Calorons, monopoles and the Dirac equation

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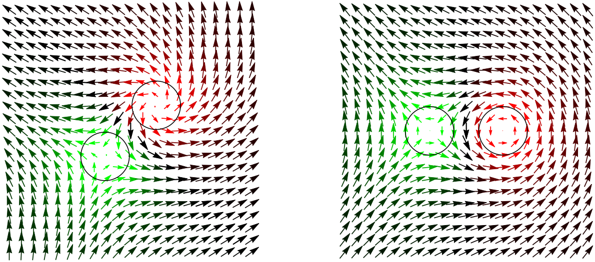
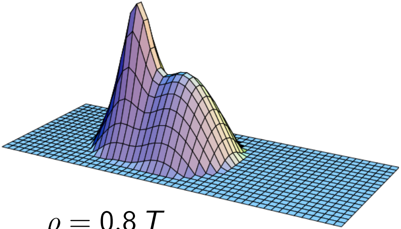
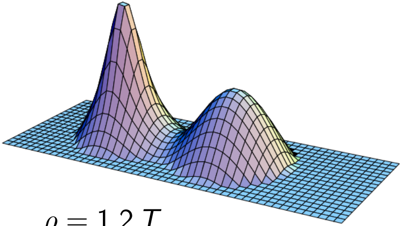


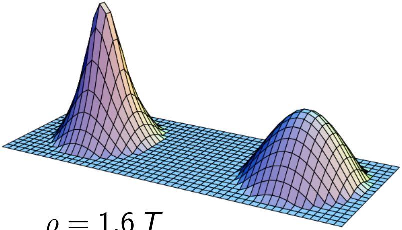
action density of SU(2) Calorons



$\rho = 0.8\ T$



$\rho = 1.2\ T$



$\rho = 1.6\ T$

from: Thomas C Kraan and Pierre van Baal
Nuclear Physics B 533 (1998) 627–659

Calorons = instantons at finite temperature

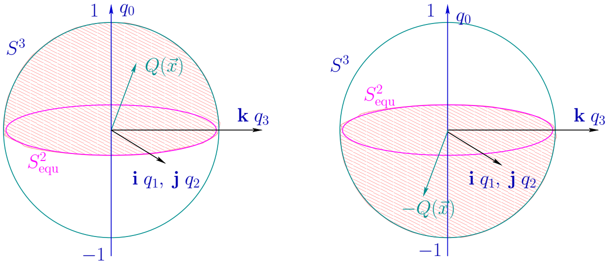
in SU(N) instantons separable in N dyons, quarks or monopoles, calorons are characterised by Polyakov matrices

$$Q(\vec{x}) := \Pi_{i=1}^{N_t} U_4(\vec{x}) = q_0(\vec{x}) - \mathrm{i}\vec{\sigma}\vec{q}(\vec{x})$$

size of monopoles depends on $q_0(\infty)$, the asymptotic (an)holonomy,

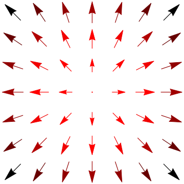
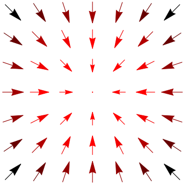
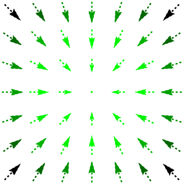
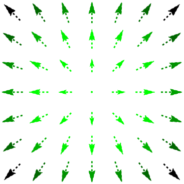
specialise:

- SU(2)
- $q_0 = 0 \cdots$ equal sizes of monopoles



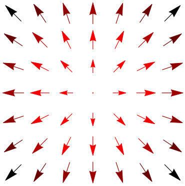
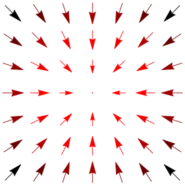
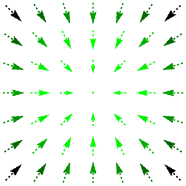
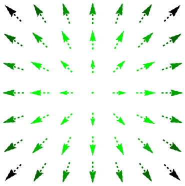
$$Q(\vec{x}) = q_0(\vec{x}) - \mathrm{i}\vec{\sigma}\vec{q}(\vec{x}) = \cos\alpha(\vec{x}) - \mathrm{i}\vec{\sigma}\vec{n}(\vec{x})\sin\alpha(\vec{x}), \quad q_0^2 + \vec{q}^2 = 1$$

4 Types of monopoles

$\mathcal{T} = 1$	$\mathcal{T} = \Pi$	$\mathcal{T} = z$	$\mathcal{T} = z\Pi$
M	$\bar{M}$	L	$\bar{L}$
$e = 1$	$e = 1$	$e = -1$	$e = -1$
$m = 1$	$m = -1$	$m = 1$	$m = -1$
			
$q_0 \geq 0$	$q_0 \geq 0$	$q_0 \leq 0$	$q_0 \leq 0$

arrows indicate imaginary part $\vec{q}(x) = \sin\alpha(x)\,\vec{n}(x)$ of quaternions $Q(x)$.

4 Types of monopoles

$\mathcal{T}=1$	$\mathcal{T}=\Pi$	$\mathcal{T}=z$	$\mathcal{T}=z\Pi$
M $e=1$ $m=1$	$\bar{M}$ $e=1$ $m=-1$	L $e=-1$ $m=1$	$\bar{L}$ $e=-1$ $m=-1$
			
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try real monopoles:

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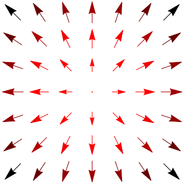	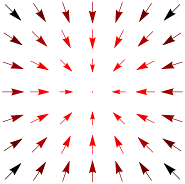	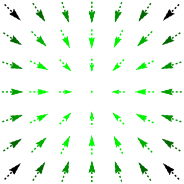	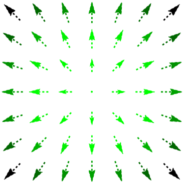
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arrows indicate imaginary part $\vec{q}(x) = \sin\alpha(x)\,\vec{n}(x)$ of quaternions $Q(x)$.
try real monopoles:
just with scalar field $Q(x)$: finite, electric, with charge and spin?

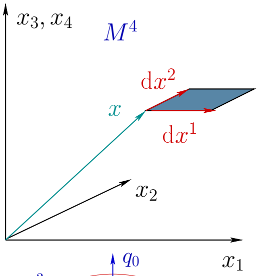
Lagrangian

$Q(x) \to \vec{\Gamma}_\mu(x) \to \vec{R}_{\mu\nu}(x) \to \mathcal{L}(x)$

Connection one-form = dual photon field

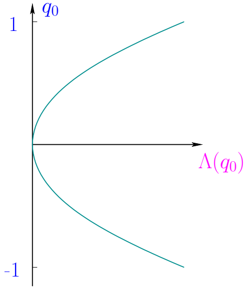
$\partial_\mu Q(x) Q^\dagger(x) =: -\mathrm{i}\vec{\Gamma}_\mu(x)\,\vec{\sigma}$

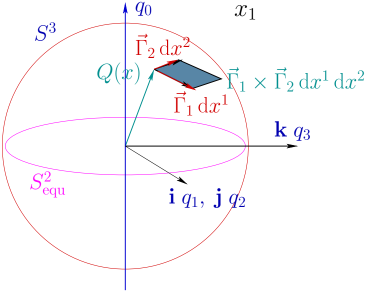
Curvature: $\vec{R}_{\mu\nu}(x) := \vec{\Gamma}_\mu(x) \times \vec{\Gamma}_\nu(x)$



Lagrangian: $\mathcal{L} = -\frac{\alpha_f \hbar c_0}{4\pi}\left(\frac{1}{4}\,\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \Lambda\right)$

potential term: $\Lambda(x) = q_0^6/r_0^4$





2D degeneracy of vacuum → two Goldstone bosons = photons

Relation to other models

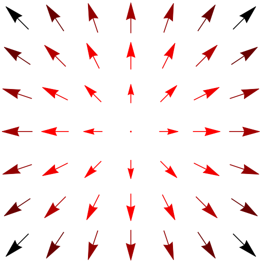
- a 3D generalisation of the Sine-Gordon model
  1+1D $\to$ 3+1D, 1 dof $\to$ 3 dofs
- a model for soft dual Dirac monopoles
  no Dirac string, no singularity in the origin
- a modification of the Skyrme model
  short range $\to$ long-range interaction

Relation to nature:

$$
{}^\star\vec{F}_{\mu\nu} := -\frac{e_0}{4\pi\varepsilon_0 c}\vec{R}_{\mu\nu} = \begin{pmatrix} 0 & \vec{B}_1 & \vec{B}_2 & \vec{B}_3 \\ -\vec{B}_1 & 0 & \frac{\vec{E}_3}{c} & -\frac{\vec{E}_2}{c} \\ -\vec{B}_2 & -\frac{\vec{E}_3}{c} & 0 & \frac{\vec{E}_1}{c} \\ -\vec{B}_3 & \frac{\vec{E}_2}{c} & -\frac{\vec{E}_1}{c} & 0 \end{pmatrix}. \qquad (1)
$$

Stable minima of energy (topological Solitons)

- hedgehog ansatz: $\vec{n}(x) = \frac{\vec{r}}{r}$,
  $\vec{q}(x) = \vec{n}(x) \sin\alpha(x),\ q_0 = \cos\alpha(x)$,
  $\alpha = \alpha(\rho), \quad \rho = r/r_0$,
  $q_0^2 + \vec{q}^2 = 1$,
  $Q(x) = q_0(x) + \mathrm{i}\vec{\sigma}\vec{q}(x)$,
  soliton covers half of S^3

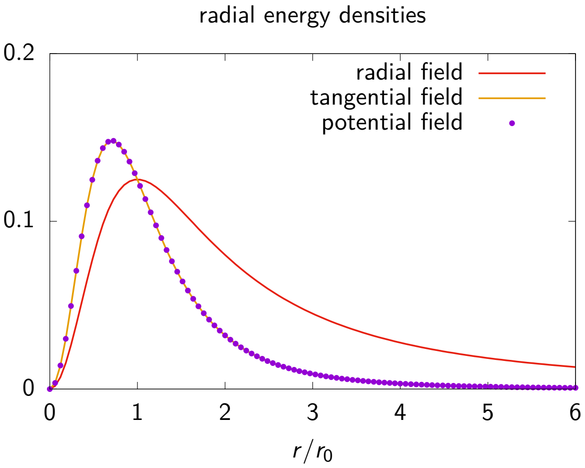


- minimisation of energy $\quad q_0 = \pm\frac{r_0}{\sqrt{\vec{x}^{\,2}+r_0^2}} \quad q_i = \pm\frac{x_i}{\sqrt{\vec{x}^{\,2}+r_0^2}}$
- energy of soliton $\quad E = \frac{\alpha_f \hbar c}{r_0}\frac{\pi}{4}$
- compare with monopoles? with (non) existing?

$$\alpha_f \hbar c = 1.44\,\mathrm{MeV\,fm},\ m_e c^2 = 0.511\ \mathrm{MeV},\ r_0 = 2.21\ \mathrm{fm}$$

Energy densities   No Divergencies!

$$\mathcal{L} = -\frac{\alpha_f \hbar c}{4\pi}\left(\frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{q_0^6}{r_0^4}\right), \quad q_0(\rho) = \cos\alpha(\rho) = \frac{1}{\sqrt{1+\rho^2}}$$



particle and field are indistinguishable

Field variables in 3+1D

describe field of rotations of spatial Dreibein in $\mathbb{M}^4 \ni x$

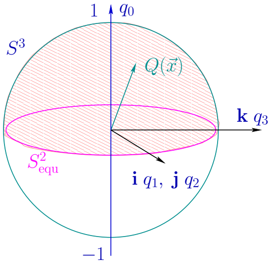
Use rotational group $D(x) \in SO(3)$

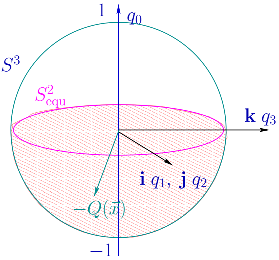
or simpler double covering group of SO(3): SU(2) $\ni Q(x)$

SO(3) versus SU(2)$\simeq \mathbb{S}^3$, $D(x) \leftrightarrow \pm Q(x)$

Field configurations $\pm\{Q(x)\}$ are identical

invariance under center-trafos z, exchanges of the hemispheres of $\mathbb{S}^3$





$$Q(\vec{x}) = q_0(\vec{x}) - \mathrm{i}\vec{\sigma}\vec{q}(\vec{x}) = \cos\alpha(\vec{x}) - \mathrm{i}\vec{\sigma}\vec{n}(\vec{x})\sin\alpha(\vec{x}), \quad q_0^2 + \vec{q}^2 = 1$$

Imagine we have only Space-Time

What can we explain?

- Non-trivial metric: $g_{\mu\nu} \quad \rightarrow \quad$ Gravitation
- Rotating frames in $\mathbb{R}^3$: $D(x) \in SO(3)$

  Topological excitations $\leftrightarrow$ Topological quantum numbers

  $\Pi_3(\mathbb{S}^3) = \mathbb{Z} \quad \leftrightarrow$ spin

  $\Pi_3(\mathbb{S}^2) = \mathbb{Z} \quad \leftrightarrow$ charge

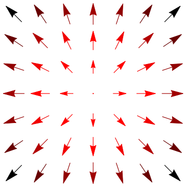
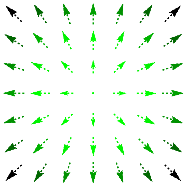
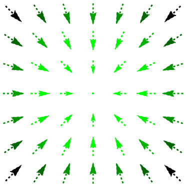
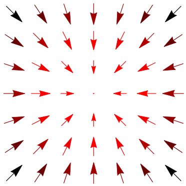
  $\Pi_2(\mathbb{S}^2) = \mathbb{Z} \quad \leftrightarrow$ photon number

  Non-topological excitations:

  dark matter?

  dark energy?

Four classes of solitons

$\mathcal{T} = 1$	$\mathcal{T} = z\Pi$	$\mathcal{T} = z$	$\mathcal{T} = \Pi$
$Z = 1$ $\mathcal{Q} = \frac{1}{2}$	$Z = 1$ $\mathcal{Q} = -\frac{1}{2}$	$Z = -1$ $\mathcal{Q} = \frac{1}{2}$	$Z = -1$ $\mathcal{Q} = -\frac{1}{2}$
			

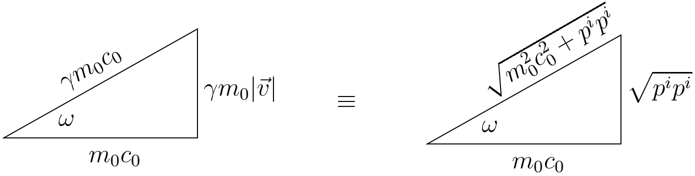
Corresponding to Dirac spinor $\psi = (e_-^\uparrow, e_-^\downarrow, e_+^\uparrow, e_+^\downarrow)$
Intrinsic parity operator: $\gamma_0 := \mathrm{diag}(1, 1, -1, -1)$

$$H_0\psi_0 := m_0 c_0^2\, \gamma_0 \psi_0$$

attributes to positrons a negative rest energy $\rightarrow\ -m_0 c_0^2$

Moving solitons

$$p^\mu = \gamma m_0 (c_0, \vec{v})^\mu, \quad \gamma = \frac{1}{\sqrt{1 - \frac{\vec{v}^2}{c_0^2}}}$$



$$\cos\omega = 1/\gamma(\vec{p}), \quad \sin\omega = |\vec{v}|/c_0$$

Electron-Positron mixing

Free solitons: invariant under center transformations z!

$\rightarrow$ mix electrons and positrons

Dirac spinor $\psi_0 = (e_-^\uparrow, e_-^\downarrow, e_+^\uparrow, e_+^\downarrow) \quad \stackrel{U_\omega}{\rightarrow} \quad \psi := U_\omega \psi_0$

$$\mathrm{i} := \begin{pmatrix} 0 & -\mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix} \quad \Rightarrow \quad \mathrm{i}^2 = -\mathbb{1}_2$$

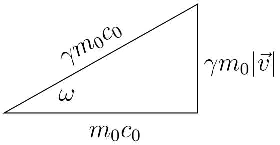
$$U_\omega := \exp\{\mathrm{i}\,\vec{\omega}\,\frac{\vec{\sigma}}{2}\} = \cos\frac{\omega}{2} + \underbrace{\begin{pmatrix} 0 & -\mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix} \vec{\sigma}}_{-\vec{\gamma}} \cdot \vec{e}_\omega \sin\frac{\omega}{2}$$

$$\gamma^0\gamma^i + \gamma^i\gamma^0 = 0 \quad \Rightarrow \quad U_\omega\gamma^0 = \gamma^0 U_\omega^\dagger$$

Foldy-Wouthuysen transformation

$$H := p^0 c_0 = \gamma(\vec{p})\, m_0 c_0^2\, \gamma_0 \quad \xrightarrow{U_\omega} \quad H_\omega := \underbrace{U_\omega H}_{H U_\omega^\dagger} U_\omega^\dagger = H (U_\omega^\dagger)^2$$

$$H_\omega = \gamma(\vec{p})\, m_0 c_0^2\, \gamma_0 \exp\{-\mathrm{i}\, \vec{\omega}\, \vec{\sigma}\} = \gamma(\vec{p})\, m_0 c_0^2\, \gamma_0 (\cos\omega + \vec{\gamma}\, \vec{e}_\omega \sin\omega)$$



$$\frac{\gamma_0}{c_0} \;\Big|\; p^0 c_0 = H_\omega = m_0 c_0^2\, \gamma_0 + \gamma_0 \gamma^i p^i c_0$$

$$\gamma^\mu p_\mu = m_0 c_0$$

$$\gamma_0 H \psi_0 = \gamma(\vec{p})\, m_0 c_0^2\, \psi_0 \quad \xrightarrow{U_\omega} \quad \gamma^\mu p_\mu\, \psi = m_0 c_0\, \psi$$

$$S_{\text{free}} := \int \mathrm{d}^4 x\, \bar{\psi} (\gamma^\mu p_\mu - m_0 c_0)\, \psi$$

Interaction with electromagnetic fields

to get interaction term: $q\, j^\mu\, A_\mu = q\, \underbrace{c_0\, \bar\psi \gamma^\mu \psi}_{j^\mu}\, A_\mu$

MTP: dual non-Abelian vector field $\vec{C}_\mu = -\frac{e_0}{4\pi\varepsilon_0 c}\vec{\Gamma}_\mu \quad \rightarrow$

$\rightarrow \quad A_\mu$ Abelian vector field in QED

two steps:

(1) non-Abelian $\,^\star\vec{F}_{\mu\nu} \quad \rightarrow \quad ^\star f_{\mu\nu}$ Abelian

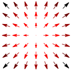
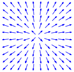
Extended monopoles $\rightarrow$ point-like monopoles

(2) dual transformation:

$$\underbrace{\begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3/c & -E_2/c \\ -B_2 & -E_3/c & 0 & E_1/c \\ -B_3 & E_2/c & -E_1/c & 0 \end{pmatrix}}_{^\star f_{\mu\nu}} \rightarrow \underbrace{\begin{pmatrix} 0 & E_1/c & E_2/c & E_3/c \\ -E_1/c & 0 & -B_3 & B_2 \\ -E_2/c & B_3 & 0 & -B_1 \\ -E_3/c & -B_2 & B_1 & 0 \end{pmatrix}}_{f_{\mu\nu}=\partial_\mu A_\nu - \partial_\nu A_\mu}$$

back to Dirac (Electrodynamic) limit

artificial separation between particles and field

  bald $\to$ cowlick(hair whorl)  

$$r_0 \to 0 \quad \Longleftrightarrow \quad q_0 = \cos\alpha = 0 \quad \Longleftrightarrow \quad \alpha = \frac{\pi}{2}$$

Dual Dirac monopoles (dual Wu-Yang monopoles)

soliton has singularity in the center

$$Q(x) = -\mathrm{i}\vec{\sigma}\vec{n}(x),\ \vec{\Gamma}_\mu(x) = \vec{n}(x)\times\partial_\mu\vec{n}(x),\ \vec{R}_{\mu\nu}(x) = \partial_\mu\vec{n}(x)\times\partial_\nu\vec{n}(x)$$

$$^*f_{\mu\nu}(x) = -\frac{e_0}{4\pi\varepsilon_0 c}\vec{R}_{\mu\nu}\vec{n} = -\frac{e_0}{4\pi\varepsilon_0 c}\vec{n}(x)[\partial_\mu\vec{n}(x)\times\partial_\nu\vec{n}(x)]$$

$$\mathcal{L}_{\mathrm{ED}} = -\frac{1}{4\mu_0}{}^*f_{\mu\nu}(x)\,{}^*f^{\mu\nu}(x)$$

$$Q_{\mathrm{el}}(\mathcal{S}) = -\frac{e_0}{4\pi}\oint_{\mathcal{S}(u,v)} \mathrm{d}u\mathrm{d}v\ \vec{n}[\partial_u\vec{n}\times\partial_v\vec{n}]$$

world-lines of singularities

Interaction with E and B fields

world-lines of singularities

$$\frac{1}{4\pi}\frac{1}{2}\varepsilon^{\kappa\lambda\mu\nu}\partial_\lambda\{\vec{n}(\partial_\mu\vec{n}\times\partial_\nu\vec{n})\} =: \underbrace{\sum_{i=1}^{N} Z_i \int \mathrm{d}\tau\, \frac{\mathrm{d}X_i^\kappa(\tau)}{\mathrm{d}\tau}}_{\frac{q}{-e_0 c_0} j^\kappa}\ \delta^4(x - X_i(\tau)).$$

$$\mu_0\, q\, j^\kappa = -\partial_\lambda \underbrace{\frac{e_0 c_0 \mu_0}{4\pi}\frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}\,\vec{n}[\partial_\mu\vec{n}\times\partial_\nu\vec{n}]}_{f^{\kappa\lambda}} = \partial_\lambda f^{\lambda\kappa}.$$

This is inhomogeneous Maxwell equations!

possible? $\quad f_{\mu\nu} \stackrel{?}{=} \partial_\mu a_\nu - \partial_\nu a_\mu \quad$ homogeneous Maxwell equations

Problem of magnetic currents g^μ

$$g^\mu := c\,\partial_\nu {}^*f^{\nu\mu} \quad \text{with} \quad \partial_\mu g^\mu = 0. \tag{2}$$

origin of magnetic currents: less degrees of freedom (dofs) in MTP

QED: $A_\mu(x) \cdots 4$ dofs

MDP: $\vec{n}(x) \cdots 2$ dofs, $\quad \vec{n}^2(x) = 1$

least action principle $\Rightarrow$ equations of motion:

$$\partial_\mu \vec{n}\, g^\mu = 0 \quad \Rightarrow \quad {}^*f_{\mu\nu} g^\nu = 0. \tag{3}$$

No dual Coulomb and Lorentz forces on magnetic currents

do be shown: g^ν: No Coulomb and Lorentz forces on j^μ

does not influence dynamics of j^μ

$\Rightarrow g^\mu$ can be neglected $\quad \Rightarrow \partial_\nu {}^*f^{\nu\mu} = 0 \cdots$ homogeneous Maxwell Eqs

Coulomb and Lorentz forces

Canonical energy-momentum tensor $\Theta^{\mu}{}_{\nu}$ in electrodynamic limit
$T^{\mu}{}_{\nu}(x) = -\frac{1}{\mu_0}{}^*f_{\nu\sigma}(x){}^*f^{\mu\sigma}(x) - \frac{1}{4\mu_0}{}^*f_{\lambda\sigma}(x){}^*f^{\lambda\sigma}(x)\,\delta^{\mu}_{\nu}$
is symmetric

We split total force density
$f^{\nu} = \partial_{\mu}\Theta^{\mu\nu} = f^{\mu}_{\mathrm{charges}} + \partial^{\nu} T^{\mu\nu} = 0$

interaction is a consequence of topology
Coulomb and Lorentz forces: $f^{\mu}_{\mathrm{charges}} = f^{\mu\sigma} j_{\sigma} q$

$$f^{0}_{\mathrm{charges}} = \frac{q}{c}\,\mathbf{j}\,\mathbf{E},$$
$$\mathbf{f}_{\mathrm{charges}} = p(\rho\,\mathbf{E} + \mathbf{j} \times \mathbf{B}).$$

Derivation of Coulomb and Lorentz forces

$$
\begin{aligned}
f^{\mu}_{\text{charges}} &:= -\partial^{\nu} T^{\mu}_{\ \nu} = \frac{1}{\mu_0}\partial^{\nu}\left({}^{\star}f_{\nu\rho}\,{}^{\star}f^{\mu\rho}\right) - \frac{1}{4\mu_0}\partial^{\mu}\left({}^{\star}f_{\rho\nu}\,{}^{\star}f^{\rho\nu}\right) = \\
&= \frac{1}{\mu_0}\Big[\underbrace{\partial^{\nu}\,{}^{\star}f_{\nu\rho}}_{\frac{1}{c_0}g_{\rho}}\,{}^{\star}f^{\mu\rho} + \underbrace{{}^{\star}f_{\nu\rho}\,\partial^{\nu}\,{}^{\star}f^{\mu\rho}}_{-\,{}^{\star}f_{\nu\rho}\partial^{\rho}\,{}^{\star}f^{\mu\nu}} + \frac{1}{2}\,{}^{\star}f_{\nu\rho}\,\partial^{\mu}\,{}^{\star}f^{\rho\nu}\Big] = \\
&= \frac{1}{\mu_0 c_0}\underbrace{{}^{\star}f^{\mu\rho}g_{\rho}}_{0} + \frac{1}{2\mu_0}\,{}^{\star}f_{\nu\rho}\Big[\underbrace{\partial^{\nu}\,{}^{\star}f^{\mu\rho} + \partial^{\rho}\,{}^{\star}f^{\nu\mu} + \partial^{\mu}\,{}^{\star}f^{\rho\nu}}_{-\mu_0\, q\, \epsilon^{\mu\nu\rho\sigma} j_{\sigma}}\Big] = \\
&= -\frac{q}{2}\epsilon^{\mu\nu\rho\sigma}\,{}^{\star}f_{\nu\rho}j_{\sigma} = f^{\mu\sigma}j_{\sigma}q.
\end{aligned}
$$

Complete the separation of charges and fields

$f_{\mu\nu} := \partial_\mu a_\nu - \partial_\nu a_\mu, \quad \mathcal{L} := -\mathcal{L}_{\mathrm{ED}} = -\frac{1}{4\mu_0} f_{\mu\nu} f^{\mu\nu}$

$f_{\mu\nu} := \mathcal{F}_{\mu\nu} + F_{\mu\nu}, \quad a_\mu = \mathcal{A}_\mu + A_\mu$

$F_{\mu\nu} \cdots$ external fields
$\mathcal{F}_{\mu\nu} \cdots$ internal field contributions, $\quad \partial_\mu \mathcal{F}^{\mu\nu} = \mu_0\, q\, j^\nu$
infinite self-energy of point-like particles aborbed in free Lagrangian

$$S = -\frac{1}{4\mu_0} \int \mathrm{d}^4x\, f_{\mu\nu} f^{\mu\nu} = -\frac{1}{4\mu_0} \int \mathrm{d}^4x\, \{\mathcal{F}_{\mu\nu} + F_{\mu\nu}\}^2$$

$$S_{\mathrm{free}} = -\frac{1}{4\mu_0} \int \mathrm{d}^4x \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \equiv \int \mathrm{d}^4x\, \bar{\psi}(\gamma^\mu p_\mu - m_0 c_0)\, \psi$$

$$S_{\mathrm{int}} := -\frac{1}{2\mu_0} \int \mathrm{d}^4x\, F_{\mu\nu} \mathcal{F}^{\mu\nu} = \frac{1}{\mu_0} \int \mathrm{d}^4x\, A_\nu \partial_\mu \mathcal{F}^{\mu\nu} =$$

$$= \int \mathrm{d}^4x\, q\, A_\nu j^\nu = \int \mathrm{d}^4x\, c_0\, q\, A_\nu \bar{\psi} \gamma^\nu \psi.$$

$$S = \int \mathrm{d}^4x \left\{ \bar{\psi} [\gamma^\mu (p_\mu - qA_\mu) - m_0 c_0] \psi - \frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} \right\}.$$

Summary: MTP$\rightarrow$Dirac equation

MTP: $\begin{cases} \text{Everything is finite} \\ \text{same dofs for particles and fields} \end{cases}$

Dirac Equation: Point-like charges

- solitons$\rightarrow$ $\text{QM} \begin{cases} \text{Hilbert space vectors} \\ \text{Borns rule} \\ \text{canonical commutation relations} \end{cases}$
- artificial separation of fields and point-like charges
- absorb infinite self-energies into the experimental masses

Observe:

finite size of solitons modifies the $1/$r-potential

How does this modify the eigenstates of Schrödinger and Dirac equation?

Comparison to Lamb shift?

Aftermath

Physics is measurements of distances of objects and times of events.

This may indicate, that

Physics is geometry and not algebra.

Finally, one should use the algebra to describe the geometry.

General Relativity:

Wheeler: “Spacetime tells matter how to move;

matter tells spacetime how to curve.

My addition for Electrodynamics:

$\cdots$ Charges and electromagnetic fields tell space how to rotate.

Everything on earth is finite, besides $\cdots$

Thanks

Comparison to Maxwell's electrodynamics

1. The Lagrangian is Lorentz covariant, thus the laws of special relativity are respected.
2. Charges have Coulombic fields fulfilling Gaußes law.
3. Charges interact via $\frac{1}{r^2}$ electric fields, they feel Coulomb and Lorentz forces.
4. A local U(1) gauge invariance is respected.
5. There are two dofs of massless excitations for photons.

In distinction to Maxwell's electrodynamics

1. Electric charges are quantised, like the magnetic charges of Dirac monopoles. Charge is a topological quantum number.
2. By topological construction, mirror properties of particles and antiparticles.
3. The mass of solitons is completely due to field energy and finite.
4. The self-energy of charges is finite and does not need regularisation and renormalisation.
5. Charges and their fields are described by the same SO(3) dofs.
6. SO(3) dofs interpreted as orientations of spatial Dreibeins.
7. Gauge symmetry a geometrical phenomenon,basis changes on S^3.
8. Spin has usual quantisation properties and combination rules.
9. 4 basic configurations of solitons, quantum numbers of Dirac spinors.

In distinction to Maxwell's electrodynamics

10. Solitons and antisolitons have opposite internal parity.
11. Solitons are characterised by a chirality quantum number which can be related to the sign of the magnetic quantum number.
12. Spin contributes to angular momentum due to internal rotations.
13. The canonical energy-momentum tensor is automatically symmetric.
14. Static charges are described by the spatial components of vector fields. Moving charges need time-dependent fields.
15. r-dependence of charge by finite size of solitons $\rightarrow$ running coupling.
16. Local U(1) gauge invariance explained, bases choice on $\mathbb{S}^2$.
17. Photon number $\rightarrow$ Gaußian linking number of fibres on $\mathbb{S}^2$.
18. Photon number changes by interaction with charges.

Rather unexpected

1. Spin and magnetic moment are dynamical properties only.
2. Electric and magnetic field vectors are perpendicular to each other
3. Existence of unquantised magnetic currents is allowed.
4. α-waves in $q_0 = \cos\alpha$ contribute to (dark) matter density.
5. α-waves lead to additional forces on particles and are a possible origin of quantum fluctuations.
6. Potential term allows mechanism of cosmic inflation
7. Potential term contributes to dark energy.