Theta Vacuum and Strong CP Problem

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Outline

Objective

Gradient Flow

Running Coupling and Confinement

Vacuum Structure at Finite θ

Conclusions

Objective

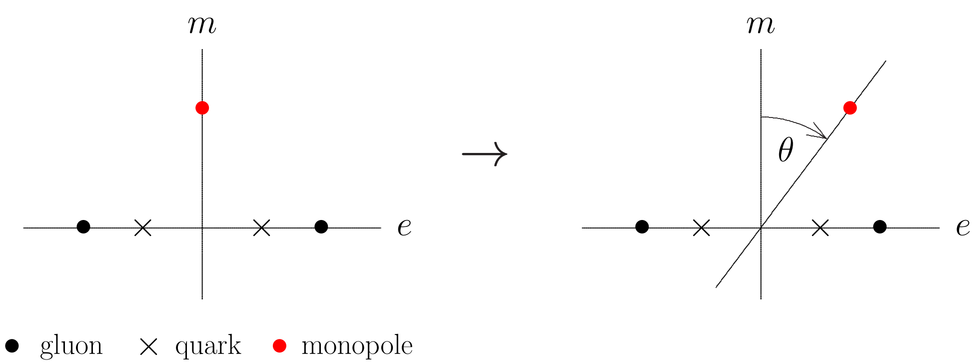
- QCD decribes the strong interactions remarkably well, from the smallest distances probed so far to hadronic scales where quarks and gluons confine to hadrons. Yet it faces a problem. The theory allows for a CP-violating term S_θ in the action. In Euclidean space-time it reads

$$S = S_{\rm QCD} + S_\theta \; : \quad S_\theta = i\,\theta\,Q\,, \quad Q = \frac{1}{32\pi^2}\int d^4x\; F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} \,\in\, \mathbb{Z}\,,$$

  where Q is the toplogical charge, and θ is an arbitrary phase with values $-\pi < \theta \leq \pi$. A nonvanishing value of θ would result in an electric dipole moment (EDM) d_n of the neutron. The current experimental upper limit is $|d_n| < 1.8 \times 10^{-13} e\,\mathrm{fm}$, which suggests that θ is anomalously small. This feature is referred to as the strong CP problem, which is considered as one of the major unsolved problems in the elementary particles field

- The prevailing paradigm is that QCD is in a single confinement phase for $|\theta| < \pi$. The Peccei-Quinn solution of the strong CP problem, for example, is realized by the shift symmetry $e^{i\,\delta\,Q_5}:\; \theta \to \theta + \delta$, trading the theta term S_θ for the hitherto undetected axion

- However, it is known from the case of the massive Schwinger model that a θ term may change the phase of the system. Callan, Dashen and Gross have claimed that a similar phenomenon will occur in QCD. The statement is that the color fields produced by quarks and gluons will be screened by instantons for $|\theta| > 0$. 't Hooft has argued that the relevant degrees of freedom responsible for confinement are color-magnetic monopoles. Confinement occurs when the monopoles condense in the vacuum, by analogy to superconductivity. In the θ vacuum the monopoles acquire a color-electric charge proportional to θ. Due to the joint presence of gluons and monopoles a rich phase structure is expected to emerge



Idea: Isolate the relevant dynamical variables at the hadronic scale by gauge fixing SU(3) $\rightarrow$ U(1)$\times$U(1)

For $|\theta| > 0$ quarks and gluons will be screened by forming bound states with the monopoles

- In this talk I will investigate the long-distance properties of the theory in the presence of the θ term, S_θ, and show that CP is naturally conserved in the confining phase

Gradient Flow

QCD exhibits a striking change in behavior over different length scales. To reveal the macroscopic properties of the theory, we are faced with a multi-scale problem, involving the passage from the short-distance perturbative regime to the long-distance confining regime. Such multi-scale behavior is typically addressed by renormalization group (RG) techniques bridging the different regimes

A promising framework is provided by the gradient flow (GF), which evolves the gauge field along the gradient of the action. The flow of SU(3) gauge fields is defined by the diffusion equation

$$\partial_t B_\mu(t,x) = D_\nu G_{\mu\nu}(t,x)\,, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu] \qquad B_\mu(t=0,x) = A_\mu(x)$$

The scale is set by $\boxed{\mu = 1/\sqrt{8t}}$ $\quad \sqrt{8t} \,\widehat{=}\,$ smoothing range over which B_μ is averaged — Lüscher

Formally, GF is an infinitesimal realization of the coarse-graining step of momentum space RG transformations (à la Wilson, Polchinski, Wetterich) and, as such, keeps the long-distance physics unchanged

Lüscher
Makino, Morikawa, Suzuki
Carosso, Hasenfratz, Neil

Running coupling

Number one choice for studying physical system over several length scales

The expectation value $\langle E(t) \rangle$ of the energy density

$$E(t,x) = \frac{1}{4} G^a_{\mu\nu}(t,x)\, G^a_{\mu\nu}(t,x)$$

has the perturbative expansion

$$\langle E(t) \rangle = \frac{3}{4\pi t^2} \alpha_{\overline{MS}}(\mu) \left[1 + k_1 \alpha_{\overline{MS}}(\mu) + k_2 \alpha_{\overline{MS}}(\mu)^2 + \cdots \right] \qquad t = 1/8\mu^2$$

$$\equiv \frac{3}{4\pi t^2} \alpha_{GF}(\mu)$$

Thus

$$\boxed{\alpha_{GF}(\mu) = \frac{4\pi^2}{3} t^2 \langle E(t) \rangle}$$

$$\Lambda_{GF} = \exp\left\{ \frac{2\pi}{11} k_1 \right\} \Lambda_{\overline{MS}}$$

Lüscher

For a start we may restrict our investigations to the Yang-Mills (YM) theory. If the strong CP problem is resolved in the YM theory, then it is expected to be resolved in QCD as well. We use the plaquette action to generate representative ensembles of fundamental gauge fields on three different volumes

$$S = \beta \sum_{x,\,\mu<\nu} \Big(1 - \frac{1}{3}\,\mathrm{Re\,Tr}\,U_{\mu\nu}(x)\Big)$$

	16^4	24^4	32^4
#	4000	5000	5000

$\beta = 6.0 \quad a = 0.082\,\mathrm{fm}$

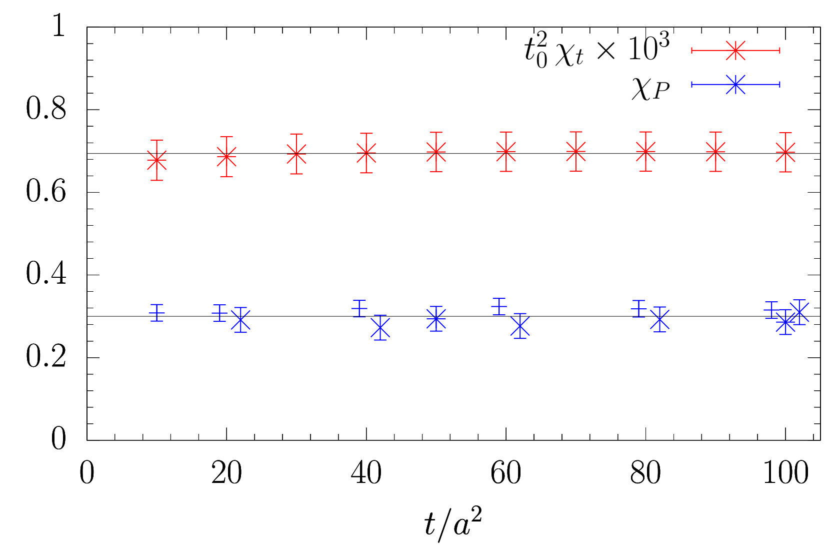
In the YM theory quantities that can be computed precisely are limited. Two examples:

- Topological susceptibility

$$\chi_t = \frac{\langle Q^2\rangle - \langle Q\rangle^2}{V}$$

- Normalized Polyakov susceptibility

$$\chi_P = \frac{\langle |P|^2\rangle - \langle |P|\rangle^2}{\langle |P|\rangle^2}, \quad P = \frac{1}{V_3}\sum_{\mathbf{x}} P(\mathbf{x})$$



Both quantities, χ_t and χ_P, are independent of the flow time t, as expected

The Polyakov loop (nonlocal operator) requires normalization, to be interpreted as free energy of static quarks

$$\sqrt{t_0}\,\chi_t^{\frac{1}{4}} = 0.162(3)$$

$$\chi_P = 0.289(7)$$

Literature:

$$\sqrt{t_0}\,\chi_t^{\frac{1}{4}} = 0.161(4)$$

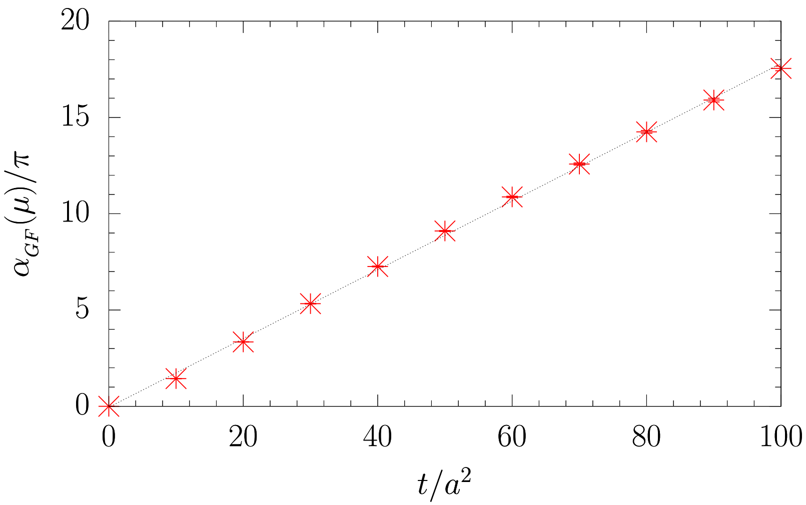
arXiv:1506.06052

2D Gaussian distribution:

$$\chi_P = 4/\pi - 1 = 0.273$$

Running Coupling and Confinement

Confinement is intimately connected with the IR behavior ($\mu \to 0$) of the running coupling $\alpha_{GF}(\mu)$



$$\frac{\partial\, \alpha_{GF}(\mu)}{\partial\, \ln\, \mu} \equiv \beta_{GF}(\alpha_{GF}) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -\, 2\, \alpha_{GF}(\mu)$$

$$\frac{\Lambda_{GF}}{\mu} = (4\pi b_0 \alpha_{GF})^{-\frac{b_1}{2\, b_0^2}} \exp\left\{ -\frac{1}{8\pi b_0 \alpha_{GF}} - \int_0^{\alpha_{GF}} d\alpha \, \frac{1}{\beta_{GF}(\alpha)} + \frac{1}{8\pi b_0 \alpha^2} + \frac{b_1}{2 b_0^2 \alpha} \right\}$$

$$\alpha_{GF}(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_{GF}^2}{\mu^2}$$

To make contact with phenomenology, it is desirable to transform the GF coupling α_{GF} to a common scheme. A preferred scheme in the YM theory is the V scheme: $V(q) = -4\pi C_F \alpha_V(\mu)/q^2$

$$\frac{\Lambda_{GF}}{\Lambda_V} = \exp\left\{-\int_0^{\alpha_{GF}} d\alpha \frac{1}{\beta_{GF}(\alpha)} + \int_0^{\alpha_V} d\alpha \frac{1}{\beta_V(\alpha)}\right\}$$

$$\beta_V(\alpha_V) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -2\,\alpha_V(\mu)$$

$$\alpha_V(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_V^2}{\mu^2}$$

$$\frac{\Lambda_V}{\Lambda_{\overline{MS}}} = 1.60\,, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_{GF}} = 0.534$$

The linear growth of $\alpha_V(\mu)$ with $1/\mu^2$ is commonly dubbed infrared slavery. The static quark-antiquark potential can be described by the exchange of a single dressed gluon

$$V(r) = -\frac{1}{(2\pi)^3}\int d^3\mathbf{q}\; e^{i\,\mathbf{qr}}\;\frac{4}{3}\,\frac{\alpha_V(q)}{\mathbf{q}^2 + i0} \underset{r \gg 1/\Lambda_V}{=} \sigma\, r$$

where $\sigma = \frac{2}{3}\Lambda_V^2$, giving the string tension $\sqrt{\sigma} = 445(19)$ MeV

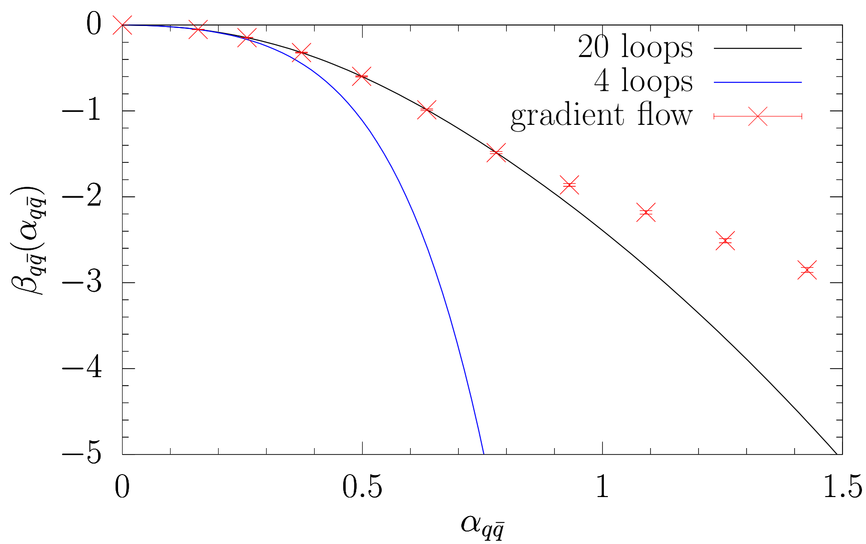
$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.217(7)$$

Literature:

$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.220(3)$$

arXiv:1905.05147

It is interesting to compare the nonperturbative GF beta function with the perturbative beta function known up to twenty loops



20 loops arXiv:1309.4311

4 loops arXiv:1012.3037

In general

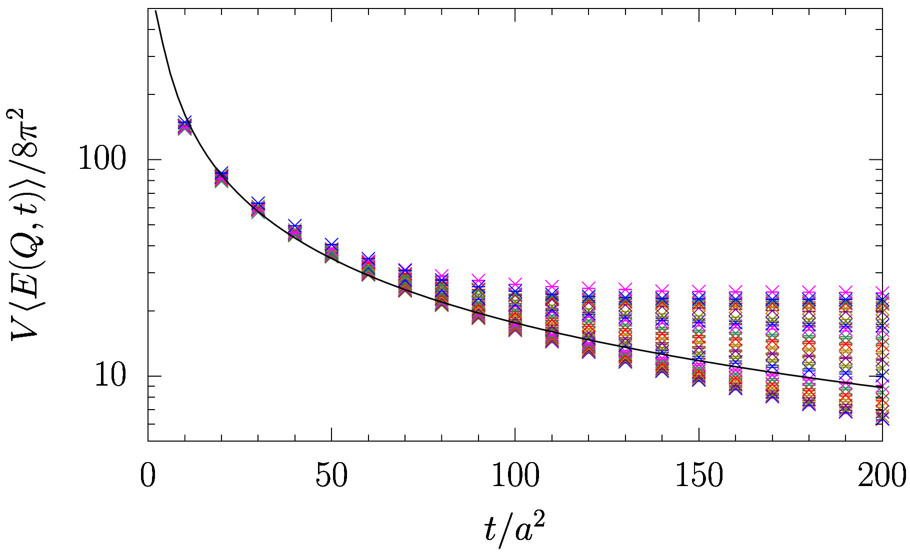
$$\alpha_S \xrightarrow[\mu \to 0]{} \begin{cases} 0 \\ \infty \end{cases} \text{possible}$$

$$\frac{\Lambda_{q\bar{q}}}{\Lambda_V} = 0.655$$

As was to be expected, the perturbative beta function gradually approaches the nonperturbative beta function with increasing order

Vacuum Structure at Finite θ

With increasing flow time the initial gauge field ensemble splits into effectively disconnected topological sectors of charge Q, at ever smaller flow time as β is increased



$$Z(\theta) = \int \mathcal{D}A_\mu \, e^{-S+i\theta Q}$$

$$= \sum_Q e^{i\theta Q} \int_Q \mathcal{D}A_\mu \, e^{-S}$$

$$= \sum_Q e^{i\theta Q} \, P(Q)$$

$V\,\langle E(Q,t)\rangle/8\pi^2 \equiv S_Q \simeq |Q|$, while the ensemble average vanishes like $1/t$

$$Q = \int d^4x \, \partial_\mu \omega_\mu \,, \quad \partial_t \omega_\mu = (1/8\pi^2) D_\rho G_{\nu\rho} \tilde{G}_{\mu\nu}$$

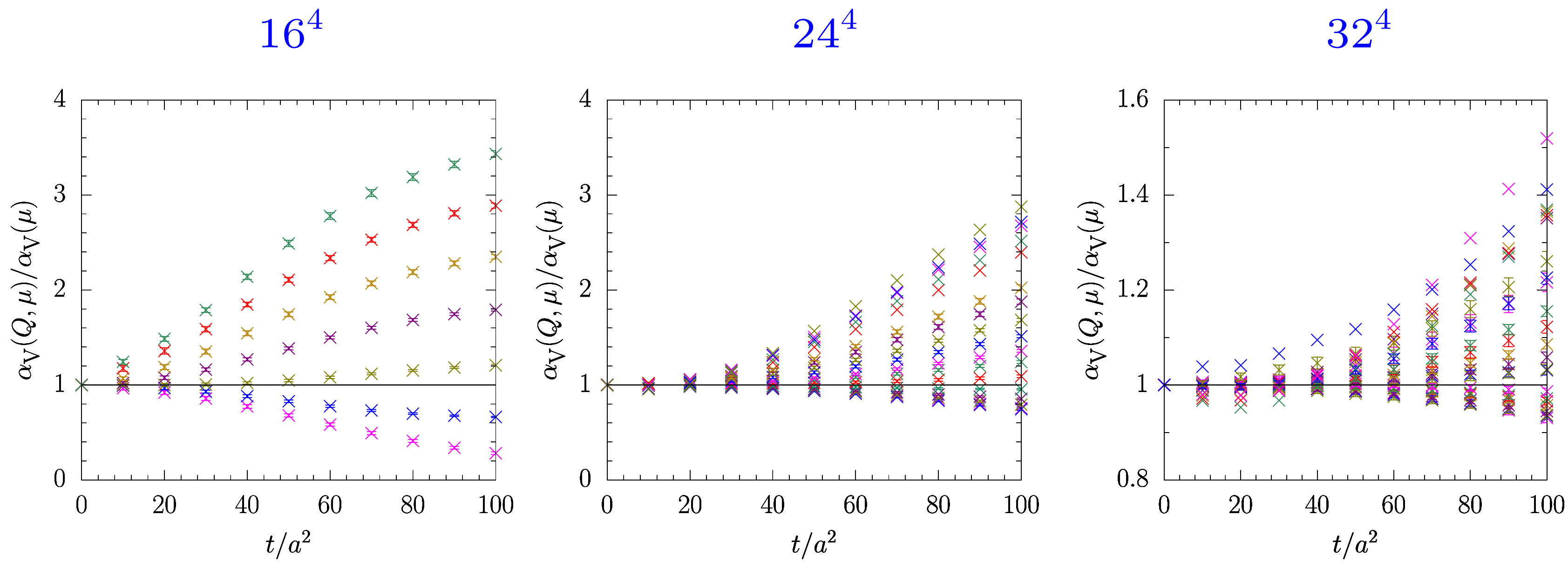
gauge invariant

$$\Rightarrow \; \partial_t Q = 0$$

Running coupling α_V

If the general expectation is correct and the color fields are screened for $|\theta| > 0$, we should, in the first place, find that the running coupling constant is screened in the infrared

From $\langle E(Q,t)\rangle$ we obtain $\alpha_V(Q,\mu)$ in the individual topological sectors $|Q|$ from bottom to top



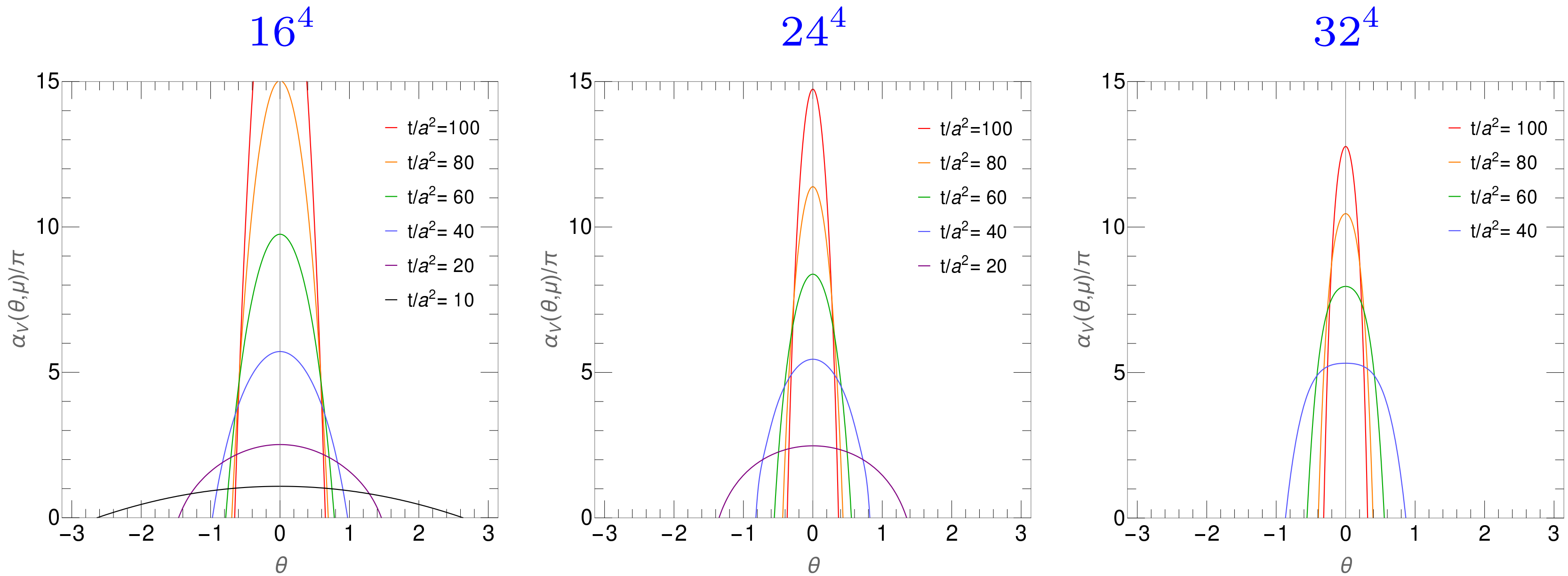
Interestingly, $\alpha_V(Q,\mu)$ vanishes in the infrared for $Q = 0$, while the ensemble average $\alpha_V(\mu)$ is represented by $|Q| \simeq \sqrt{2\langle Q^2\rangle/\pi}$

The transformation of $\alpha_V(Q,\mu)$ from Q to the θ vacuum is achieved by the discrete Fourier transform

$$\alpha_V(\theta,\mu) = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q} P(Q) \alpha_V(Q,\mu)$$

$$Z(\theta) = \sum_Q e^{i\,\theta\,Q} P(Q)$$

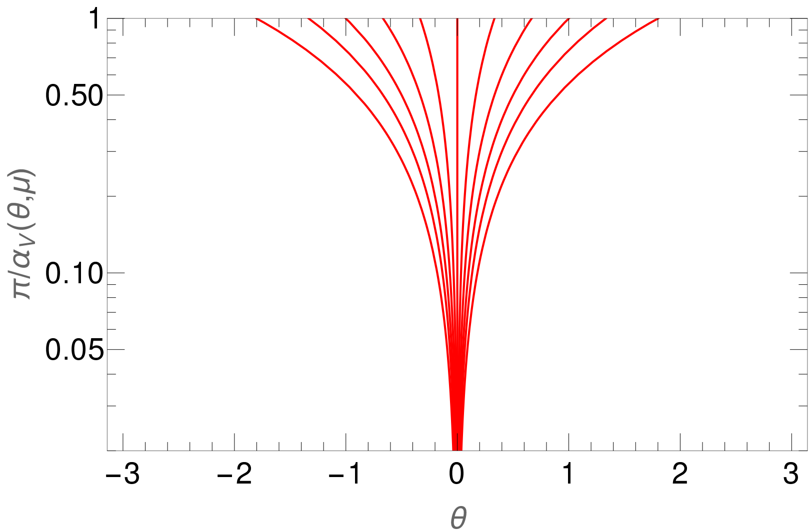
- Z_θ analytic at $\theta = 0$ Vafa-Witten
- $e^{i\theta Q} P_{\theta=0}(Q) = P_\theta(Q)$
- Limits set by convergence of the Fourier sum



The color charge is totally screened for $|\theta| \gtrsim 0$ in the infrared, while it becomes gradually independent of θ as we approach the perturbative regime

Behavior for $|\theta| > 0$ similar to $\alpha_s(T,\mu)$ at $T > T_c$

The IR behavior of the running coupling constant as a function of θ and flow time t can be quantified by deriving RG equations for $\pi/\alpha_V(\theta,\mu)$

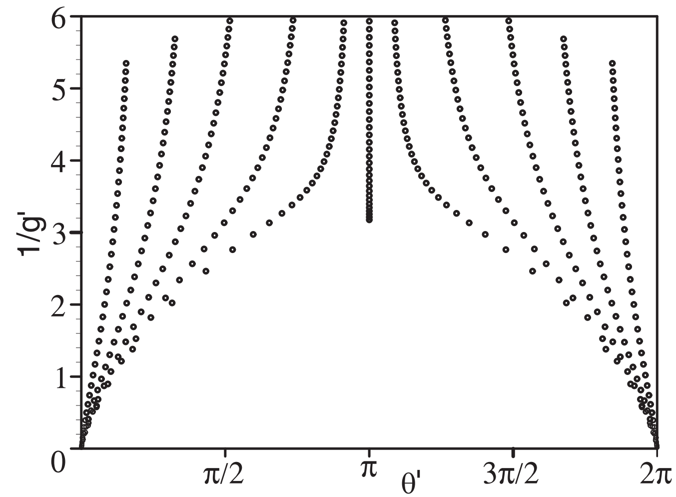


$$\frac{\partial(\pi/\alpha_V)}{\partial \ln t} \simeq -\frac{\pi}{\alpha_V} + D\theta^2 \quad \frac{\partial \theta}{\partial \ln t} \simeq -\frac{1}{2}\theta$$

IR fixed point

Renormalization of θ is a generic property of instanton fluctuations Knizhnik & Morozov

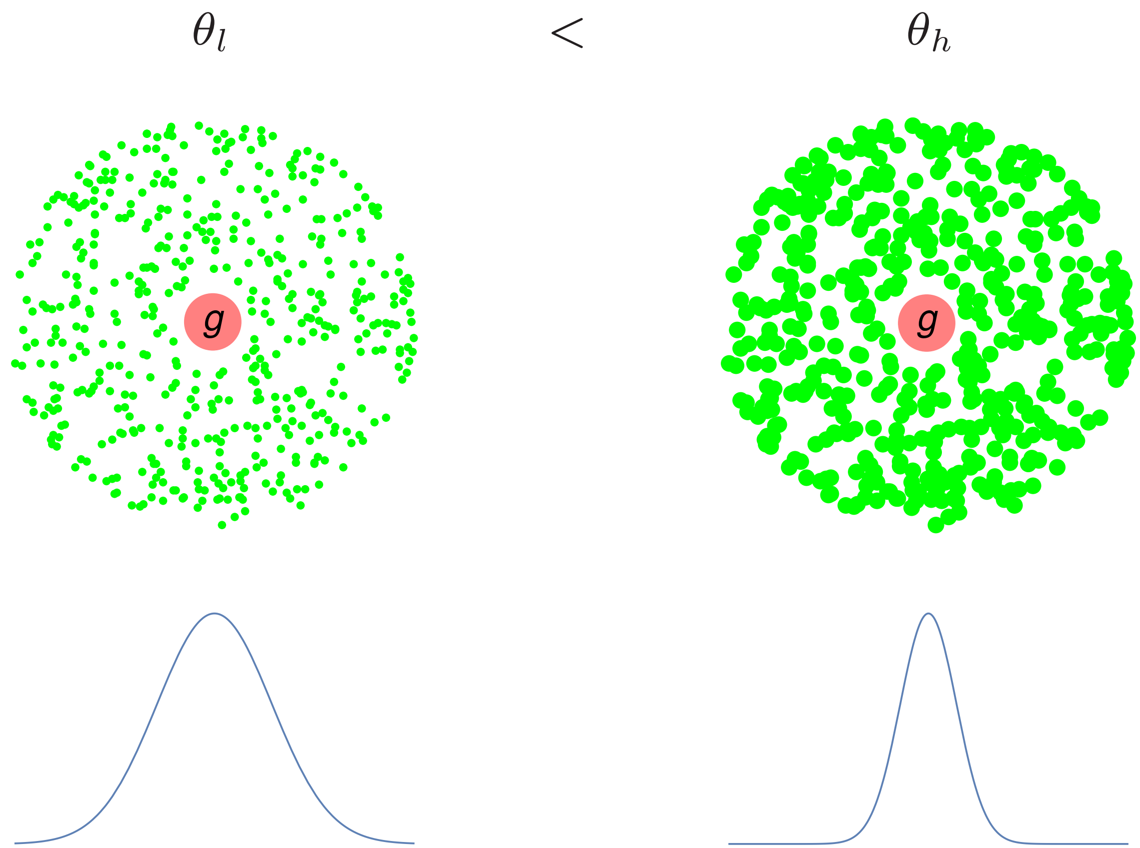
Another example is the scaling theory of the integer quantum Hall effect, in which θ stands in analogy to the Hall conductivity Levine, Libby & Pruisken



$$\mathbf{J} = \sigma \mathbf{E}\,; \quad \sigma_{xx} \sim 1/g^2\,, \quad \sigma_{xy} \sim \theta$$

Has served as a model for the solution of the strong CP problem

't Hooft

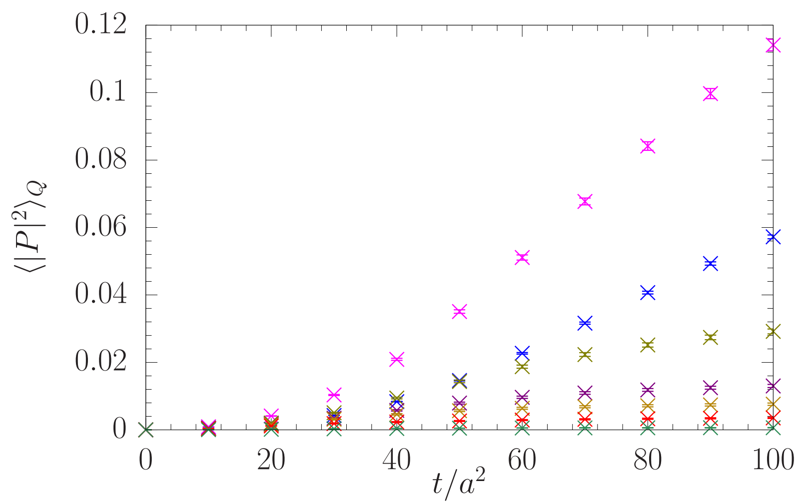


The density of color-electric charge ρ_e is proportional to the density of color-magnetic charge ρ_m times θ, i.e. $\rho_e \propto \rho_m \times |\theta|$. Thus, the screening length will be the larger the smaller $|\theta|$ is

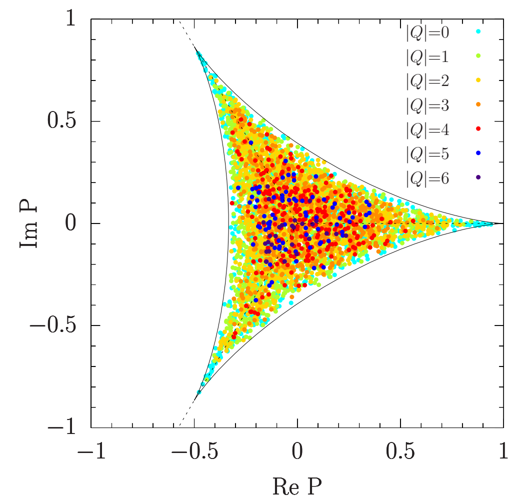
Polyakov loop

The Polyakov loop P describes the propagation of a single static quark travelling around the periodic lattice

16^4



$t/a^2 = 60$



From $Q = 0$ (top) to 6 (bottom)

$\langle P \rangle = 0$ in each sector. That implies center symmetry throughout. P rapidly populates the entire theoretically allowed region for small values of $|Q|$, while it stays small for larger values of $|Q|$

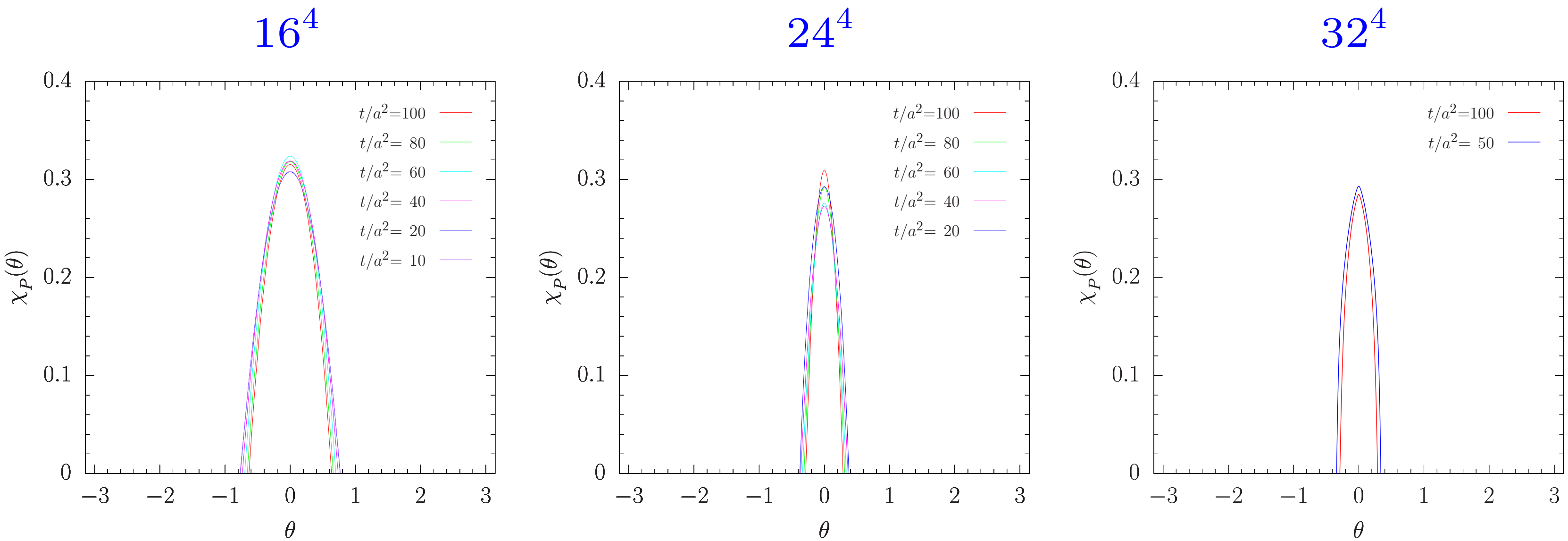
The transformation of the Polyakov loop expectation values to the θ vacuum is again achieved by the discrete Fourier transform

$$\langle |P|^2 \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P|^2 \rangle_Q$$

$$\langle |P| \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P| \rangle_Q$$

The connected part of $\langle |P|^2 \rangle_\theta$ is described by the normalized Polyakov loop susceptibility

$$\chi_P(\theta) = \frac{\langle |P|^2 \rangle_\theta - \langle |P| \rangle_\theta^2}{\langle |P| \rangle_\theta^2}$$



The Polyakov loop gets totally screened for $|\theta| \gtrsim 0$. The normalized Polyakov loop susceptibility is independent of flow time t (even for $\theta \neq 0$!)

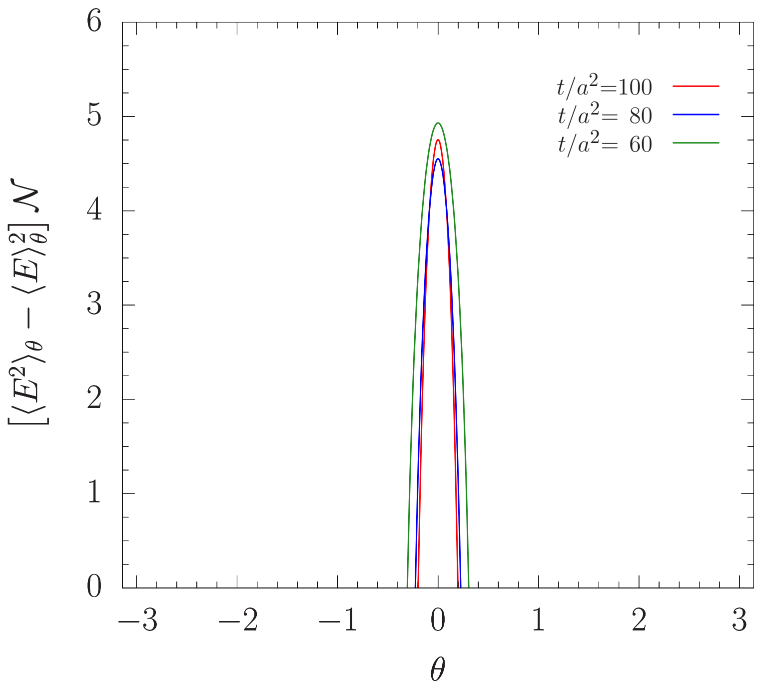
Mass gap

$$\langle E^2\rangle = \frac{1}{T}\sum_t \langle E(0)E(t)\rangle \qquad [\langle E^2\rangle - \langle E\rangle^2]\,\mathcal{N} = \sum_{n>0,t} \frac{1}{2m_n}\,|\langle 0|E|n\rangle|^2\, e^{-m_n t}$$

$$E(t) = \frac{1}{V_3}\sum_{\vec{x}} E(\vec{x},t) \qquad \simeq \frac{1}{m^2_{0^{++}}}\,|\langle 0|E|0^{++}\rangle|^2 \;\propto\; \xi^2$$

Correlation length

24^4



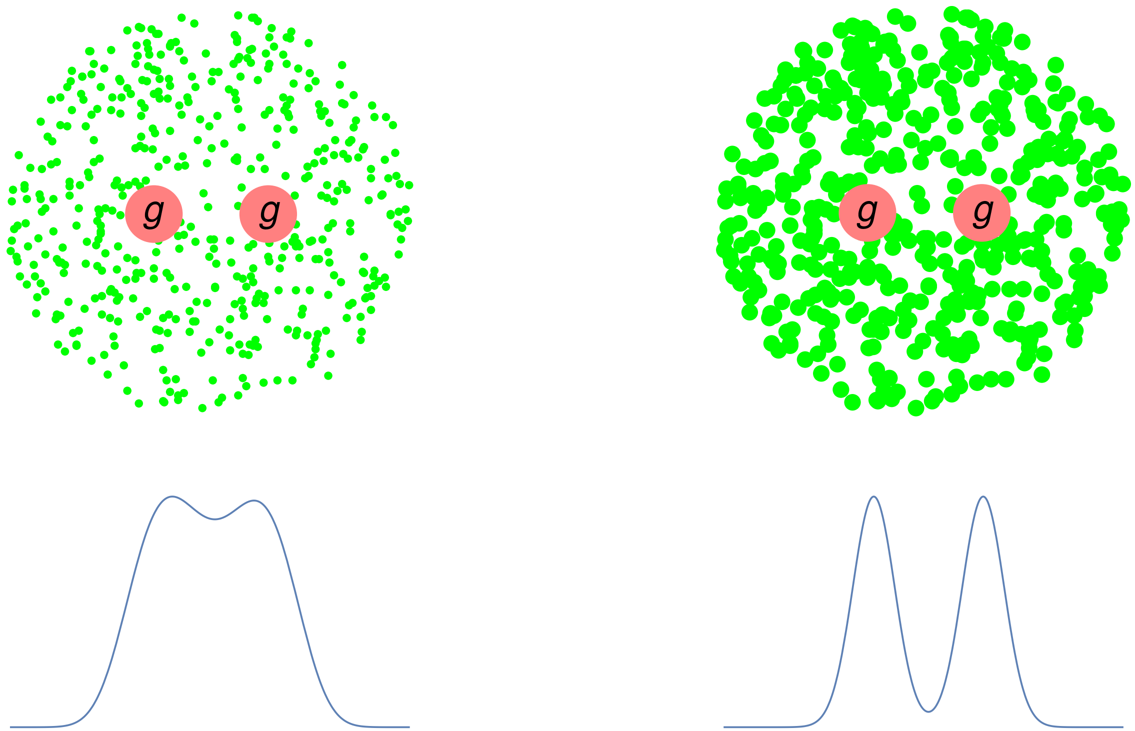
$\langle E^2\rangle_\theta - \langle E\rangle^2_\theta$ Independent of flow time t

$\xi \simeq 0$ for $|\theta| \gtrsim 0$

No mass gap

't Hooft

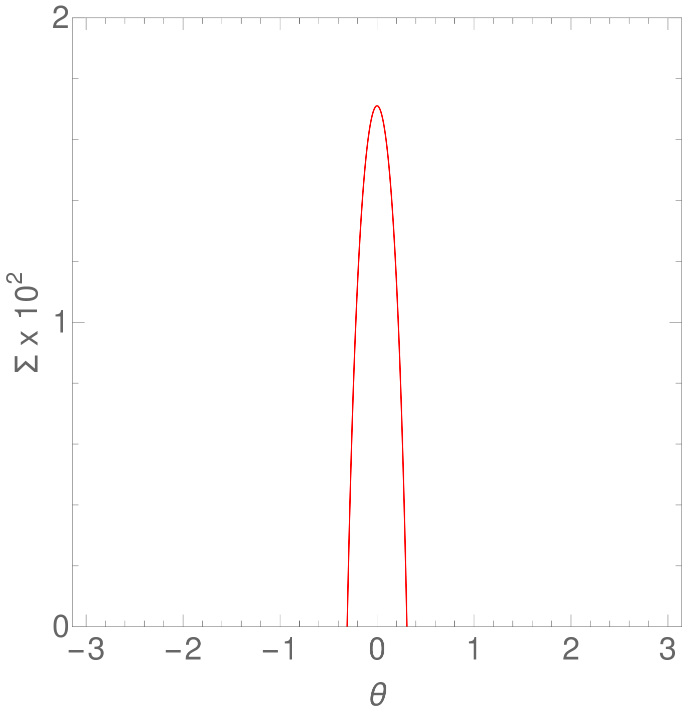
$\theta_l \quad < \quad \theta_h$



For the glueball to dissipate and the Polyakov loop to be totally screened, the screening length must be smaller than the glueball radius respectively the radius of the heavy-quark bound state. On the larger lattices this appears to be the case for $|\theta| > 0.2 - 0.3$ Similar to finite temperature: $|\theta| \to T - T_c$

$\boldsymbol{\Sigma}(\theta)$ Chiral Condensate Preliminary

24^4



$$\Sigma = \pi \rho(0)$$

Staggered Dirac D

$$\Sigma(Q) \propto \sqrt{|Q|}$$

Diakonov
Schäfer & Shuryak
Follana et al

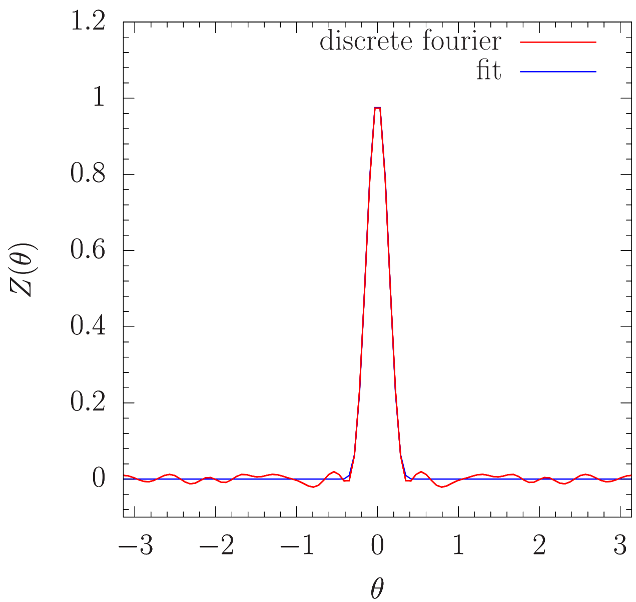
$$\Sigma(\theta) = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q)\, \Sigma(Q)$$

Thus, predictions of nonvanishing electric dipole moment d_n for $|\theta| > 0$ from ChPT not valid

Errors

Source of errors

- Convergence of the (discrete) Fourier series $\sum_Q \exp\{i\theta Q\}\, P(Q) \cdots$
- Statistics
- Topological charge generally limited to $|Q| \leq |Q|_{\max}$, $\quad |Q|_{\max} \propto \sqrt{V}$



$Z(\theta), \alpha_V(\theta), \chi_P(\theta), \cdots$ are positive functions of θ

After the quantities I showed have dropped to 'zero' at $|\theta| \gtrsim 0$, they start to oscillate around zero with frequency $\nu \approx |Q|_{\max}$ due to the truncated Fourier series

Various techniques to filter unphysical high-frequency modes are discussed in the literature. We fit the tail of the distributions to a smooth function. Alternatively, one can employ a low-pass filter, which practically gives the same result

Conclusions

★ The gradient flow proved a powerful tool for tracing the gauge field over successive length scales and showed its potential for extracting low-energy quantities. A key point is that the path integral splits into disconnected topological sectors for $t \gtrsim 0$, which is expected to occur at ever smaller flow times with decreasing lattice spacing. Comparing results on different volumes enabled us to control the accuracy of the calculation

★ The novel result is that color charges are screened, and confinement is lost, for $|\theta| > 0$ due to nonperturbative effects, limiting the vacuum angle to $\theta = 0$ at macroscopic distances, which rules out any strong CP violation at the hadronic level

★ Screening is a gradual process, which is completed once the vacuum has attained a sufficient level of color-electric charge density, which for hadrons seems to be the case for $|\theta| \gtrsim 0.2 - 0.3$. This result does not come as a surprise. One simply did not have the tools to address the problem

★ The nontrivial phase structure of QCD has far-reaching consequences for anomalous chiral transformations. In particular, the confining QCD vacuum will be unstable under the anomalous Peccei-Quinn transformation, $U_{\rm PQ}(1) = e^{i\delta Q_5}$, resulting in the shift symmetry $\theta \to \theta + \delta$, which thwarts the axion conjecture

It is surprising that the seminal work of Schwinger, Coleman, Callan, Dashen, Gross, 't Hooft, Witten, Seiberg, etc. has been completely ignored