String spectrum, glueballs and topology in SU(N) gauge theories

Michael Teper (Oxford) - ECT 2022

- glueball spectra $\longrightarrow$ topology and its freezing: in $SU(N)$ gauge theories $\forall N$;

- flux tube spectra $\longrightarrow$ world sheet string action

glueballs and topology with A. Athenodorou: arXiv:2106.00364;2007.06422

extra methods for topology: arXiv:2202.02528

flux tube spectra with A. Athenodorou: arXiv:2112.11213;2205.03642;...

energies from Euclidean correlators:

$$C(t) = \frac{\langle \phi^\dagger(t)\phi(0)\rangle}{\langle \phi^\dagger(0)\phi(0)\rangle} = \sum_{n=0} |c_n|^2 \exp\{-E_n t\} \stackrel{t=an_t\to\infty}{\longrightarrow} |c_0|^2 \exp\{-aE_0 n_t\}$$

glueballs: $\phi(t)$ contractible loop, quantum numbers R^{PC}, momentum $p = 0$

flux tubes: $\phi(t)$ non-contractible winding loop, quantum numbers, $p = 0$

continuum extrapolation:

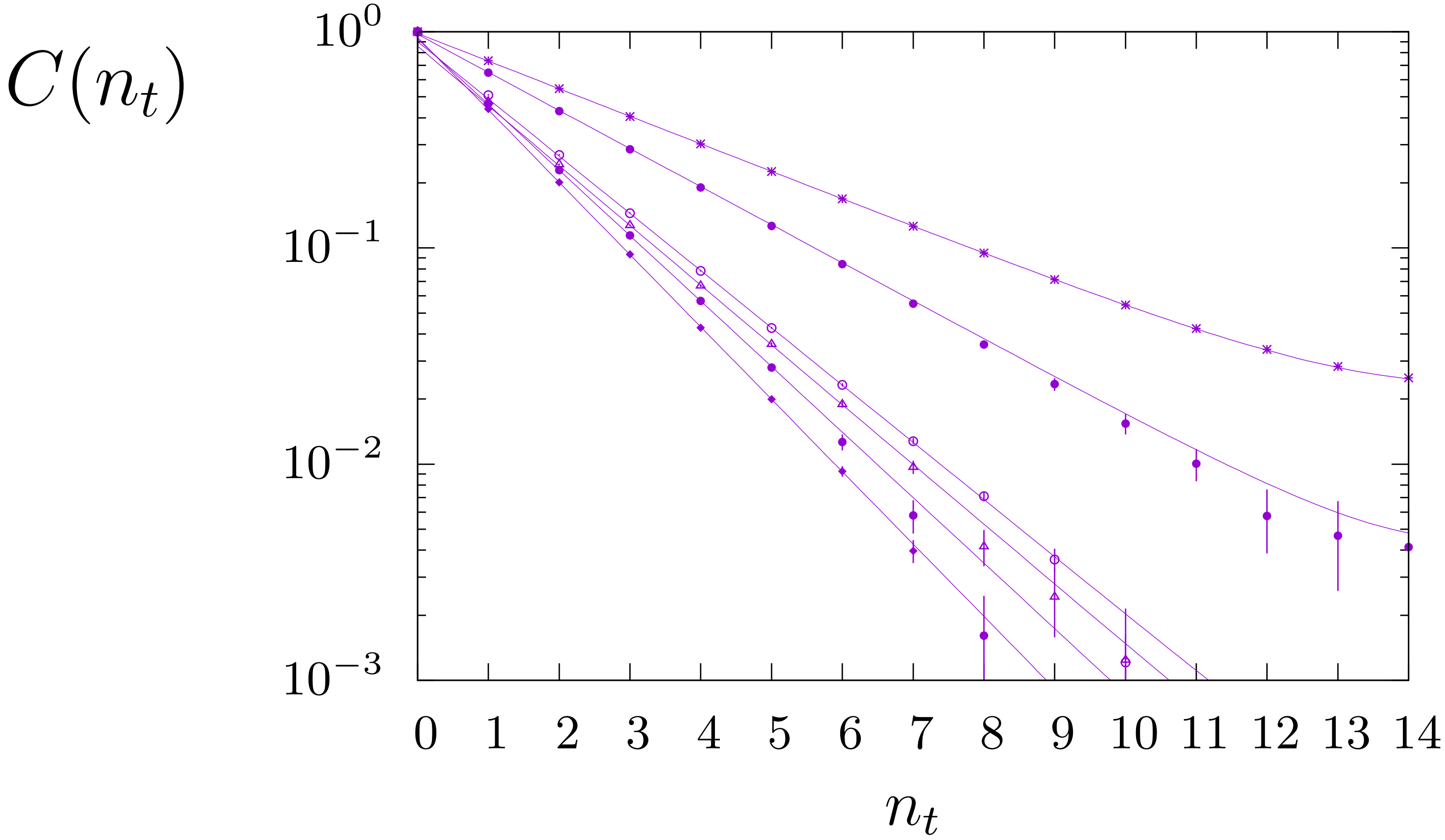
$$\frac{aM(a)}{a\mu(a)} \stackrel{a\to 0}{=} \frac{M(a)}{\mu(a)} \simeq \frac{M(0)}{\mu(0)} + ca^2\mu(a)^2$$

we use string tension as ourscale, $\mu = \sqrt{\sigma}$

$N = \infty$ extrapolation:

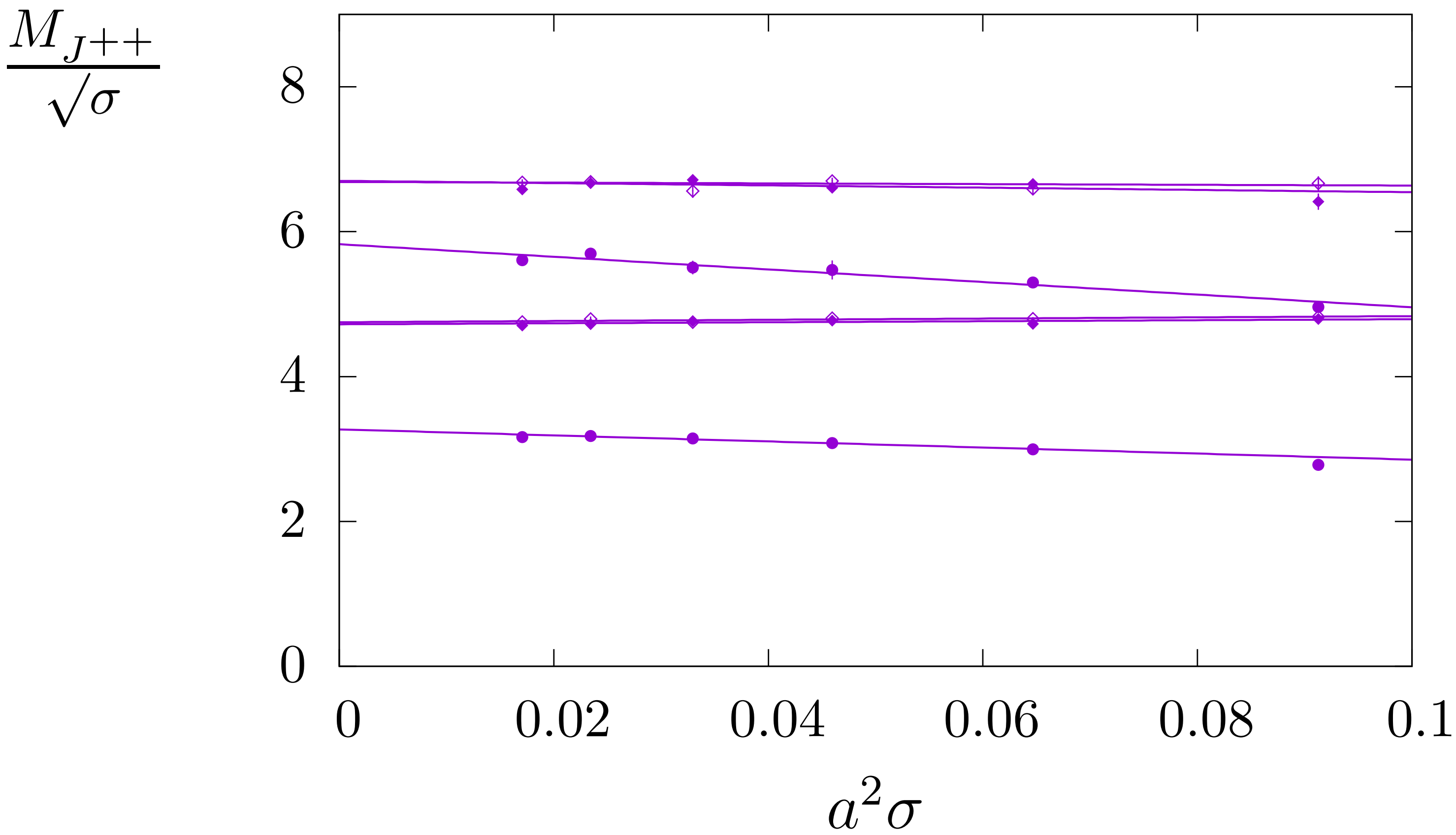
$$\left.\frac{M_i}{\sqrt{\sigma}}\right|_N = \left.\frac{M_i}{\sqrt{\sigma}}\right|_\infty + \frac{c_i}{N^2} + O\left(\frac{1}{N^4}\right)$$

SU(8) , $20^3 30$, $a\sqrt{\sigma} = 0.1325$



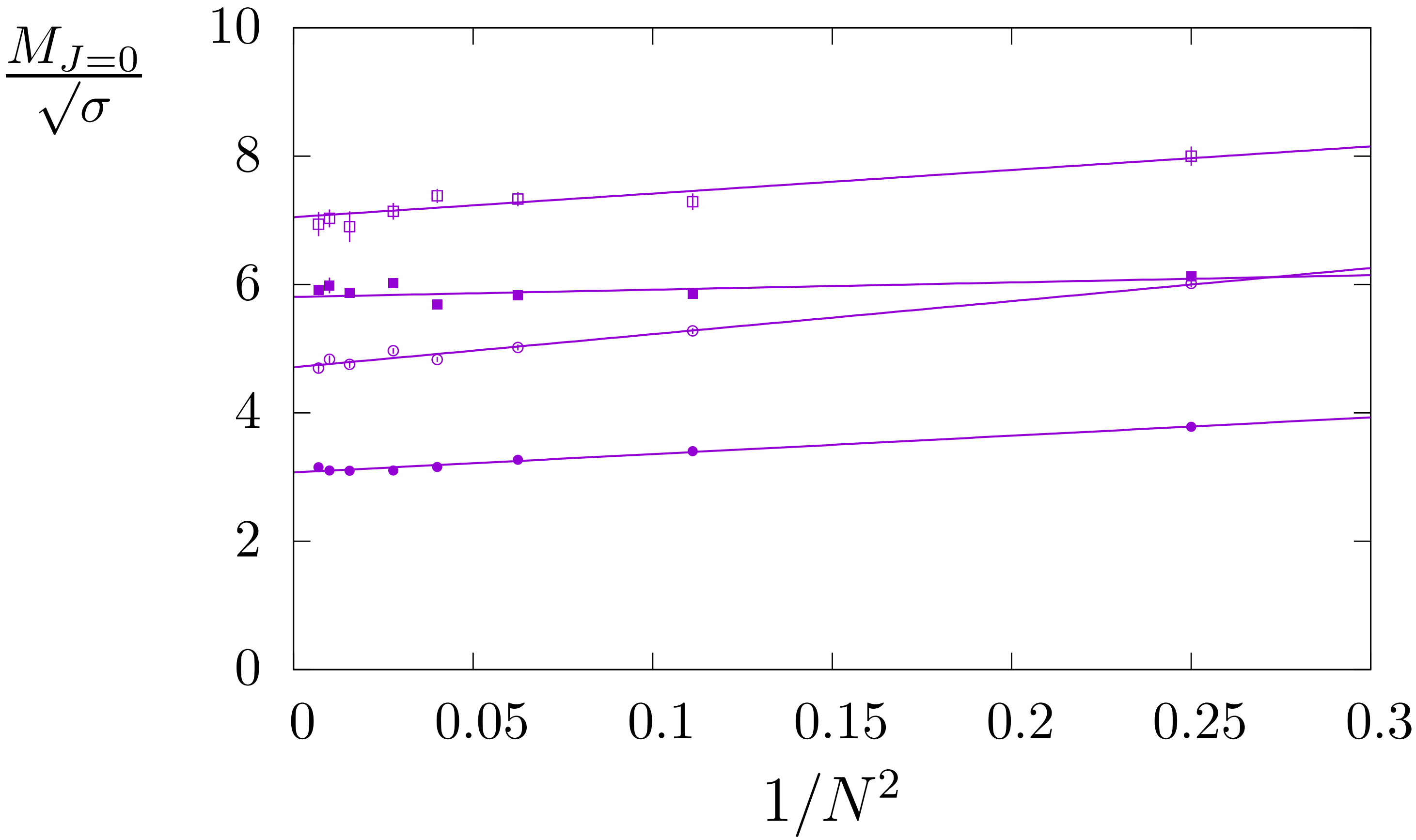
l_f $*$; $J^{PC} = 0^{++}(A_1^{++})$ ● ; $2^{++}(E^{++})$ ○ ; $0^{-+}(A_1^{-+})$ △ ; $1^{+-}(T_1^{+-})$ ♦

e.g. SU(4): some continuum extrapolations



$A_1^{++} \to 0^{++}$ ($\bullet$), $E^{++} \to 2^{++}$ ($\blacklozenge$) and $T_2^{++} \to 2^{++}$ ($\lozenge$).

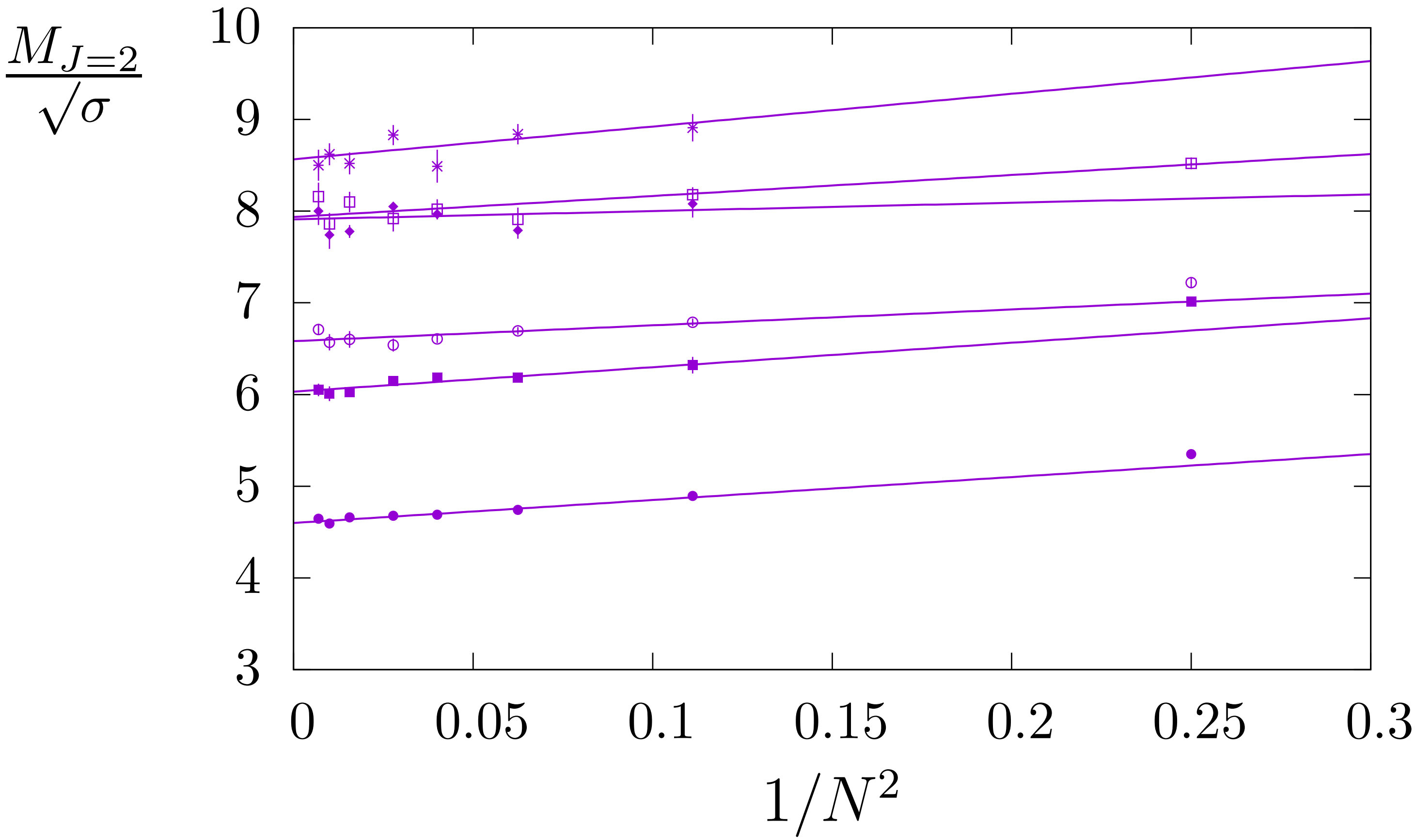
NOTE: doublet E^{++} + triplet $T_2^{++} \longrightarrow$ five components of $J^{PC} = 2^{++}$ glueball some $N \to \infty$ extrapolations



$J^{PC} = 0^{++}$ ground ($\bullet$) and first excited ($\blacksquare$); 0^{-+} ground ($\circ$) and first excited ($\square$).

With extrpolations to $N = \infty$ from $N = 2 - 12$.

some more $N \to \infty$ extrapolations



$J^{PC} = 2^{++}$ ground (●) and first excited (○) tensors; 2^{-+} ground (■) and first excited (□) pseudotensors; lightest 2^{+-} ($*$) and the lightest 2^{--} (♦).

Topological freezing

basic idea: $Q \to Q-1$ involves an instanton shrinking from $\rho \sim O(1)$fm to $\rho \sim a$ and then disappearing within a hypercube, so upper bound is probability of finding very small I with $\rho \sim a \times few$:

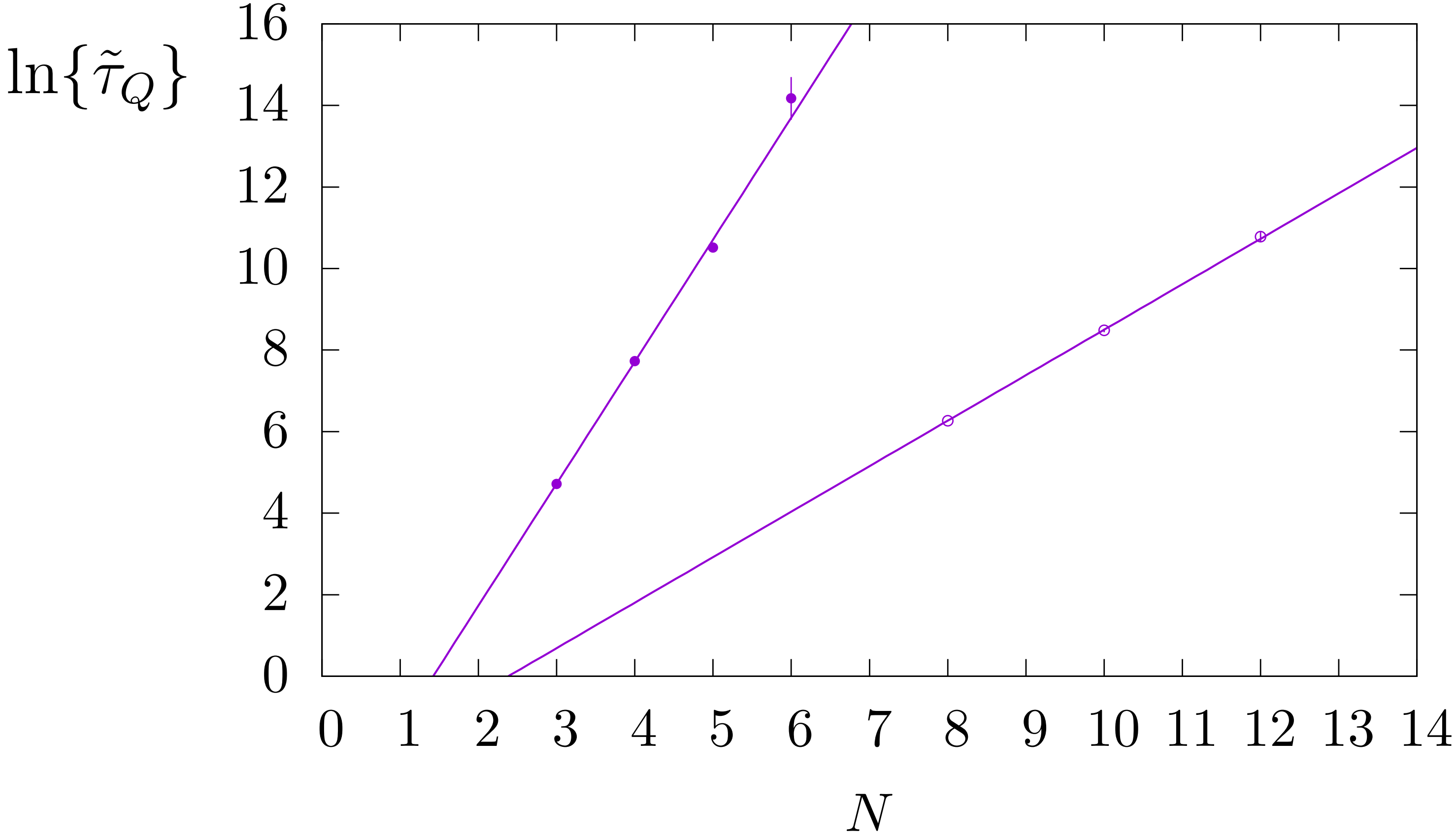
$$D(\rho) \propto \frac{1}{\rho^5}\frac{1}{g^{4N}} \exp\left\{-\frac{8\pi^2}{g^2(\rho)}\right\} \stackrel{N\to\infty}{\propto} \frac{1}{\rho^5}\left\{\exp\left\{-\frac{8\pi^2}{g^2(\rho)N}\right\}\right\}^N \stackrel{\rho\sim a}{\propto} (a\Lambda)^{\frac{11N}{3}-5}.$$

so let: τ_Q = average number sweeps for $Q \to Q \pm 1$

$\Longrightarrow$

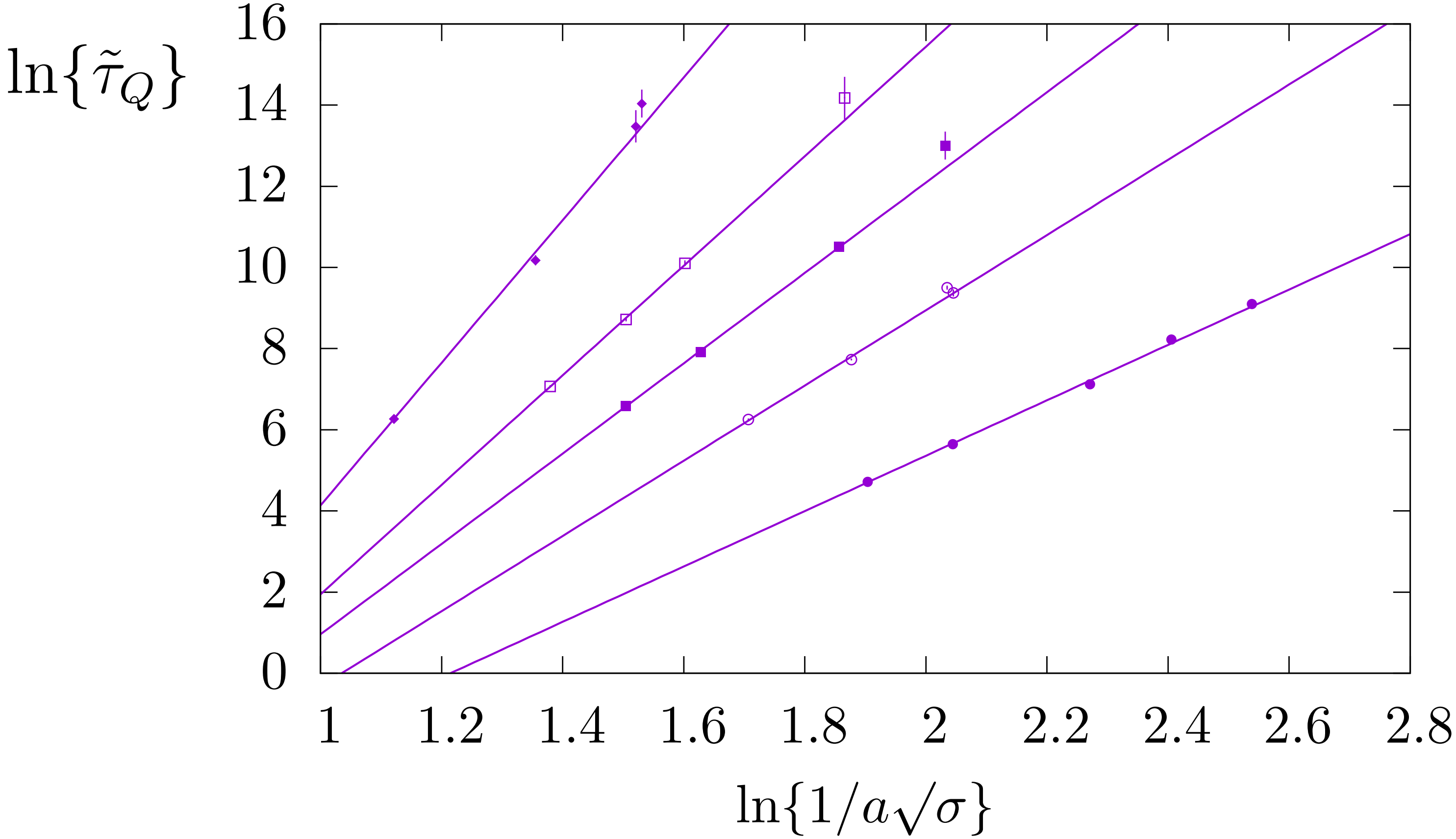
$\tau_Q \twoheadrightarrow \infty$ for $a \to 0$ at fixed N *or* for $N \to \infty$ at fixed a

τ_Q vs N with fits $\tau_Q = b\exp\{cN\}$:



$a\sqrt{\sigma} \sim 0.15$ ($\bullet$) and $a\sqrt{\sigma} \sim 0.33$ ($\circ$).

τ_Q vs a with fits $\tau_Q = b\{1/a\sqrt{\sigma}\}^c$:



$SU(3)$ (●), $SU(4)$ (○), $SU(5)$ (■), $SU(6)$ (□), $SU(8)$ (♦) on volume $= (3/\sqrt{\sigma})^4$.

does the freezing matter here?

- not for large N: $\frac{\langle C(t) Q^2 \rangle_c}{\langle C(t) \rangle \langle Q^2 \rangle} \sim 1 + O(1/N^2)$ (Witten's interlaced θ-vacua)

- for $N \leq 5$ and most $N = 6$ no freezing issue in our calculations

- for $N \geq 8$ freezing, but explicit check $\Rightarrow$ no visible effect

- improvement: multiple parallel sequences starting with different Q with a 'reasonable' distribution

BUT: cannot calculate Q-dependent properties, e.g. susceptibility, for $N \geq 8$ (or even 6) dealing better with freezing:

- very large (physical) volumes - computing time!

- open (non-periodic) boundary - only partial success

- introduce a suitable defect (M. Hasenbusch 1706.04443)
- see C. Bonnano et al 2205.06190

Of course changes in Q are a lattice artifact, albeit a useful one!

Calculating Q on the lattice

$Q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\{F_{\mu\nu}(x)F_{\rho\sigma}(x)\}$

$\overset{lattice}{\Longrightarrow}$

$Q_L(x) \equiv \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\{U_{\mu\nu}(x)U_{\rho\sigma}(x)\} = a^4 Q(x) + O(a^6)$

on smooth fields. On rough fields (symmetrise above) $Q_L(x)$

$Q_L(x) = a^4 Z(\beta)Q(x) + \eta(\beta) + O(a^6)$

so remove pert fluctuations by smoothening fields locally so that Q does not change - replace heat bath by action minimisation ('cooling'

$Q_L(x) \xrightarrow{cool} Q(x)$

when $\rho_I/a \to \infty$; small corrections otherwise

Calculating Q on the lattice – some other algorithms

•zero modes of Dirac operator
same as cooling up to $\rho \sim a$ lattice artifacts: e.g. Cundy et al: hep-lat/0203030

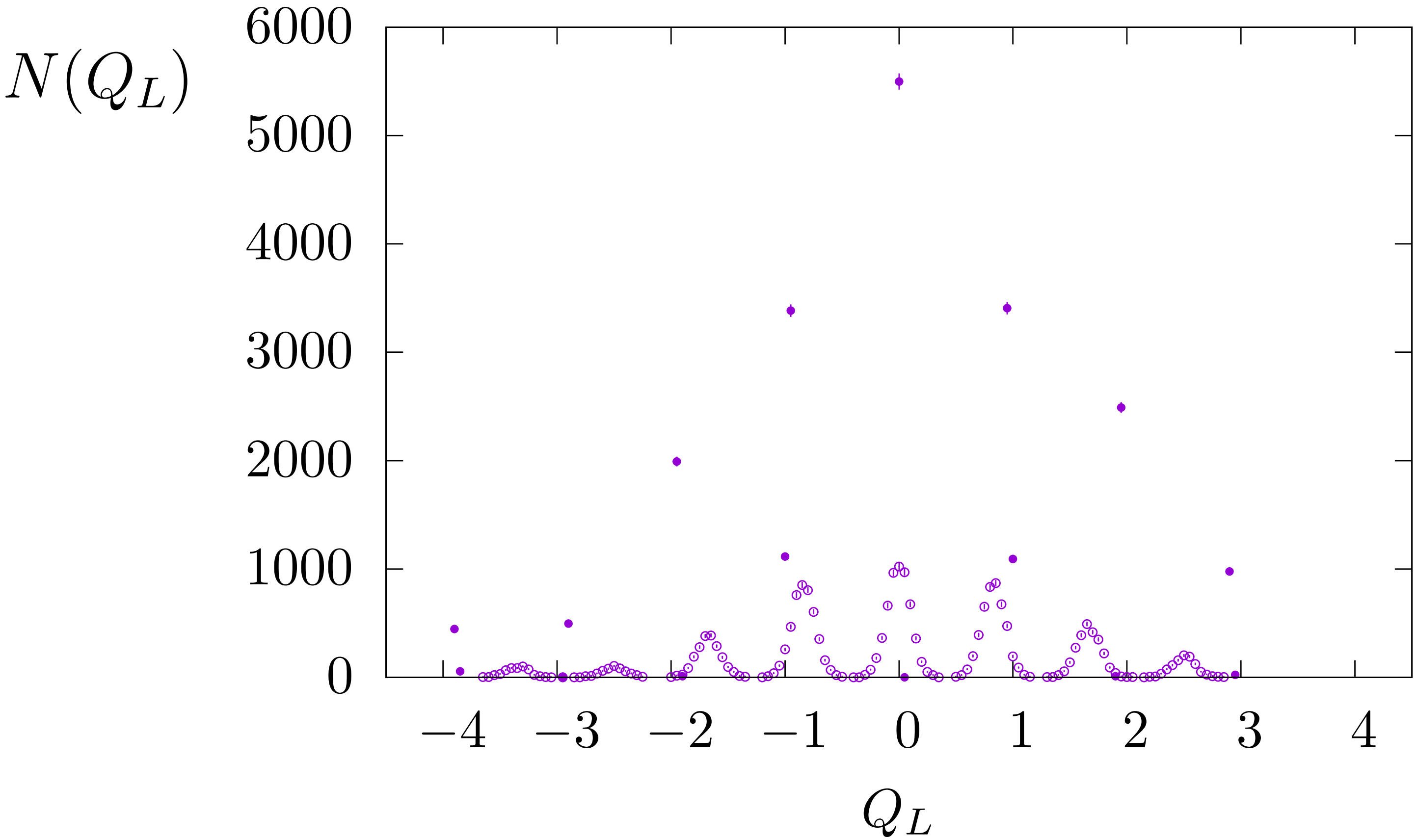
•Wilson flow
'continuous cooling', result same as cooling: e.g. Alexandrou et al: 1708.00696

•variations on cooling
e.g. any saddle point of action: Garcia Perez,van Baal hep-lat/9403026

etc ....
back to cooling for now

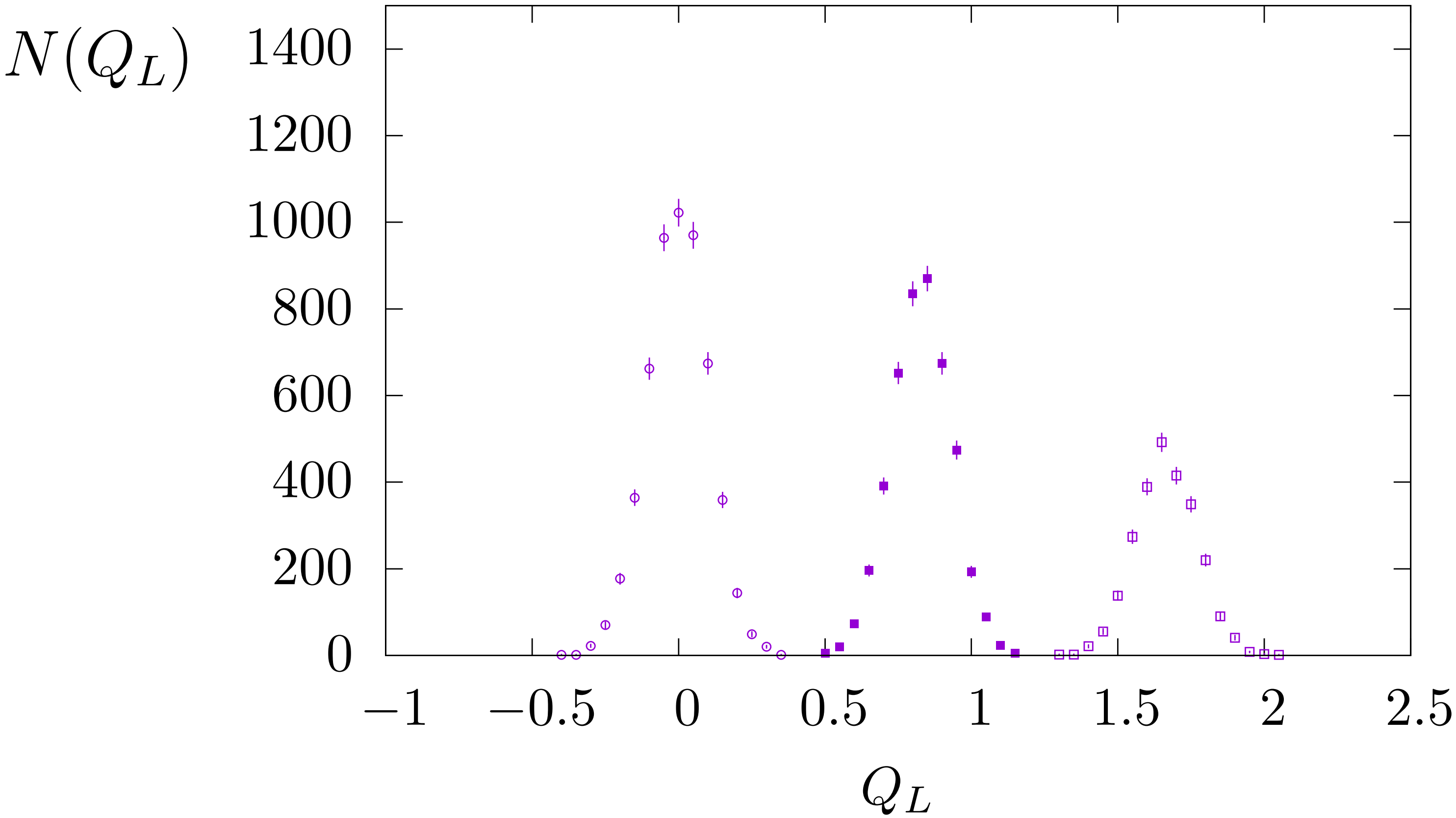
$SU(8)$ lattice fields on a $20^3 30$ lattice with $a\sqrt{\sigma} \simeq 0.133$:

Q_L after 2 ($\circ$) and 20 ($\bullet$) cooling sweeps.

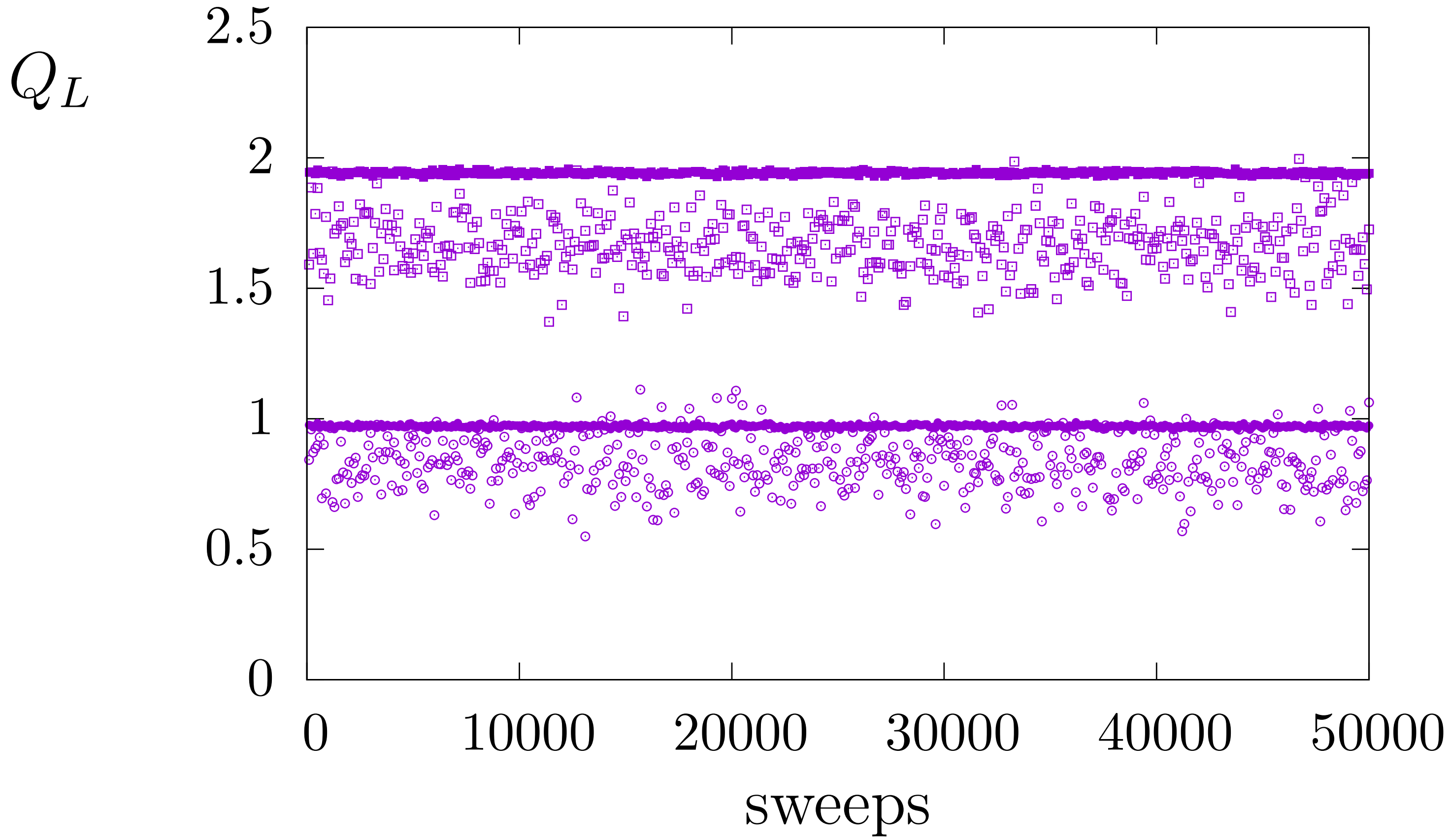


$SU(8)$ lattice fields on a $20^3 30$ lattice with $a\sqrt{\sigma} \simeq 0.133$:

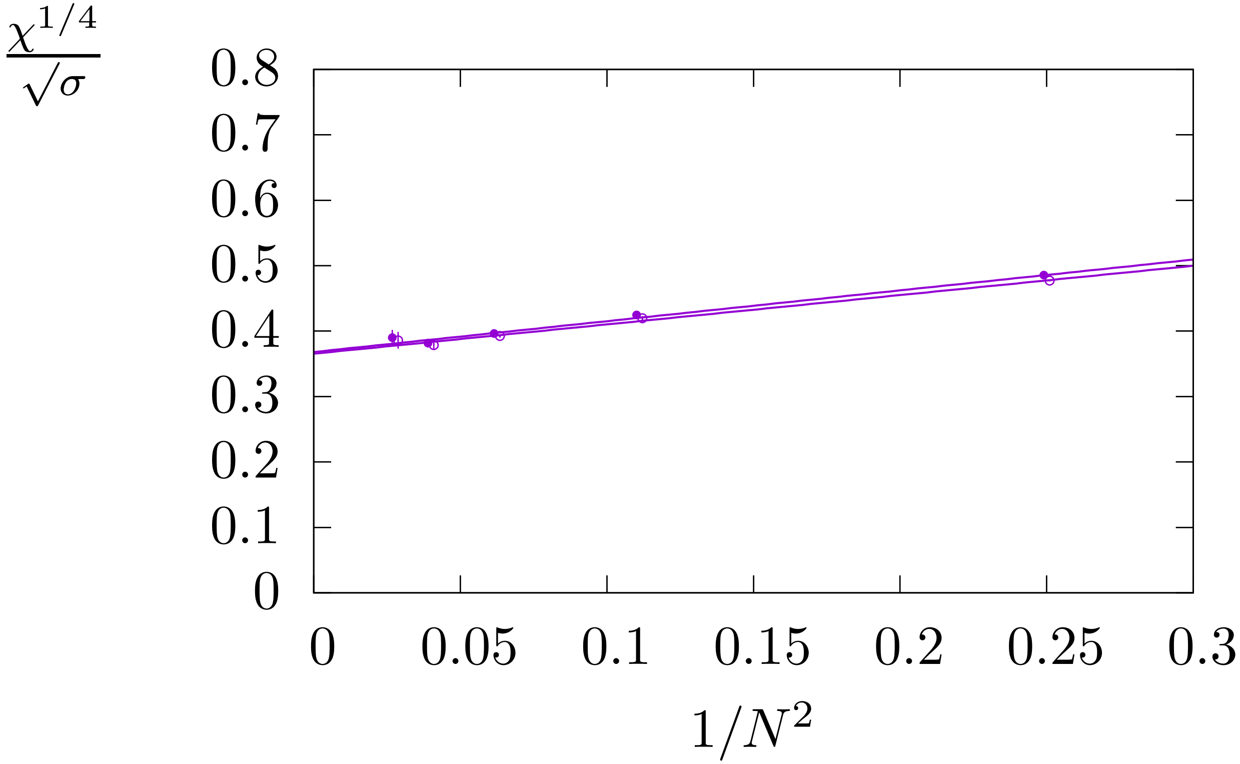
Q_L after 2 cooling sweeps for fields with $Q_L = 0, 1, 2$ ($\circ, \blacksquare, \square$) after 20 cooling sweeps.



two sequences of $SU(8)$ lattice fields on a $20^3 30$ lattice with $a\sqrt{\sigma} \simeq 0.133$: Q_L after 2 ($\circ, \square$) and 20 ($\bullet, \blacksquare$) cooling sweeps.



topological susceptibility: $\frac{\chi^{1/4}}{\sqrt{\sigma}} = 0.368(3) + \frac{0.47(2)}{N^2}$



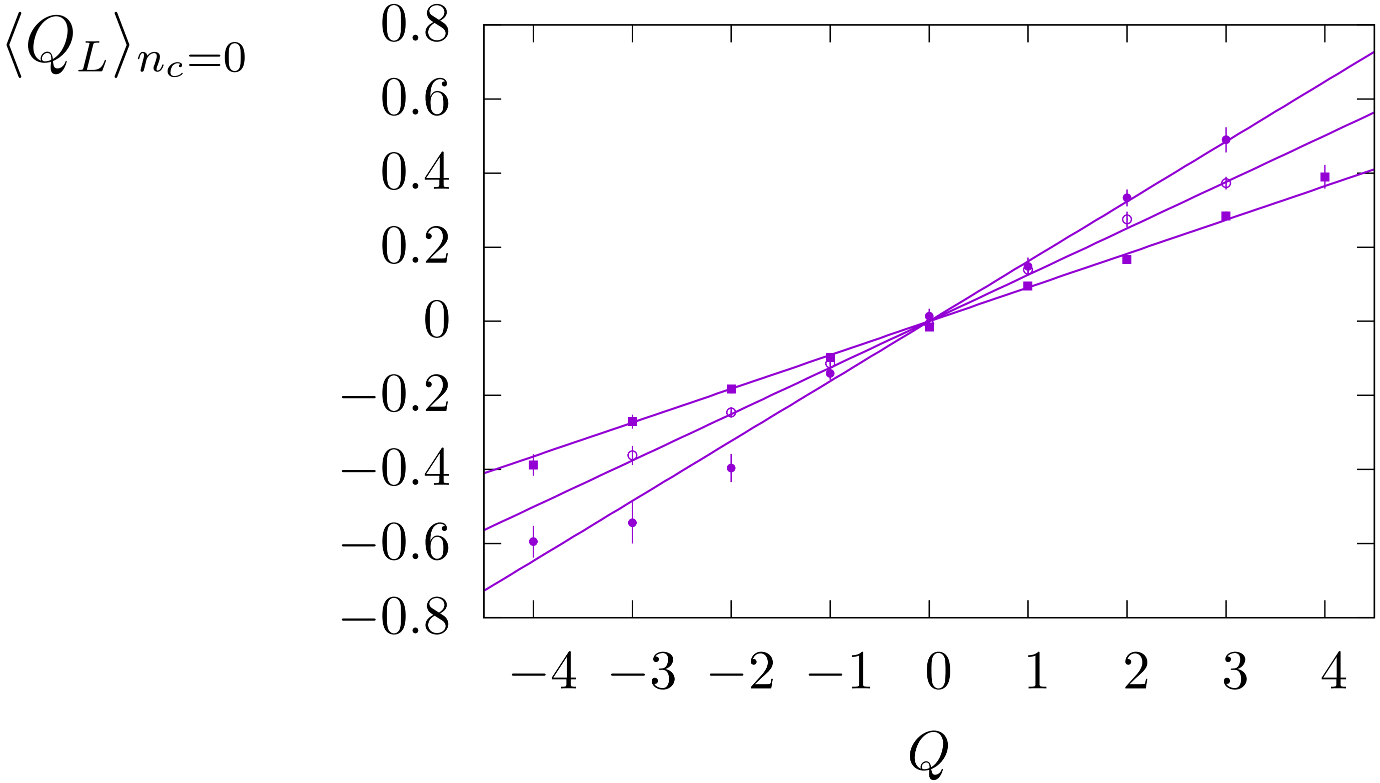
also $SU(3)$:

$\frac{\chi^{1/4}}{\sqrt{\sigma}} = 0.4246(36)|_{su3} \longrightarrow \chi^{1/4} = 206(4)\text{MeV}$

using $r_0\sqrt{\sigma} = 1.160(6)$ and $r_0 = 0.472(5)\text{fm}$ (Sommer: 1401.3270)

$Z_Q(\beta)$ for $SU(8)$ lattice fields at $\beta = 2N/g^2 = 44.10,\ 45.50,\ 46.70$

corresponding to $a\surd\sigma \sim 0.326(\blacksquare), 0.219(\circ), 0.166(\bullet)$



Note that $Z_Q(\beta) \sim 0.1 - 0.2$ in this range of β

•We have many mutually consistent methods for calculating the total topological charge Q of a lattice field

•But calculating the charge density $Q(x)$ is more tricky: alterred by any smoothing

$\Longrightarrow$Some extra methods ... MT 2202.02528 and Phys.Lett. B232 (1989) 227
'repetition', blocking, smearing

Problem: given a lattice field $\{U_l\}_0$, how to calculate its physical density $Q(x)$?

'Repetition' - relatively simple and unambiguous if (computationally) expensive

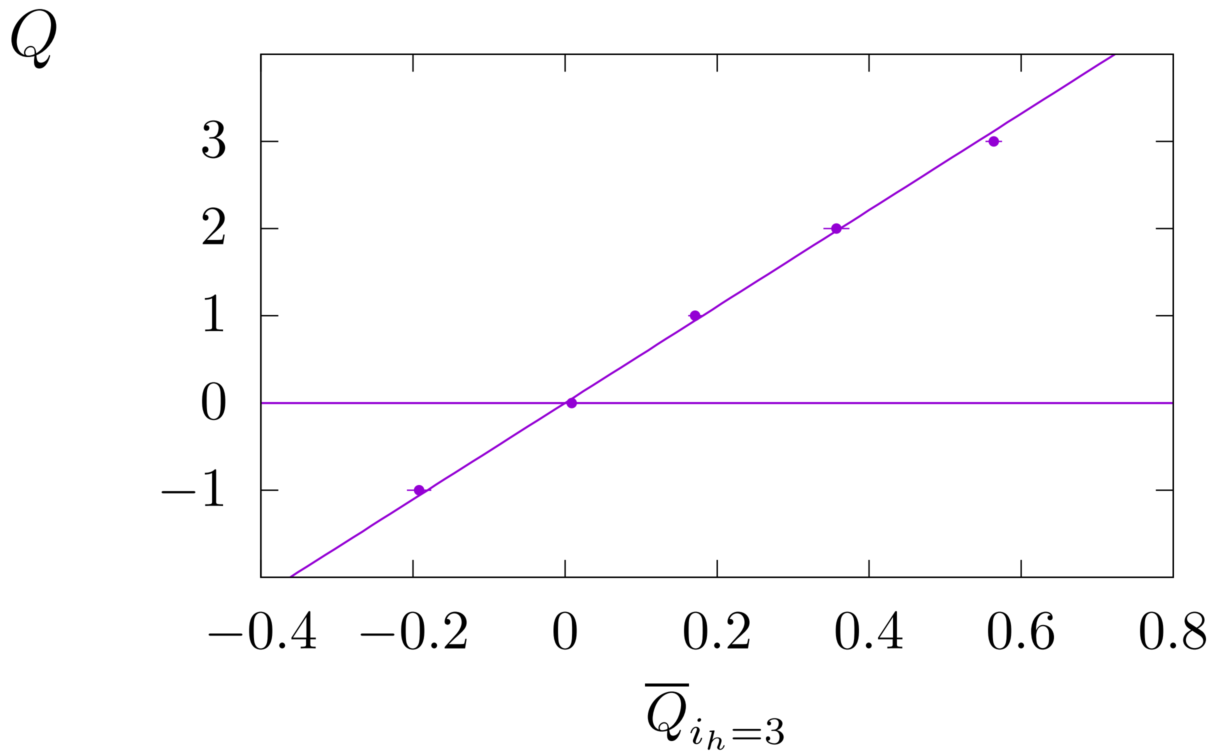
$\{U_l\}_0 \to \{U_l\}_{i_h}$ with i_h heat bath sweeps at same β

repeat with different random numbers $\to$ generate an ensemble of n_r such fields $\{U_l\}^j_{i_h}; j = 1, ..., n_r$ each just i_h heat bath sweeps from $\{U_l\}_0$

calculate the average density:
$\overline{Q}_{i_h}(x) = \frac{1}{n_r} \sum_{j=1}^{n_r} Q^j_{i_h}(x)$

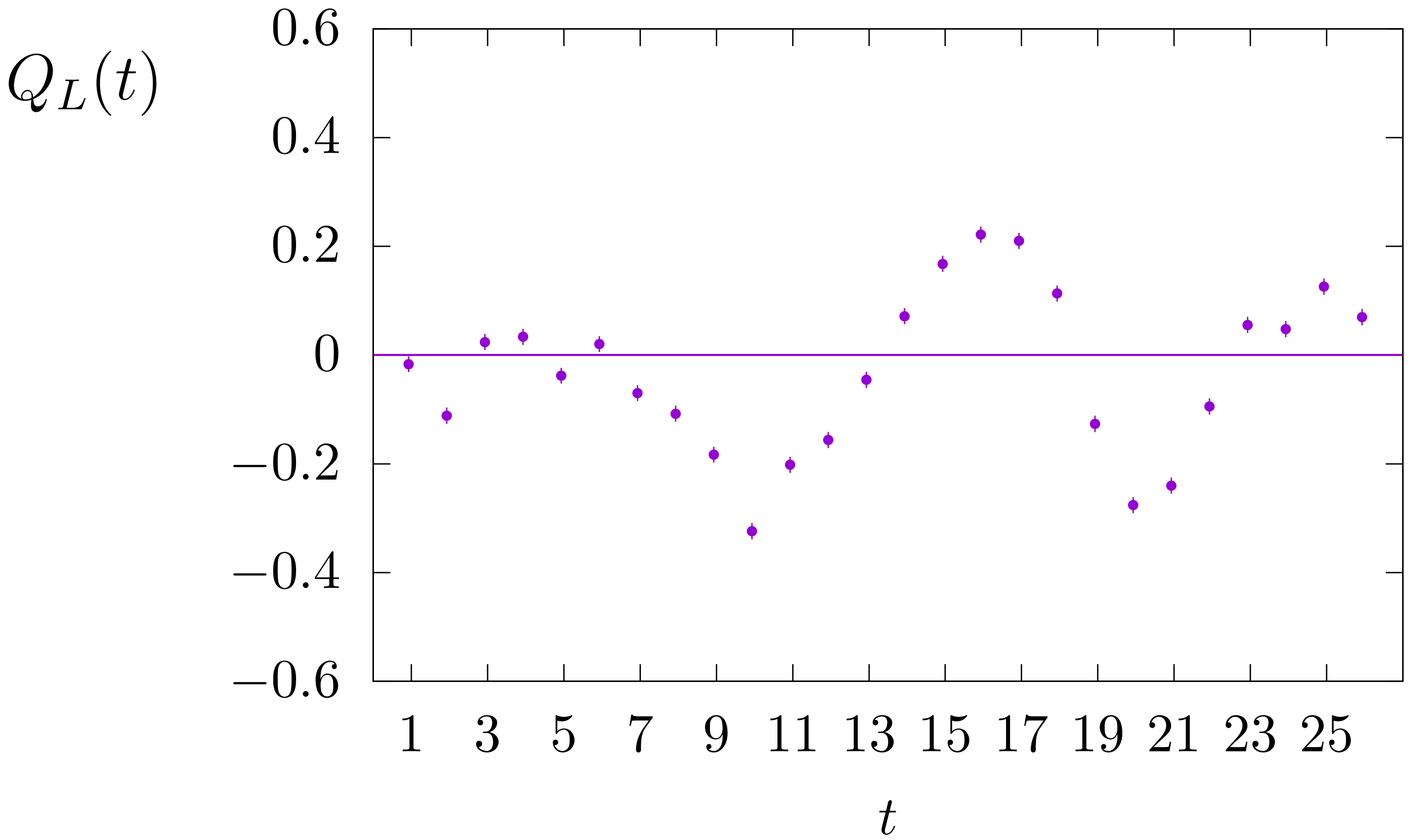
for i_h very small, e.g. $i_h = 3$, this will average the most UV fluctuations but not those on physical length scales, for example:

Q averaged over 10^4 repetitions of 3 heat bath sweeps starting from five separate starting fields with $Q = -1, 0, 1, 2, 3$, generated at $\beta = 6.235$ on a $18^3 26$ lattice:

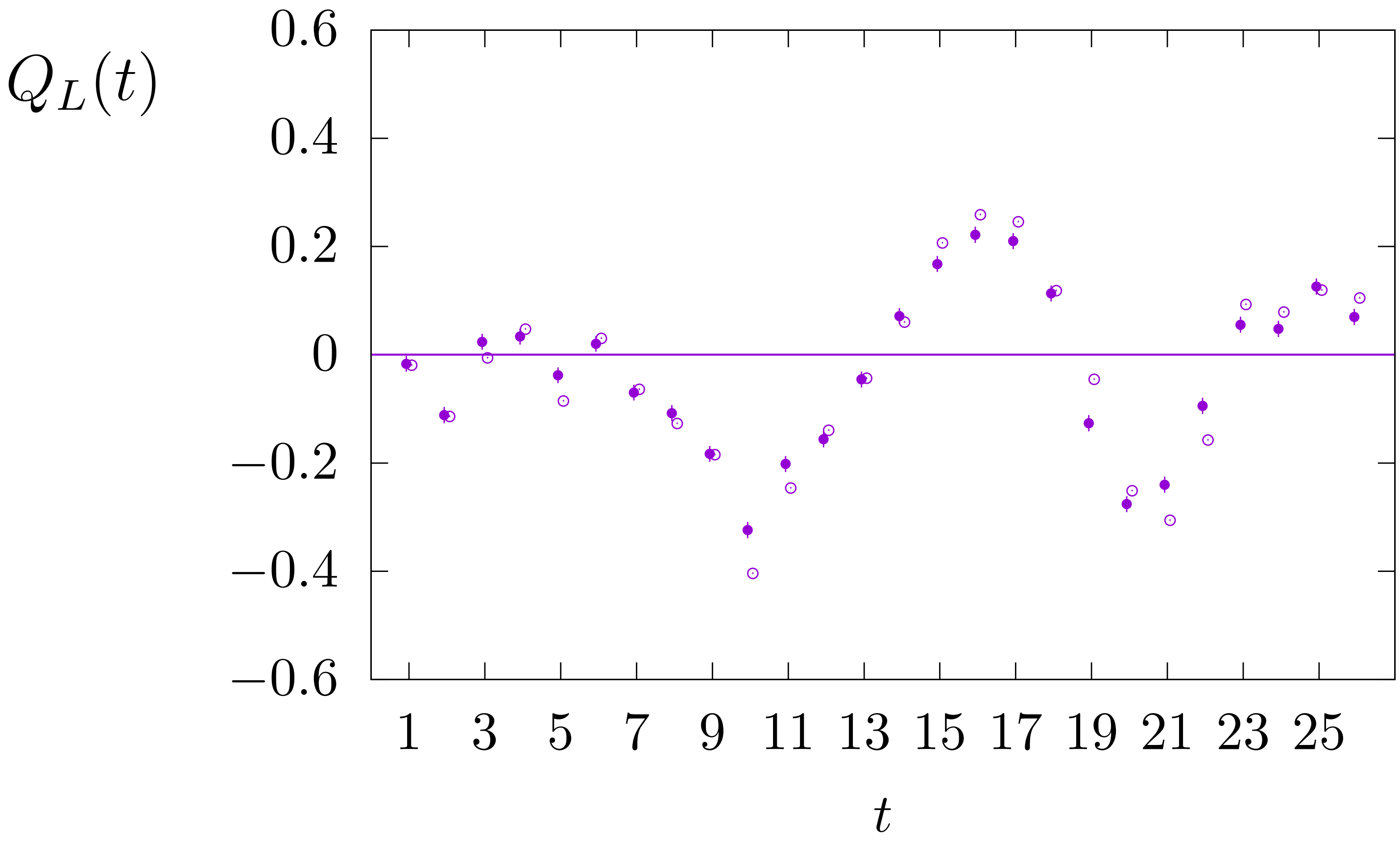


Diagonal line is $\overline{Q}_L = Z(\beta)Q$ with correct $Z(\beta = 6.235) = 0.1808$

Profile of $\overline{Q}_L$ ($\bullet$) from 10^4 fields each 3 heat bath sweeps from a single $Q = -1$ $SU(3)$ lattice field generated at $\beta = 6.235$.(Normalised)



Profile in t of $\overline{Q}_L$ ($\bullet$) from 10000 fields each 3 heat bath sweeps from a single $Q = -1$ $SU(3)$ lattice field generated at $\beta = 6.235$, compared to profile of original $Q = -1$ field after 2 cooling sweeps ($\circ$).(Normalised to same Q.)



spectrum of confining flux tubes in $SU(N)$ gauge theories in $D = 3 + 1$:

•energy spectrum of a flux tube winding around a spatial torus, length l

•relevant quantum numbers: spin J around axis, parity $P_\perp$ perpendicular to axis, parity $P_{||}$ along axis, momentum $p = 2\pi q/l$ along axis

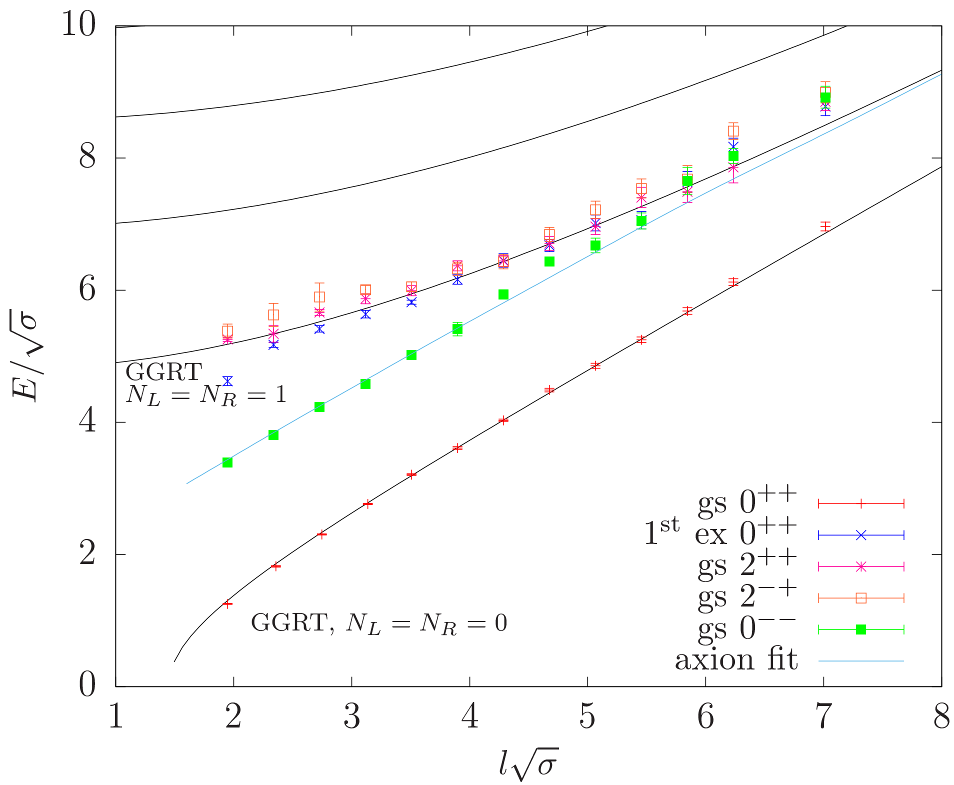
•quantised fluctuations $\rightarrow$ massless 'phonons' on string

•what is effective action? anything other than phonons?

•e.g. GGRT = Nambu-Goto spectrum for $D = 26$ bosonic strings:

$$E_{N_L,N_R}(q,l) = \sigma l \sqrt{1 + \frac{8\pi}{(l\sqrt{\sigma})^2}\left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{(l\sqrt{\sigma})^2}\right)^2}$$

flux tube in $SU(3)$ $\beta = 6.0625$: lightest 2 energy levels (1+4 states) :



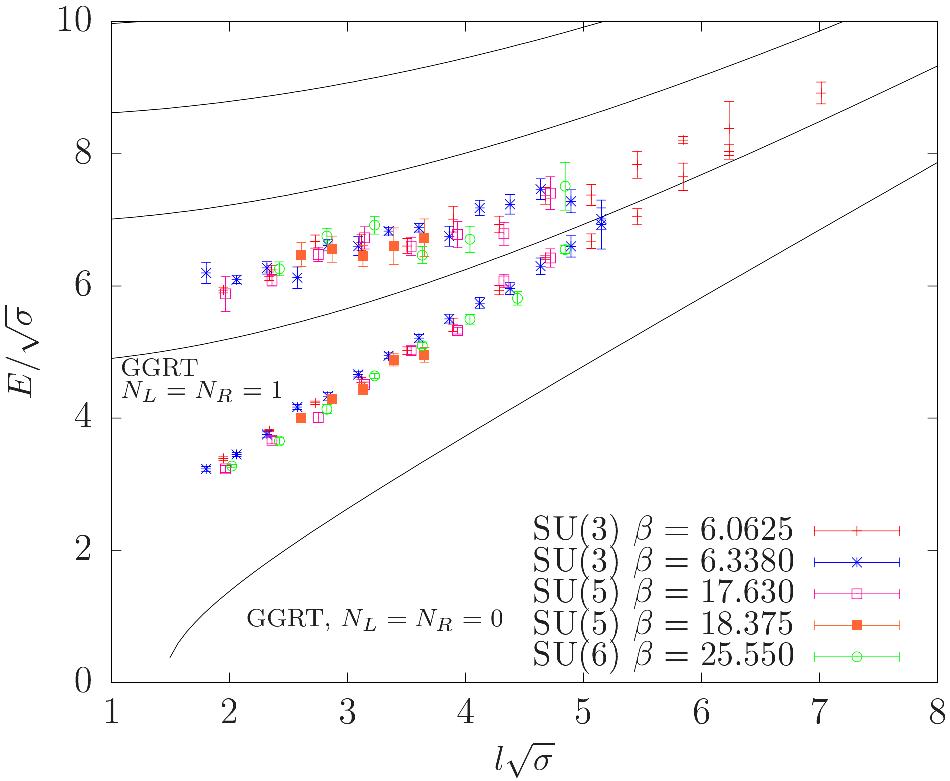
lines are GGRT energy levels; note 'deconfinement' scale at $l_c\sqrt{\sigma} \simeq 1.56$

• why is Nambu-Goto so good at large $l\surd\sigma$?
expand $E_{NG}(l) \Longrightarrow$ terms are universal up to $O(1/l^5)$ for ground state, up to $O(1/l^3)$ for excited states
e.g. Aharony,Komargodski 1302.6257

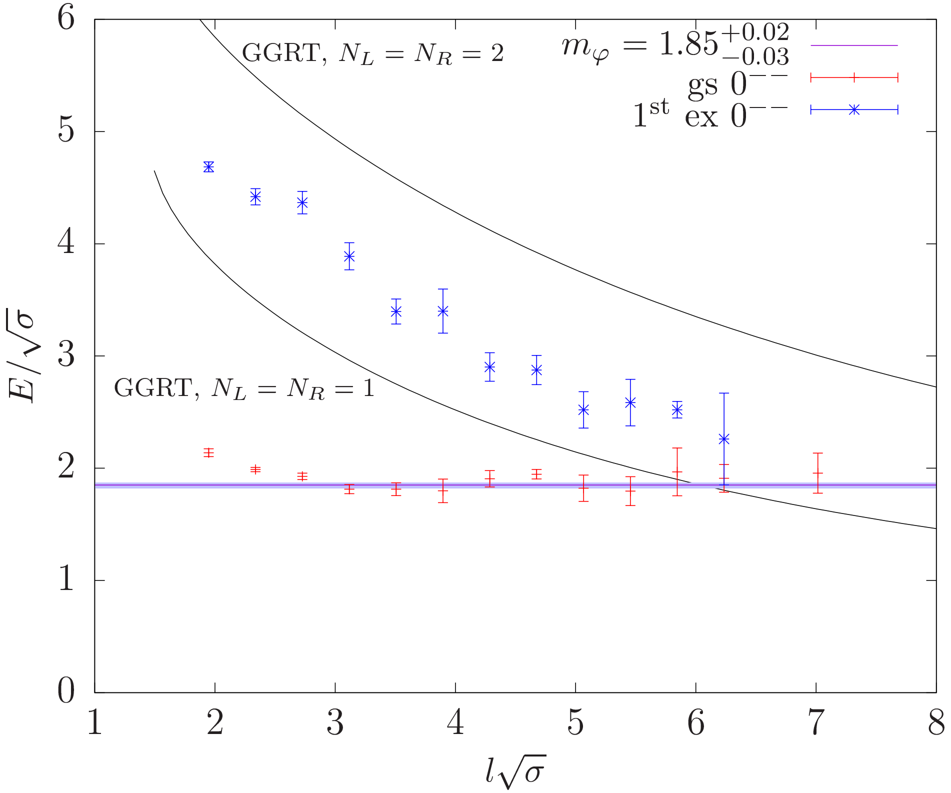
• why is Nambu-Goto so good at small $l\surd\sigma$?
integrable phonon scattering + Bethe ansatz for scattering finite V
'spatial-thermal gas'
Dubovsky, Flauger, Gorbenko et al e.g 1404.0037

• is the lightest 0^{--} an outlier stringy state or a massive excitation on the lightest flux tube?

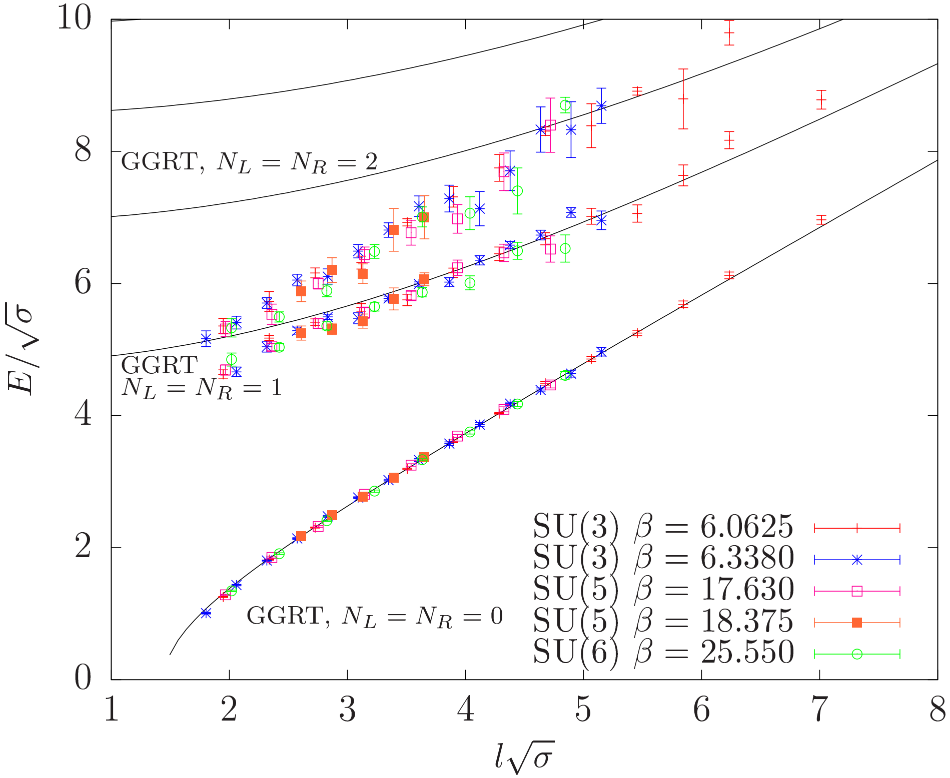
energies of ground and first excited 0^{--} flux tube states



$\Longrightarrow$ lightest 0^{--} is indeed an extra 'axionic' non-phonon state: Dubovsky, Gorbenko 1511.01908 energies of ground and first excited 0^{--} flux tubes minus absolute ground state



ground, first and second excited 0^{++} flux tubes



energy of second excited 0^{++} flux tube minus absolute ground state