A gauge invariant mechanism of color confinement in SU3 based on violation of non-Abelian Bianchi identity

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1. Color confinement in QCD as an Abelian dual Meissner effect

1. 1974-75: Idea of dual superconductor (electric $\leftrightarrow$ magnetic) as the color-confinement mechanism ('tHooft-Mandelstam): Something color magnetic must be condensed in QCD.

2. 1981: 'tHooft idea of monopole in QCD: A partial gauge-fixing $SU(3) \to U(1) \times U(1)$ and Abelian projection: Monopoles appear as a topological object coming from the singularity of the gauge-fixing matrix. Numerical data supporting this idea are shown especially on the basis of maximally Abelian gauge. But this idea has serious problems:(1) gauge dependence, For example, in Polyakov-loop gauge, monopoles are always only time-like, hence can not contribute to confinement. (2) Abelian charge confinement, not non-Abelian color confinement,

Gauge invariance problem, i.e., how to explain why SU3 color singlets alone can survive in the Abelian framework is most important!

3. In 2010, an important relation was found by Bonati et al. which shows that violation of non-Abelian Bianchi identity (VNABI) $J_\nu^\alpha(x)$ exists behind any Abelian projection scheme: Reference C. Bonati et al,, P.R.D81, 085022 (2010)

$$\begin{aligned}\mathrm{Tr}(\Phi^\alpha(x)J_\nu^\alpha(x)) &= \mathrm{Tr}(\Phi^\alpha(x)D_\mu^\alpha G_{\mu\nu}^{\alpha*}(x)) \\ &= \partial_\mu F_{\mu\nu}^{\alpha*}. \end{aligned} \tag{1}$$

$G_{\mu\nu}^{\alpha*}$: a non-Abelian dual field strength,
$\Phi^\alpha(x)$: an adjoint operator characterizing the Abelian projection scheme
$F_{\mu\nu}^{\alpha*}$: the 'tHooft tensor.

They asserted further that the above relation leads to gauge invariance of 'tHooft Abelian projection schemes. But regretabbly it is not correct. Gauge invariance is not proved only through (1).

The key to solve the gauge invariance problem is to find a gauge-independent color magnetic quantity, a magnetic monopole in QCD. Bonati et als' work suggests that VNABI $J_\nu^\alpha(x)$ may be the clue.

2. Abelian magnetic monopoles of the Dirac type in QCD

The Jacobi identities $\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0$ where $D_\mu \equiv \partial_\mu - igA_\mu$.
Calculate explicitly:

$$\begin{aligned} [D_\rho, D_\sigma] &= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\ &= -igG_{\rho\sigma} + [\partial_\rho, \partial_\sigma] \end{aligned}$$

If $[\partial_\rho, \partial_\sigma]$ is neglected, we get $D_\nu G^*_{\mu\nu} = 0 \rightarrow$ Non-Abelian Bianchi identity (NABI):

When define an Abelian-like field strength:
$f_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\sigma^a/2$. if A^a_μ are regular $\rightarrow \partial_\nu f^*_{\mu\nu} = 0$:
Abelian-like Bianchi identity:

What happens if $[\partial_\rho, \partial_\sigma]$ is not neglected?

Jacobi identity + $[D_\nu, G_{\rho\sigma}] = D_\nu G_{\rho\sigma}$

$$
\begin{aligned}
J_\mu \equiv D_\nu G^*_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma} = -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu, [\partial_\rho, \partial_\sigma]] \\
&= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho, \partial_\sigma] A_\nu = \partial_\nu f^*_{\mu\nu} \equiv k_\mu
\end{aligned}
$$

$k^a_\mu \neq 0 \rightarrow$ color magnetic Abelian-like monopole: $\partial_\mu k_\mu = 0$

$$
\begin{gathered}
[\partial_\rho, \partial_\sigma] A_\nu \neq 0 \\
\Downarrow
\end{gathered}
$$

Line singularities existing in original gauge fields $A_\mu(x)$ themselves!!! are the origin of Abelian monopoles in QCD.

Since the monopoles defined here comes from the (line) singularities of the gauge field themselves, they are much the same as those discussed by Dirac in QED with magnetic monopoles in 1931.

8 monopoles exist in $SU(3)$.

3. Gauge invariance of the Abelian confinement picture

Assume that original non-Abelian gauge fields contain a line-like singularity leading us to the existence of the Abelian-like monopoles (violation of non-Abelian Bianchi identity).

$$\Downarrow$$

When such Abelian monopoles make condensation, 8 Abelian electric fields are squeezed due to the dual Meissner effect leading to the color confinement.

$SU(2)$: $V(x) = e^{i\alpha_i(x)\sigma_i}$
Consider a $U(1)_3 : \quad e^{i\alpha_3(x)\sigma_3} \in SU(2)$. Quarks have a charge $\pm g/2$ and are described as

$$q = \begin{pmatrix} u_3 \\ d_3 \end{pmatrix} \tag{2}$$

When $k_\mu^3(x)$ make condensation, electric charged states with respect to $U(1)_3$ are confined.

But $\bar{\mathbf{u}}_3\mathbf{u}_3 - \bar{\mathbf{d}}_3\mathbf{d}_3$ **is not confined in** $U(1)_3$.

Note one can choose any other $U(1)$, $U(1)_1$ described by $e^{i\alpha_1(x)\sigma_1}$ or $U(1)_2$ described by $e^{i\alpha_2(x)\sigma_2}$. Then the quark field having an electric charge $\pm g/2$ w.r.t these $U(1)$ is expressed in terms of (u_3, d_3) as

$$\begin{pmatrix} u_1 \\ d_1 \end{pmatrix} = \begin{pmatrix} \frac{(u_3+d_3)}{\sqrt{2}} \\ \frac{(u_3-d_3)}{\sqrt{2}} \end{pmatrix}. \qquad \begin{pmatrix} u_2 \\ d_2 \end{pmatrix} = \begin{pmatrix} \frac{(iu_3+d_3)}{\sqrt{2}} \\ \frac{(iu_3-d_3)}{\sqrt{2}} \end{pmatrix}.$$

For example, $\bar{u}_3 u_3 - \bar{d}_3 d_3 = \bar{u}_1 d_1 + \bar{d}_1 u_1$. Hence such states are charged with respect to $U(1)_1$ and confined if the Abelian monopoles $k_\mu^1(x)$ make condensation.

Hence in this scheme , $SU(2)$ is not broken and all three Abelian monopoles are assumed to condense, so that only $SU(2)$ singlets alone can exist in physical states.

The situations are more easily seen when one writes the quark-gluon coupling term in $SU(2)$ QCD as

$$
\begin{aligned}
\bar{q}\gamma^{\mu}\frac{\sigma^{a}}{2}qA_{\mu}^{a} &= \frac{1}{2}(\bar{u}_3\gamma_{\mu}d_3+\bar{d}_3\gamma_{\mu}u_3)A_{\mu}^{1}-i\frac{1}{2}(\bar{u}_3\gamma_{\mu}d_3-\bar{d}_3\gamma_{\mu}u_3)A_{\mu}^{2} \\
&\quad +\frac{1}{2}(\bar{u}_3\gamma_{\mu}u_3-\bar{d}_3\gamma_{\mu}d_3)A_{\mu}^{3} \qquad (3) \\
&= \frac{1}{2}(\bar{u}_1\gamma_{\mu}u_1-\bar{d}_1\gamma_{\mu}d_1)A_{\mu}^{1}+\frac{1}{2}(\bar{u}_2\gamma_{\mu}u_2-\bar{d}_2\gamma_{\mu}d_2)A_{\mu}^{2} \\
&\quad +\frac{1}{2}(\bar{u}_3\gamma_{\mu}u_3-\bar{d}_3\gamma_{\mu}d_3)A_{\mu}^{3}, \qquad (4)
\end{aligned}
$$

In case of $SU(3)$, all 8 magnetic currents $k_{\mu}(x)^{a}$ are assumed to condense, so that only $SU(3)$ singlets appear in physical states.

Gauge invariance and non-Abelian color confinement can be explained by means of $Abelian$ dual Meissner effect.

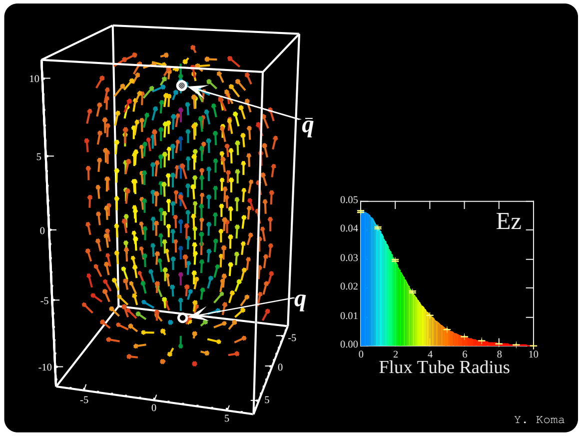


Figure 1: electric flux

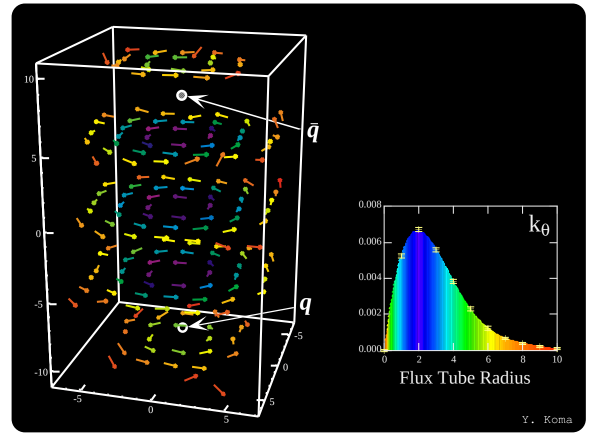


Figure 2: solenidal magnetic current

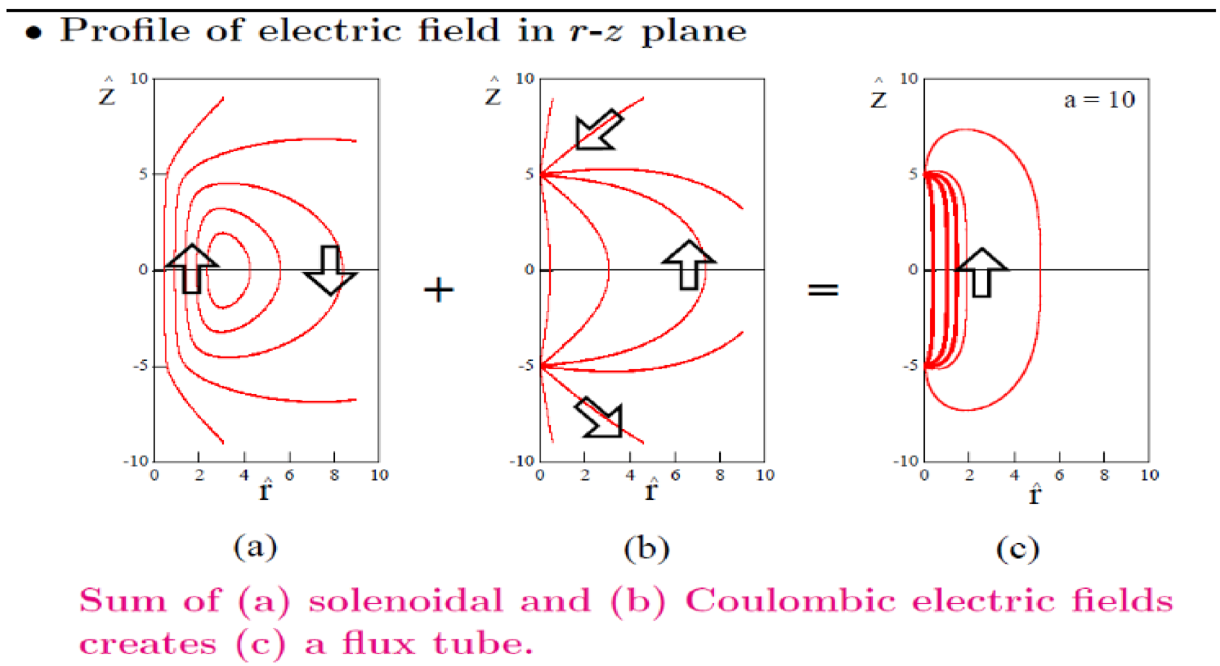


Figure 3: The dual Meissner effect

4. Numerical supports of the new confinement scheme

Theoretical expectations from the Abelian dual Meissner picture:

1. **(Perfect Abelian dominance)** : The linear part of non-Abelian static potential is totally reproduced by the simple color average over those of 8 Abelian static potentials. Without additional gauge fixing, all string tensions of the 8 Abelian potential are the same, so that $\sigma_F = \sigma_a$ for any color component.

2. **(Perfect monopole dominance)** :An Abelian static potential is composed of two contributions, that is, the Coulomb interaction and the solenoidal monopole current. The linear part is only from the solenoidal monopole current. Hence $\sigma_a = \sigma_m$.

3. **(The dual Ampère's law and Ginzburg-Landau parameter)** The Abelian electric flux is squeezed by the monopole solenoidal current, that is, the dual Ampère law holds.

4. **(Existence of the continuum limit of Abelian monopoles satisfying the Dirac quantization condition)**

(1) Perfect Abelian and monopole dominances

Abelian link fields on lattice without any additional gauge-fixing

Maximize $R = \sum Re\,\mathrm{Tr}\,e^{i\theta_1(s,\mu)\lambda_1}U^\dagger(s,\mu) \Longrightarrow$

$$
\begin{aligned}
\theta_1(s,\mu) &= \tan^{-1}\frac{U_1(s,\mu)}{U_0(s,\mu)}, \quad (\mathrm{SU2}: \quad U(s,\mu) = U_0(s,\mu) + i\vec{\sigma}\cdot\vec{U}(s\mu)) \\
&= \tan^{-1}\frac{Im(U_{12}(s,\mu)+U_{21}(s,\mu))}{Re(U_{11}(s,\mu)+U_{22}(s,\mu))}, \quad (\mathrm{SU3})
\end{aligned}
$$

Abelian monopoles on lattice satisfying the Dirac quantization condition

Calculate Abelian plaquette variables $\theta_1(s,\mu\nu) = \partial_\mu\theta_1(s,\nu) - \partial_\nu\theta_1(s,\mu)$:

$$
\theta_1(s,\mu\nu) = \bar{\theta}_1(s,\mu\nu) + 2\pi n_1(s,\mu\nu) \quad (|\bar{\theta}_1(s,\mu\nu)| < \pi)
$$

Since $n_1(s,\mu\nu)$ can be regarded as the number of the Dirac string, Abelian monopoles are defined following DeGrand-Toussaint:

$$
\begin{aligned}
k_\mu^1(s) &= -(1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha\bar{\theta}_1(s+\hat{\mu},\beta\gamma) \\
&= (1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha n_1(s+\hat{\mu},\beta\gamma)
\end{aligned}
$$

Evaluate each static potential through Polyakov-loop correlators.

$$V(R) = -\frac{1}{aN_t} \ln\langle P(0)P^*(R)\rangle \ .$$

$$P_{\rm F} = {\rm Tr}\,\Pi_{k=0}^{N_t-1} U(s+k\hat{4},4) \ ,$$

$$P_{\rm A} = \exp[i \sum_{k=0}^{N_t-1} \theta_1(s+k\hat{4},4)] = P_{\rm ph}\cdot P_{\rm mon} \ ,$$

$$P_{\rm ph} = \exp\{-i \sum_{k=0}^{N_t-1} \sum_{s'} D(s+k\hat{4}-s')\partial'_{\nu}\bar{\Theta}_1(s',\nu 4)\} \ ,$$

$$P_{\rm mon} = \exp\{-2\pi i \sum_{k=0}^{N_t-1} \sum_{s'} D(s+k\hat{4}-s')\partial'_{\nu} n_1(s',\nu 4)\}$$

Perfect Abelian dominance can be proved using the Lüscher's multilevel method.

Table 1: Simulation parameters: $N_{\rm sub}$ is the sublattice size divided and $N_{\rm iup}$ is the number of internal updates in the multilevel method.

β	$N_s^3 \times N_t$	$a(\beta)$ [fm]	$N_{\rm conf}$	$N_{\rm sub}$	$N_{\rm iup}$
5.60	$16^3 \times 16$	0.2235	6	2	10000000
5.70	$12^3 \times 12$	0.17016	6	2	5000000
5.80	$12^3 \times 12$	0.13642	6	3	5000000

Table 2: The string tension σa^2, the Coulombic coefficient c, and the constant μa.

$\beta = 5.6, 16^3 \times 12$	σa^2	c	μa
$V_{\rm NA}$	0.239(2)	-0.39(4)	0.79(2)
$V_{\rm A}$	0.25(2)	-0.3(1)	2.6(1)
$\beta = 5.7, 12^3 \times 12$			
$V_{\rm NA}$	0.159(3)	-0.272(8)	0.79(1)
$V_{\rm A}$	0.145(9)	-0.32(2)	2.64(3)
$\beta = 5.8, 12^3 \times 12$			
$V_{\rm NA}$	0.101(3)	-0.28(1)	0.82(1)
$V_{\rm A}$	0.102(9)	-0.27(2)	2.60(3)

Perfect monopole dominance:

The Lüscher's multilevel method does not work. To evaluate $< P_{\rm mon} P^*_{\rm mon} >$, we need tremendous number of vacuum configurations.

Table 3: Typical simulation parameters. $N_{\rm RGT}$ is the number of random gauge transformations.

	β	$N_s^3 \times N_t$	$a(\beta)$ [fm]	$N_{\rm conf}$	$N_{\rm RGT}$
$SU2$	2.43	$24^3 \times 8$	0.1029(4)	7,000	4,000
$SU3$	5.6	$24^3 \times 4$	0.2235	910000	400

Table 4: Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa.

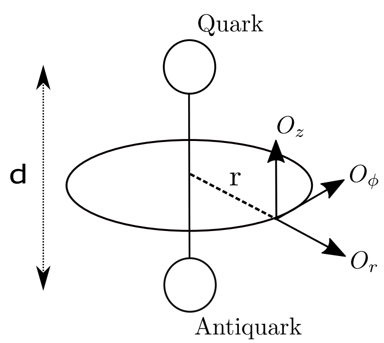
$SU(3)$ $(24^3 \times 4)$					
	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.178(1)	0.86(4)	0.99(1)	5 - 9	1.23
$V_{\rm A}$	0.16(3)	0.9(11)	2.5(3)	5 - 9	1.03
$V_{\rm mon}$	0.17(2)		2.9(1)	4 - 7	1.08
$V_{\rm ph}$	$-0.0007(1)$	0.046(3)	0.945(1)	3 - 10	7.22e-08
$SU(2)$ $(24^3 \times 8)$					
$V_{\rm NA}$	0.0415(9)	0.47(2)	0.46(8)	4.1 - 7.8	0.99
$V_{\rm A}$	0.041(2)	0.47(6)	1.10(3)	4.5 - 8.5	1.00
$V_{\rm mon}$	0.043(3)	0.37(4)	1.39(2)	2.1 - 7.5	0.99
$V_{\rm ph}$	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.46649(6)	7.7 - 11.5	1.02

(2) The Abelian dual Meissner effect

Table 5: Simulation parameters for the measurement of E_i^a and k^2 (Left). $\nabla \times \vec{E}$, $\partial_4 \vec{B}$, k_i^a (Right). id is the distance between two Polyakov loops. Nconf, Nran and Ns are numbers of configurations, random gauge copies and smearing, respectively.

E_i^a			
id	Nconf	Nran	Ns
d=5	80000	100	120
d=6	80000	100	120
k^2			
id	Nconf	Nran	Ns
d=5	960000	0	120
d=6	960000	0	120

the dual Ampère's law			
id	Nconf	Nran	Ns
d=3	20000	100	90
k_ϕ^a			
d=3	11200	3000	90



$$\rho_{conn}(O(r)) = \frac{\langle \mathrm{Tr}(P(0)LO(r)L^\dagger)\mathrm{Tr}P^\dagger(d)\rangle}{\langle \mathrm{Tr}P(0)\mathrm{Tr}P^\dagger(d)\rangle} - \frac{1}{3}\frac{\langle \mathrm{Tr}P(0)\mathrm{Tr}P^\dagger(d)\mathrm{Tr}O(r)\rangle}{\langle \mathrm{Tr}P(0)\mathrm{Tr}P^\dagger(d)\rangle},$$

Figure 4: The Abelian color electric field around static quarks for $d = 5$ at $\beta = 5.6$ on $24^3 \times 4$ lattices. (Left) The squared monopole density at $d = 5$.(Right)

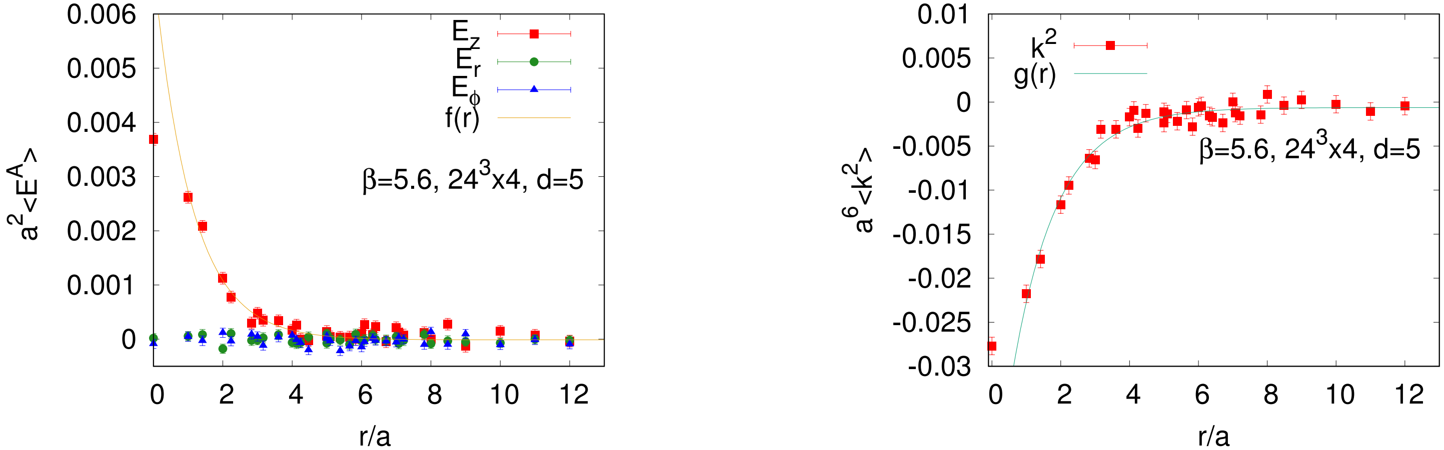
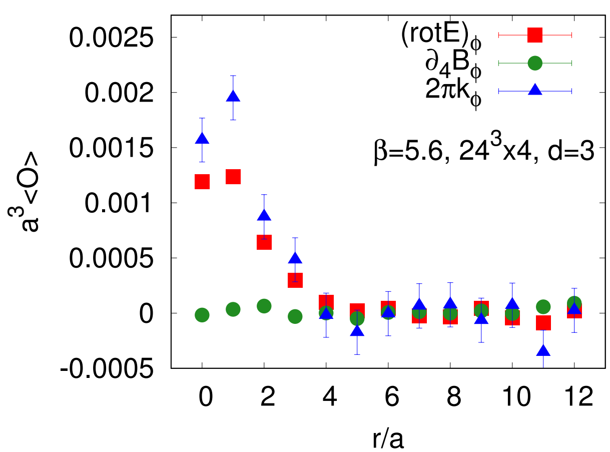


Figure 5: The dual Ampère's law with $d = 3$ at $\beta = 5.6$ on $24^3 \times 4$ lattices. The Ginzburg-Landau parameter $\kappa = \lambda/\xi$ (Right).



d	$\sqrt{2}\kappa$
3	0.87(5)
4	0.93(7)
5	0.83(9)
6	0.9(2)

$SU(2)$: $\sqrt{2}\kappa \sim 1.1$

5. Existence of the continuum limit

Does the continuum limit of $k^a(s,\mu)$ exist?

The monopole density in the continuum limit in pure SU2 and SU3 QCD.

The lattice vacuum is contaminated with large amount of lattice artifact monopoles. To reduce lattice artifacts, various techniques smoothing the vacuum are introduced.

1. Tadpole improved (SU2) and Iwasaki (SU3) action:
48^4 at $\beta = 3.0 \sim 3.9$ in SU2 and at $\beta = 2.3 \sim 3.5$ in SU3

2. Introduction of various smooth gauge-fixings
1) Maximal center gauge(MCG): Maximization of $R = \sum_{s,\mu}(\mathrm{Tr}U(s,\mu))^2$
SU(3) $\rightarrow$ Z(3)

2) Maximal Abelian and $U(1)^2$ Landau gauge (MAU1)

3) Direct Laplacian center gauge (DLCG) only in SU2

3) Maximal Abelian Wilson loop gauge (AWL) only in SU2: Maximization of
$R = \sum_{s,\mu\neq\nu}\sum_a(cos(\theta^a_{\mu\nu}(s))$

3. The blockspin transformation of monopoles

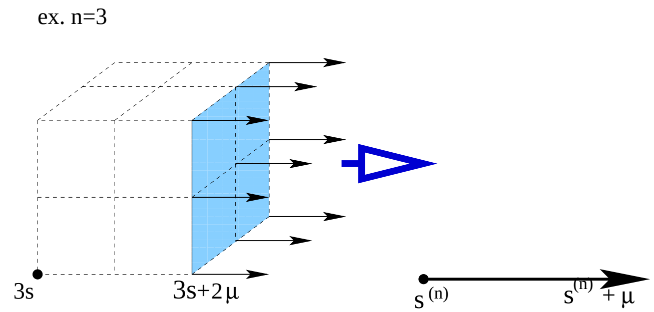


Figure 6: Blockspin definition of monopoles:
T.L. Ivanenko et al., Phys. Lett. **B252**, (1990) 631

Monopole is defined on a a^3 cube and
the n-blocked monopole is defined on a cube
with a lattice spacing $b = na$

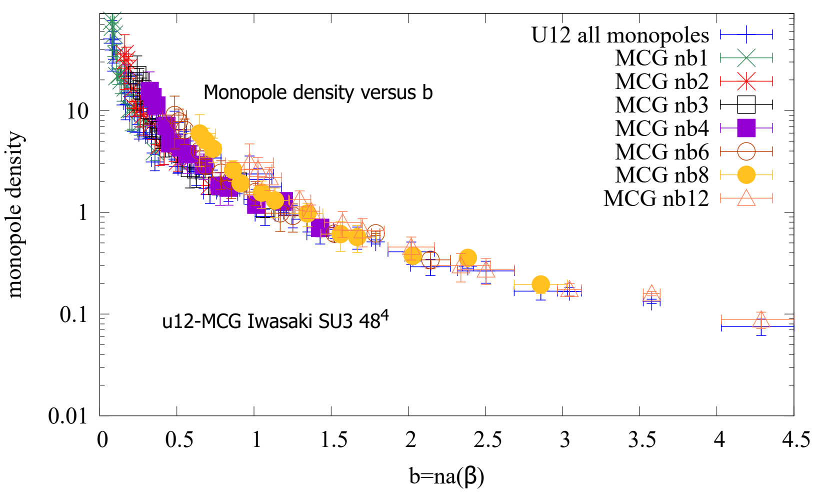
$$k_\mu^{(n)}(s_n) = \sum_{i,j,l=0}^{n-1} k_\mu(ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma})$$

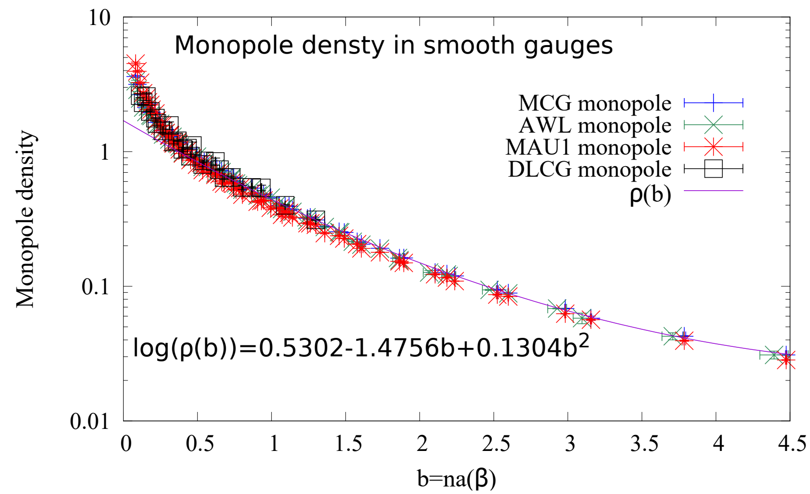
$n = 1, 2, 3, 4, 6, 8, 12$ blockings are adopted on 48^4 lattice.

Evaluate a gauge-invariant density of the n-blocked monopole:

$$\boldsymbol{\rho}(a(\beta), n) = \frac{\sum_{\mu, s_n} \sqrt{\sum_a (k_\mu^{(n)a}(s_n))^2}}{4\sqrt{3} V_n b^3}$$

Figure 7: Comparison of the VNABI (Abelian-like monopoles) densities versus $b = na(\beta)$ in SU3: MCG, MAU12 (Left) and in SU2: MCG, AWL, DLCG and MAU1 (Right). A uniform curve is obtained for all gauges.





Summary

1. Clear scaling behaviors are observed up to the 12-step blockspin transformations for (SU2) $\beta = 3.0 \sim 3.9$ and (SU3) $\beta = 2.3 \sim 3.5$. The density $\rho(a(\beta), n)$ is a function of $b = na(\beta)$ alone, i.e. $\rho(b)$. $n \to \infty$ means $a(\beta) \to 0$ for fixed $b = na$. Existence of the continuum limit!

2. When the vacuum becomes smooth enough shown here in (SU2) MCG, DLCG, AWL, MAU1 and in (SU3) MCG, MAU1, the same $\rho(b)$ is obtained. Gauge independence! This is naturally expected in the continuum limit.

Similar beautiful scaling behaviors are observed also in **the monopole effective action** $S(k_\mu^{(n)})$ in SU2. From the scaling results of the monopole density and the infrared monopole action, we can say that the new monopoles of the Dirac type have the continuum limit.

6. Future outlook

1. There is in principle no problem concerning the existence of this new color magnetic monopoles in full QCD with light dynamical quarks. To study these Abelian new monopoles of the Dirac type in full QCD is important.

   - What is the scaling behavior with respect to monopole density when small dynamical quarks exist?
   - Could they explain all mass generation in QCD such as hadron masses?
   - What is an infrared effective monopole action in full QCD
   - Is it rewritten by a kind of the dual Abelian Higgs model?
   - Could the monopoles explain also chiral symmetry breaking?

2. What is the origin of the assumed singularity in original gauge fields? Is there any clue in ' beyond the standard model' ?