Recent results on the confining string in non-abelian Lattice Gauge Theories[1] .

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[1] 1) F. Caristo, M. Caselle, N. Magnoli, A. Nada, M. Panero and A. Smecca,
Fine corrections in the effective string describing SU(2) Yang-Mills theory in three dimensions,
JHEP 03 (2022) 115 [arXiv:2109.06212].

2) C. Bonati, M. Caselle and S. Morlacchi,
The Unreasonable effectiveness of effective string theory: The case of the 3D SU(2) Higgs model,
Phys. Rev. D **104** (2021) 054501 [arXiv:2106.08784]

Summary:

Introduction and motivations

- Understanding confinement is probably the most important open problem in QCD
- A characteristic feature of color confinement is the formation of chromoelectric flux tubes (confining strings) connecting quarks.
- The properties of these confining strings can be studied numerically looking for instance at the interquark potential in Lattice Gauge Theories. The recent progress in the precision and reliability of simulations allows now to study the fine details of these strings.
- At the same time in the last few years there has been a remarkable progress on the theoretical side. In particular the so called "Low Energy Universality" theorem.
- The combination of numerical and theoretical improvements makes it possible to explore the confining string at unprecedented levels of precision.

Introduction and motivations

- It is well known that the long distance behaviour of the interquark potential at zero temperature in pure LGTs is well described by the Nambu-Goto Effective String Theory (EST)

- At shorter distances and/or higher temperature corrections beyond the Nambu-Goto action are expected, their study is of great importance to understand confinement: Nambu-Goto can be considered as a sort of mean field model for the interquark potential, the fine details of the gauge theory (the gauge group, the physical mechanisms behind confinement, the physical degrees of freedoms which originate the EST) are encoded in these higher order terms.

- A perfect laboratory to address this issue is the (2+1) dimensional SU(2) model, which can be simulated at a reasonable cost and has a second order deconfinement transition which allows to use universality to constrain the model in the vicinity of the critical point.

Lattice regularization and quark confinement.

Only a truly non-perturbative approach such as lattice regularization can describe the deconfinement transition and the confined phase of non-abelian gauge theories.
For SU(N) pure gauge theories on the lattice the dynamics are described by the standard Wilson action

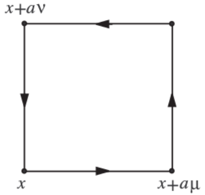
$$S_W = \beta \sum_{p=sp,tp} (1 - \frac{1}{N} \mathrm{ReTr} U_p)$$

where U_P is the product of four U_μ SU(N) variables on the space-like or time-like plaquette P and $\beta = \frac{2N}{g^2}$.
The partition function is

$$Z = \int \prod_{x,\mu} \mathrm{d} U_\mu(x) e^{-S_W}$$

the expectation value of an observable A

$$\langle A \rangle = \frac{1}{Z} \int \prod_{n,\mu} \mathrm{d} U_\mu(n)\, A(U_\mu(n))\, e^{-S_W}$$

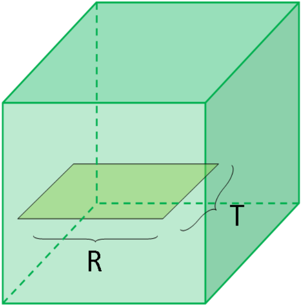


Lattice determination of the interquark potential.

In pure lattice gauge theories the interquark potential is usually extracted from two (almost) equivalent observables

- Wilson loop expectation values $< W(R,T) >$ ("zero temperature potential")

$$V(R) = \lim_{T\to\infty} -\frac{1}{T} \log < W(R,T) >$$

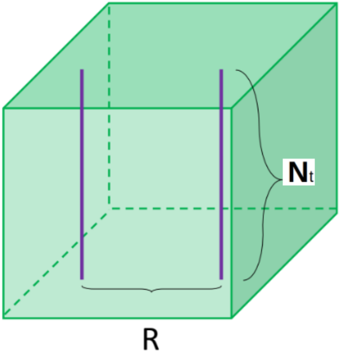


- Polyakov loop correlators $< P(0)P(R)^\dagger >$ ("finite temperature potential")

$$< P(0)P(R)^\dagger > \sim \sum_{n=0}^{\infty} c_n \; e^{-N_t E_n}$$

where N_t is the inverse temperature, i.e. the length of the lattice in the compactified imaginary time direction

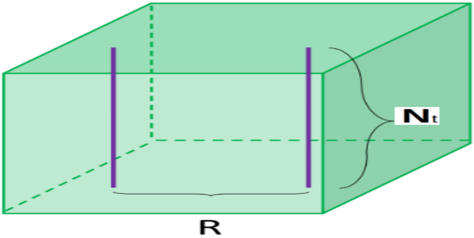
$$V(R, N_t) = -\frac{1}{N_t} \log < P(0)P(R)^\dagger >$$



- In the low temperature limit ($N_t \to \infty$) $V(R)$ is dominated by E_0 and coincides with the result obtained with the Wilson loop

$$E_0 = V(R) = - \lim_{N_t \to \infty} \frac{1}{N_t} \log < P(0) P(R)^\dagger >$$

- $V(R, N_t)$ flattens out as the temperature increases (i.e. as N_t decreases) and becomes constant for a critical value $(N_t)_c$. the Deconfinement Temperature. The Polyakov loop is an order parameter of this phase transition.



The "area law".

- In the large R limit $< P(0)P(R)^\dagger >$ is dominated by the "area law"

$$< P(0)P(R)^\dagger >\sim e^{-\sigma R N_t}$$

which implies

$$V(R) = \sigma R$$

where σ is a function of the temperature: $\sigma(N_t)$

- $\sigma(N_t)$ decreases as the temperature increases and vanishes at the Deconfinement Temperature. I will define the $T = 0$ value of $\sigma(N_t)$ as $\sigma_0 = \lim_{N_t \to \infty} \sigma(N_t)$.

Effective String Theory: the Nambu-Goto action.

- Confinement is associated to the creation of a thin flux tube joining the quark antiquark pair. In this framework the "area law" represents the classical contribution to the interquark potential on top of which we expect to have quantum corrections. The theory which describes these quantum fluctuations is known as "effective string theory" (EST).

- The simplest choice fulfilling Poincaré invariance for the EST is the Nambu-Goto string in which quantum corrections are evaluated summing over all the possible surfaces bordered by the two Polyakov loops with a weight proportional to their area.
- In the low T (large N_t) limit the interquark potential is dominated by the lowest state

$$V(R) \sim E_0(R) = \sqrt{\sigma_0^2 R^2 - 2\pi\sigma_0 \frac{D-2}{24}} \,,$$

$$V(R) \;\sim\; \sigma_0 R - \frac{\pi(D-2)}{24R} - \frac{1}{2\sigma_0 R^3}\left(\frac{\pi(D-2)}{24}\right)^2 + \ldots.. \,,$$

The Lüscher term.

The first correction is known as "Lüscher term" and turns out to be in remarkable agreement with numerical simulations. First high precision test in d=4 SU(3) LGT almost twenty years ago. [1]

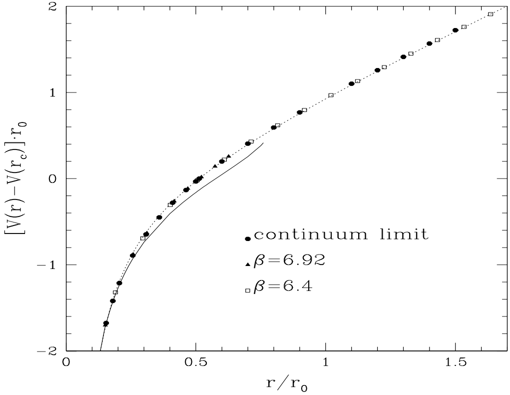


Figure: The static potential. The dashed line represents the bosonic string model and the solid line the prediction of perturbation theory.

[1] S. Necco and R. Sommer, Nucl.Phys. B622 (2002) 328

Beyond the Lüscher term: $1/R^3$ corrections.

High precision fit in the SU(2) case in 2+1 dimensions (A. Athenodorou, B. Bringoltz, M. Teper JHEP 1105:042 (2011))

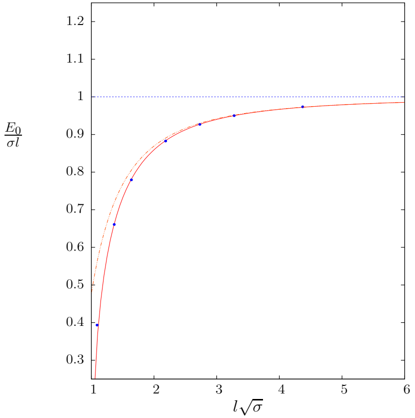


Figure 6: Energy of absolute ground state for SU(2) at $\beta = 5.6$. Compared to full Nambu-Goto (solid curve) and just the Lüscher correction (dashed curve).

Confining String: Low Energy Universality.

- The Nambu-Goto action is only the leading-order approximation of the actual EST describing the infrared dynamics of a confining gauge theories. Why should we trust the $1/R^3$ correction?
- It can be shown that, due to Poincaré and parity invariance, the Nambu-Goto action for $d = 3$ LGTs is exact up to the order $1/R^5$ included [1] : "**Low energy universality**"
- The terms beyond the Nambu-Goto approximation (i.e. starting from $1/R^7$) are likely to encode important physical information. They should depend on the particular gauge group of the LGT and could shed light on the mechanisms underlying confinement and on the physical degrees of freedoms from which the confining string arises.

[1] M. Lüscher and P. Weisz, JHEP **0407** (2004) 014
O. Aharony and Z. Komargodski, JHEP **1305** (2013) 118
S. Dubovsky, R. Flauger and V. Gorbenko, JHEP **1209** (2012) 044

Geometrical description.

An intuitive geometrical description of this result is obtained writing the effective action as the most general mapping fulfilling Poincare' invariance in the target manifold:

$$X^\mu : \mathcal{M} \to \mathbb{R}^D, \qquad \mu = 0, \cdots, D-1$$

- $\mathcal{M}$: two-dimensional surface describing the worldsheet of the string
- $\mathbb{R}^D$: (flat) D dimensional target space $\mathbb{R}^D$ of the gauge theory.

Main Result [1] : "**Low energy universality**"

- The first few terms of the action compatible with Poincaré and parity invariance are suitable combinations of geometric invariants constructed from the induced metric $g_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$.
- These terms can be classified according to their weight, i.e. the difference between the number of derivatives minus the number of fields X^μ.

[1] M. Lüscher and P. Weisz, JHEP **0407** (2004) 014
O. Aharony and Z. Komargodski, JHEP **1305** (2013) 118
S. Dubovsky, R. Flauger and V. Gorbenko, JHEP **1209** (2012) 044

Geometrical description.

- The only term of weight zero is the Nambu-Goto action

$$S_{\rm NG} = \sigma \int d^2\xi \sqrt{g} \ ,$$

where $g \equiv \det(g_{\alpha\beta})$.

- This term has a natural geometric interpretation: it measures the area swept out by the worldsheet in space-time.
- Fixing the physical gauge one finds (choosing an euclidean metric)

$$S = \sigma \int d^2\xi \sqrt{det(\eta_{\alpha\beta} + \partial_\alpha X \cdot \partial_\beta X)}$$

$$\sim \sigma RT + \frac{\sigma}{2} \int d^2\xi \left[\partial_\alpha X \cdot \partial^\alpha X + \frac{1}{8}(\partial_\alpha X \cdot \partial^\alpha X)^2 - \frac{1}{4}(\partial_\alpha X \cdot \partial_\beta X)^2 + \dots \right] ,$$

Geometrical description.

- At weight two, two new contributions appear:

$$S_{2,\mathcal{R}} = \gamma \int d^2\xi \sqrt{g}\mathcal{R} \ ,$$
$$S_{2,K} = \alpha \int d^2\xi \sqrt{g} K^2,$$

where $\mathcal{R}$ denotes the Ricci scalar constructed from the induced metric, and $K \equiv \Delta(g)X$ is the extrinsic curvature, where $\Delta(g)$ is the Laplacian in the space with metric $g_{\alpha\beta}$.

However both these terms can be neglected!

- $\mathcal{R}$ is topological in two dimensions and, since in the long string limit in which we are interested we do not expect topologically changing fluctuations, its contribution is constant and can be neglected.
- K^2 is proportional to the equation of motion of the Nambu-Goto Lagrangian and can be eliminated by a suitable field redefinition.

Confining String: higher order terms.

- Thus the first non trivial terms appear at level four and contribute to the interquark potential with terms proportional to $1/R^7 \rightarrow$ **Low Energy Universality**
- The coefficient of this term which we call in the following k_c is a new non-universal constant which together with σ defines the EST and likely depends on the peculiar features of the model: gauge group, space time dimensions.... It is thus of mandatory importance to measure it with MC simulations.
- However, this goal is hampered by the existence of boundary terms (which encode the interaction of the flux tube with the Polyakov loops at the boundaries) which contribute at the order $1/R^4$.
- The solution to this problem is to use the "open-closed" string duality and look to the interquark potential in the high temperature regime, in the neighbourhood of the deconfinement transition, but still in the confining phase.
  It can be shown that in this regime the boundary term becomes subleading and can be neglected.

The boundary contribution to the interquark potential

The leading boundary correction to the Effective String action turns out to be:

$$S_{b,2}^{(1)} = \int d\xi_0 \left[b_2 \partial_1 \partial_0 X \cdot \partial_1 \partial_0 X \right] .$$

Its contribution to the interquark potential can be evaluated performing a gaussian functional integration[1]. In the usual low temperature limit in which $N_t >> R$ one finds:

$$\langle S_{b,2}^{(1)} \rangle = -b_2 \frac{\pi^3 N_t}{60 R^4} E_4(e^{-\frac{\pi N_t}{R}})$$

where E_4 denotes the fourth-order Eisenstein series:

$$E_4(q) \equiv 1 + \frac{2}{\zeta(-3)} \sum_{n=1}^{\infty} \frac{n^3 q^n}{1 - q^n}$$

and $\zeta(s)$ is the Riemann ζ function.

[1] O. Aharony and M. Field JHEP01(2011)065

The boundary contribution to the interquark potential

- We end up with the following expression for the interquark potential

$$V(R) = \sigma_0 R - \frac{\pi(D-2)}{24R} - \frac{1}{2\sigma_0 R^3}\left(\frac{\pi(D-2)}{24}\right)^2 - b_2 \frac{\pi^3(D-2)}{60R^4} + O(1/R^5) \ ,$$

  where b_2 is a new physical parameter, similar to the string tension σ_0, which depends on the theory that we study and should be determined by simulations and comparison with experiments.
- This $1/R^4$ term shadows the bulk corrections to the Nambu-Goto action and makes it very difficult to detect it in MC simulations.
- The solution to this problem is to use the "open-closed" string duality and look to the interquark potential in the high temperature regime, in the neighbourhood of the deconfinement transition, but still in the confining phase.
  It can be shown that in this regine the boundary term becomes subleading and can be neglected.

The boundary contribution to the interquark potential

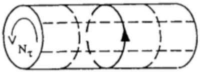
Using the modular properties of the Eisenstein function,

$$E_4\left(e^{-\frac{\pi N_t}{R}}\right) = \left(\frac{2R}{N_t}\right)^4 E_4\left(e^{-\frac{4\pi R}{N_t}}\right),$$

it is easy to see that in the $R \gg N_t$ ("high-temperature") regime, the boundary correction becomes

$$\langle S_{b,2}^{(1)}\rangle = -b_2 \frac{4\pi^3}{15 N_t^3} E_4\left(e^{-\frac{4\pi R}{N_t}}\right),$$

which does not contain terms linear in R and thus it does not contribute to the temperature-dependent string tension.



Effective String Theory at Finite Temperature

- For a generic Poincaré invariant EST in $D = 2 + 1$, we have[2] for $R >> N_t$

$$\langle P(0)P^\dagger(R)\rangle = \sum_{n=0}^{\infty} |v_n(N_t)|^2 \frac{E_n}{\pi} K_0(E_n R)$$

- In the large R limit the sum is dominated by the lowest state and from the exponential decay of the modified Bessel function we may extract the correlation length $E_0 = \frac{1}{\xi}$

- For the Nambu-Goto string

$$\langle P(0)P^\dagger(R)\rangle = \sum_{n=0}^{\infty} \omega_n N_t \sqrt{\frac{\sigma_0}{\pi}} K_0(E_n R)$$

with $E_n = \sigma_0 N_t \sqrt{1 + \frac{8\pi}{\sigma_0 N_t^2}\left(n - \frac{1}{24}\right)} \Longrightarrow E_0 = \sigma_0 N_t \sqrt{1 - \frac{\pi}{3\sigma_0 N_t^2}}$

[2] M. Lüscher and P. Weisz, *JHEP* 2004.07(2004), p. 014

Interquark Potential and Deconfinement Transition

- In the large R limit $\langle P(0)P^\dagger(R)\rangle$ is dominated by the lowest state E_0
- For the Nambu-Goto string

$$\langle P(0)P^\dagger(R)\rangle = e^{-N_t V(R,N_t)} \sim K_0(E_0 R)$$

with $E_0 = \sigma_0 N_t \sqrt{1 - \frac{\pi}{3\sigma_0 N_t^2}}$

$$E_0 = \sigma_0 N_t - \frac{\pi}{6N_t} - \frac{\pi^2}{72\sigma_0 N_t^3} - \frac{\pi^3}{432\sigma_0^2 N_t^5} - \frac{5\pi^4}{10368\sigma_0^3 N_t^7} + ...$$

- Low Energy Universality: Corrections to the Nambu-Goto action appear only at the order $\frac{1}{\sigma_0^3 N_t^7}$.
- Problem: detecting such a correction requires a very precise determination of E_0.

- Solution: Svetitsky-Yaffe mapping[3]

- For some choices of the gauge group the finite temperature Deconfinement Transition of a pure Yang-Mills Theory is critical and is described by a CFT. This is the case, in particular for the (2+1) dimensional SU(2) LGT whose deconfinement transition belongs to the same universality class of the 2d Ising model.

- The confining phase is mapped into the *symmetric* phase of the perturbed CFT (i.e the high T phase of the Ising realization, even if for the Yang-Mills theory it corresponds to the low T regime!!).

- $\langle P(0)P^{\dagger}(R)\rangle$ in the vicinity of the critical point is equivalent to the spin-spin correlator of the 2d Ising model $\langle s(0)s(R)\rangle_t$ which is know exactly.

[3] B. Svetitsky and L. G. Yaffe, *Nucl. Phys.* **B210** (1982) 423

$\langle s(0)s(R)\rangle$ correlator in the 2d Ising model

For the 2d Ising model, thanks to the exact integrability, the $\langle s(0)s(R)\rangle$ correlator is known exactly[4]. (apart from the non-universal normalization constants C_1 and C_2):

- For $R < \xi$ it coincides with the Conformal Perturbation result:

$$\langle s(0)s(R)\rangle = \frac{C_1}{r^{\frac{1}{4}}}\left[1 + \frac{t}{2}\ln\left(\frac{e^{\gamma_E}t}{8}\right) + \frac{1}{16}t^2 + \frac{1}{32}t^3\ln\left(\frac{e^{\gamma_E}t}{8}\right) + O(t^4)\right]$$

where $t = \frac{R}{\xi}$

- For $R > \xi$ is the typical expression for a free isolated particle in $d = 2$

$$\langle s(0)s(r)\rangle = C_2 K_0(R/\xi)$$

where K_0 denotes the modified Bessel function: $K_0(x) \sim \frac{e^{-x}}{\sqrt{x}}$ for large x

- Notice the shift from $r^{-\frac{1}{4}}$ at short distance to $r^{-\frac{1}{2}}$ at large distance

[4] T. T. Wu, B. M. McCoy, C. A. Tracy and E. Barouch, *Phys. Rev. B* **13** (1976) 316

SU(2) LGT in (2+1) dimensions. Summary of simulations

$N_t \times N_s^2$	β	T/T_c	n_{conf}
9×96^2	11.3048	0.80	2.5×10^5
	11.72873	0.83	2.5×10^5
9×160^2	12.15266	0.86	2.5×10^5
7×96^2	9.228023	0.83	2.5×10^5
	9.561566	0.86	2.5×10^5

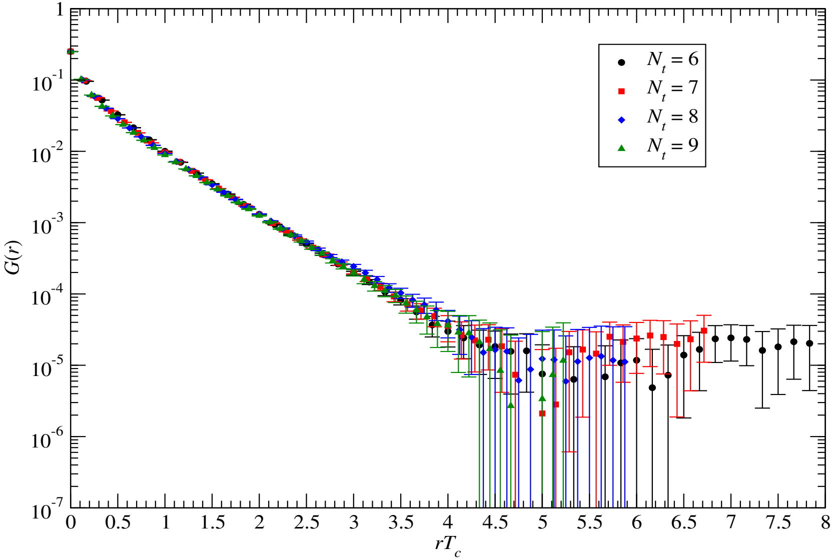
$N_t \times N_s^2$	β	T/T_c	n_{conf}
8×96^2	10.10736	0.80	2.5×10^5
	10.486386	0.83	2.5×10^5
	10.865412	0.86	2.5×10^5
6×96^2	8.258494	0.86	2.5×10^5
	8.546581	0.89	2.5×10^5

β	N_t	N_s	T/T_c	n_{conf}
$\beta = 9$	6	160	0.935	2.0×10^5
	7	96	0.801	2.0×10^5
	8	96	0.701	2.0×10^5
	9	96	0.623	2.0×10^5
	10	96	0.561	2.0×10^5
	11	96	0.510	2.0×10^5
	12	96	0.468	2.0×10^5

β	N_t	N_s	T/T_c	n_{conf}
$\beta = 12.15266$	8	240	0.960	2.0×10^5
	9	160	0.853	2.0×10^5
	10	96	0.768	2.0×10^5
	11	96	0.698	2.0×10^5
	12	96	0.640	2.0×10^5
	13	96	0.591	2.0×10^5
	14	96	0.549	2.0×10^5

β	N_t	N_s	T/T_c	n_{conf}
$\beta = 13.42445$	9	240	0.947	2.0×10^5
	10	160	0.852	2.0×10^5
	11	160	0.775	2.0×10^5
	12	96	0.710	2.0×10^5
	13	96	0.655	2.0×10^5
	14	96	0.609	2.0×10^5
	15	96	0.568	2.0×10^5

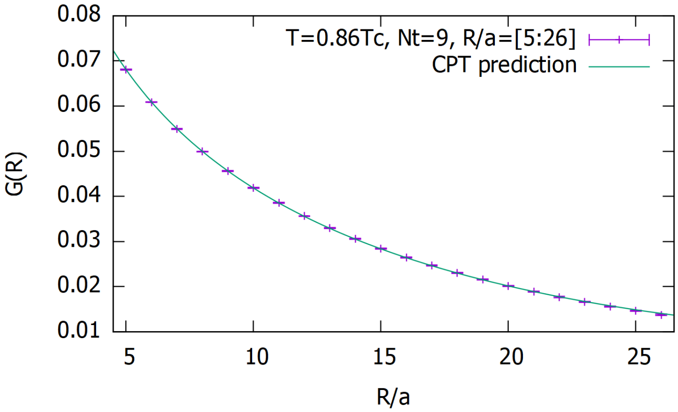
SU(2) LGT in (2+1) dimensions: Scaling test



Testing CPT Predictions

- $f(x) = \frac{C_1}{x^{\frac{1}{4}}}\left[1 + \frac{x}{2\xi}\ln\left(\frac{e^{\gamma_E}x}{8\xi}\right) + \frac{1}{16}\left(\frac{x}{\xi}\right)^2 + \frac{1}{32}\left(\frac{x}{\xi}\right)^3\ln\left(\frac{e^{\gamma_E}x}{8\xi}\right)\right]$

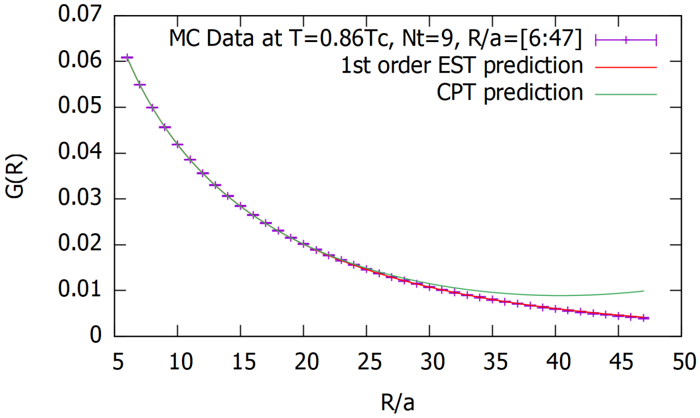
  $\xi = 22.05(6)$ $\quad \chi^2_{red} = 0.92$ in the range $R/a = [5:26]$



CPT vs EST

- $f(x) = C_2 \cdot K_0(b \cdot x) \Longrightarrow \xi = 22.13(17)$

  $\chi^2_{red} = 0.33$ in the range $R/a = [6:47]$



Deviations from Nambu-Goto

- Deviations from Nambu-Goto can be better appreciated looking at the T dependence of E_0 at fixed β. We chose three values of β: $\beta = 9$, $\beta = 12.15266$ and $\beta = 13.42445$ and simulated several values of N_t.

- The correlation length diverges at the critical point as $\xi \sim |T - T_c|^{-\nu}$. Two predictions for the critical exponent:
  1. Nambu-Goto: $\nu = 1/2$

$$E_0 = \sigma_0 N_t \sqrt{1 - \frac{\pi}{3\sigma_0 N_t^2}} \rightarrow \xi = \xi_0 \sqrt{1 - T^2/T_c^2} = \xi_0 \sqrt{1 - \frac{N_{t,c}}{N_t}}$$

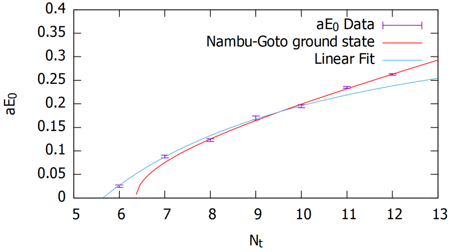
  2. Svetitsky-Yaffe conjecture: $\nu = 1$

- For the linear divergent behaviour we fit the string ground state with the function

$$f(N_t) = k_s \left(1 - \frac{N_{t,c}}{N_t}\right)$$

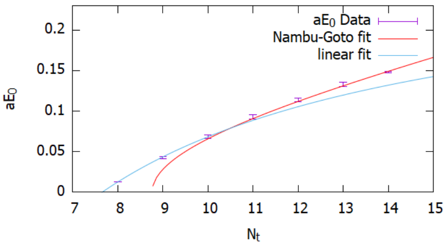
Deviations From Nambu-Goto

At $\beta = 9$:



And $N_{t,c} = 5.64(2)$

At $\beta = 12.15266$:



And $N_{t,c} = 7.65(1)$

Deviations from Nambu-Goto

- Due to the Poincarè invariance and to consistency requirements, the first correction to the Nambu-Goto energy spectrum is expected to be of the order $\sim \frac{1}{\sigma_0^3 N_t^7}$

- We fit the temperature dependence of the string ground state E_0 with the function

  $f(N_t) = \text{Taylor}_4(E_{0,NG}) + \frac{k_c}{\sigma_0^3 N_t^7}$
  where:
  $\text{Taylor}_4(E_{0,NG}) = \sigma_0 N_t - \frac{\pi}{6N_t} - \frac{\pi^2}{72\sigma_0 N_t^3} - \frac{\pi^3}{432\sigma_0^2 N_t^5} - \frac{5\pi^4}{10368\sigma_0^3 N_t^7}$

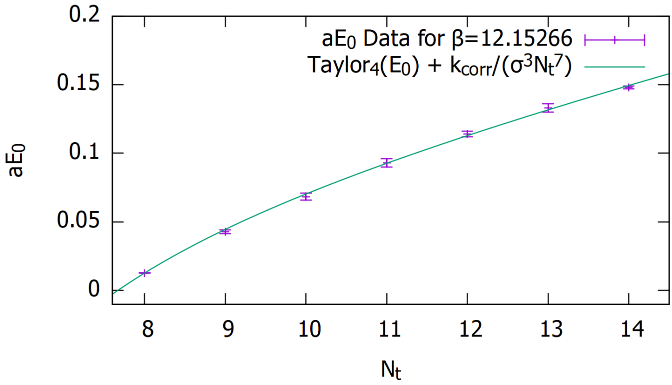
- k_c is not completely free. It can be constrained using a bootstrap analysis[1]. Setting

$$\gamma_3 = -\frac{225}{32\pi^6} k_c$$

  one find: $\gamma_3 \geq -\frac{1}{768} \simeq -0.0013$

[1] J. Elias Miró, A. L. Guerrieri, A. Hebbar, J. Penedones and P. Vieira, Phys. Rev. Lett. **123** (2019) 221602
J. Elias Miró and A. Guerrieri, JHEP **10** (2021), 126

Deviations from Nambu-Goto



β	$N_{t,min}$	$N_{t,max}$	k_c	σ_0	χ^2_{red}	literature
9	6	12	0.040(8)	0.02603(19)	1.60	0.02583(3)
12.15266	8	14	0.054(5)	0.01366(5)	0.89	0.01371(29)
13.42445	9	15	0.053(8)	0.01104(5)	1.33	0.01108(23)

- A strong consistency check of the whole picture is that the adimensional ratio k_c should not change with β as we approach the continuum limit.
- The three values for k_c are almost compatible within the errors: the correction shows the correct scaling behaviour in the continuum limit!
- As our final result for k_c, we have:

$$k_c = 0.050(8)$$

from which we find

$$\gamma_3 = -\frac{225}{32\pi^6} k_c = -0.00037(6),$$

- which is well inside the bound $\gamma_3 \geq -\frac{1}{768} \simeq -0.0013$ derived in refs.[1]

[1] J. Elias Miró, A. L. Guerrieri, A. Hebbar, J. Penedones and P. Vieira, Phys. Rev. Lett. **123** (2019) 221602
J. Elias Miró and A. Guerrieri, JHEP **10** (2021), 126

Comments

- The value is confirmed by an independent set of simulations[5] of zero momentum correlators at $\beta = 16$.

- We start to see a dependence on the gauge group. Independent estimates based on the glueball spectrum[6] suggest a positive value of γ_3 for the $SU(6)$ LGT in (2+1) dimensions.

- Preliminary results on the 3d gauge Ising model suggest again a negative value for γ_3, larger in magnitude than the $SU(2)$ but still compatible with the bootstrap bound.

- Using Conformal Perturbation Theory this type of analysis could be extended to other gauge groups and to the $(3+1)$ dimensions.

[5] A. Athenodorou and M. Teper, JHEP **10** (2016), 093

[6] S. Dubovsky, R. Flauger and V. Gorbenko, J. Exp. Theor. Phys. **120** (2015), 399-422
A. Athenodorou, B. Bringoltz and M. Teper, JHEP **05** (2011), 042

Conclusions

- The interquark potential in the SU(2) LGT in (2+1) dimensions near the deconfinement transition is perfectly fitted by the 2d Ising spin-spin correlator in the whole range of distances that we studied, down to $0.8T_c$.
- This allows to reach an increased precision in the determination of the ground state energy E_0 of the string.
- A reliable estimate of the first correction to Nambu-Goto can be extracted from the data. As expected from low energy universality, this correction is proportional to $1/(\sigma_0^3 N_t^7)$. Its value is negative and it is compatible with the bootstrap constraints.
- This term depends on the gauge group of the model and can be cosidered as the "first correction beyond the mean field approximation" of the effective string. It likely encodes information on the confining mechanism at the basis of the formation of the flux tube.
- It would be interesting to explore this type of correction for other gauge groups and also in $(3+1)$ dimensions

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Coupling SU(2) with bosonic matter[7] .

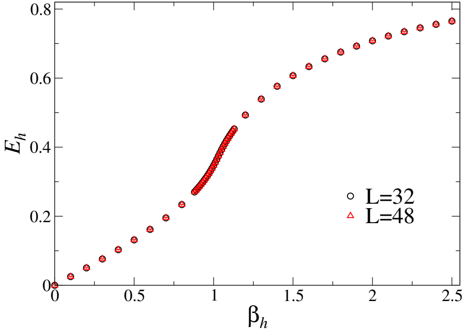
- It is interesting to see what happens of the EST if we couple the theory with matter
- Fermions are too difficult to handle, but bosonic matter can be simulated easily
- to avoid any subtelty let us start with bosonic matter in the fundamental representation.

$$S = -\beta_h \sum_{x,\mu} \mathrm{Re}\,\mathrm{Tr}\left(\varphi_x^\dagger U_{x,\mu}\varphi_{x+\hat{\mu}}\right) - \frac{\beta}{2}\sum_{x,\mu>\nu} \mathrm{Re}\mathrm{Tr}\, U_{\mu\nu}(x)\ .$$

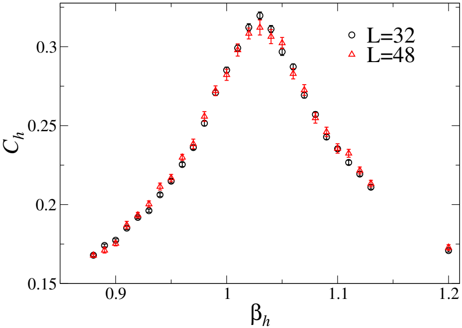
with the constraint $\mathrm{Tr}\ \varphi_x^\dagger \varphi_x = 1$.
We have a very simple phase diagram: a confining phase and a Higgs-like pahse separated by a crossover (non phase transition)

[7] C. Bonati, M. Caselle and S. Morlacchi, Phys. Rev. D **104** (2021) no.5, 054501

Phase diagram at $\beta = 9$

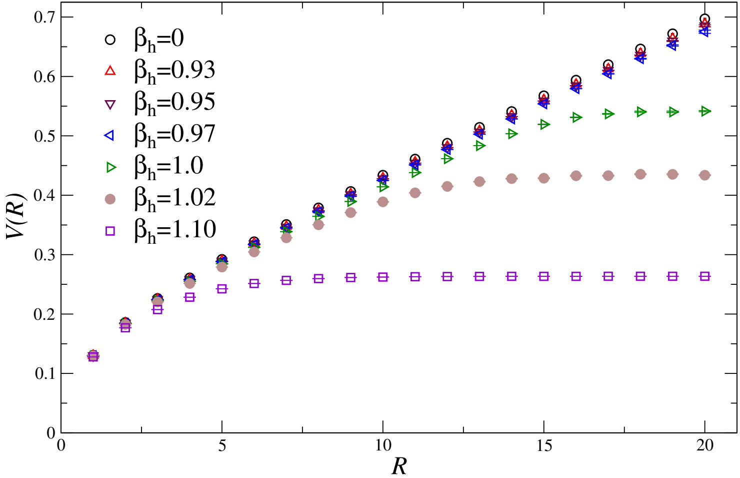


$E_h = \langle \mathrm{Re}\left(\varphi_x^\dagger U_{x,\mu} \varphi_{x+\hat{\mu}}\right) \rangle$



$C_h = L^3 \left(\langle \mathrm{Re}\left(\varphi_x^\dagger U_{x,\mu} \varphi_{x+\hat{\mu}}\right)^2 \rangle - E_h^2 \right)$

Potential as a function of β_h



Potential as a function of β_h

Main result:

Even in the presence of string breaking, the confining part of the interquark potential is well described by the Effective String Theory.

$$V(R) = \sigma_0 R - \frac{\pi(D-2)}{24R} - \frac{1}{2\sigma_0 R^3}\left(\frac{\pi(D-2)}{24}\right)^2 - b_2 \frac{\pi^3(D-2)}{60R^4} + O(1/R^5) \; ,$$

β_h	σ	$\tilde{b}_2$	E_{sb}	R_{sb}
0	0.02583(3)	0.020(3)		
0.93	0.02541(1)	0.019(2)		
0.95	0.02523(3)	0.019(3)		
0.97	0.02493(3)	0.020(1)	$0.683^{+0.018}_{-0.009}$	~ 20
1.00	0.02386(8)	0.019(3)	0.548(2)	~ 15
1.02	0.0214(8)	0.006(15)	0.440(2)	~ 10.5