Understanding the topological fluctuations near the chiral crossover transition in QCD

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References:

- R. Larsen, S.S., E. Shuryak, Phys. Rev. D (Letter) 105, L071501 (2022).
- R. Larsen, S.S., E. Shuryak, Phys. Rev. D 102, 034501 (2020).

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- In QCD with 2+1 dynamical quark flavors which transforms in the fundamental representation of $SU(3)$, the deconfinement transition is a smooth crossover.
- In addition, the chiral symmetry is also simultaneously restored. The crossover temperature is known to unprecedented accuracy $156.5(1.5)$ MeV. [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]
- Why do deconfinement and chiral symmetry restoration occur simultaneously?
- What is its microscopic origin?

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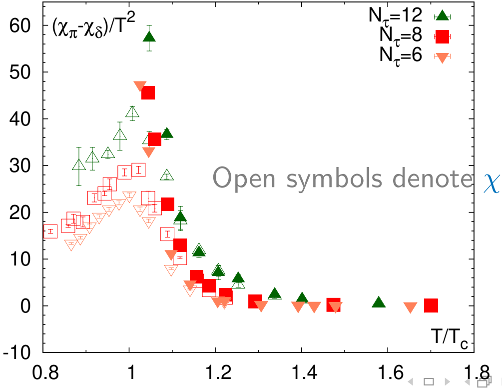
Relation between chiral symmetry and confinement

- When chiral symmetry is restored

$$\chi_{\langle\bar{\psi}\tau_2\gamma_5\psi\rangle} - \chi_{\langle\bar{\psi}\gamma_5\psi\rangle} = 2\ \chi_{5,disc} = 2\ \chi_{disc}\ .$$

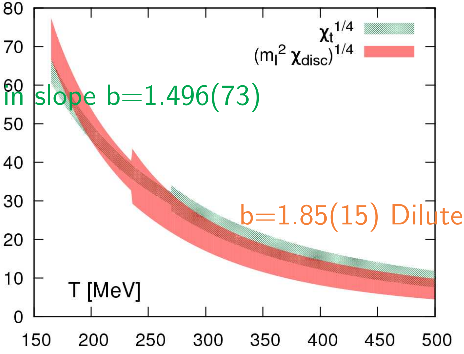
- The topological susceptibility is related to this quantity
$\chi_t = \ m^2\ \chi_{disc}$. [L. Giusti, G. C. Rossi, M. Testa, 04, HotQCD 1205.3535]



Open symbols denote χ_{disc}

- Characterizing, $\chi_t^{1/4}(T) \sim (T_c/T)^b$

[Petreczky, Schadler, S.S. 16].



Abrupt change in slope b=1.496(73)

b=1.85(15) Dilute instanton gas

[See also C. Bonati et. al., 15, 18, Borsanyi et. al., 18, F. Burger et. al, 18]

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  [A. A. Belavin, A. M. Polyakov, A. S. Schwartz and Yu. S. Tyupkin, 75, Harrington & Shepard, 78].

- Lattice is a 4-dimensional torus. Gauge theories on a torus is extremely rich [t'Hooft, 80]. The unstable $Q = 1$ self-dual solution on a torus [Braam & van Baal, 89] goes over to the usual caloron solution when the three space dimensions goes to infinity.

- On sufficiently large volume lattices one recovers the usual caloron solutions. [A. Gonzalez-Arroyo & P. van Baal, 99].

- Compactifying QCD on a large torus does not make the instantons disappear. The topological fluctuations still exist and produce instantons as the boundary effects are sub-dominant.

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Understanding the topological constituents

- Instantons were shown to cause color-confinement in 2+1 D [Polyakov, 77].
- In 3+1 D the potential is not long-ranged to ensure confinement.
- Interacting ensemble of instanton (liquid) explains many properties related to chiral symmetry breaking. [Shuryak, 82, Shuryak & Schaefer 96].
- At finite T, instantons characterized by the holonomy and Q. [Gross, Pisarski, Yaffe, 83].
- Immediately above T_c, a range of temperature where the Polyakov loop (at spatial infinity) has non-trivial eigenvalues.

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Topology at finite temperature

- It was proposed that instantons can dissociate into components in a dense regime which could explain quark confinement. [C. G. Callan, Jr., R. F. Dashen and D. J. Gross, 77].
- Instantons with a non-trivial holonomy dissociate into dyons with non-trivial color electric +magnetic charges.

  [Kraan & van Baal, 98, Lee & Lu, 98].

- For $SU(N_c)$ gauge theories each instanton have N_c such dyon constituents.
- Topological charge of a dyon $= 1/N_c$ of the instanton.

- Dyons can directly interact with the holonomy potential. It can drive towards the confining values? [Diakonov, 2006]

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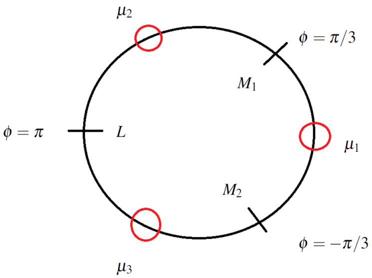
- Do dyons really exist? If yes, how to identify them and find its density. [Garcia-Perez et. al., 99, Gattringer, 02, Ilgenfritz, Mueller-Preussker et. al., 13, 15].
- How robust is the identification of the dyons?
- Can we identify different species of dyons in the hot QCD medium.
- Can there be a semi-classical description of dyons?
- How to convincingly show the connection between topological fluctuations and confinement without relying on dyons, vortices,...[For other approaches see talk by D. Leinweber]

Dyon-zero modes in SU(3): A typical case

- Holonomy $L = \frac{1}{3} Tr\ \mathrm{e}^{\mathrm{i}\ \mathrm{diag}(\mu_1,\mu_2,\mu_3)} \rightarrow$ the action of the $i-th$ dyon action is characterized by $\mu_{i+1} - \mu_i$.
- The zero mode of Dirac operator with b.c $\psi(t+\beta) = \mathrm{e}^{\mathrm{i}\phi}\psi(\mathrm{t})$ have a normalizable solution for ith-dyon background if $\phi\ \epsilon\ [\mu_{i+1} - \mu_i]$



Dyon zero modes in SU(3)

The density at any spacetime point x is:

$$\rho(x) = -\frac{1}{4\pi^2}\partial_\mu^2 f_x(\phi,\phi) \ ,$$

where

$$\left[\left(\frac{1}{i}\partial_\phi - \tau\right)^2 + r^2(x,\phi) + \sum_{m=1}^{3} \delta(\phi-\mu_m)\frac{|x_m - x_{m+1}|}{2\pi}\right] f_x = \delta(\phi-\phi') \, .$$

- distances between center of the m-th and $(m+1)$-th dyon given as $x_m - x_{m+1}$ where $m = 1, 2, 3$.
- $r^2(x,\phi) = r_m^2(x), \phi \in [\mu_m, \mu_{m+1}]$ is the distance between the observation point x and the center of the m-th dyon.

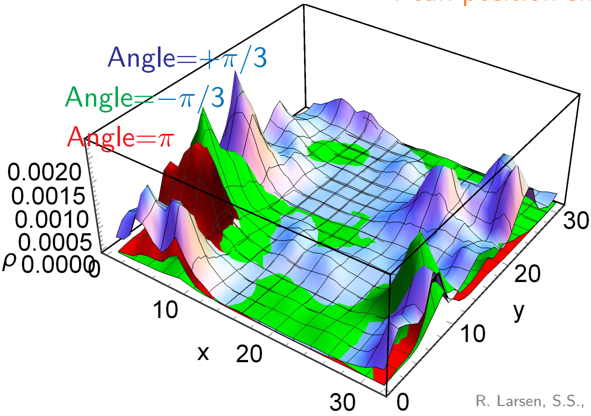
[A. Gonzalez-Arroyo et. al. Phys.Rev. D60 (1999) 031901]

Numerical set-up

- We use 2+1 flavor Möbius domain wall fermion configurations with physical quark mass+ Iwasaki gauge action [HotQCD collab. 14].
- Chiral pseudo-critical $T_c = 155 \pm 9$ MeV.
- Lattice size $32^3 \times 8$ which implies spatial volumes $4/m_\pi$.
- Use overlap fermions to detect the zero-modes due to an exact index theorem.
- The GW relation and the matrix-sign function achieved with a precision 10^{-10}.
- Configurations are smooth enough $\rightarrow$ the topological charge measured using fermion and bosonic definitions match [V. Dick et. al. 16].
- For simplicity we focus on $|Q| = 1$ configurations.

Snapshot of QCD vacuum at $\sim 1.1\,T_c$

Peak position shifts → Instanton-dyons?



R. Larsen, S.S., E. Shuryak, Phys. Lett. B. 794, 2019.

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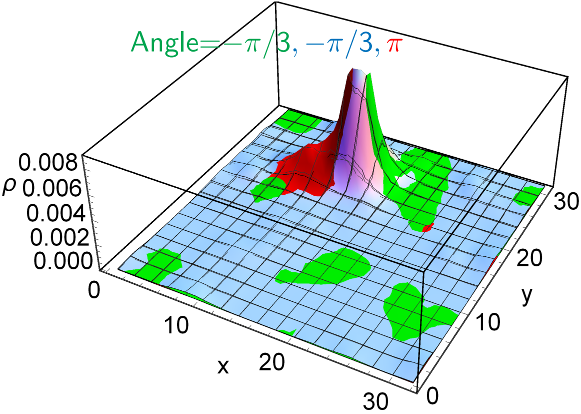


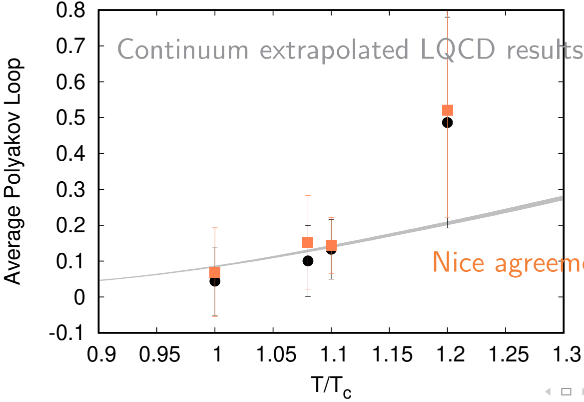
The fermion zero modes insensitive to temporal periodicity phase

$\rightarrow$ Instanton-dyon or caloron?

R. Larsen, S.S., E. Shuryak, Phys. Lett. B. 794, 2019, PRD, 2020.

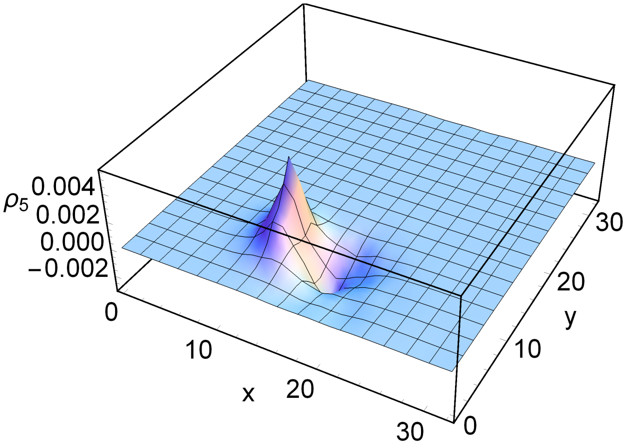
Comparing with the semi-classical theory

- Analytic solutions of the instanton-dyons are known [Kraan & van Baal, Lee and Lu, 98].
- Choose an initial trial value of Polyakov loop → Fit it to analytic profiles assuming a weakly interacting ensemble of dyons [R. Larsen, S.S., E. Shuryak, 20]



What do Dirac near-zero modes tell us?

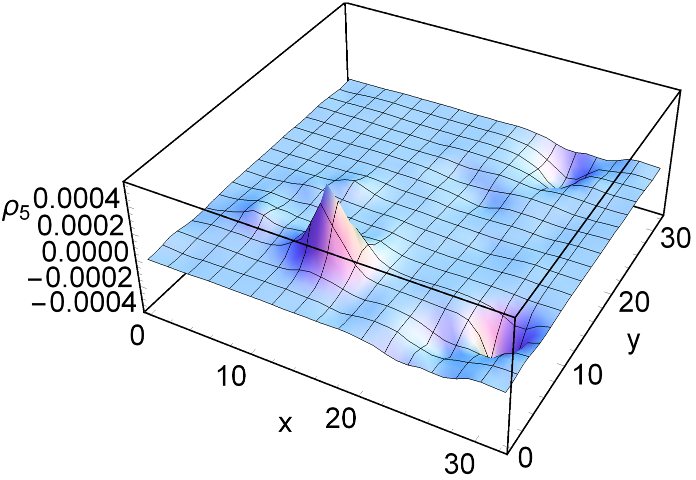
- L-dyon-pairs are rarer and only those which are near-by appear at high T as expected.



R. Larsen, S.S., E. Shuryak, PRD 2020.

What do Dirac near-zero modes tell us?

- *M*-dyons appear for all separations!



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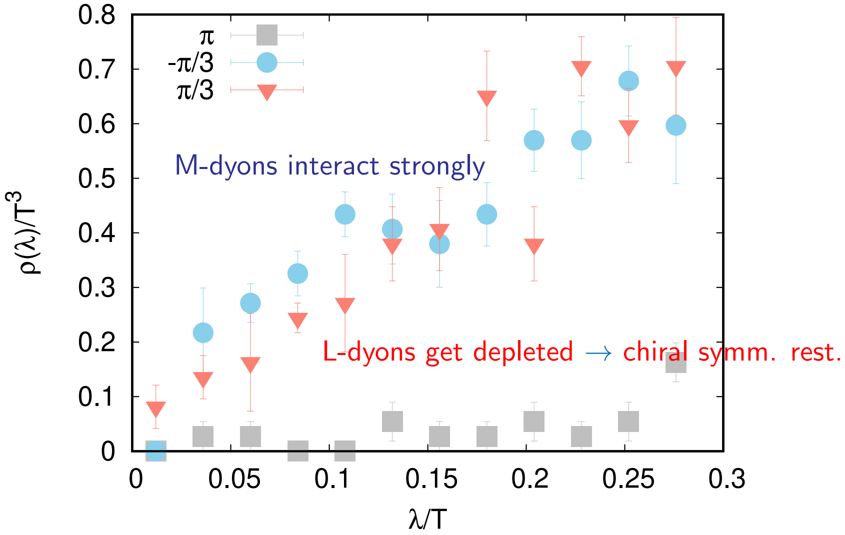

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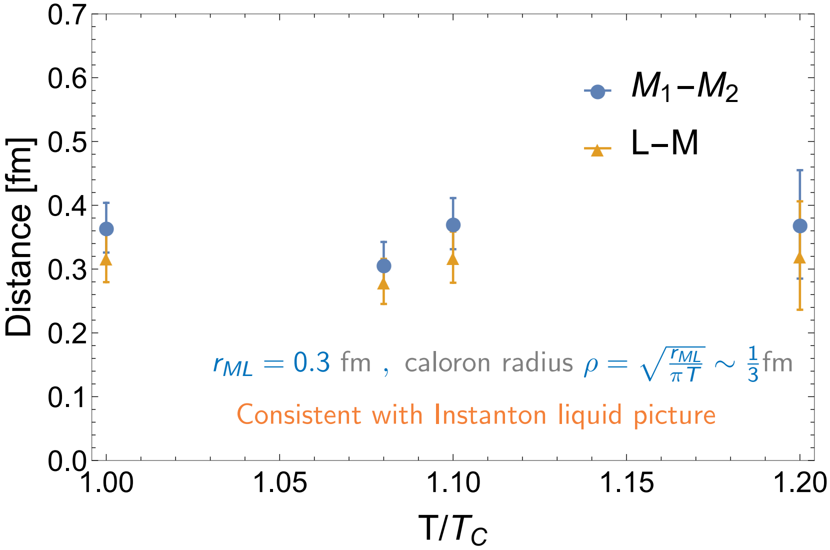
Characterizing the instanton-dyons

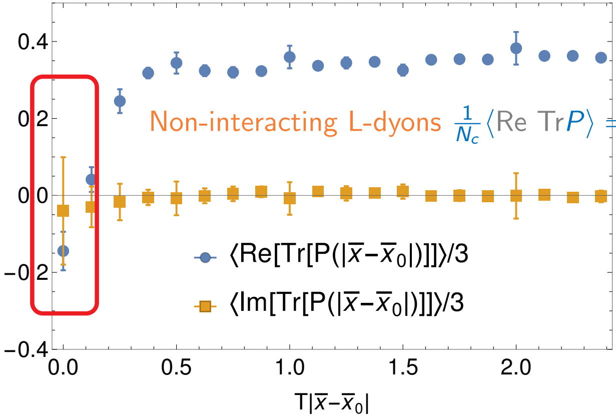


R. Larsen, S.S., E. Shuryak, PRD 2020.

Understanding the nature of dyon-interactions at $1.2\ T_c$

- For non-interacting dyons

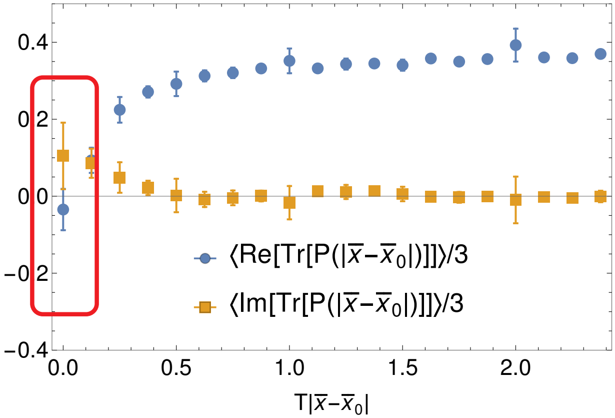
$$P(\vec{x}_i) = \mathrm{diag}\left[e^{i2\pi\mu_{i-1}}, e^{i\pi(\mu_i+\mu_{i+1})}, e^{i\pi(\mu_i+\mu_{i+1})}\right]$$



Non-interacting L-dyons $\frac{1}{N_c}\langle \mathrm{Re\ Tr}P\rangle = -\frac{1}{3}$, $\mathrm{Im\ Tr}P = 0$

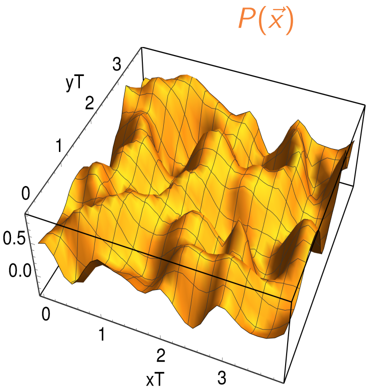
R. Larsen, S.S., E. Shuryak, PRD(L) 2022.

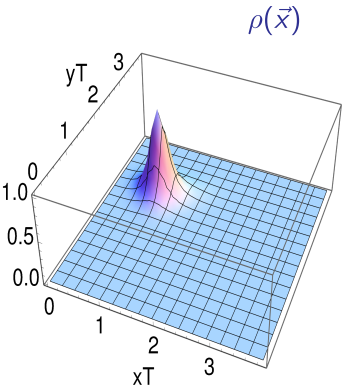
- The M-dyons are strongly interacting. Do not follow the usual non-interacting expectations
$\frac{1}{N_c}\langle \mathrm{Re\ Tr}P\rangle = \frac{1}{6}\ ,\ \frac{1}{N_c}\langle \mathrm{Im\ Tr}P\rangle = \pm\frac{1}{\sqrt{12}}$.



R. Larsen, S.S., E. Shuryak, PRD(L) 2022.

Correlation between topological & Polyakov loop fluctuations at 1.1 T_c





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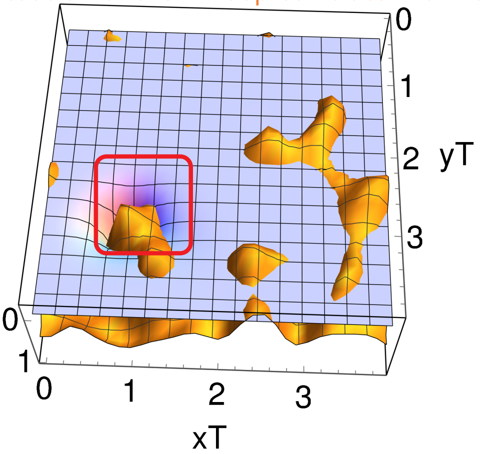
The most localized flutuation of the P. loop coincides with zero mode

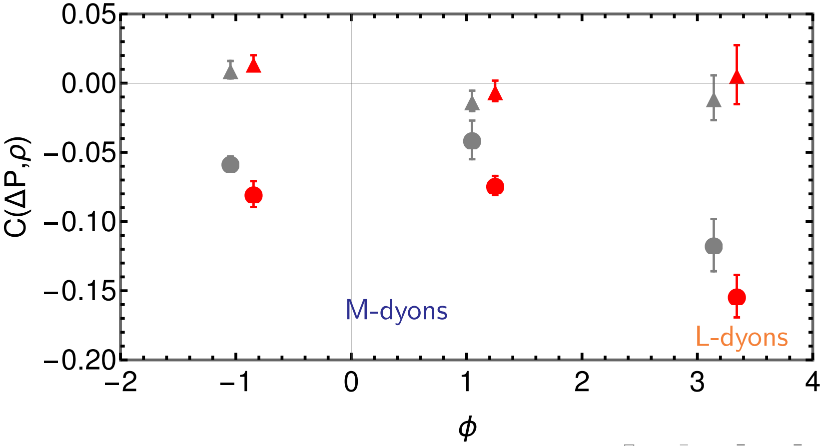


R. Larsen, S.S., E. Shuryak, PRD(L) 2022.

Correlation between topological & Polyakov loop fluctuations

$$C(\Delta P, \rho) = \frac{1}{3} \int d^3\vec{x}\ \rho(\vec{x})\ \left[\mathrm{Tr}P(\vec{x}) - \langle \mathrm{Tr}P(\vec{x})\rangle\right]$$

Two diff. temperatures $1.1, 1.2\,T_c$



Summary

- Lattice techniques which are essential for obtaining bulk thermodynamic properties of gauge theories are now giving us more insights about its microscopic constituents.

- In QCD with physical quark masses just above T_c, the topological fluctuations repel strongly the short-distance fluctuations of the Polyakov loop.

- There are tantalizing hints that instanton-dyons may play a role $\rightarrow$ now it is possible to identify the different species, their separation and interactions.

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