# Role of inhomogeneities in the flattening of the quantum effective potential

Gergely Markó[*], with Gergely Endrődi[*] and Tamás G. Kovács[†,⋆]

[*]Fakultät für Physik, Universität Bielefeld, Bielefeld.

[†]Eötvös Loránd University, Budapest. [⋆]Institute for Nuclear Research, Debrecen.

Based on: PRL 127 (2021) 23, 232002

24th of May, 2022, Trento, ECT* workshop ”Gauge Topology, Flux Tubes and Holographic Models: the Intricate Dynamics of QCD in Vacuum Extreme Environments”

- Spontaneous symmetry breaking
- Constrained Monte Carlo simulations
- Summary

# Spontaneous symmetry breaking

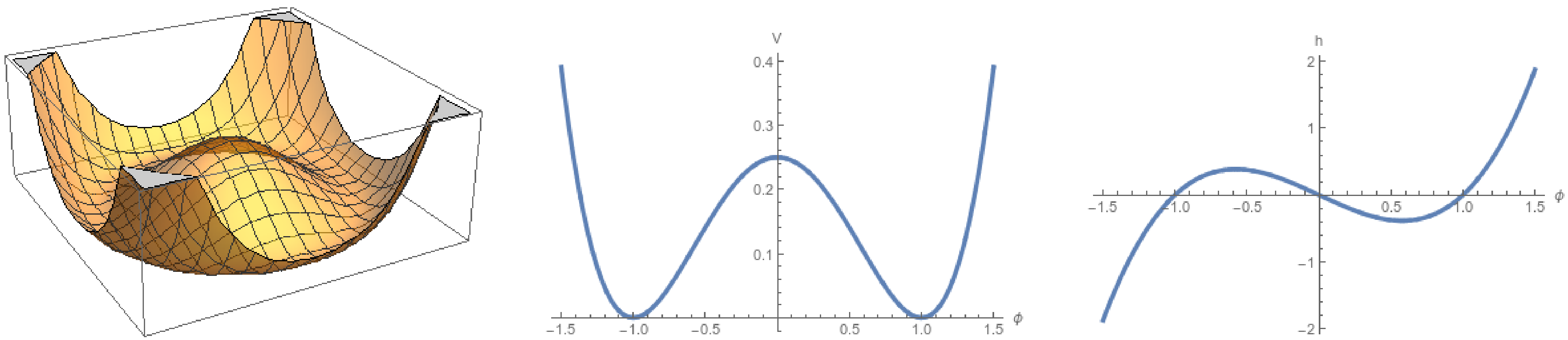
- More than one vacua exist connected by transformations by the broken symmetry.
- Spontaneous breaking is defined as a **double-limit**: 1) volume, 2) explicit breaking: $\bar{\phi}_{\min} = \lim_{h\to 0}\lim_{V\to\infty} \langle\phi\rangle_{V,h}$.
- Crossing from one direction of $h$ to another through $h = 0$ is a **first order transition**.
- The **effective potential** between the different vacua is **flat**, but cannot be accessed by usual simulations.
- Discrete symmetry: **domains** of different vacua, if the order parameter is constrained inside the flat region.
- What happens if the broken symmetry is continuous? Expectation: regions of different vacua will change continuously.
- How exactly? What configurations dominate the effective potential?
- What happens to translational invariance?

# Spontaneous symmetry breaking

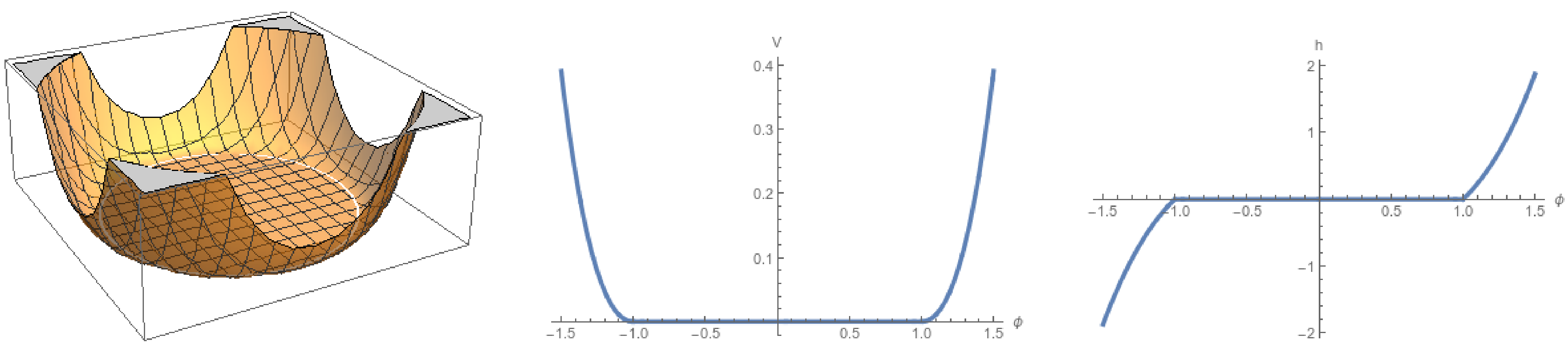
- Consider the 3D $O(2)$ symmetric scalar model

$$S = \int_{d^3x} \frac{(\partial_\mu \vec{\varphi})^2}{2} + \frac{m^2 \vec{\varphi}^2}{2} + \frac{g}{4!}(\vec{\varphi}^2)^2 \,.$$

- Classically:




- QFT:



# Accessing the flat region, constraint potential

Define the **constraint** effective potential

$$\exp\left(-V\Omega(\bar{\phi})\right) = \int \mathcal{D}\varphi \exp\left(-S[\varphi]\right) \delta\left(\int \varphi - V\bar{\phi}\right) .$$

- In the **infinite volume** limit (and only there) agrees with the standard effective potential (Legendre-transform).

  O'Raifeartaigh et al., NPB **271** (1986)

- Markov chain Monte Carlo techniques can be constructed which **satisfy** the constraint.

  Fodor et al., PoS LATTICE2007 056 (2007)

- Analogous to changing from **canonical** (fixed $h$) to **microcanonical** (fixed $\bar{\phi}$) ensemble.

- $h$ can be recovered as

$$h = \frac{d\Omega(\bar{\phi})}{d\bar{\phi}} = m^2\bar{\phi} + \frac{g}{6V}\left\langle \int_x \varphi^3(x) \right\rangle_{\bar{\phi}} .$$

# Constrained simulations

- We carry out constrained MC simulations on 3D lattices of size $L^3$ with **periodic** boundary conditions.

- In analogy to using $\vec{h} = (h, 0)$ we constrain both field directions:

$$V^{-1} \int \varphi_1 = \bar{\phi} \quad \text{and} \quad V^{-1} \int \varphi_2 = 0\,.$$

- We only study volume dependence towards the **infinite volume** limit, not the lattice spacing dependence.

- We expect spin-wave like configurations, which break translational symmetry.

# Constrained simulations

Looking at **typical configurations**:

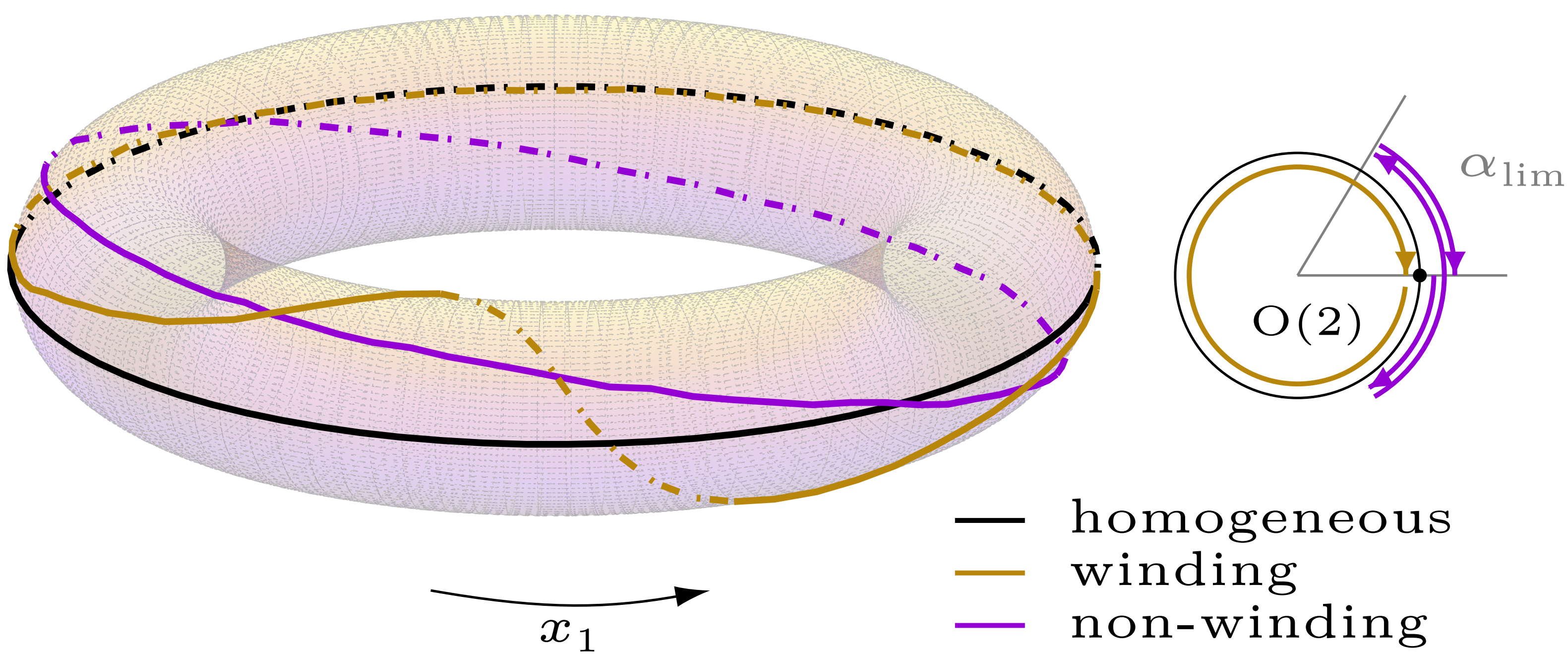
- Spin-wave only in one direction.
- Slice averages

$$\vec{s}(x) = \frac{1}{L^2} \sum_{y,z \in L} \vec{\varphi}(x,y,z)\,.$$

show pronounced inhomogeneities:

**Two dominant** types of configurations emerge for $\bar{\phi} < \bar{\phi}_{\rm min}$:

- a **winding ($w = 1$)**: the field winds around the full $O(2)$ space;
- a **non-winding ($w = 0$)**: the field oscillates in $O(2)$ space.



# Simulational problem: ”topological freezing”

Problem:

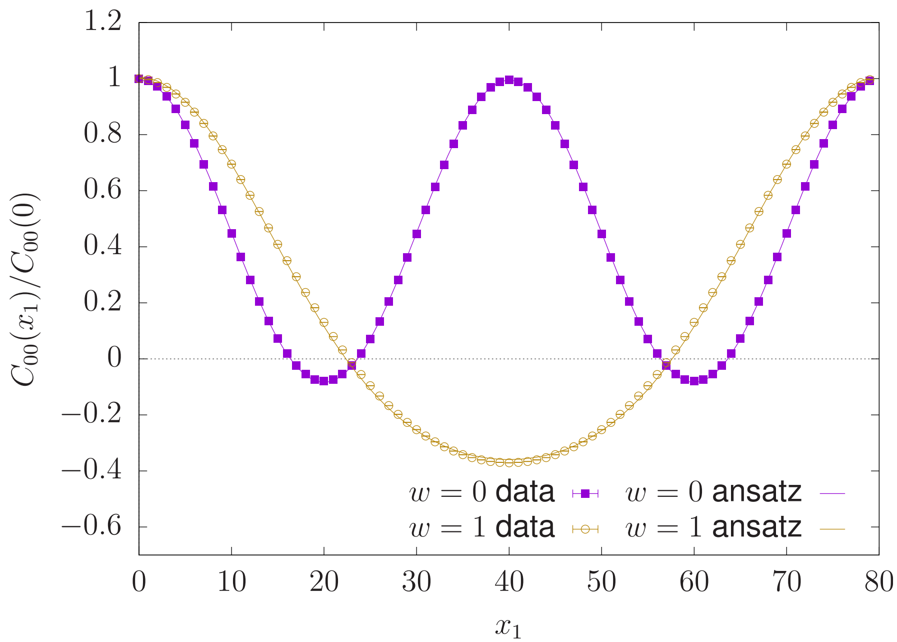
- Configurations with different $w$ are far in configuration space.
- Local changes are not enough to transform $w$.
- Transitions are rare in Markov time even if $S_{w=0} \approx S_{w=1}$.

Remedy:

- Change in dominance happens rapidly $\rightarrow$ only affects small range of $\bar{\phi}$ around some $\bar{\phi}_c$.
- Preparing initial conditions $\rightarrow$ measure observables in fixed $w$ sectors.
- Measure $\bar{\phi}_c$ based on the generalization of the surface tension.

# Observables: correlators

Ensemble averaged slice **correlators**: $C_{ij}(\tau) = \frac{1}{L}\left\langle \sum_{x\in L} s_i(x+\tau)s_j(x) \right\rangle_{\bar{\phi}}$ .



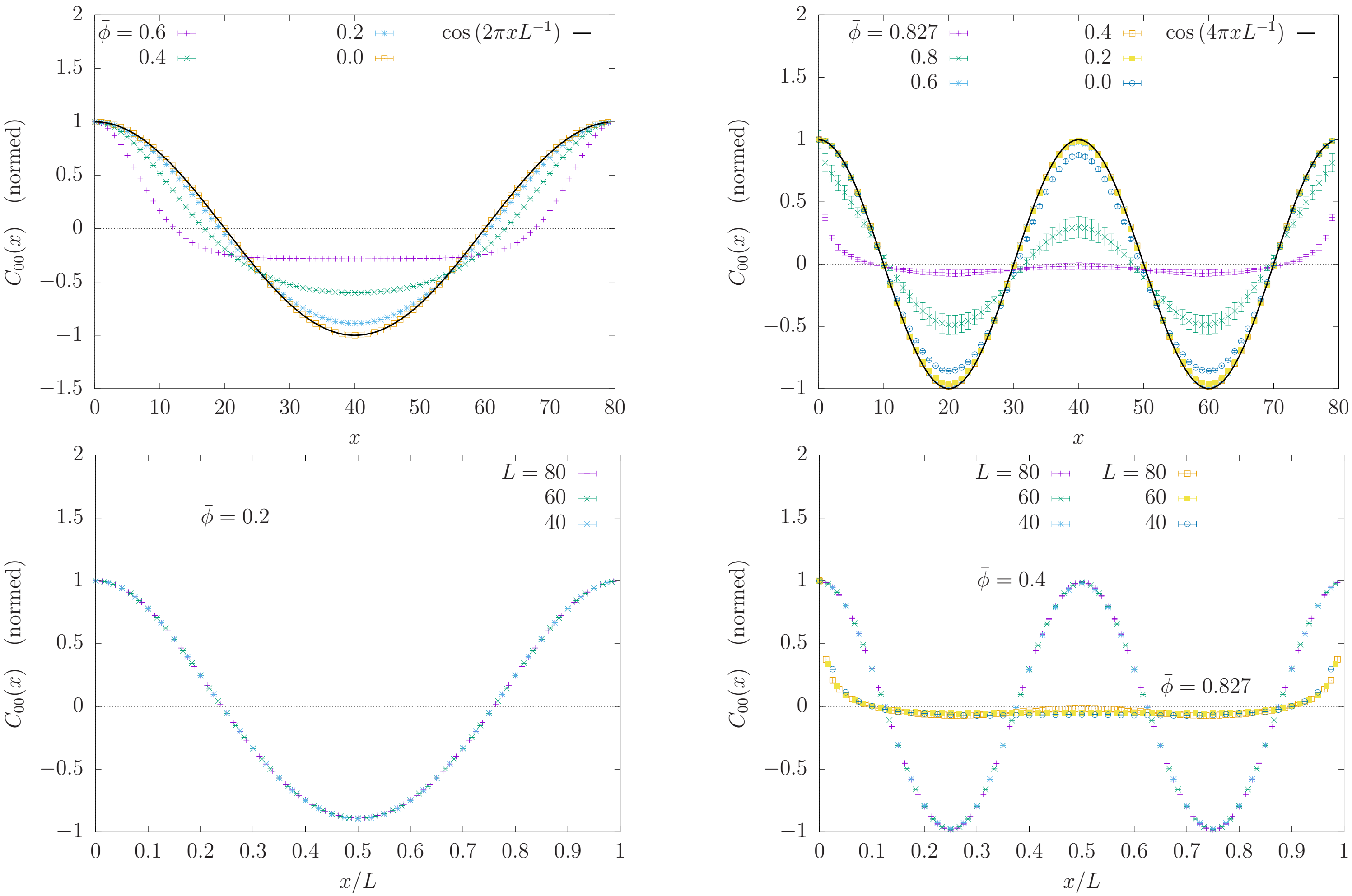

- Suppressed UV fluctuaions $\longrightarrow$ more pronounced macroscopic inhomogeneities.
- Classically motivated ansatz: constant length ($\bar{\phi}_{\min}$), angle depends on $x_1$:

$$\alpha_w(x_1) = \frac{2\pi w x_1}{L} + \alpha_{\lim} \sin\left(\frac{2\pi x_1}{L}\right) .$$

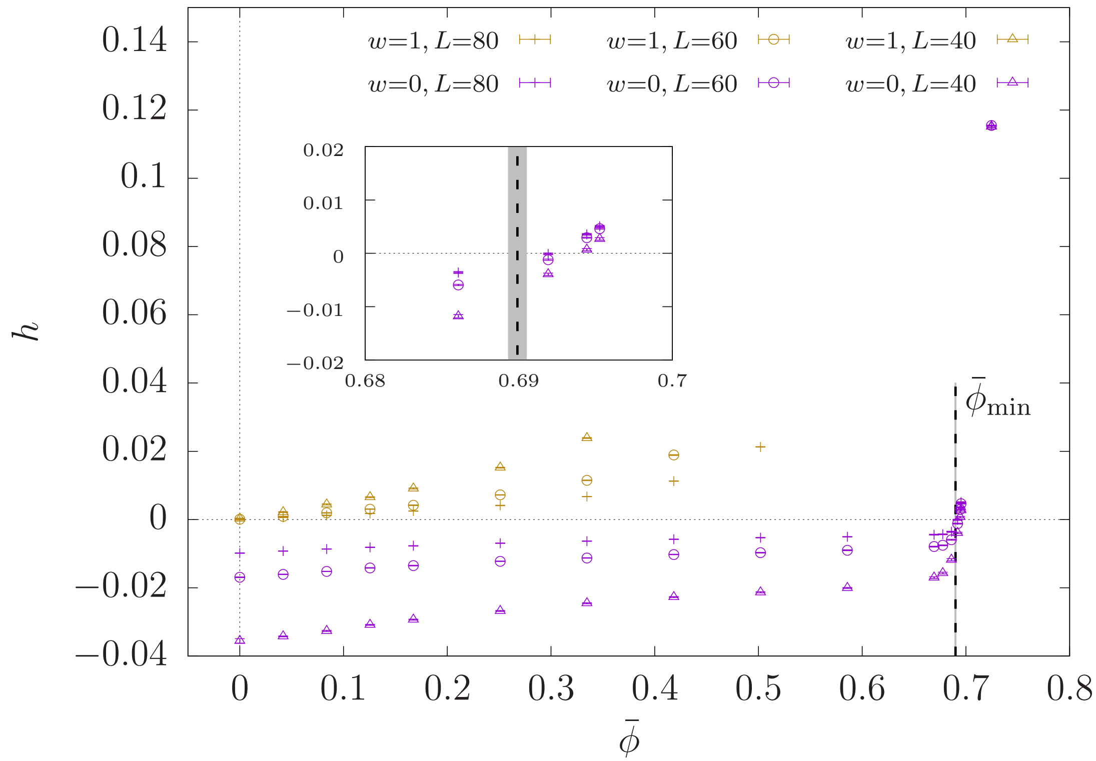
- Trivial volume dependece: length scale proportional to $L$ to minimize kinetic energy.

# Observables: correlators

- Non-winding cfgs **continously connect** to homogeneous cfgs and dominate for $\bar{\phi} \lessapprox \bar{\phi}_{\min}$.
- Winding cfgs **do not exist** for $\forall\bar{\phi}$, but dominate for small $\bar{\phi}$.
- Sharp transition between the two ($\bar{\phi}_c$).



# Observables: magnetic field



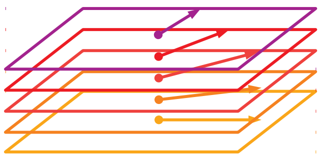

- Controlled $V \to \infty$ limit for $\bar{\phi}_{\min}$ from
$$h_{w=0}(\bar{\phi}_{\min,\mathrm{V}}, V) = 0$$
- At infinite volume both sets of curves tend to zero.
- Integral method for the potential:

$$\Omega_w(\bar{\phi}) = \int_0^{\bar{\phi}} d\phi h_w(\phi) + c_w \, ,$$

- Setting the integration constants is simple for $w = 0$, hard for $w = 1$.
- Transition around $\bar{\phi}_{\mathrm{c}}$ will be sharp due to the large difference between $h_0$ and $h_1$.
- Setting $c_1$ and finding $\bar{\phi}_{\mathrm{c}}$, both through generalizied surface tension.

# Differential surface tension

- Different vacuua $\equiv$ different points on the circle.
- We can think of **rotating cfgs** as **connecting** different vacuua.
- $\sum$ **kinetic energy** between roughly homogeneous slices



$$E_{\text{kin}}(\bar{\phi}) = \frac{1}{2L} \sum_x \langle [\partial_x \vec{s}(x)]^2 \rangle_{\bar{\phi}} \,.$$

- Excess energy comes from kinetic energy of macroscopic inhomogeneities:

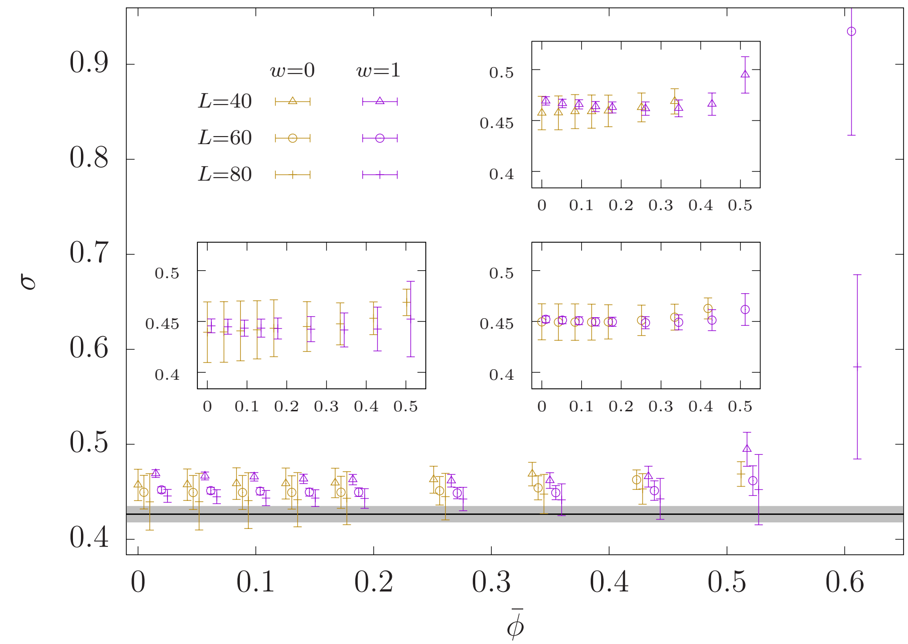
$$\Omega_{\text{inhom}} - \Omega_{\text{hom}} = \frac{\sigma}{\bar{\phi}^2_{\text{min}}} \left[ E_{\text{kin}}(\bar{\phi}) - E_{\text{kin}}(\bar{\phi}_{\text{min}}) \right] \,.$$

- Non-winding cfgs become homogeneous at $\bar{\phi}_{\text{min}}$, set $\Omega_{\text{w}=0}(\bar{\phi}_{\text{min}}) = 0$ to obtain $c_0$.
- For the winding case, $c_1$, use that

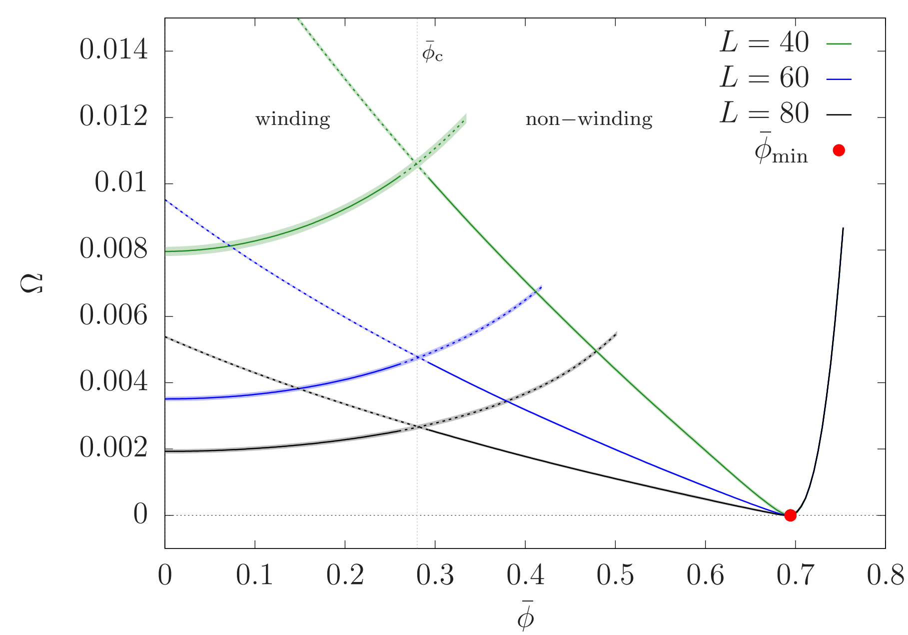
$$\Omega_0(\bar{\phi}_{\text{c}}) \stackrel{!}{=} \Omega_1(\bar{\phi}_{\text{c}}) \Leftrightarrow E_{\text{kin},0}(\bar{\phi}_{\text{c}}) \stackrel{!}{=} E_{\text{kin},1}(\bar{\phi}_{\text{c}}) \,.$$

# Differential surface tension

- Consistent with a constant.
- Independent of topology.
- Thermodynamic limit can be carried out: $\sigma = 0.427(8)$
- Approaching $\bar{\phi}_{\min}$ requires a $\dfrac{0}{0}$ type limit $\Rightarrow$ errors blow up.




- Constraint effective potential can be reconstructed almost everywhere.
- Except around $\bar{\phi}_c$ (transition $w = 0 \to 1$).
- $\Omega$ becomes flat as $V \to \infty$ indeed.



# Summary

- One can construct and simulate the constraint potential which coincides with the effective potential in the $V \to \infty$ limit.
- The flat region is dominated by inhomogeneous spin wave configurations.
- The dominant configurations can be topologically classified (winding or not).
- We extended the definition of the surface tension to spontaneously broken continuous symmetry systems $\to$ applications?
- Can this be extended to fermionic systems? Non-local constraint before Grassmann integration!