GAUGE TOPOLOGY, FLUX TUBES AND HOLOGRAPHIC MODELS: THE INTRICATE DYNAMICS OF QCD IN VACUUM AND EXTREME ENVIRONMENTS

FBK FONDAZIONE BRUNO KESSLER | ECT* EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

Maria Paola Lombardo

INFN Firenze

Andrey Yu. Kotov, MpL and Anton Trunin

e-print: 2111.15421 [hep-lat]
e-print: 2111.03406 [hep-lat]
Phys.Lett. B823 (2021) 136749
Symmetry 13 (2021) 10, 1833

23 May 2022 — 27 May 2022

STRONG 2020 | CINECA | INFN

Issues/questions:

Strongly coupled Quark Gluon Plasma

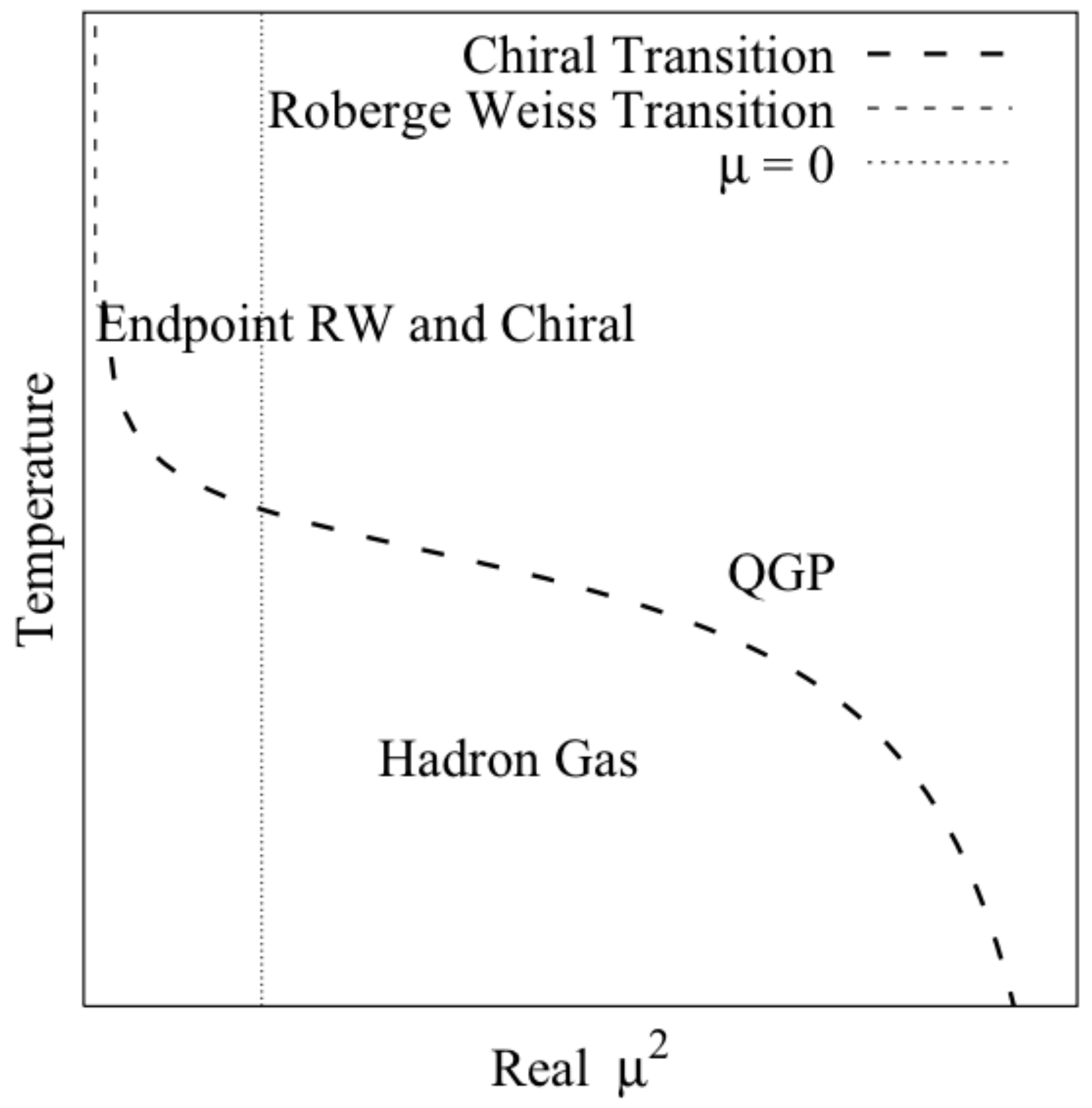
Nature of the crossover to weakly coupled plasma

A new phase within the QGP?

How can topology help diagnose this phase?

Role of the known critical points?

Strongly coupled QGP and singularities



Di Renzo, D'Elia, MpL 2007

T_{RW} approx. 207 MeV

Talk by F. Di Renzo

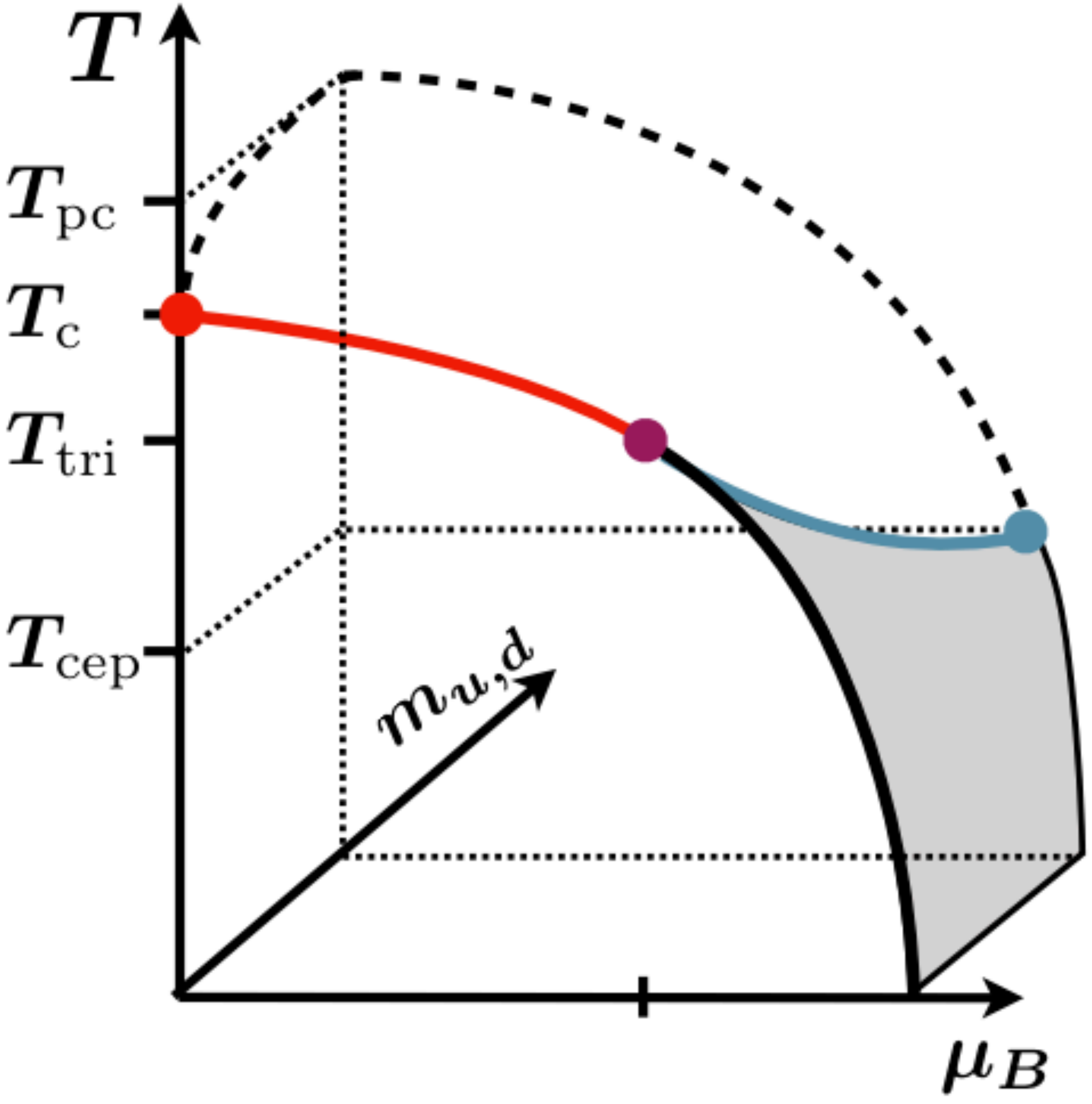
Speculation: sQGP at least for $T < T_{RW}$

$T = T_{RW}$ $\quad n(\mu_I) = A\mu_I({\mu_I^c}^2 - \mu_I^2)^\alpha$ ←

simple, but true (?)!

How far from the critical point does a system "feel" a singularity?

How "far"
in mass
and temperature
does Tc influence the
QGP?



Karsch 2019

Possible answer:
within the scaling window
of the theory

Byproducts of the study of the scaling window:
.Universality class at Tc — which, in turn, gives information on topology
.Value of Tc, upper bound to Tcep

Outline

Topology at high Temperature

Scaling window around Tc

(*Speculations of a possible further threshold at T > Tpc*)

What do we know about $\chi_{top}(T) \equiv \left.\frac{\partial^2 F(\theta, T)}{\partial \theta^2}\right|_{\theta=0}$

Topological Susceptibility and θ-dependence

The QCD axion, precisely

Giovanni Grilli di Cortona[a], Edward Hardy[b],
Javier Pardo Vega[a,b] and Giovanni Villadoro[b]

[a] *SISSA International School for Advanced Studies and INFN Trieste, Via Bonomea 265, 34136, Trieste, Italy*

[b] *Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34151, Trieste, Italy*

$$\frac{\chi_{top}(T)}{\chi_{top}} \overset{\text{NLO}}{=} \frac{m_\pi^2(T) f_\pi^2(T)}{m_\pi^2 f_\pi^2} = \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle}$$

$T \to 0$

$$F(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\theta}{2}\right)}$$

$$= \langle \bar{q}q \rangle \sqrt{m_u + m_d + 2 m_u m_d \cos\theta}$$

$T \to \infty$

$$\sim C \left(\frac{T_c}{T}\right)^\beta \cos(\theta) \qquad \beta = 7 + n_f/3$$

$$\frac{F(\theta)_T}{F(\theta)} = 1 + \frac{3}{2} \frac{T^4}{f_\pi^2 m_\pi^2(\theta)} J_0\left[\frac{m_\pi^2(\theta)}{T^2}\right]$$

$$J_0[\xi] \equiv -\frac{1}{\pi^2} \int_0^\infty dq\, q^2 \log\left(1 - e^{-\sqrt{q^2+\xi}}\right)$$

QCD and instantons at finite temperature

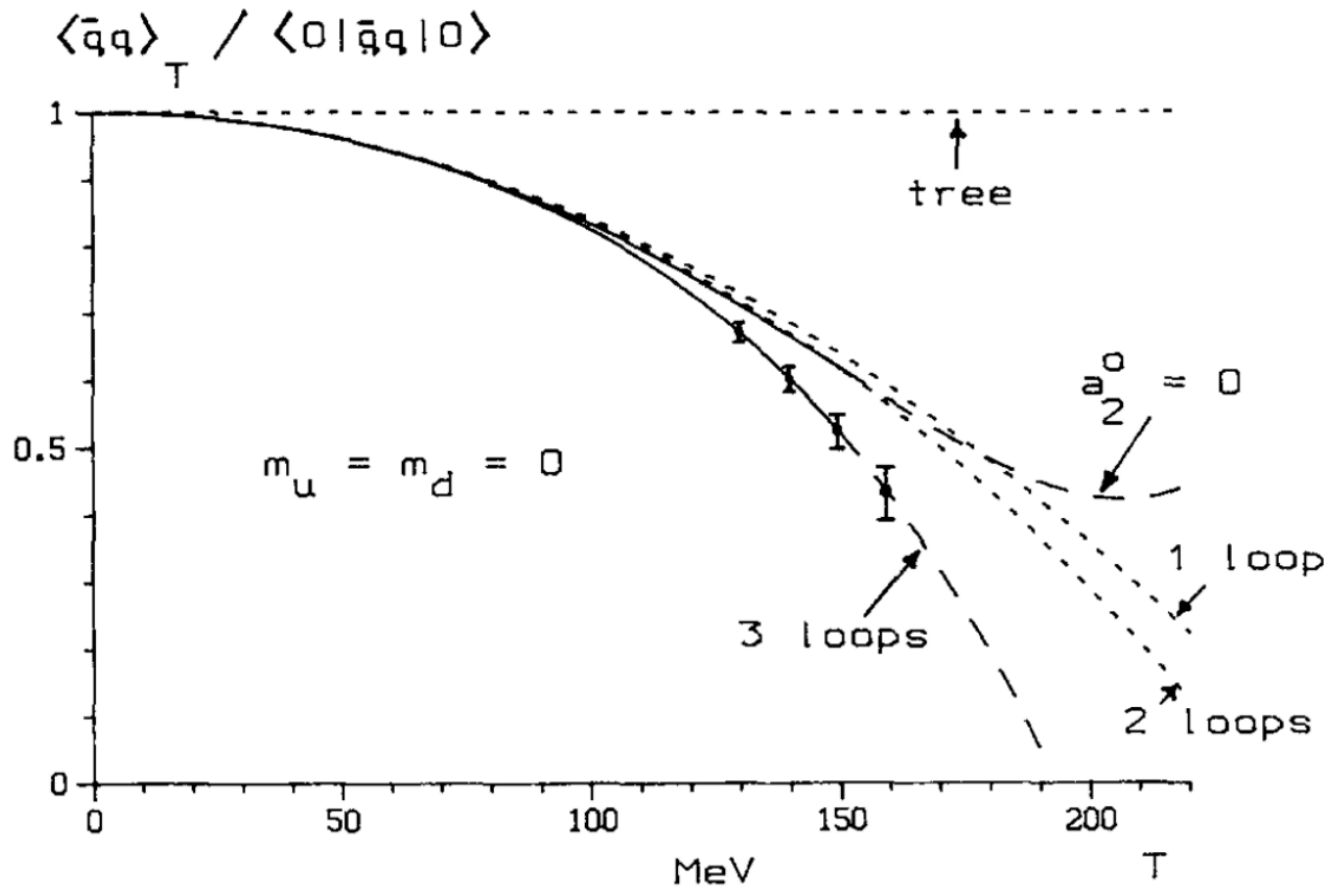
David J. Gross
Department of Physics, Princeton University, Princeton, New Jersey 08544
Robert D. Pisarski
J. W. Gibbs Laboratories, Yale University, New Haven, Connecticut 06520
Laurence G. Yaffe*
Department of Physics, Princeton University, Princeton, New Jersey 08544

Slide by G. Villadoro

Finite temperature

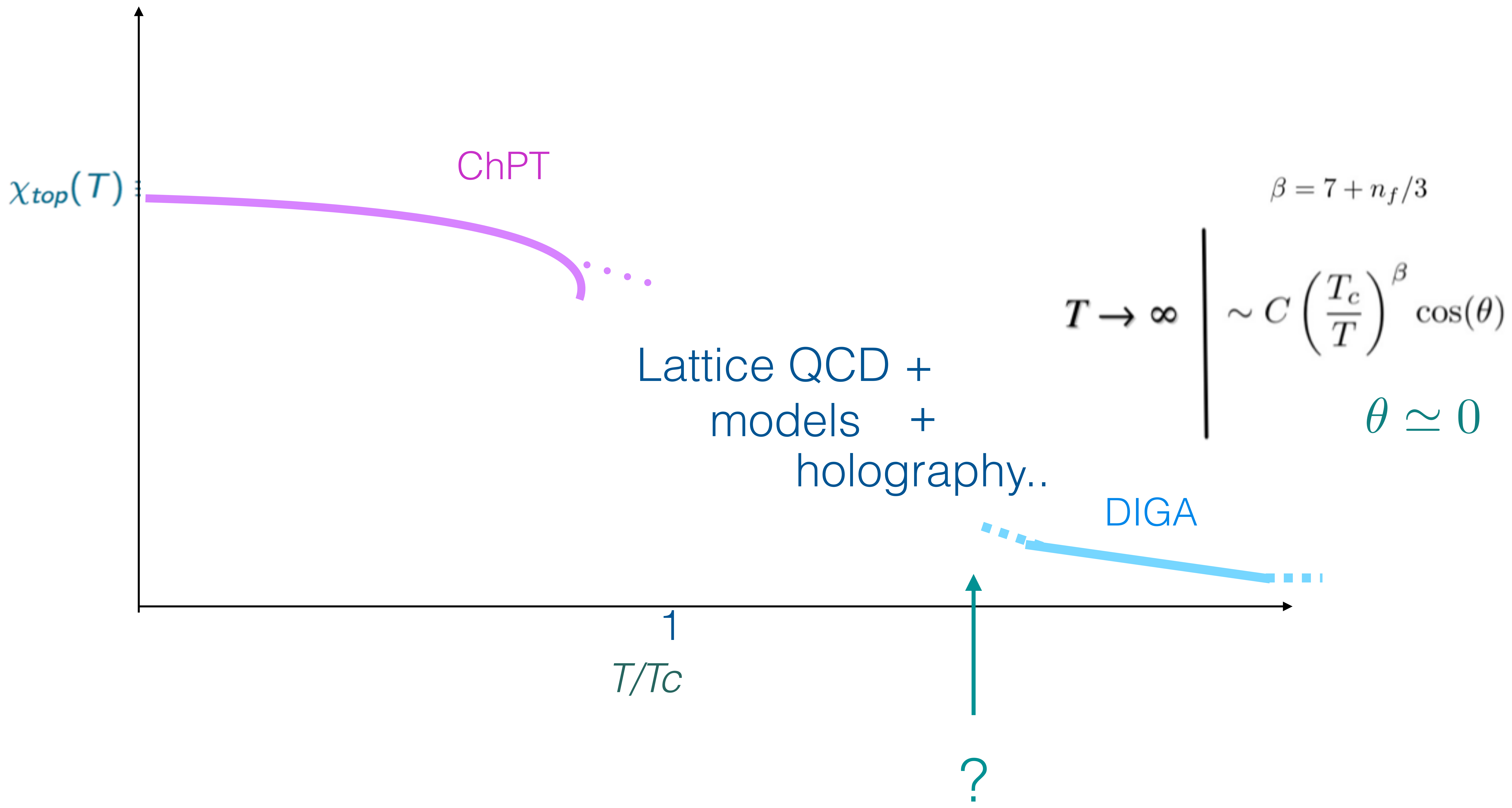
$$\frac{\chi_{top}(T)}{\chi_{top}} \overset{\text{NLO}}{=} \frac{m_\pi^2(T) f_\pi^2(T)}{m_\pi^2 f_\pi^2} = \frac{\langle \bar{q}q\rangle_T}{\langle \bar{q}q\rangle}$$

$$\langle \bar{q}q\rangle \overset{m\to 0}{=} \langle 0|\bar{q}q|0\rangle \left(1 - \frac{T^2}{8F^2} - \frac{T^4}{384F^4} - \frac{T^6}{288F^6}\ln\frac{\Lambda_q}{T} + O(T^8)\right)$$



Gasser-Leutwyler 1987-1989

What do we know about $\chi_{top}(T) \equiv \left.\frac{\partial^2 F(\theta, T)}{\partial \theta^2}\right|_{\theta=0}$



Lattice Topology Michael Mueller-Preussker(2015)

- Gluonic: **Luscher(2010), Bonati,d'Elia e al (2014),Alexandrou et al . (2015)**

$$Q = \frac{a^4}{32\pi^2}\varepsilon_{\mu\nu\rho\sigma}\sum_n \mathrm{Tr}[F^{\mu\nu}_{lat}(n)F^{\rho\sigma}_{lat}(n)],$$

Need smooth configurations,using smearing,cooling, gradient flow..

$$\dot{V}_\mu(n,\tau) = -g^2[\partial_{n,\mu}S_G(V(\tau))]V_\mu(n,\tau), \qquad V_\mu(n,0) = U_\mu(n),$$

Pros: Easy
Cons: suffers very much from lattice artifacs

C. Bonanno's talk

- Fermionic: **Atiyah Singer(1971,1984)**

$$Q = \frac{1}{32\pi^2}\varepsilon_{\mu\nu\rho\sigma}\int \mathrm{Tr}[F^{\mu\nu}(x)F^{\rho\sigma}(x)]\, d^4x = n_+ - n_-$$

Pros: not affected but UV fluctuations
Cons: very high computational cost

- Fermionic - simple but approximate: **Kogut et al.(1996),Petreczky, Sharma(2016)**

$$\chi_{top} = \frac{\langle Q^2\rangle}{V} = m_l^2\,\chi_{5,disc}$$

$$\chi_{top}(T \gtrsim T_c) = m_l^2\,\chi_{disc} = m_l^2\,\frac{V}{T}\left(\langle(\bar\psi\psi)^2\rangle_l - \langle\bar\psi\psi\rangle_l^2\right).$$

S.Sharma's talk

Setup

Twisted mass - Maximal twist Dynamical strange and charm

Nf = 2 +1 + 1, $m_\pi^{phys} < m_\pi < 470$ MeV $a = 0.06 - 0.09$ fm

Fixed scale approach - Temperature range 130 MeV$< T < 500$ MeV

Observables: Chiral condensate and Susceptibility,

[light mesons' screening masses,,η']

Statistics for physical pion mass

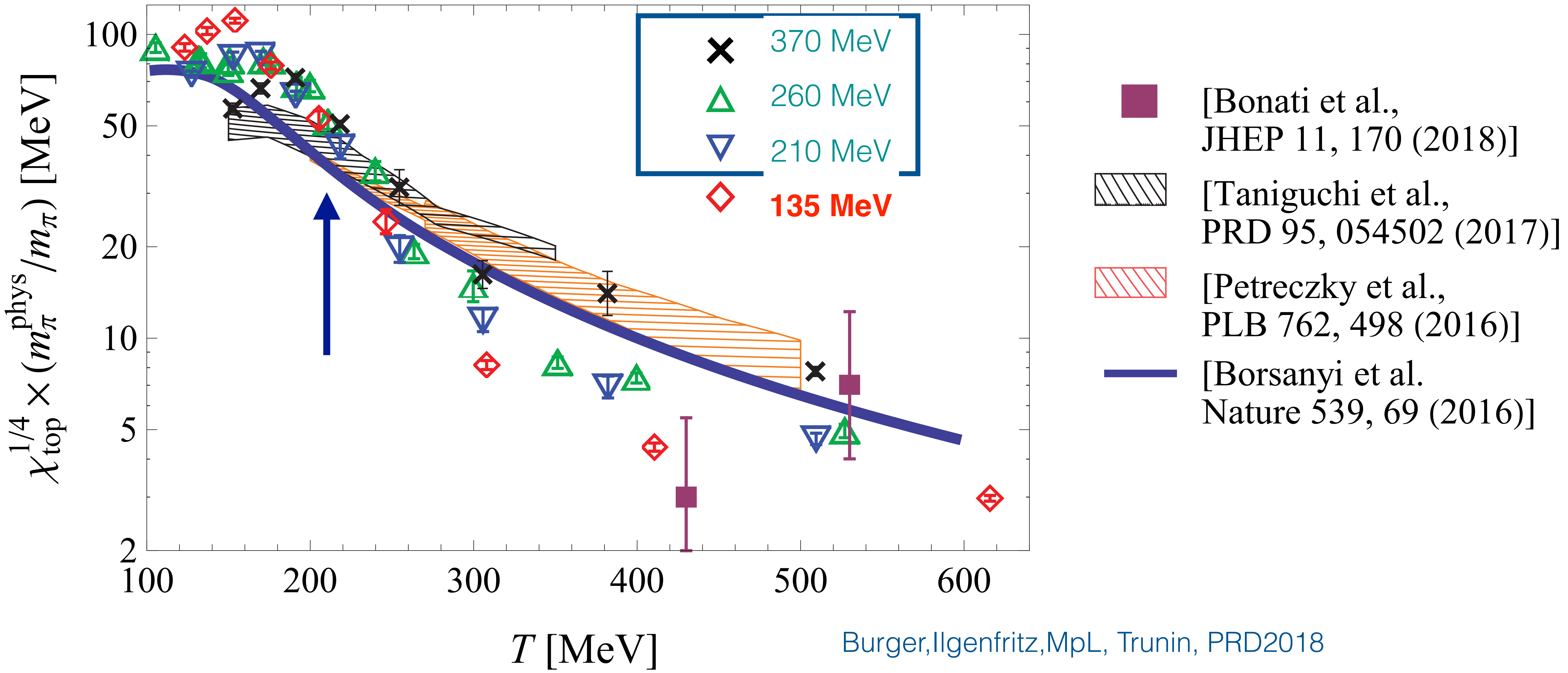
N_t	T [MeV]	# conf	N_t	T [MeV]	# conf
20	123(1)	782	10	246(1)	592
18	137(1)	892	8	308(2)	498
16	154(1)	534	6	411(2)	195
14	176(1)	359	4	616(3)	472
12	205(1)	337			

Heavier masses: Burger, Ilgenfritz, MpL, Trunin *Phys.Rev.D* 98 (2018) 9, 094501

Results for physical pion mass + Rescaled heavier masses

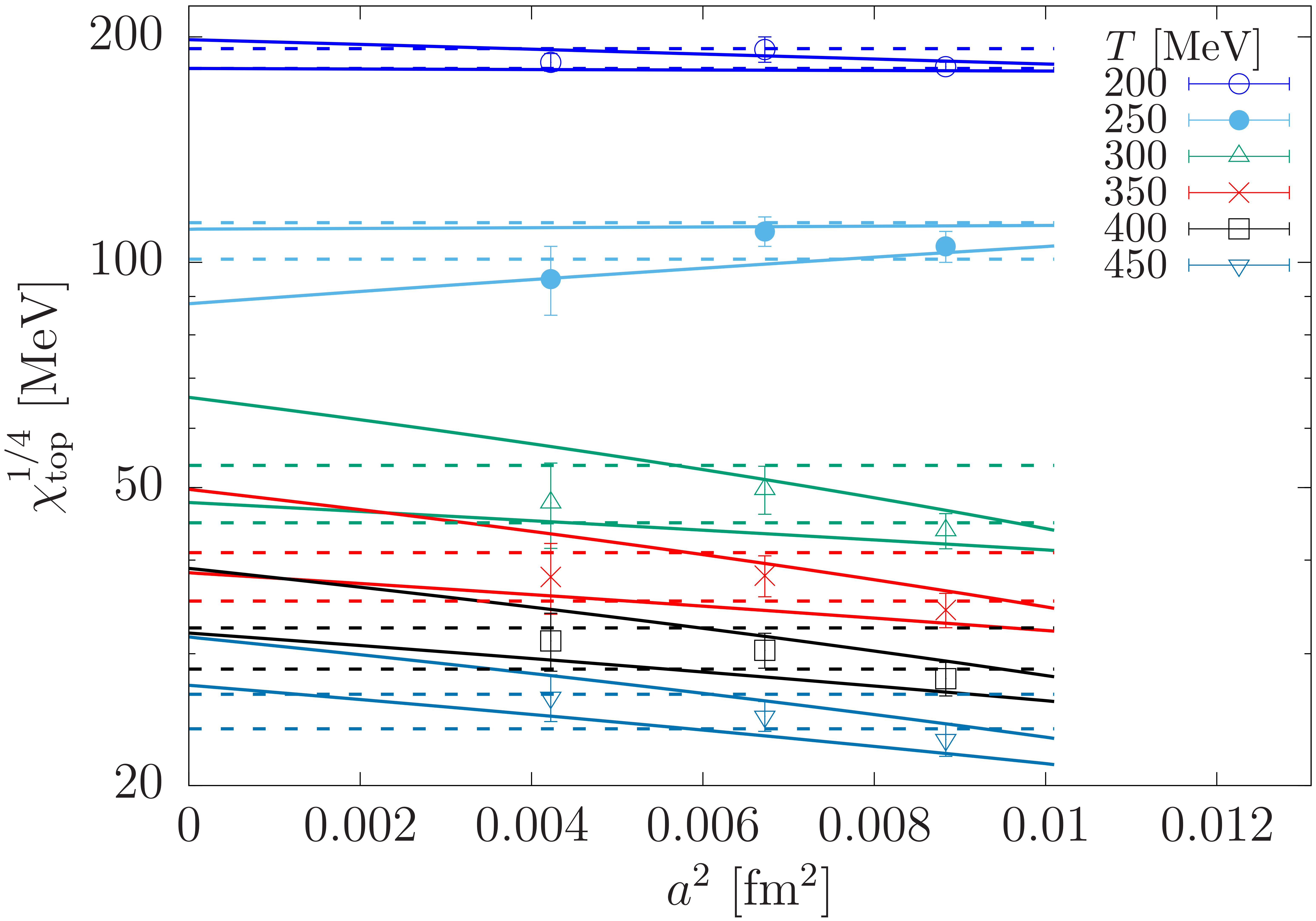
$T^{4-\beta_0}\left(\frac{m}{T}\right)^{N_f}$ Diga

m_π m_π



Burger,Ilgenfritz,MpL, Trunin, PRD2018

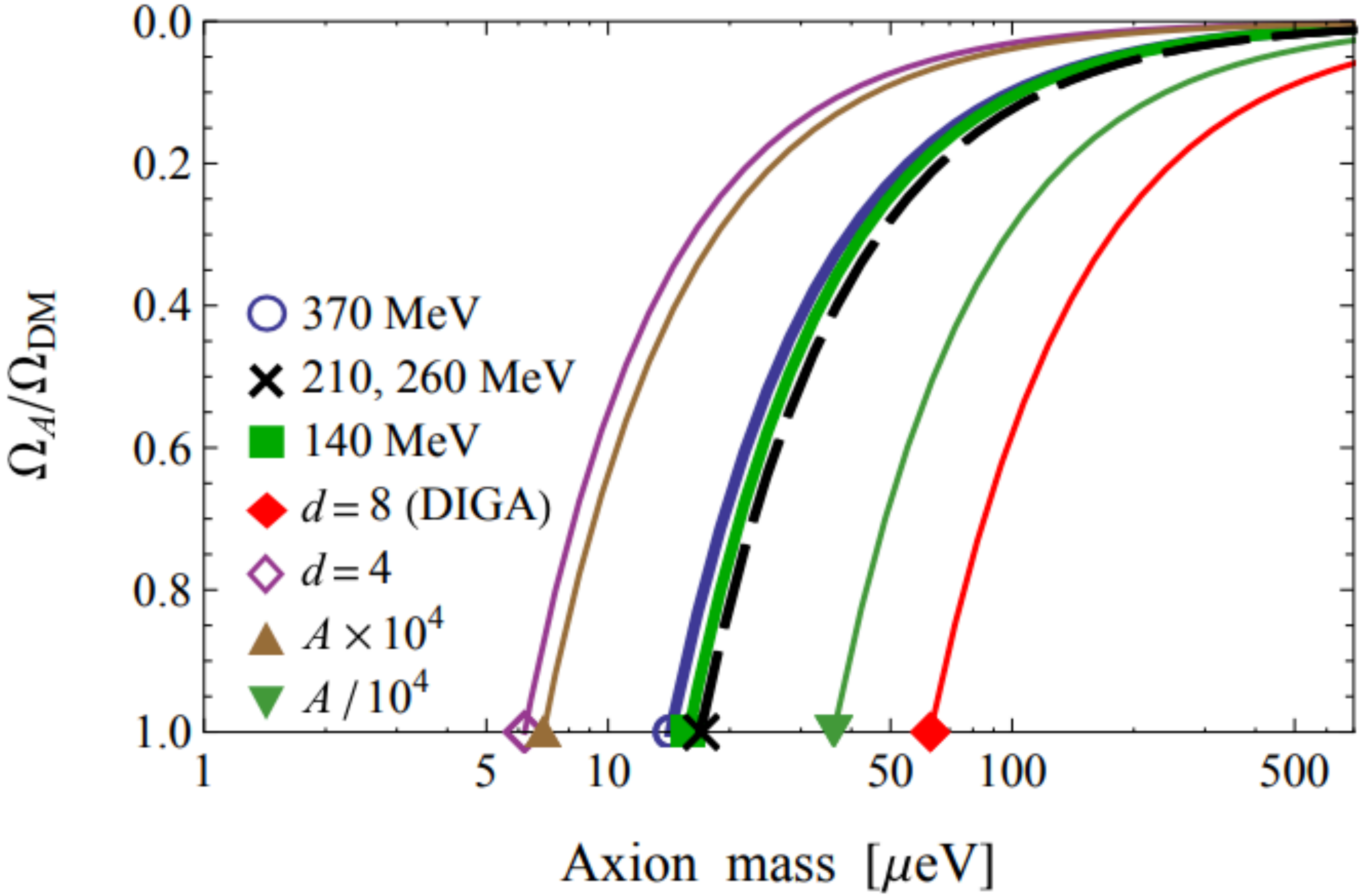
Kotov, Trunin, MpL, arXiv 2021



Continuum limit

Burger et al. (2018)

[as an aside]



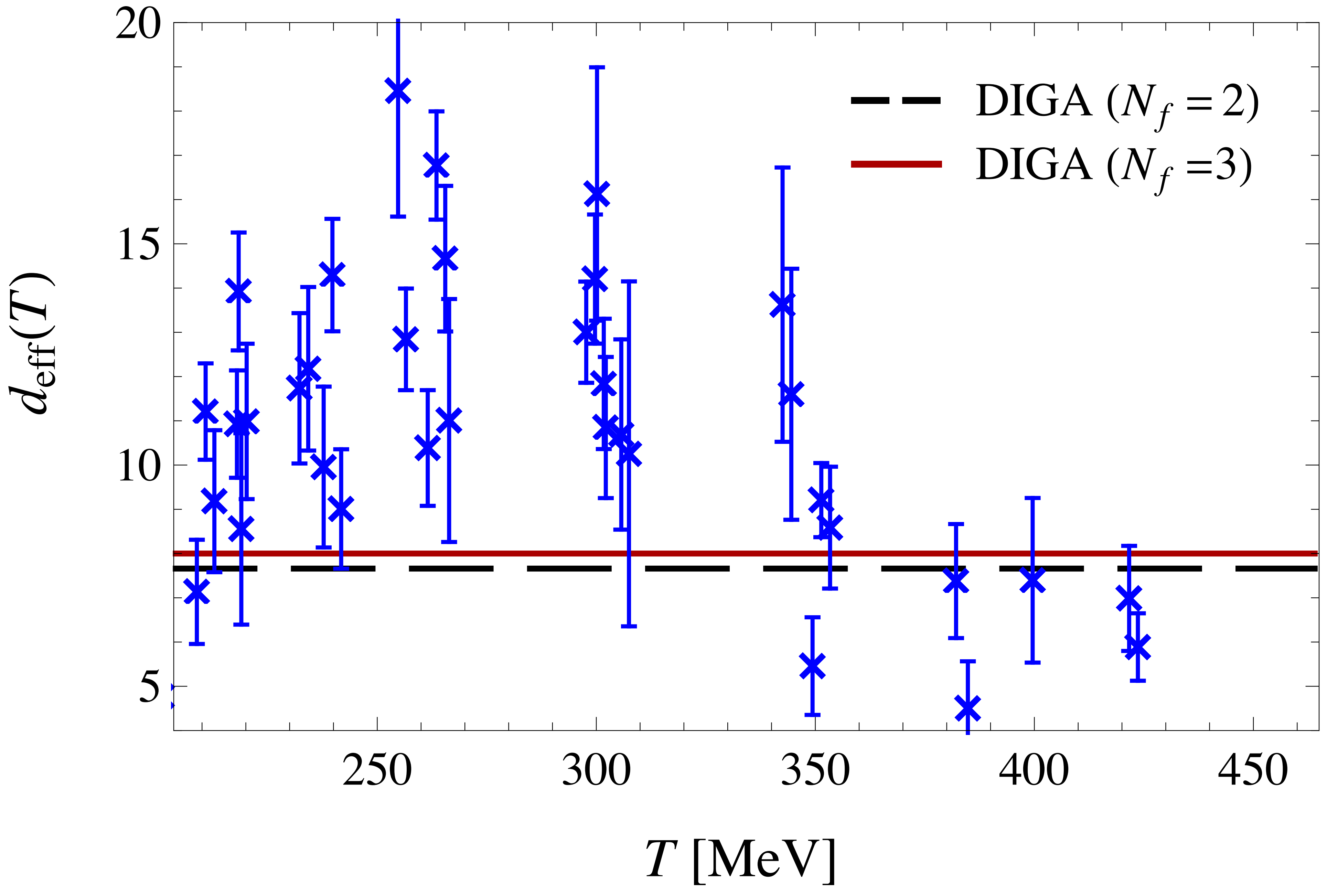
0.0
0.2
0.4
0.6
0.8
1.0
$\Omega_A/\Omega_{\rm DM}$
370 MeV
210, 260 MeV
140 MeV
$d=8$ (DIGA)
$d=4$
$A\times10^4$
$A/10^4$
1
5
10
50
100
500
Axion mass [μeV]

Power-law decay?

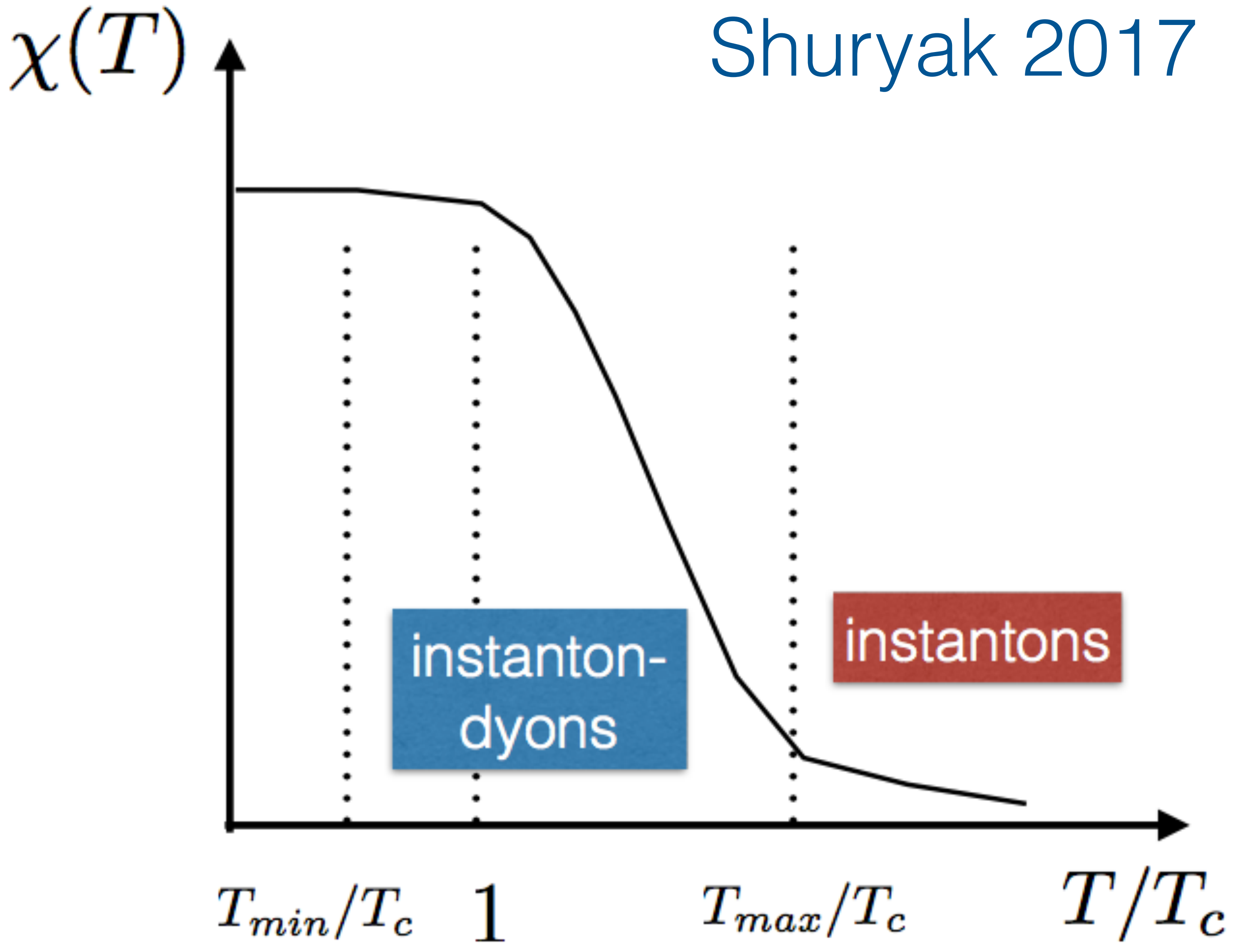
$$\chi^{0.25}(T) = aT^{-d(T)}$$

For instanton gas $\quad d(T) \equiv const \simeq (7 + \frac{N_f}{3})$

$$d(T) = -T \frac{d}{dT} \ln \chi^{0.25}(T)$$



Possibly consistent with instant -dyon?
Shuryak 2017



Faster decrease before DIGA sets in

QCD - Summary of b parameter

$$\chi(T) = A\,T^{b}$$

Y. Taniguchi, K. Kanaya, H. Suzuki and T. Umeda (2017) (d),

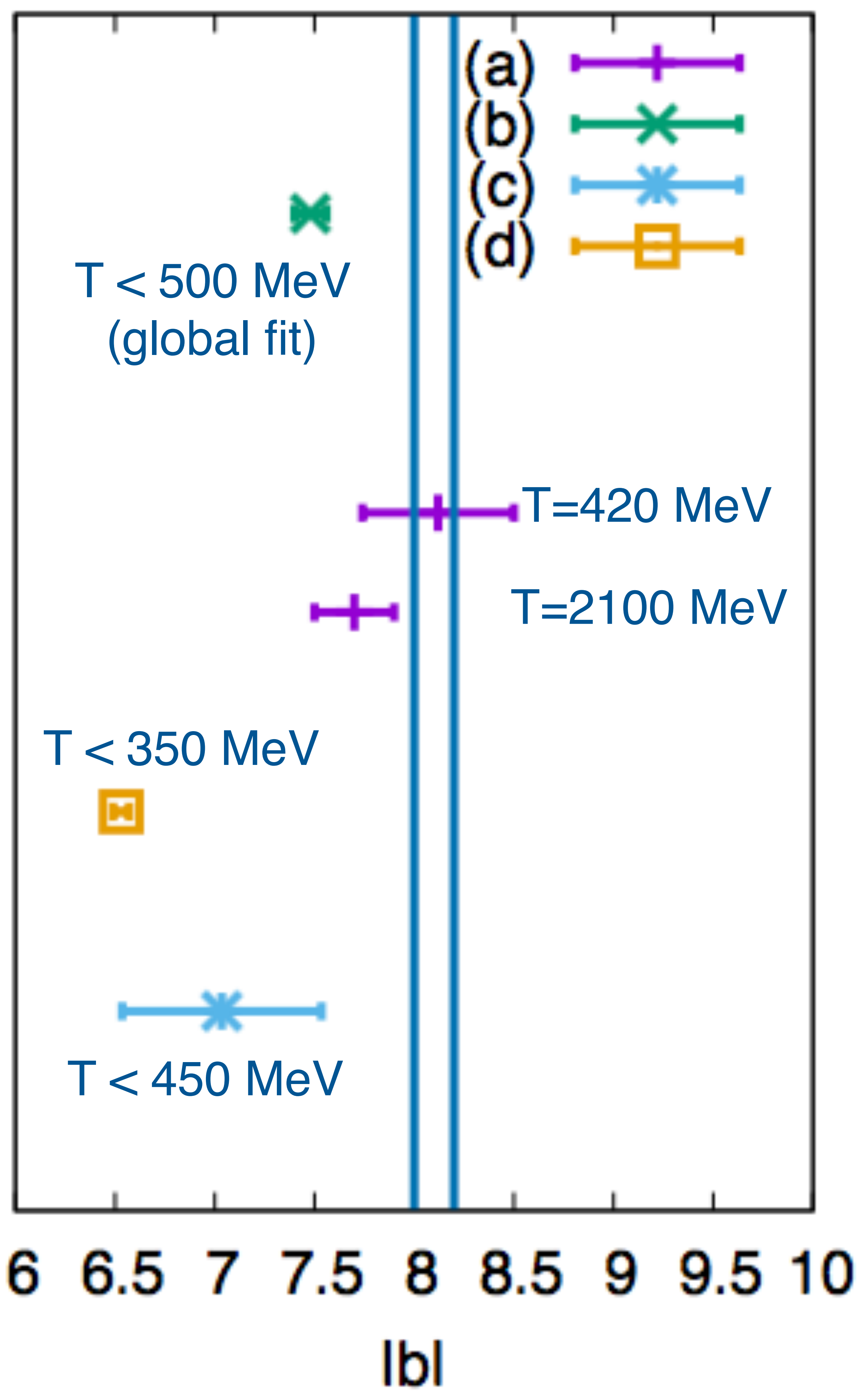
Borsanyi et al. (2016) (a)

Petreczky, Schlaeder, Scharma (2016) (b)

Burger et al. (2018) (c)

DIGA, Nf = 3

DIGA, Nf=4



For T > 300 MeV the DIGA exp is approached *from below*

Tc < T < 250 MeV ??

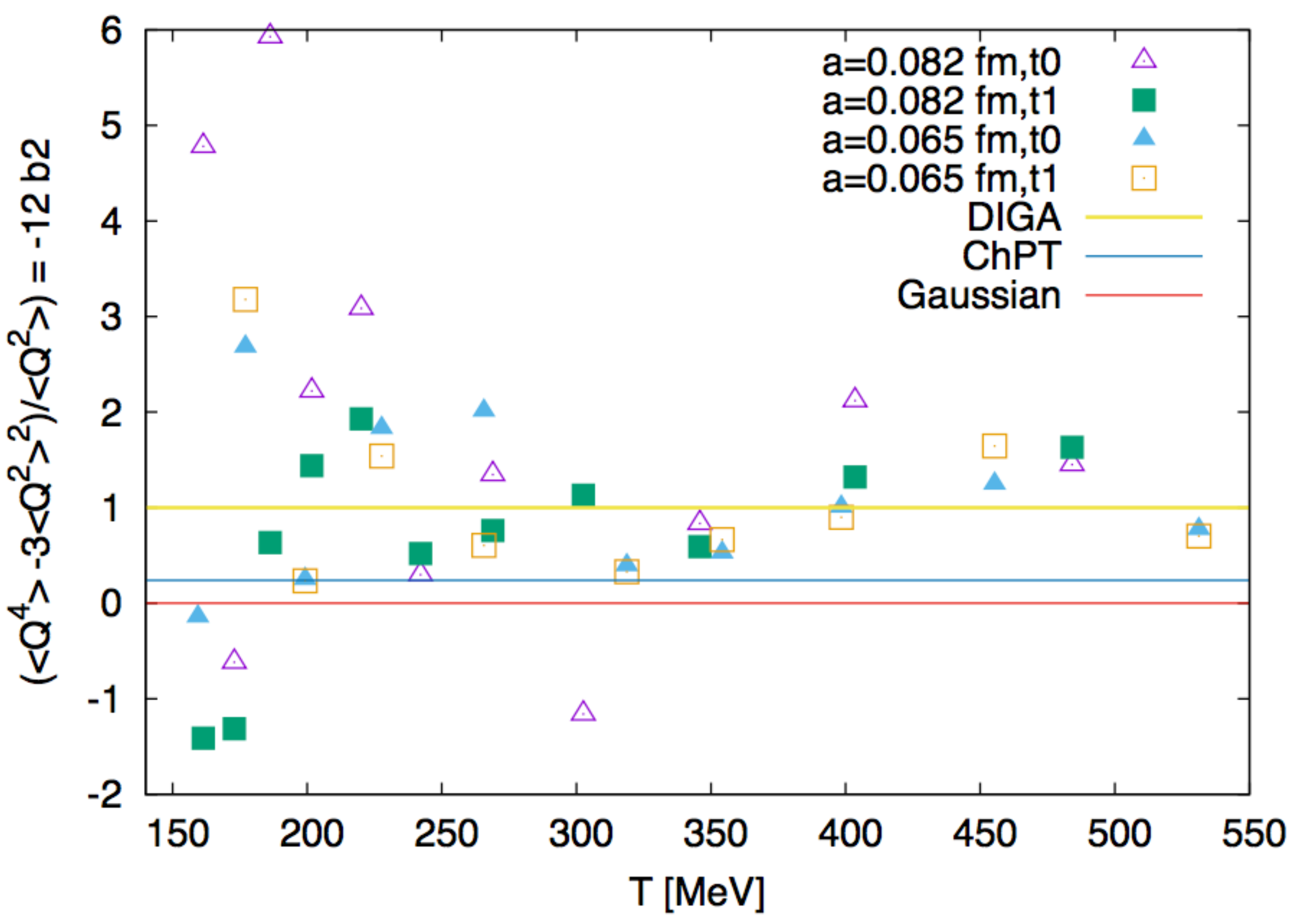
Further evidence of DIGA behaviour

$$T \to \infty \quad \Big| \sim C \left(\frac{T_c}{T}\right)^{\beta} \cos(\theta)$$

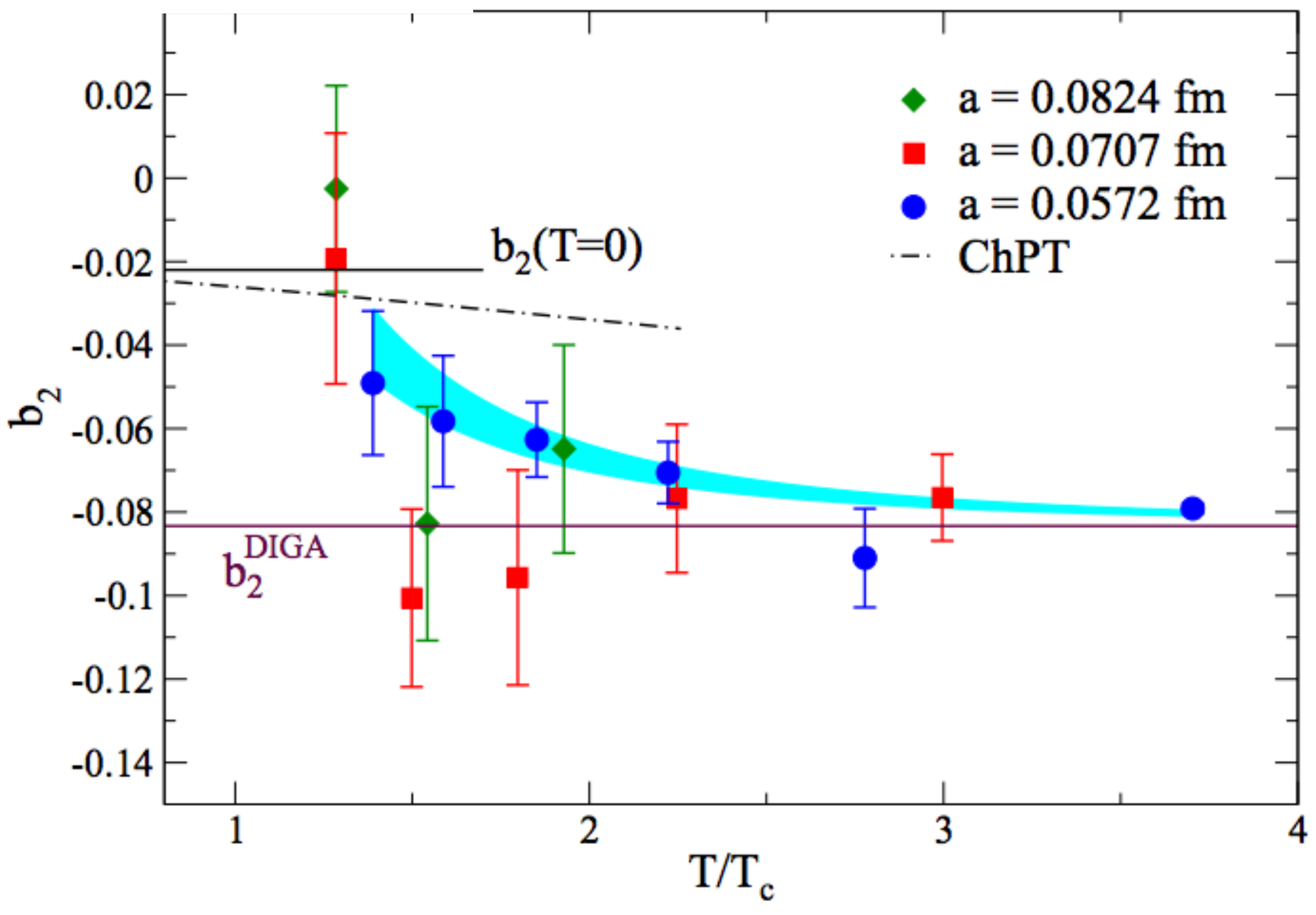
T > 250-300 MeV

$$C_n = (-1)^{n+1} \frac{d^{2n}}{d\theta^{2n}} F(\theta, T)\Big|_{\theta=0} = \langle Q^{2n} \rangle_{conn}.$$

d'Elia, Vicari 1301.7640

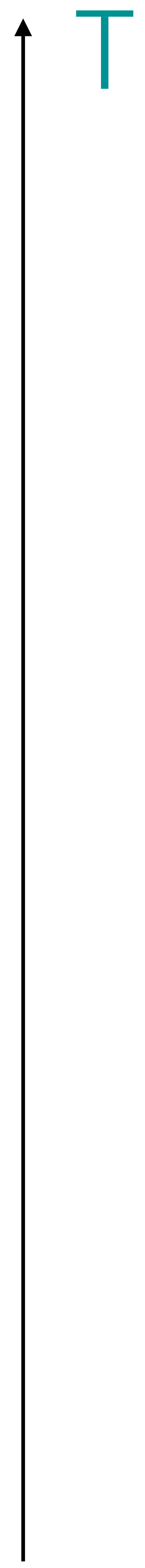


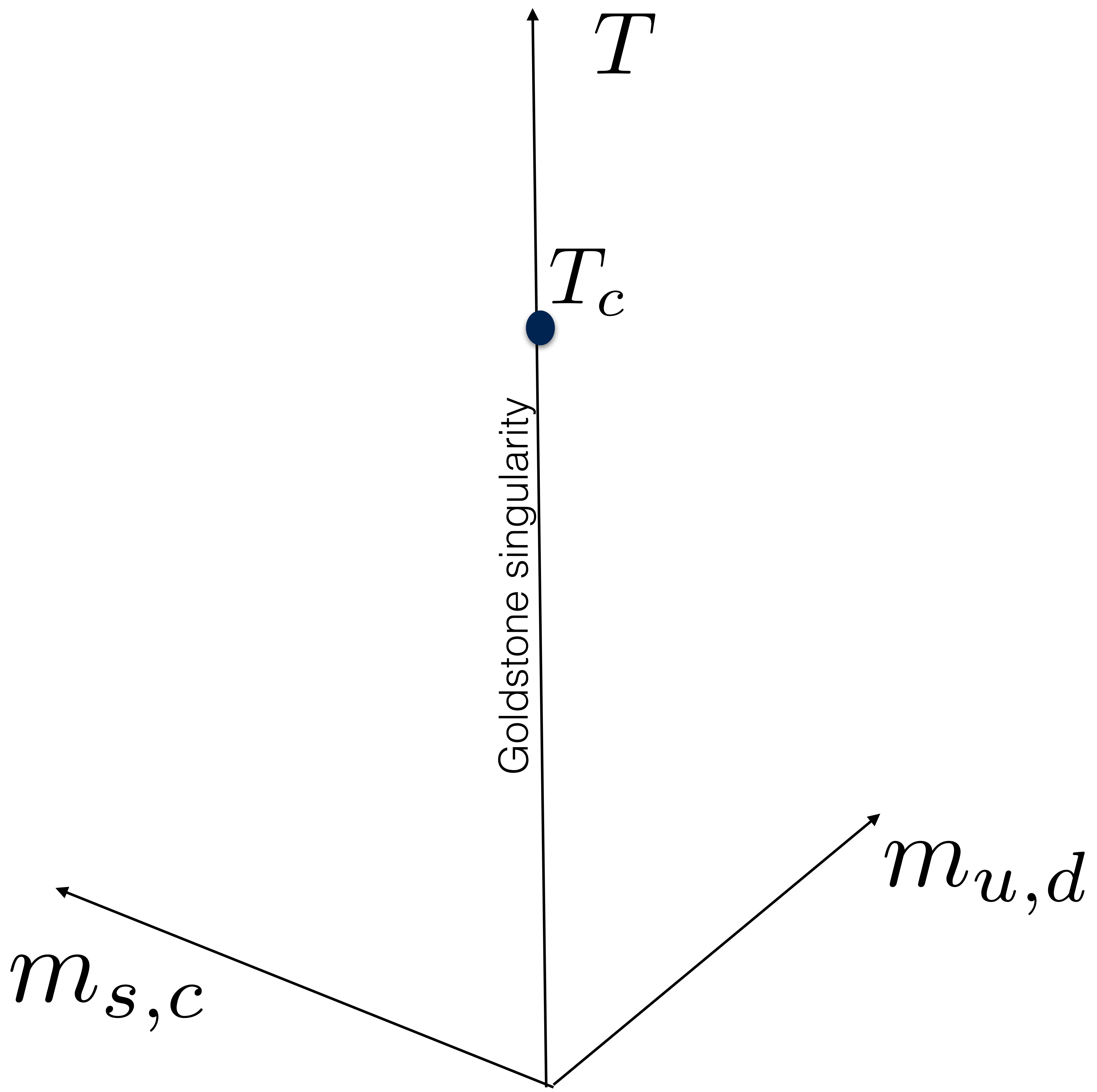
Trunin at al (2018)



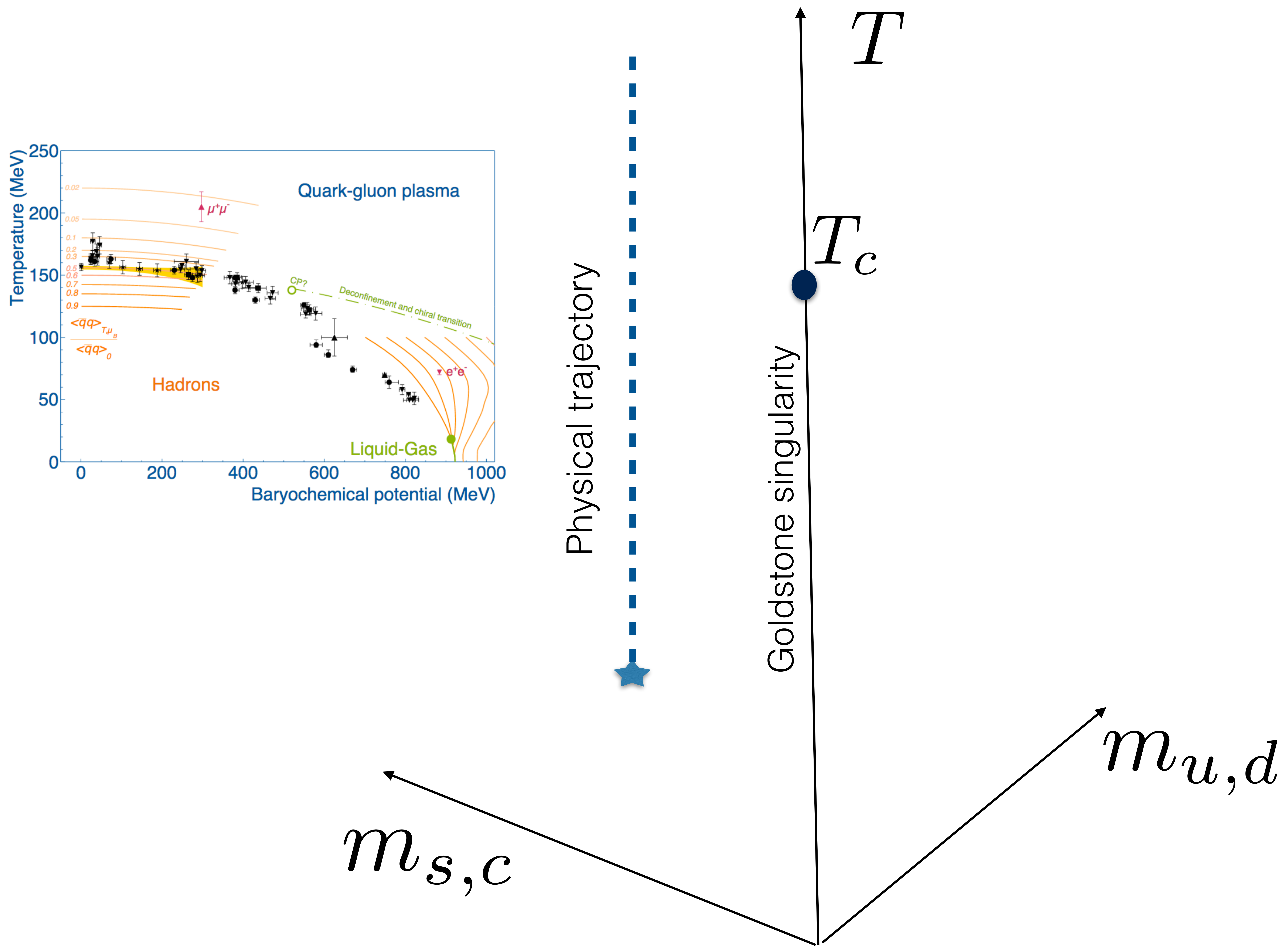
Bonati et al. (2016)

Scaling window around Tc


T



T
T_c
Goldstone singularity
$m_{u,d}$
$m_{s,c}$



Temperature (MeV)
250
200
150
100
50
0
0
200
400
600
800
1000
Baryochemical potential (MeV)
Quark-gluon plasma
Hadrons
Liquid-Gas
CP?
Deconfinement and chiral transition
Physical trajectory
Goldstone singularity
T
T_c
$m_{u,d}$
$m_{s,c}$

Symmetries of QCD

$$\mathcal{L} = \sum_{a=1}^{n} \bar{q}_{La} \partial\!\!\!/ q_{La} + \bar{q}_{Ra} \partial\!\!\!/ q_{Ra} - m(\bar{q}_{La} q_{La} + \bar{q}_{Ra} q_{Ra}) + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a + \mathcal{L}_{gauge}$$

With $m = 0$, invariant under

$q_L \to V_L q_L q_R \to V_R q_R$, with $V \in U(n)$

Global symmetry:

$$U(n)_L \times U(n)_R \cong SU(n) \times \ SU(n) \times U(1)_V \times U(1)_A$$

.

baryon number

Spontaneously Broken, $(n^2 - 1)$ GB

Explicitely broken

Experimental Evidence

$$N_f = ?$$

T=0, no difference, just different #Goldstones

$m_{u,d} = 0$

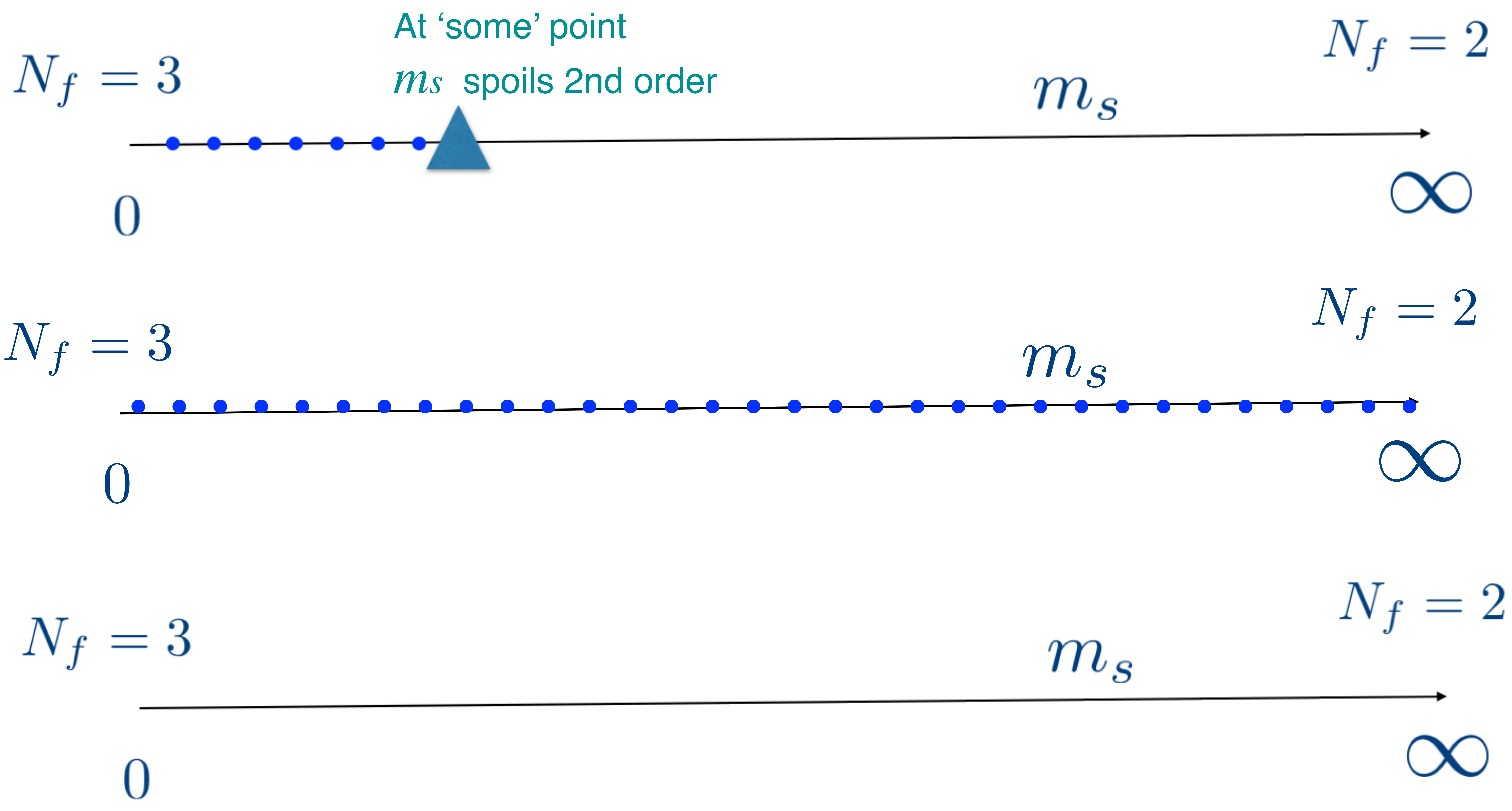
$N_f = 3$

m_s

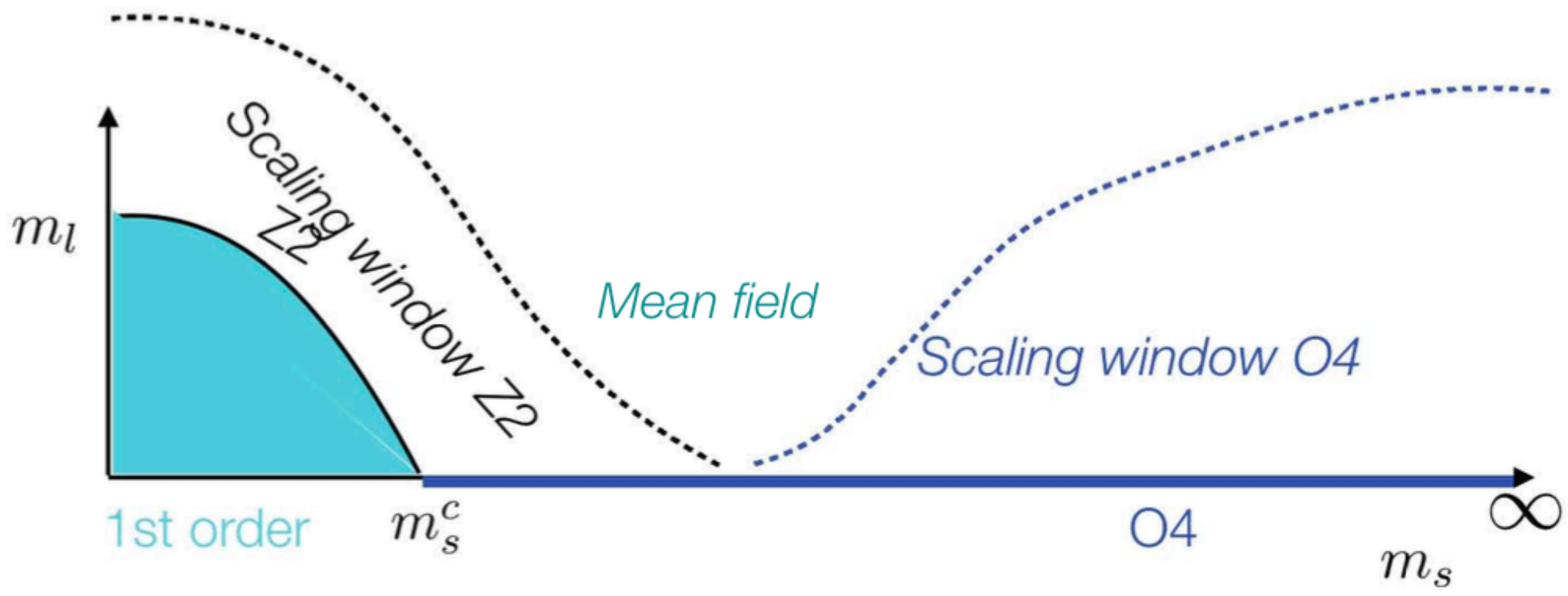
$N_f = 2$

0 → ∞

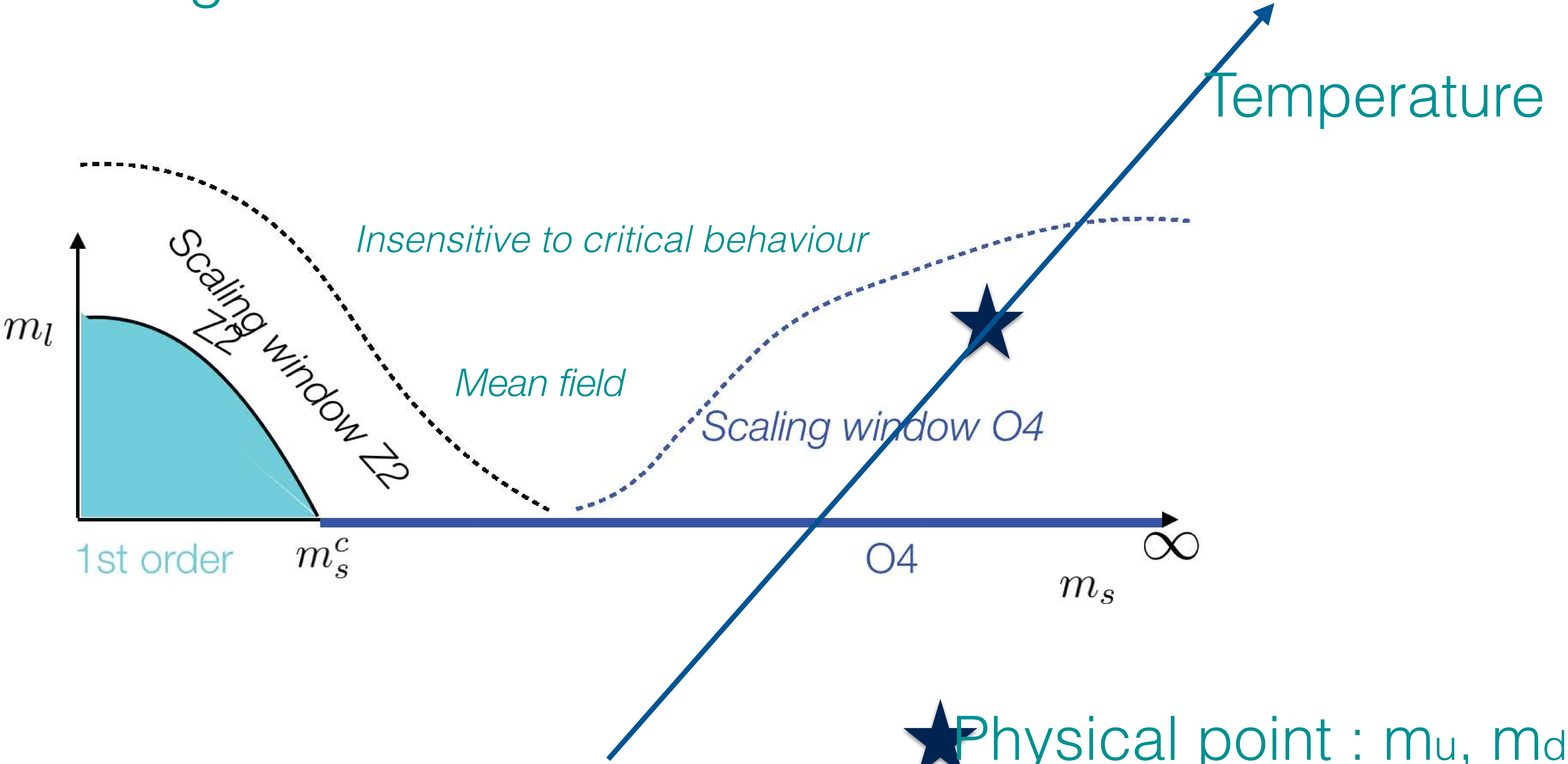
Strange mass as interpolator between Nf=3 and Nf=2



Switching on the light mass: a possible Scenario



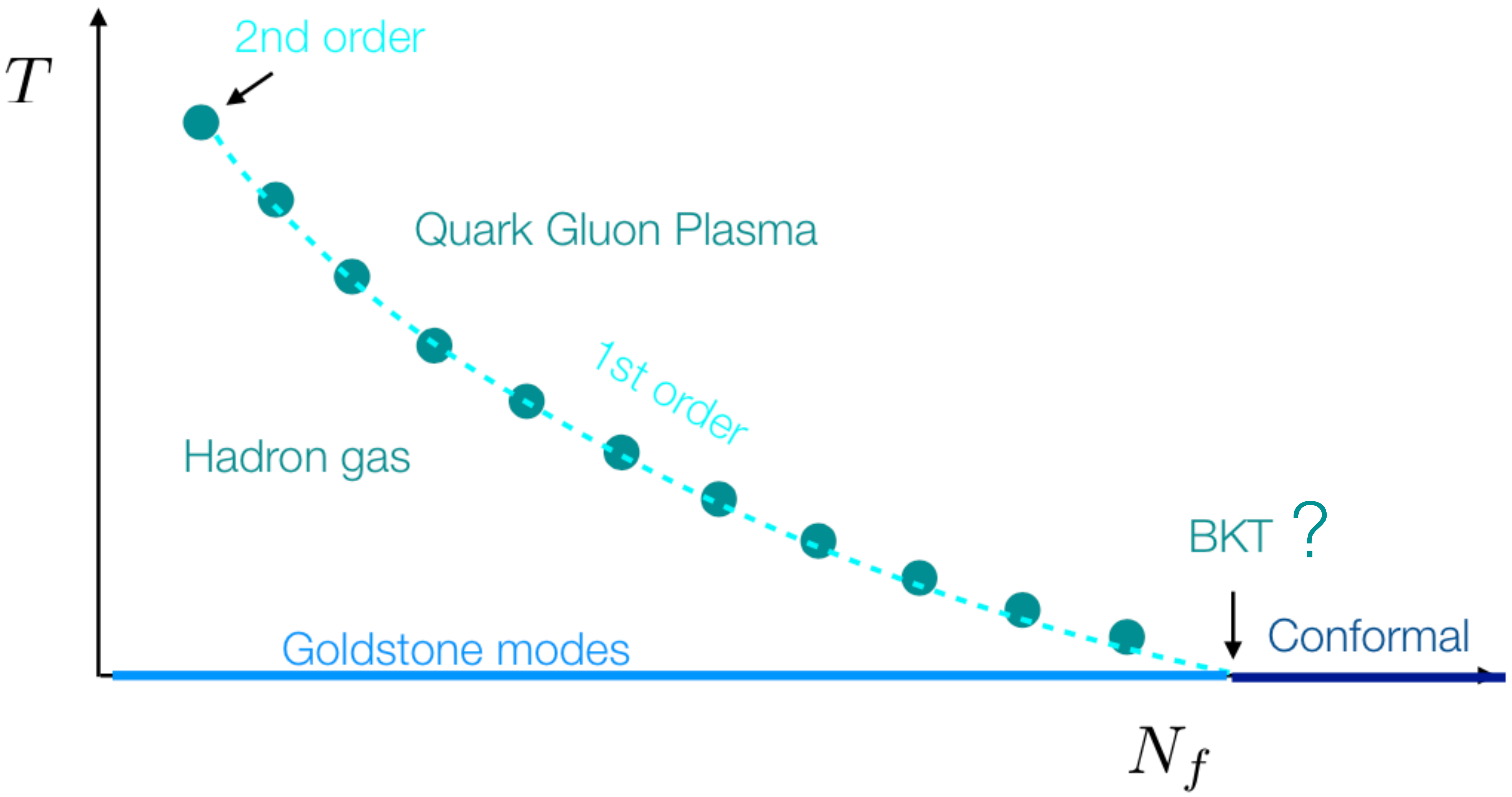
Where is the scaling window in QCD in mass and T?



★Physical point : mu, md, m

Switching on temperature -

AdS/CFT



The magnetic equation of State:

$$h = M^{\delta} f(t/M^{1/\beta})$$

$M \equiv \bar{\psi}\psi$, $h \equiv m_q$, $t \equiv T - T_c$, m_q is the quark mass and T_c is the critical temperature

Three strategies to identify the scaling behaviour:

- direct comparison with the Equation of State
- the study of the dependence of the pseudo-critical temperatures on the breaking field, also known as scaling of pseudo-critical temperatures
- definition of RG invariant quantities, which do not depend on the breaking field at the critical point.

Byproduct: critical temperature in the chiral limit

Significant source of scaling violations:

additive linear mass corrections to $\bar{\psi}\psi$

Playing with the order parameter

also mentioned in the PhD thesis by Wolfgang Unger

'Beating' the regular terms/additive renormalization for more stringent universality checks

$$\Delta_3 \equiv (\bar\psi\psi - m\chi_L) \equiv (\bar\psi\psi - m\frac{\partial\bar\psi\psi}{\partial m}) \qquad \equiv m(\chi_T - \chi_L)$$

Advantage wrt standard subtracted condensate: admits EoS

Note: Transverse and longitudinal susceptibilities

$$\chi_T = \frac{\bar\psi\psi}{m}$$

$$\chi_L = \frac{\partial\bar\psi\psi}{\partial m}.$$

$$R_\pi \equiv \chi_T^{-1}/\chi_L^{-1}$$

$$\frac{1}{R_\pi(t,m)} = \delta - \frac{x}{\beta}\frac{f'(x)}{f(x)},$$

$$R_\pi(0,m) = \frac{1}{\delta}$$

Kocic, Kogut, MpL;
Karsch, Laermann

Equation of State for Δ_3

- linear terms in m drop in $\Delta_3 \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m})$

Use: $M = h^{1/\delta} f_G(t/h^{1/\beta\delta})$ (parametrization in:

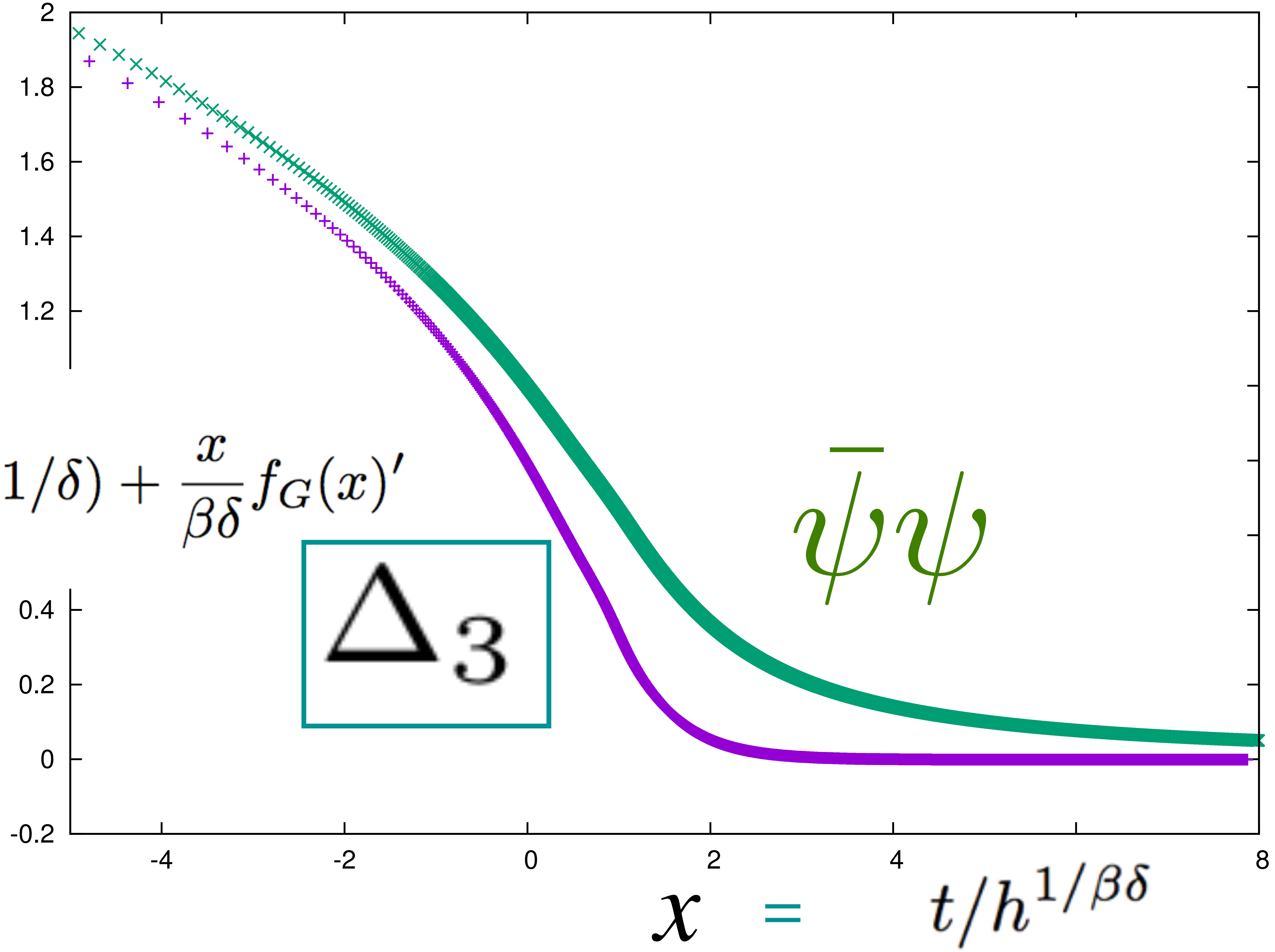
J.Engels and F.Karsch, Phys. Rev. D 85, (2012)

To get EoS for Δ_3

$$\Delta_3 = m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) - 1/\delta m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) + m^{1/\beta\delta+1} f_G'((t/m^{1/\beta\delta})$$

$$\frac{\Delta_3}{m^{1/\delta}} = f_G(x)(1-1/\delta) + \frac{x}{\beta\delta} f_G(x)'$$

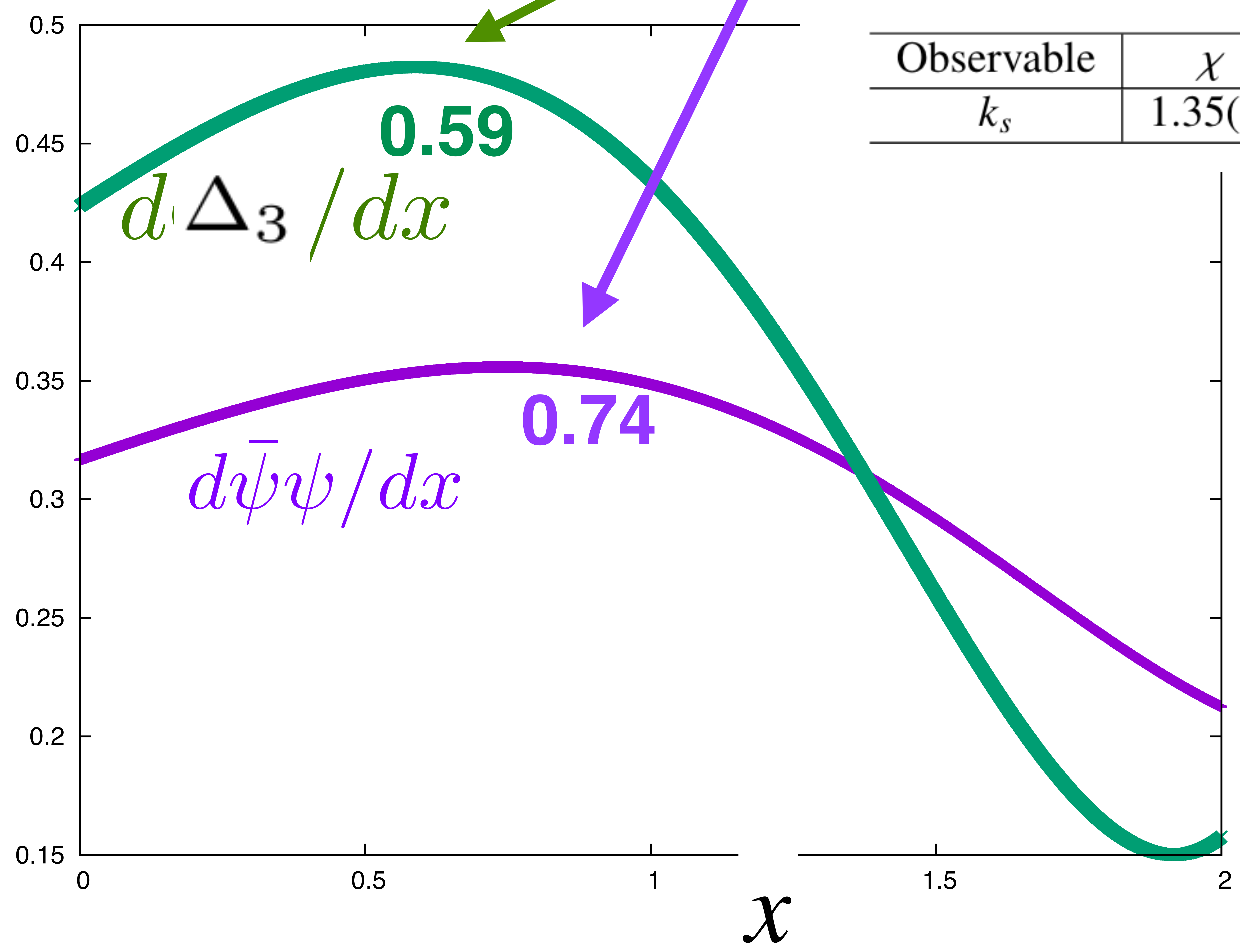
$$\frac{\Delta_3}{m^{1/\delta}} = f_G(x)(1 - 1/\delta) + \frac{x}{\beta\delta} f_G(x)'$$



Derivatives: give scaling of pseudo critical temperature Tc with mass

$$T_c = T_c(0) + k_s m_\pi^{2/\beta\delta}$$

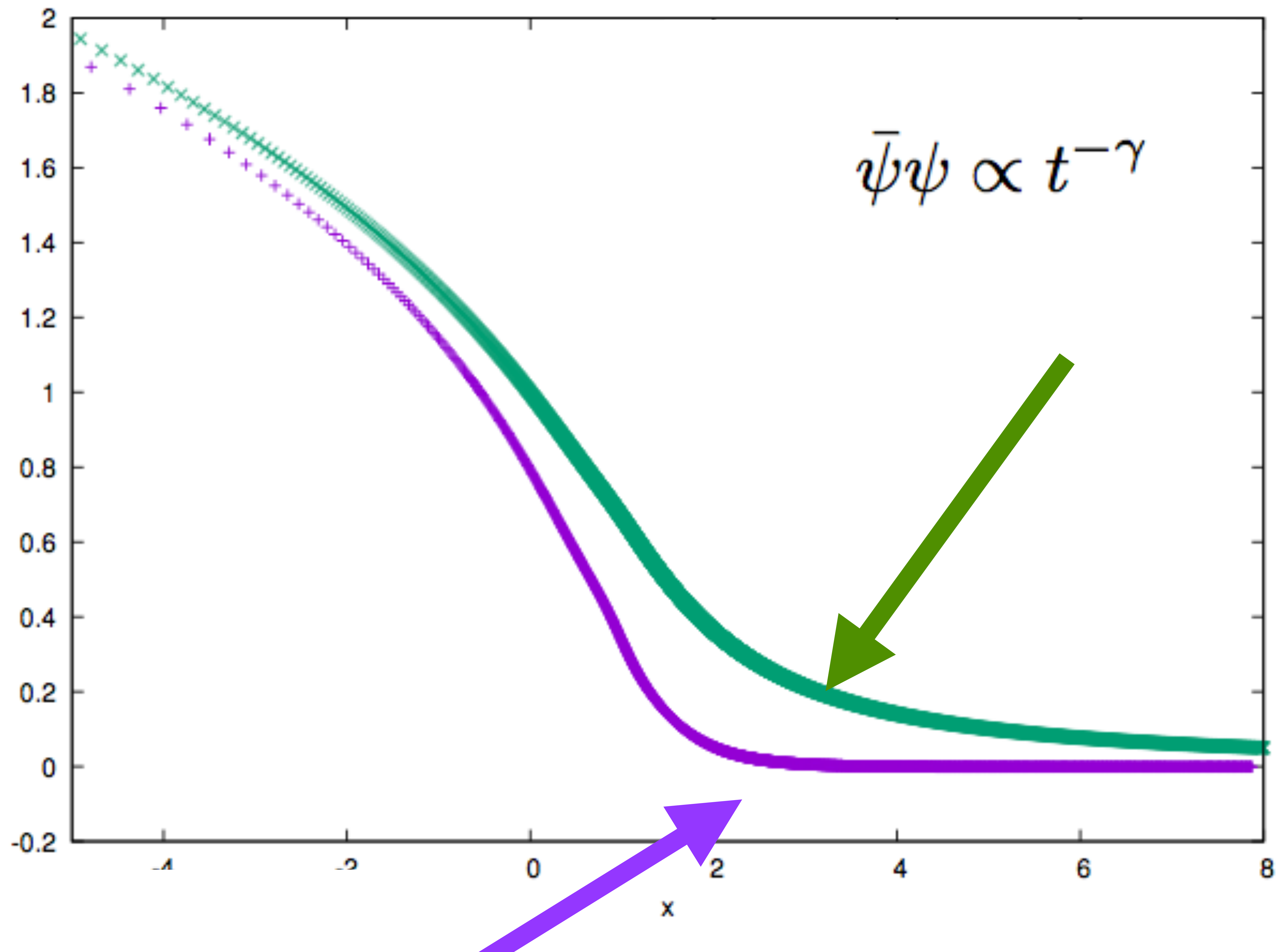
Observable	χ	$\bar{\psi}\psi$	Δ_3
k_s	1.35(3)	0.74(4)	0.59(1)



Asymptotic behavior - high T expansion

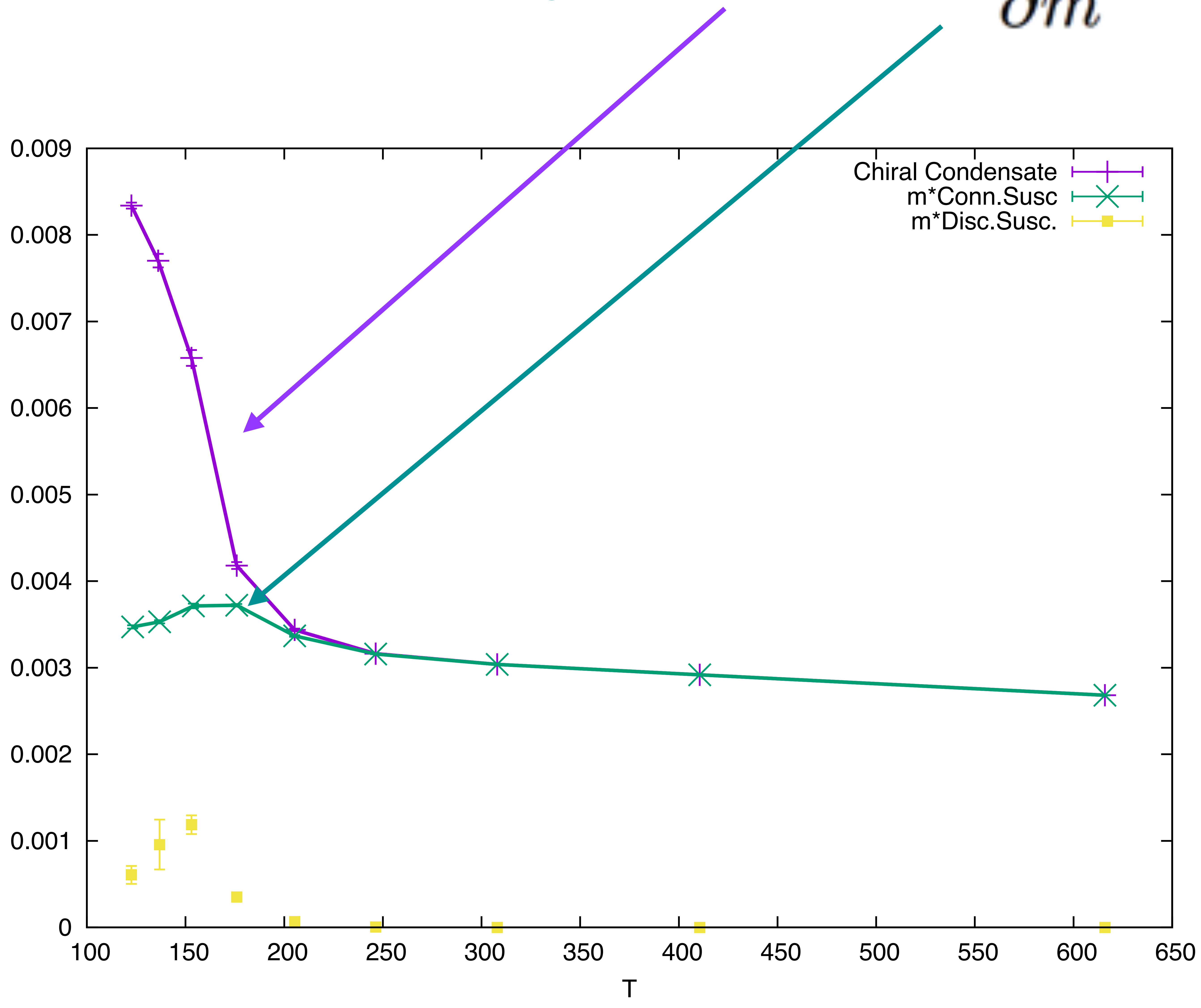
$$f_G(x) = x^{-\gamma} \sum_{n=0}^{\infty} d_n x^{-2n\Delta}$$

again, linear term drops in Δ_3

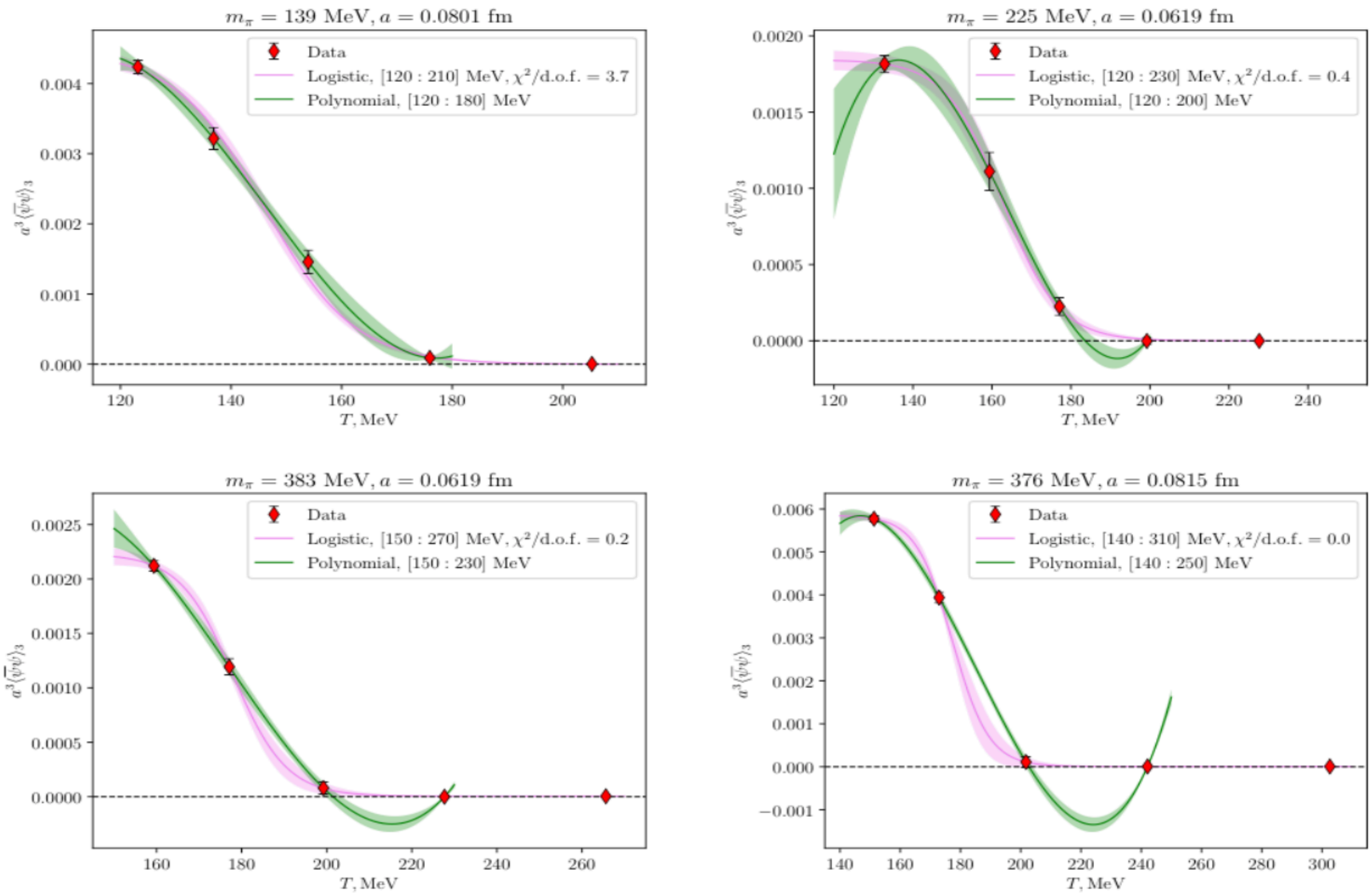


$$\Delta_3 \propto t^{-\gamma-2\beta\delta}$$

Building $\Delta_3 \equiv (\bar\psi\psi - m\frac{\partial\bar\psi\psi}{\partial m})$

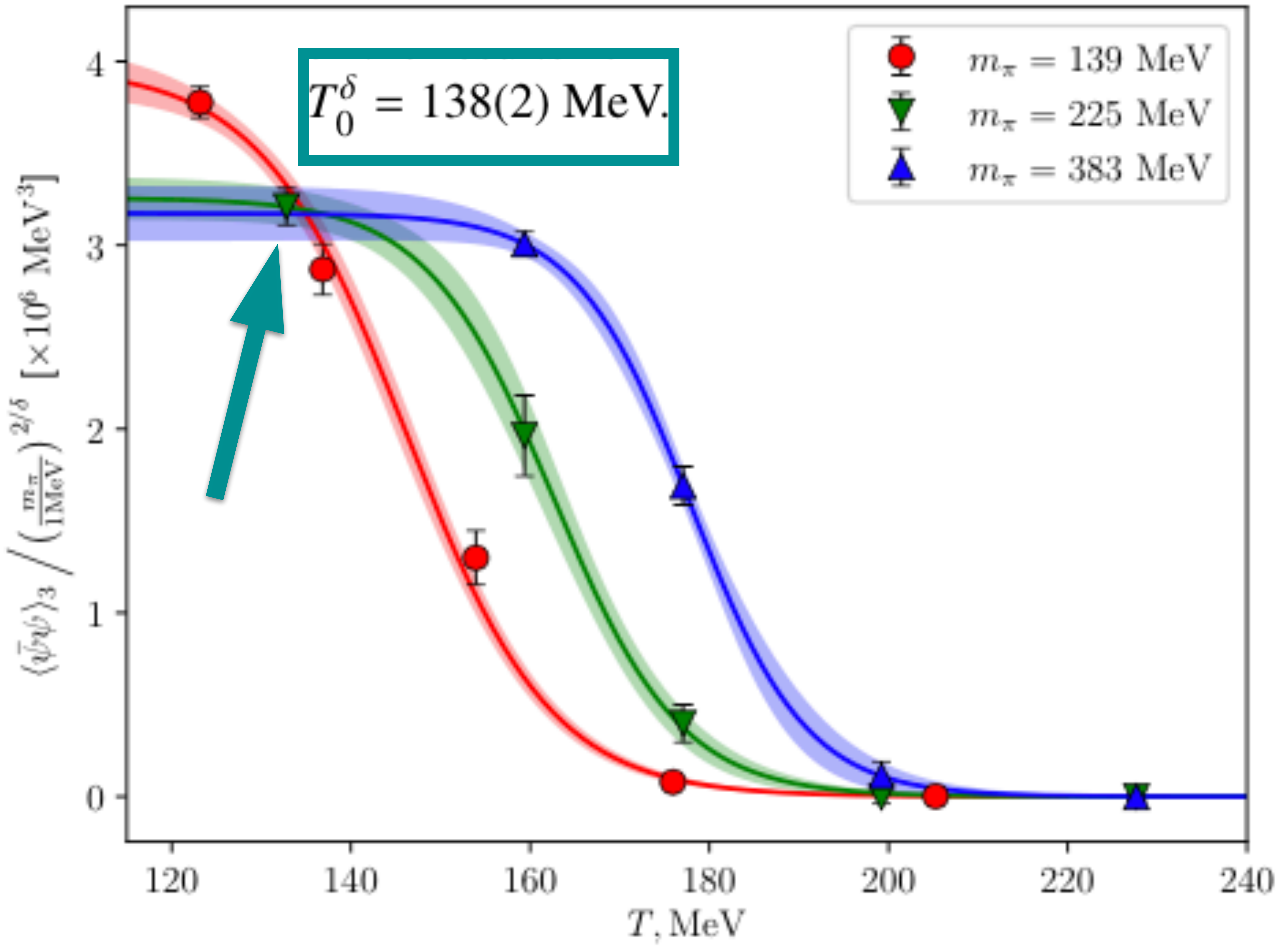


Bare Δ_3



$m_\pi = 139$ MeV, $a = 0.0801$ fm
Data
Logistic, [120 : 210] MeV, χ^2/d.o.f. = 3.7
Polynomial, [120 : 180] MeV
$a^3\langle\bar{\psi}\psi\rangle_3$
T, MeV
$m_\pi = 225$ MeV, $a = 0.0619$ fm
Data
Logistic, [120 : 230] MeV, χ^2/d.o.f. = 0.4
Polynomial, [120 : 200] MeV
$a^3\langle\bar{\psi}\psi\rangle_3$
T, MeV
$m_\pi = 383$ MeV, $a = 0.0619$ fm
Data
Logistic, [150 : 270] MeV, χ^2/d.o.f. = 0.2
Polynomial, [150 : 230] MeV
$a^3\langle\bar{\psi}\psi\rangle_3$
T, MeV
$m_\pi = 376$ MeV, $a = 0.0815$ fm
Data
Logistic, [140 : 310] MeV, χ^2/d.o.f. = 0.0
Polynomial, [140 : 250] MeV
$a^3\langle\bar{\psi}\psi\rangle_3$
T, MeV

Scaling at the critical point: searching for $< \bar{\psi}\psi >_3 (T = T_0) = A m_\pi^{2/\delta}$

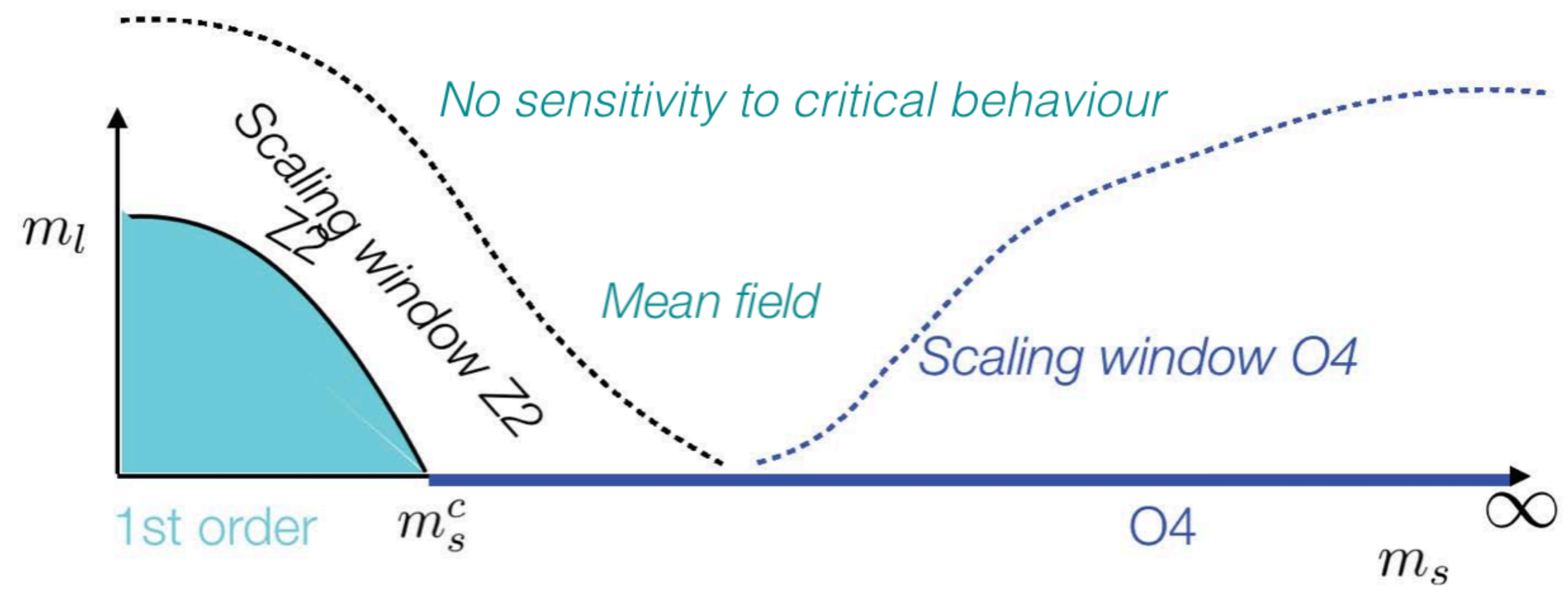


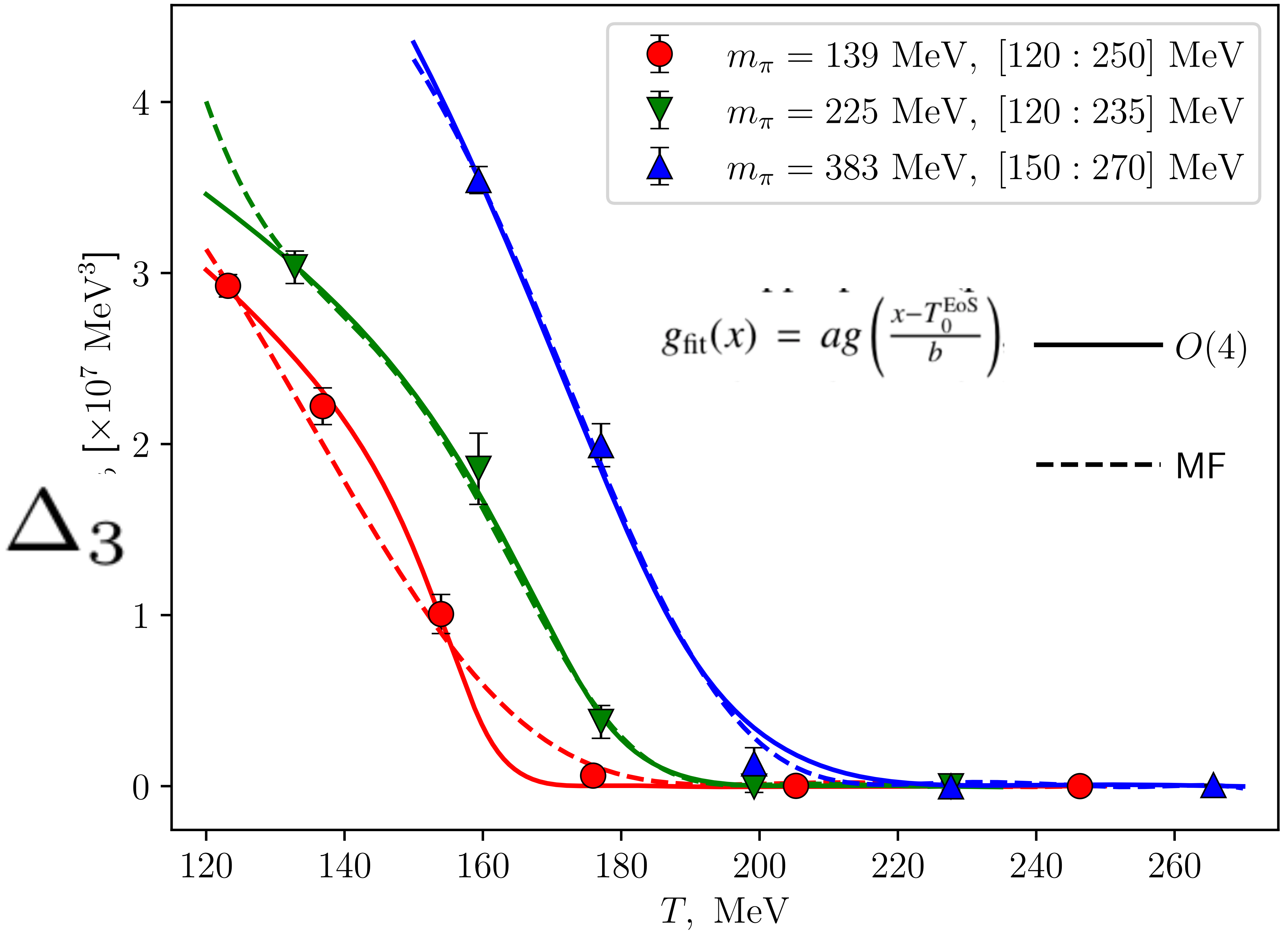
Searching for the scaling window in mass

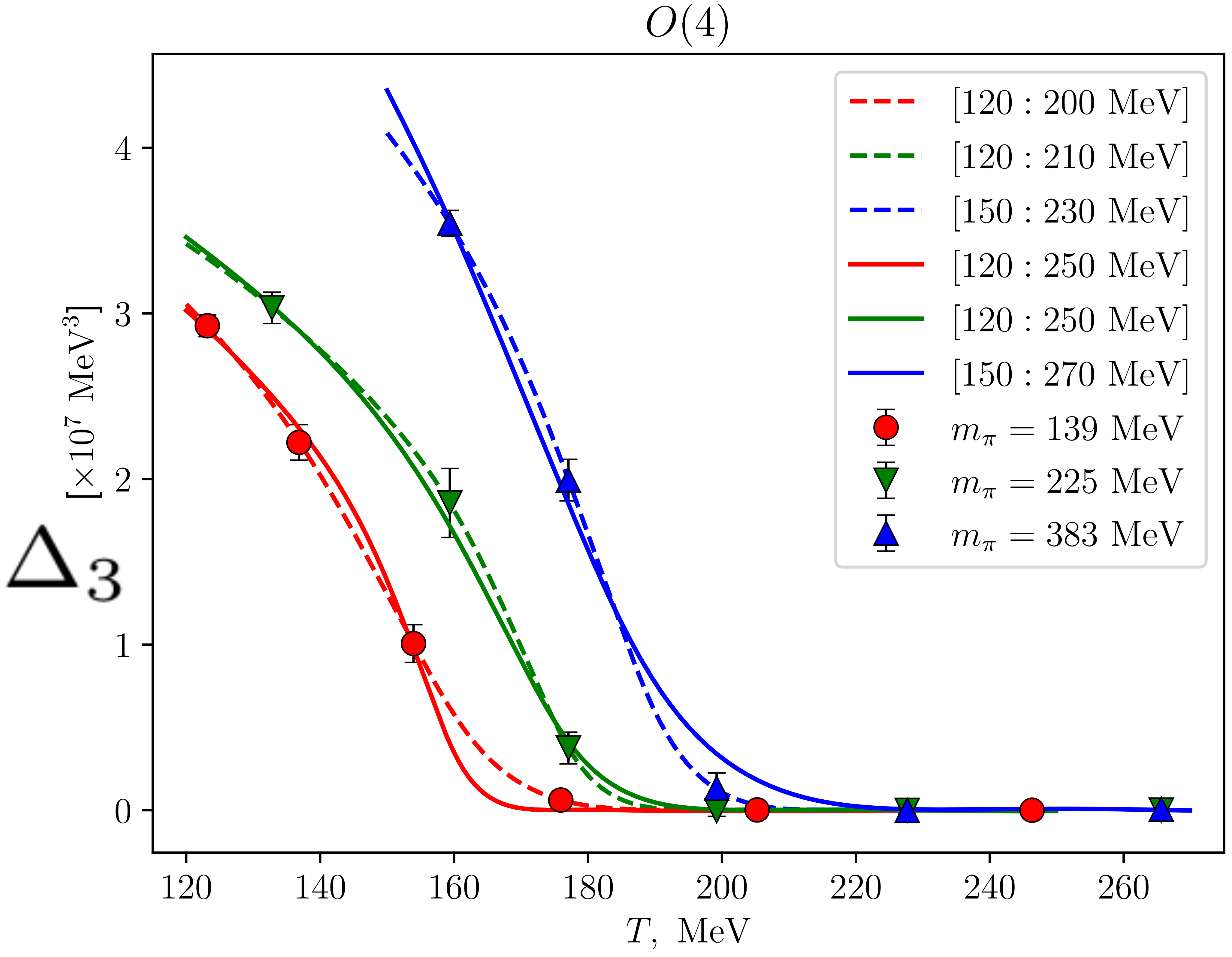
O(4) or mean field?

Unrealistic T_0 from O4 at high mass

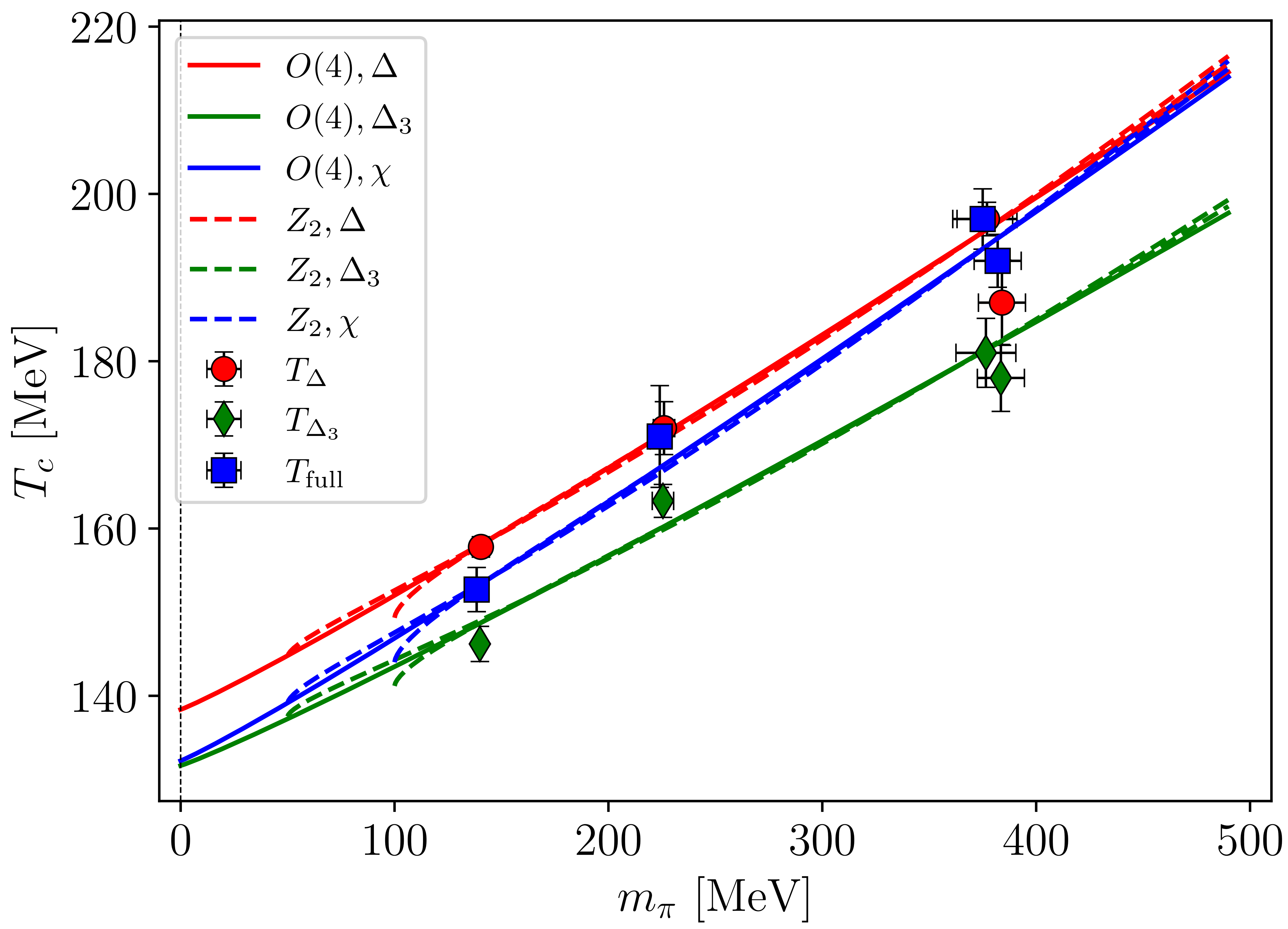
T_{EOS} = 142(2),159(3), 174(2) MeV







Scaling of the pseudo critical temperatures



Consistent (not a proof) with O4

Robust extrapolation:

$$T_0 \equiv T_c(m_\pi \to 0) = 134^{+6}_{-4} \text{ MeV}$$

Check O4:

$$T_c(m_\pi) = T_0 + A z_p m_\pi^{2/\beta\delta}$$

Observable	T_0 [MeV]	$z_p/z_{\bar\psi\psi_3}$	$z_p/z_{\bar\psi\psi_3}$ $O(4)$	z_p $O(4)$
χ	132(4)	1.24(17)	2.45(4)	1.35(3)
$\langle\bar\psi\psi\rangle$	138(2)	1.15(24)	1.35(7)	0.74(4)
$\langle\bar\psi\psi\rangle_3$	132(3)	1	1	0.55(1)

O4 vs Z2

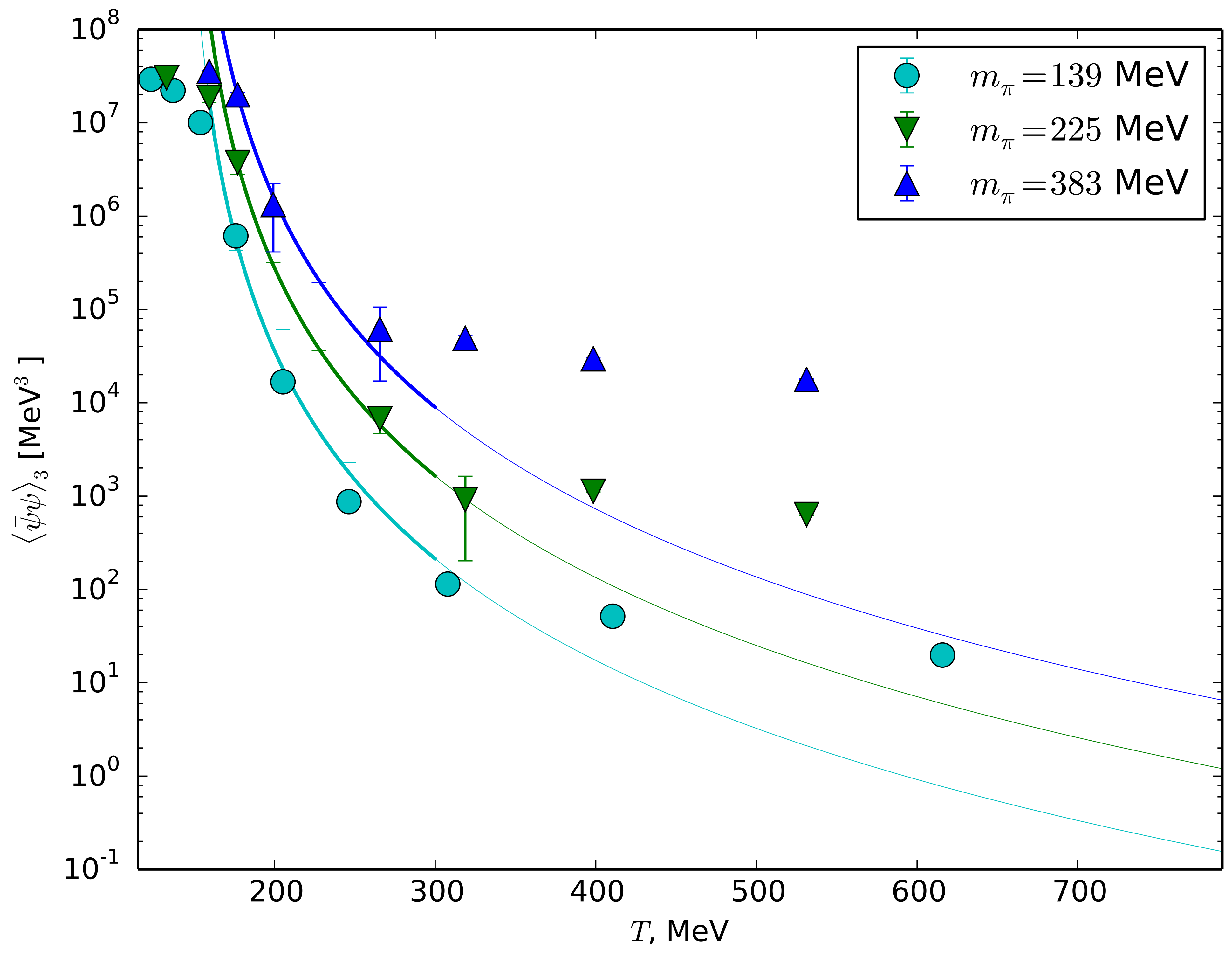
$$T_c(m_\pi) = T_0 + B(m_\pi^2 - m_c^2)^{1/\beta\delta}$$

m_c = 100 MeV still OK

m_c = 0 still OK, indistinguishable from O4

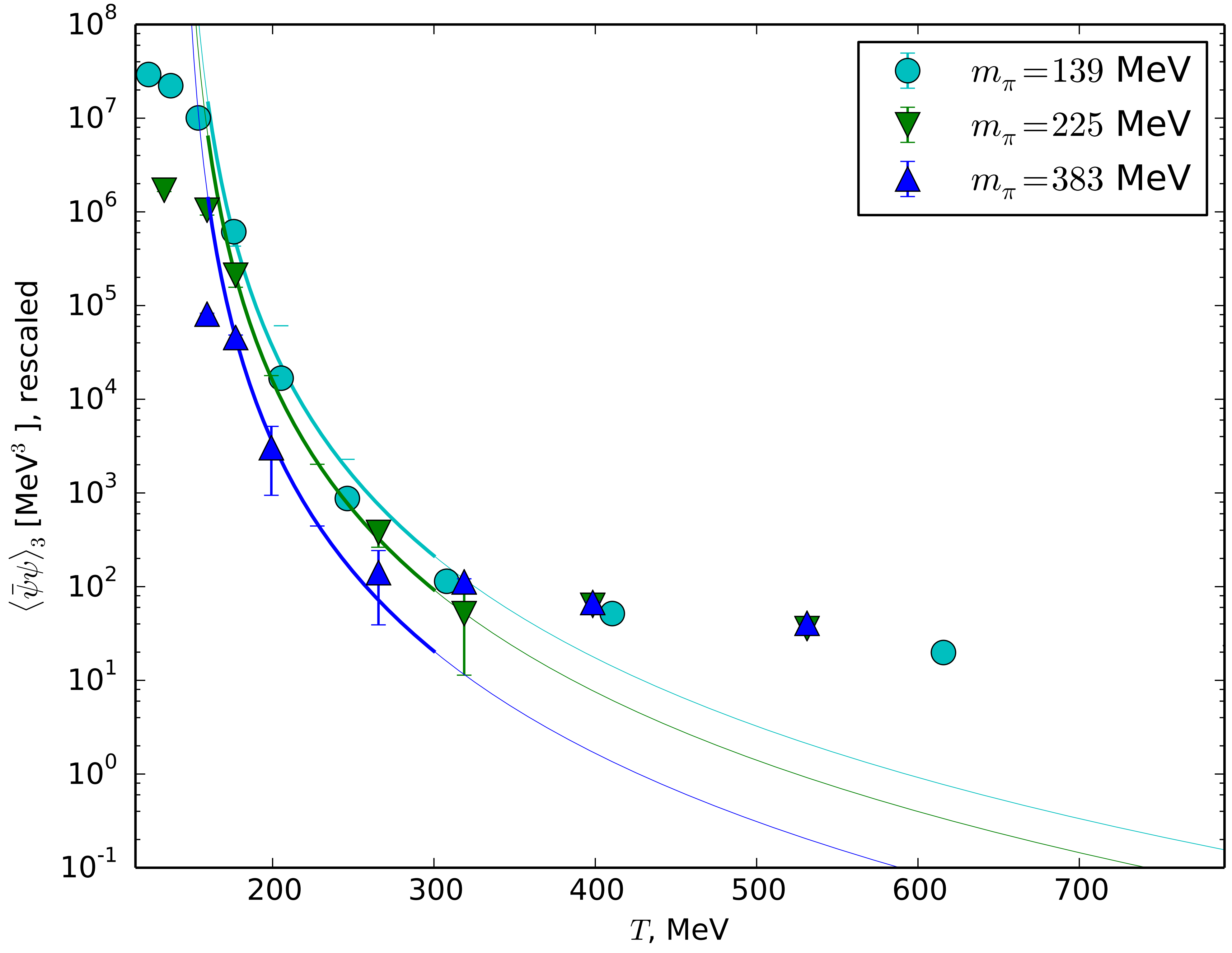
Searching for the scaling window in temperature

‘Forgotten’ microscopic dynamics



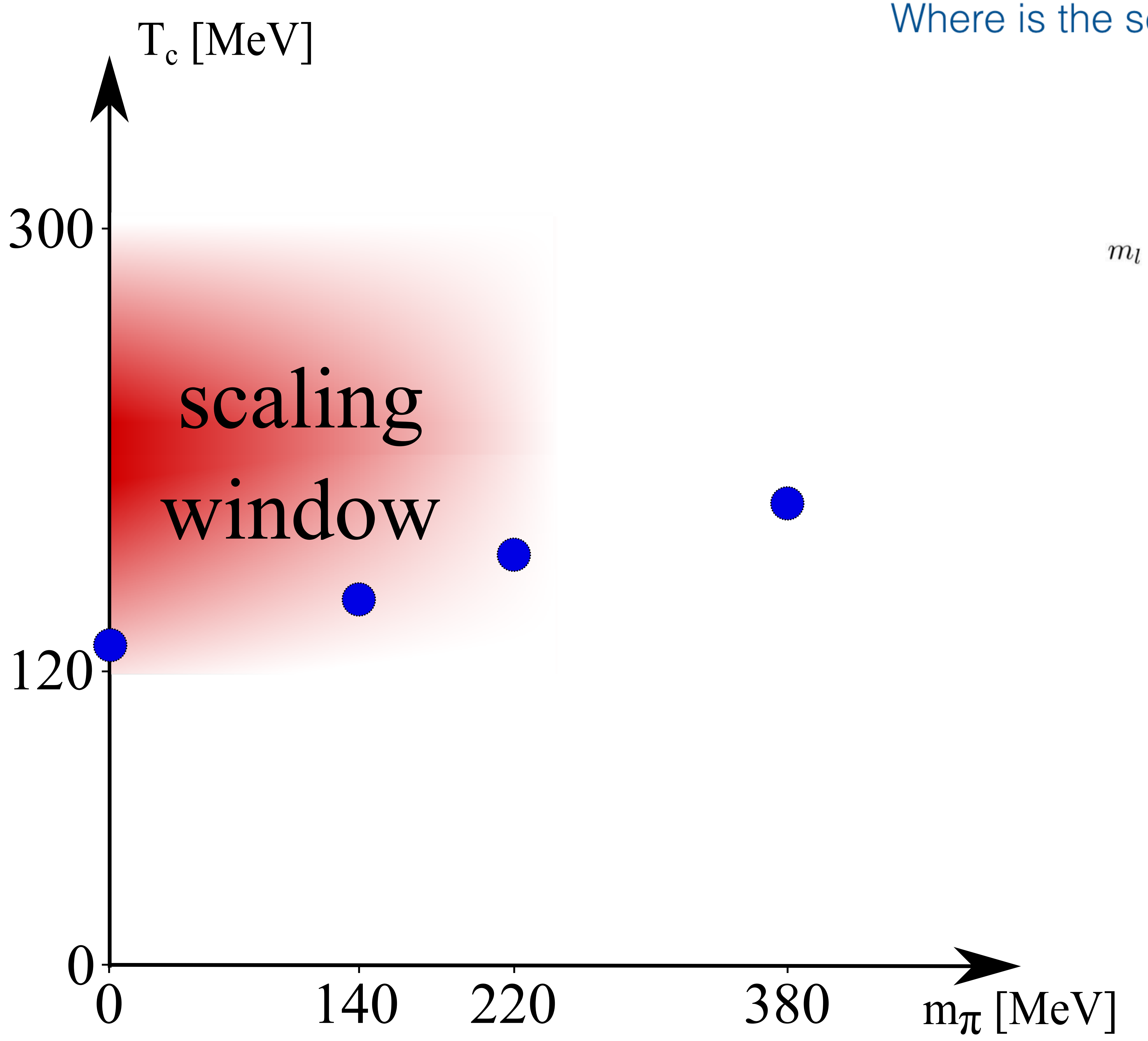
$\Delta_3 \propto t^{-\gamma-2\beta\delta}$ T < 300 MeV

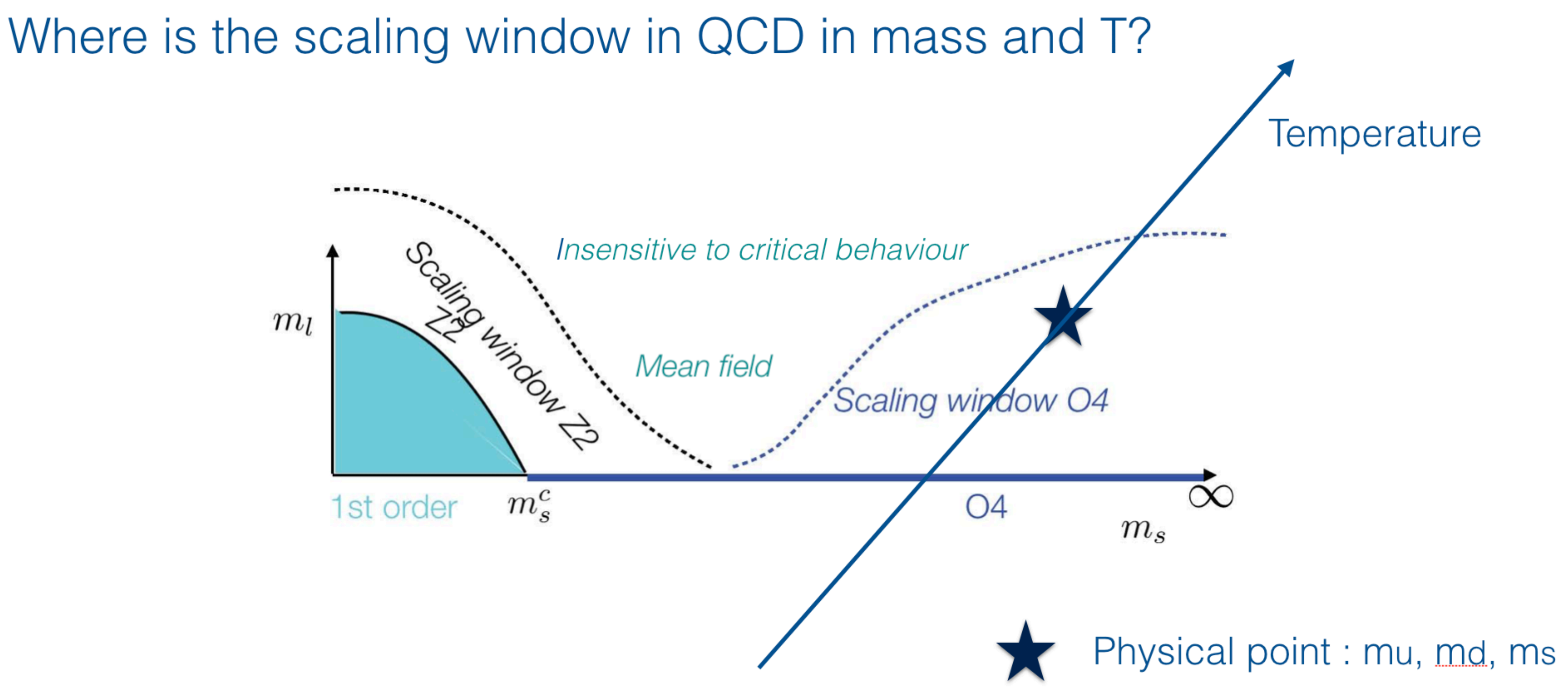
‘Forgotten’ critical behaviour..



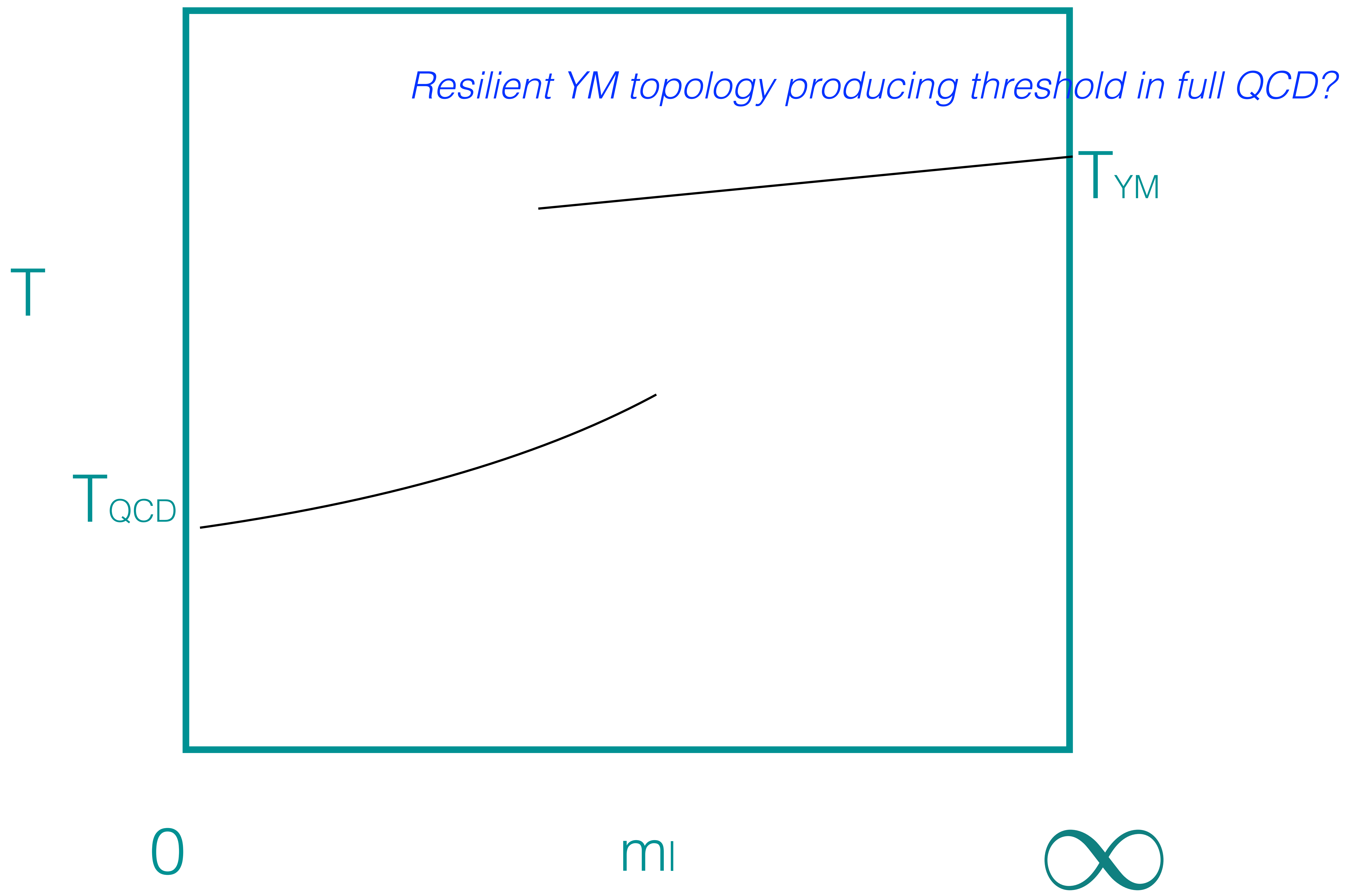
$\Delta_3 \propto m_\pi^6$ T > 300 MeV

A sketch of the scaling window for physical strange mass



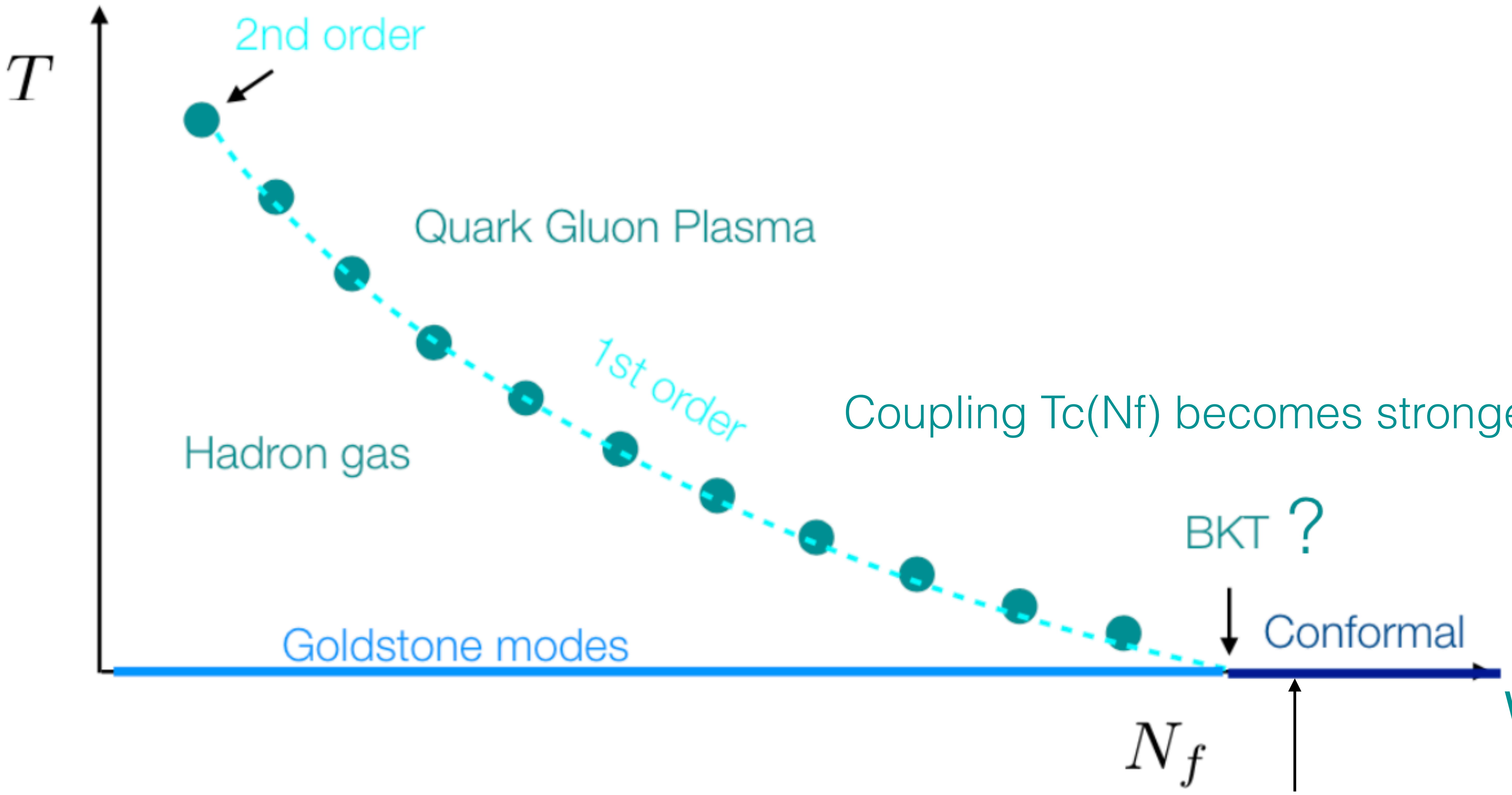


.. a speculation…



another speculation

IR phase? **Alexandru and Horvath**



Coupling Tc(Nf) becomes stronger with Nf

Miura, MpL

Weakly int. conformal

Strongly interacting conformal

Summary

Three different 3D O(4) scaling checks produce T0 in the chiral limit:
-Conformal scaling Tc = 138(2) MeV
-EoS analysis Tc = 142(2) MeV
-Mass dependence of the pseudo critical temperatures Tc = 134 (+6,-4) MeV

Consistency with 3D O4 scaling at physical pion masses, and temperatures $T < \simeq 300$ MeV
No memory of criticality for $T > \simeq 300 \text{ MeV} > T_{RW}$

The upper limit of the scaling window in temperature $T \simeq 300$ MeV is in the same range as the observed crossover for the topological susceptibility to a DIGA behaviour as seen in the fall-off exponent and b2,

Other indications of crossover in the plasma: Alexandru and Horvath (2019-2021);
Glozman et al; Glozman, Philipsen, Pisarski (2016-2022)

In short, consistent indications of a broad crossover from strong to weakly coupled QGP between 200 and 300 MeV. The sQGP is influenced by the critical point(s).