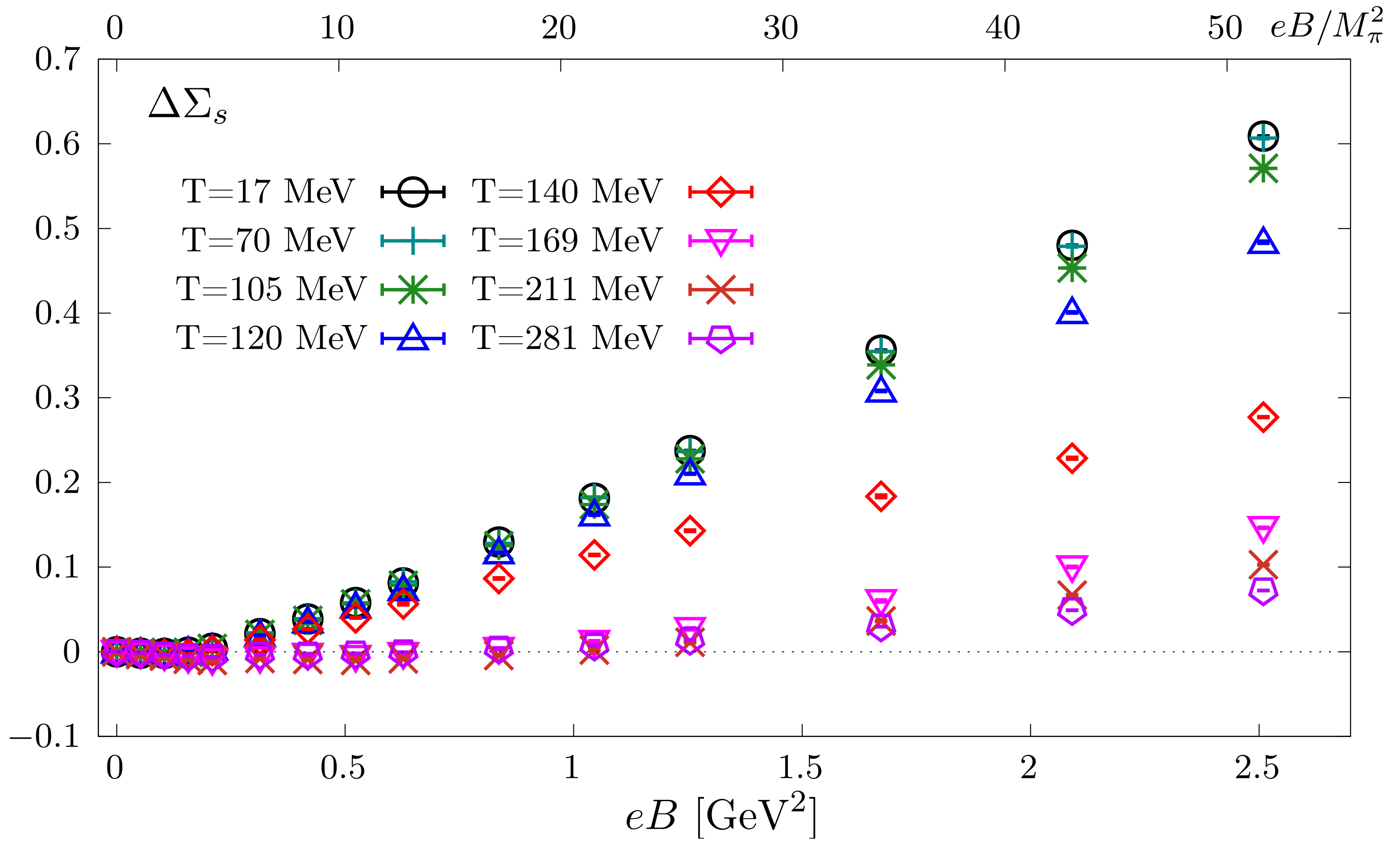
# Valence and sea contributions to screening masses

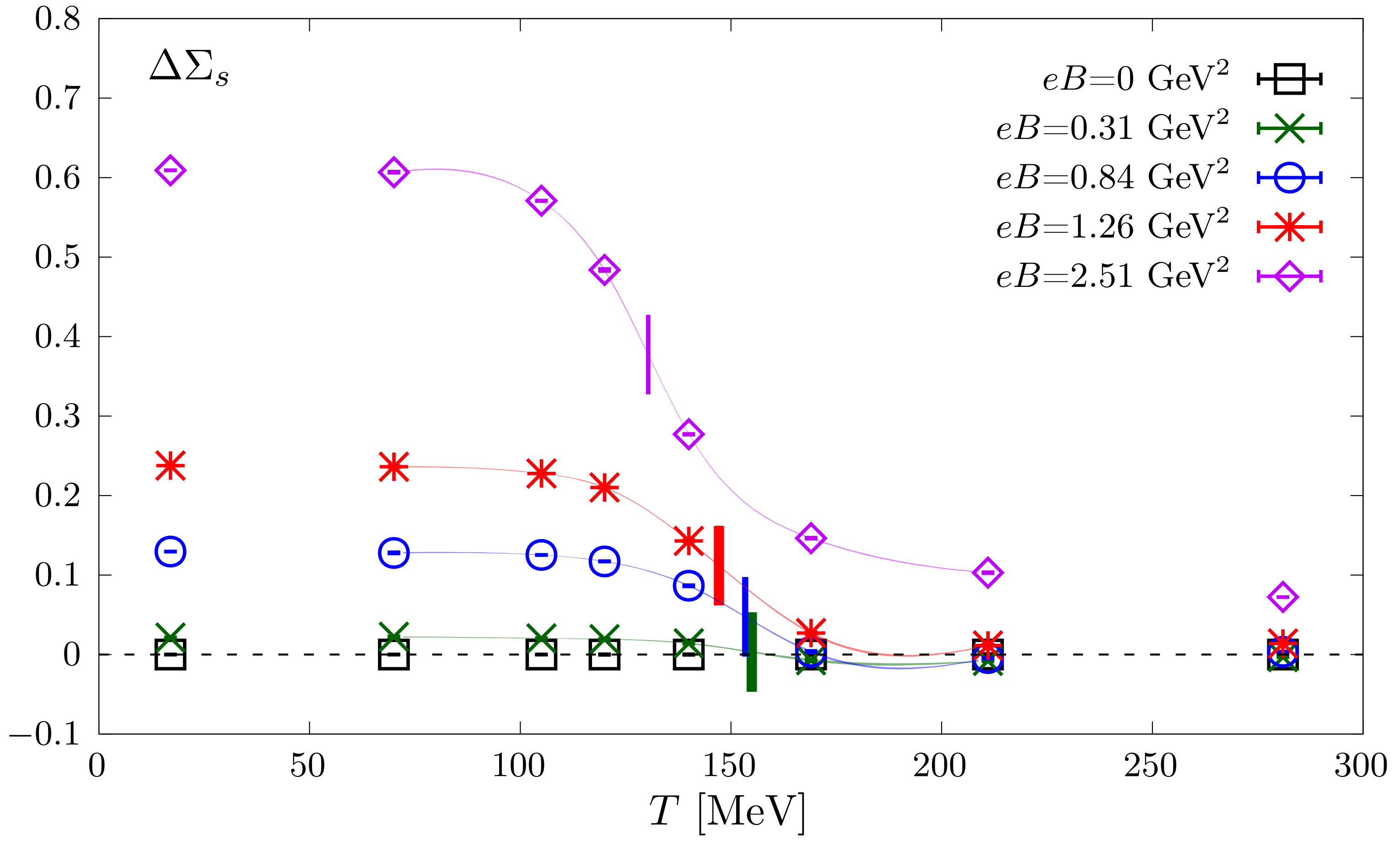
Only sea effects affect $T_{pc}$

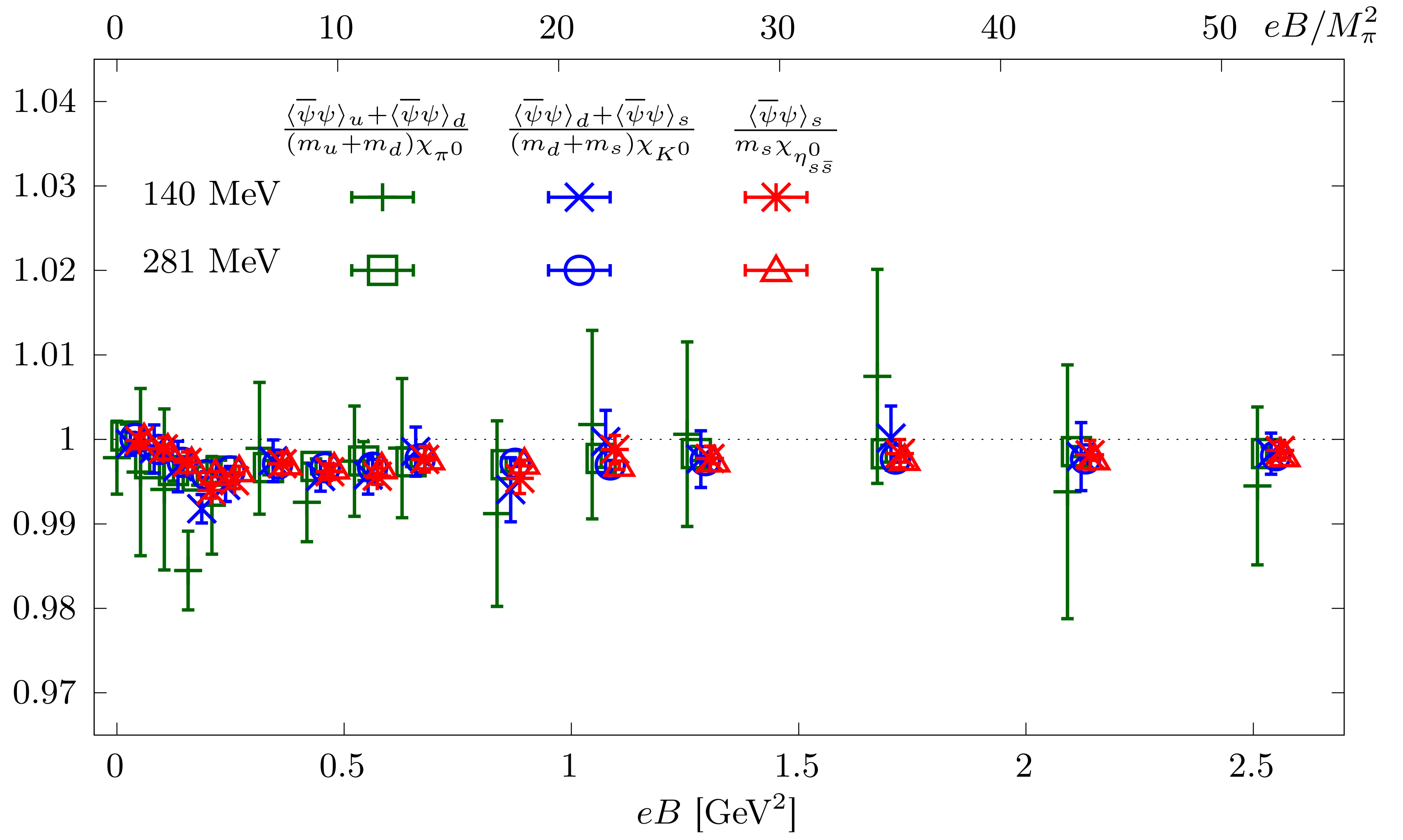


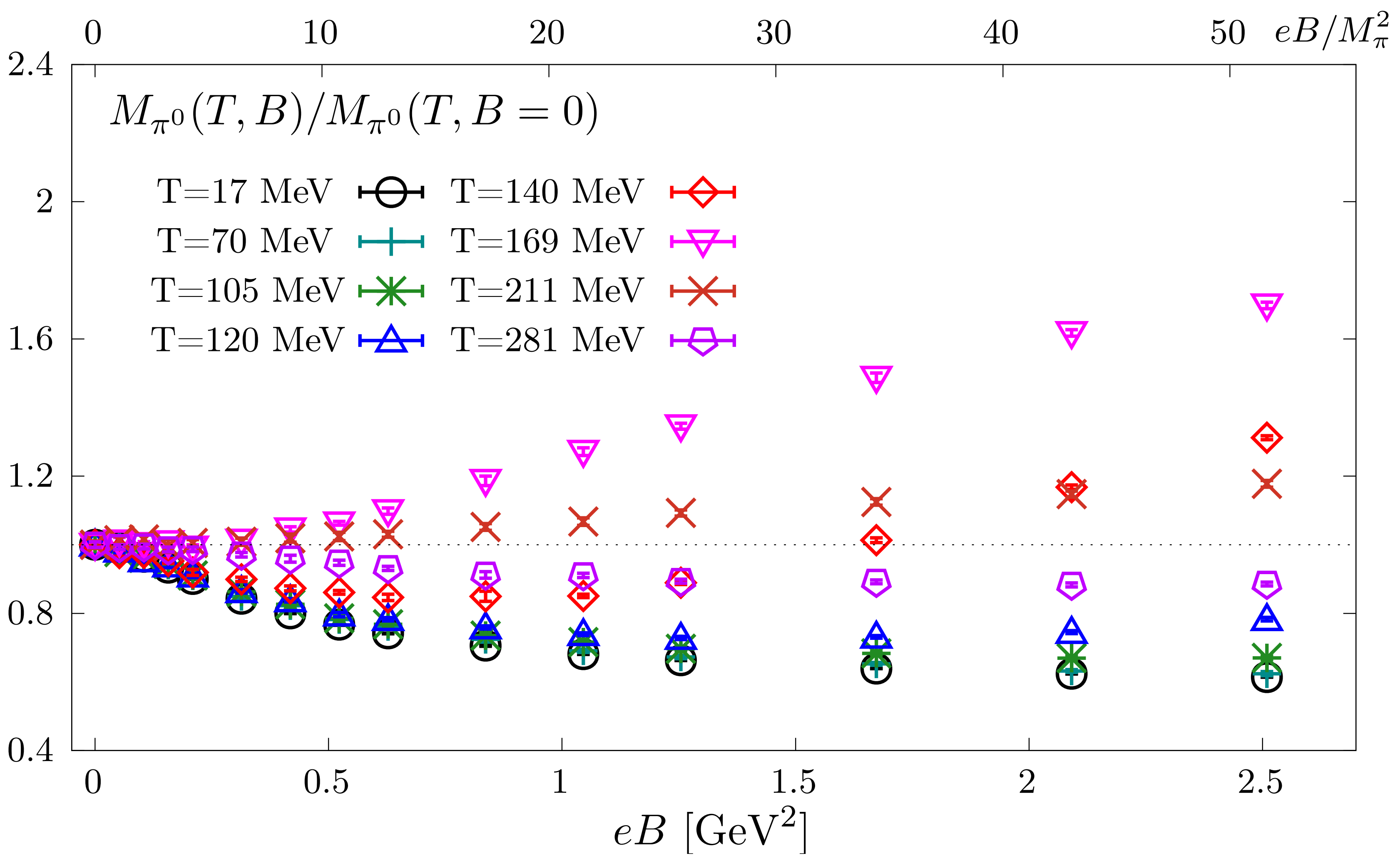

As long as sea effects exits, $T_{pc}$ deceases as eB grows

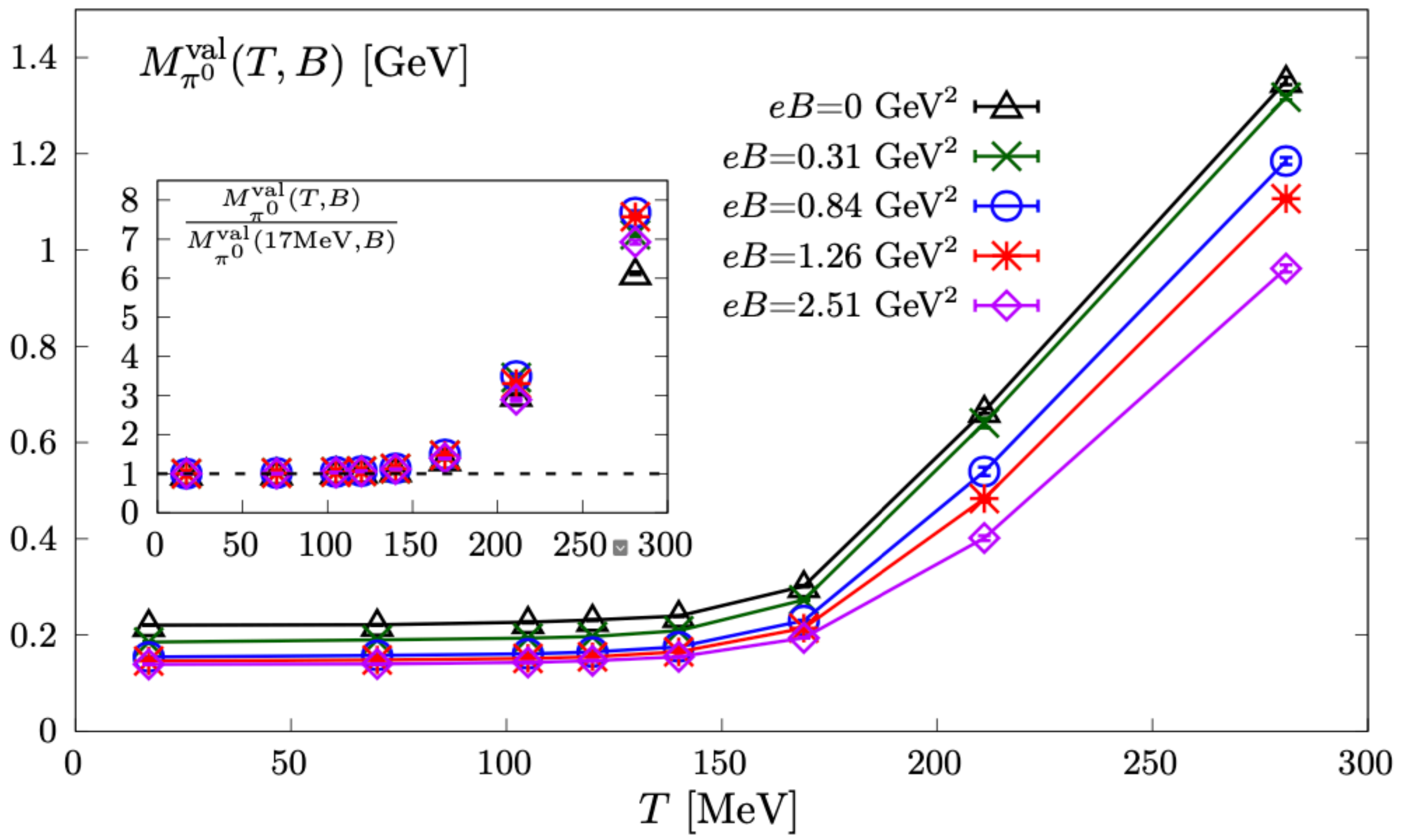
# Summary for T≠0 results

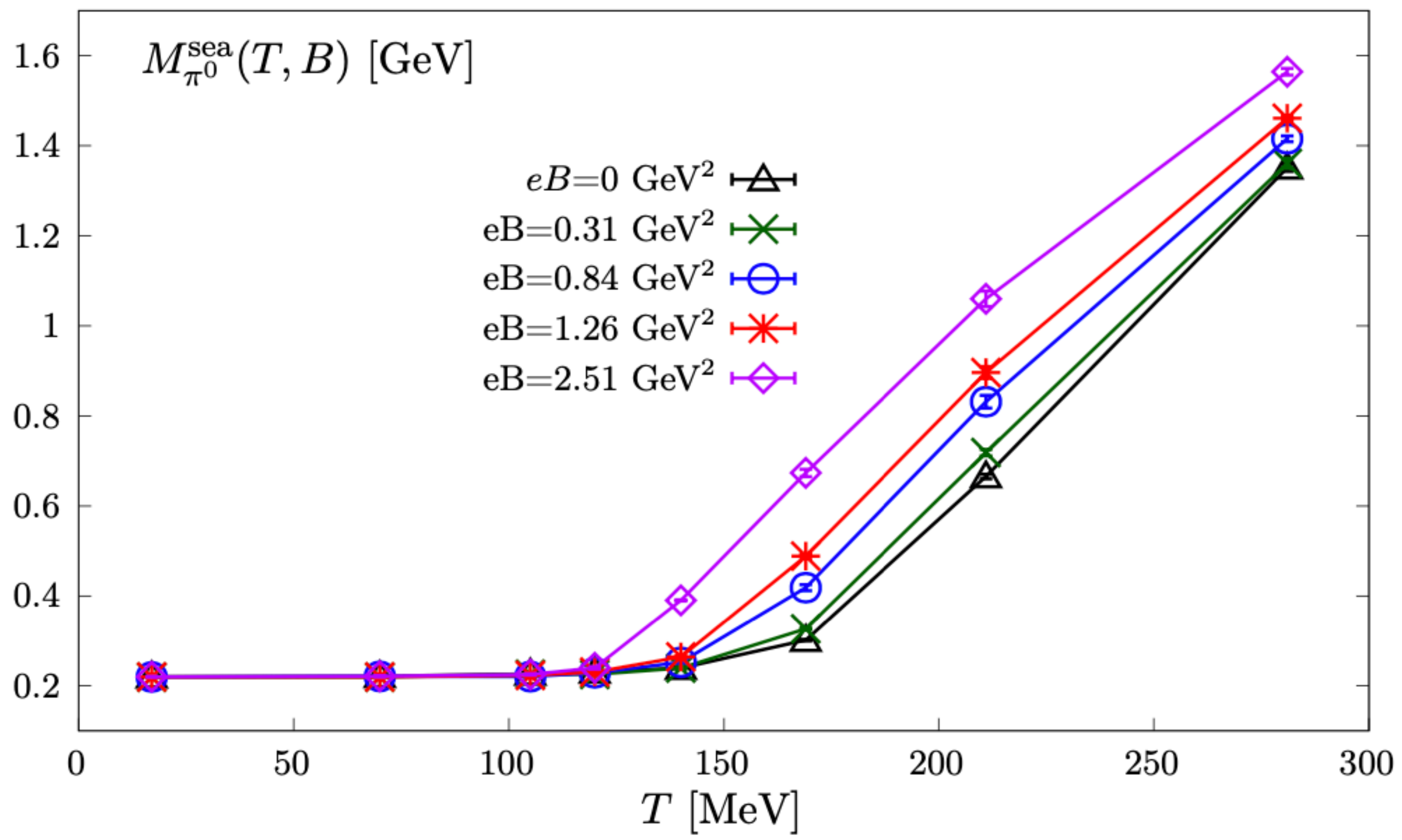


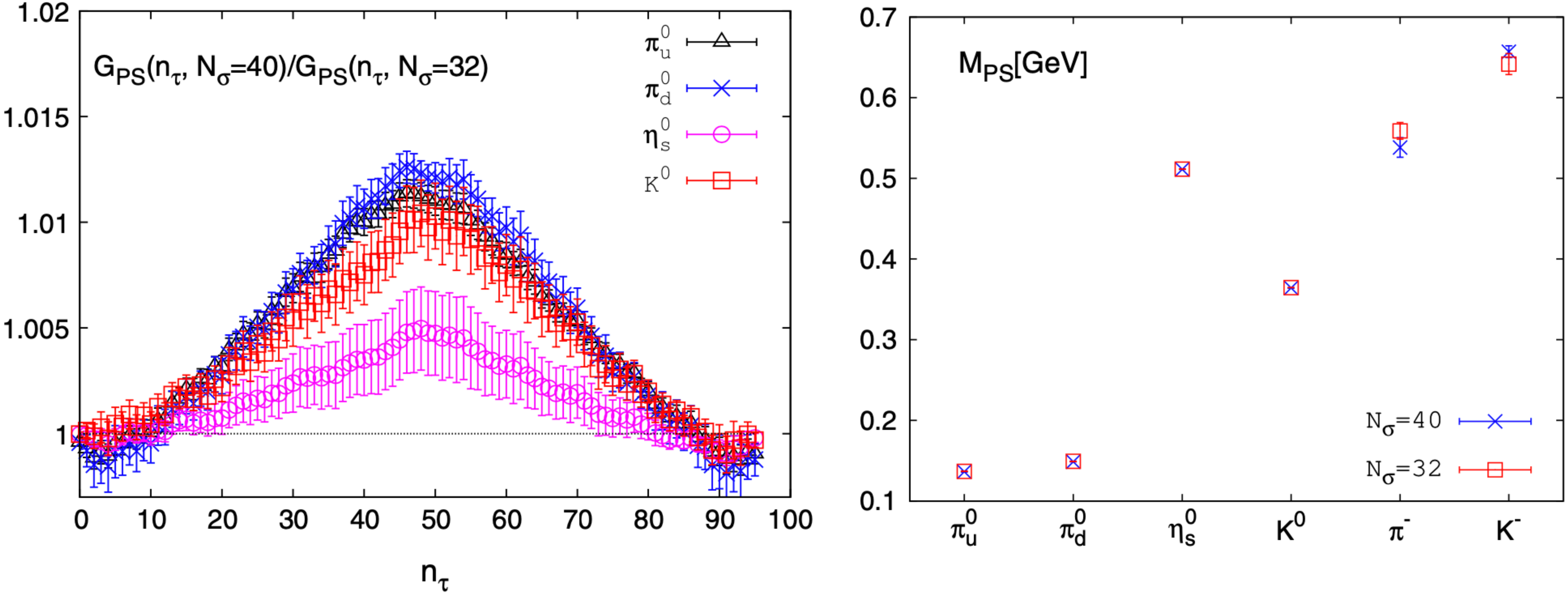



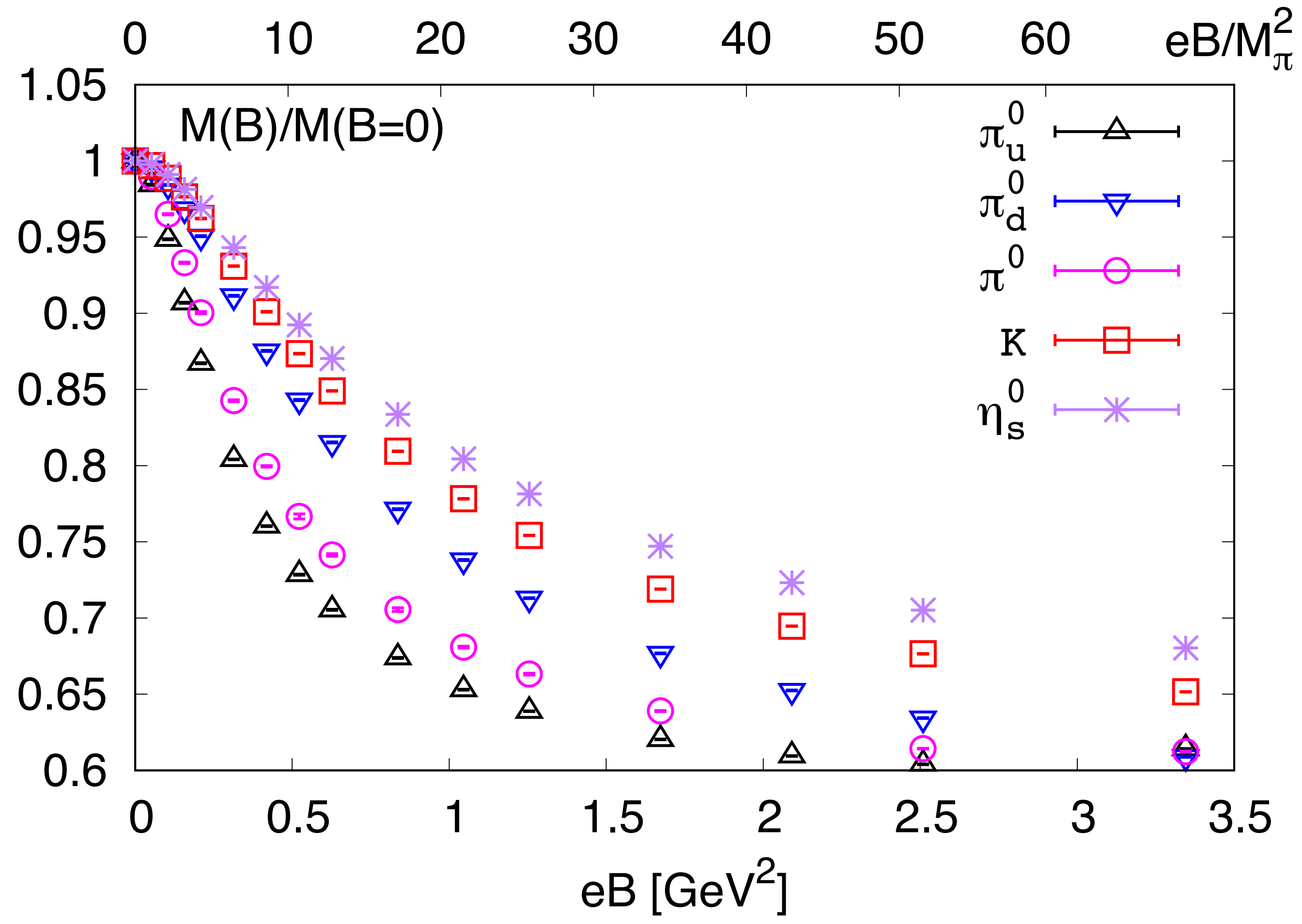
# Thanks for your attention!

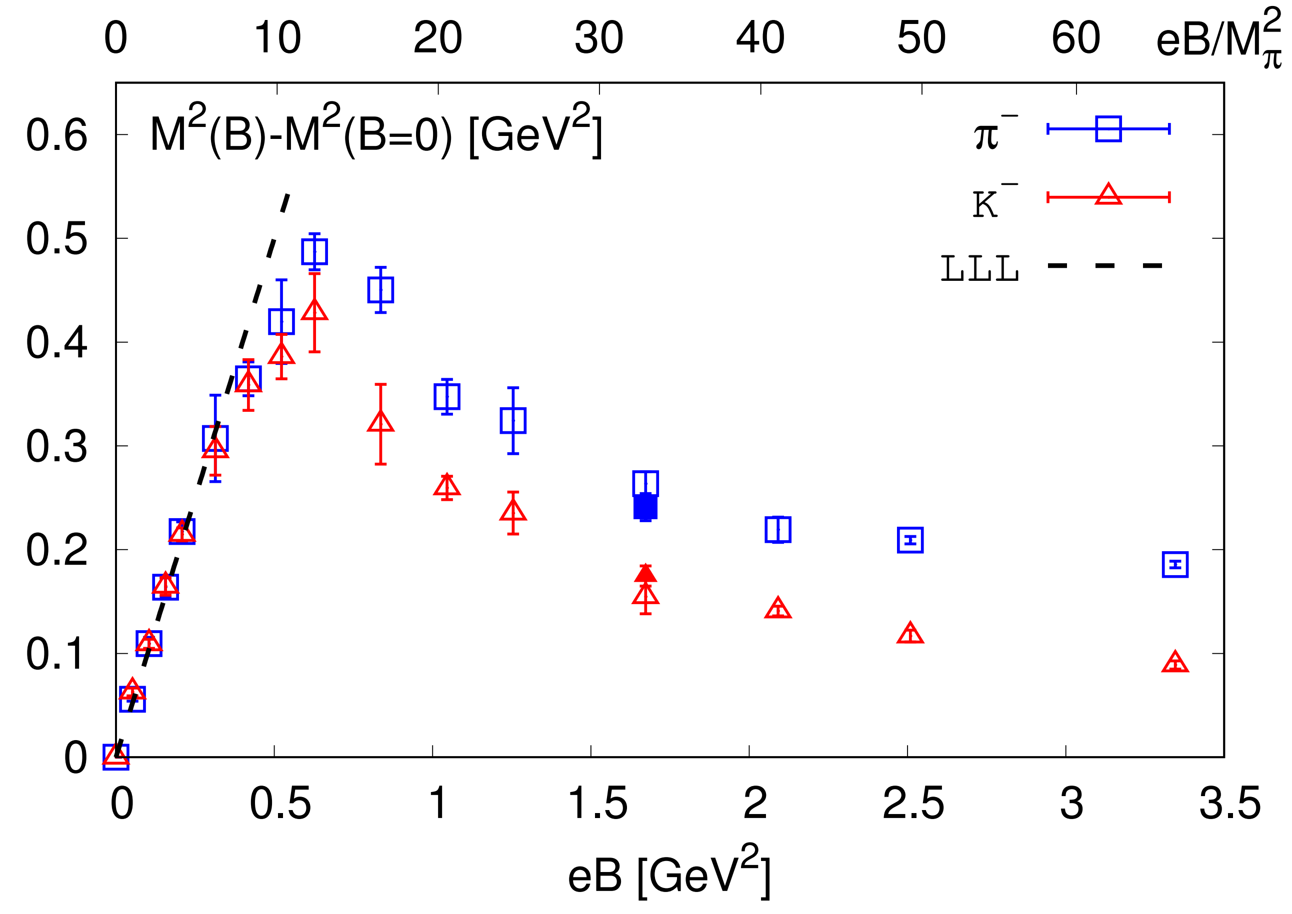
# Volume dependence at T=0



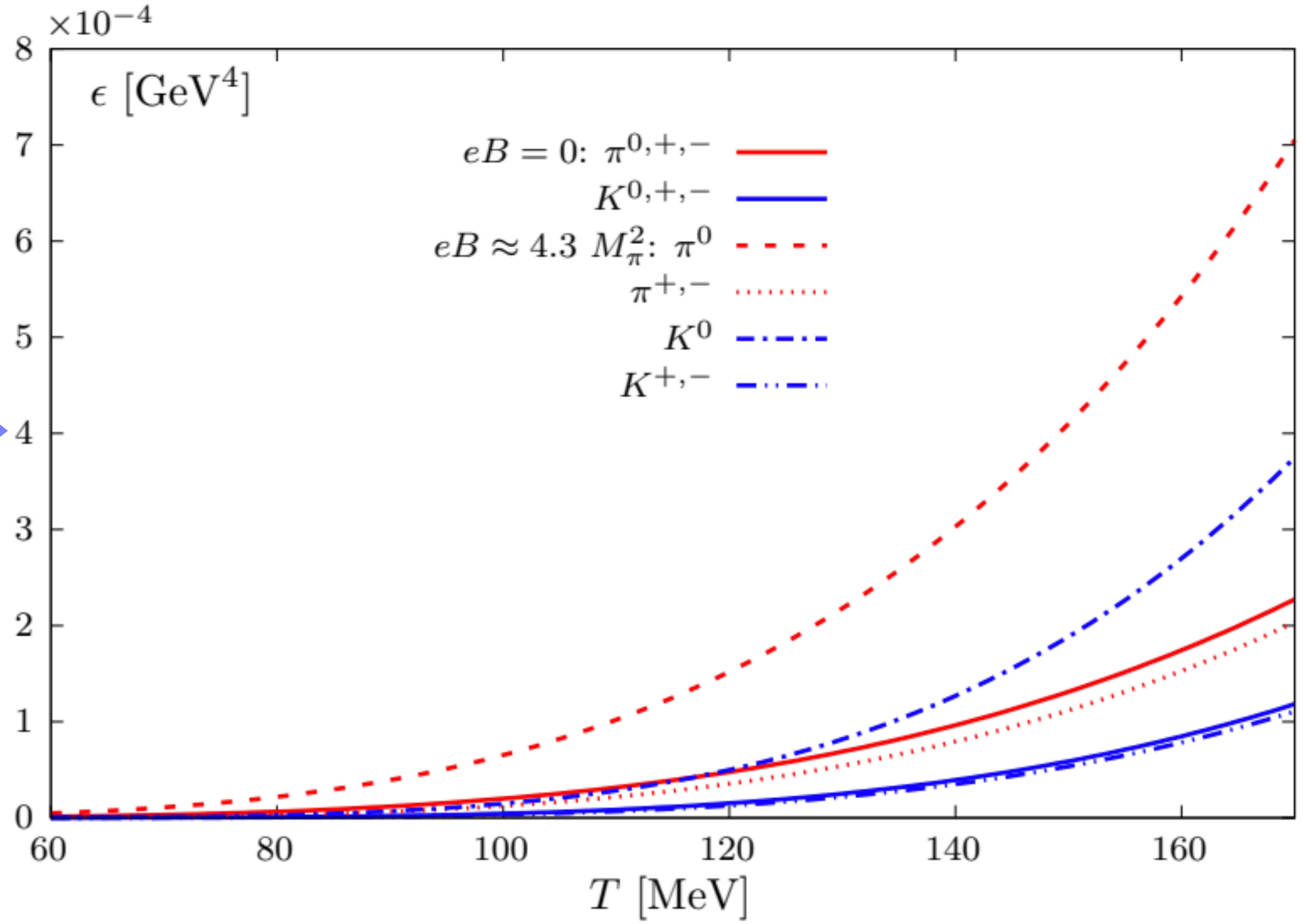

HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505

# Contributions to pressure and energy density from individual hadrons in Hadron resonance gas model




HRG




HTD, S.-T. Li, Q. Shi, A. Tomiya, X.-D. Wang, Y. Zhang, arXiv: 2011.04870