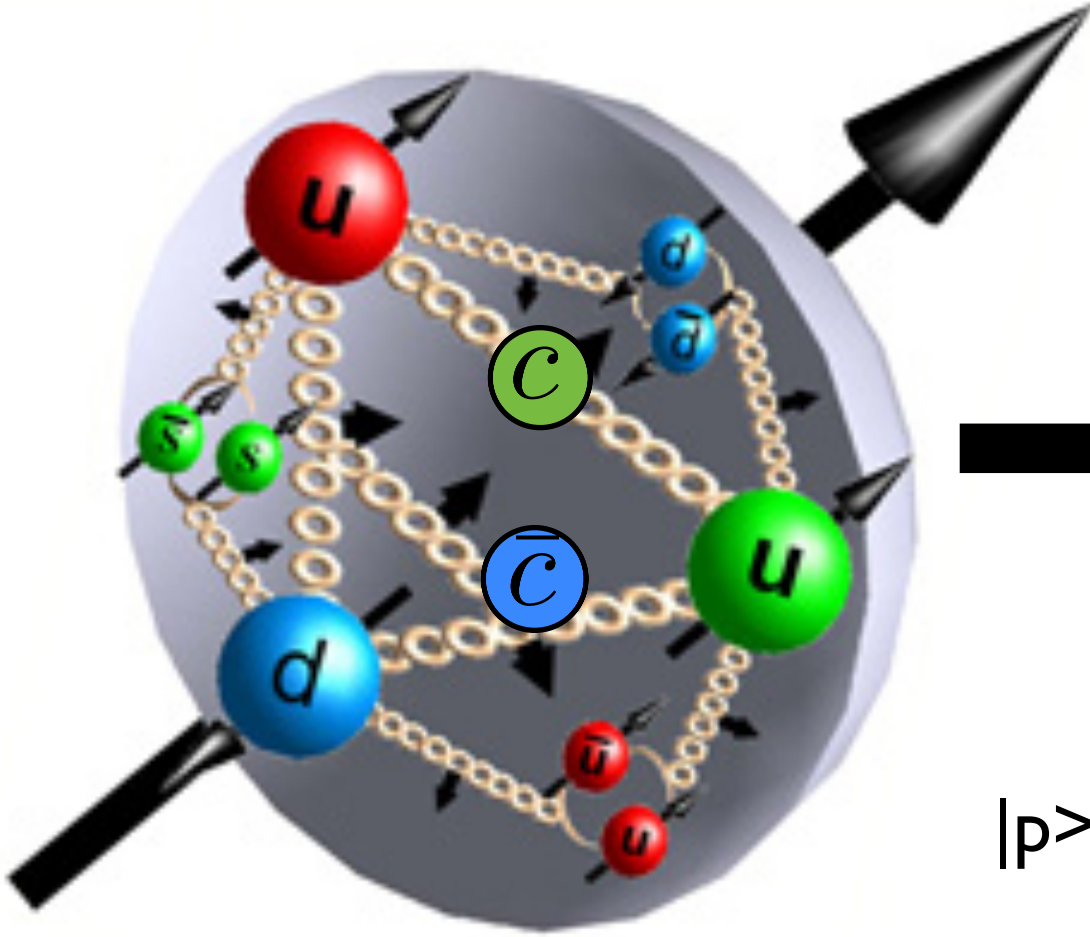
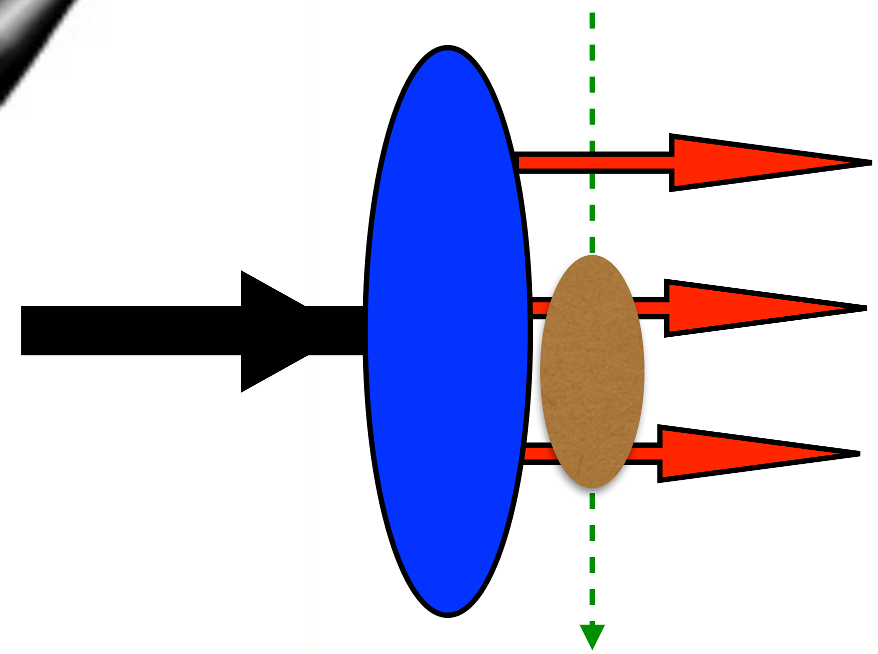
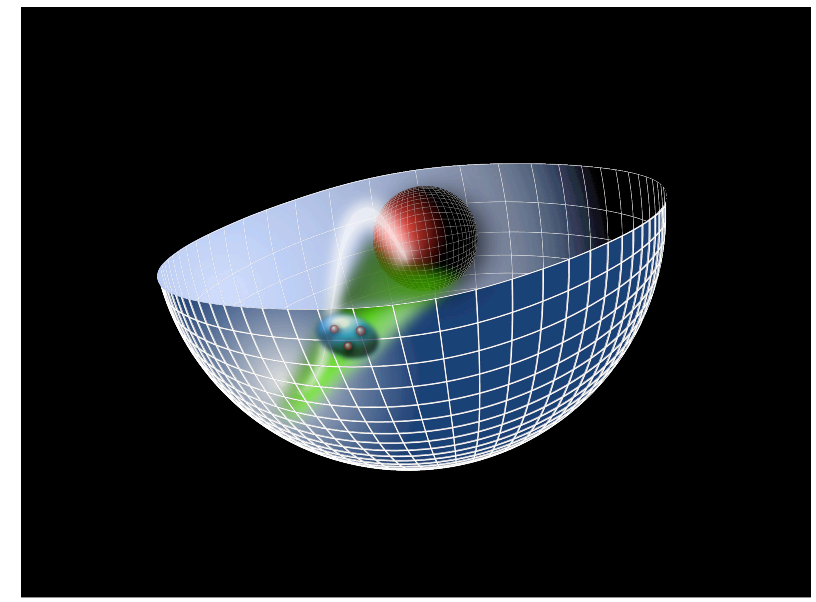
Light-Front Holography — A Novel Approach to QCD Color Confinement, Hadron Spectroscopy and Dynamics



Fixed $\tau = t + z/c$





|p>=|u[ud]> quark-diquark cluster

with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, Ivan Schmidt, F. Navarra, Jennifer Rittenhouse West, G. Miller, Keh-Fei Liu, Tianbo LIu, Liping Zou, S. Groote, S. Koshkarev, Xing-Gang Wu, Sheng-Quan Wang, Cedric Lorcè, R. S. Sufian, A. Deur, R. Vogt, G. Lykasov, S. Gardner, S. Liuti

STRONG 2020

Trento ECT*

Gauge Topology, Flux Tubes and Holographic Models

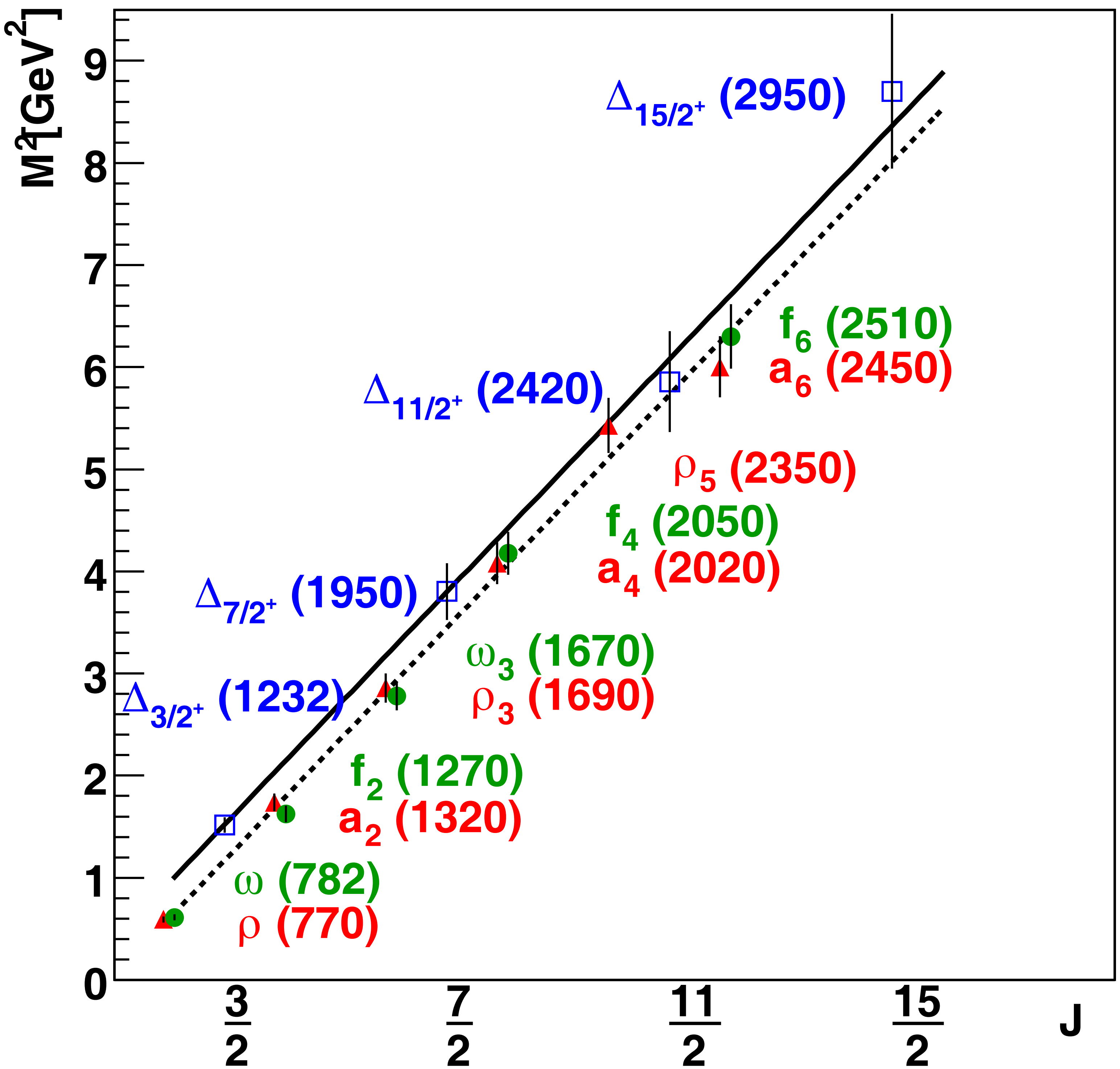
Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

May 23, 2022

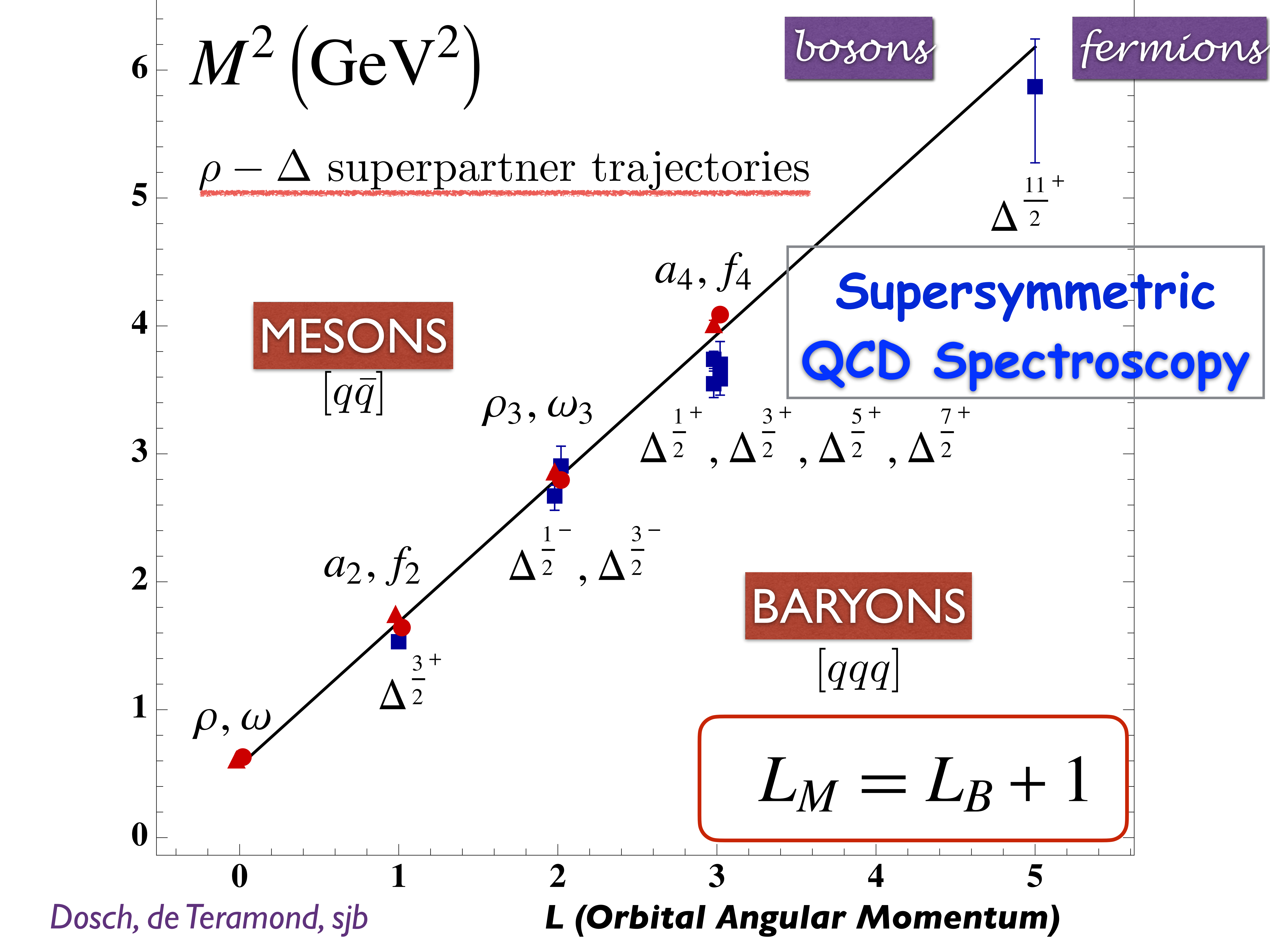
Mesons and Baryons: Same Regge Slope $M^2 \propto J$!

M²[GeV²]



The leading Regge trajectory: Δ resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with J = L+S.

E. Klempt and B. Ch. Metsch



M^2 (GeV^2)
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
Supersymmetric
QCD Spectroscopy
$L_M = L_B + 1$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta \frac{3}{2}^+$
$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$
$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$
$\Delta \frac{11}{2}^+$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

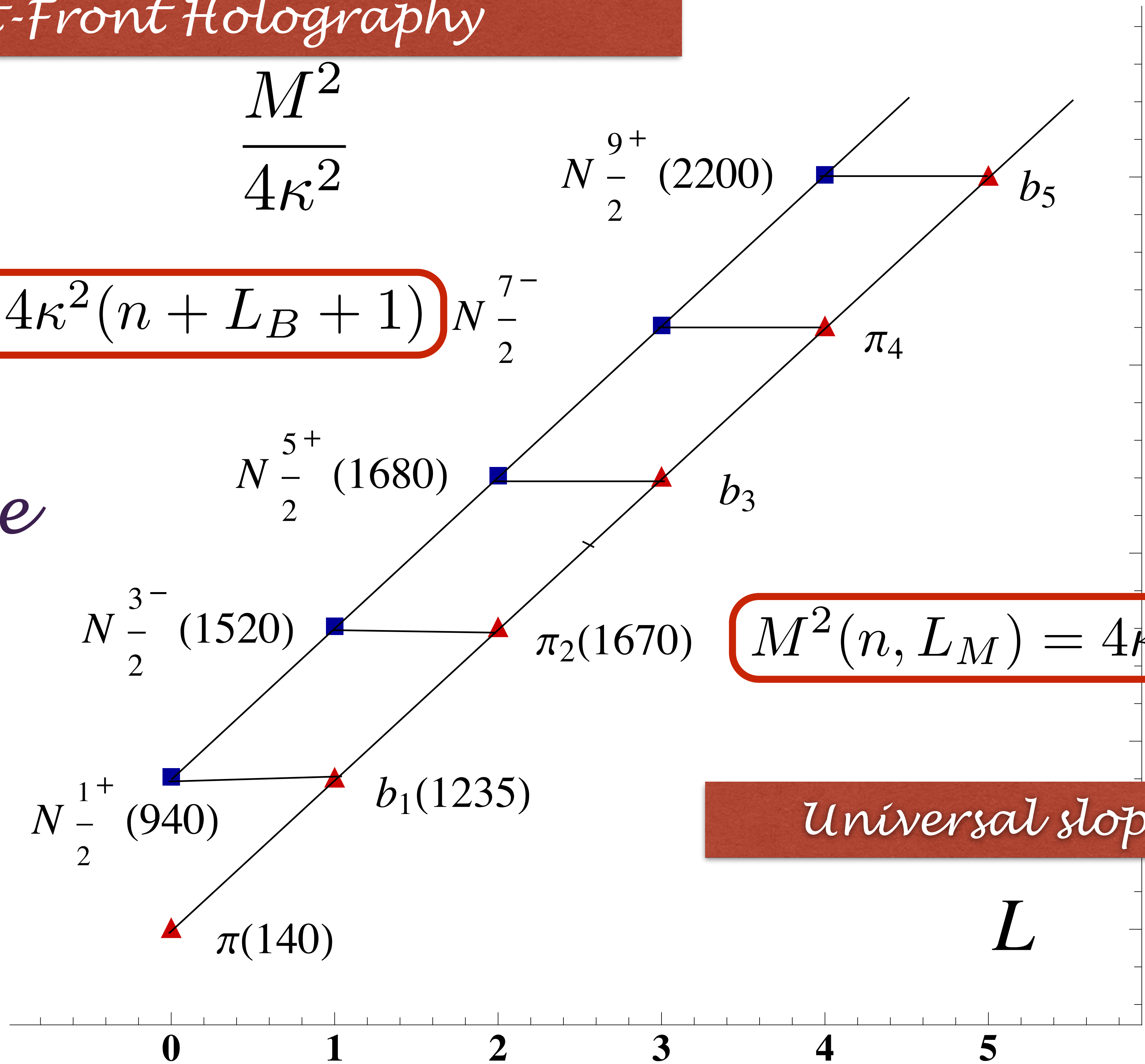
Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

$$\frac{M^2}{4\kappa^2}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

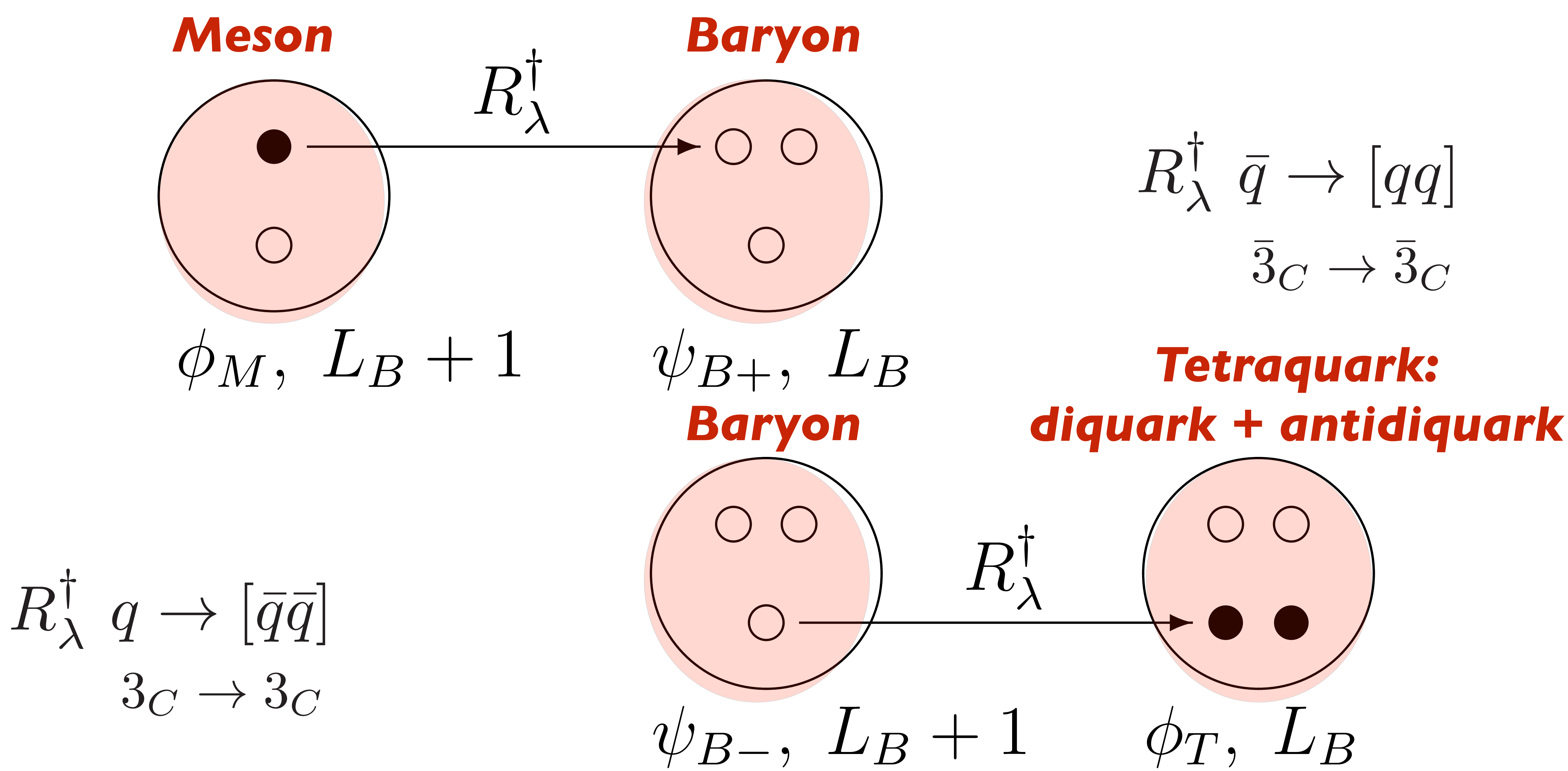
Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

- **Color Confinement, Analytic form of confinement potential**
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L**
- **Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- *Color Confinement*
- *Origin of the QCD Mass Scale*
- *Meson and Baryon Spectroscopy*
- *Exotic States: Tetraquarks, Pentaquarks, Gluonium,*
- *Universal Regge Slopes: n, L, Mesons and Baryons*
- *Almost Massless Pion: GMOR Chiral Symmetry Breaking*

  $M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}\left((m_u + m_d)^2\right)$

- *QCD Coupling at all Scales* $\alpha_s(Q^2)$
- *Eliminate Scale Uncertainties and Scheme Dependence*

$$\mathscr{L}_{QCD} \to \psi_n^H(x_i, \overrightarrow{k}_{\perp i}, \lambda_i)$$

Valence and Higher Fock States

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale

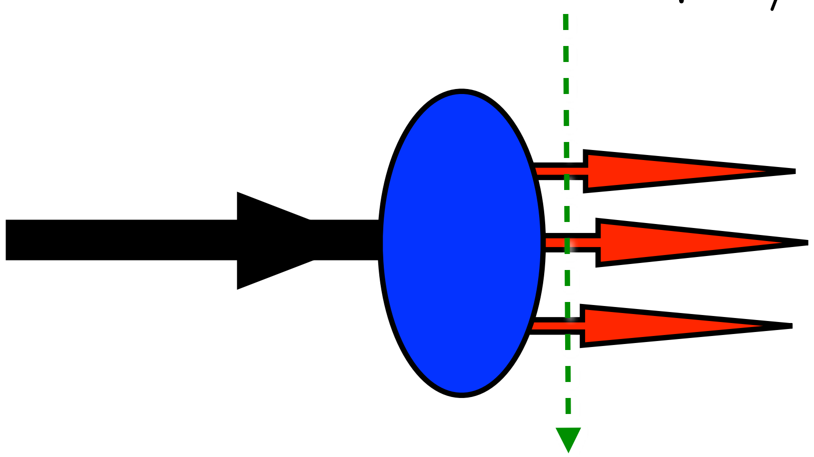
AdS/QCD
Light-Front Holography
Superconformal Algebra

No parameters except for quark masses!

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

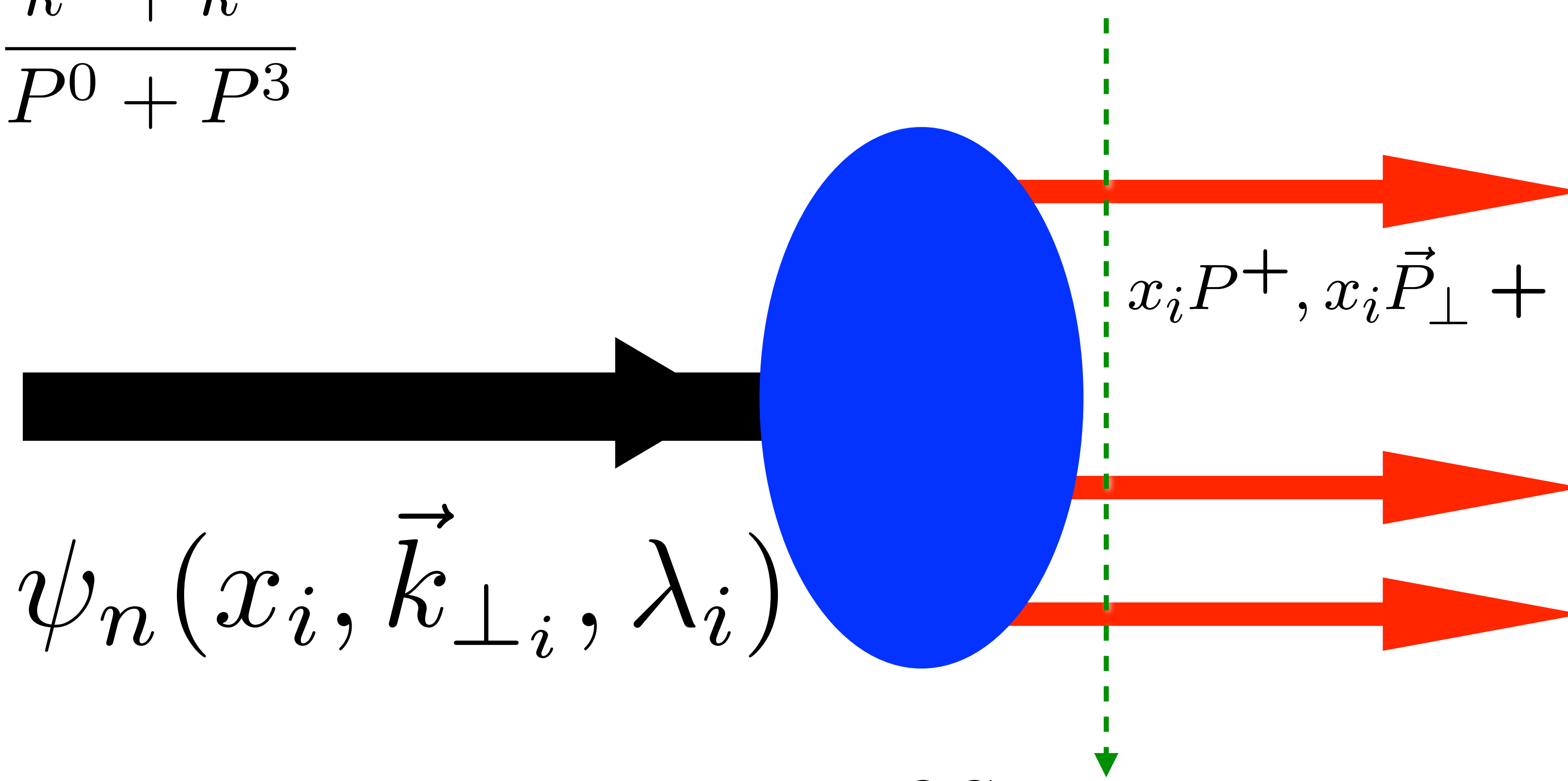
Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$P^+, \vec{P}_\perp$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

Eigenstate of LF Hamiltonian

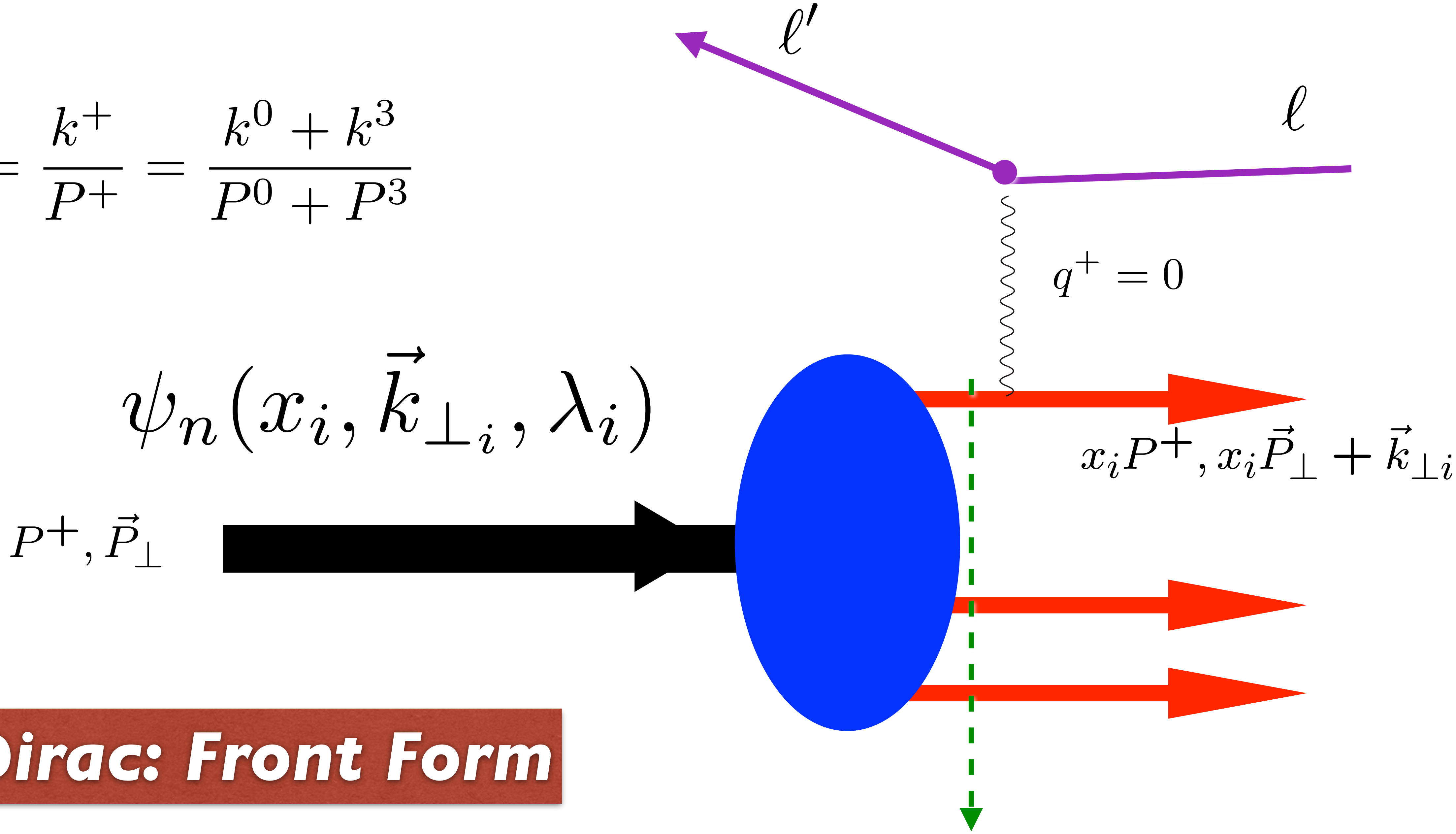
$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

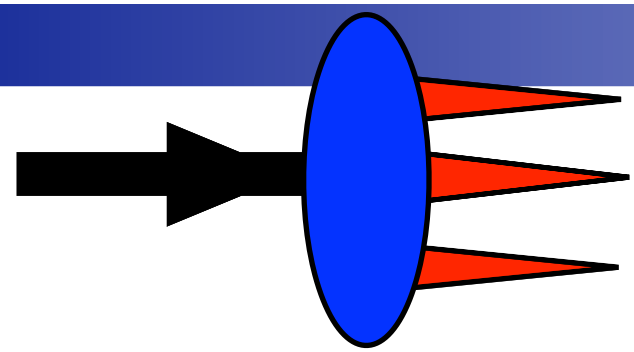
Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions underly hadronic observables

Lorce, Pasquini



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

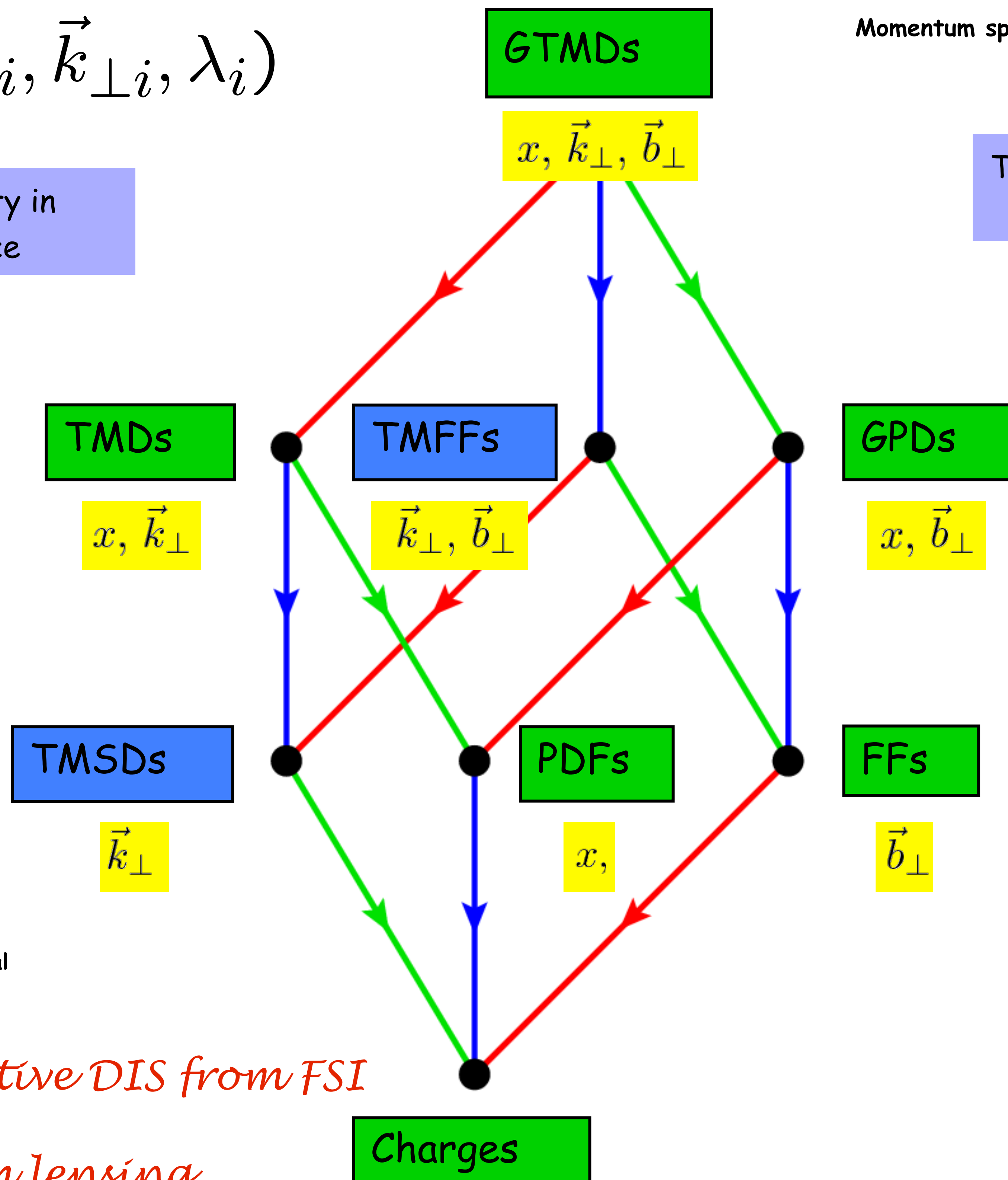
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in momentum space

Transverse density in position space

Weak transition form factors

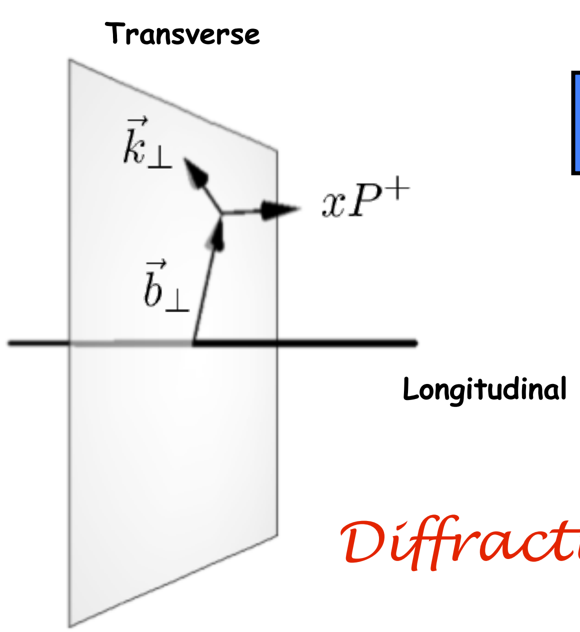


DGLAP, ERBL Evolution Factorization Theorems

$\int d^2 b_\perp$

$\int dx$

$\int d^2 k_\perp$



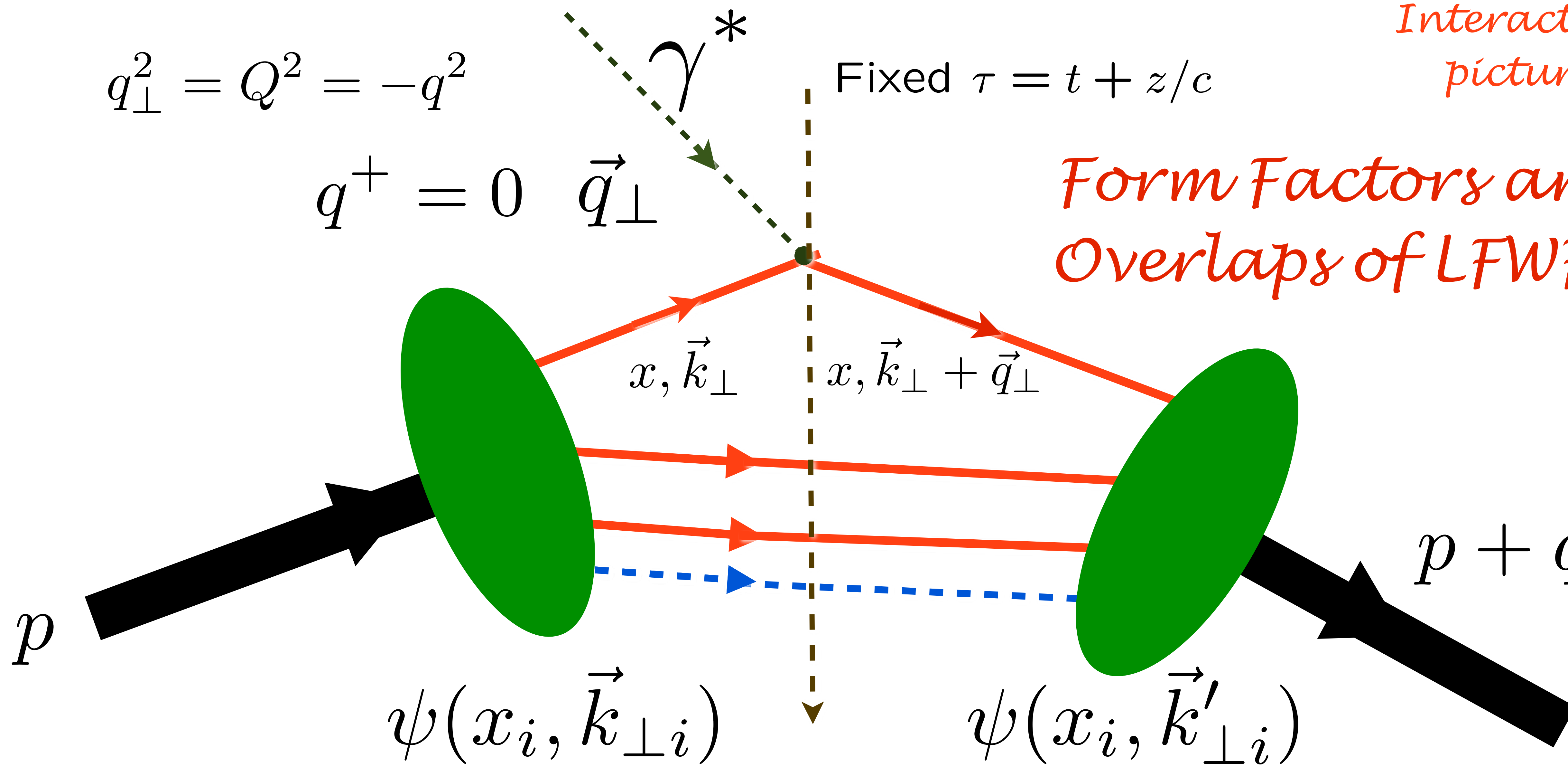
Diffractive DIS from FSI

Sivers, T-odd from lensing

$$< p+q|j^+(0)|p > = 2p^+ F(q^2)$$

Front Form

Interaction picture



Form Factors are Overlaps of LFWFs

struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Transverse size $\propto \frac{1}{Q}$

Advantages of the Dirac's Front Form for Hadron Physics
Poincare' Invariant

Physics Independent of Observer's Motion

- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent: no boosts, no pancakes!**

Penrose, Terrell, Weisskopf

- **Same structure function measured at an e p collider and the proton rest frame**
- **No dependence of hadron structure on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial up to zero modes**
- **Implications for Cosmological Constant**

Roberts, Shrock, Tandy, sjb

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i \left[\frac{m^2 + k^2_\perp}{x}\right]_i + H^{int}_{LF}$$

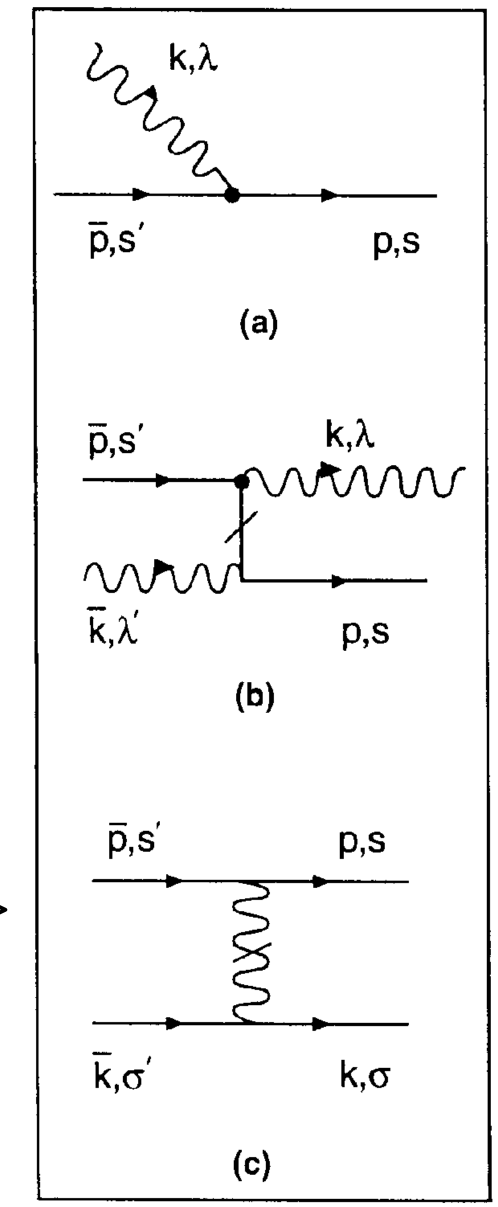
H^{int}_{LF}: Matrix in Fock Space

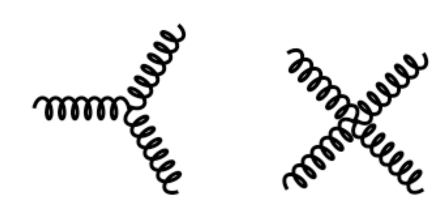
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass





H^{int}_{LF}

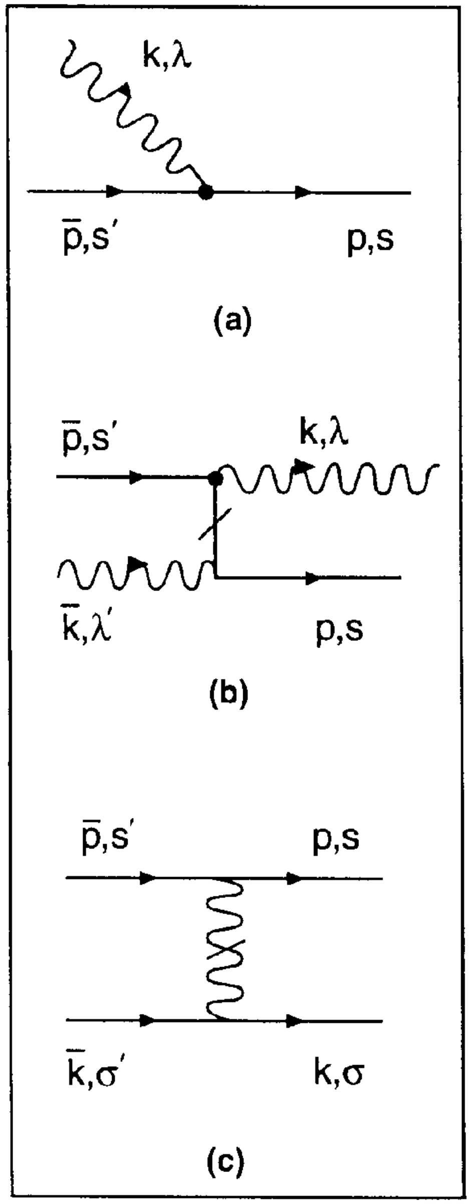
Light-Front QCD

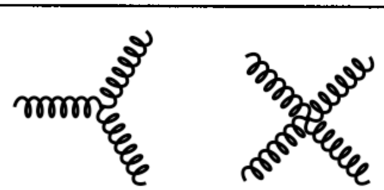
Heisenberg Equation

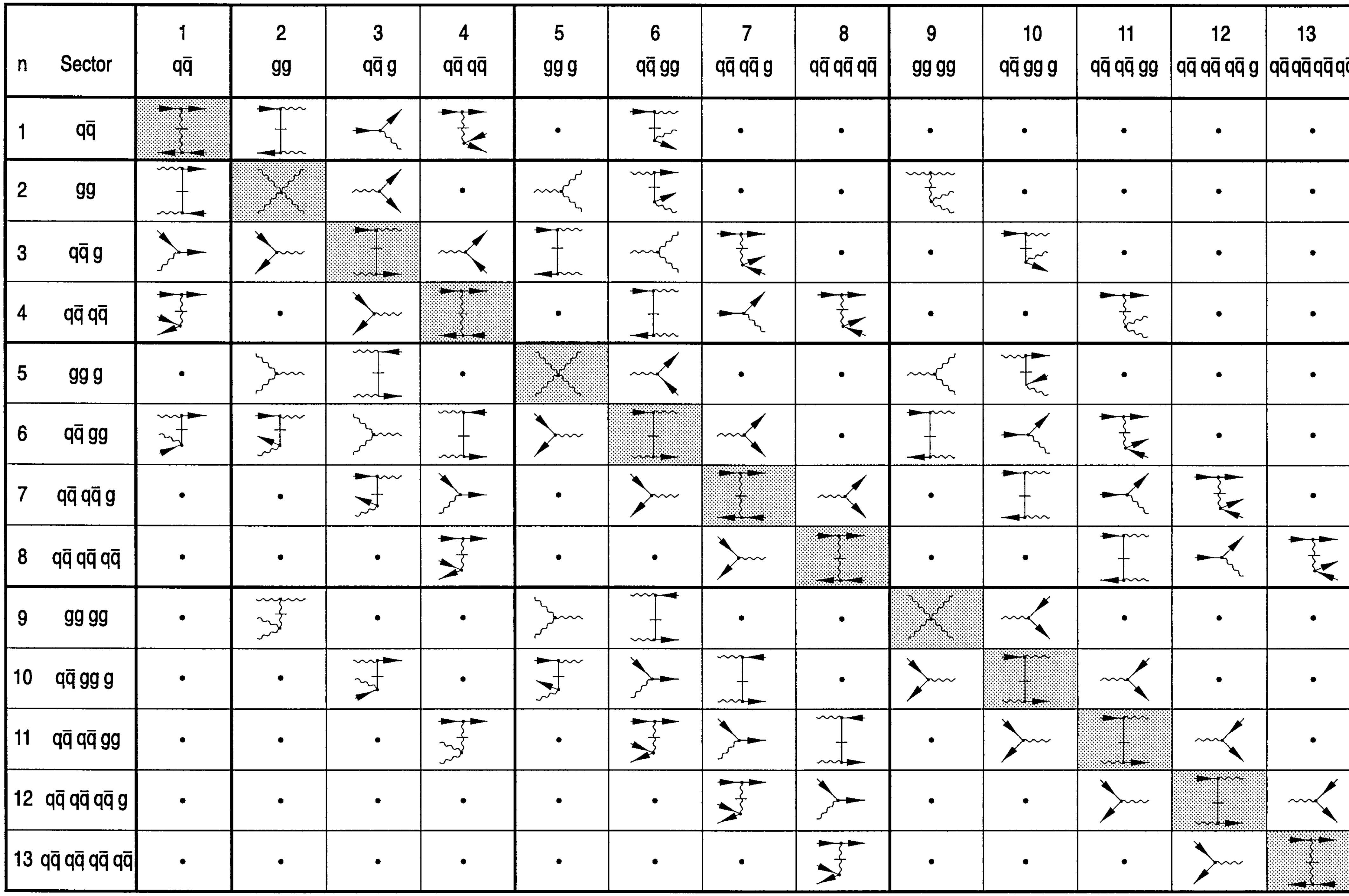
$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 \, |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb







Minkowski space; frame-independent; no fermion doubling; no ghosts
trivial vacuum

Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + \cdots$$

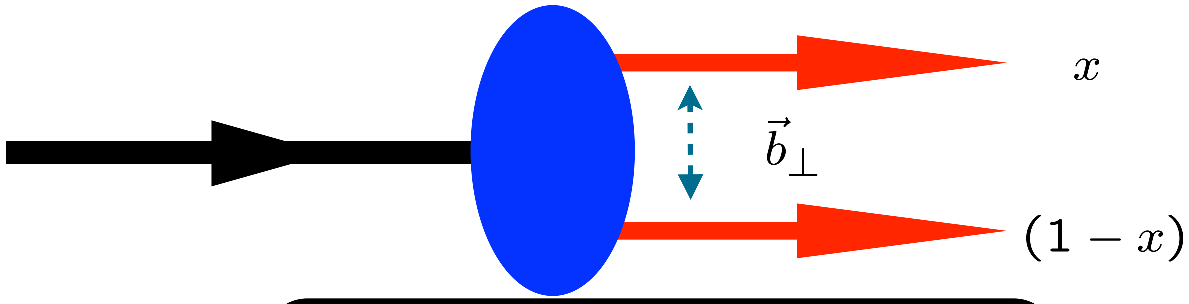
- *"History": Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes! Cluster Decomposition*
- *Wick Theorem applies, but few amplitudes since all $k^+ > 0$.*
- *J_z Conservation at every vertex* $$| \sum_{initial} S^z - \sum_{final} S_z | \leq n$$ *at order g^n*
- *Unitarity is explicit*

K. Chiu, Lorcé, sjb

- *Loop Integrals are 3-dimensional* $$\int_0^1 dx \int d^2 k_\perp$$
- *hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x) b_\perp^2$$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\rm eff}^{LF}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \, \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Azimuthal Basis ζ, ϕ

Single variable Equation

$m_q = 0$

AdS/QCD:

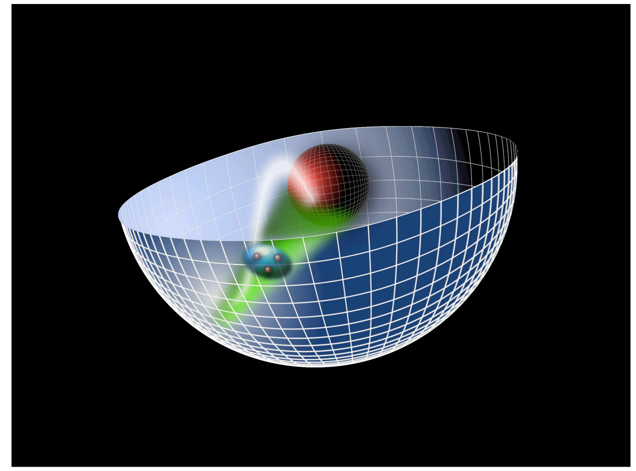
$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential!

Semiclassical first approximation to QCD

Sums an infinite # diagrams

de Tèramond, Dosch, sjb



AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

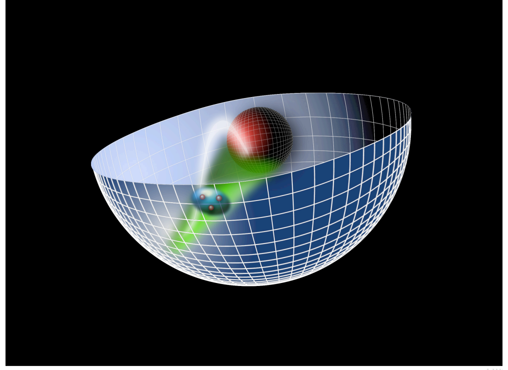
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

GeV units external to QCD: Only Ratios of Masses Determined

Maldacena

AdS5



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

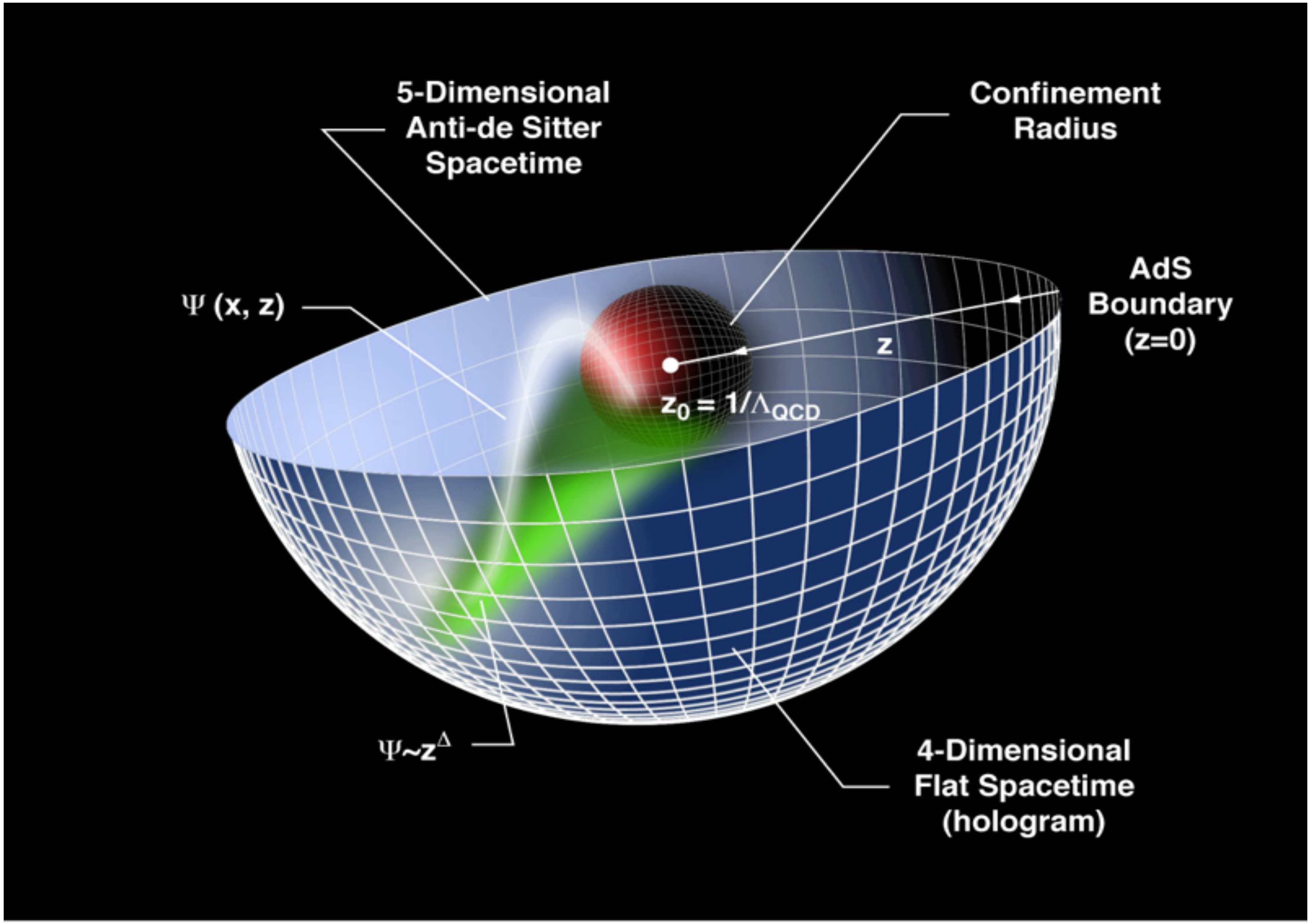
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Applications of AdS/CFT to QCD

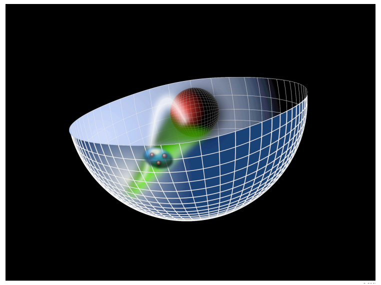


Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond and H. Guenter Dosch

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu}x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

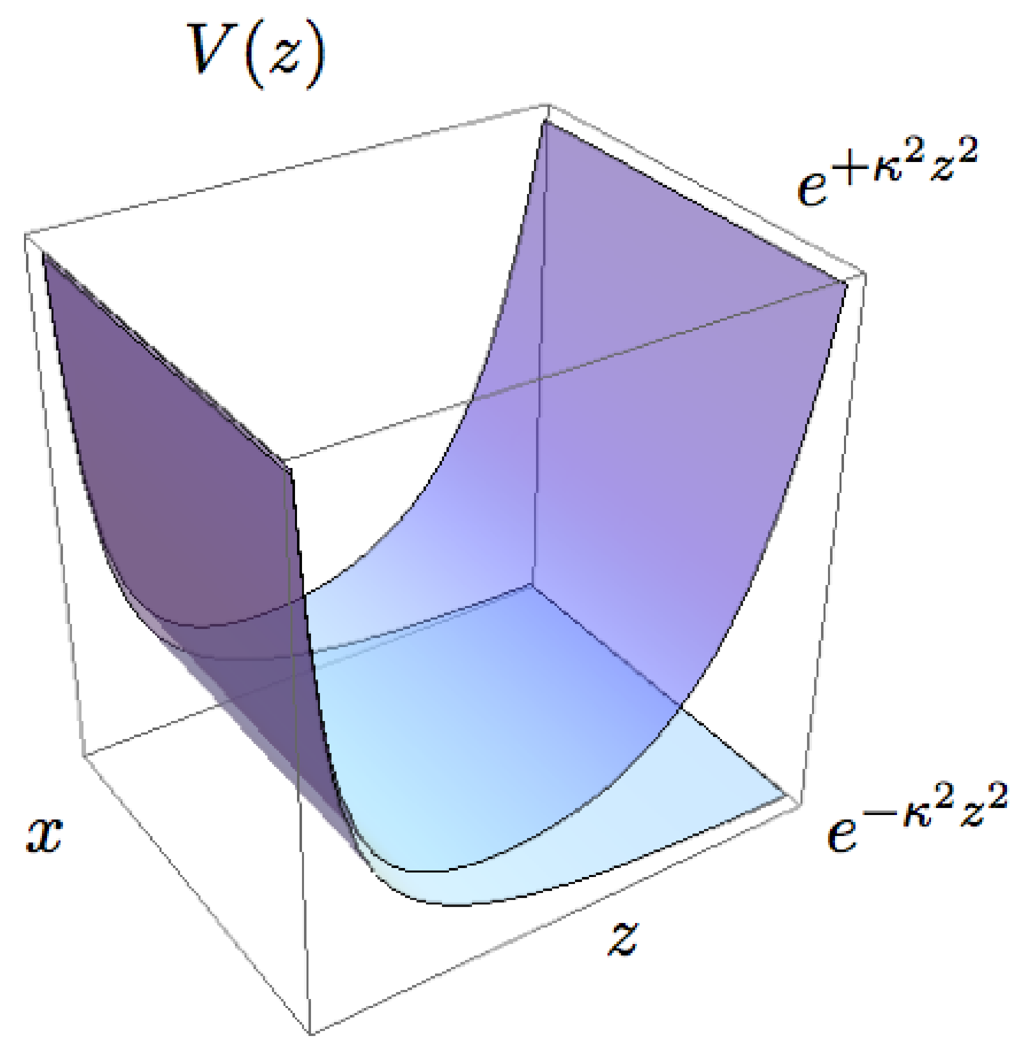
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS$_5$

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Light-Front Holography

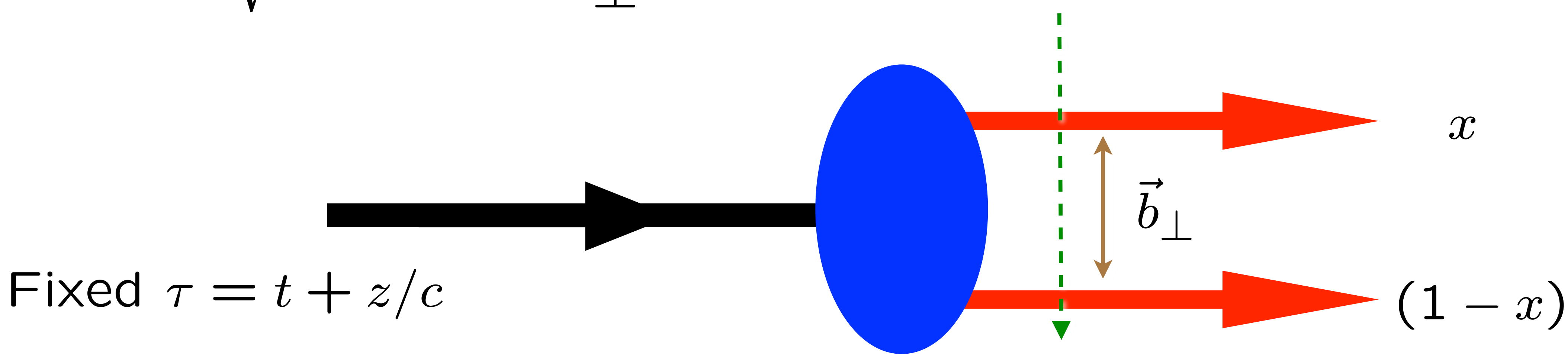
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

G. de Teramond, H. G. Dosch, sjb

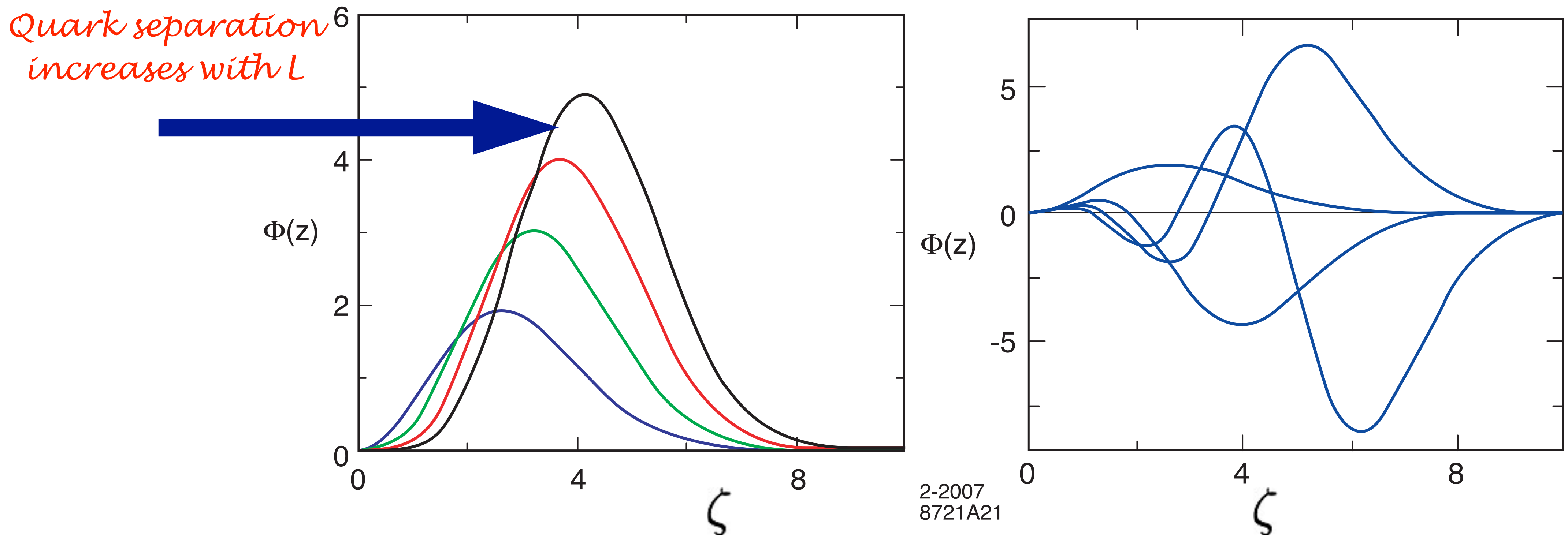
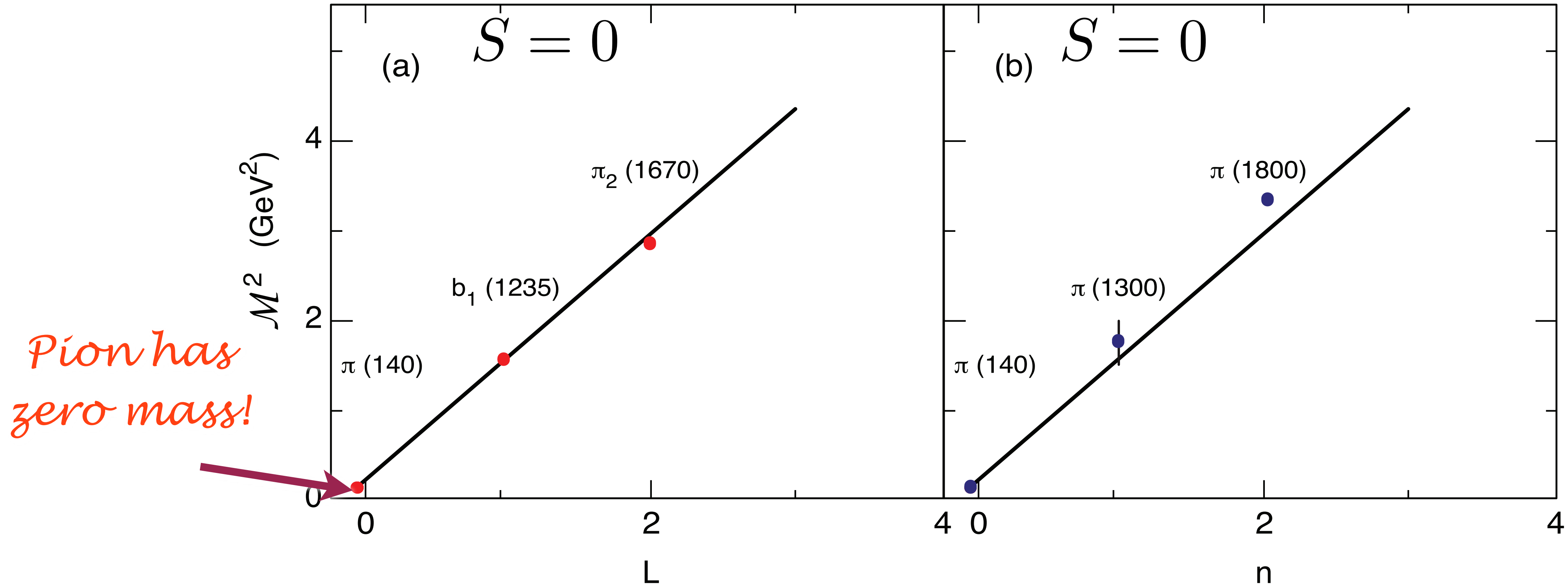


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

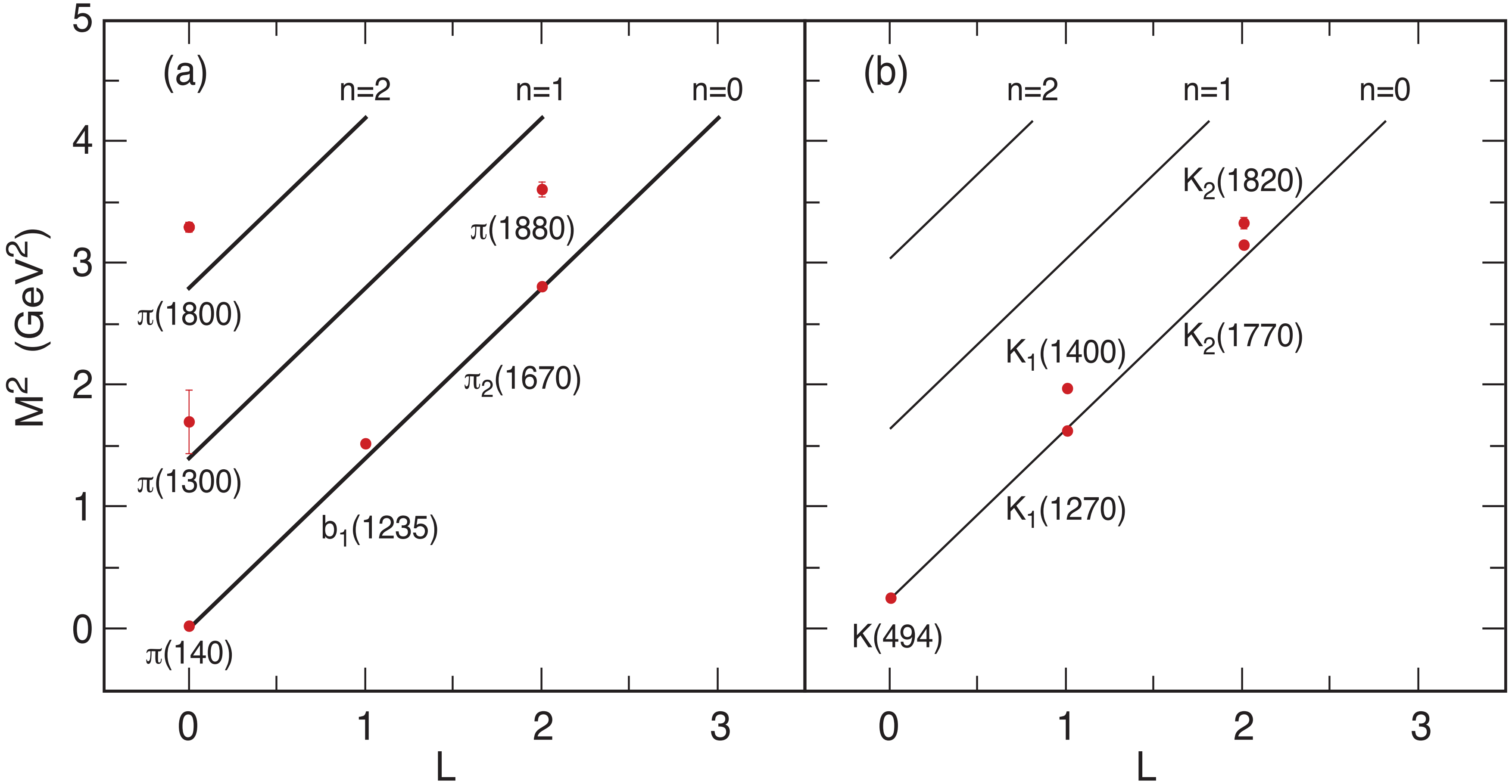
Soft Wall Model

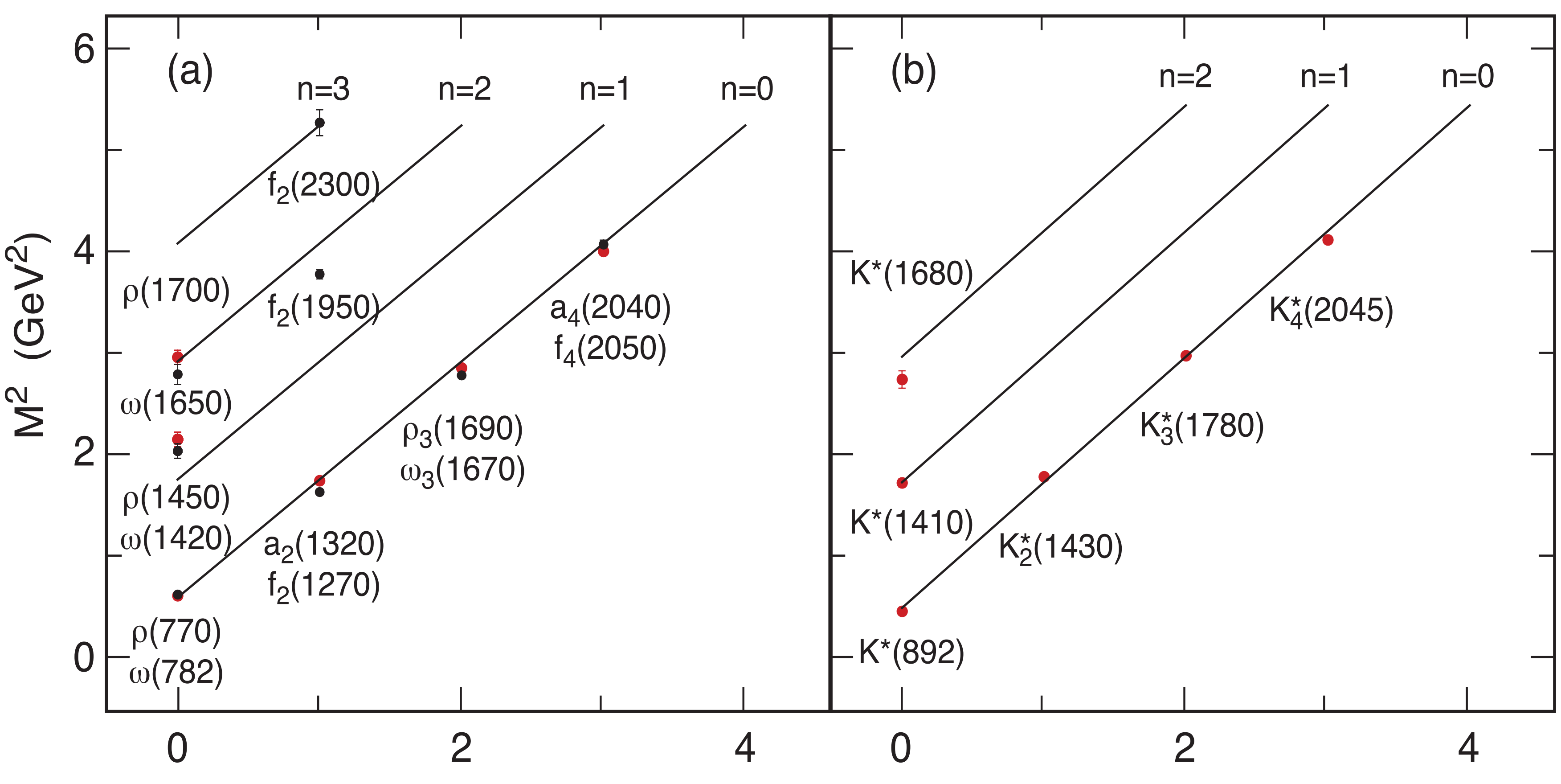
Same slope in n and L!

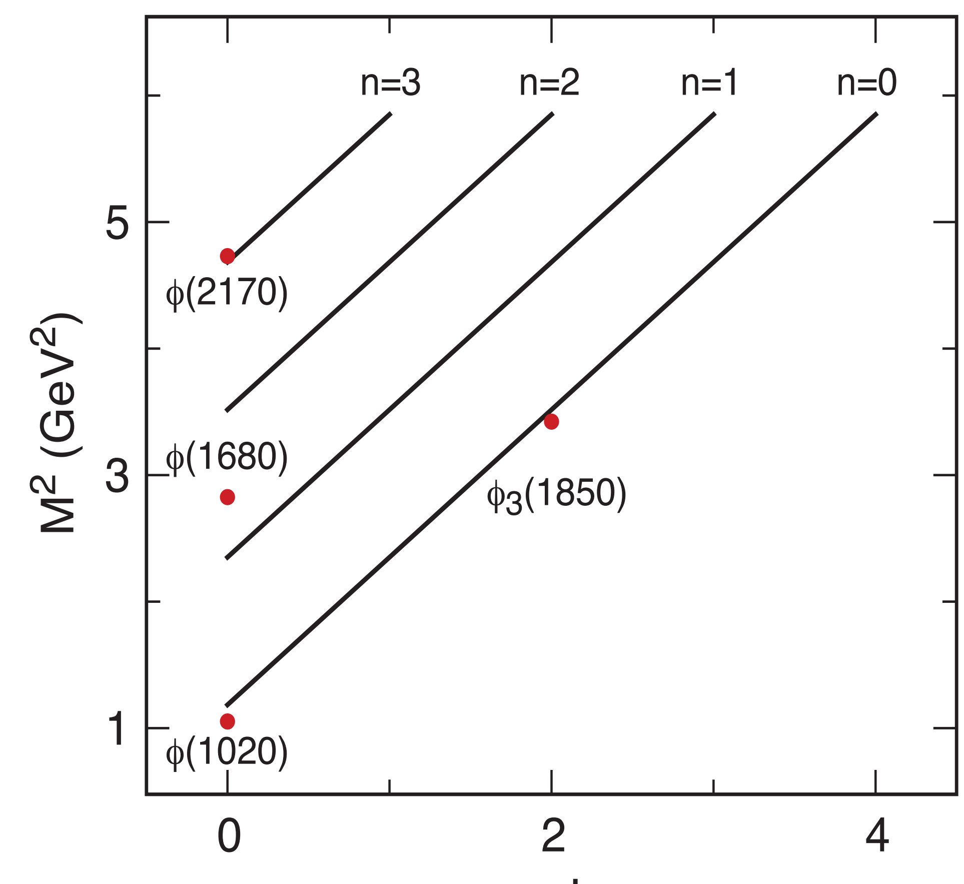


Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$m_q = 0$

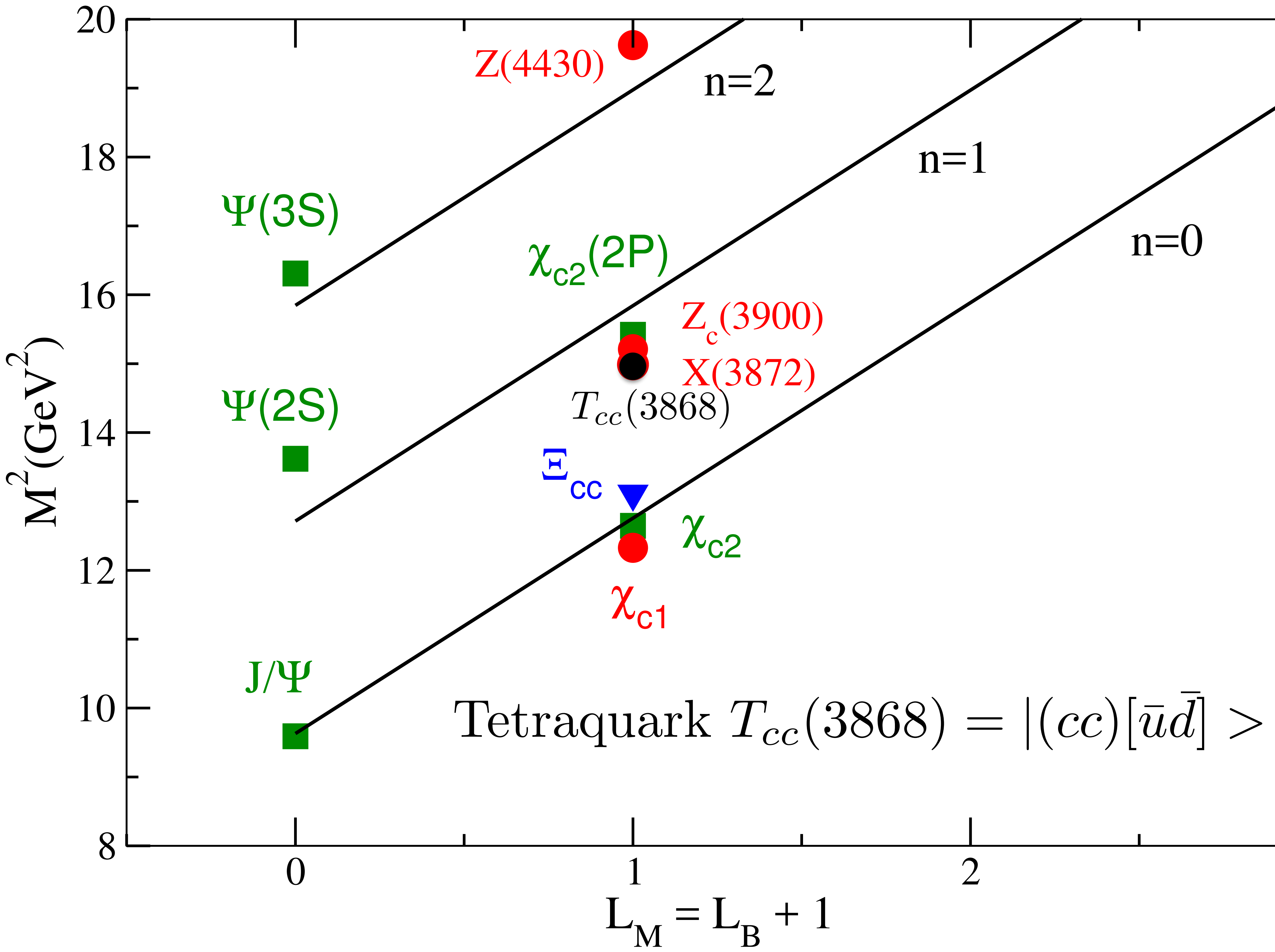






$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L



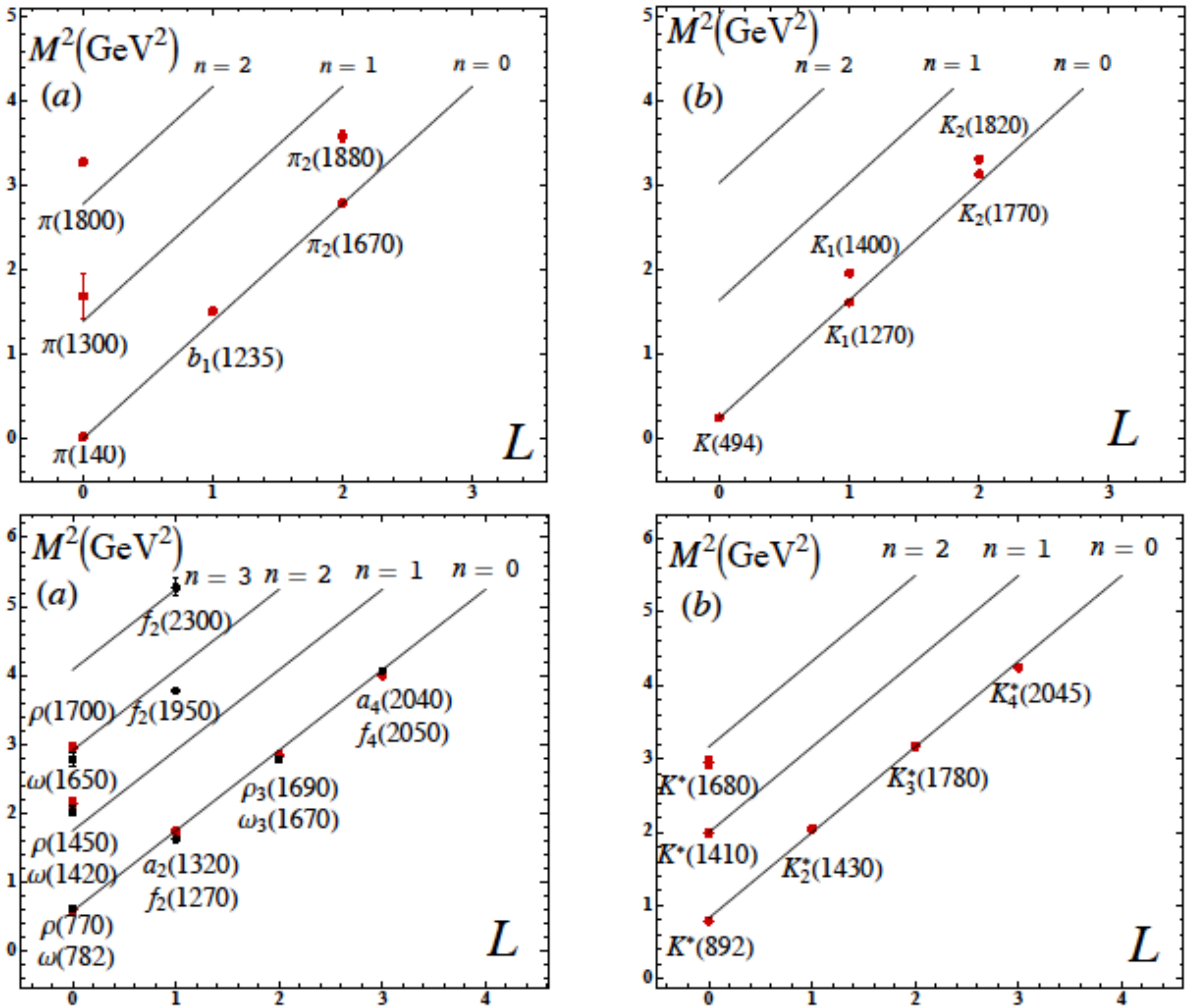
$Mesons: GreenSquare, Baryons(BlueTriangle), Tetraquarks(RedCircle)$

De Tèramond, Dosch, sjb

$$m_u = m_d = 46\ \text{MeV}, \quad m_s = 357\ \text{MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism



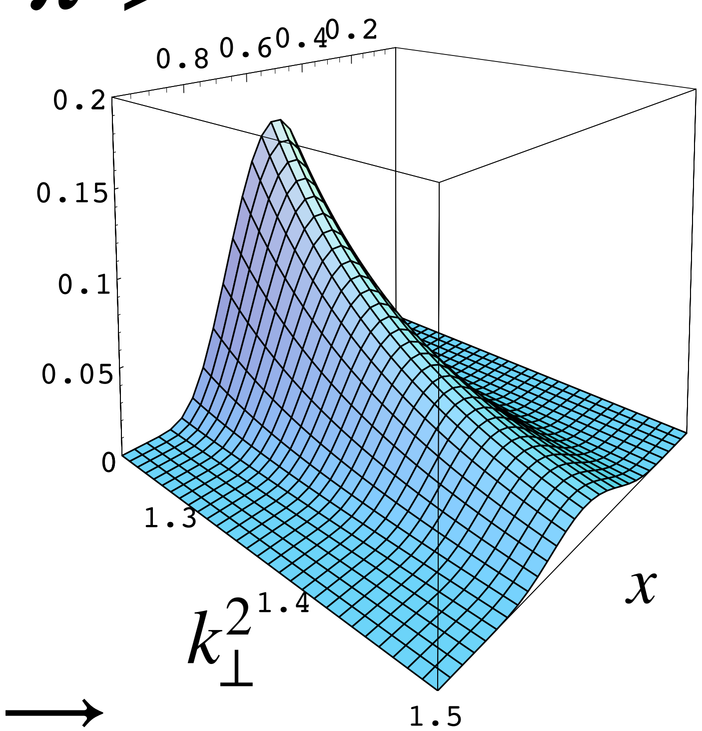
Effective mass from $m(p^2)$

Roberts, et al.

The Pion's Valence Light-Front Wavefunction

- Relativistic Quantum-Mechanical Wavefunction of the pion eigenstate $H^{QCD}_{LF} | \pi > = m^2_\pi | \pi >$

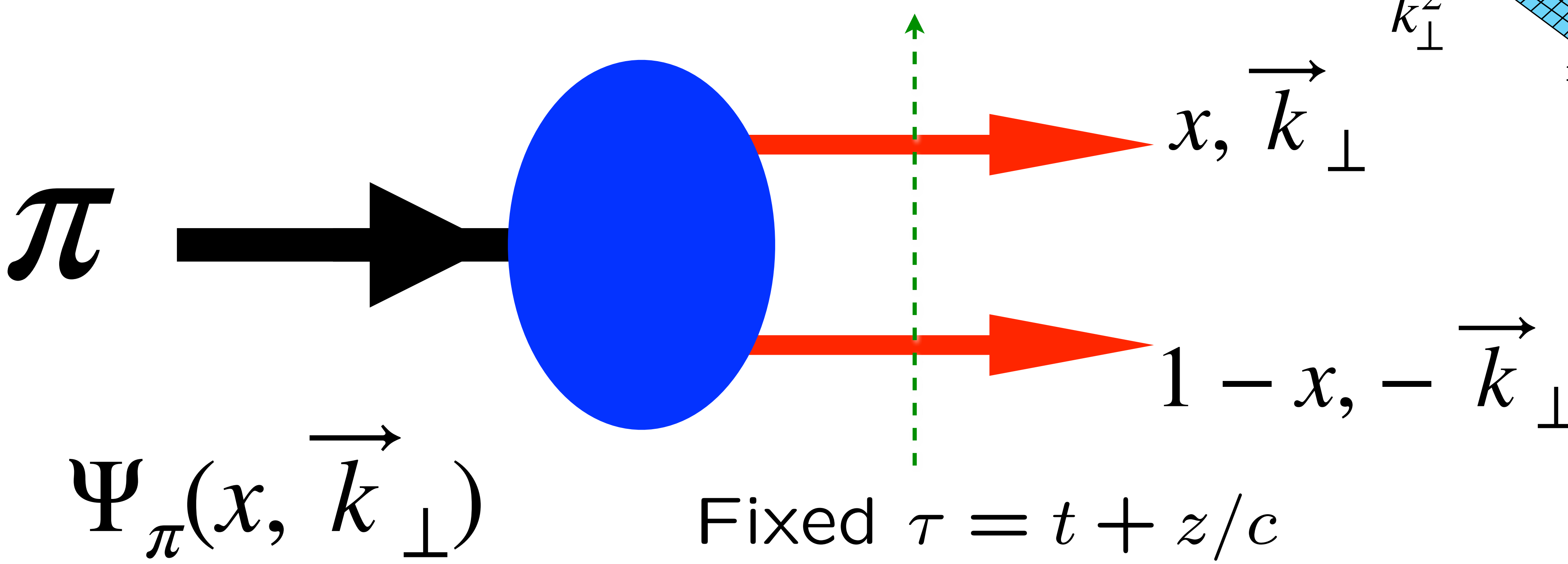
$$\Psi_\pi(x, \vec{k}_\perp) = < q(x, \vec{k}_\perp)\bar{q}(1-x, -\vec{k}_\perp) | \, \pi >$$



- Independent of the observer's or pion's motion

- No Lorentz contraction; causal

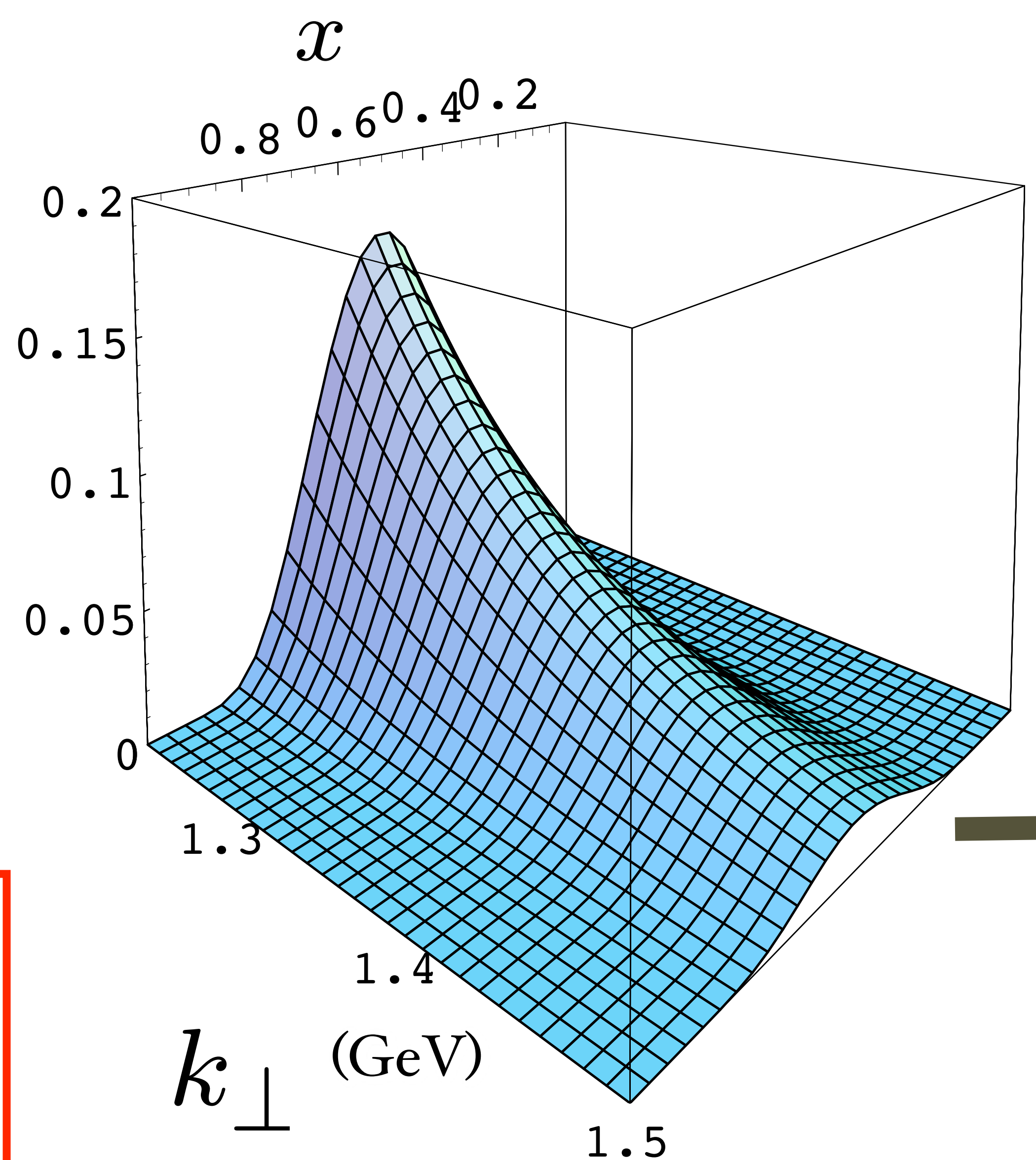
- ***Confined*** quark-antiquark bound state



Prediction from AdS/QCD: Meson LFWF

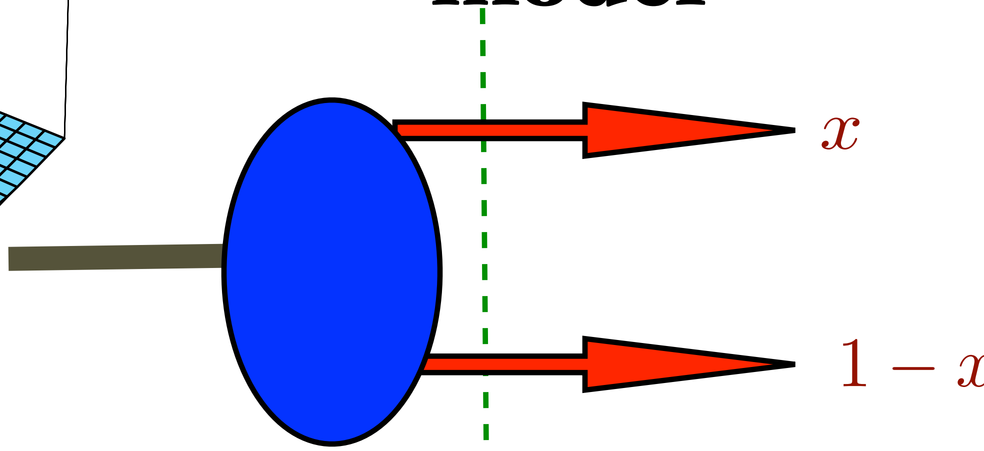
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

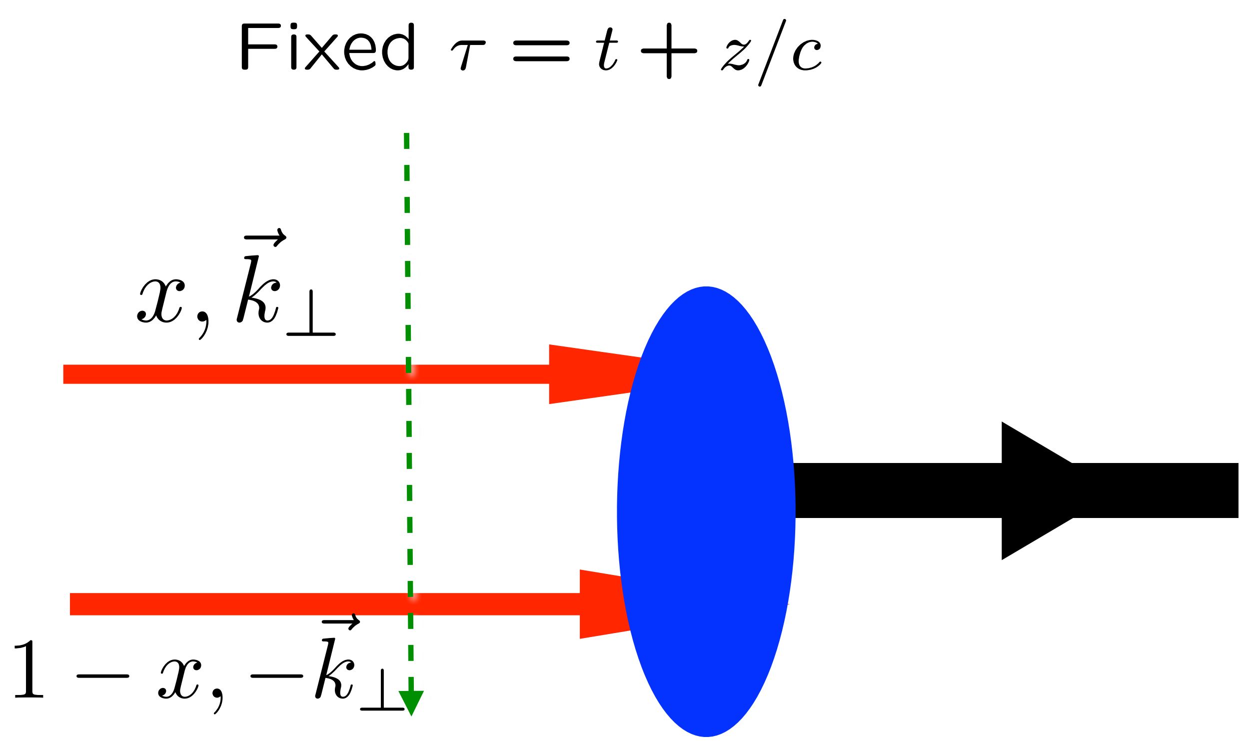
$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

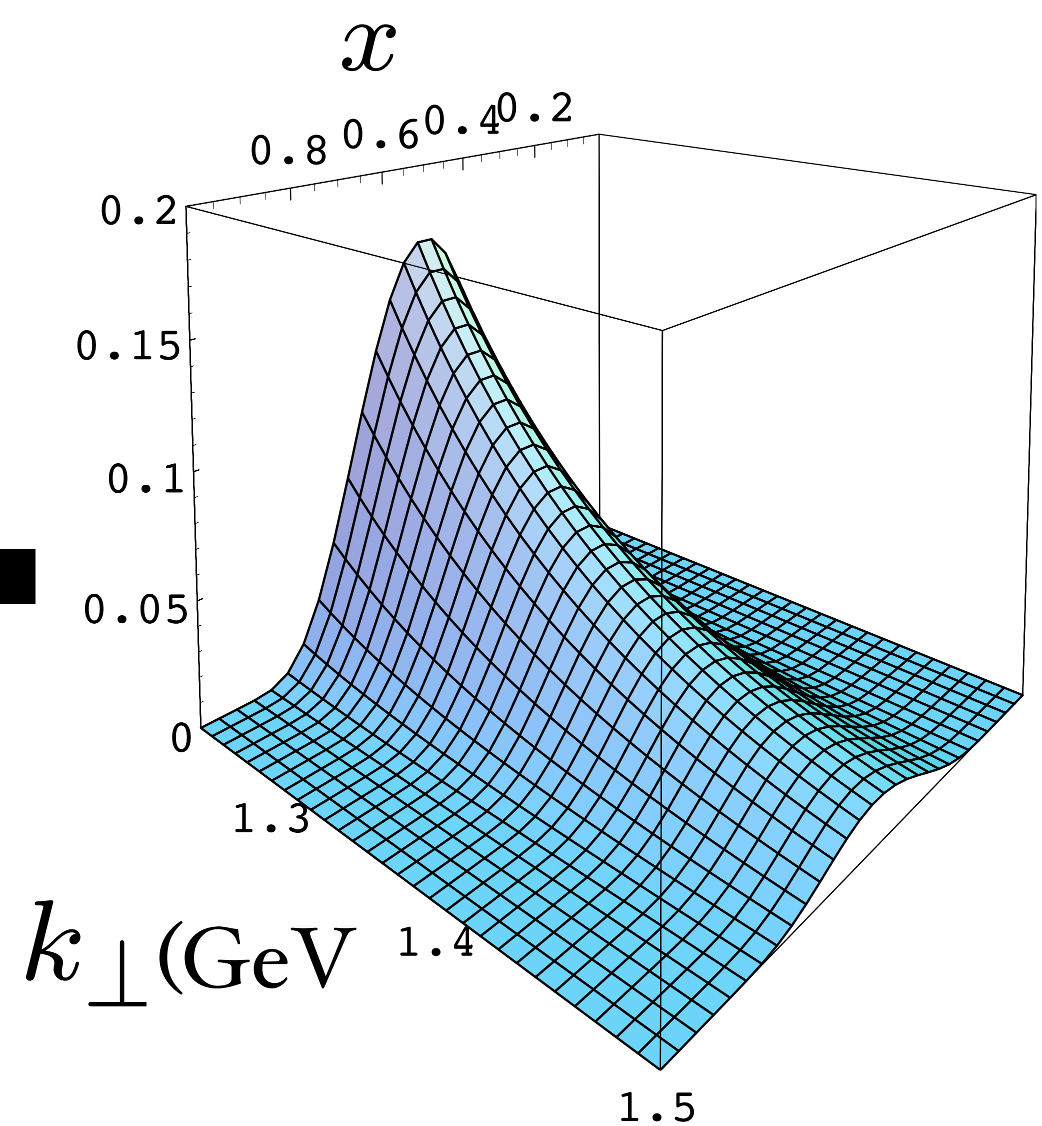
Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$



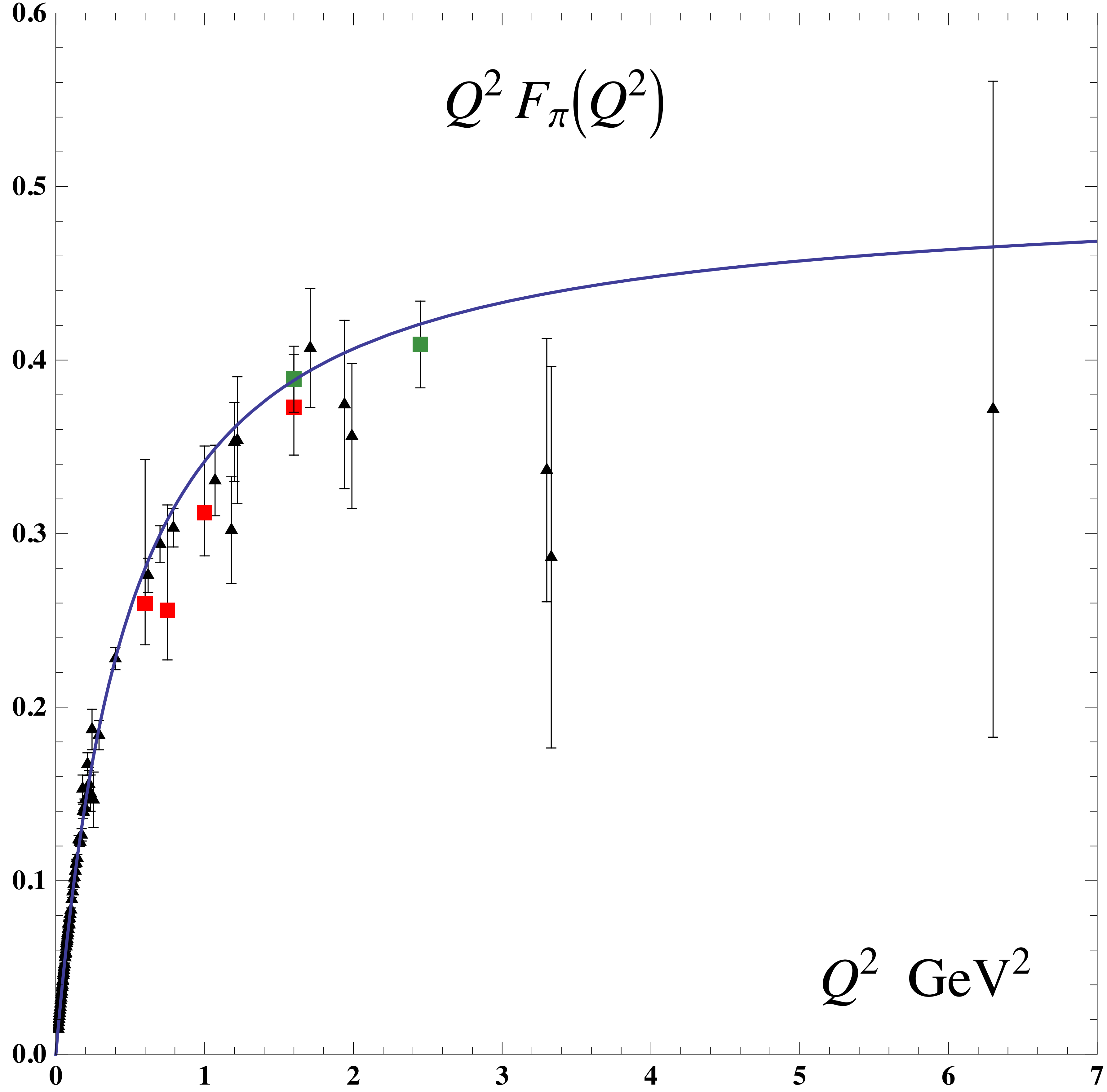


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

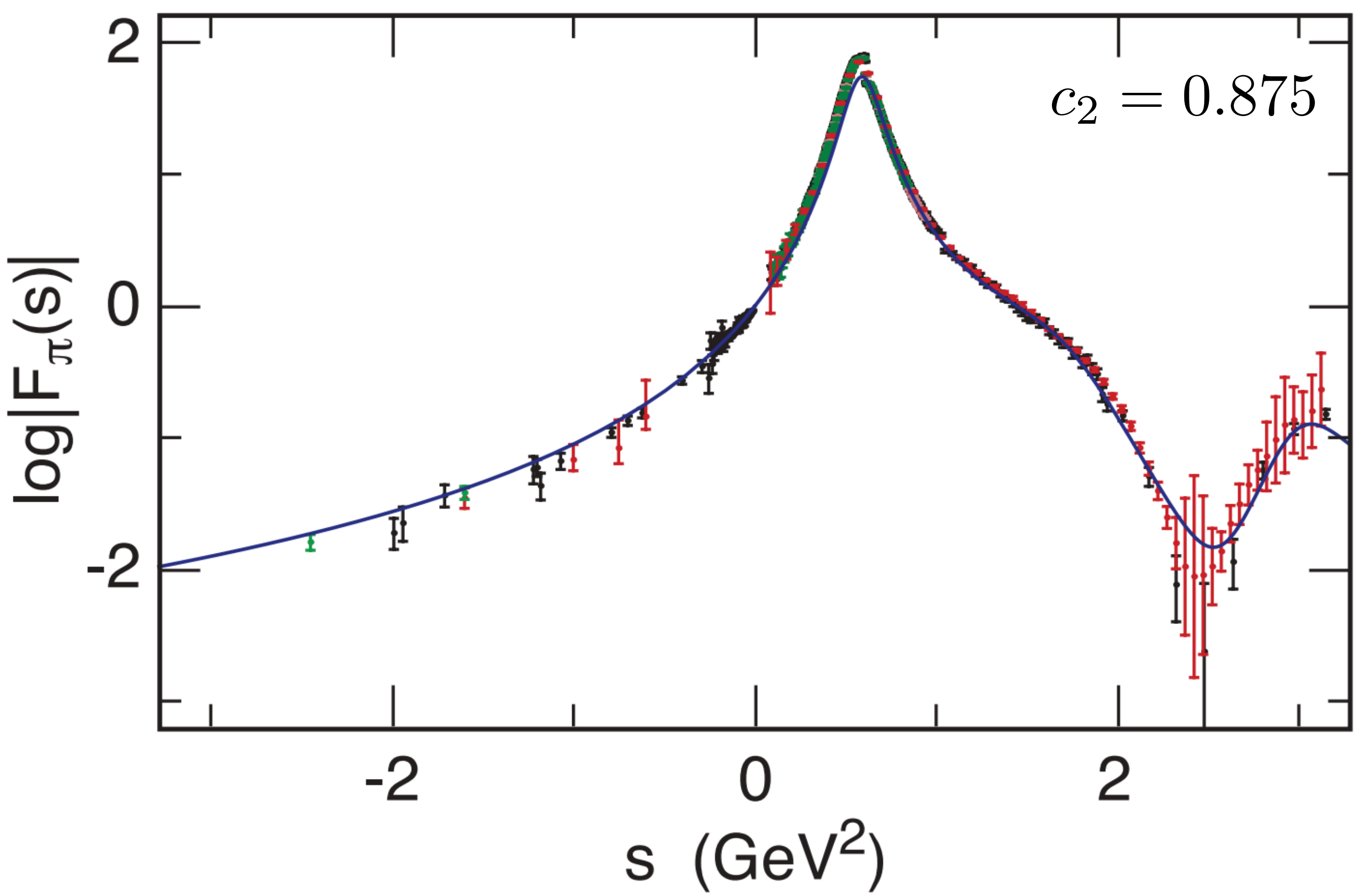
Proceeds in LF time τ within casual horizon
Instant time violates causality



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Pion EM Form Factor

Pion form factor compared with data



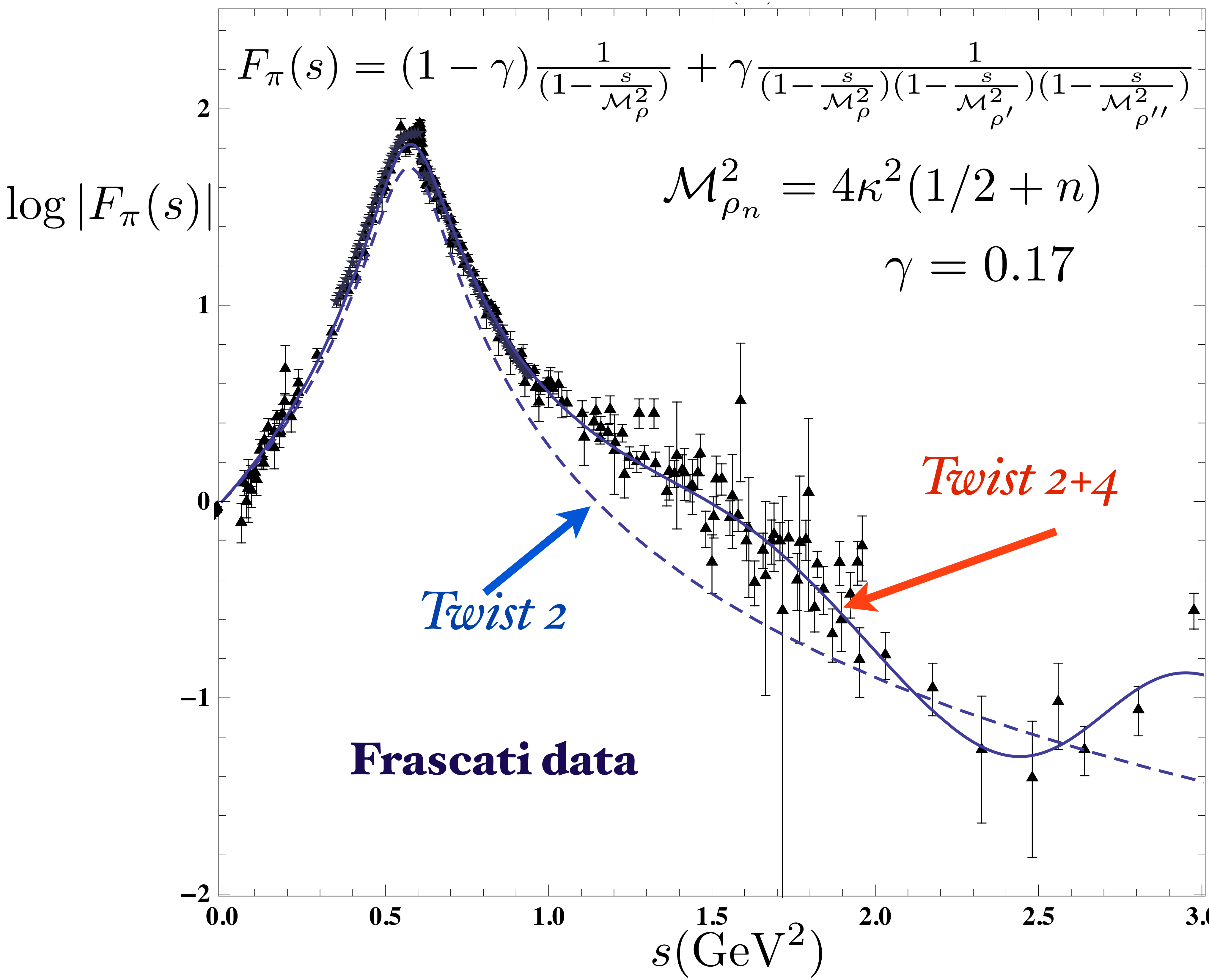
$$F_\pi(t) = \sum_\tau P_\tau F_\tau(t) \qquad \sum_\tau P_\tau = 1$$

Truncated at twist-$\tau = 4$

$$F_\pi(t) = c_2 F_{\tau=2}(t) + (1 - c_2) F_{\tau=4}(t)$$

G.F. de Téramond and S.J. Brodsky, Proc. Sci. LC2010 (2010) 029.
S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015). [Sec. 6.1.5]

Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



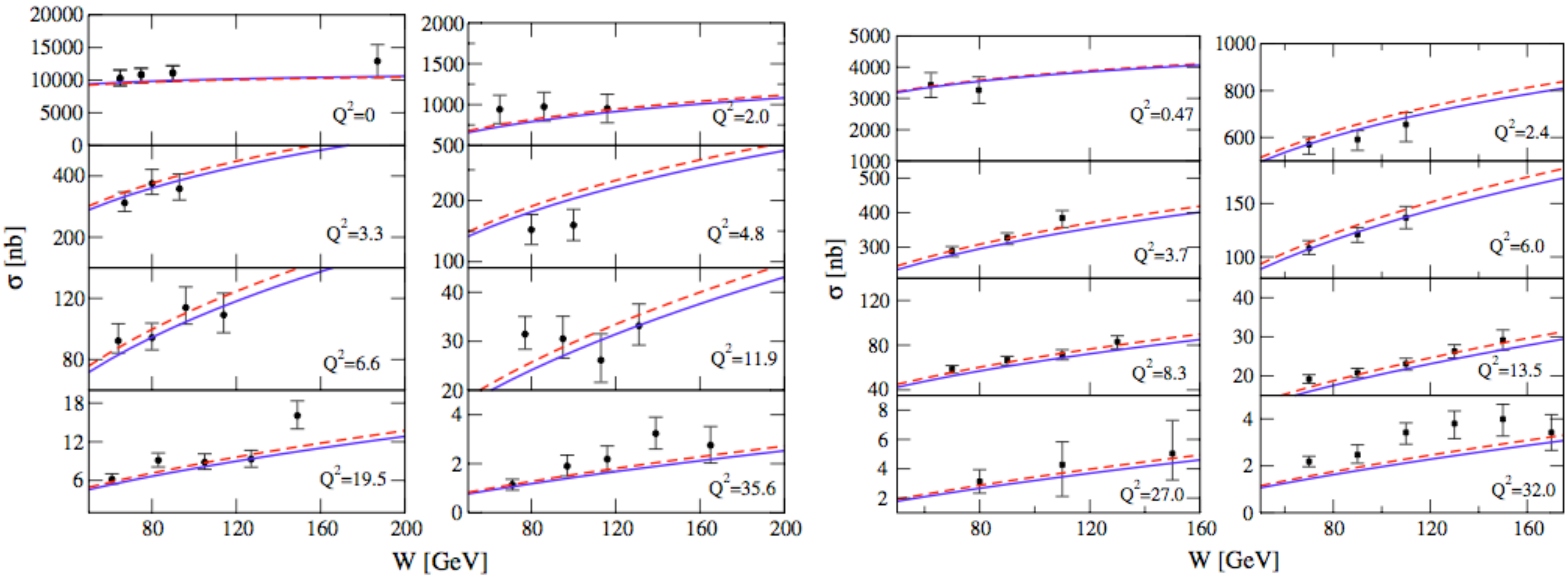
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

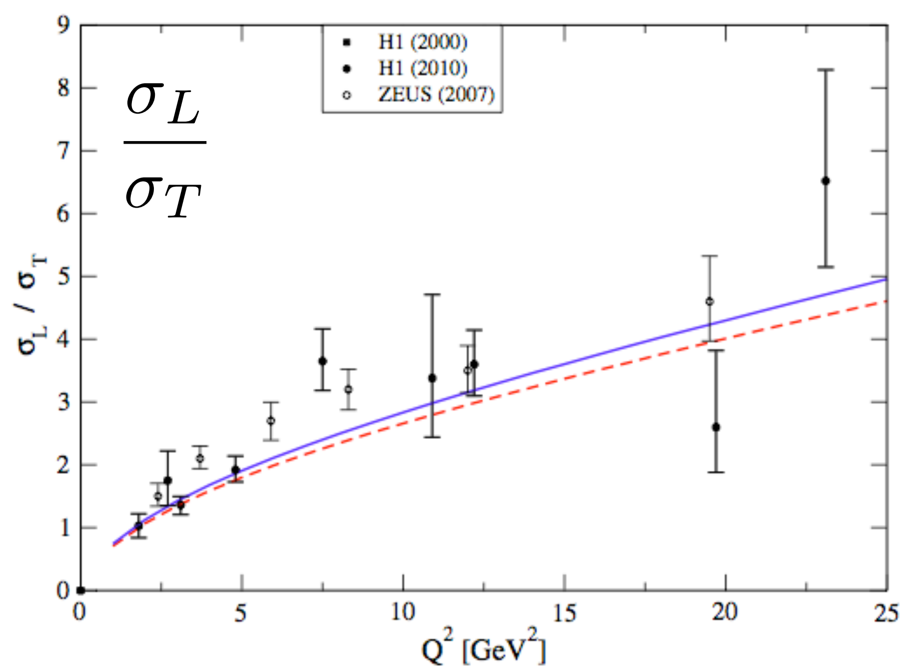
14% four-quark probability

G. de Teramond & sjb

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



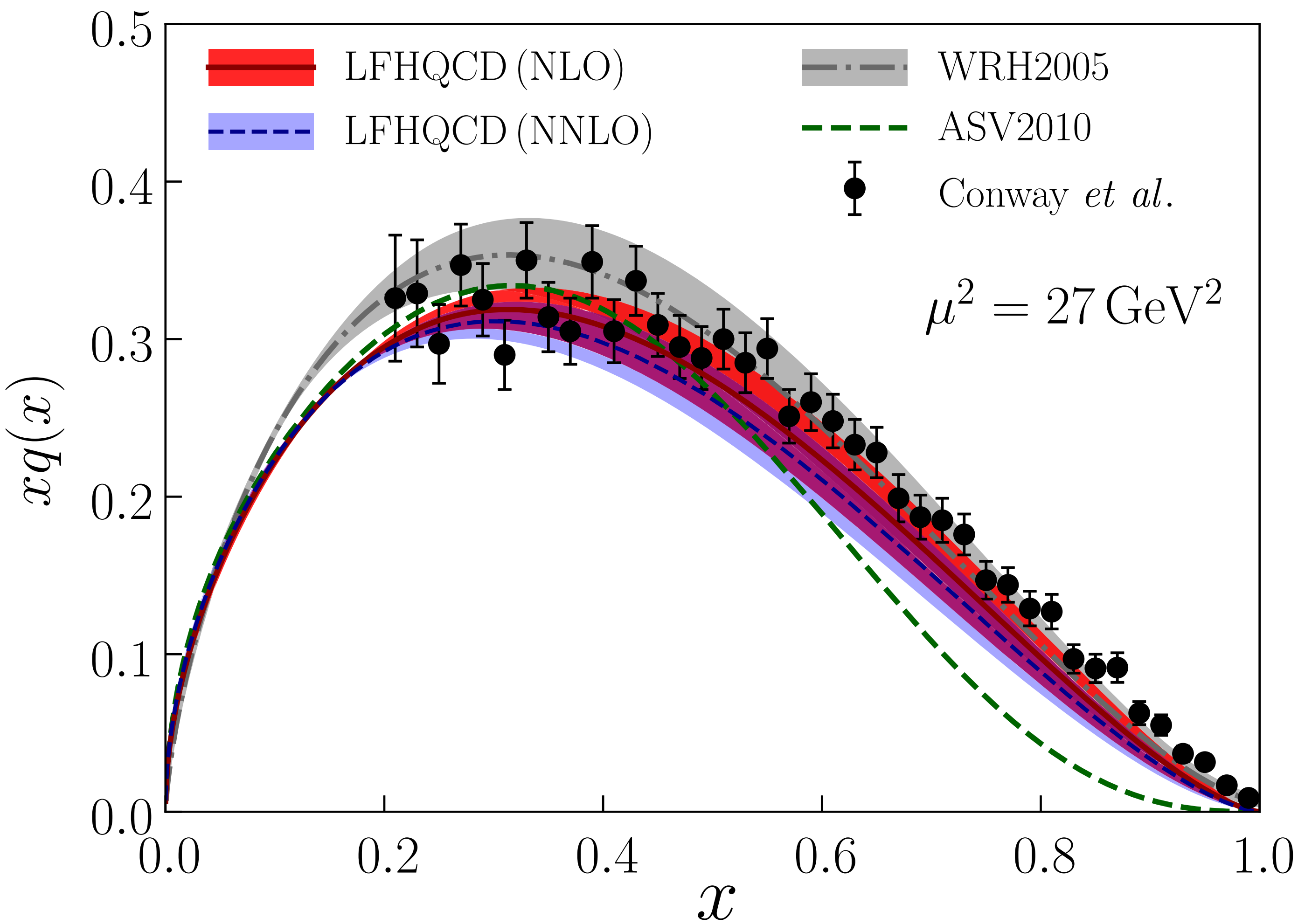
(a) H1 (b) ZEUS



J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

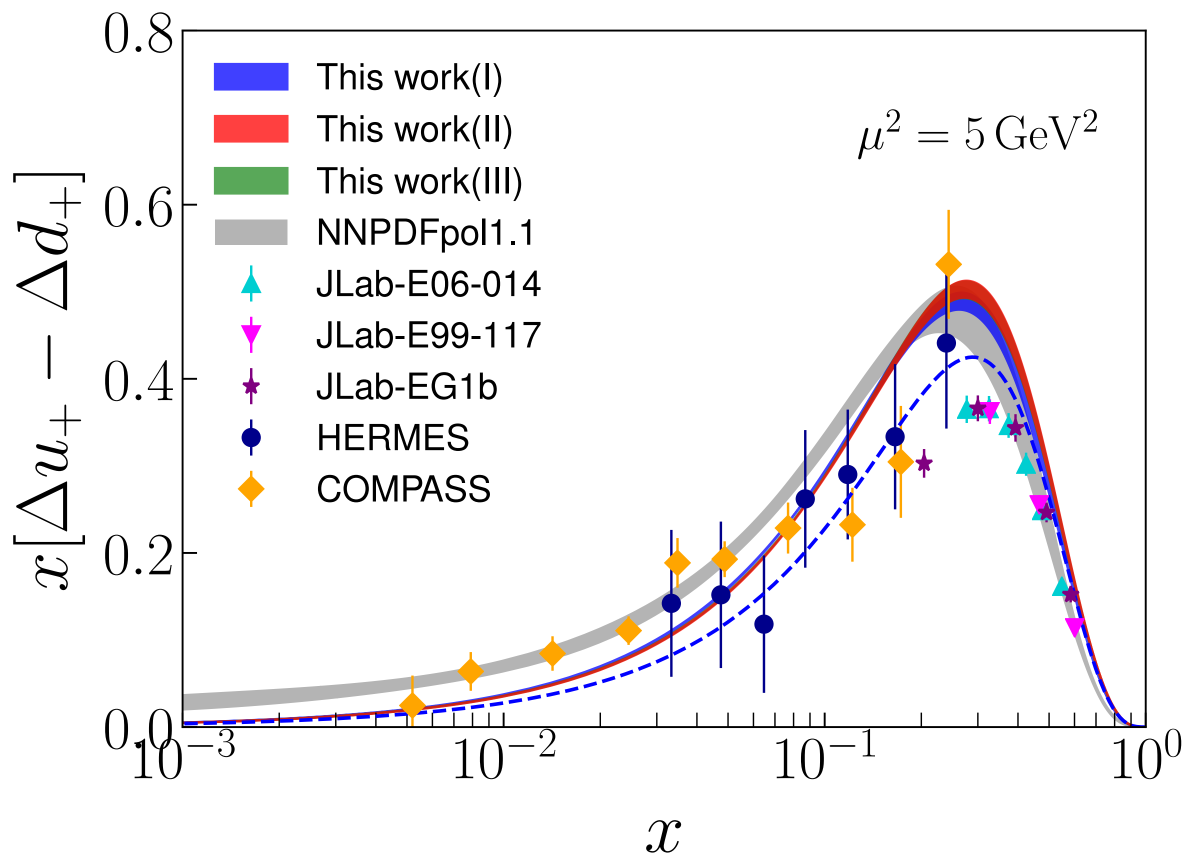


Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1 \pm 0.2$ GeV at NLO and the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV at NNLO.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Te´ramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur ***PHYSICAL REVIEW LETTERS 120, 182001 (2018)***

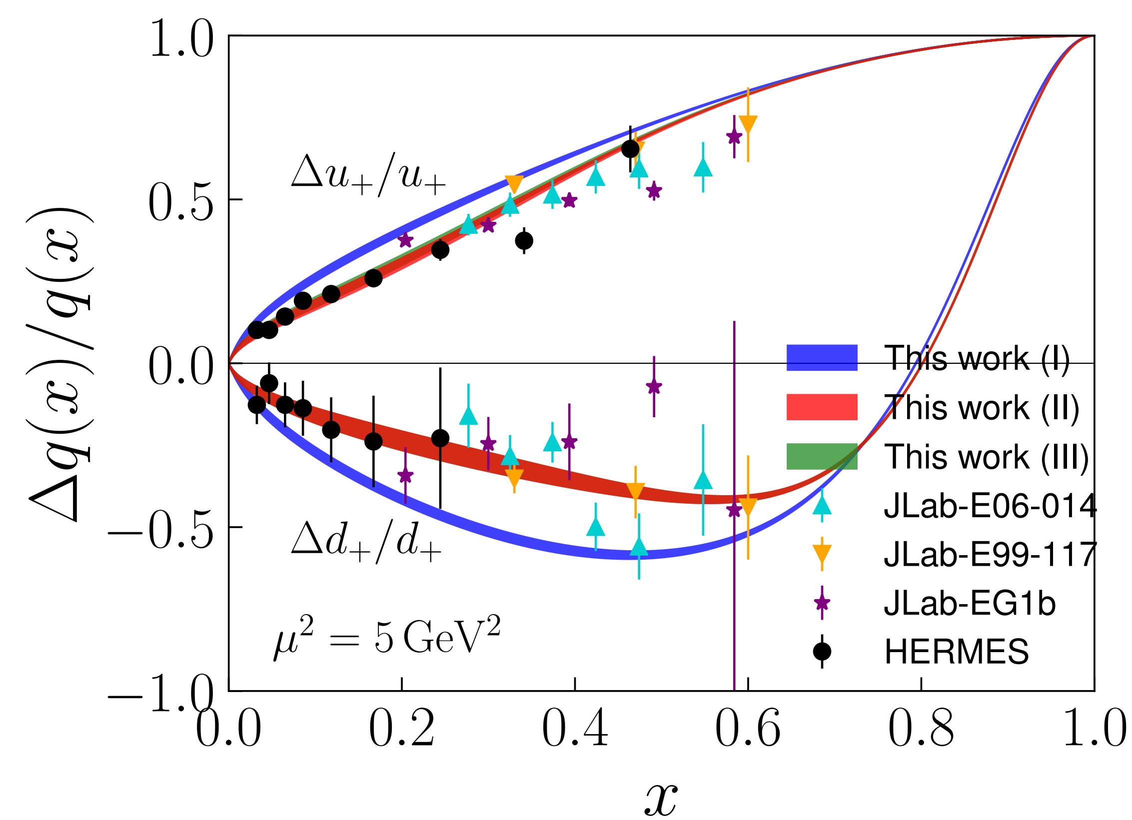
Tianbo Liu, Raza Sabbir Sufian, Guy F. de T´eramond, Hans Gunter Dösch, Alexandre Deur, sjb



Polarized distributions for the isovector combination x[Δu₊ (x) − Δd₊ (x)]

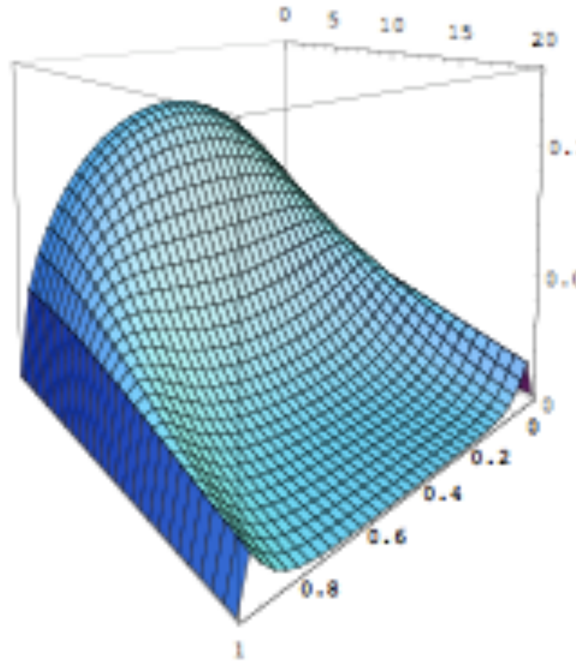
$$d_+(x) = d(x) + \bar{d}(x) \qquad u_+(x) = u(x) + \bar{u}(x)$$

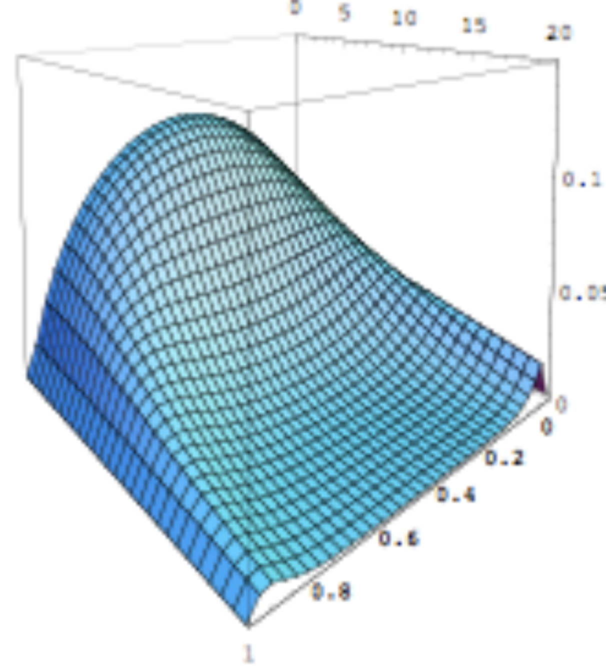
$$\Delta q(x) = q_\uparrow(x) - q_\downarrow(x)$$



$|\pi^+> = |u\bar{d}>$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



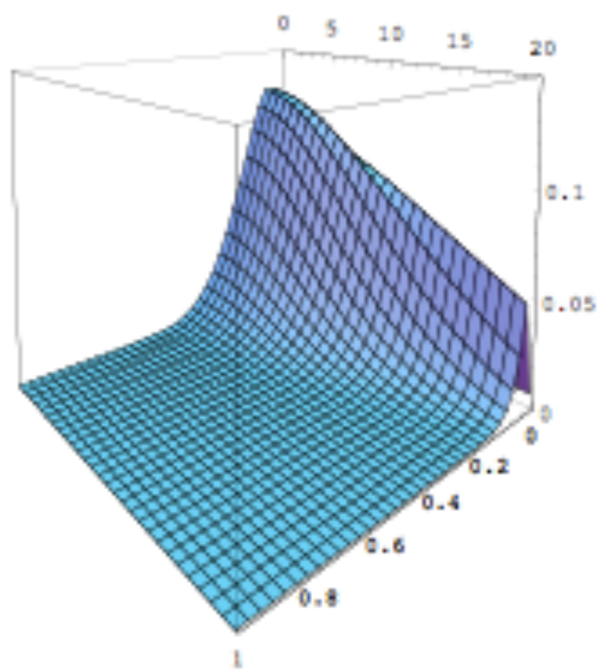


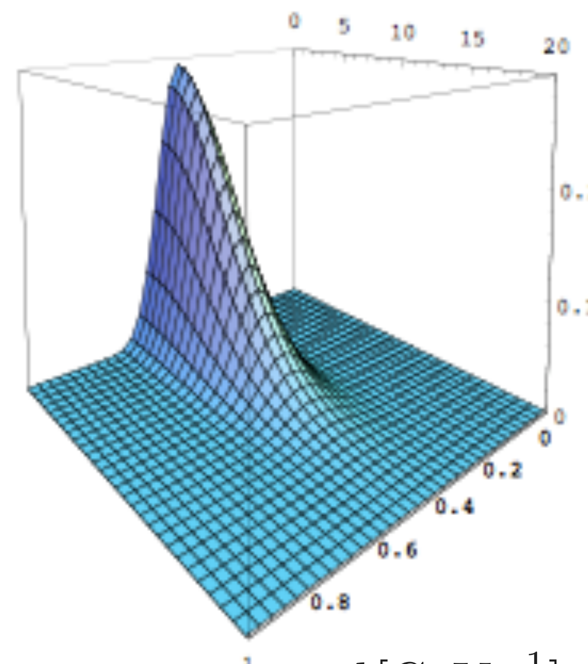
$|K^+> = |u\bar{s}>$

$m_s = 95\ MeV$

$|D^+> = |c\bar{d}>$

$m_c = 1.25\ GeV$

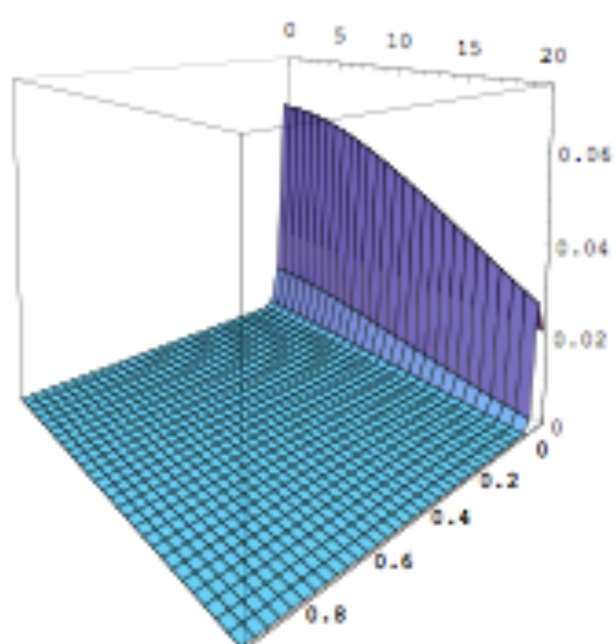


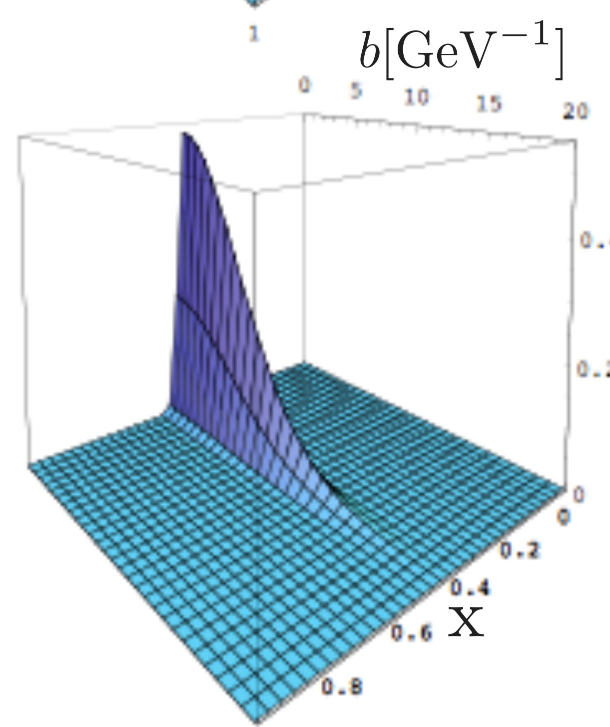


$|\eta_c> = |c\bar{c}>$

$|B^+> = |u\bar{b}>$

$m_b = 4.2\ GeV$





$|\eta_b> = |b\bar{b}>$

$\kappa = 375\ MeV$

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau)>=i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

New term

$$G=uH+vD+wK$$

$$G=H_\tau=\frac{1}{2}\left(-\frac{d^2}{dx^2}+\frac{g}{x^2}+\frac{4uw-v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw-v^2=\kappa^4=[M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\psi(\zeta)=\mathcal{M}^2\psi(\zeta)$$

$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

- **Dosch, de Teramond, sjb**

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

↕

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

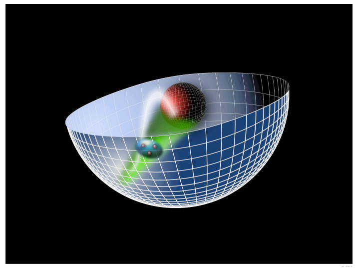
Unique confinement potential!

LFHQCD: Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time τ*

- *Causality: Information within causal horizon: Light-Front*

- *Light-Front Holography: AdS₅ = LF (3+1)*



$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$

- *Introduce Mass Scale κ while retaining the Conformal Invariance of the Action (dAFF)*

- *Unique Dilaton in AdS₅:* $e^{+\kappa^2 z^2}$

- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$

- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

Consider $R_w = Q + wS;$ $\quad w$: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \big(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}\big)$$

$$G_{22} = \big(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}\big)$$

Identify $f - \frac{1}{2} = L_B \, , \quad w = \kappa^2$ $\qquad \lambda = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

de Téramond, Dosch, Lorcé, sjb

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\Big)\psi_J^+ = M^2\psi_J^+$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\Big)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\Big)\phi_J = M^2\phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

S=0, P=+

Same κ!

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

LF Holography

Baryon LFWFs

Superconformal Quantum Mechanics

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

- Eigenvalues

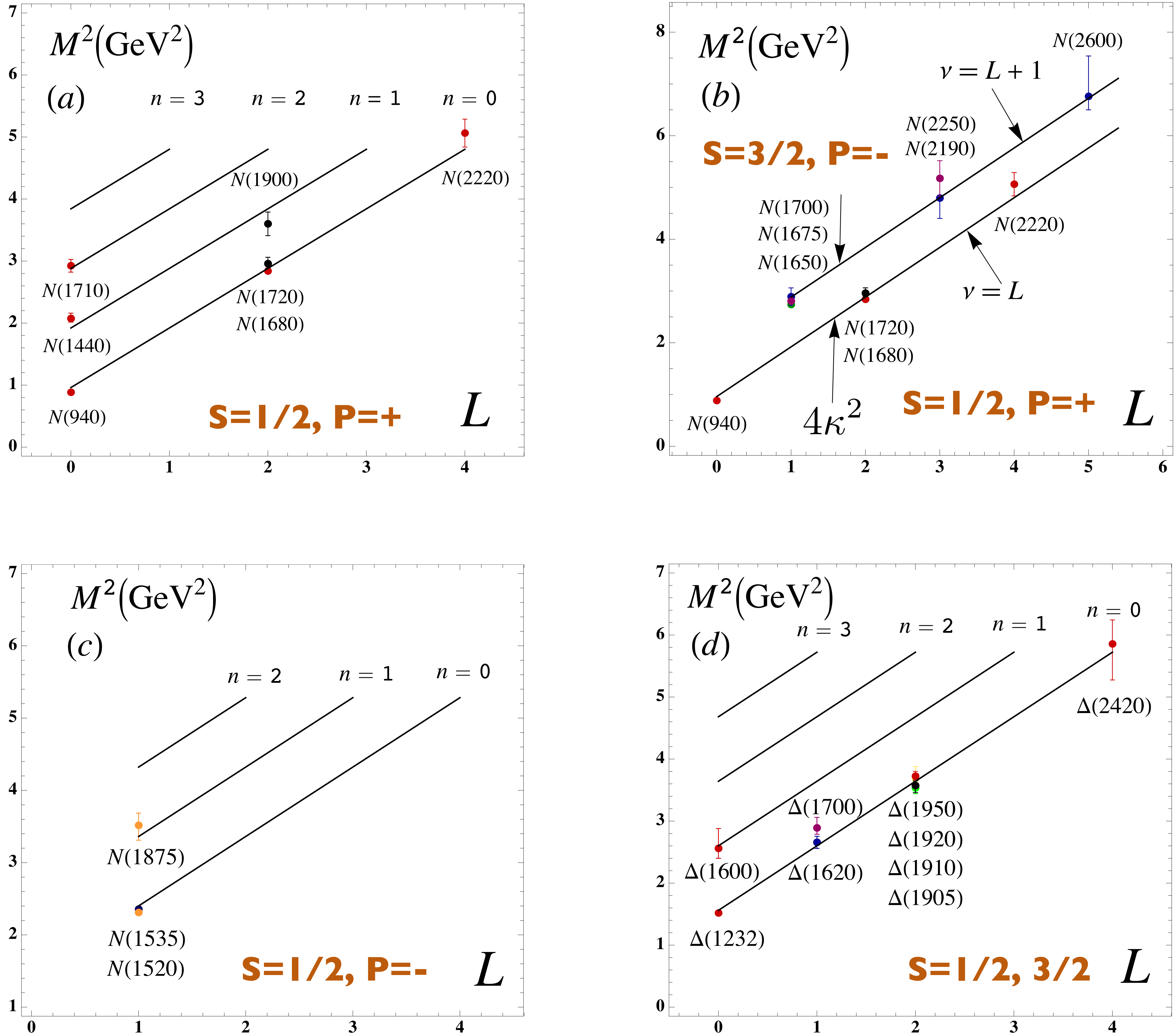
$$\int_0^\infty d\zeta \int_0^1 dx \psi_+^2(\zeta^2, x) = \int_0^\infty d\zeta \int_0^1 dx \psi_-^2(\zeta^2, x) = \frac{1}{2}$$

Quark Chiral Symmetry of Eigenstate!

Nucleon: Equal Probability for L=0,1

$$J^z = +1/2 : \quad \frac{1}{\sqrt{2}}[\,|S_q^z = +1/2, L^z = 0 > + |S_q^z = -1/2, L^z = +1 >\,]$$

Nucleon spin carried by quark orbital angular momentum



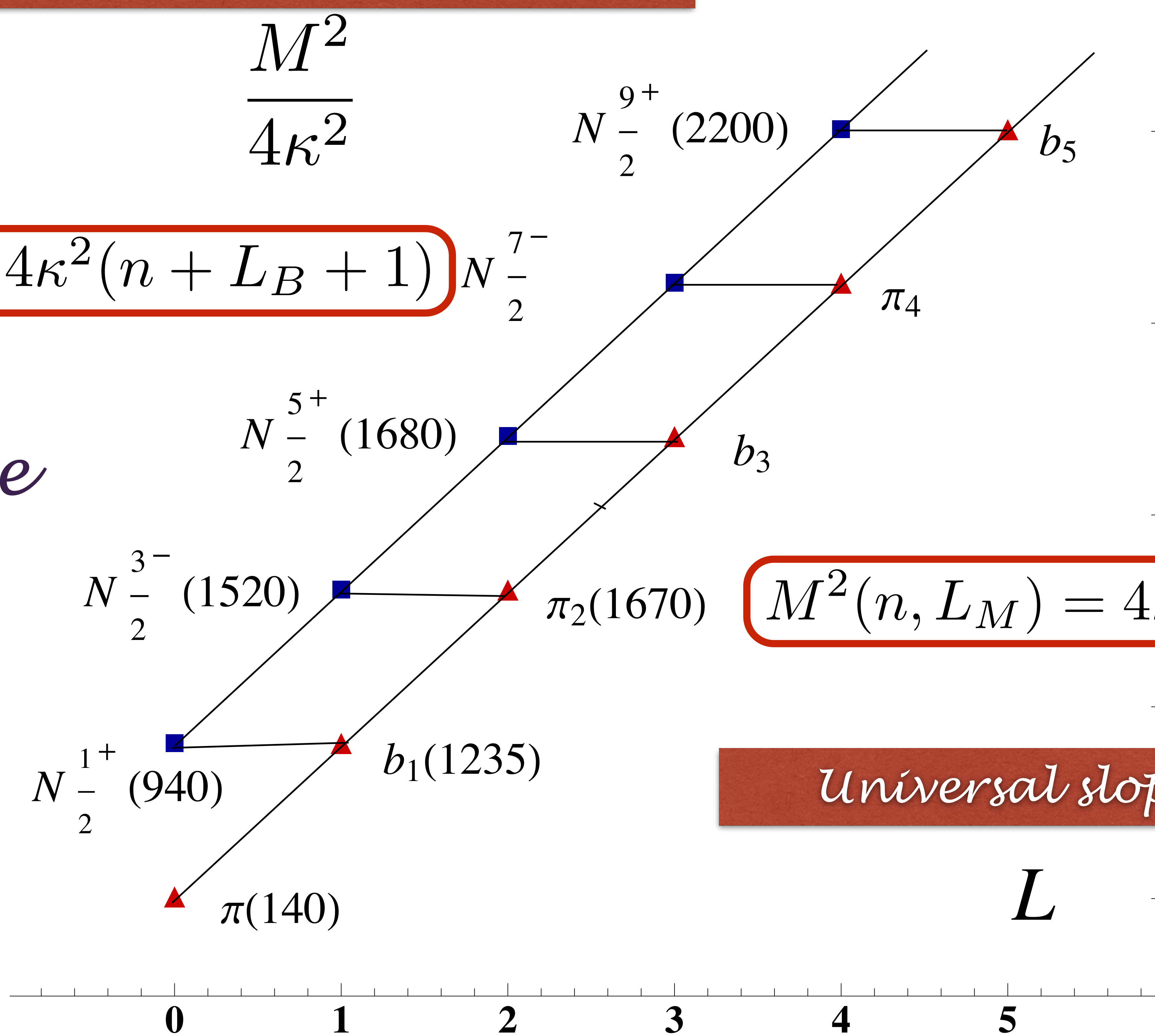
$M^2(\mathrm{GeV}^2)$
(a)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
$N(1900)$
$N(2220)$
$N(1710)$
$N(1720)$
$N(1680)$
$N(1440)$
$N(940)$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
$N(2600)$
$\nu = L + 1$
$N(2250)$
$N(2190)$
S=3/2, P=-
$N(1700)$
$N(1675)$
$N(1650)$
$N(2220)$
$\nu = L$
$N(1720)$
$N(1680)$
$N(940)$
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
$n = 2$
$n = 1$
$n = 0$
$N(1875)$
$N(1535)$
$N(1520)$
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
$\Delta(2420)$
$\Delta(1700)$
$\Delta(1950)$
$\Delta(1920)$
$\Delta(1910)$
$\Delta(1905)$
$\Delta(1600)$
$\Delta(1620)$
$\Delta(1232)$
S=1/2, 3/2
L

Superconformal Quantum Mechanics
Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

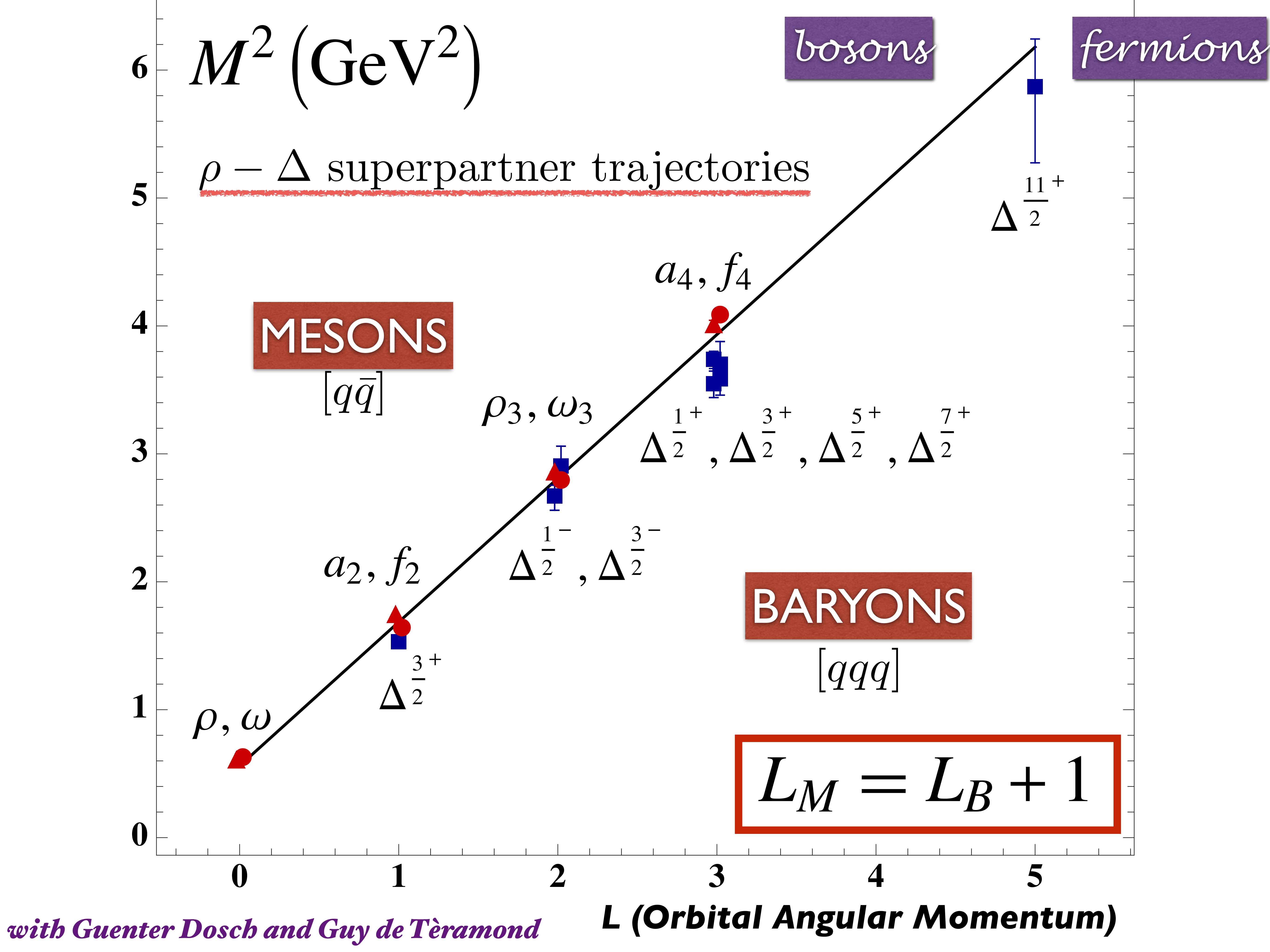


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

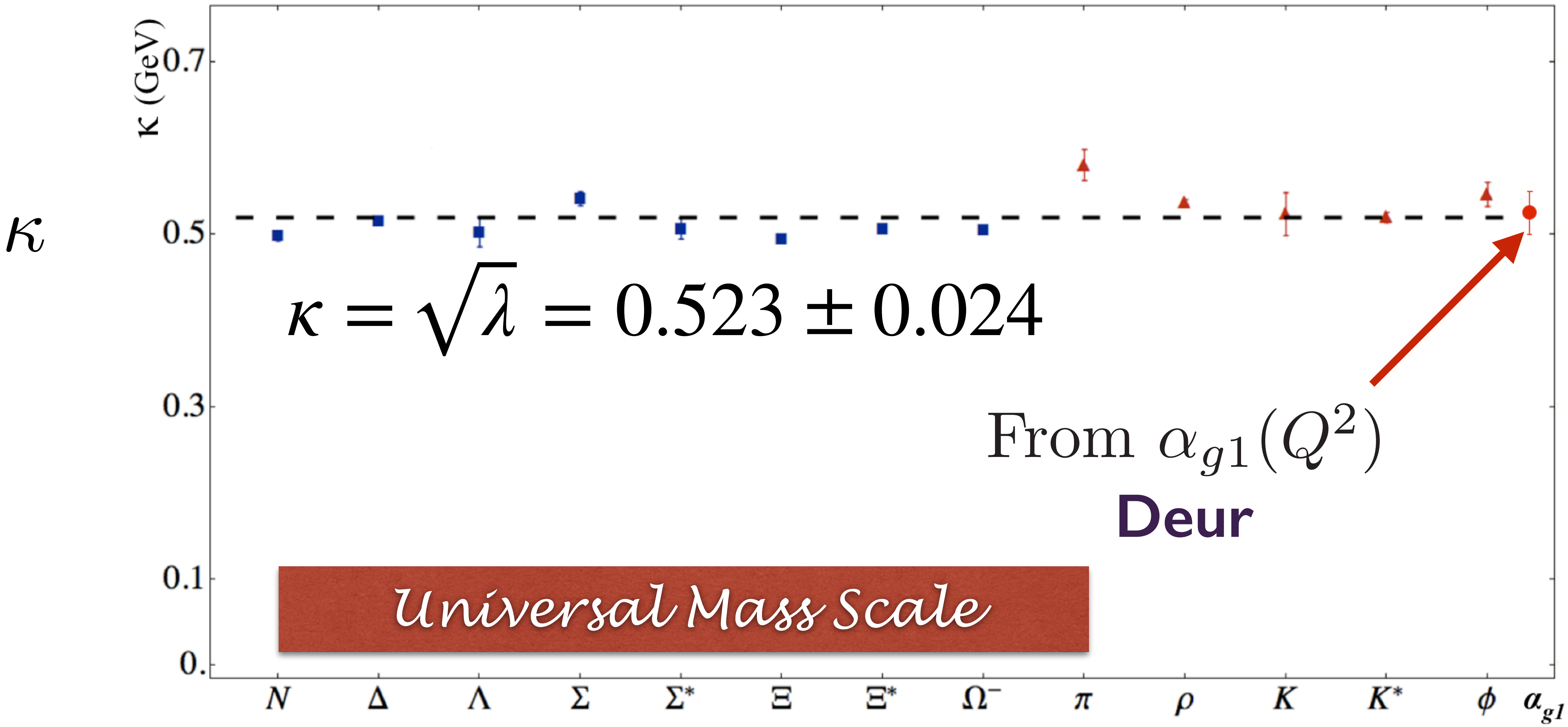


$M^2\ (\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta^{\frac{3}{2}^+}$
$\Delta^{\frac{1}{2}^-}, \Delta^{\frac{3}{2}^-}$
$\Delta^{\frac{1}{2}^+}, \Delta^{\frac{3}{2}^+}, \Delta^{\frac{5}{2}^+}, \Delta^{\frac{7}{2}^+}$
$\Delta^{\frac{11}{2}^+}$
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
with Guenter Dosch and Guy de Tèramond

$\lambda = \kappa^2$

de Téramond, Dosch, Lorcé, sjb

$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$



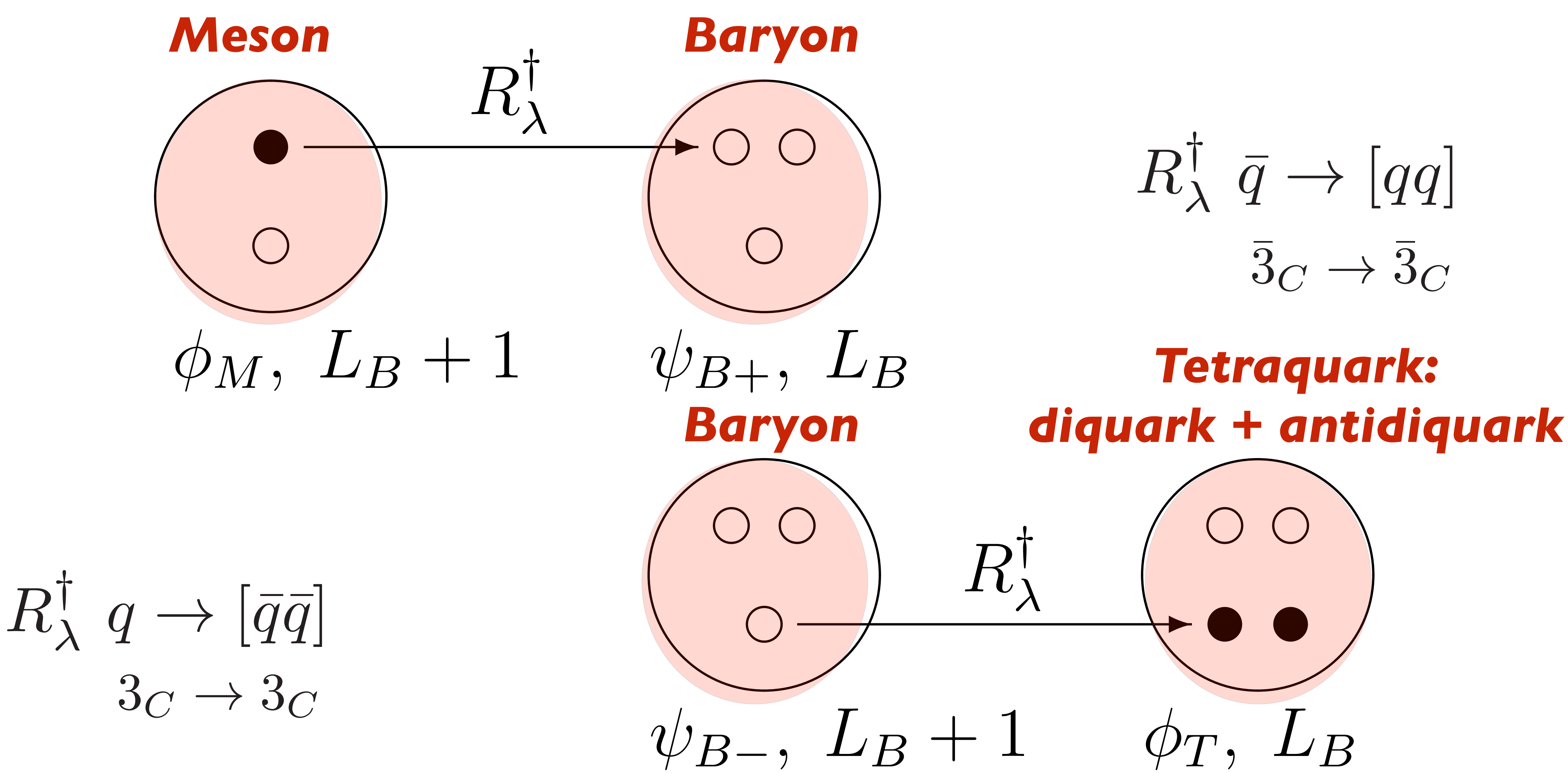
Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

Superconformal Algebra

2X2 Hadronic Multiplets

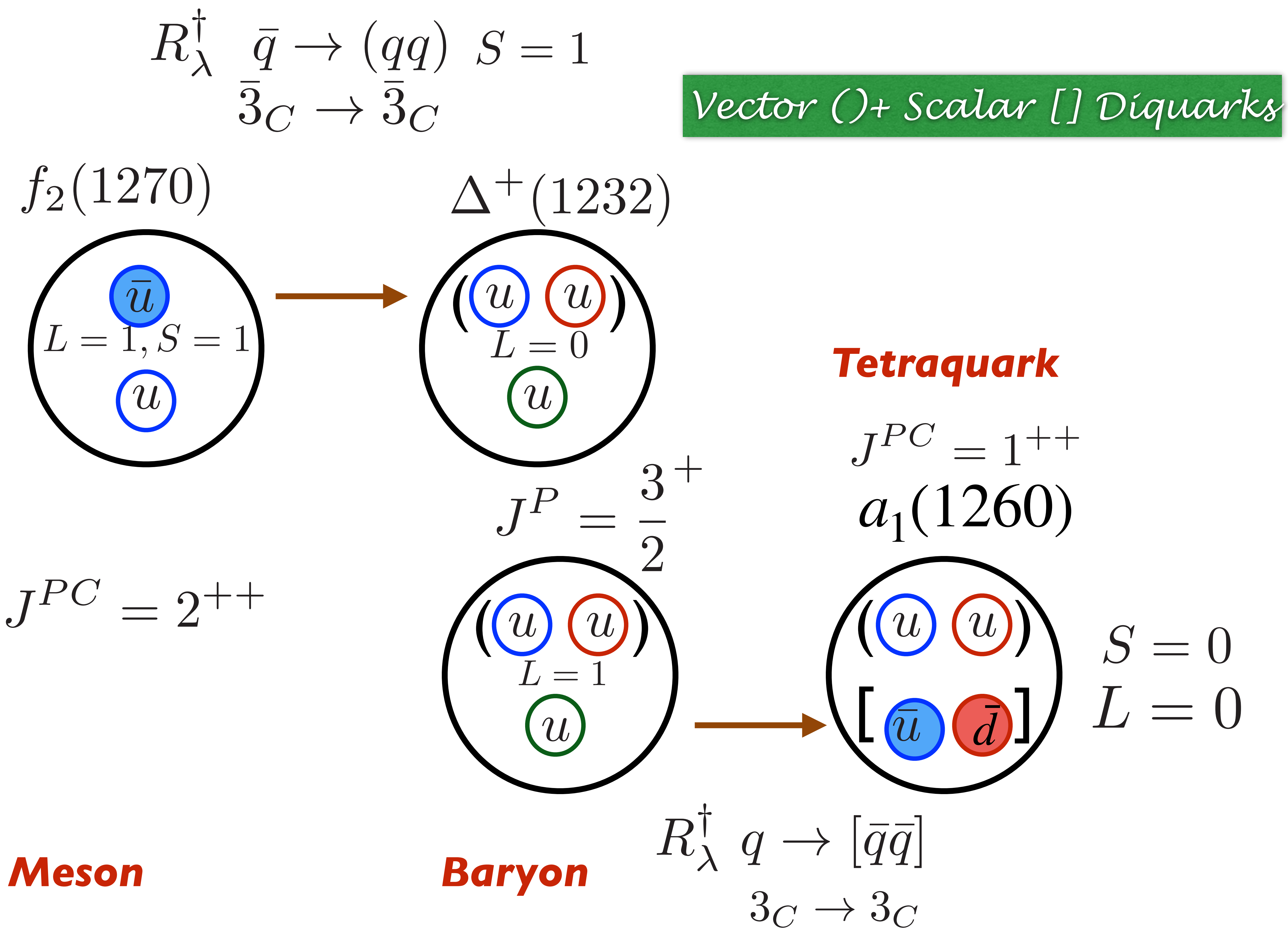
Bosons, Fermions with Equal Mass!



Proton: |u[ud]> Quark + Scalar Diquark

Equal Weight: L=0, L=1

Superconformal Algebra 4-Plet



Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770)$, $\omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320)$, $f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690)$, $\omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)$?
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040)$, $f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)$?
$\bar{q}s$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$K_1^*(\sim 1700)$?
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)$?
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)$?
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)$?
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)$?
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)$?

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- ***Universal quark light-front kinetic energy***

Equal: Virial Theorem

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$

- ***Universal quark light-front potential energy***

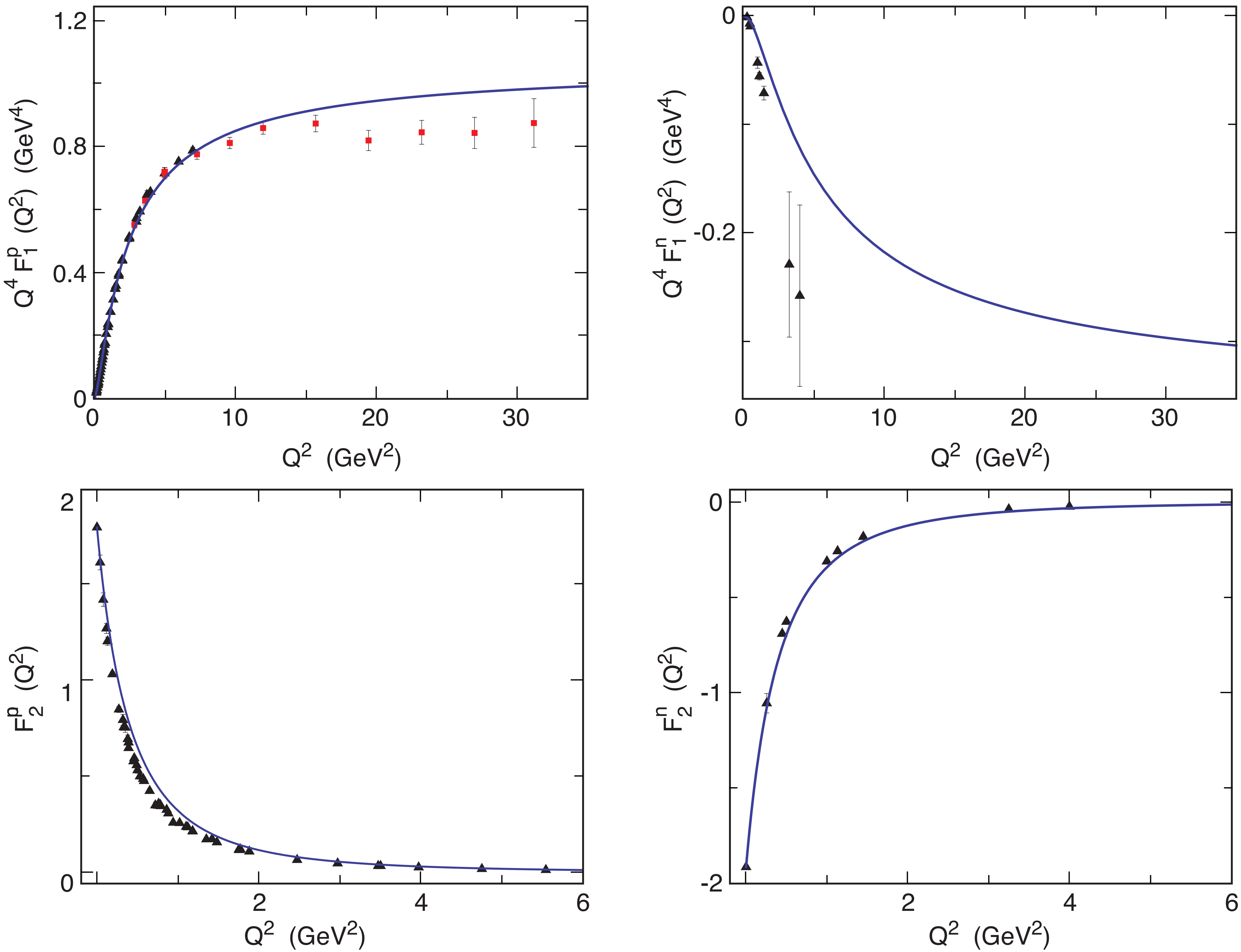
$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- ***Universal Constant Contribution from AdS and Superconformal Quantum Mechanics***

$$\Delta\mathcal{M}^2_{spin} = 2\kappa^2(L + 2S + B - 1)$$

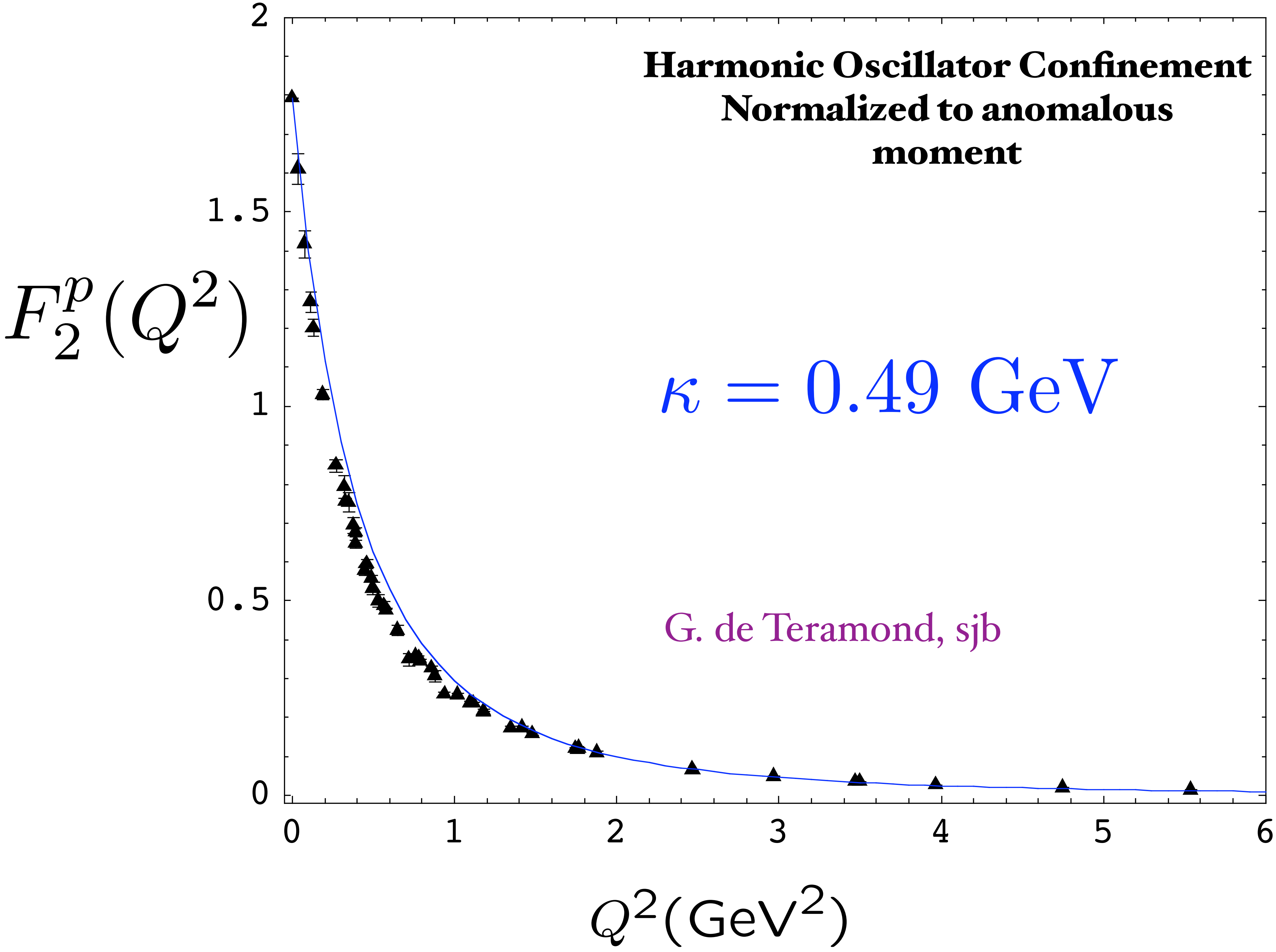
hyperfine spin-spin

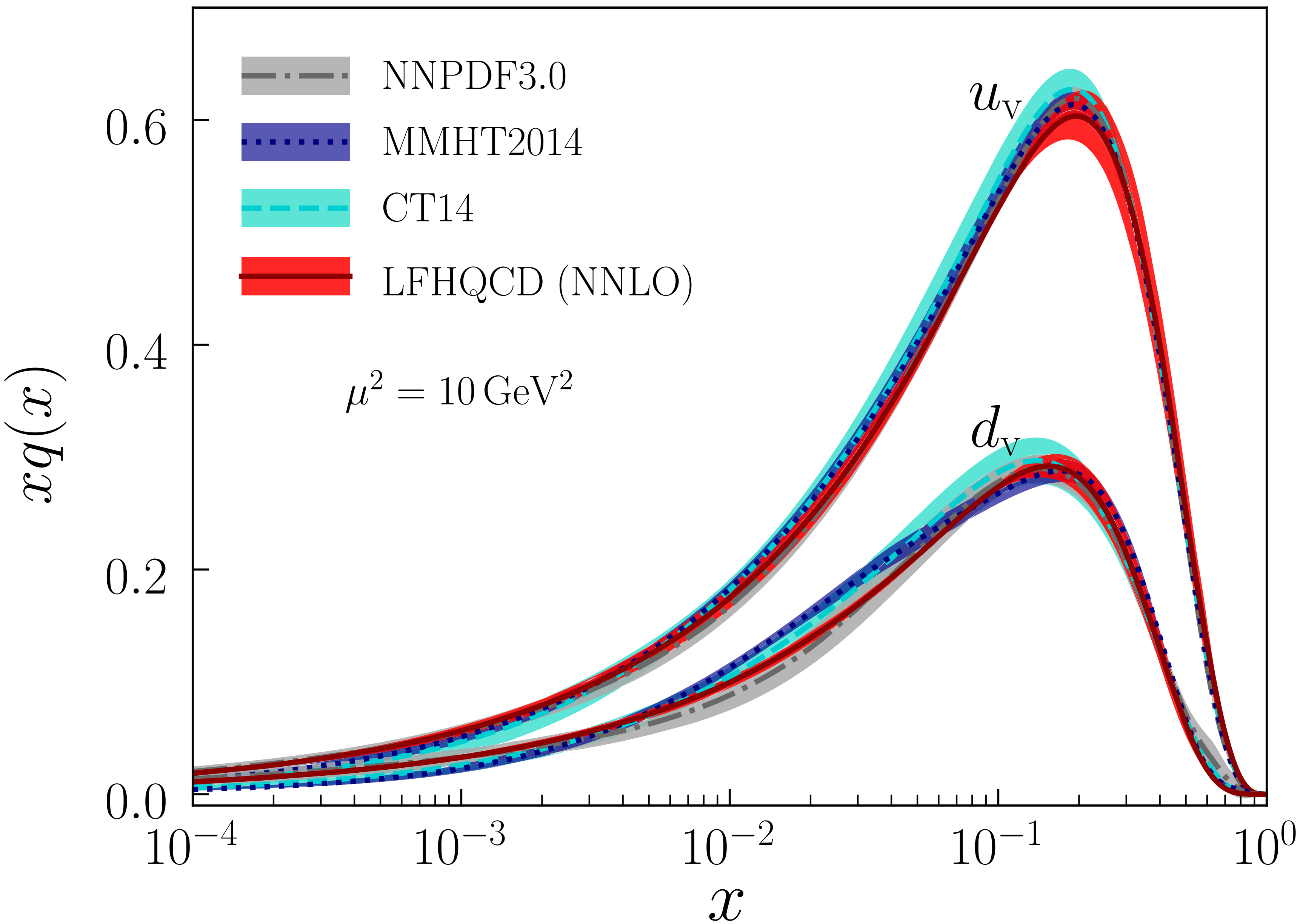
Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs





Comparison for $xq(x)$ in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV.

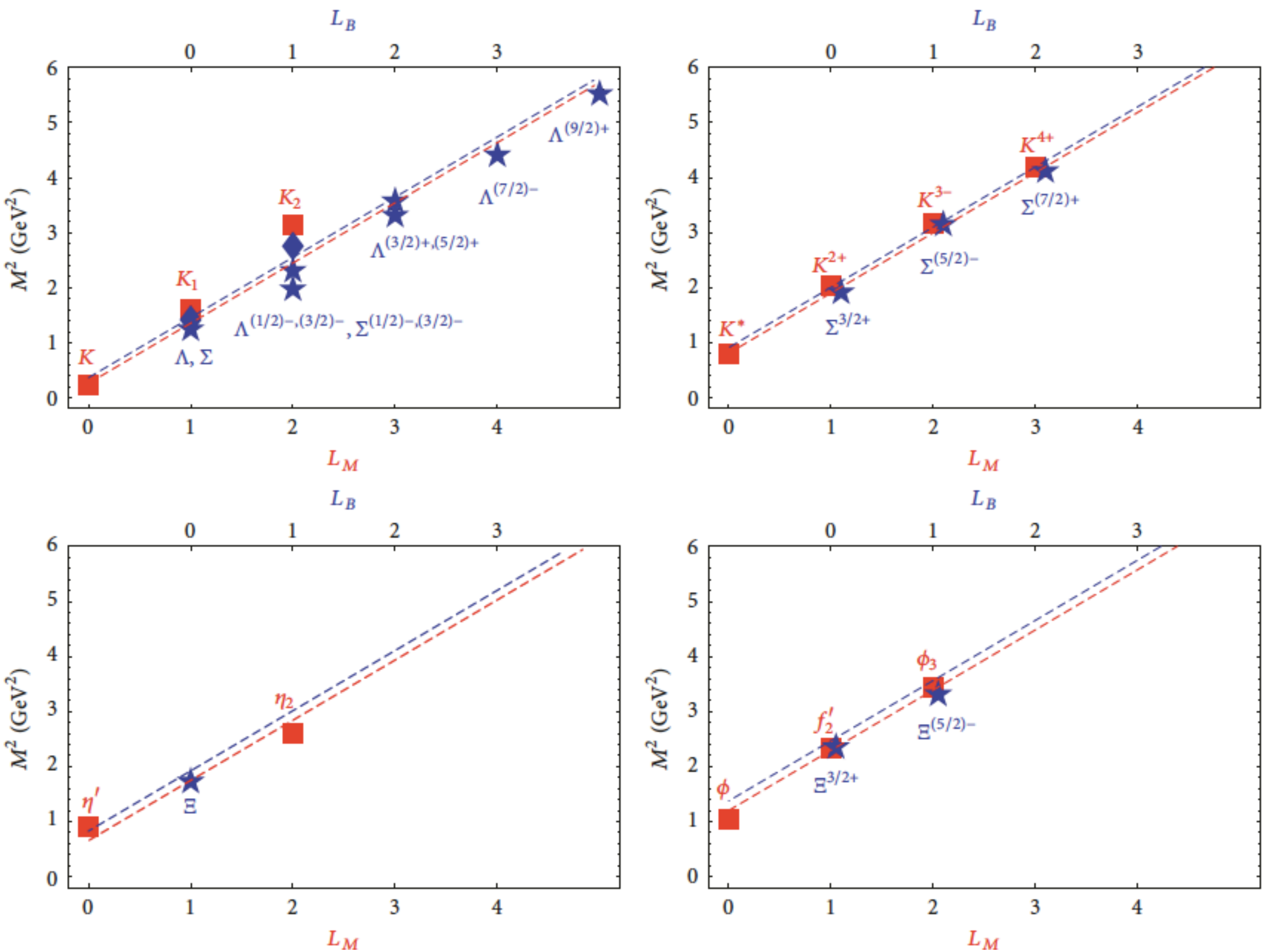
Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Te´ramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur

PHYSICAL REVIEW LETTERS 120, 182001 (2018)

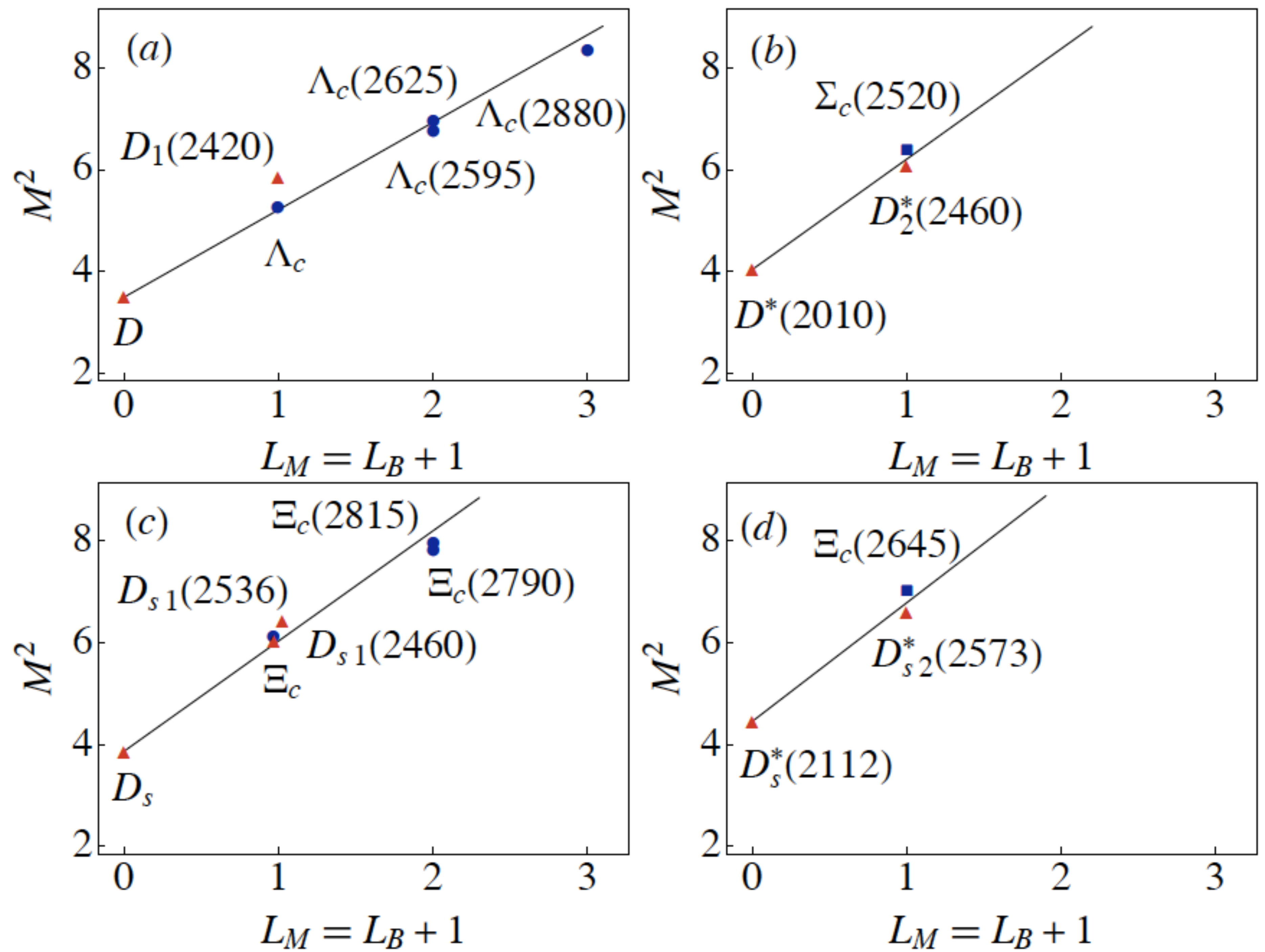
de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum



de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

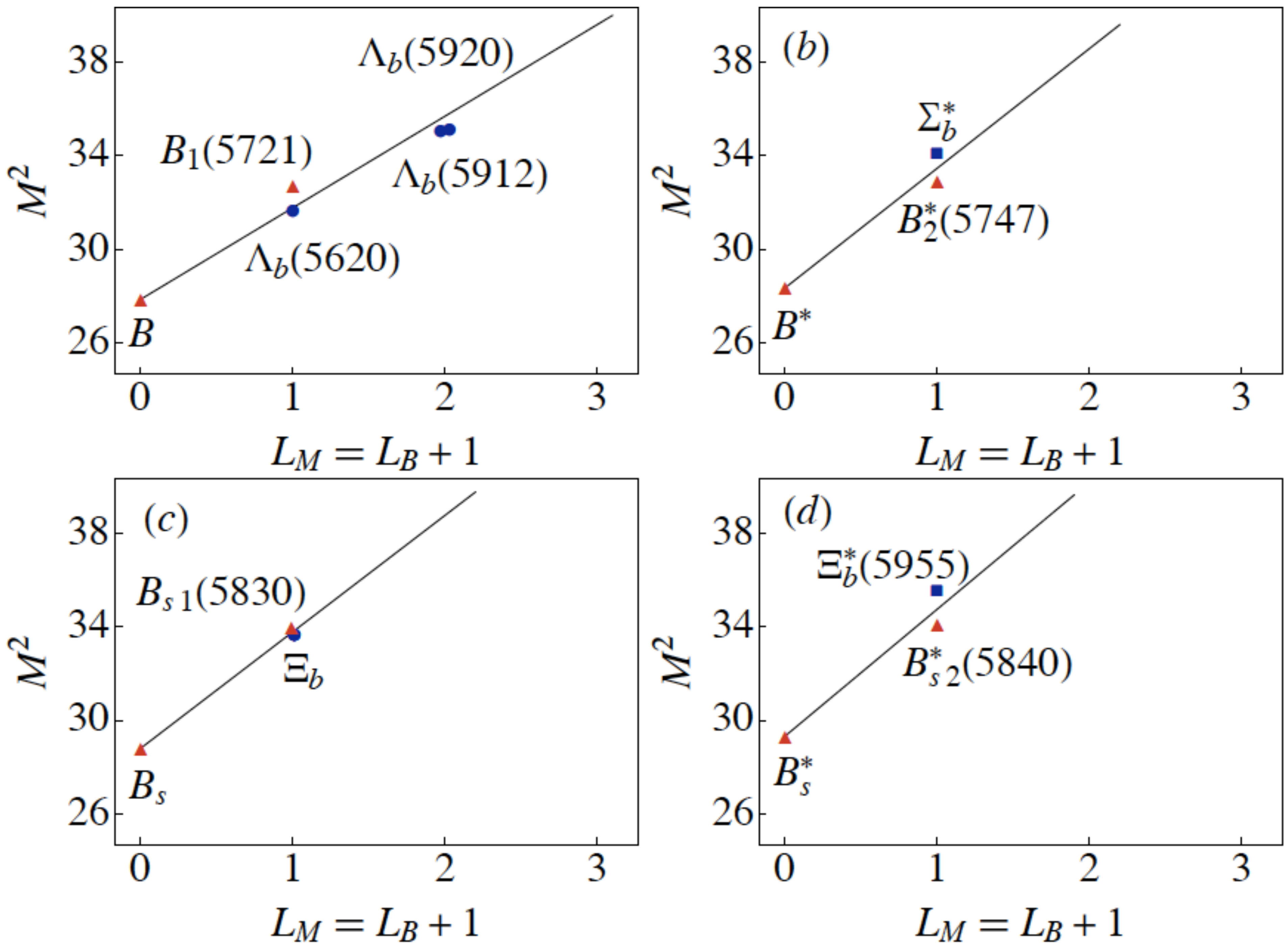
predictions

beautiful agreement!

M. Nielsen, sjb

de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

Heavy-light and heavy-heavy hadronic sectors

- Extension to the heavy-light hadronic sector

  [H. G. Dosch, GdT, S. J. Brodsky, PRD **92**, 074010 (2015), PRD **95**, 034016 (2017)]

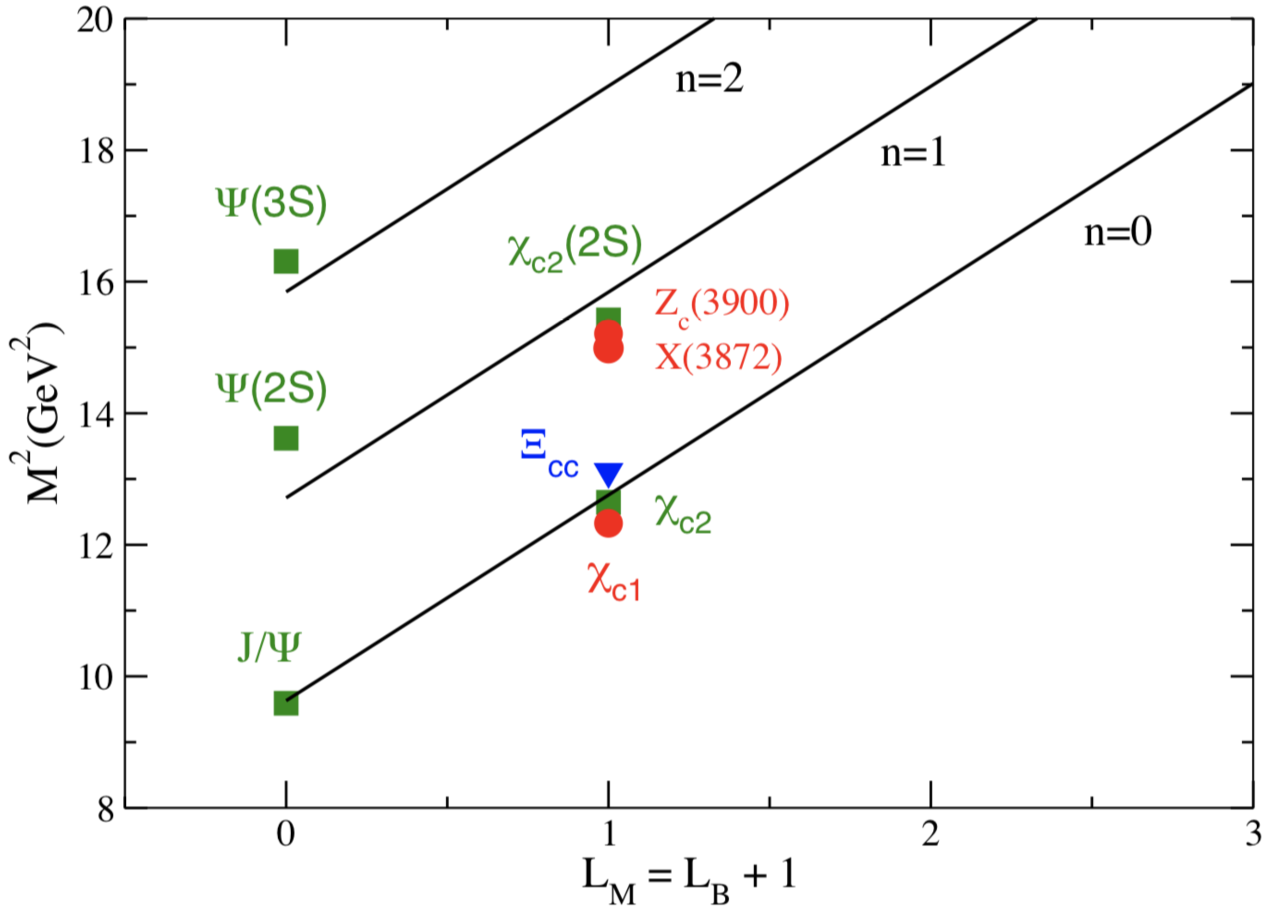
- Extension to the double-heavy hadronic sector

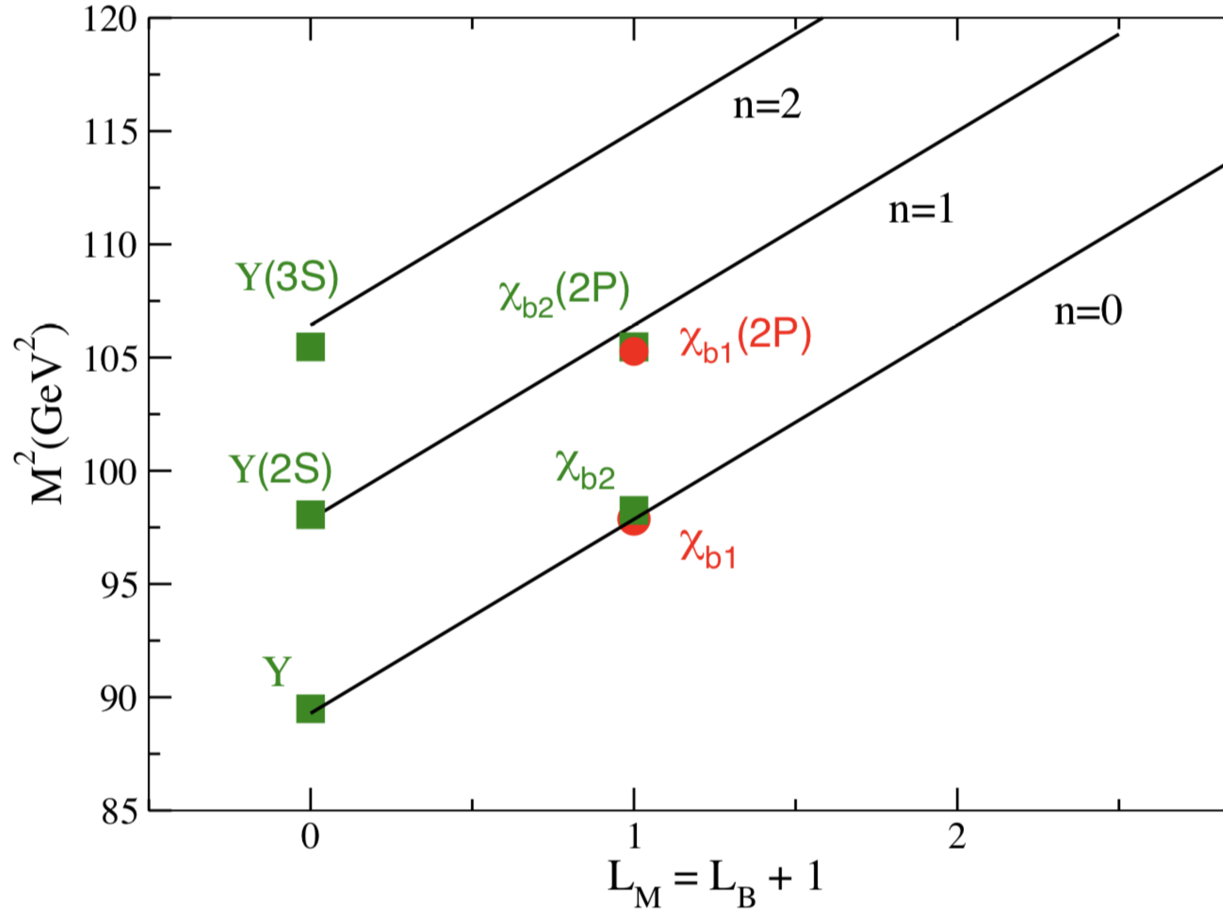
  [M. Nielsen and S. J. Brodsky, PRD, 114001 (2018)]

  [M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch, F. S. Navarra, L. Zou, PRD **98**, 034002 (2018)]

- Extension to the isoscalar hadronic sector

  [L. Zou, H. G. Dosch, GdT,S. J. Brodsky, arXiv:1901.11205 [hep-ph]]





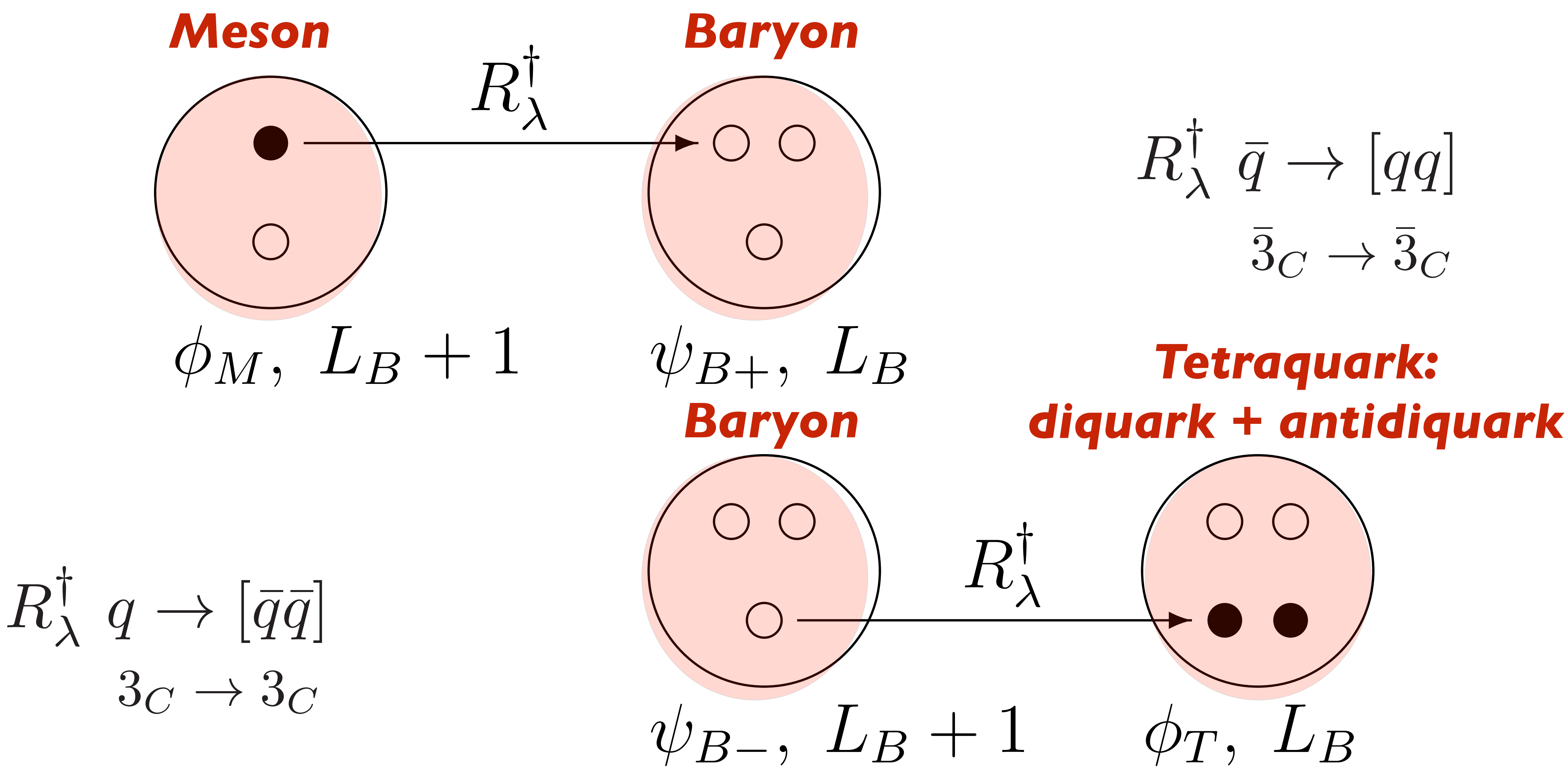
Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_C$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

Superconformal Algebra

Four-Plet Representations

Bosons, Fermions with Equal Mass!



Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Other Consequences of $[ud]_{\bar{3}_C, I=0, J=0}$ diquark cluster

QCD Hidden-Color Hexadiquark in the Core of Nuclei

J. Rittenhouse West, G. de Teramond, A. S. Goldhaber, I. Schmidt, sjb

$$|\Psi_{HDQ}>=|[ud][ud][ud][ud][ud][ud]>$$

mixes with

$${}^4He|npnp>$$

Increases alpha binding energy, EMC effects

Diquarks Can Dominate Five-Quark Fock State of Proton

$$|p>=\alpha|[ud]u> +\beta|[ud][ud]\bar{d}>$$

J. Rittenhouse West, sjb (to be published)

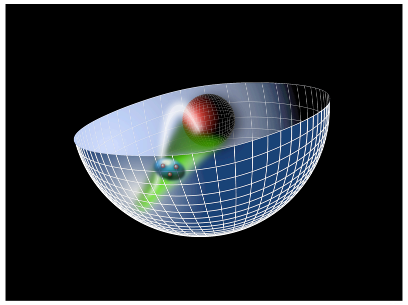
Natural explanation why $\bar{d}(x) >> \bar{u}(x)$ in proton

Underlying Principles

- *Polncarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time* τ

- *Causality: Information within causal horizon: Light-Front*

- *Light-Front Holography:* AdS_5 *= LF (3+1)*



$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$

- *Introduce mass scale* κ *while retaining the Conformal Invariance of the Action (dAFF)*

"Emergent Mass"

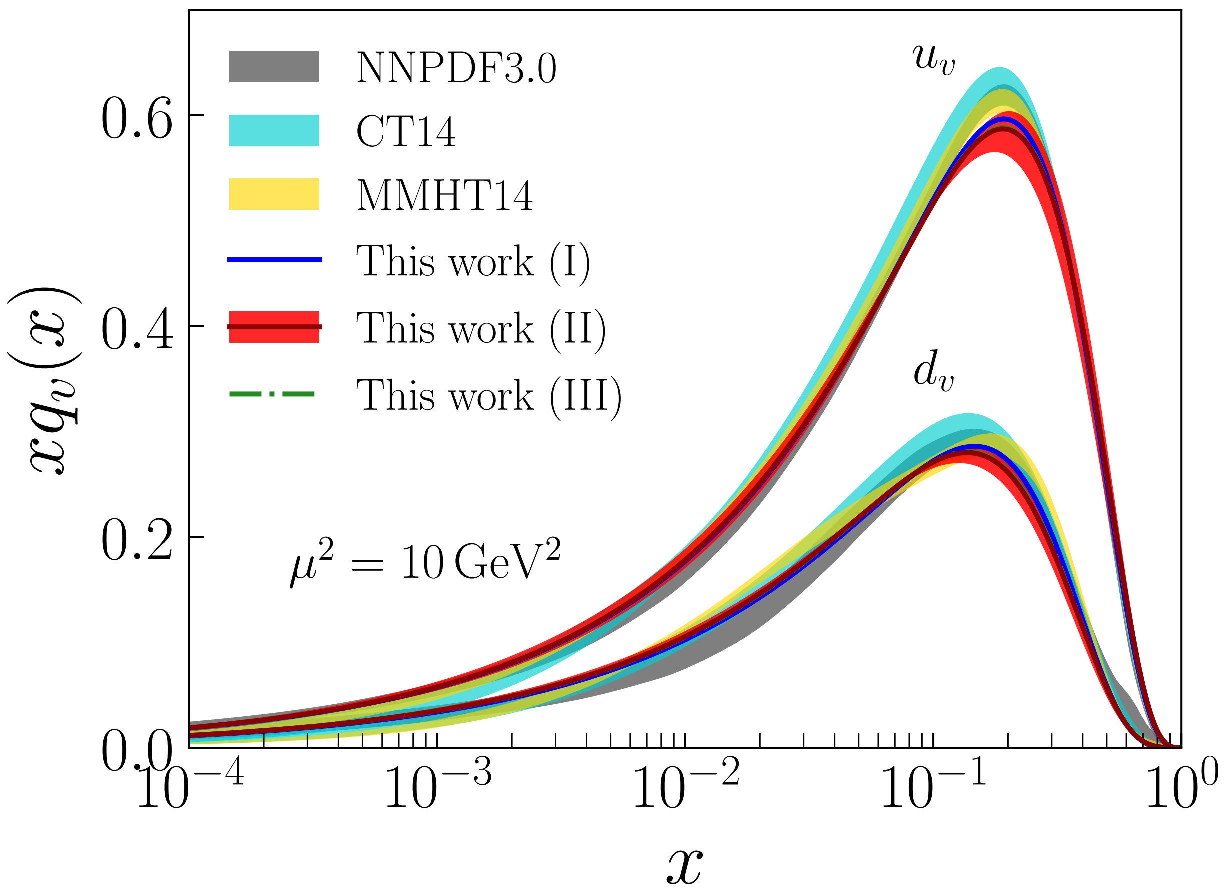
- *Unique Dilaton in* AdS_5*:* $e^{+\kappa^2 z^2}$

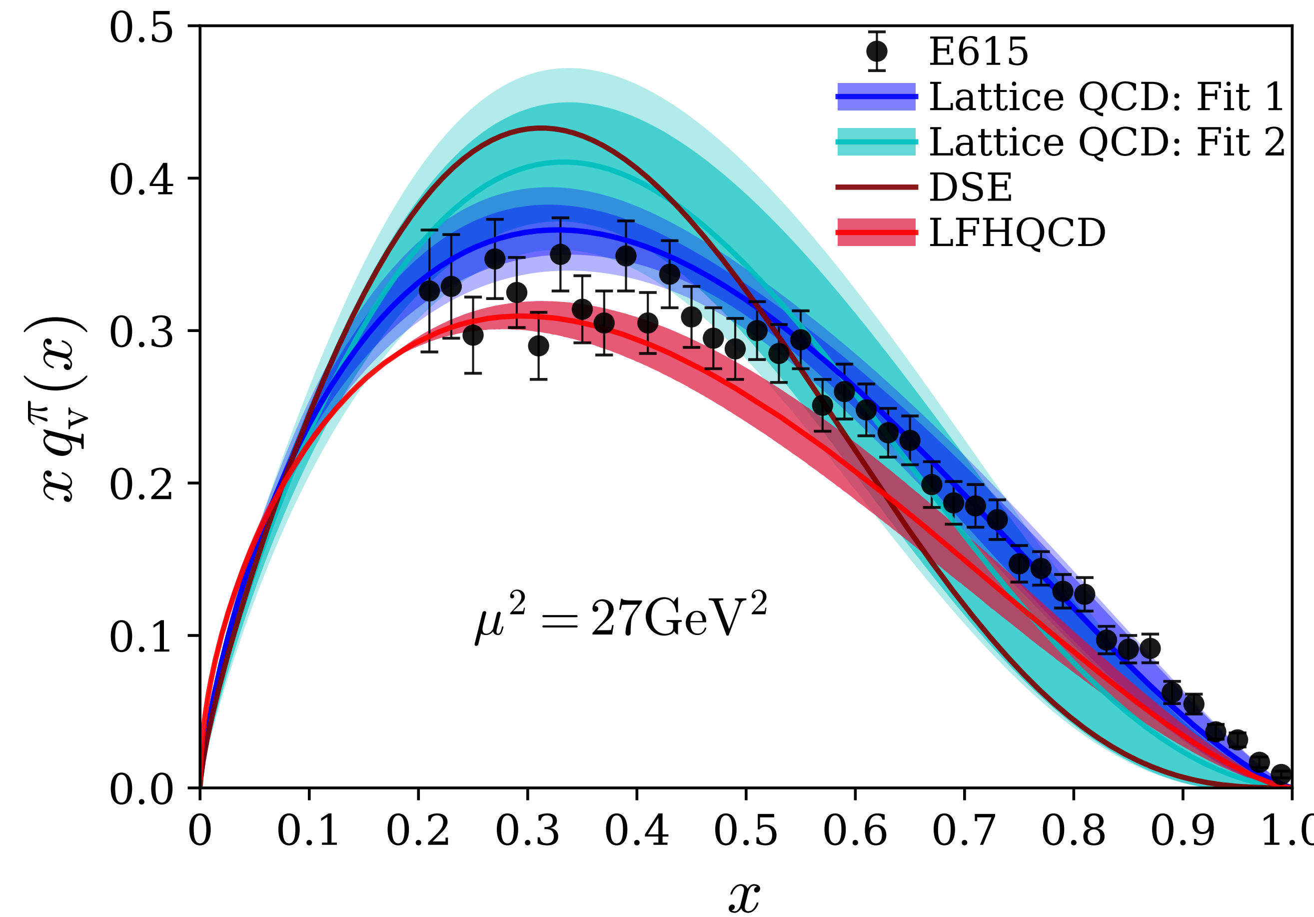
- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$

- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

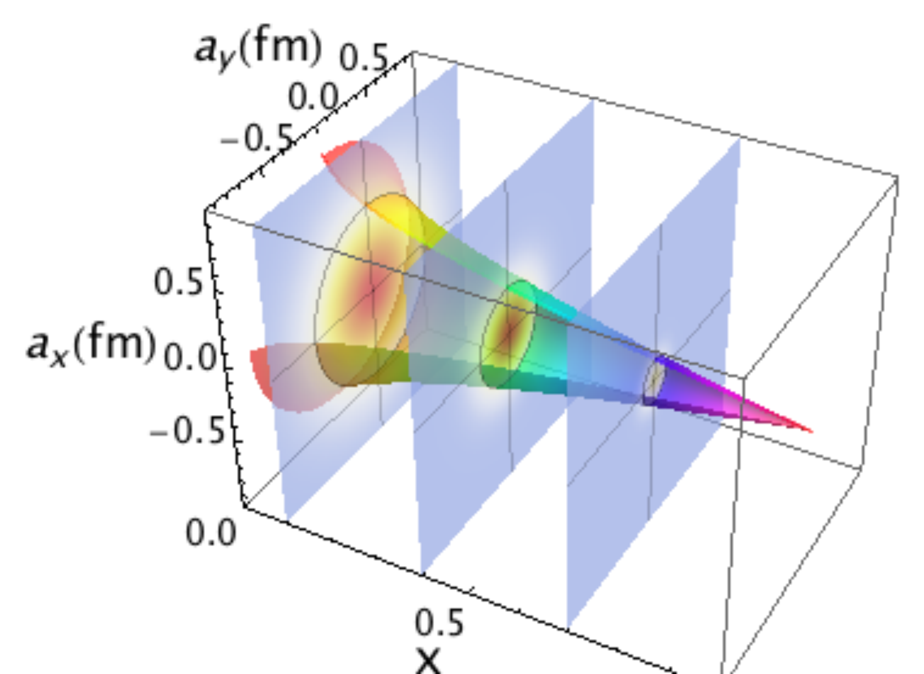
Unpolarized GPDs and PDFs (HLFHS Collaboration, 2018)





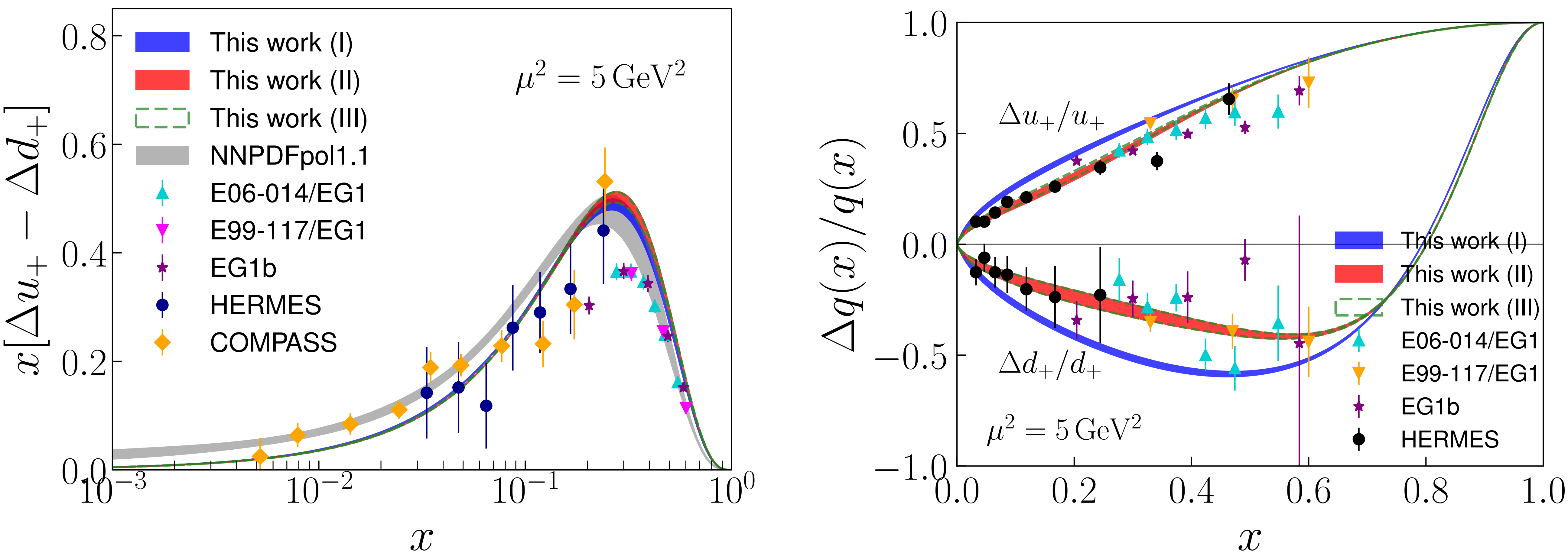
- Transverse impact parameter quark distribution

$$u(x, \mathbf{a}_\perp) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{a}_\perp \cdot \mathbf{q}_\perp} H^u(x, \mathbf{q}_\perp^2)$$



Polarized GPDs and PDFs (HLFHS Collaboration, 2019)

- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients c_τ are fixed for the vector current
- Helicity retention between quark and parent hadron (pQCD prediction): $\lim_{x\to 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron: $\lim_{x\to 0} \frac{\Delta q(x)}{q(x)} = 0$



Transverse and Longitudinal LF Confinement

$$M_H^2 = M_{||}^2 + M_{\perp}^2$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\perp}(\zeta)\right)\phi(\zeta) = M_{\perp}^2\phi(\zeta),$$

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_{||}(x)\right)\chi(x) = M_{||}^2\,\chi(x),$$

Longitudinal contribution for nonzero quark mass

S. S. Chabysheva and J.R.Hiller,

Constraint: Rotational symmetry in non-relativistic heavy-quark limit.

Transverse Confinement

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_\perp(\zeta)\right)\phi(\zeta) = M_\perp^2 \phi(\zeta),$$

$$U_\perp(\zeta) = \lambda^2\zeta^2 + 2\lambda(J-1). \qquad \zeta^2 = b_\perp^2 x(1-x)$$

$$M_\perp^2(n,J,L) = 4\lambda\left(n + \frac{J+L}{2}\right),$$

and eigenfunctions

de Teramond, Dosch, sjb

$$\phi_{n,L}(\zeta) = \lambda^{(1+L)/2}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2)$$

$M_\pi = 0$ in chiral $(m_q = 0)$ limit

Longitudinal Confinement

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_{\|}(x)\right)\chi(x) = M_{\|}^2\,\chi(x)$$

$$U_{\|}(x) = -\sigma^2 \partial_x \left(x(1-x)\,\partial_x\right)$$

Li, Maris, Zhao, Vary

$$U_{||} = \sigma^2 x(1-x)\tilde{z}^2$$

Ioffe length $\tilde{z}$: conjugate to LF $x = \frac{k^+}{P^+}$

G.A. Miller, sjb

$\frac{\gamma^+\gamma^+}{{k^+}^2}$ LF interaction in $A^+ = 0$ gauge

de Teramond, sjb

Same potential: t' Hooft Equation $\mathrm{QCD}(1{+}1)_{N_C \to \infty}$

Longitudinal dynamics and chiral symmetry breaking in holographic light-front QCD

Guy F. de Téramond[1,*] and Stanley J. Brodsky[2,†]

[1] *Laboratorio de Física Teórica y Computacional, Universidad de Costa Rica, 11501 San José, Costa Rica*

[2] *SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA*

(Dated: April 18, 2021)

The breaking of chiral symmetry in holographic light-front QCD is encoded in its longitudinal dynamics with its chiral limit protected by the superconformal algebraic structure which governs its transverse dynamics. The scale in the longitudinal light-front Hamiltonian determines the confinement strength in this direction: It is also responsible for most of the light meson ground state mass consistent with the Gell-Mann-Oakes-Renner constraint. Longitudinal confinement and the breaking of chiral symmetry are found to be different manifestations of the same underlying dynamics like in 't Hooft large N_C QCD$(1+1)$ model.

Longitudinal Confinement

$$U_{||} = \sigma^2 x(1-x)\tilde{z}^2$$

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar q}^2}{1-x}\right)\chi(x) + \frac{g^2 N_C}{\pi} P\int_0^1 dx' \frac{\chi(x)-\chi(x')}{(x-x')^2} = M_{\parallel}^2\,\chi(x),$$

$$\sigma = g\sqrt{\pi N_C/3} = \text{const},$$

$$\chi(x) \sim x^{\frac{2m_q}{\sigma}}(1-x)^{\frac{2m_{\bar q}}{\sigma}}$$

$$M_\pi^2 = g\sqrt{\pi N_C/3}\,(m_u+m_d) + \mathcal{O}\big((m_u+m_d)^2\big)$$

GMOR relation

de Teramond, sjb

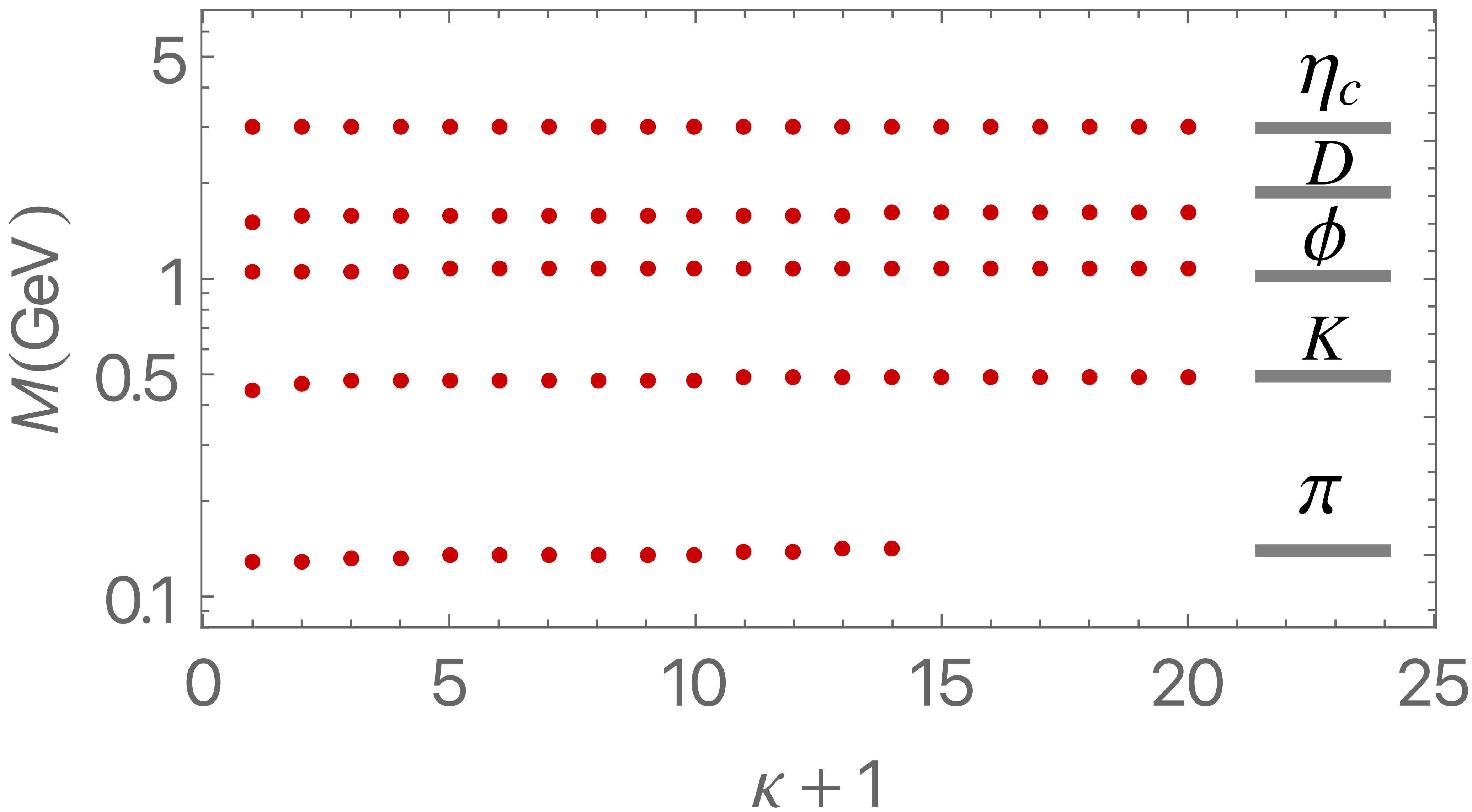
Expand in complete orthonormal basis

$$\chi_\kappa^{\alpha,\beta}(x) = N x^{\alpha/2}(1-x)^{\beta/2} P_\kappa^{(\alpha,\beta)}(1-2x).$$

$$M_\parallel^2 = \sigma^2 \int_0^1 dx\, \chi(x)\Big(- \partial_x \left(x(1-x)\partial_x\right) + \frac{1}{4}\Big[\frac{\alpha^2}{x} + \frac{\beta^2}{1-x}\Big]\Big)\chi(x) = \sigma^2 \sum_\kappa C_\kappa^2\, \nu^2(\kappa,\alpha,\beta),$$

where $\nu^2(\kappa,\alpha,\beta) = \frac{1}{4}(\alpha+\beta+2\kappa)(2+\alpha+\beta+2\kappa)$, with $\alpha = 2m_q/\sigma$ and $\beta = 2m_{\bar{q}}/\sigma$.

Mode expansion



Convergence of ground state meson masses with increasing κ
The horizontal grey lines in the figure are the observed masses.

$$M_\pi^2 = \sigma(m_u + m_d) + \mathcal{O}\big((m_u + m_d)^2\big) \, ,$$

in the limit $m_u, m_d \to 0$. It has the same linear dependence in the quark mass as the Gell-Mann-Oakes-Renner (GMOR) relation

$$M_\pi^2 f_\pi^2 = -\tfrac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}\big((m_u + m_d)^2\big)$$

where the "vacuum condensate" $\langle \overline{\psi}\psi \rangle \equiv \frac{1}{2}\langle \bar{u}u + \bar{d}d \rangle$ plays the role of a chiral order parameter. The same linear dependence arises for the $(3+1)$ effective LF Hamiltonian, since the constraints from the superconformal algebra require that the contribution to the pion mass from the transverse LF dynamics is identically zero.

Interpret $< \bar{\psi}\psi >$ as an *in-hadron condensate*

Roberts, Shrock, Tandy, sjb

Roberts, Richards, Horn, Chang

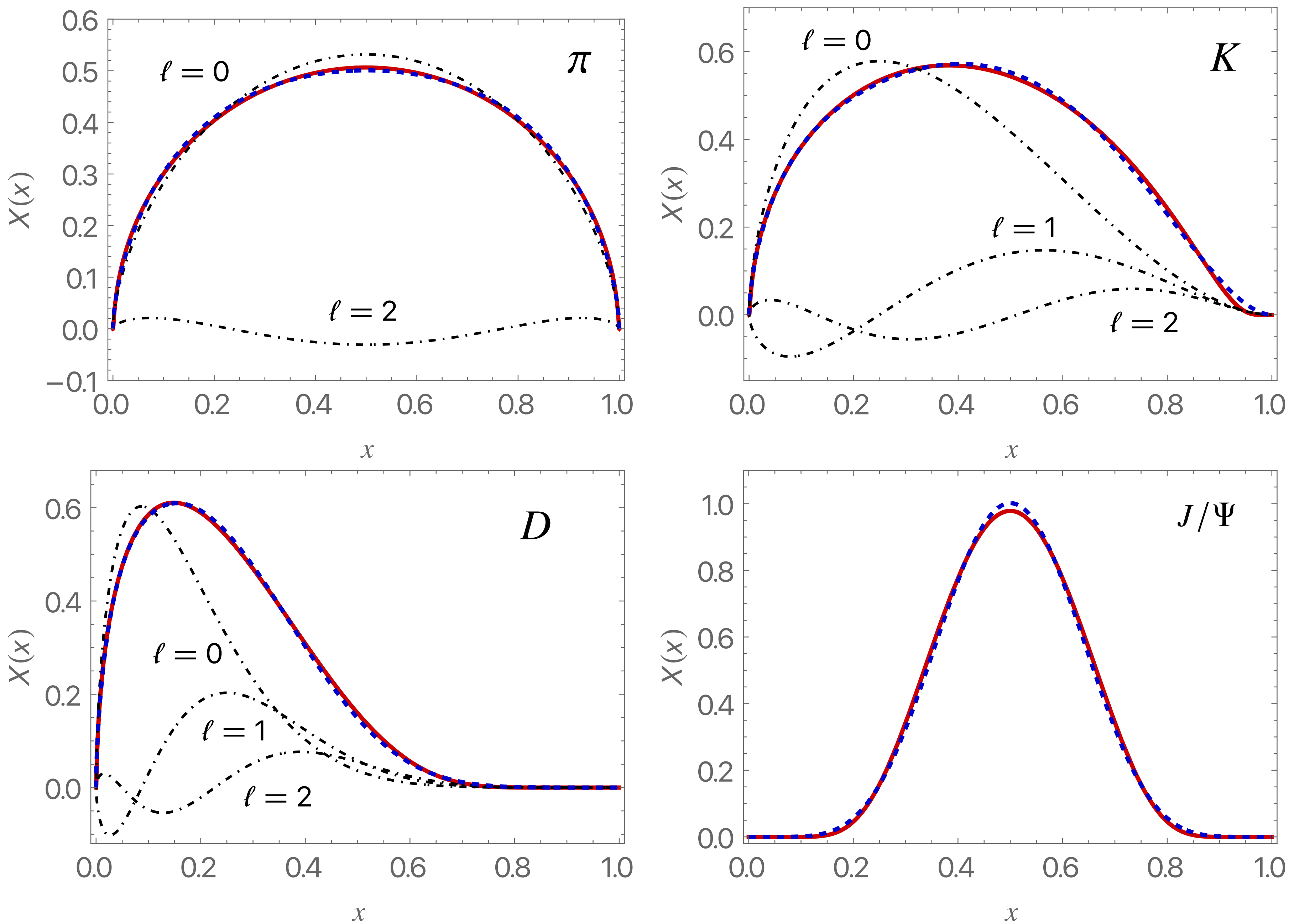


FIG. 3. Light-front distribution amplitudes $X(x)$ for the π, K, D and J/Ψ mesons: the red curve is the invariant mass result, dot dashed black curves are individual modes in the expansion (16), dashed blue curve represent the sum of modes in the figure. Notice that the J/Ψ result is well described by the zero mode alone.

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

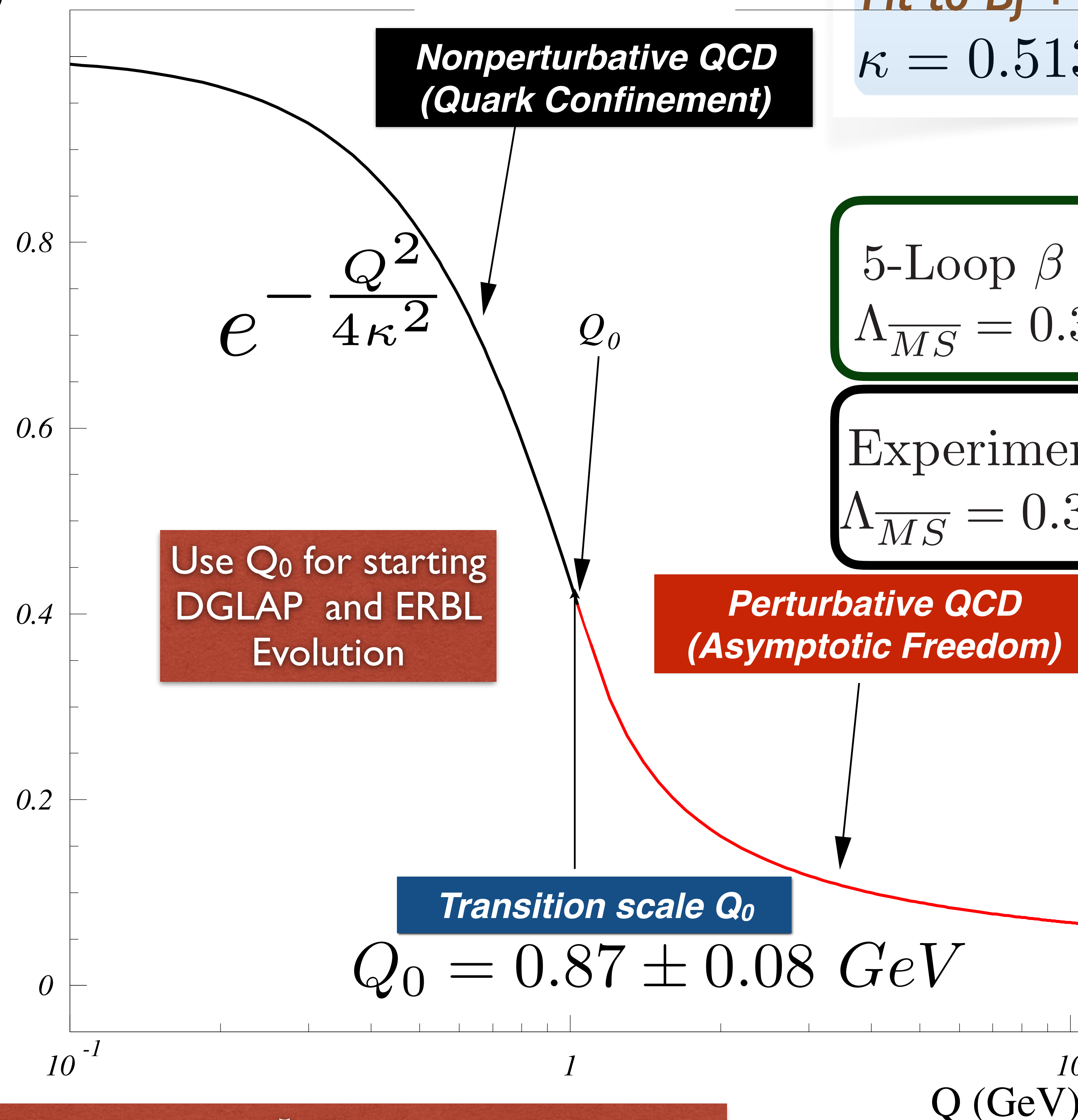
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

$m_\rho = \sqrt{2}\kappa$

$m_p = 2\kappa$

All-Scale QCD Coupling

Deur, de Tèramond, sjb



Fit to Bj + DHG Sum Rules:
$\kappa = 0.513 \pm 0.007\ GeV$

5-Loop β Prediction:
$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$

Experiment:
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$

$\lambda \equiv \kappa^2$

$\overline{MS}$ scheme

Reverse Dimensional Transmutation!

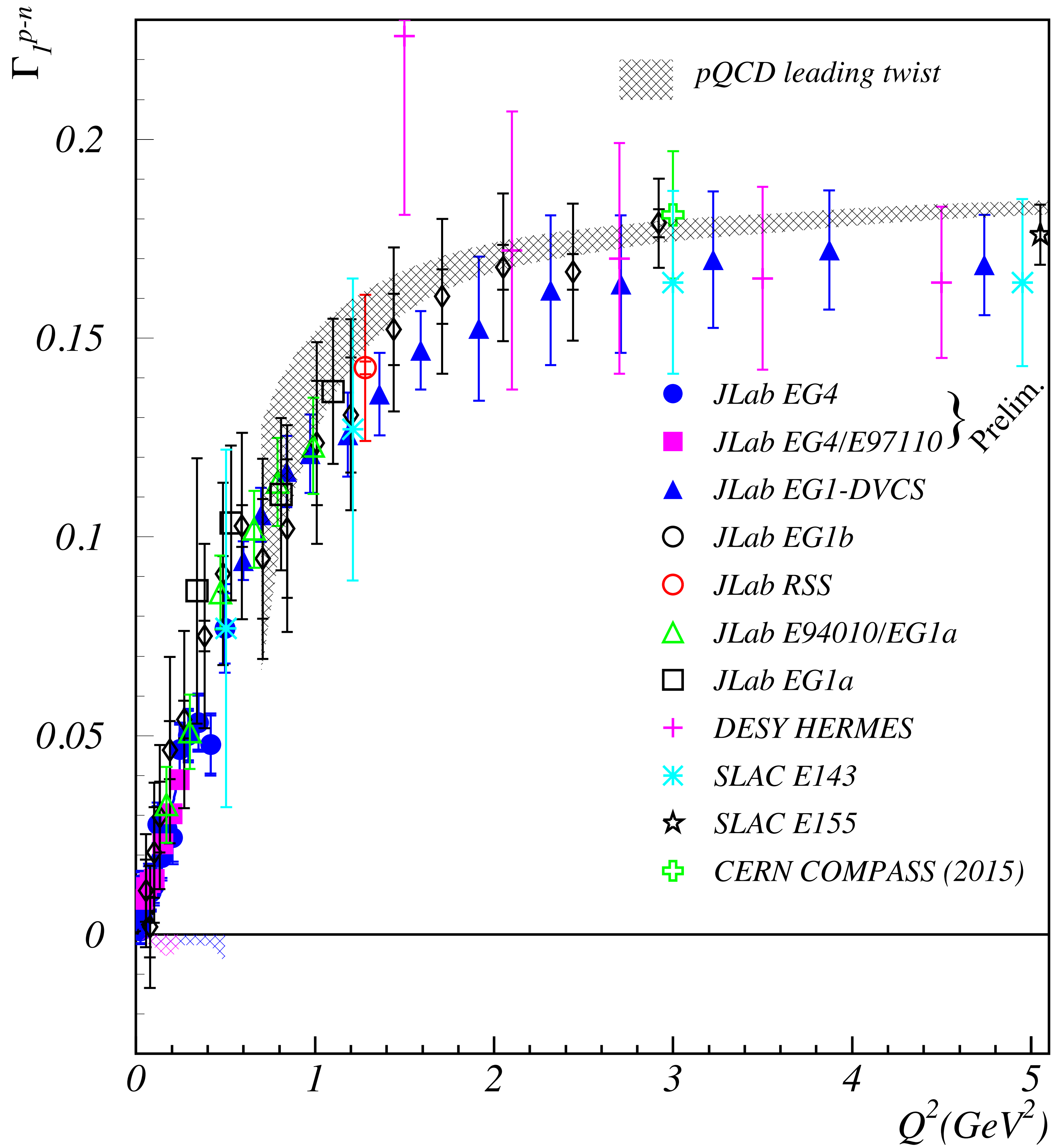
Bjorken sum rule defines effective charge

$\alpha_{g1}(Q^2)$

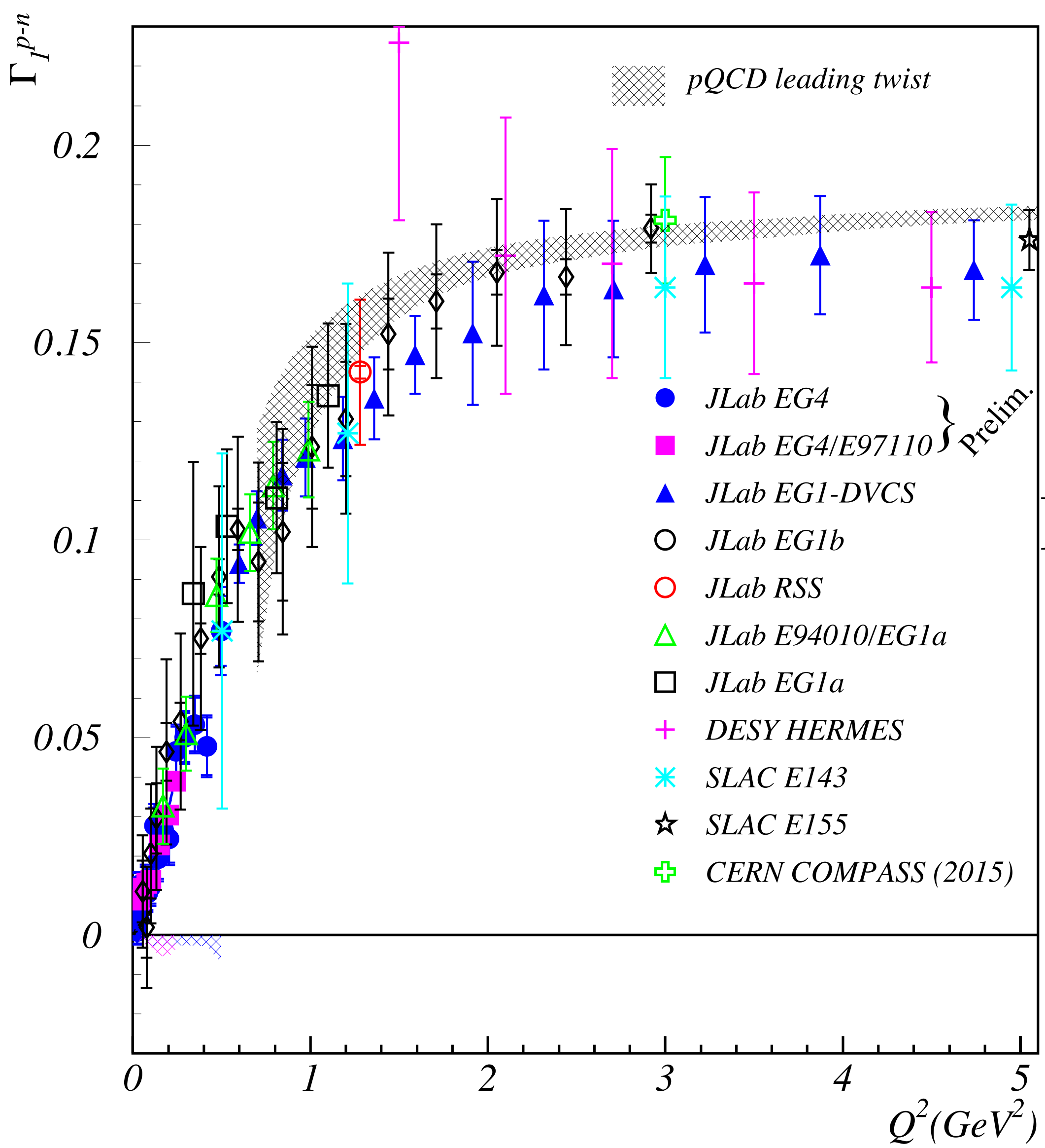
$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

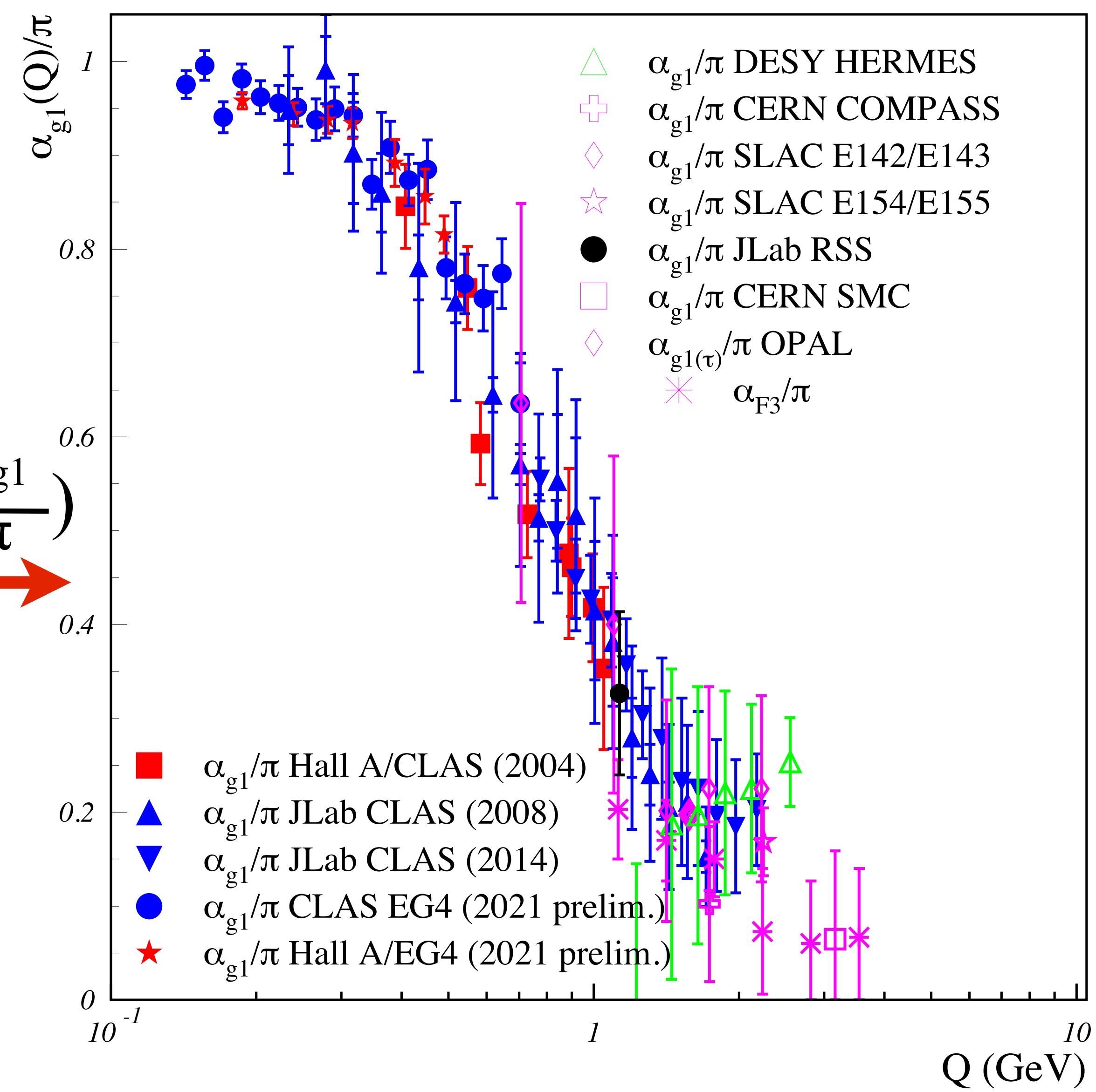
Bjorken sum $\Gamma_1^{p\text{-}n}$ measurements



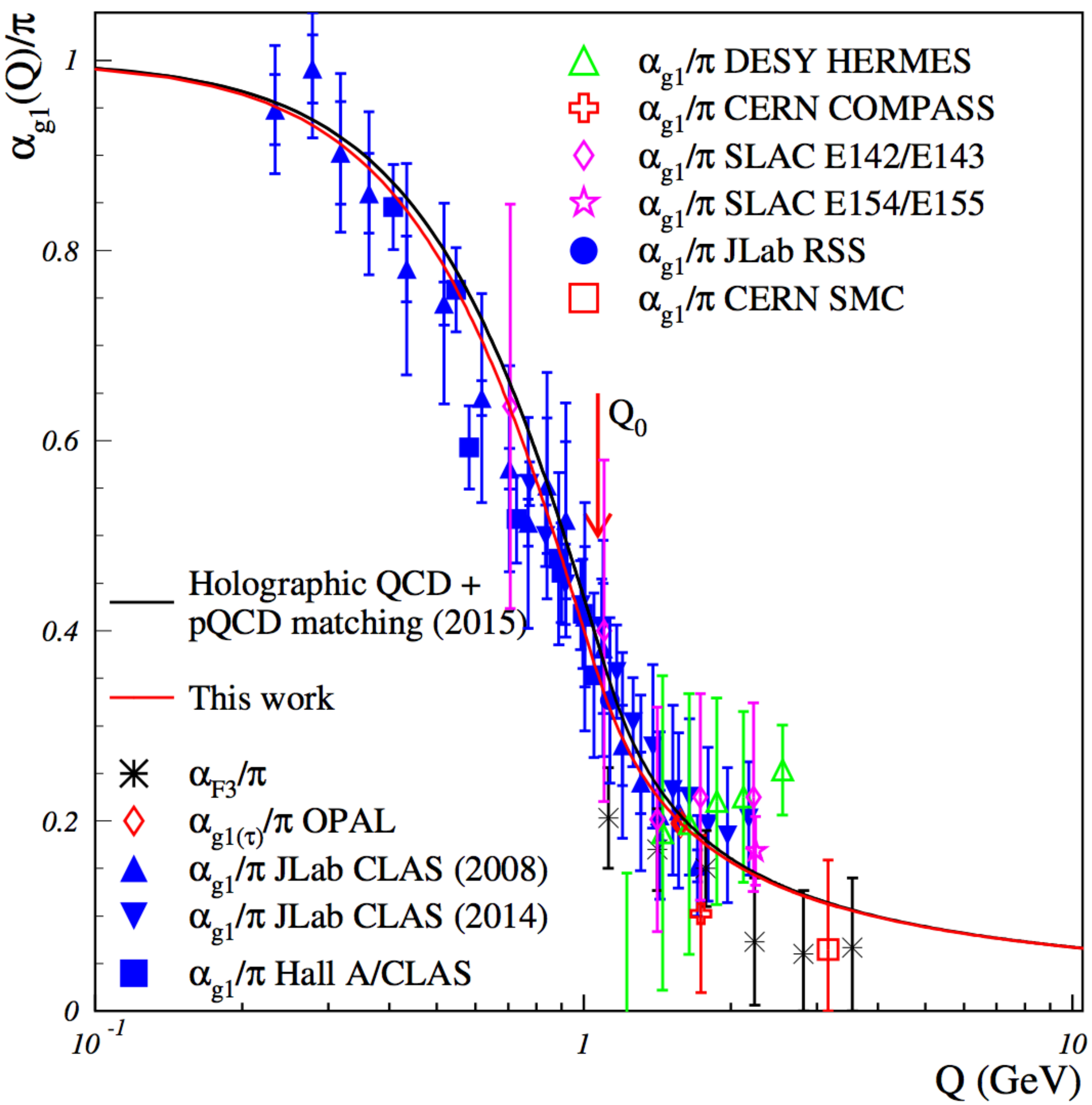
Bjorken sum $\Gamma_1^{p\text{-}n}$ measurements



$$\Gamma_1^{p\text{-}n} \triangleq \frac{1}{6} g_A \left(1 - \frac{\alpha_{g1}}{\pi}\right)$$



Running Coupling from AdS/QCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A}\int_0^1 dx\, g_1^{p-n}(x, Q^2)$$

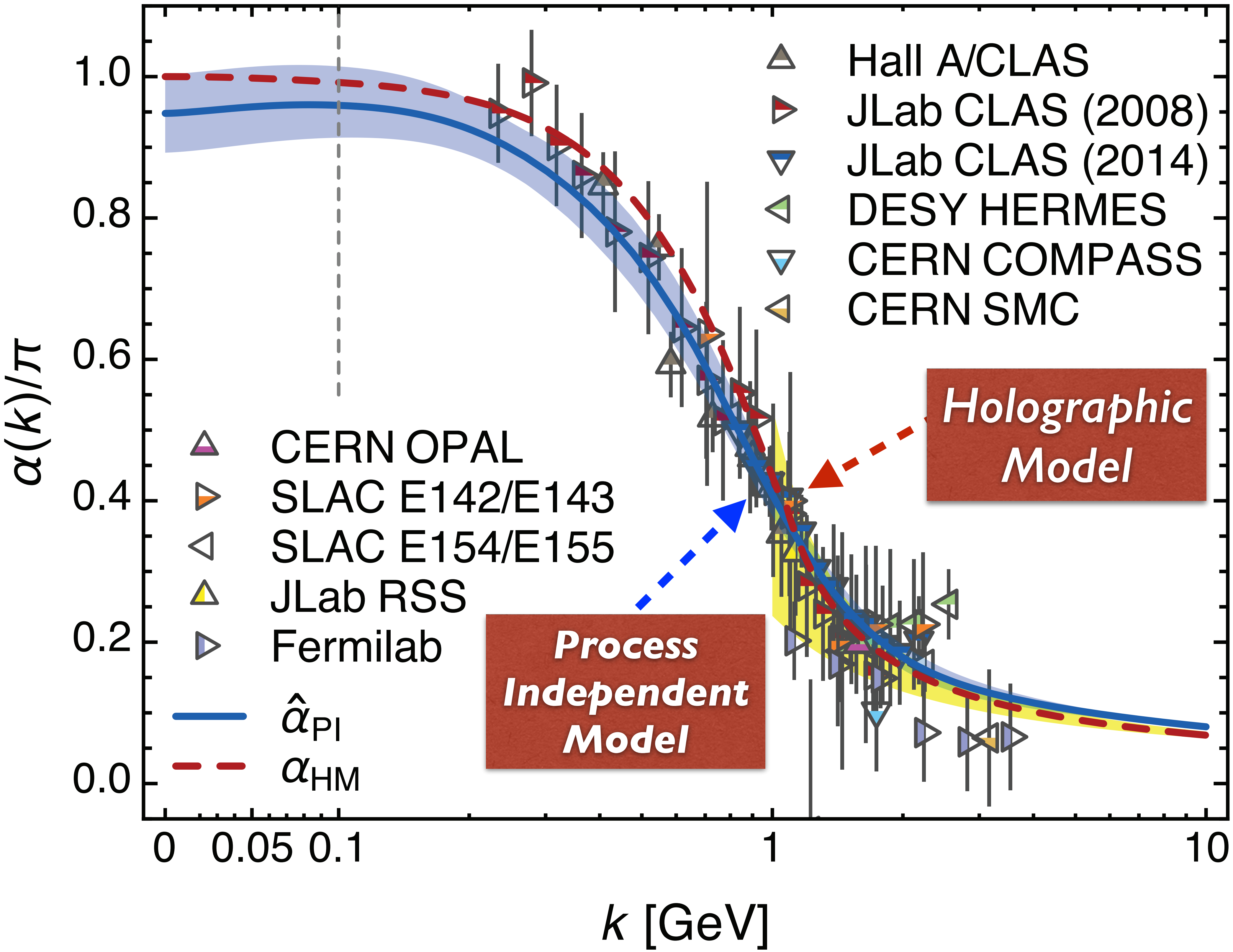
Effective coupling in LFHQCD (valid at low-Q^2)

$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp\left(-Q^2/4\kappa^2\right)$$

Imposing continuity for α and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond,
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

Analytic, defined at all scales, IR Fixed Point



Process-independent strong running coupling

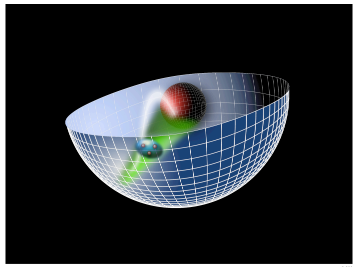
Daniele Binosi,[1] Cédric Mezrag,[2] Joannis Papavassiliou,[3] Craig D. Roberts,[2] and Jose Rodríguez-Quintero[4]

Novel Effects Derived from Light-Front Wavefunctions

- Color Transparency
- Intrinsic heavy quarks at high x $c(x), b(x)$
- Asymmetries $s(x) \neq \bar{s}(x), \; \bar{u}(x) \neq \bar{d}(x)$
- Spin correlations, counting rules at x to 1
- Diffractive deep inelastic scattering $ep \to epX$
- Nuclear Effects: Hidden Color

LFHQCD: Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time* τ
- *Causality: Information within causal horizon: Light-Front*
- *Light-Front Holography:* AdS_5 *= LF (3+1)*



$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b^2_\perp x(1-x)$$

- *Introduce Mass Scale* κ *while retaining the Conformal Invariance of the Action (dAFF)*
- *Unique Dilaton in* AdS_5*:* $e^{+\kappa^2 z^2}$
- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

Compute Hadron Structure, Spectroscopy, and Dynamics from Light-Front Holography

- *Color Confinement*
- *Origin of the QCD Mass Scale*
- *Meson and Baryon Spectroscopy*
- *Exotic States: Tetraquarks, Pentaquarks, Gluonium,*
- *Universal Regge Slopes: n, L, Mesons and Baryons*
- *Almost Massless Pion: GMOR Chiral Symmetry Breaking*
  $M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}\big((m_u + m_d)^2\big)$
- *QCD Coupling at all Scales* $\alpha_s(Q^2)$
- *Eliminate Scale Uncertainties and Scheme Dependence: PMC*

$$\mathscr{L}_{QCD} \rightarrow \psi_n^H(x_i, \overrightarrow{k}_{\perp i}, \lambda_i)$$ *Valence and Higher Fock States*

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770)$, $\omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320)$, $f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690)$, $\omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)$?
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040)$, $f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)$?
$\bar{q}s$	$0^{-(+)}$	$\bar{K}(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$\bar{K}_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$\bar{K}_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$\bar{K}_1^*(\sim 1700)$?
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)$?
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)$?
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)$?
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)$?
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)$?

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

$$3_C \times 3_C = \bar{3}_C + 6_C$$

Bound!

- Diquark: Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states $\bar{3}_C \times 3_C = 1_C$
- $$\sigma(TN) \simeq 2\sigma(pN) - \sigma(\pi N)$$

$$2\left[\sigma([\{qq\}N) + \sigma(qN)\right] - [\sigma(qN) + \sigma(\bar{q}N)] = [\sigma(\{qq\}N) + \sigma(\{qq\}N)]$$

Candidates $f_0(980) I = 0, J^P = 0^+$, partner of proton

$a_1(1260) I = 0, J^P = 1^+$, partner of $\Delta(1233)$

Test twist=4, power-law fall-off of form factors

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- OPE: Constituent Counting Rules
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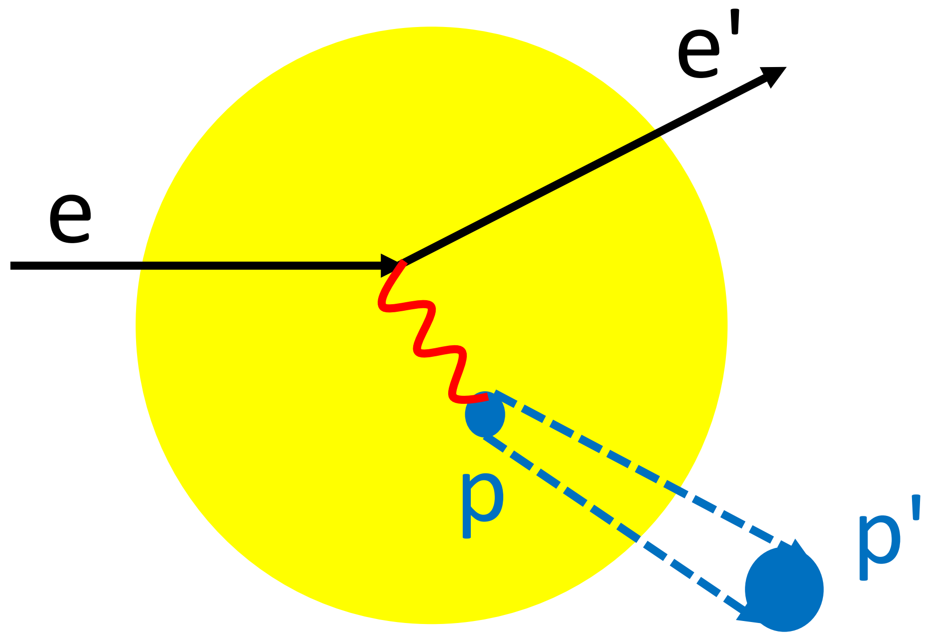
- Systematically improvable: Basis LF Quantization (BLFQ)

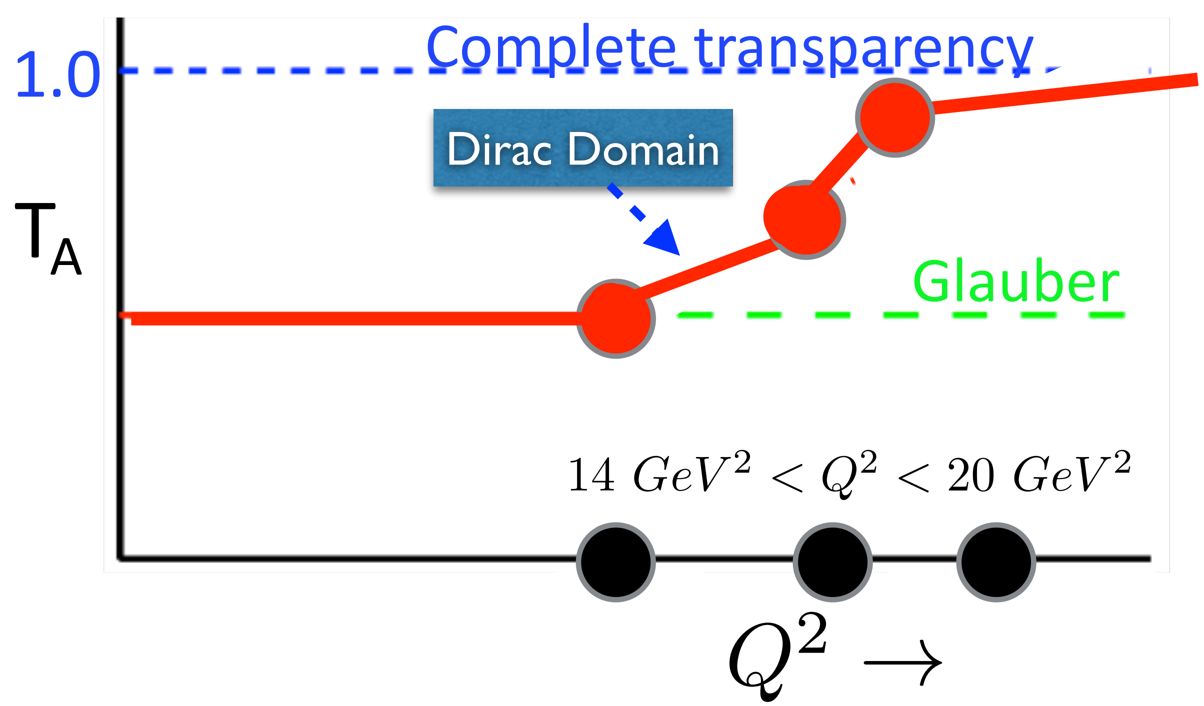
Color transparency: fundamental prediction of QCD

A.H. Mueller, sjb

$$e + A \to e' + p' + X$$



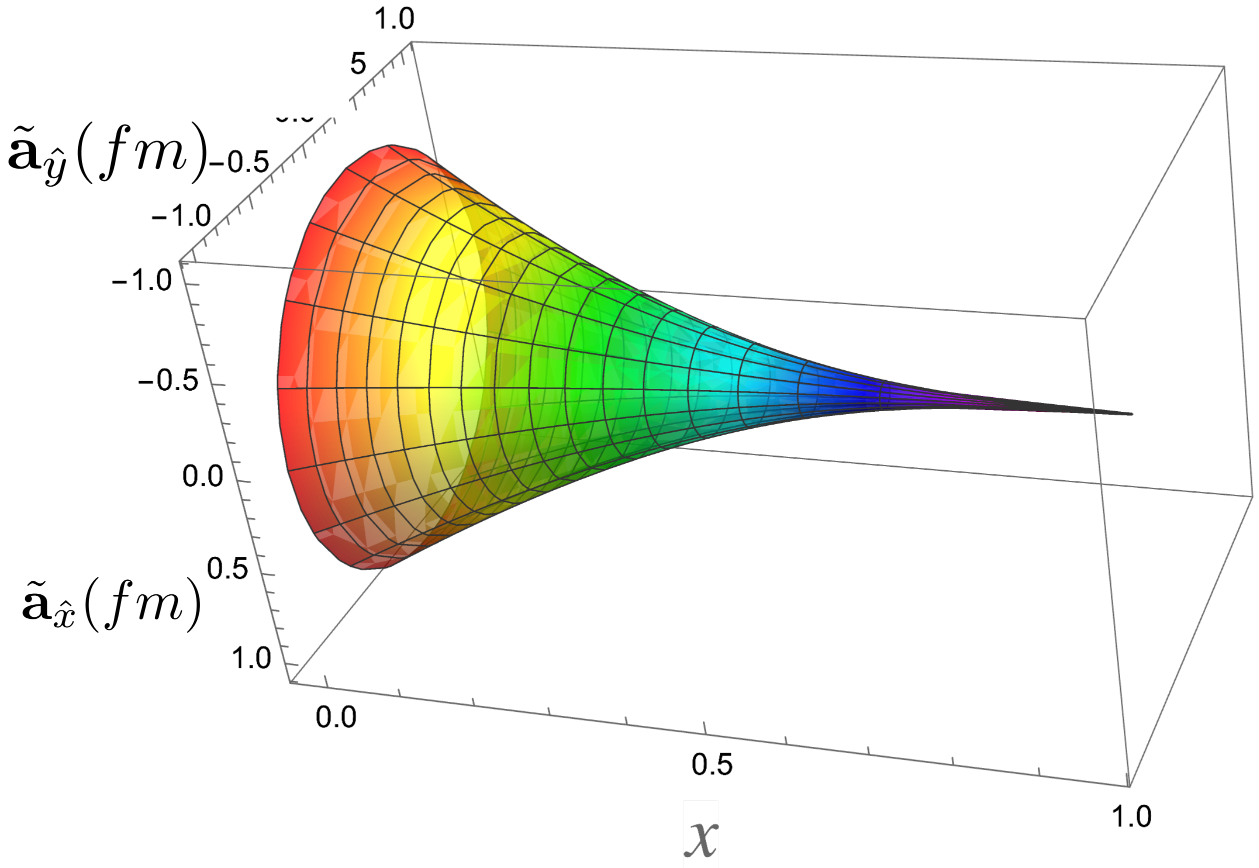
- Not predicted by strongly interacting hadronic picture → arises in picture of quark-gluon interactions
- QCD: color field of singlet objects vanishes as size is reduced
- Signature is a rise in nuclear transparency, T_A, as a function of the momentum transfer, Q^2



$$T_A = \frac{\sigma_A}{A\ \sigma_N}$$ σ_A (nuclear cross section), σ_N (free nucleon cross section)

with Guy de Tèramond

Two-Stage Color Transparency for Proton



$$< \tilde{\mathbf{a}}^2_\perp(x) >= \frac{\int d^2\mathbf{a}_\perp \mathbf{a}^2_\perp q(x,\mathbf{a}_\perp)}{\int d^2\mathbf{a}_\perp q(x,\mathbf{a}_\perp)}$$

At large light-front momentum fraction x, and equivalently at large values of Q^2, the transverse size of a hadron behaves as a point-like color-singlet object. This behavior is the origin of color transparency in nuclei.

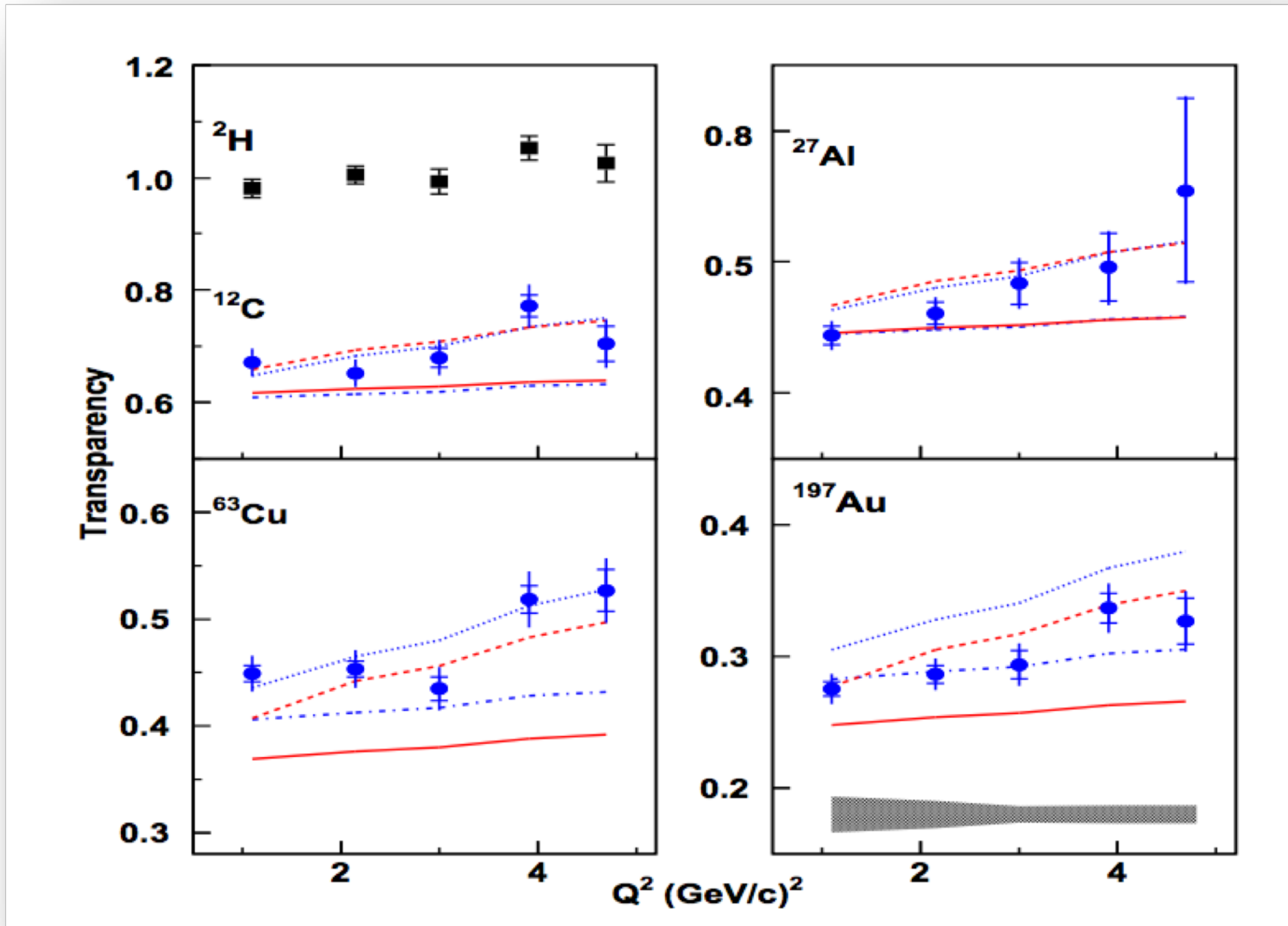
Although the dependence of the transverse impact area as a function of x is universal, the behavior in Q^2 depends on properties of the hadron, such as its twist.

$$\langle \mathbf{a}^2_\perp(Q^2)\rangle_\tau \to \frac{4(\tau-1)}{Q^2}.$$

Mean transverse size as a function of Q and Twist

Hall C E01-107 pion electro-production

$A(e,e'\pi^+)$

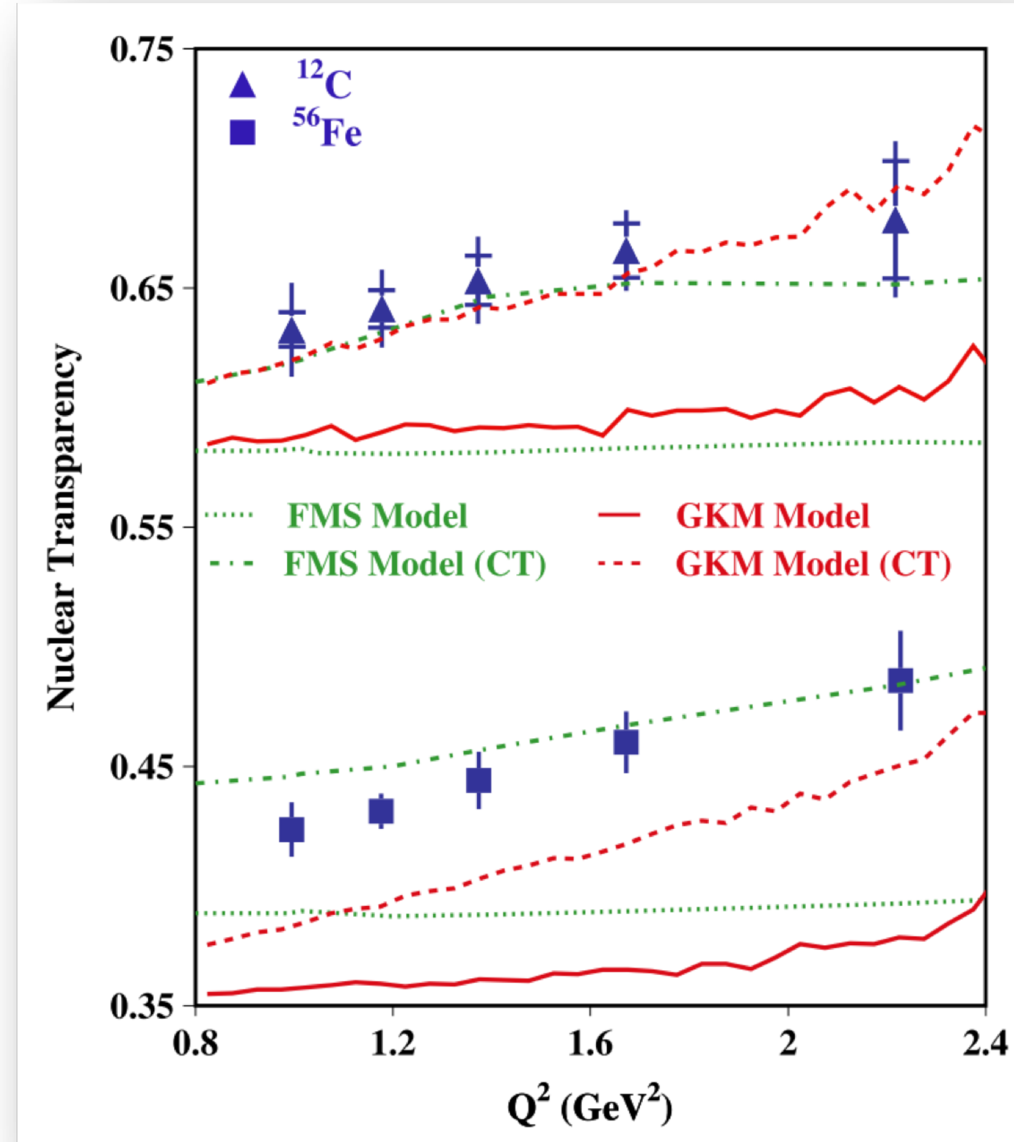


B.Clasie *et al.* PRL 99:242502 (2007)

X. Qian *et al.* PRC81:055209 (2010)

CLAS E02-110 rho electro-production

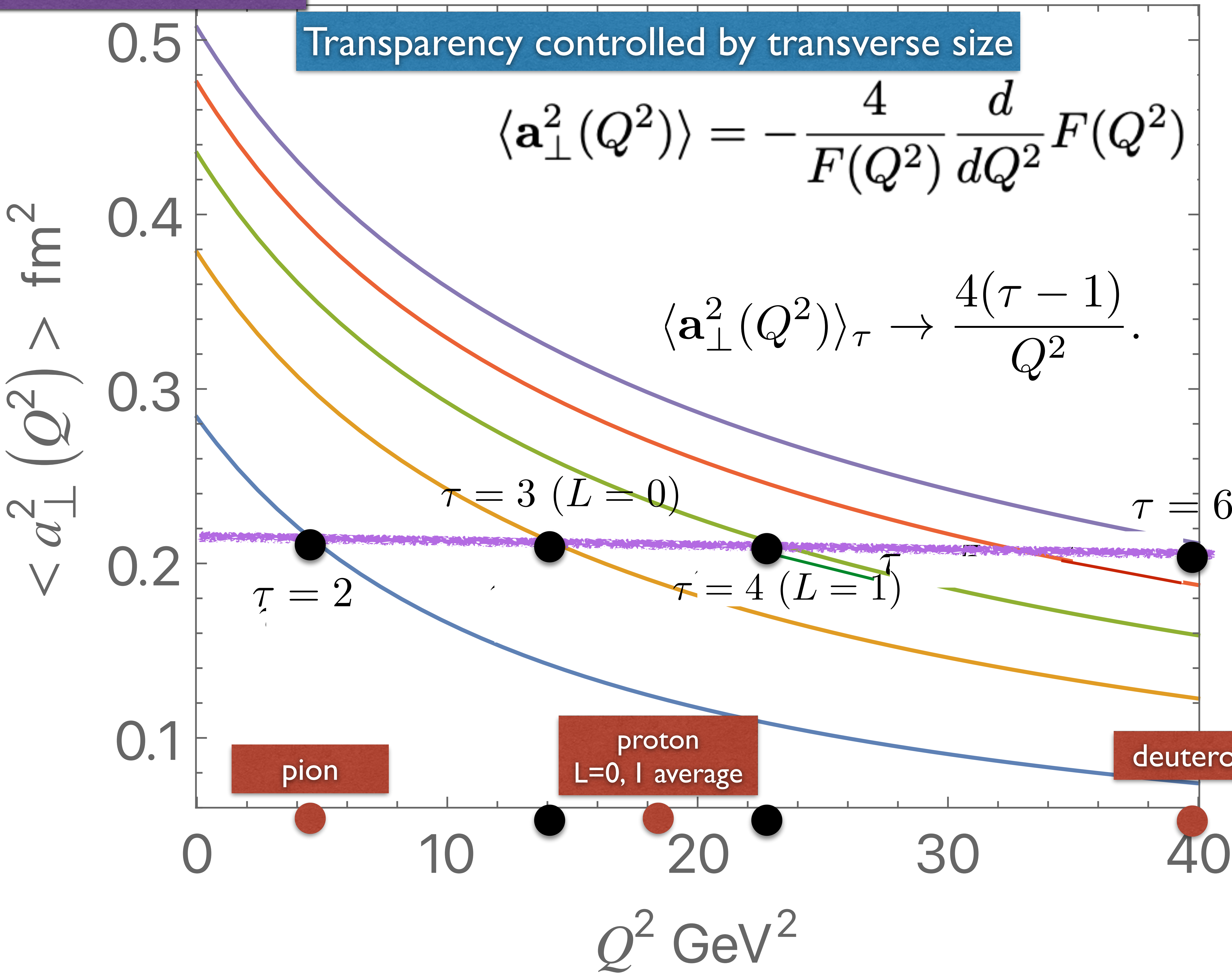
$A(e,e'\rho^0)$



L. El Fassi *et al.* PLB 712,326 (2012)

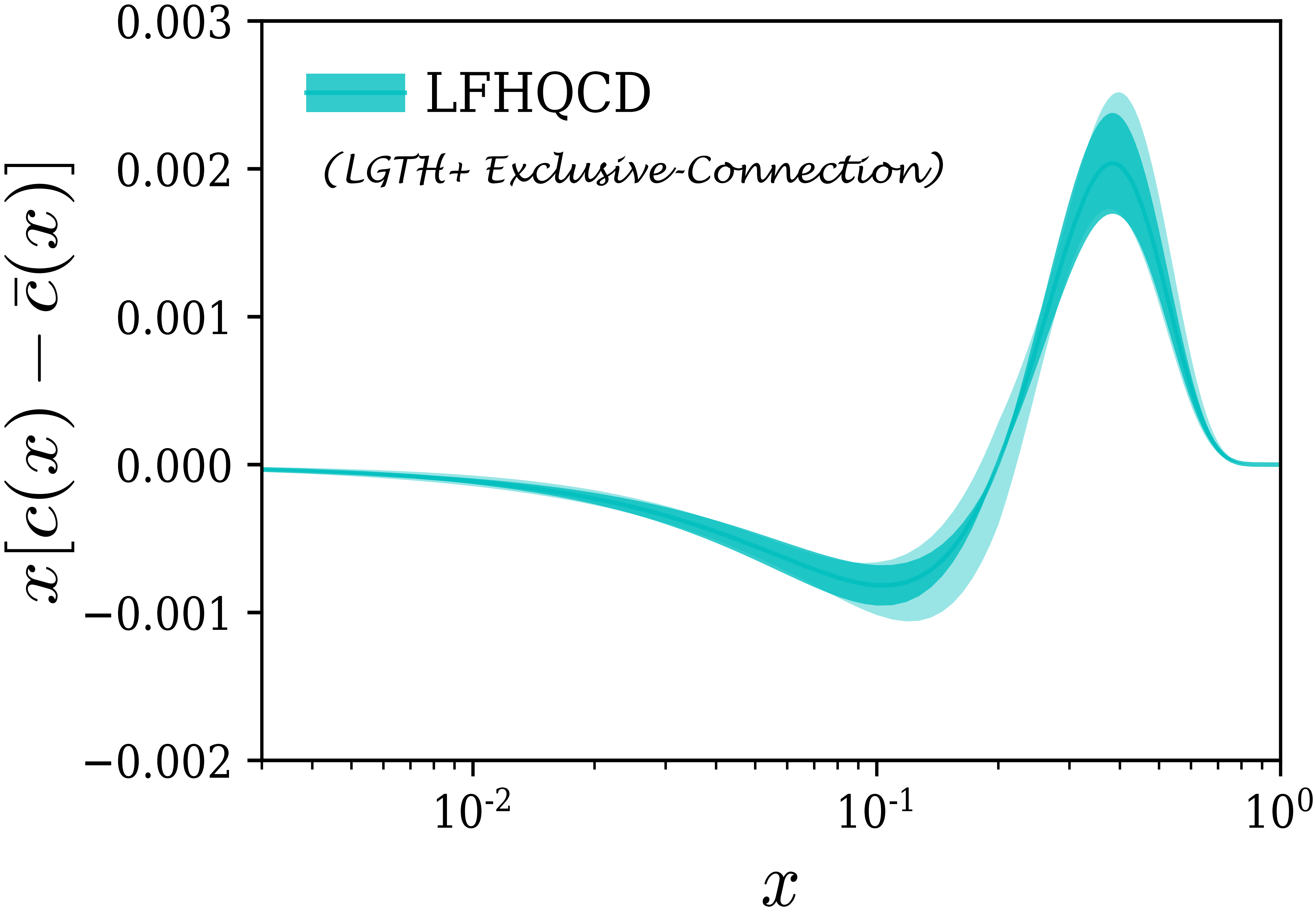
$$< a_\perp^2(Q^2 = 4\ GeV^2) >_{\tau=2} \simeq < a_\perp^2(Q^2 = 14\ GeV^2) >_{\tau=3} \simeq < a_\perp^2(Q^2 = 22\ GeV^2) >_{\tau=4} \simeq 0.24\ fm^2$$

5% increase for T_π in ^{12}C at $Q^2 = 4\ GeV^2$ implies 5% increase for T_p at $Q^2 = 18\ GeV^2$



Proton has equal probability for $\tau = 3$ and $\tau = 4$

with Guy de Tèramond



The distribution function $x[c(x) - \bar{c}(x)]$ obtained from the LFHQCD formalism using the lattice QCD input of charm electromagnetic form factors $G^c_{E,M}(Q^2)$. The outer cyan band indicates an estimate of systematic uncertainty in the $x[c(x) - \bar{c}(x)]$ distribution obtained from a variation of the hadron scale κ_c by 5%.

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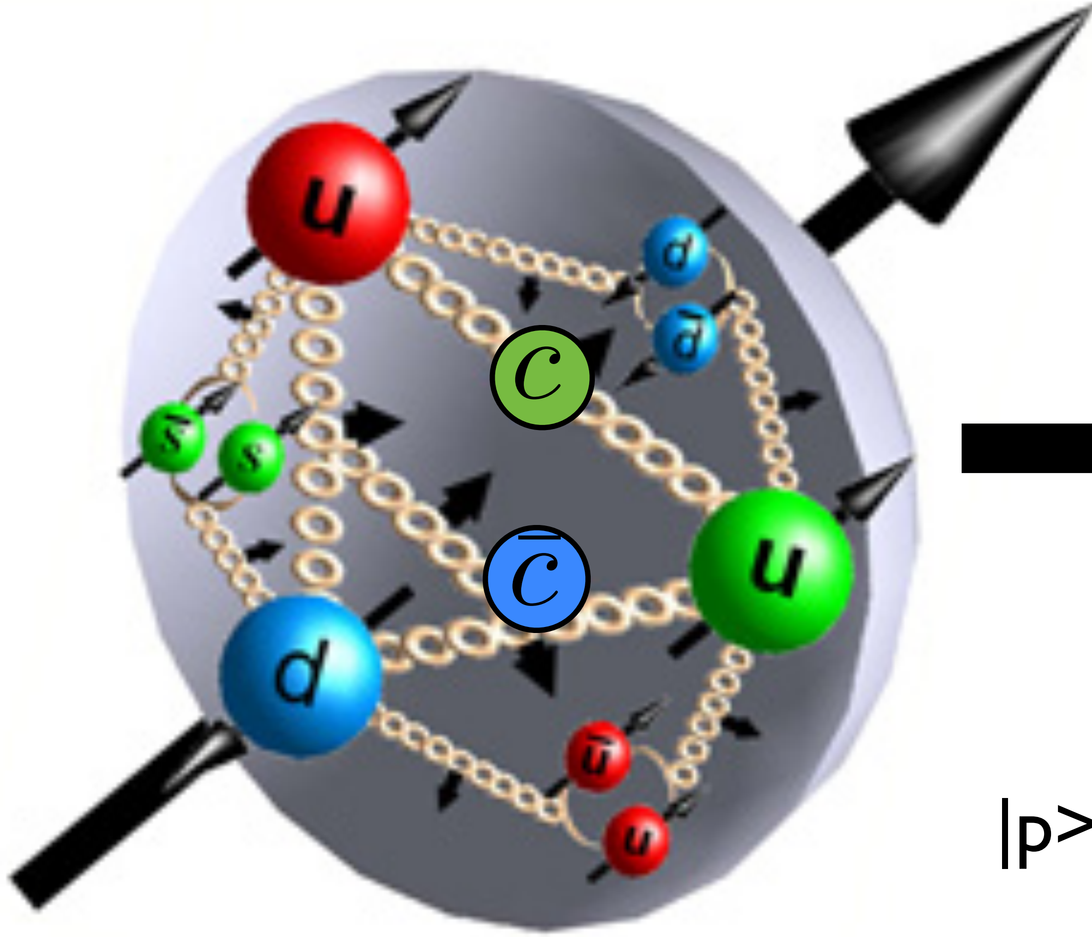
- **Systematically improvable: Basis LF Quantization (BLFQ)**

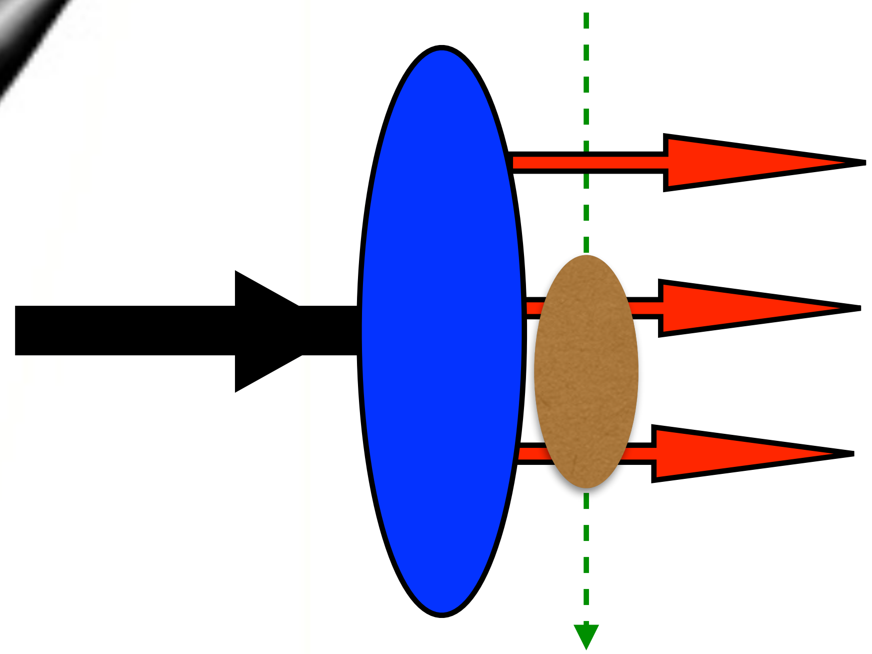
Invariance Principles of Quantum Field Theory

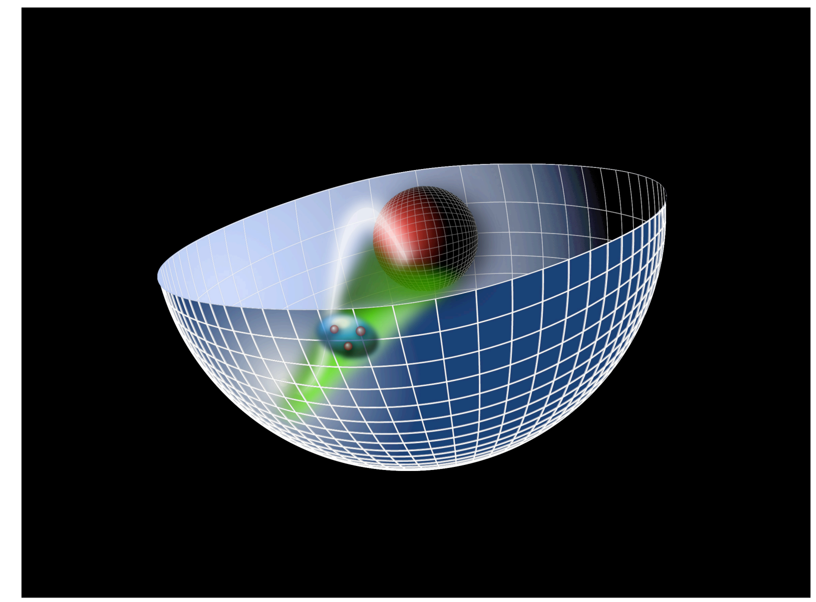
- *PoIncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form*
- *Causality: Information within causal horizon: Front Form*
- *Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge*
- *Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme —*
  Principle of Maximum Conformality (PMC)
- *Mass-Scale Invariance:*
  Conformal Invariance of the Action (DAFF)

Light-Front Holography — A Novel Approach to QCD Color Confinement, Hadron Dynamics and Spectroscopy



Fixed $\tau = t + z/c$





|p>=|u[ud]> quark-diquark cluster

with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, Ivan Schmidt, F. Navarra, Jennifer Rittenhouse West, G. Miller, Keh-Fei Liu, Tianbo LIu, Liping Zou, S. Groote, S. Koshkarev, Xing-Gang Wu, Sheng-Quan Wang, Cedric Lorcè, R. S. Sufian, A. Deur, R. Vogt, G. Lykasov, S. Gardner, S. Liuti

STRONG 2020

Trento ECT*

Gauge Topology, Flux Tubes and Holographic Models

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

May 23, 2022