Confinement, chiral symmetry breaking, holography and the 3D image of the pion

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Gauge Topology, Flux Tubes And Holographic Models

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FONDAZIONE BRUNO KESSLER

ECT* EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

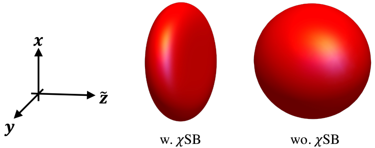
STRONG 2020

Outline

- The two faces of the pion
- Chiral sum rule on the light front
- Holographic QCD
- 3D image of the pion
- Summary

Based on: YL, Maris, Vary, arXiv:2203.14447 [hep-th]

Pion light-front amplitude in 3D

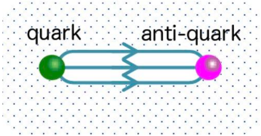


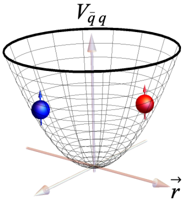
Janus, the mythological two-faced Roman god, is the god of beginnings, gates, transitions, time, duality, doorways, passages, frames, and endings.

Introduction

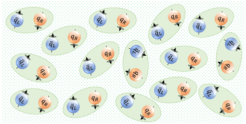
Two distinguished non-perturbative properties of QCD:

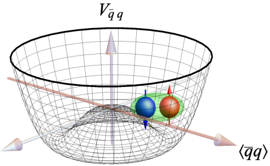
Confinement





Chiral symmetry breaking





Implication to hadron physics:

- Quark confinement
- Nambu-Goldstone boson
  - Gell-Mann-Oakes-Renner relation
  - Low-energy theorems
- Pion, the Janus meson, holds the key

What is the structure of the pion to accommodate these two seemingly opposing faces?

Parton structure of the pion

Light-front formalism is mandatory to describe the internal parton distributions.

zitterbewegung

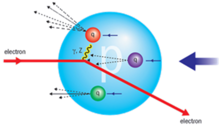
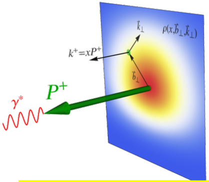


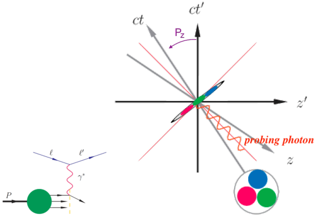
Image credit: CERN Courier

Experiment: DIS

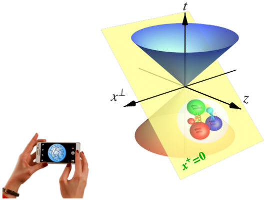


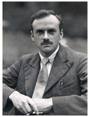
Theory: light-front distributions

infinite momentum frame ($P_z \to \infty$)



light front quantization ($x^+ = 0$)





$$x^{\pm} = x^0 \pm x^3$$

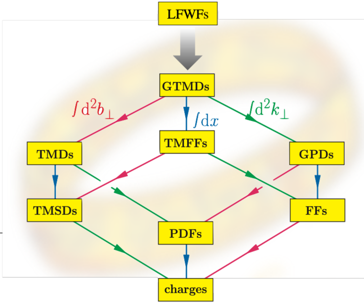
Light front wave functions

[Brodsky, Phys. Rep. '98]

$$|P(p)\rangle = \sum_{s,\bar{s}} \int \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \psi_{s\bar{s}/P}(x,\vec{k}_\perp) \frac{1}{\sqrt{N_C}} \sum_i b^\dagger_{si}(p_1) d^\dagger_{\bar{s}i}(p_2)|0\rangle + \cdots$$

where $x = p_1^+/p^+$, and $\vec{k}_\perp = \vec{p}_{1\perp} - x\vec{p}_\perp$.

- LFWFs are relativistic (Minkowskian) & frame independent, $r_h \sim M_h^{-1}$
- Direct access to hadron structures, e.g. parton distributions
  - *local matrix elements (e.g. form factors):* $\langle h'|O(x)|h\rangle$
  - *light-like correlators (e.g. GPDs, LCDAs):* $\langle h'|O(x)O(y)|h\rangle_{x^+=y^+}$
  - *scattering amplitudes, time-dependent problems, intrinsic densities, spectral densities, entropy & entanglement, ...*

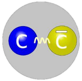


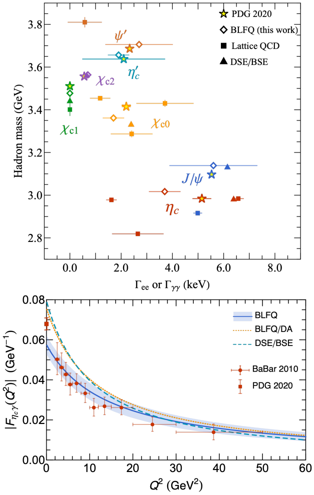
Access to the light front amplitudes:

- OPE + moments reconstruction [Chang '13]
- Nakanishi representation & light-front projection of BSA [Nakanishi '63; de Pauli '22]
- un-Wick rotation [Maris '20; Eichmann '21]
- IMF and large momentum effective theory [Ji '13]
- Schrödinger-Einstein equation [Hornbostel '90; Vary, '10]

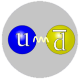
The state of the art examples from basis light-front quantization

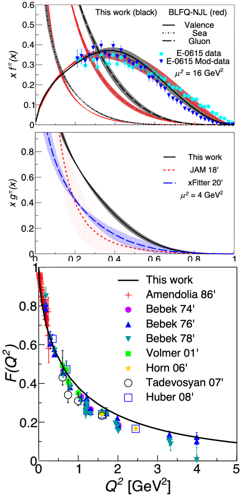
[Li, PRD '22]



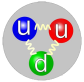


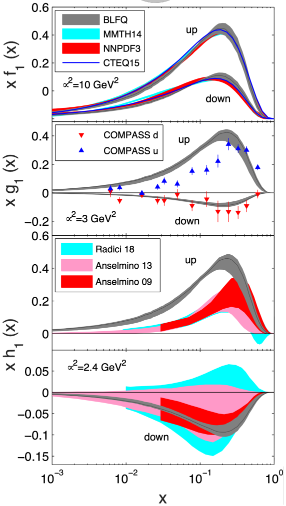
[Lan, PRL '19]





[Mondal, PRD '20]





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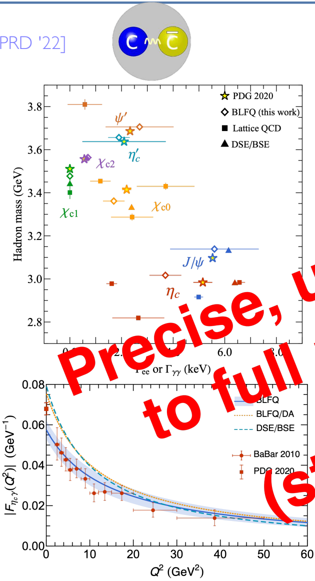

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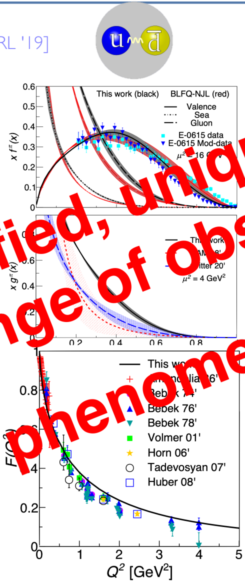
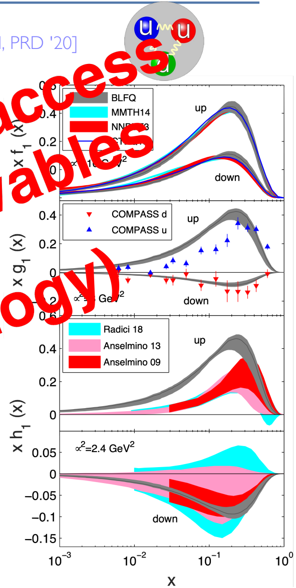


[Mondal, PRD '20]



Precise, unified, unique access to full range of observables (still phenomenology)

Chiral symmetry breaking on the light front

Nature Review | Physics (2022)

Artificial dynamical effects in quantum field theory

Stanley J. Brodsky, Alexandre Deur and Craig D. Roberts

Abstract | In Newtonian mechanics, studying a system in a non-Galilean reference frame can lead to inertial pseudoforces appearing, such as the centrifugal force that seems to arise in dynamics analysed in a rotating frame. Likewise, artificial effects may arise in relativistic quantum field theory (QFT) if a system is studied in a framework that violates Poincaré invariance. In this Perspective, we highlight how

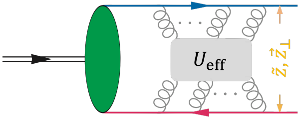
Some myths around the chiral symmetry breaking on the light front:

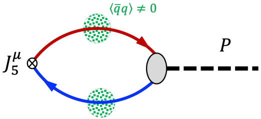
- Light front vacuum is trivial -- no χ condensate/NGB
  Answer: LF vacuum is not trivial -- there is chiral condensate on the LF [Wu, JHEP '04; Beane, AP '13]
- Light front spinors are chiral spinors -- no χSB
  Answer: only true in free theory [Burkardt PRD '97]
- χSB is a collective pheno./property of vacuum/zero modes -- no effect on hadrons (FB phys.)
  Answer: $Q_5|0\rangle_{LF} = 0$ -- *in-vacuum condensate* $\to$ *in-hadron condensate* [Maris '97; Brodsky '13; Casher '74]

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]





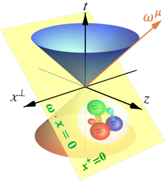
Covariant light-front dynamics

The *most general covariant structure* of the pion valence LFWF: [Carbonell, PR '98]

$$\psi_{s\bar{s}/P}(x,\vec{k}_\perp) = \bar{u}_s(p_1)\Big[\gamma_5\,\phi_1(x,k_\perp) + \hat{f}_\chi \frac{\gamma_5 \not\omega}{\omega\cdot p}\phi_2(x,k_\perp)\Big]v_{\bar{s}}(p_2),$$

where ω is the null vector ($\omega^2=0$) indicating the orientation of the quantization surface, $\not\omega = \gamma^+ \sim \not p$.

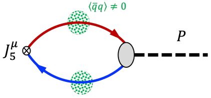
- ω dependent terms are needed to maintain rotational invariance: $L^{\mu\nu}_{\text{int}} = i\omega^{[\mu}\partial/\partial\omega_{\nu]}$
- Conformal symmetry: $\omega^\mu \to \xi\omega^\mu \Rightarrow$ Lorentz structure $\gamma_5\not\omega/\omega\cdot p$
- Need a $\hat{f}_\chi$ is in mass dimension
- In QCD, χSB $\Rightarrow$ $f_\pi \neq 0$ $\Rightarrow$ $\hat{f}_\chi \neq 0$ (previous works chose $\hat{f}_\chi = 0, m_q, M_\pi$, wrong!)



$$\psi_{\uparrow\uparrow/P} = \psi^*_{\downarrow\downarrow/P} = -\frac{k_\perp e^{-i\arg\vec{k}_\perp}}{\sqrt{x(1-x)}}\phi_1; \quad \psi_{\uparrow\downarrow-\downarrow\uparrow/P} = -\hat{f}_\chi\sqrt{8x(1-x)}\phi_2 + O(m_q);$$

Decay constant: $\langle 0|J_5^+(0)|P(p)\rangle = ip^+ f_P$

$$\frac{f_P}{2\sqrt{2N_C}} = \int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\psi_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp).$$



$J_5^\mu = i\partial^\mu\phi_\pi + J^\mu_{5,N}$

Chiral sum rule

Partially conserved axial-vector current (PCAC):

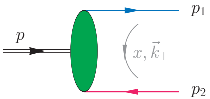
$$\partial_\mu J_5^\mu = 2im_q\bar{q}\gamma_5 q \quad \Rightarrow \quad \langle 0|\partial_\mu J_5^\mu - 2im_q\bar{q}\gamma_5 q|P(p)\rangle = 0.$$

where, $J_5^\mu(x) = \bar{q}(x)\gamma^\mu\gamma_5 q(x)$ is the axial-vector current; $q(x)$ is the quark field operator.

$$q(x) = \sum_s \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2p^+}\Big\{b_s(p)u_s(p)e^{ip\cdot x} + d_s^\dagger(p)v_s(p)e^{-ip\cdot x}\Big\}\Big|_{x^+=0},$$

$$\overset{m_q\to 0}{\Rightarrow} \quad \boxed{\int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2 k_\perp}{(2\pi)^3}\frac{k_\perp^2}{x(1-x)}\psi^{(0)}_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp) = 0}$$

- Gell-Mann-Oakes-Renner relation: $f_P^{(0)2}M_P^2 = 2m_q g_P^{(0)} + O(m_q^2)$, where $g_P = \langle 0|j_5|P(p)\rangle$
- Exact relation -- no Fock sector truncation
- Wave functions contain self-energy -- no assumption of quarks as physical eigenstates
- Uncertainty principle: $\frac{1}{2}\Delta x^+\,\Delta P^- \gtrsim 1$
- Additional sum rules from further light-front current algebra [Beane, '13; Hobbs, '16]



Covariant formalism

Similar to the Dyson-Schwinger equations/Bethe-Salpeter equations [Maris, Roberts, Tandy PLB 1998]

- Axial-vector Ward-Takahashi Identity leads to exact relation between pion BSA and the quark self-energy
- Hints to dynamical mass generation
- Maris-Tandy model (RL) shows similar mechanism with Nambu-Jona-Lasinio model, but with a more delicate matching with the UV
- Euclidean $\to$ Minkowskian: moments, Nakanishi representation, un-Wick rotation, ...

[Chang '13; de Paula '22; Maris, '20]

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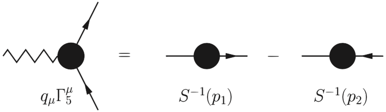
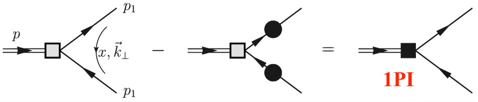
covariant formalism



$$S^{-1}(p) = i\not{p}A(p^2) + B(p^2),$$
$$\Gamma_\pi(k,P) = i\gamma_5 E_\pi(k,P) + \not{P}F_\pi(k,P) + \cdots$$

$$\boxed{f_\pi \Gamma_\pi(k,0) = B(k^2)}$$

light-front Hamiltonian formalism



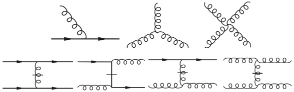
$$\Gamma_\pi(p_1,p_2,p) = \widetilde{\Gamma}_\pi(p_1,p_2,p) - \Sigma(p_1 - \omega\tau_1)\frac{\not{p}_1 + m}{x(s - M_\pi^2)}\widetilde{\Gamma}_\pi(p_1,p_2,p) - \widetilde{\Gamma}_\pi(p_1,p_2,p)\frac{\not{p}_2 - m}{(1-x)(s-M_\pi^2)}\Sigma(p_2 - \omega\tau_2)$$

First approximation to QCD

[Review: Brodsky, Phys. Rep. '98]

Light-front QCD in light cone gauge $A^+ = 0$

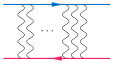
$$H_{\text{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^{\,2} + m_i^2}{x_i} + U + \sum_{ij} V_{ij}^{(\text{QCD})} - \delta_{ij} U_i$$

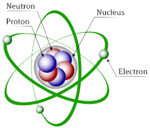
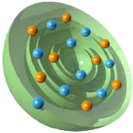
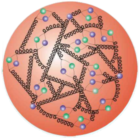


$$\Downarrow$$

Light-front Schrödinger wave equation (LFSWE)

$$\Big(\frac{\vec{k}_\perp^{\,2} + m_q^2}{x} + \frac{\vec{k}_\perp^{\,2} + m_{\bar{q}}^2}{1-x} + U\Big)\psi(x, \vec{k}_\perp) = M^2 \psi(x, \vec{k}_\perp)$$



Atoms	Nuclei	Hadrons
		
Coulomb interaction	NN, NNN interactions	QCD interactions
↓	↓	↓
Bohr Model 	Shell Model 	?
$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$	$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$	

Separation of variables

In LFD, there is a natural separation of the transverse and longitudinal d.o.f.'s:

$$\Big\{ \underbrace{\frac{\vec{k}_\perp^2}{x(1-x)}}_{\text{chiral limit },\perp} + \underbrace{\frac{(1-x)m_q^2 + x m_{\bar{q}}^2}{x(1-x)}}_{\text{mass term },\parallel} + U \Big\} \psi(x, \vec{k}_\perp) = M^2 \psi(x, \vec{k}_\perp)$$

- Separation ansatz: $U = U_\perp(\zeta_\perp) + U_\parallel(\tilde{z}) \Rightarrow M^2 = M_\perp^2 + M_\parallel^2$, $\psi(x, \vec{\zeta}_\perp) = \varphi(\vec{\zeta}_\perp)\chi(x)$

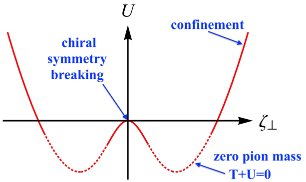
  Here, $\vec{\zeta}_\perp = \sqrt{x(1-x)}\vec{r}_\perp$, $\tilde{z} = \frac{1}{2}P^+ x^- = i\partial/\partial x|_{\vec{\zeta}_\perp}$. [Chabysheva, AP 2012; Miller & Brodsky, PRC 2020]

  $$\Big[-\nabla_\zeta^2 + U_\perp(\vec{\zeta}_\perp)\Big]\varphi(\vec{\zeta}_\perp) = M_\perp^2 \varphi(\vec{\zeta}_\perp), \quad \Big[\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel(\tilde{z})\Big]\chi(x) = M_\parallel^2 \chi(x)$$

  Brodsky et al. took $\chi_\pi(x) = 1 \Rightarrow \phi_\pi(x) = (8 f_\pi/\pi)\sqrt{x(1-x)}$. ['t Hooft, NPB '76]

- Chiral sum rule becomes:

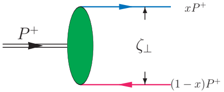
  $$f_P \nabla_{\zeta_\perp}^2 \varphi_P(\vec{\zeta}_\perp = 0) = 0 \quad \Rightarrow \quad U_\perp(\vec{\zeta}_\perp = 0) = 0$$



Light-front holography

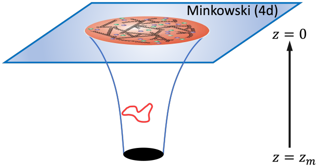
[Review: Brodsky, Phys. Rep. '15]

LFH is a unique mapping between LFQCD_{3+1} and string motion in AdS/QCD



- Based on LFQCD
- $\zeta_\perp = \sqrt{x(1-x)}r_\perp$,
- Conjugate to off-shell energy μ

LFH		soft-wall AdS/QCD
$\zeta_\perp$	$\leftrightarrow$	z,
ϕ_J	$\leftrightarrow$	$\widetilde{\phi}_{Jm}$
U_J	$\leftrightarrow$	$\frac{1}{4}\Phi'^2 + \frac{1}{2}\Phi'' + \frac{2J-3}{2z}\Phi'$
$(\mu R)^2$	$\leftrightarrow$	$m^2 - (J-2)^2$

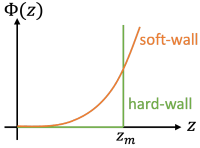


- Based on gravity/gauge duality
- $z \to 0$: CFT
- $z \to \infty$: low-energy QCD pheno.
- $z^{-1} \sim \mu_R$, RG scale

$$\left[-\frac{1}{\zeta_\perp}\frac{\mathrm{d}}{\mathrm{d}\zeta_\perp}\left(\zeta_\perp\frac{\mathrm{d}}{\mathrm{d}\zeta_\perp}\right) + \frac{m^2}{\zeta_\perp^2} + U_J(\zeta_\perp)\right]\widetilde{\varphi}_{Jm}(\zeta_\perp) = M^2\widetilde{\varphi}_{Jm}(\zeta_\perp)$$

$$\left[-\frac{z^{3-2J}}{e^{\Phi(z)}}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{e^{\Phi(z)}}{z^{3-2J}}\frac{\mathrm{d}}{\mathrm{d}z}\right) + \frac{\mu^2R^2}{z^2}\right]\varphi_J(z) = M^2\varphi_J(z)$$

where $\varphi_J(z) = \left(\frac{R}{z}\right)^{J-\frac{3}{2}} e^{-\frac{1}{2}\Phi(z)}\phi_J(z)$, $\widetilde{\varphi} = \widetilde{\phi}/\sqrt{\zeta_\perp}$, and $(\mu_{\text{eff}}R)^2 = \left(\mu(z)R\right)^2 - Jz\Phi'(z)$
$+J(5-J)$.



Light-front holography: form factor

[Review: Brodsky, Phys. Rep. '15]

- On the light front, the form factor is obtained from the Drell-Yan-West formula

$$F_\pi(Q^2) = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\zeta_\perp \rho_\pi(\vec{\zeta}_\perp) e^{i\sqrt{\frac{1-x}{x}}\vec{\zeta}_\perp\cdot\vec{q}_\perp} + \cdots$$
$$= \int \zeta_\perp \mathrm{d}\zeta_\perp \rho_\pi(\zeta_\perp) \zeta_\perp Q K_1(\zeta_\perp Q) + \cdots$$

where $\rho_\pi(x, \vec{\zeta}_\perp) = N^2 \big|\widetilde{\varphi}_\pi(\zeta_\perp)\big|^2$.

- In AdS/QCD, the form factor is obtained from the 5D current $A_\mu(x^\mu, z) = e^{iq\cdot x} V(q^2, z)\epsilon_\mu(q)$

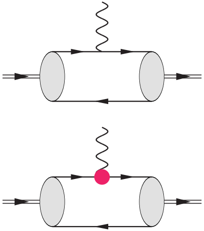
[Hong, '06; Grigoryan, '07ab&'08; Kwee '08]

$$F(Q^2) = \int \frac{\mathrm{d}z}{z^3} V(Q^2, z)\phi^2(z)$$

In hard-wall model,

$$V_{\mathrm{HW}}(Q^2, z) = zQK_1(zQ) + zQI_1(zQ)\frac{K_0(Qz_m)}{I_0(Qz_m)}.$$

It consists of a point-like part, and a second part due to the hard-wall confinement.

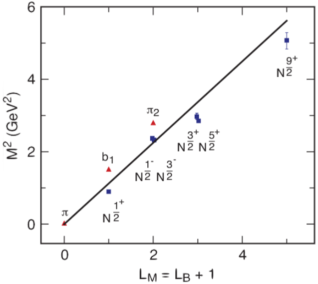


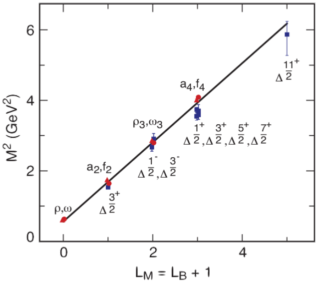
- Pole representation & QCD sum rule [Grigoryan '07; Afonin, '22]

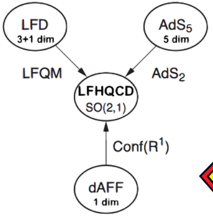
Confinement

[Review: Brodsky, Phys. Rep. '15]

- Confinement entails a dilaton field $\Phi(z)$ breaking the conformal symmetry at large z
  - $\Phi(z) \nearrow$ *with* $z \to \infty$, *and* $\Phi(z) \to 0$ *at* $z \to 0$
- Soft-wall model: $\Phi(z) \sim z^2$ at $z \to \infty$ agrees with the Regge trajectories in meson spectrum [Karch '06]
  - *Effective light-front confining potential:* $U(z) \sim z^2$ *as* $z \to \infty$
  - *Karch et al adopted* $\Phi(z) = \lambda z^2 \ \Rightarrow U(z) = \lambda^2 z^2 + 2(J-2)\lambda \ \Rightarrow$ *massless pion*
- Brodsky and de Téramond further show this choice is consistent with the superconformal symmetry
  - *Pion appears as a massless susy singlet* [Fubini, '84; de Alfaro, '76; Miyazawa '66&'68]
- What about chiral symmetry breaking?

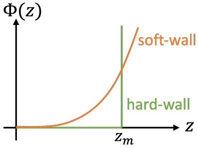






Chiral symmetry breaking in soft-wall AdS/QCD

[Erlich '05; Karch '06; Gherghetta, '09]



PRL **95,** 261602 (2005)

QCD and a Holographic Model of Hadrons

Joshua Erlich,[1] Emanuel Katz,[2] Dam T. Son,[3] and Mikhail A. Stephanov[4]

Hard-wall model: with chiral symmetry breaking, but no Regge trajectory

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PHYSICAL REVIEW D **74,** 015005 (2006)

Linear confinement and AdS/QCD

Andreas Karch,[1,*] Emanuel Katz,[2,†] Dam T. Son,[3,‡] and Mikhail A. Stephanov[4,§]

Soft-wall model: with Regge trajectory, but no chiral symmetry breaking

tion between bulk and boundary theories [12,19]. The action at quadratic order in the fields and derivatives reads

$$I = \int d^5x e^{-\Phi(z)} \sqrt{g}\left\{-|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

asymptotics $e^{z^2} \to \infty$ and $\exp\{-(3/4)z^{-2}\} \to 1$. Since the equation is linear, selecting one of the solutions in the IR (the $X < \infty$ one, of course) gives Σ simply proportional to M. This is not what one wants in a theory with spontaneous symmetry breaking such as QCD. It is clear that one has to consider higher order terms in the potential $U(X, \ldots)$ for X and all other scalar condensates. Such a potential would

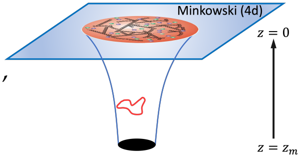
Chiral symmetry breaking in soft-wall AdS/QCD

[Gherghetta, '09; Kapusta, '10]

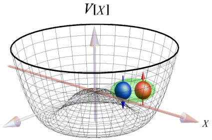
- Scalar field X dual to $\bar{q}q$, with a non-vanishing VEV: $\langle X \rangle = \frac{1}{2}\chi(z)$

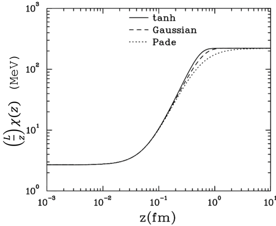
$$S = -\int \mathrm{d}^5 x \sqrt{-g} e^{-\Phi(z)} \mathrm{Tr}\Big\{ \big|DX\big|^2 - V[X] + \frac{1}{4g_5^2} F^2 \Big\},$$

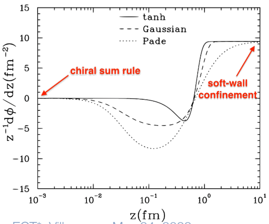
where $V[X] = -m_X^2 |X|^2 + \kappa |X|^4$ is the Higgs potential, $m_X^2 = -3$



- Eq. of motion: $\frac{\mathrm{d}}{\mathrm{d}z}\Big(\frac{e^{-\Phi}}{z^3}\frac{\mathrm{d}}{\mathrm{d}z}\chi\Big) - \frac{e^{-\Phi}}{z^5}\Big(m_X^2 \chi - \frac{\kappa}{2}\chi^3\Big) = 0$
- At the CFT boundary: $\chi(z) \sim \xi m_q z + \xi^{-1}\Sigma z^3$ at $z \to 0$ [Klebanov & Witten, '99]
- $\Phi(z) \sim z^6 \Rightarrow U \sim -z^4$ at $z \to 0$, consistent with the chiral sum rule
- Recall confinement requires $\Phi(z) \sim z^2$ at $z \to \infty$
- The Mexican-hat potential is the relic of the Higgs potential in 5D
- Similar models with χSB:
  [Babingto '04; Casero '07&'10; Jarvinen '12; Li, '13; Sui '10;]
  [Cui '16; Chelabi '16; Braga '19; Capossoli, '20;]
  [Ballon-Bayona '20&'21]







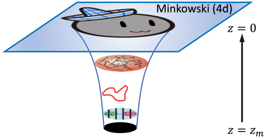
Analytic model

Let's build a chiral pion:

$$\left[-\nabla_\perp^2 + U(\zeta_\perp)\right]\widetilde{\varphi}_P(\zeta_\perp) = M_P^2\widetilde{\varphi}_P(\zeta_\perp), \qquad \nabla_\perp^2\widetilde{\varphi}(\zeta_\perp = 0) = 0, \quad U(\zeta_\perp) \to \begin{cases} \zeta_\perp^2 & \zeta_\perp \to \infty \\ -\zeta_\perp^4 & \zeta_\perp \to 0 \end{cases}$$

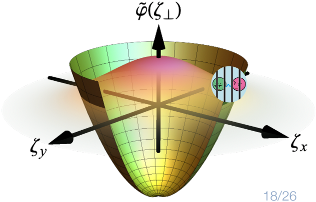
We propose the following pion wave function based on light-front holography:

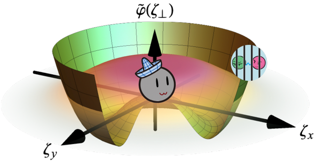
$$\widetilde{\varphi}_\pi(\zeta_\perp) = \underbrace{\left(1 + \frac{1}{2}\zeta_\perp^2 + \frac{1}{8}\zeta_\perp^4\right)}_{\text{chiral symmetry breaking}} \overbrace{e^{-\frac{\zeta_\perp^2}{2}}}^{\text{confinement}} .$$



Given the pion wave function, the potential is,

$$U(\zeta_\perp) = \frac{\widetilde{\varphi}_\pi''(\zeta_\perp) + \zeta_\perp^{-1}\widetilde{\varphi}_\pi'(\zeta_\perp)}{\widetilde{\varphi}_\pi(\zeta_\perp)} = \frac{\zeta_\perp^4(\zeta_\perp^2 - 6)}{\zeta_\perp^4 + 4\zeta_\perp^2 + 8}.$$





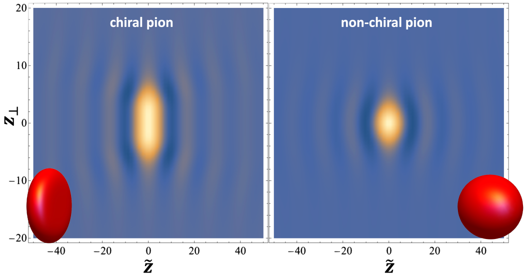
3D image of the pion

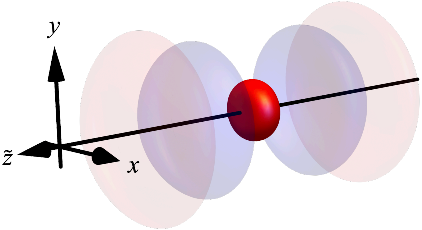
Coordinate-space wave function:

$$\widetilde{\psi}_\pi(\vec{z}_\perp, \tilde{z}) = \int_0^1 \frac{\mathrm{d}x}{2\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} e^{ix\tilde{z} - i\vec{k}_\perp \cdot \vec{z}_\perp} \psi_\pi(x, \vec{k}_\perp),$$

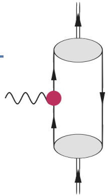
$$= \langle 0 | \bar{q}(-\tfrac{1}{2}z) \frac{\not{\omega}\gamma_5}{\omega \cdot p} q(+\tfrac{1}{2}z) | P(p) \rangle_{z^+=0}$$

where $\tilde{z} = p \cdot z$ is the Ioffe time of Miller and Brodsky. [Miller '20]





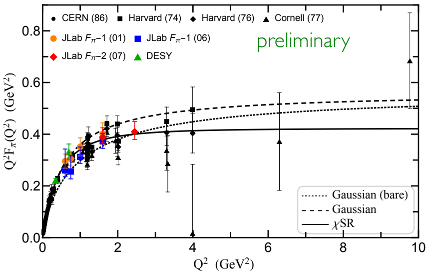
Pion form factors

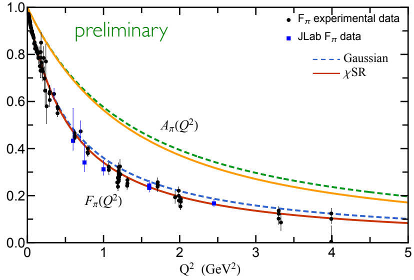


- Electromagnetic form factor: $\langle p+q|J^+(0)|p\rangle = 2p^+F_\pi(-q^2)$,
  gravitational form factor: $\langle p+q|T^{++}(0)|p\rangle = 2p^+p^+A_\pi(-q^2)$ with $q^+=0, Q^2=-q^2=q_\perp^2$

$$F_\pi(Q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \rho_\pi(\zeta_\perp) V(Q^2,\zeta_\perp), \quad A_\pi(Q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \rho_\pi(\zeta_\perp) H(Q^2,\zeta_\perp)$$

- One single parameter, $\kappa = 0.43\,\mathrm{GeV}$, sets the scale [Brodsky, '08]
- Pion electromagnetic radius $r_\pi|_{em}$ = 0.64 fm (PDG value: 0.67 fm), mass radius $r_\pi|_{mass}$ = 0.40 fm (Belle II: 0.32--0.39 fm, extracted from the GDA analysis of the $\gamma^*\gamma^* \to \pi^0\pi^0$ process) [Kumano, '18]
- χSB modify the large-Q^2 behavior: incorporating high-twist contributions automatically



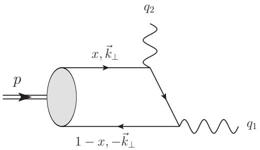


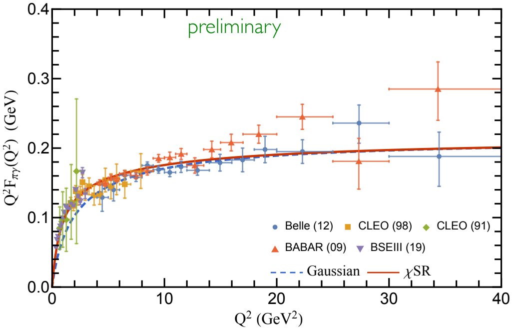
Transition form factor

Singly-virtual two-photon transition form factor:

$$F_{\pi\gamma}(Q^2) = e_f^2 2\sqrt{2N_C} \int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \frac{\psi_\pi(x, \vec{k}_\perp)}{k_\perp^2 + x(1-x)Q^2}$$

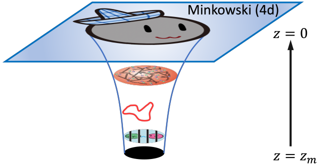
- VMD: $Q^2 \to Q'^2 = Q^2 + M_\rho^2$
- pQCD normalization: $Q^2 F_{\pi\gamma}(Q^2) \to 2f_\pi$, $Q^2 \to \infty$

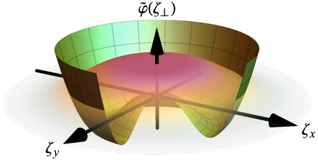


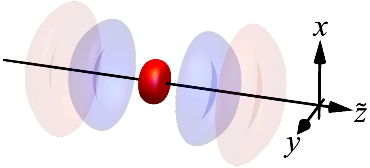


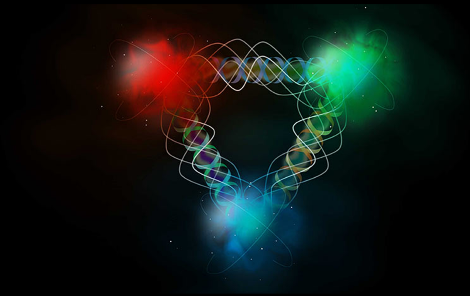
Summary

- Pion is the key to understand confinement and chiral symmetry breaking in QCD
- Light-front wave functions provide the direct access to the parton structure of the pion
- Obtained an exact sum rule for the valence sector wave function based on the most general covariant structure and PCAC
- This chiral sum rule is consistent with chiral symmetry breaking in soft-wall AdS/QCD and leads to a remarkable feature of the pion structure









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