The HISH (Holographic Inspired Stringy Hadron) model - recent developments

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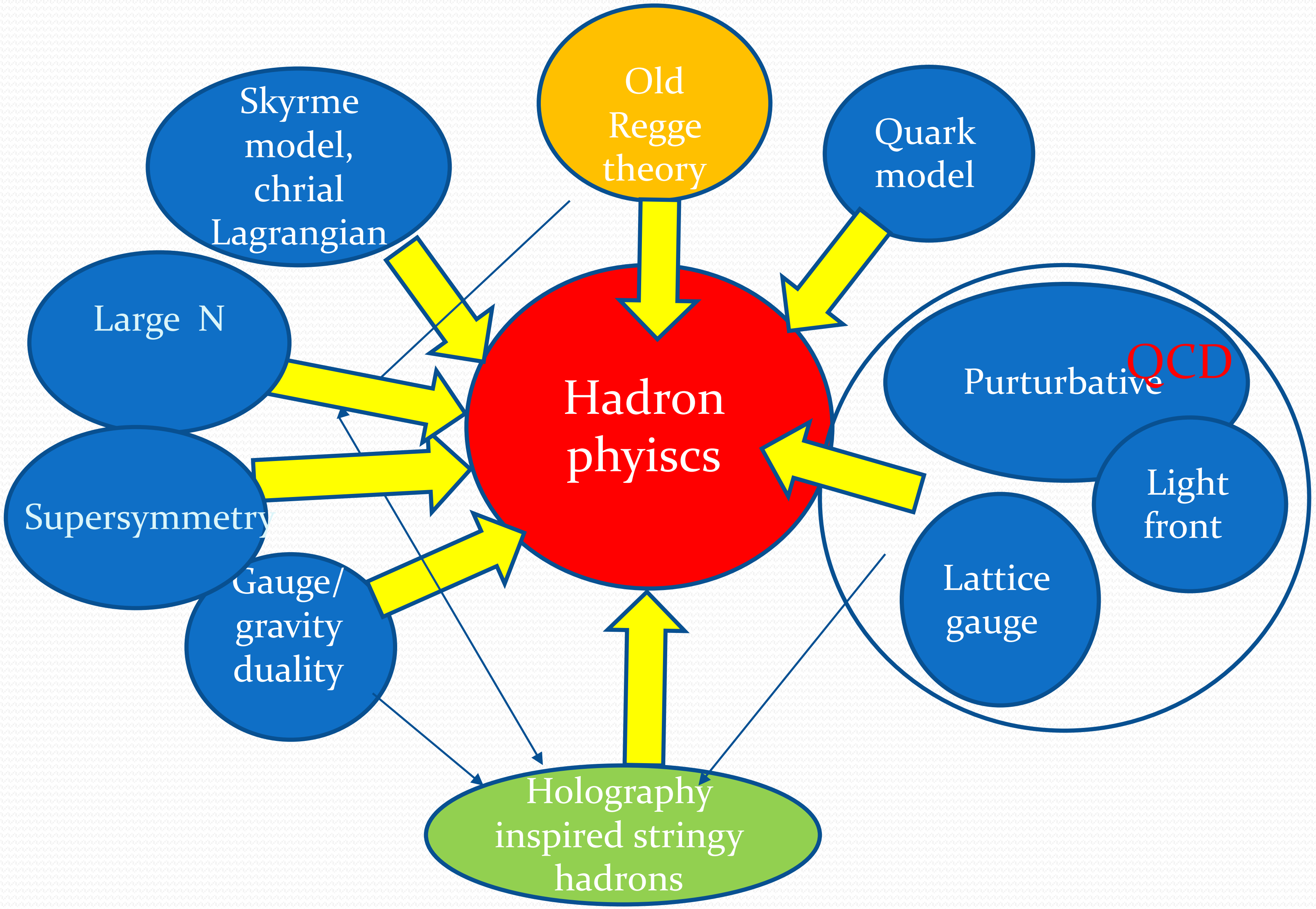
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Old Regge theory
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QCD
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Lattice gauge
Supersymmetry
Gauge/ gravity duality
Holography inspired stringy hadrons

The holography inspired stringy hadron (HISH) model

- The idea of **HISH** is to construct a **phenomenological unitary string model** that is in accordance with as much as possible **experimental data** of **hadron physics** and to **predict** properties and phenomena that have not been measured so far.
- The model is in flat four dimensions but is **inspired** by strings in **holographic backgrounds.**
- It includes **closed strings** and **open strings** that have **massive particles** on their **ends** that **carry electric charge spin** and may be a **baryonic vertex.**
- Eventually the challenge is to relate it to **QCD**

HISH- Holography Inspired Stringy Hadron

- The construction of the **HISH** model is based on the following steps.
- Step 1- Determining a prototype **confining holographic** background with flavor branes.
- Step 2- Analyzing classical strings that correspond to **mesons, baryons, glueballs and exotic hadrons** in the confining holographic background
- Step 3- ``**Mapping**" the classical holographic strings, in particular **rotating** ones, to strings with in **flat 4d** associated with **Nc=3.**

Construction of the HISH model

- Step 4- **Quantizing** the fluctuations of the classical stringy hadrons subjected to boundary conditions that correspond to adding endpoint particles with **masses, electric charges, spins and baryonic vertices**.

- Step 5- **Renormalizing** the world sheet Hamiltonian using a **contour integral** and **Casimir-like method**. Determining the **intercept.** Including the contribution of the **Liouville mode** associated with **a non-critical string**

Construction of the HISH model

- Step 6- Confronting the outcome of the model with **experimental data** extracting the **best fit values** for the **string tension(slope), endpoint masses** and **intercepts** from all the hadron (meson and baryon) **trajectories**

- Step 7- Using holography computing strong **decay processes, the total width and branching ratios.** This is based mainly on **breaking** of the **hadronic string**.

- Step 8- Analyzing stringy **glueballs and** stringy **exotic hadrons** like **tetra-quarks penta-quarks** etc

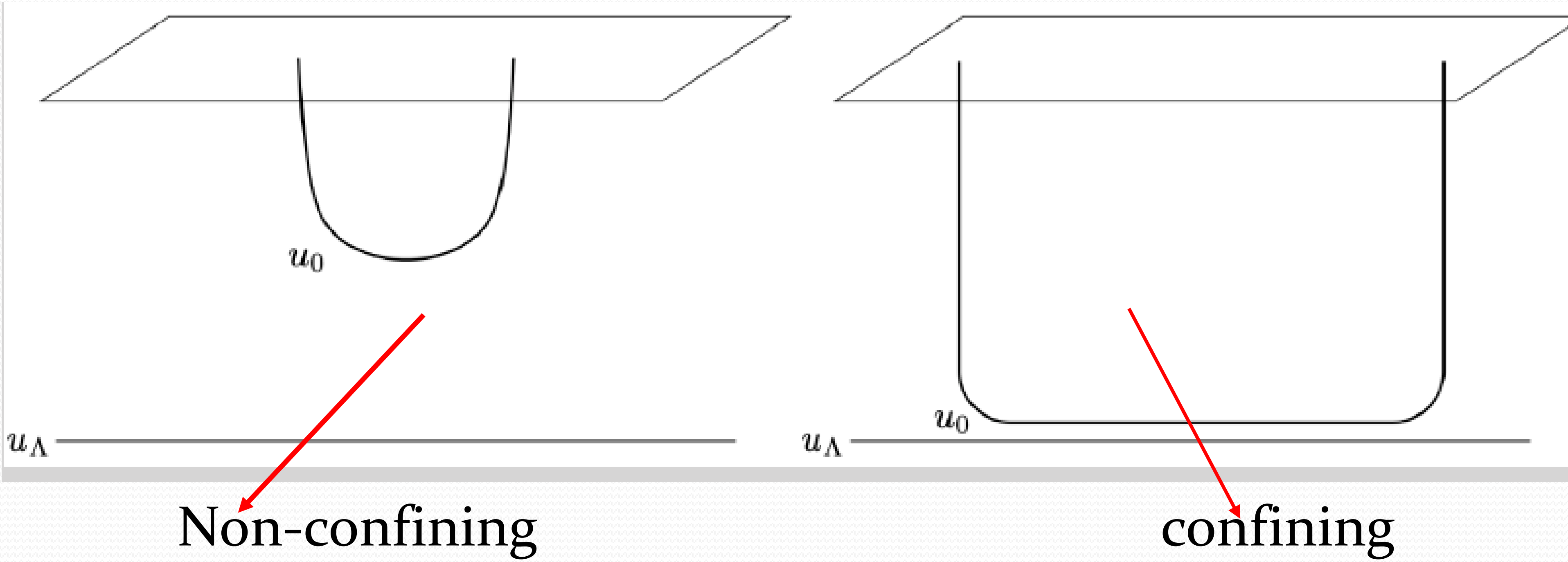
Construction of the HISH model

- Step 9- Determining **scattering amplitudes** using String amplitudes (a la Veneziano) with **weighted average scale dependent string tension**.
- Step 10- **Predicting masses** and **widths of yet unknown states** and other properties

Step 1- Holographic confining backgrounds

Holographic confining background

- We use the **Wilson-Maldacena line** as a measure of confinement. If the **renormalized action** of the **classical string** is **linear in L-** the separation between the endpoints it associates with a **confining** boundary gauge theory



Holographic confining background

- For a metric background of the form

$$ds^2 = -G_{00}(u)dt^2 + G_{x_{||}x_{||}}(u)dx_{||}^2 + G_{uu}(u)du^2 + G_{x_T x_T}(u)dx_T^2$$

- Sufficient conditions for a **confining Wilson-Maldacena line** are if either Y.Kinar, E.Schrieber J.S

(i) $f^2(u) = G_{00}G_{xx}(u)$ has a minimum at u_{min} and $f(u_{min})>0$

(ii) $g^2(u) = G_{00}G_{uu}(u)$ diverges at u_{div} and $f(u_{div})>0$

There are several such backgrounds like **Klebanov Strassler, Maldacena Nunez** and **Witten Saka Sugimoto** models

Witten's model of confining background

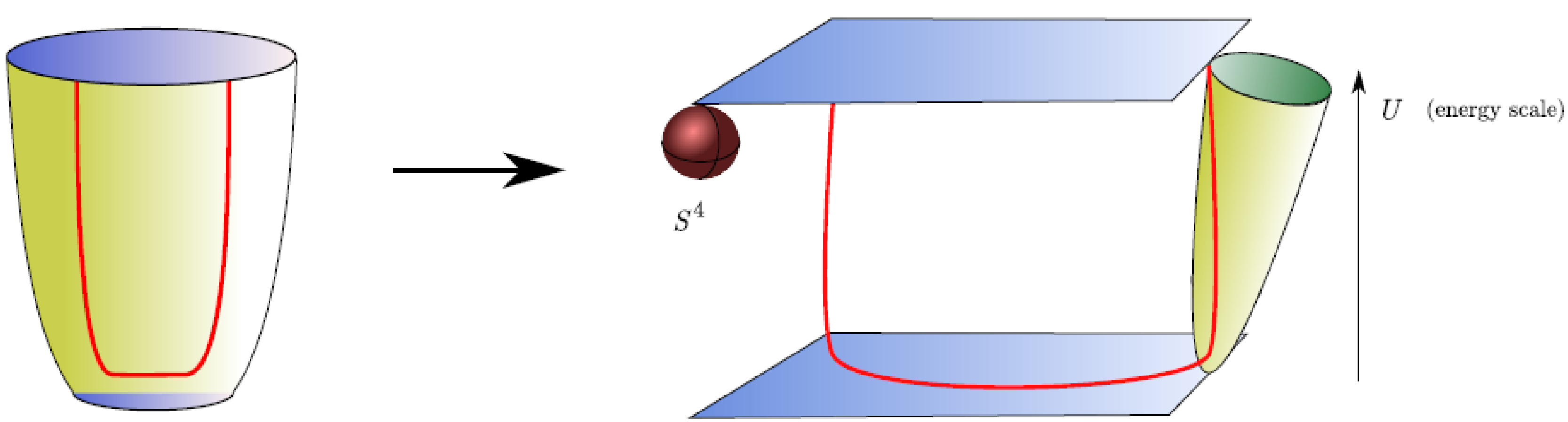
$$ds^2 = \left(\frac{U}{R_{D4}}\right)^{3/2}\left[\eta_{\mu\nu}dX^\mu dX^\nu + f(U)d\theta^2\right] + \left(\frac{R_{D4}}{U}\right)^{3/2}\left[\frac{dU^2}{f(U)} + U^2 d\Omega_4\right]$$

world-volume
our 3+1 world

$f(U) = 1 - \left(\frac{U_\Lambda}{U}\right)^3$

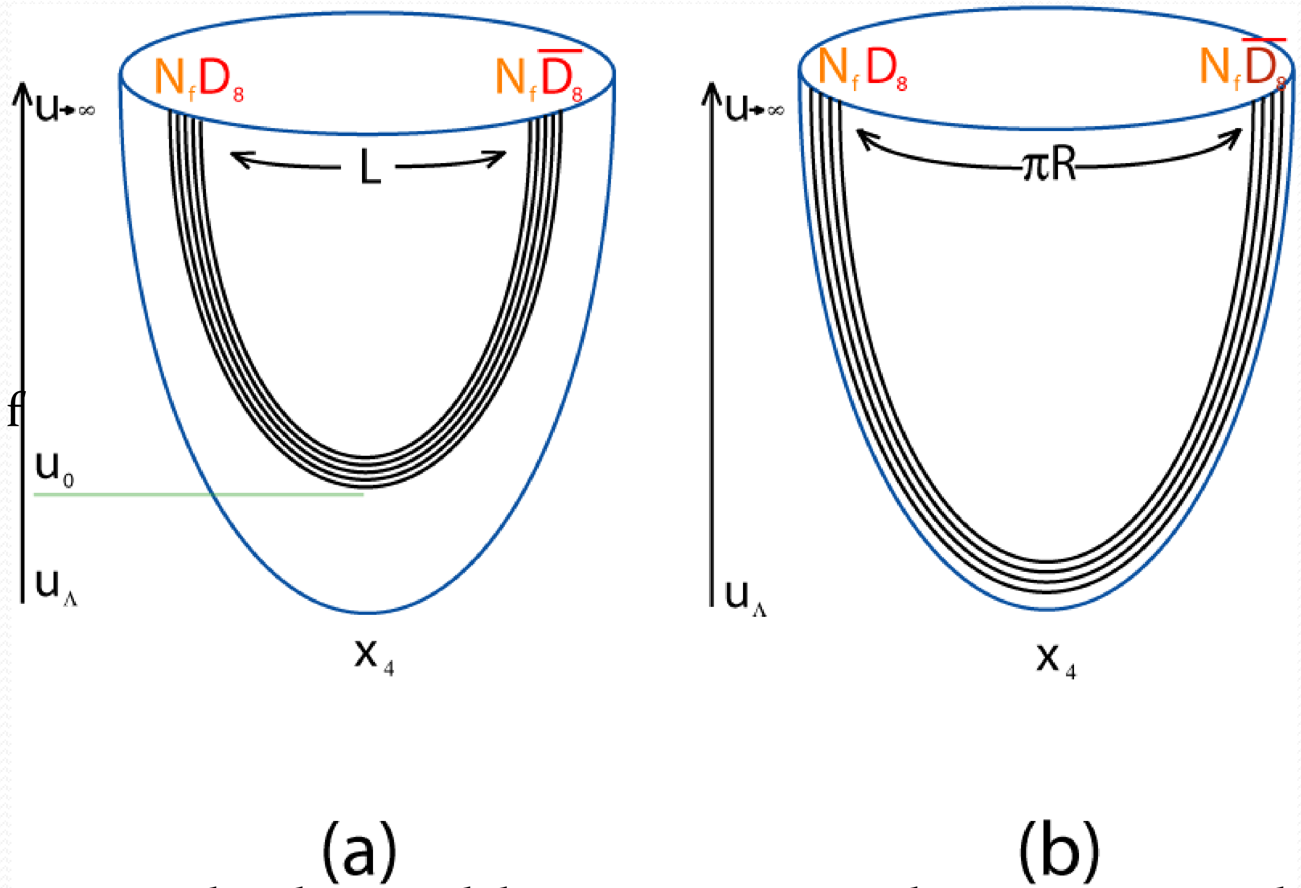
θ is a compact Kaluza-Klein circle

U: radial direction bounded from below $U \geq U_\Lambda$



Adding flavor: The Sakai Sugimoto model

- Adding Nf D8 and anti-D8 **flavor branes**
- In the cigar geometry the flavors brane have a U shape profile



Generalized SS model

Sakai Sugimoto model

Step 2- Stringy holographic Hadrons

String/field theory holography versus gravity/FT

- The **holographic duality** is an equivalence between a certain bulk **string theory** and **boundary field theories.**

- Practically most of the applications of holography are based on relating bulk fields (not strings) and operators on the dual boundary field theory .

- This is based on the usual limit of $\alpha' \rightarrow 0$ with which we go, for instance, from a closed string theory to a gravity theory .

String/QFT holography versus gravity/QFT

- There is a wide range of hadronic physical observables which cannot be faithfully described by bulk fields but rather require dual stringy phenomena like Wilson, 't Hooft and Polyakov lines

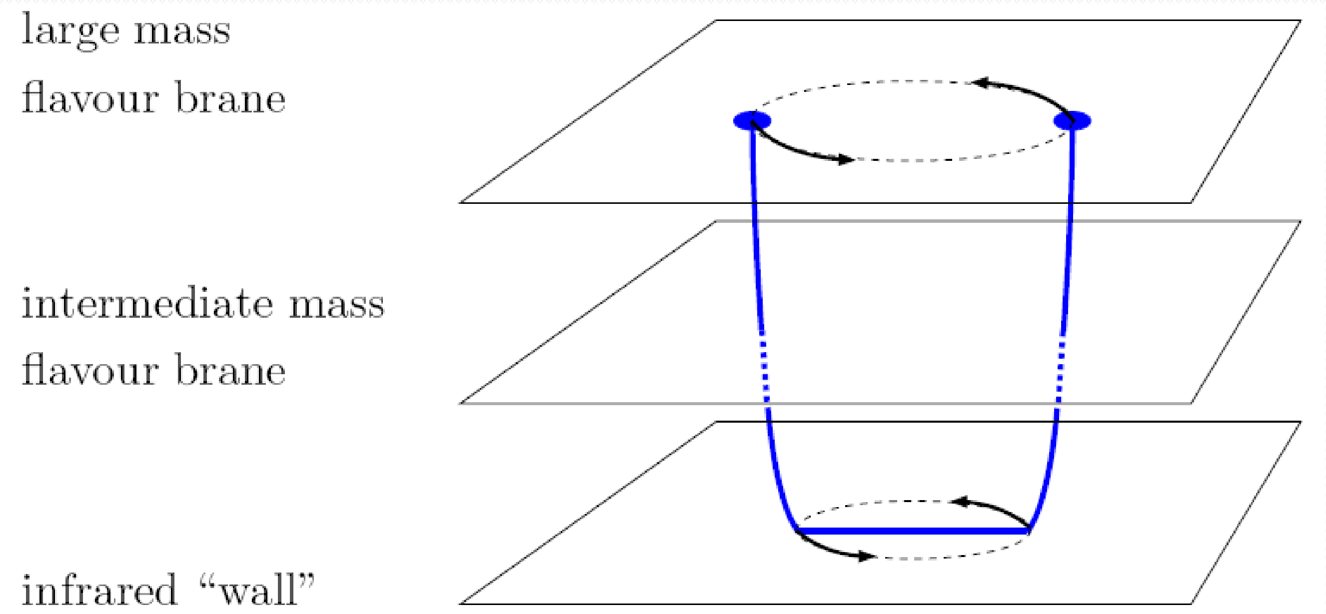
- We argue here that in fact also the **spectra, decays width** and **scattering amplitudes** of

mesons, baryons, exotics and glueballs

can be recast only as **holographic stringy hadrons**

(1) The rotating holographic stringy meson

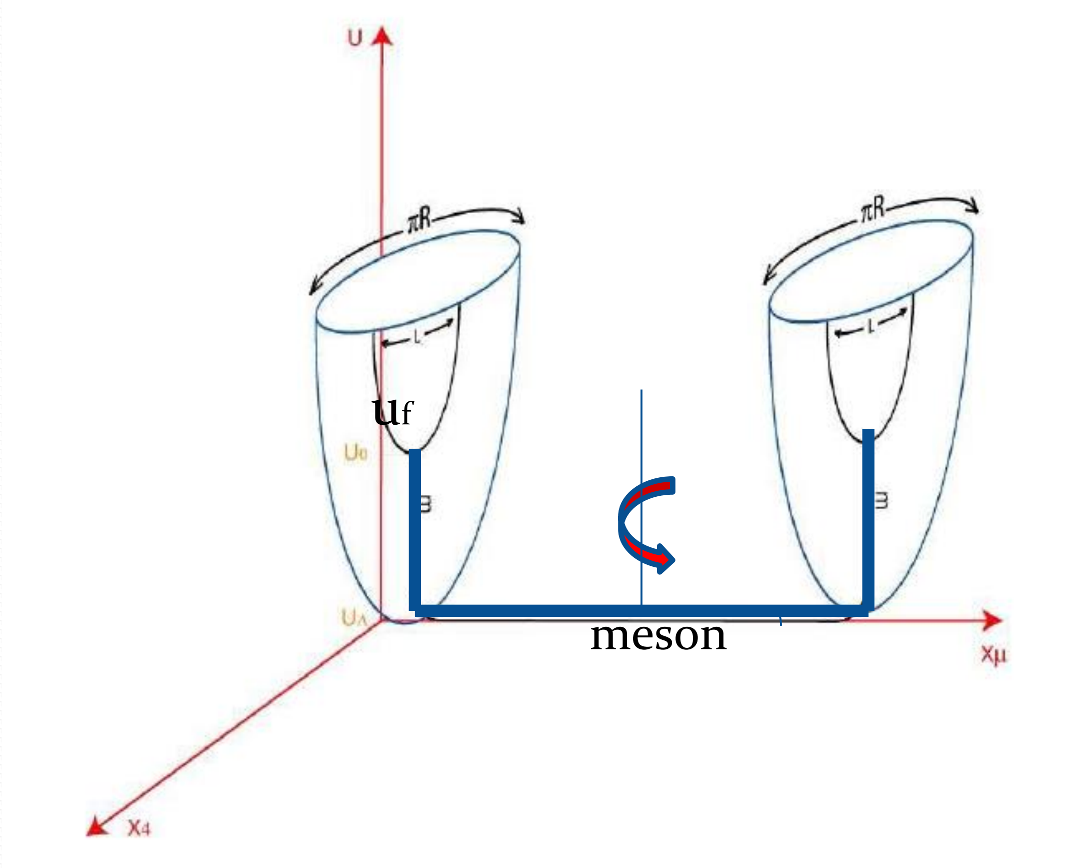
- The holographic meson with angular momentum is a rotating string connected to flavor branes



- The string is the classical solution of the **Nambu-Goto** action defined in a **confining holographic background**

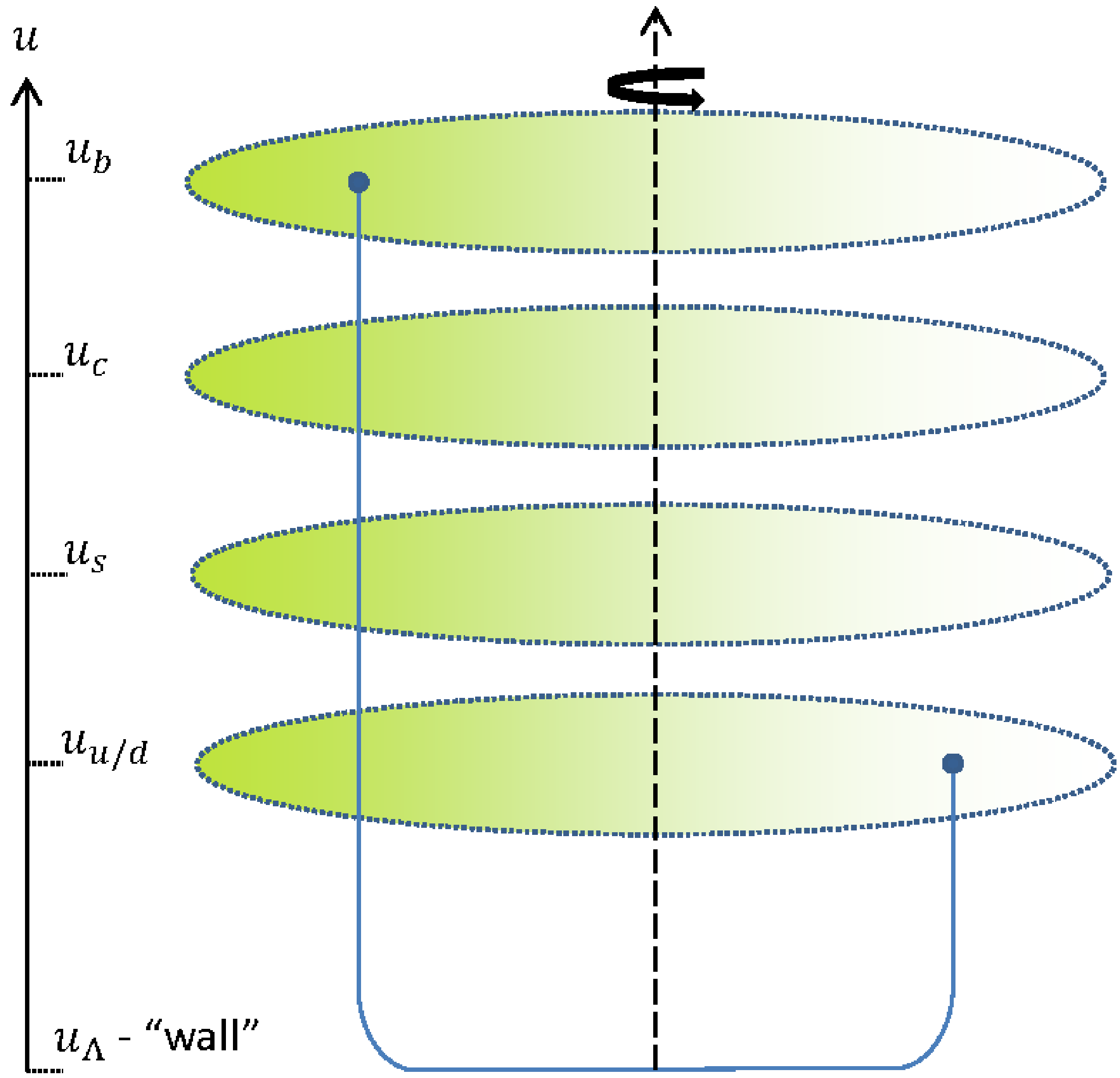
Stringy meson in holographic model

- In the generalized Sakai Sugimoto model the meson is a **rotating string** connecting the tips of



the flavor branes at two 3d space points

Example: The B meson



(2) Stringy Baryons

- How do we identify a **baryon** in holography ?

- Since a **quark** corresponds to an **end** of a **string**, the baryon has to be a structure with N_c **strings** connected to it.

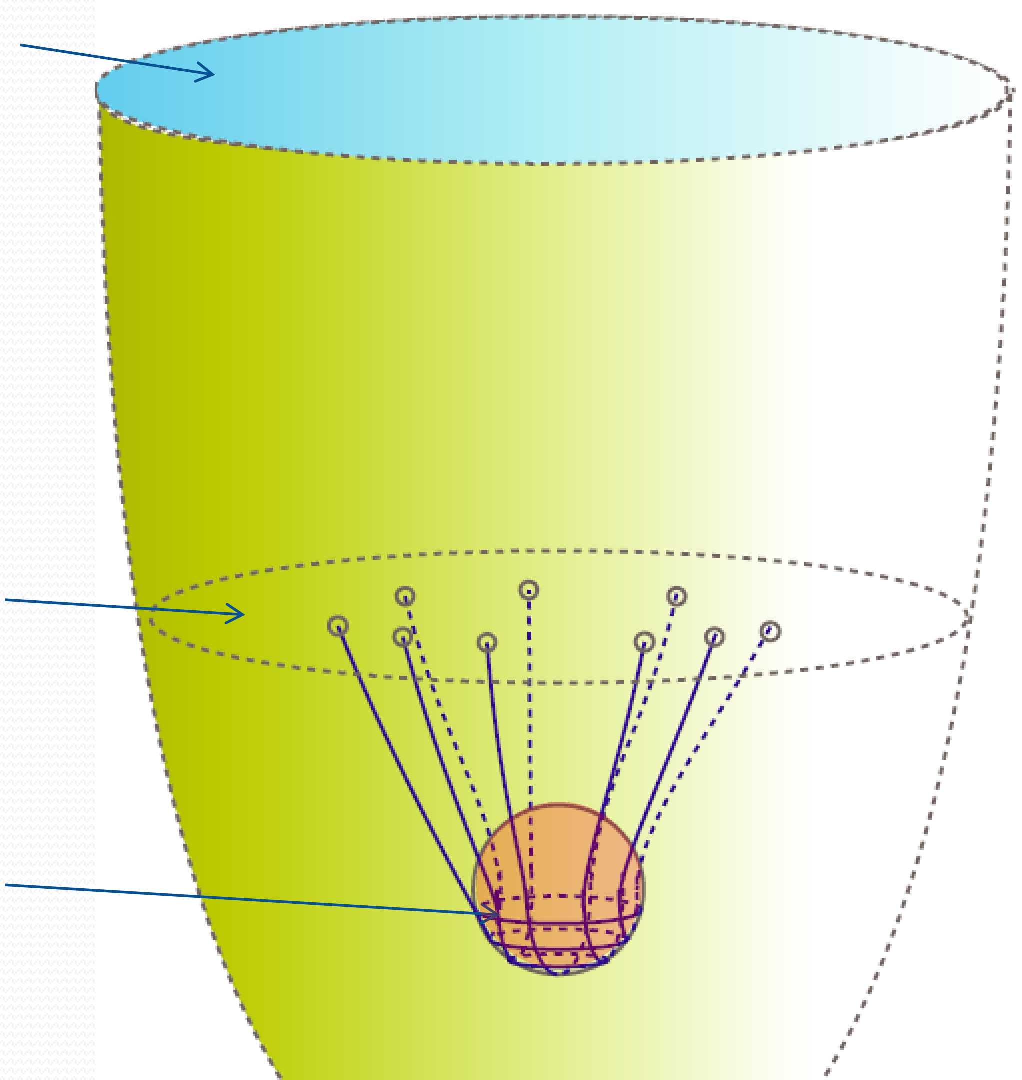
- The proposed **baryonic vertex** in holographic background is a **wrapped Dp brane over a p cycle**

Witten, Gross Ooguri

- Because of the **RR flux** in the background the wrapped brane has to be **connected to Nc strings**

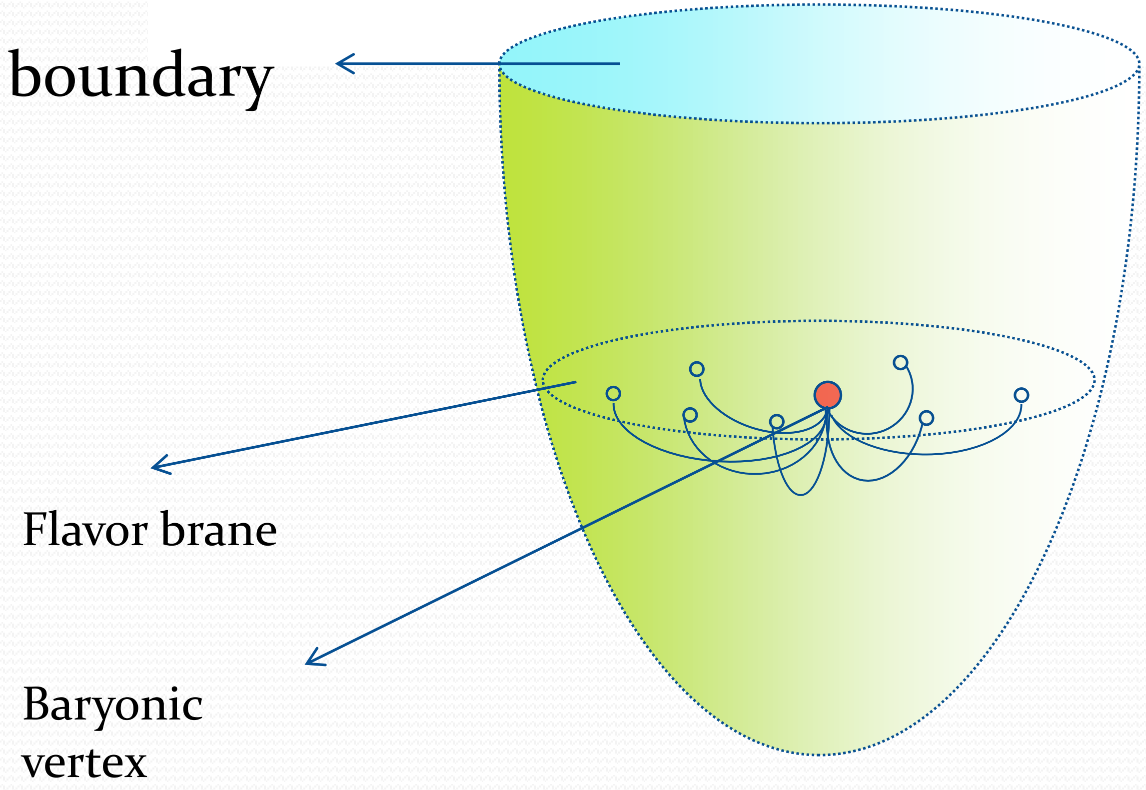
Dynamical baryon

- **Dynamical baryon** – Nc strings connecting the baryonic vertex and flavor branes.



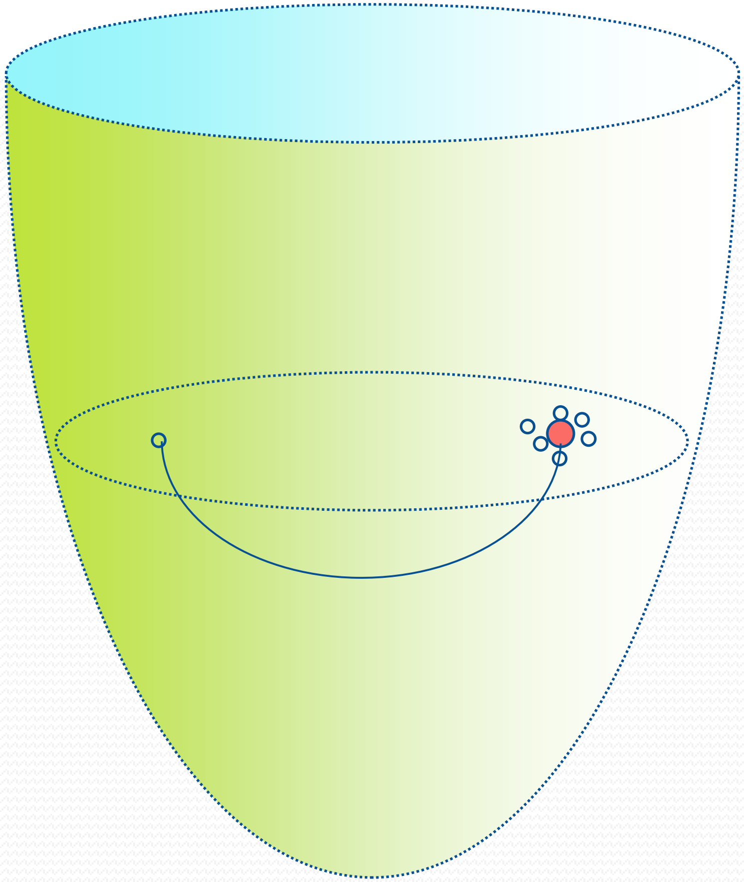
A possible baryon : Symmetric layout

- A priori there are many possible layouts, in particular the maximal **symmetric** one. The preferred one has the **lowest energy**.



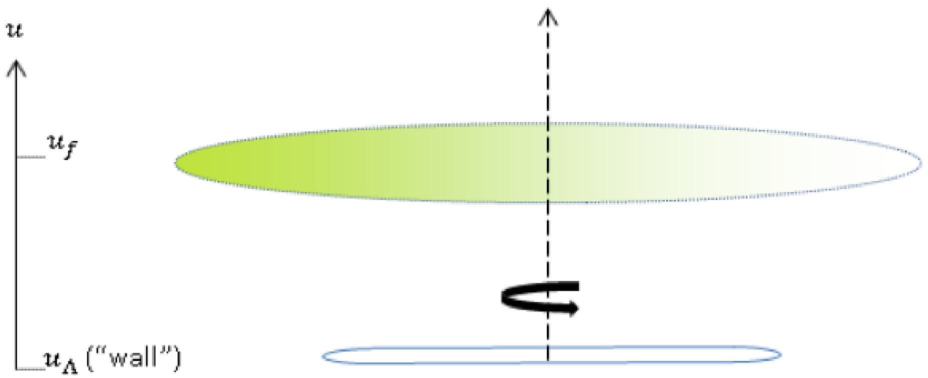
Asymmetric layout

- An **asymmetric** possible layout is that of one quark connected with a string to the baryonic vertex to which the rest of the Nc-1 quarks are attached.



(3) Glueballs as closed strings

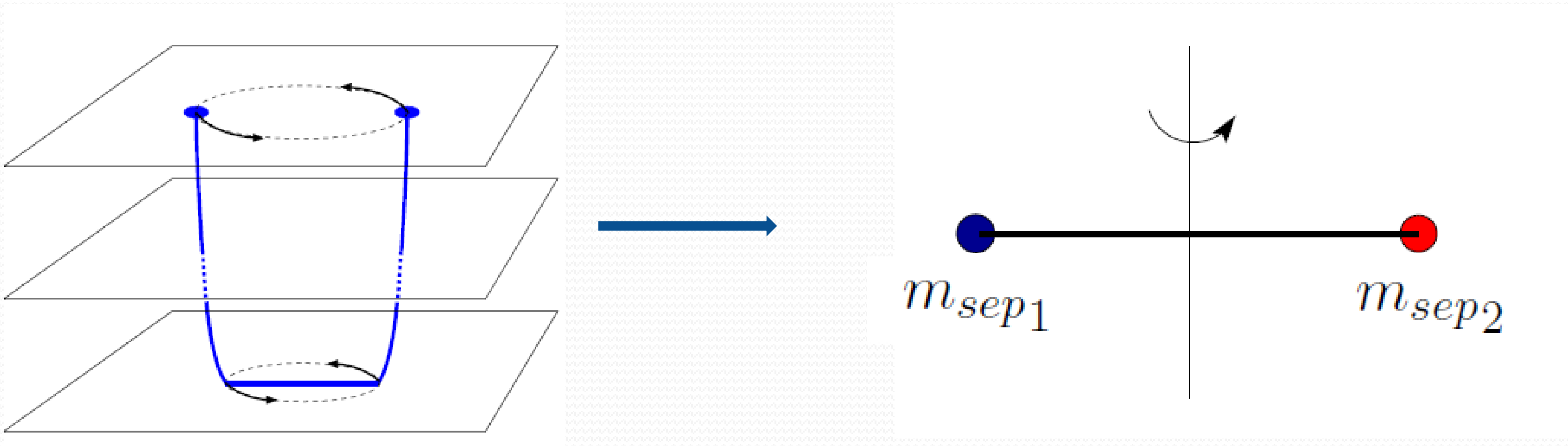
- Mesons are open strings connected to flavor branes.
- Baryons are Nc open strings connected to a baryonic vertex on one side and to a flavor brane on the other one.
- What are **glue balls**?
- Since they do not incorporate quarks it is natural to assume that they are **rotating closed strings**
- Angular momentum associates with rotation of **folded** closed strings



Step3- The Holographic Inspired stringy hadron (HISH) map

The HISH map of a stringy hadron

- The basic idea is to approximate the classical **holographic spinning** string by a string in **flat space time** with **massive endpoints.** The masses are m_{sep_1} and m_{sep_2}



The **EOMs** of the two systems are the **same** provided

String end-point mass

- We define the **string end-point quark mass**

$$m_{sep} = T\int_{u_0}^{u_f} g(u)du = T\int_{u_0}^{u_f}\sqrt{G_{00}G_{uu}}du$$

Namely the action of the vertical segments.

- The boundary equation of motion is

$$\frac{T_{eff}}{\gamma} = m_{sep}\gamma\omega^2 R_0$$

M.Kruczenski, L. Pando Zayas, D. Vaman J.S

- This simply means that the tension is balanced by the (relativistic) centrifugal force.

HMRT- HISH Modified classical Regge trajectory

- The classical solutions of a rotating string with massive endpoints modifies the original **Regge trajectories**
- The classical energy and angular momentum

$$E = \sum_{i=1,2} \left(\gamma_i m_i + T \ell_i \frac{\arcsin \beta_i}{\beta_i} \right)$$

$$J = \sum_{i=1,2} \left[\gamma_i m_i \beta_i \ell_i + \frac{1}{2} T \ell_i^2 \left(\arcsin \beta_i - \beta_i \sqrt{1 - \beta_i^2} \right) \right]$$

Small and large m_{sep} approximations

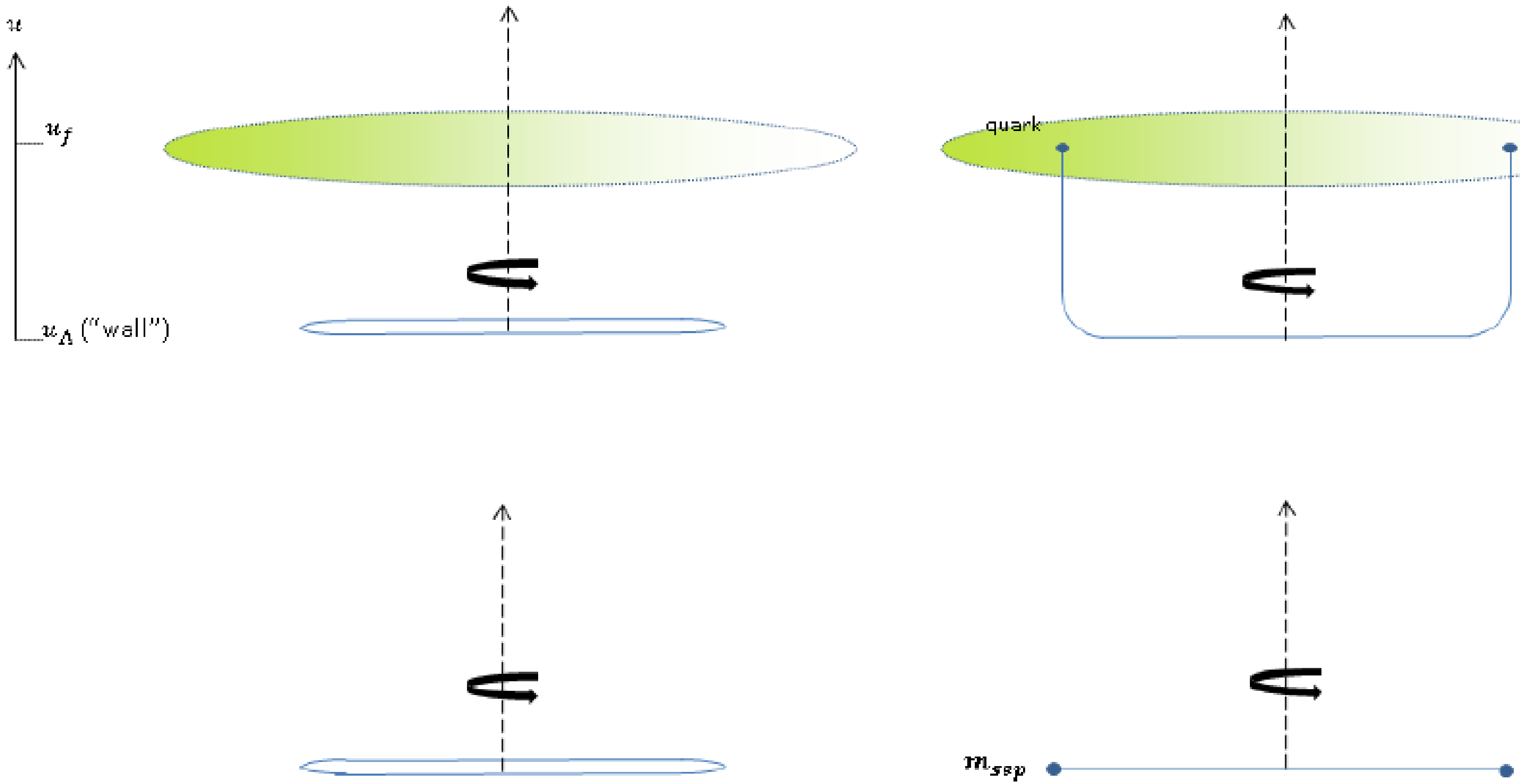
- For **small m_{sep}**, and $\beta_i \to 1$ the modified Regge trajectory is

$$J = \alpha' E^2 \times \left(\left(1 - \sum_{i=1}^{2} \left(\frac{4\sqrt{\pi}}{3} \left(\frac{m_i}{E} \right)^{3/2} + \frac{2\sqrt{\pi^3}}{10\sqrt{2}} \left(\frac{m_i}{E} \right)^{5/2} + \cdots \right) \right) \right)$$

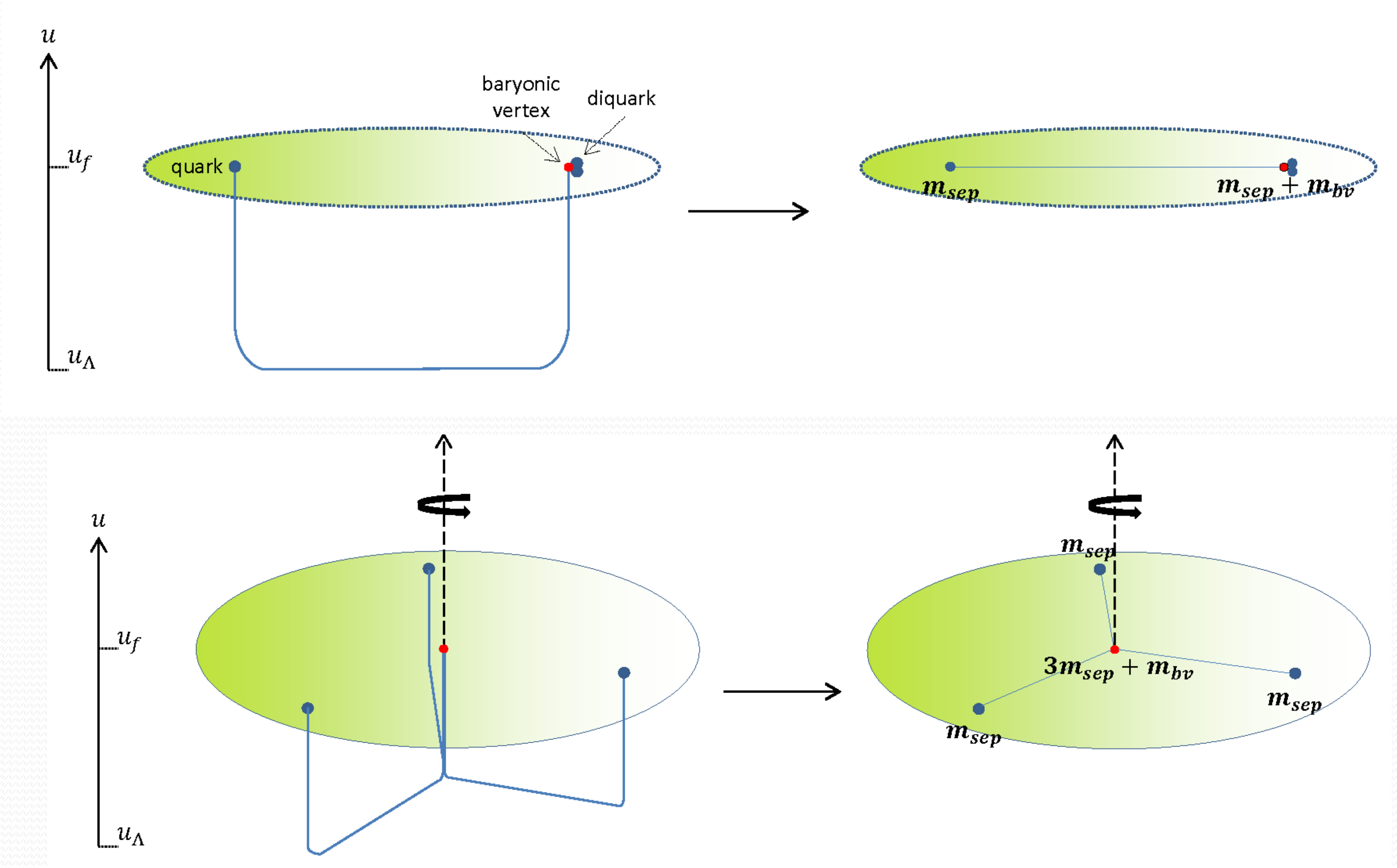
- For **large m_{sep}** and $\beta_i \to 0$,

$$J = \frac{4\pi}{3\sqrt{3}} \alpha' \sqrt{\frac{m_1 m_2}{m1 + m_2}} (E - m_1 - m_2)^{3/2} + \frac{7\sqrt{2}\pi}{27\sqrt{3}} \alpha' \frac{m_1^2 - m_1 m_2 + m_2^2}{m_1 m_2 \sqrt{(m_1 + m_2)^3}} (E - m_1 - m_2)^{5/2} + \cdots$$

(i) The Hish map of holographic mesons

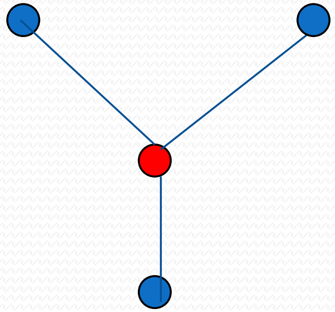


(ii) The HISH map of holographic Baryons

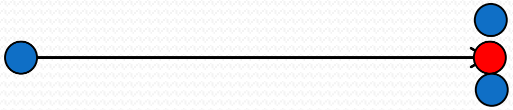


From large Nc to three colors

- Naturally the analog at Nc=3 of the **symmetric** configuration with a central baryonic vertex is the old **Y shape baryon**



- The analog of the **asymmetric** setup with one quark on one end and Nc-1 on the other is a **straight string** with **quark** and a **di-quark** on its ends.

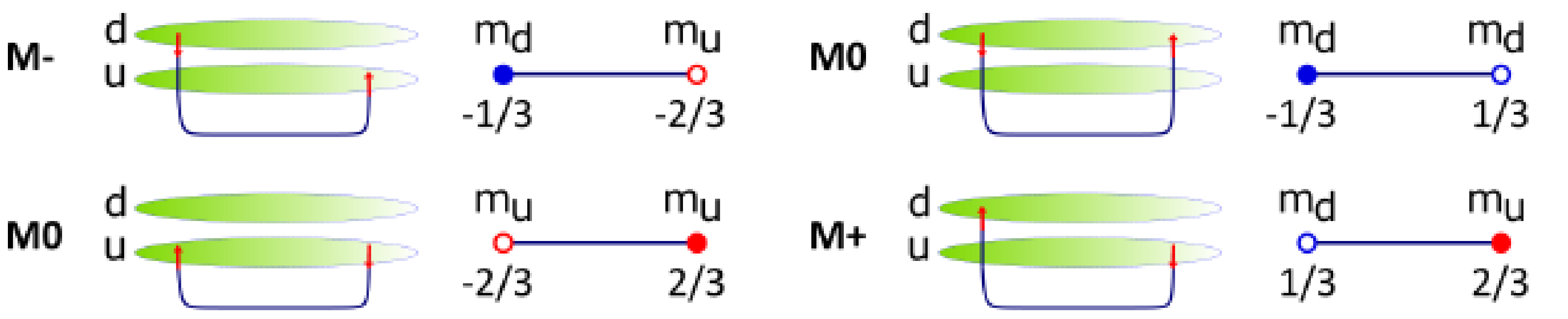


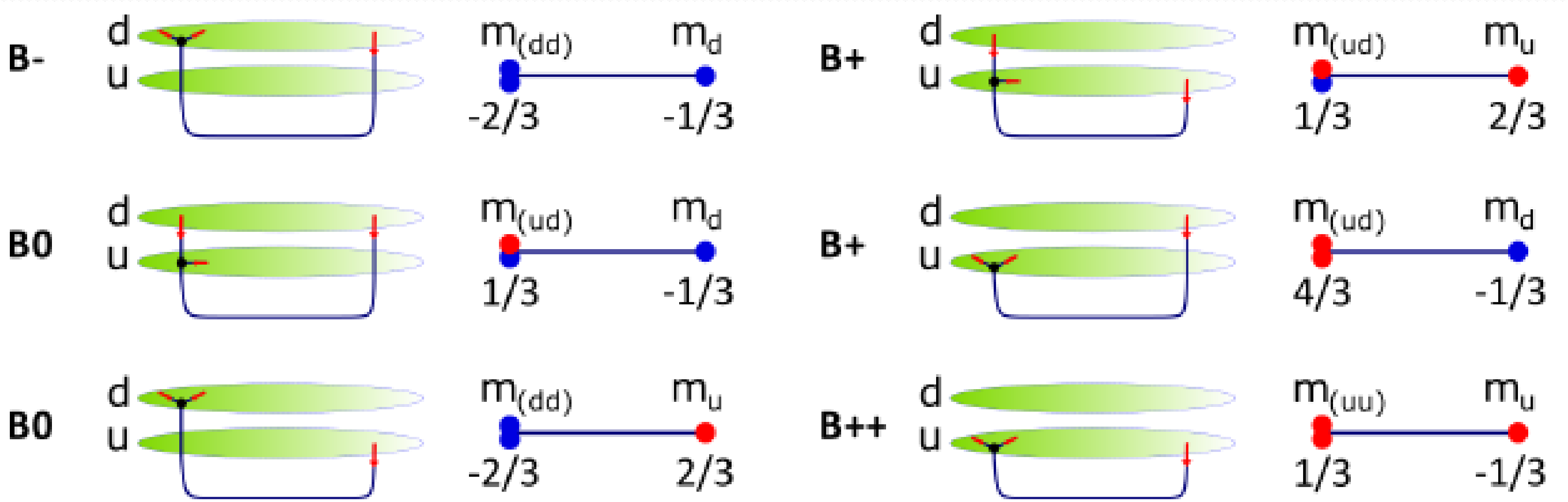
Stability of an excited baryon

- Sharov and 't Hooft showed that the classical **Y shape** three string configuration is **unstable**. An arm that is slightly shortened will eventually shrink to zero size.
- We also examined Y shape strings with massive endpoints and with a baryonic vertex in the middle.
- The analysis included numerical simulations of the motions of mesons and Y shape baryons under the influence of symmetric and asymmetric disturbance.
- We indeed detected the **instability**
- We also performed a perturbative analysis where the instability does not show up.

Charged stringy hadrons in holography and HISH

- Light stringy **mesons** and **baryons** in holography and HISH





Step4,5 - Quantization of a string with massive and charged endpoints.

Renormalization and computation of the intercept

On the quantization of the HISH

- The passage from the **classical to quantum** bosonic rotating string with **no massive endpoints** in D=26

$$J = \alpha' M^2 \qquad \rightarrow \qquad J = \alpha' M^2 + a$$

- For the excited states with excitation number n

$$n + J = \alpha' M^2 + a$$

- **a** the intercept is given by the **Casimir energy**

$$E_{Casimir} \equiv \frac{1}{2}\sum_{n=1}^{n=\infty} w_n = \frac{\pi(D-2)}{2L}\sum_{n=1}^{n=\infty} n = -\frac{(D-2)}{24}\frac{\pi}{L} = a\frac{\pi}{L}$$

Quantum modified Regge trajectory

- With massive endpoints the **intercept** is modified

$$a \equiv -\frac{D-2}{2\pi}\sum_{n=1}^{\infty}\omega_n$$

- The **eigenfrequencies** with **massive** endpoints

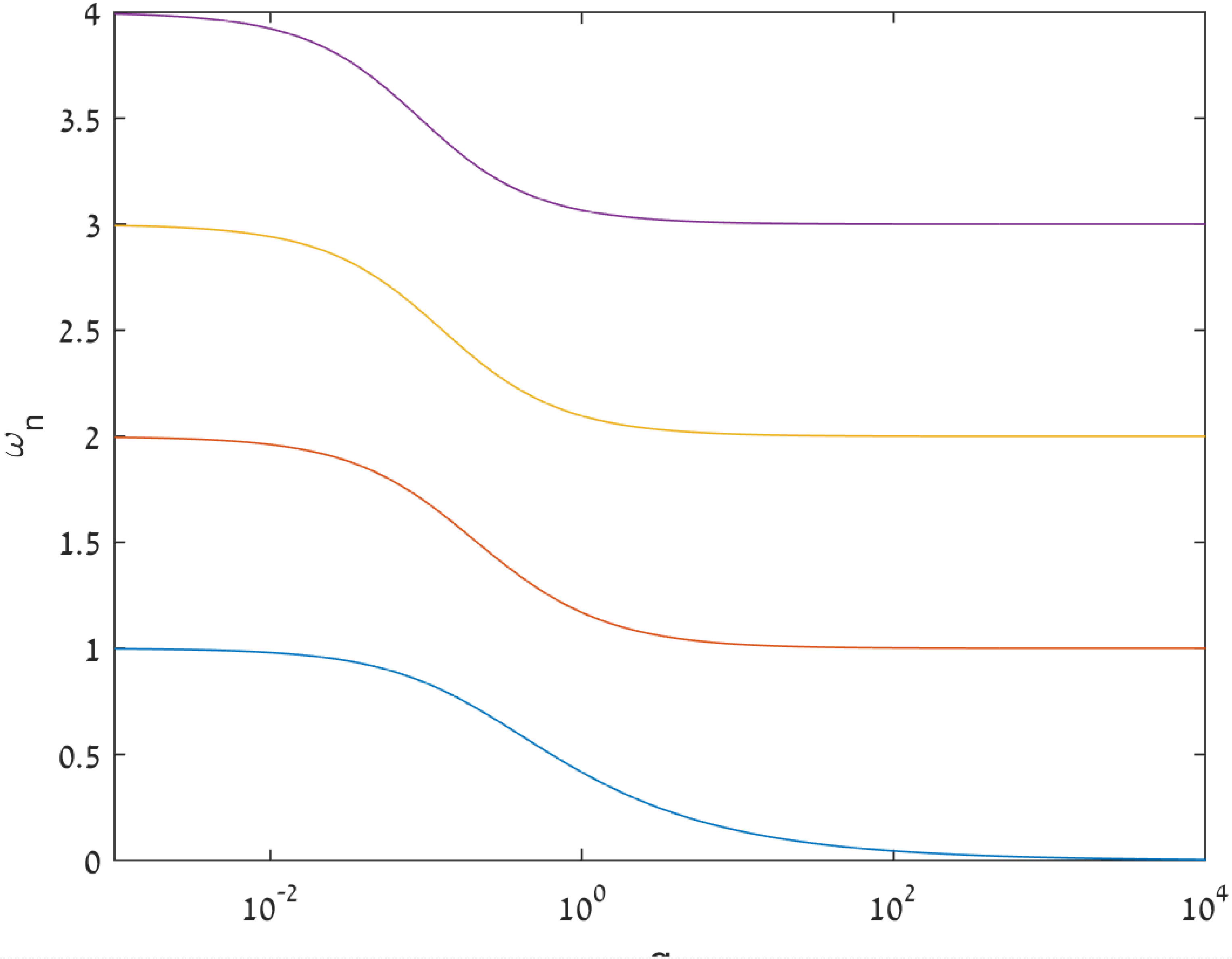
$$\tan(\omega_n) = \frac{2q\omega_n}{q^2\omega_n^2 - 1}$$

q=m/TL

- The modified intercept changes the trajectory

$$\delta J - \frac{L}{2}\delta E = a$$

The Eigenfrequencies as a function of TL/m γ



The renormalization of the sum of the eigenfrequencies

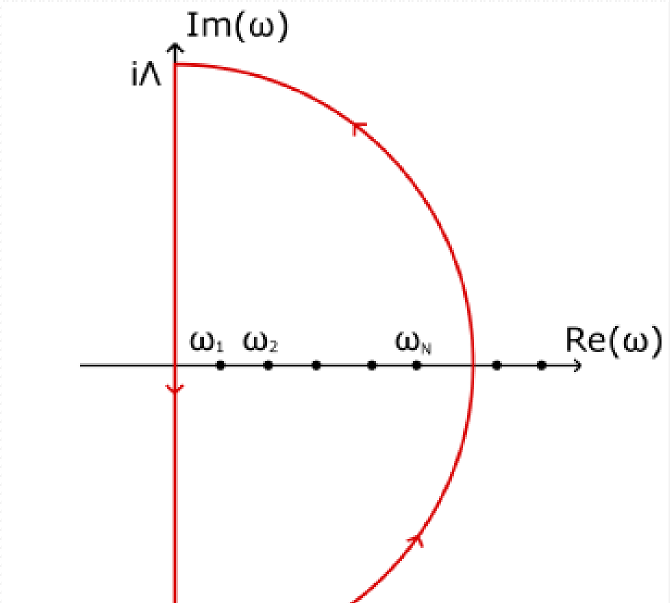
- The **zeta function** cannot be used for the massive case. We convert the infinite sum into a **contour integral** using Cauchy integral formula

$$\frac{1}{2\pi i}\oint dz z\frac{d}{dz}\log f(z) = \frac{1}{2\pi i}\oint dz z\frac{f'(z)}{f(z)} = \sum_j n_j z_j - \sum_k \tilde{n}_k \tilde{z}_k$$

- We will use a function $f(\omega)$ with only simple zeros at $\omega = \omega_n$ which are on the positive real axis

Lambiase

Nesterenko



The renormalization of the sum of the eigenfrequencies

- The sum of the eigen-frequencies is the Casimir energy

$$E_C = \frac{1}{2}\sum_{n=1}^{\infty}\omega_n = -\frac{2\beta a}{L}$$

- The semi-circle regularizes the Casimir energy

$$E_C^{(reg)} = \frac{1}{2}\sum_{n=1}^{N(\Lambda)}\omega_n$$

- We renormalize the result in the same way that we do for the **Casimir effect**. We subtract from the force of a string of length L the one of an infinite string

$$E_C^{(ren)} = \lim_{\Lambda\to\infty}\left(E_C^{(reg)}(m,T,L) - E_C^{(reg)}(m,T,L\to\infty)\right)$$

The renormalization of the sum for the massless case

- For the ordinary string with no endpoint particles

$$f(\omega) = \sin(\pi\omega\ell) = 0$$

- Usually we use the zeta function renormalization

$$\sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12}$$

- Using the contour integral method

$$E_C(m=0) = \frac{1}{2}\sum_{n=1}^{\infty}\omega_n = \frac{1}{4\pi i}\oint \omega\frac{f'(\omega)}{f(\omega)}d\omega = \frac{1}{4i}\oint \omega\ell\cot(\pi\omega\ell)d\omega$$

$$\frac{1}{4i}\oint \omega\ell\cot(\pi\omega\ell)d\omega = -\frac{1}{4}\int_{-\Lambda}^{\Lambda} y\ell\coth(\pi y\ell)dy + \frac{1}{4}\Lambda^2\ell\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2i\theta}\cot(\pi\Lambda\ell e^{i\theta})d\theta$$

The intercept for the massless case

- The regularized energy reads

$$E_C^{(reg)} = \left(-\frac{\Lambda^2\ell}{4} - \frac{1}{24\ell}\right) + \frac{1}{2}\Lambda^2\ell = \frac{\Lambda^2\ell}{4} - \frac{1}{24\ell} = \frac{\Lambda^2 L}{8} - \frac{1}{12L}$$

- The corresponding force

$$F_C^{(reg)} = -\frac{d}{dL}E_C^{(reg)} = -\frac{\Lambda^2}{8} + \frac{1}{12L^2}$$

- The renormalized force

$$F_C^{(ren)} = \lim_{\Lambda\to\infty}\left(F_C^{(reg)}(L) - F_C^{(reg)}(L\to\infty)\right) = \frac{1}{12L^2}$$

- The renormalized energy

$$E_C^{(ren)} = -\frac{1}{12L} \qquad \Rightarrow \qquad a = \frac{1}{24}$$

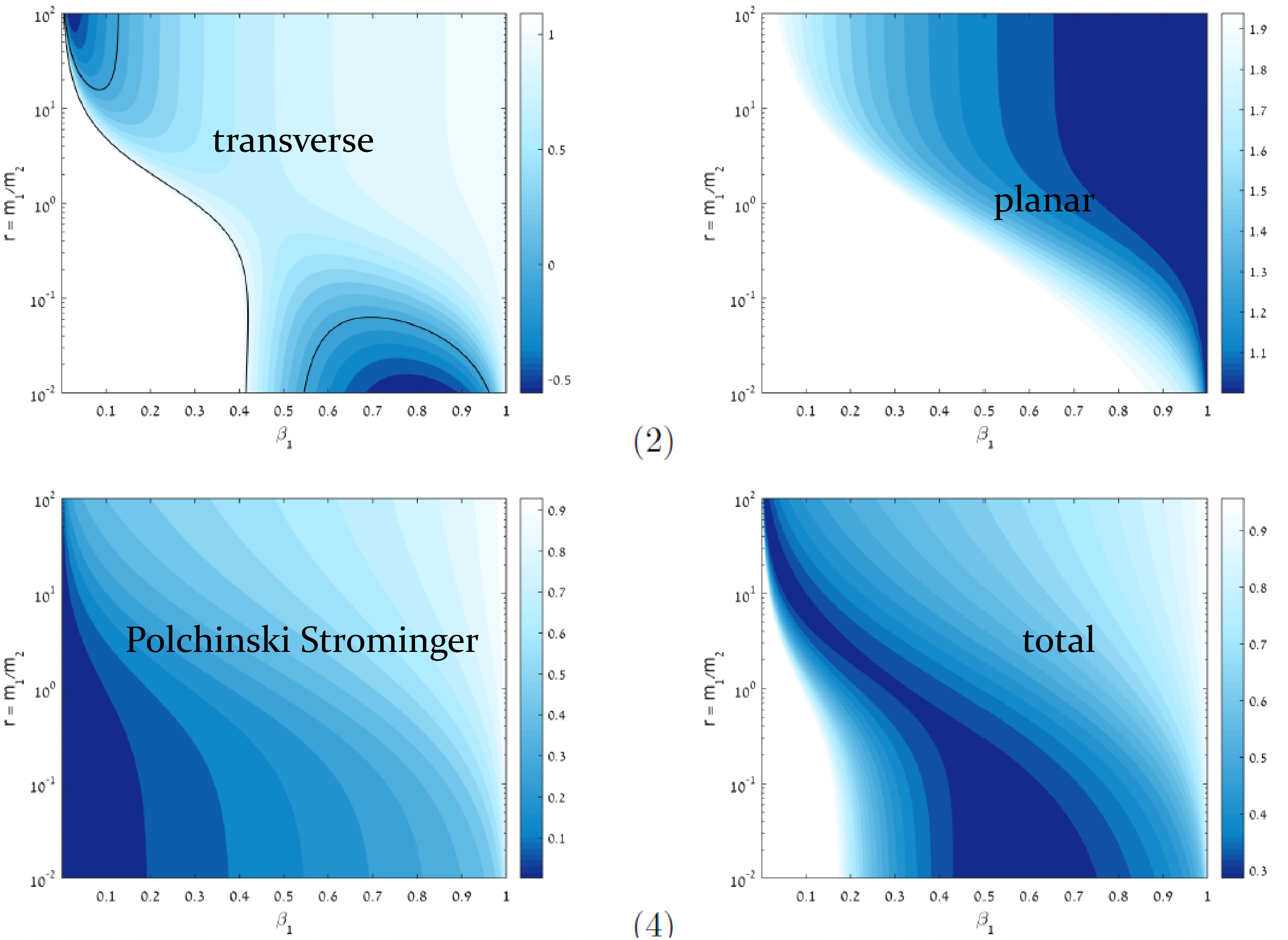
The quantum Regge trajecory

- We quantized rotating strings with small masses on their endpoints.
- It is highly non-trivial since the string is now an **interacting** one
- We convert the sum to a **contour integral** and **subtract** the **Casimir Force of a string of length L** from an **infinitely long string** instead of the zeta function renormalization
- The final result

$$a = (D-3)a_t + a_p + a_{PS}$$

$$\approx 1 - \frac{26-D}{12\pi}(\frac{2m}{TL})^{1/2} + \frac{199-14D}{240\pi}(\frac{2m}{TL})^{3/2}$$

The intercept of a string with massive endpoints



Step6- Fitting and extracting the physical parameters

Fitting and extracting the physical parameters

- The fit results for several trajectories **simultaneously**. The (J, M^2) trajectories of ρ, ω, K^*, ϕ D, and Ψ mesons
- We take the **string endpoint masses** in MeV

$$m_{u/d} = 60, m_s = 220, m_c = 1500$$

- The best fits of the **slope** and **intercept**

$$\alpha' = 0.899$$

$$a_\rho = 0.51, a_\omega = 0.52, a_{K^*} = 0.49$$

$$a_\phi = 0.44, a_D = 0.80, a_\Psi = 0.94$$

a-s

	u/d	s	c	b
u/d	-0.46	-0.29	-0.38	-0.65
s	-0.29	-0.14	-0.10	-0.08
c	-0.38	-0.10	-0.08	-
b	-0.65	-0.40	-	-0.27

	u/d	s	c	b
u/d	-0.26	-0.01	-0.25	-0.54
s	-0.01	-	-0.04	-0.31
c	-0.25	-0.04	0.00	-
b	-0.54	-0.31	-	0.00

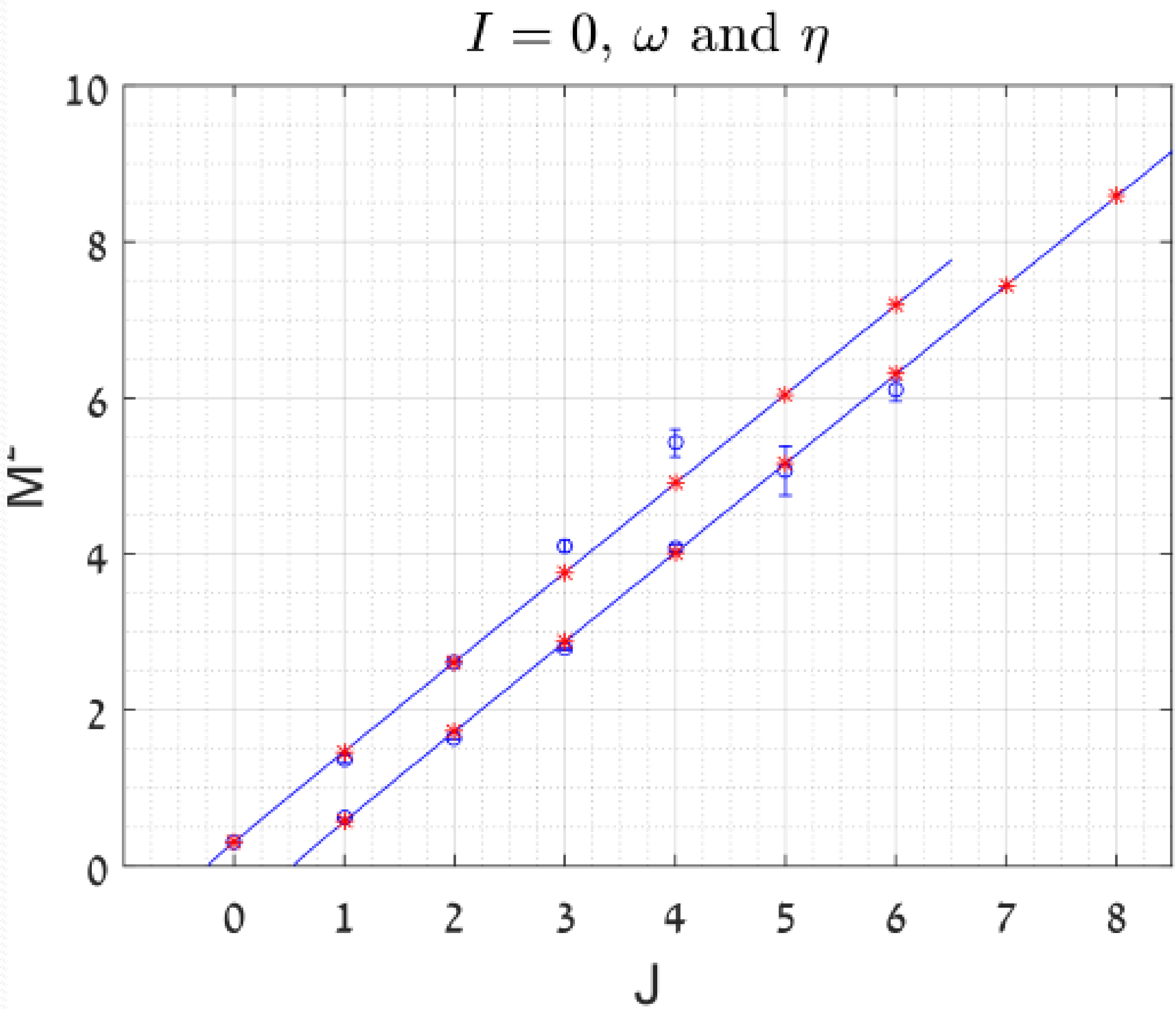
The negative intercept assumption

- In nature the intercept associated with all the hadrons whether mesons or baryons is **negative** when it is defined in relation to the **orbital** and not the total angular momentum.

$$a\text{-}s<0$$

- For instance ρ has a 0.5 and S=1 s so for L=J-S we get a= -0.5
- The **negative intercept** means a **repulsive** Casimir force that acts on the massive endpoints and balances the tension. This **prevents tachyonic states**
- To account for it we study strings with different masses, electric charges and spins at their ends.
- We can get negative a-s intercept but not yet in a fully satisfactory manner
- At present the intercepts are read from the data

Fitted trajectories of mesons



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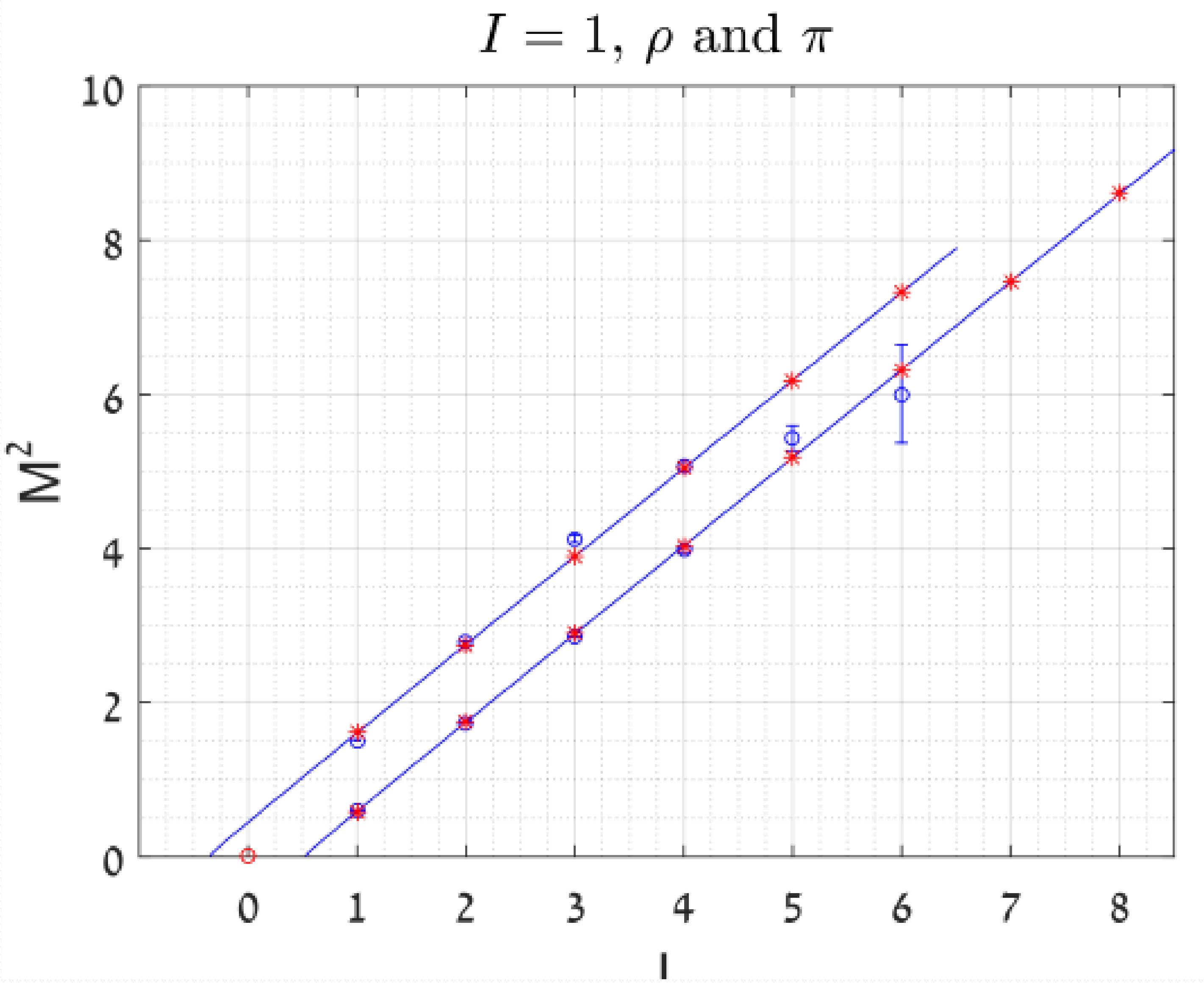

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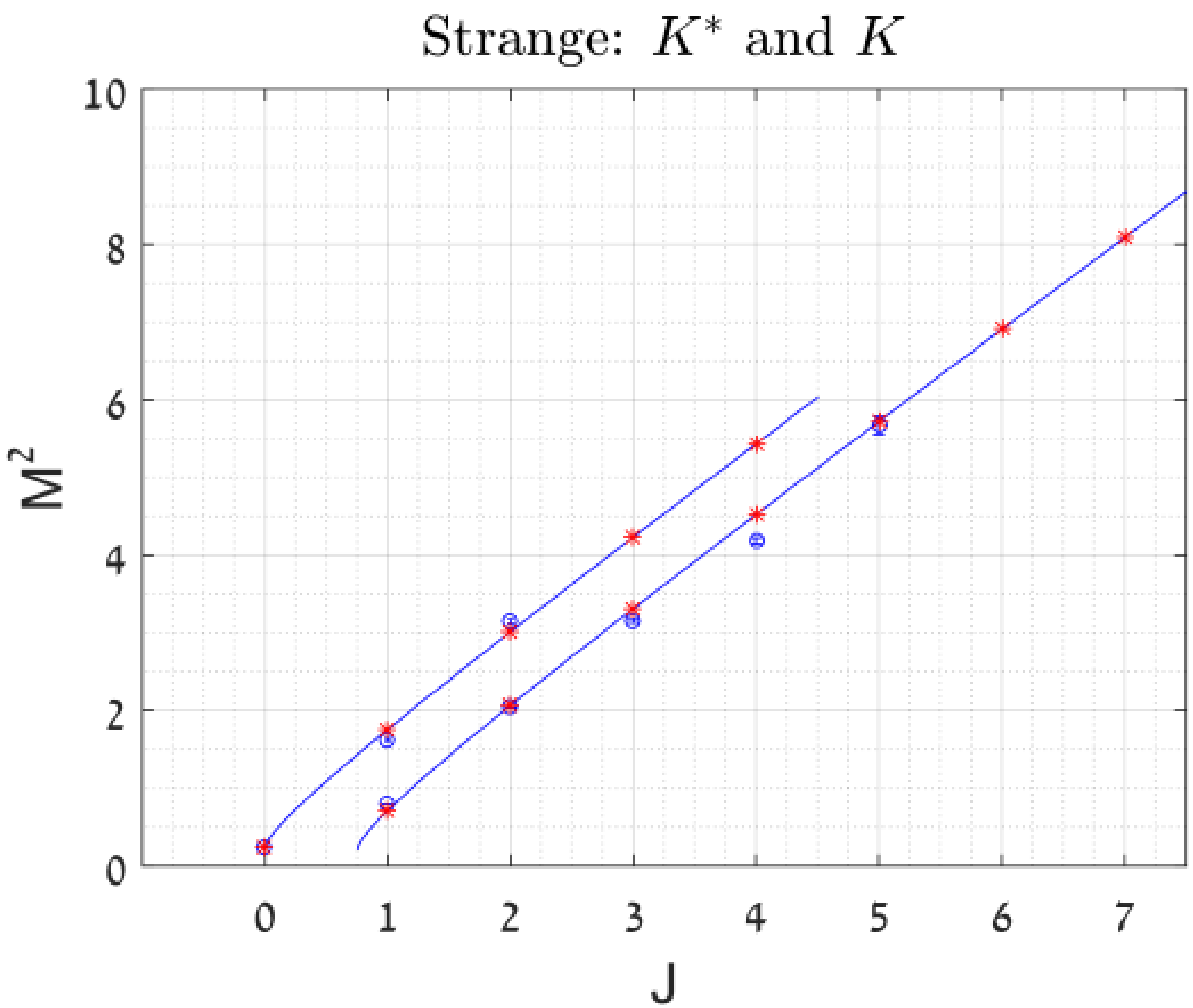

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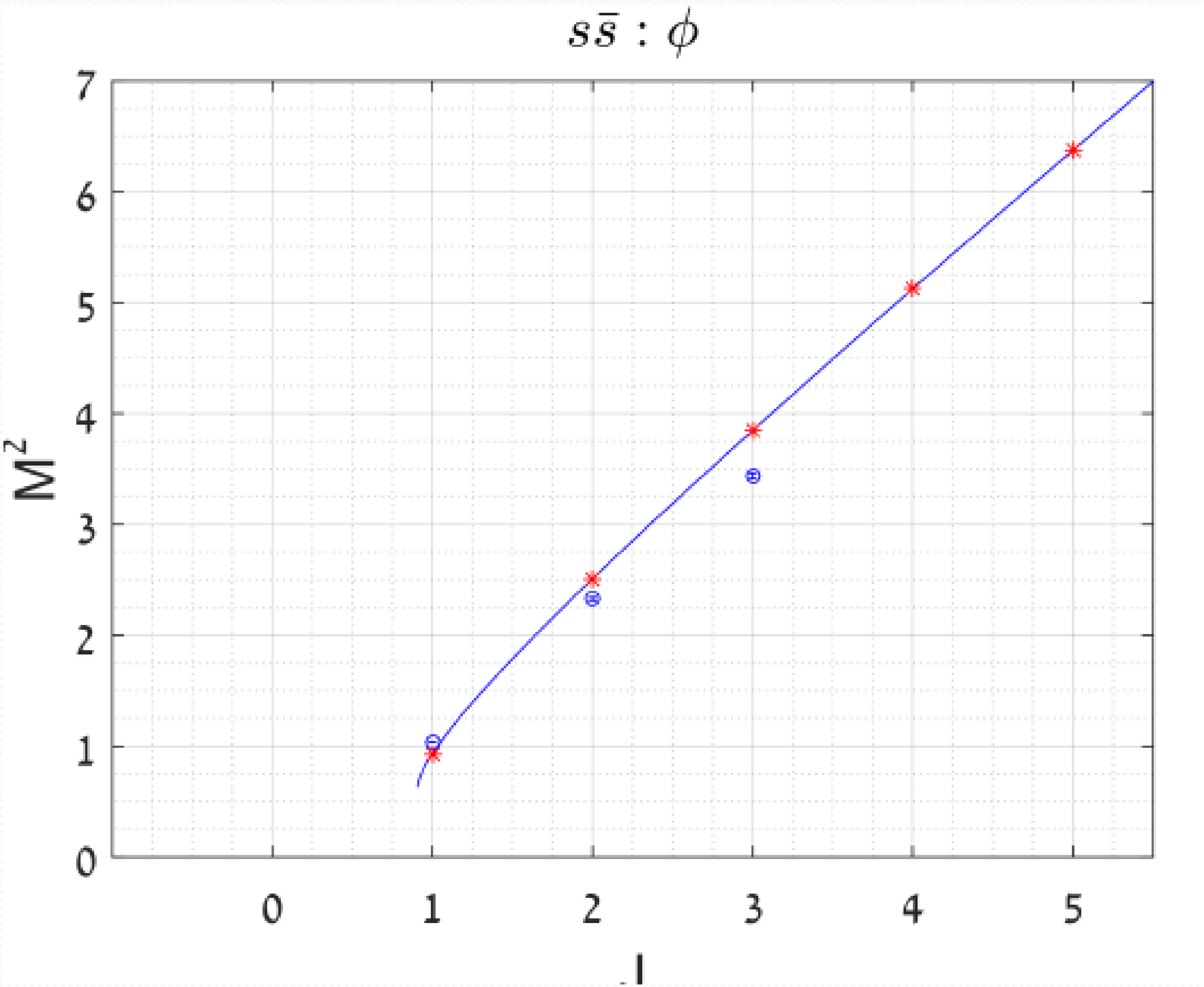

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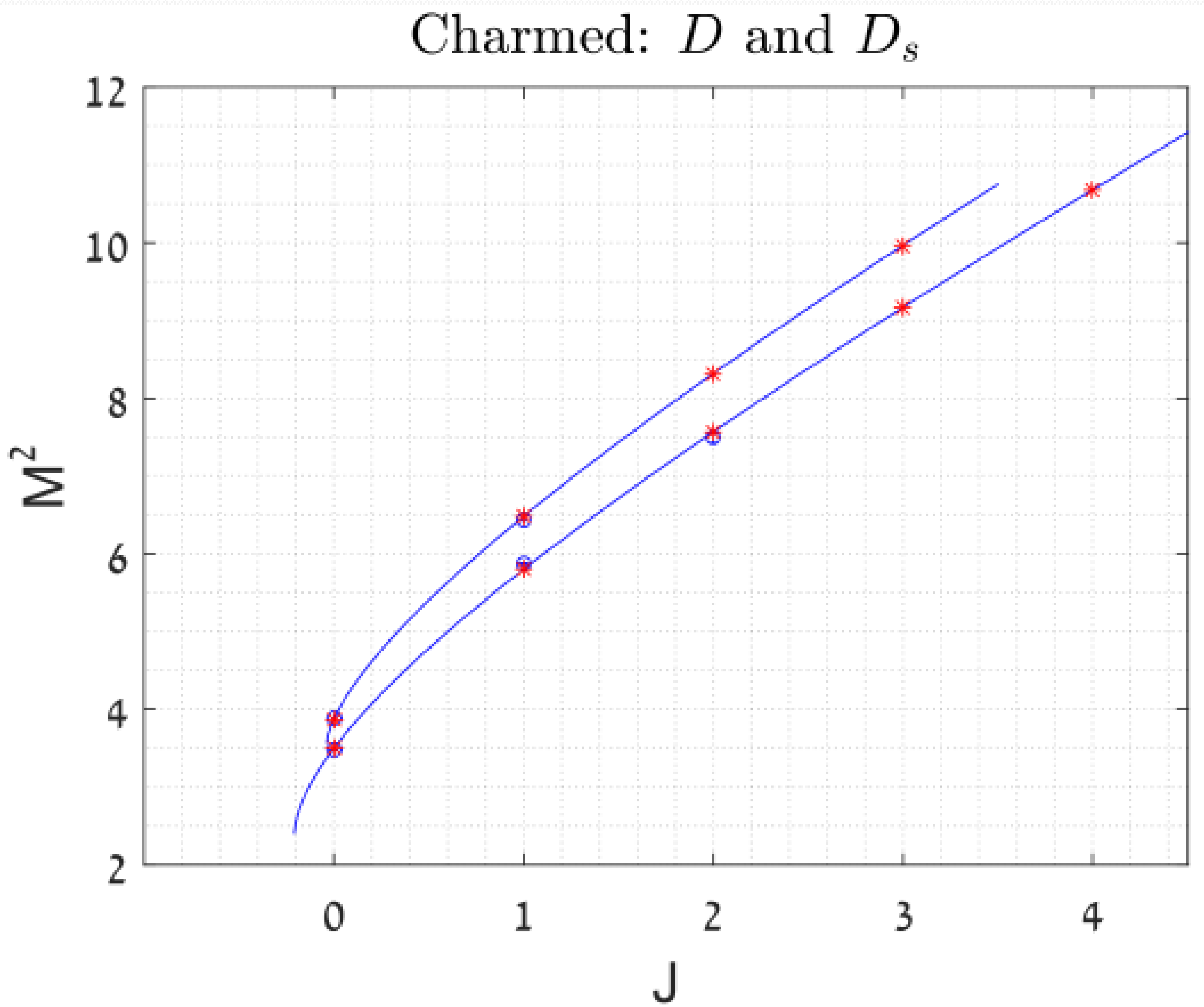

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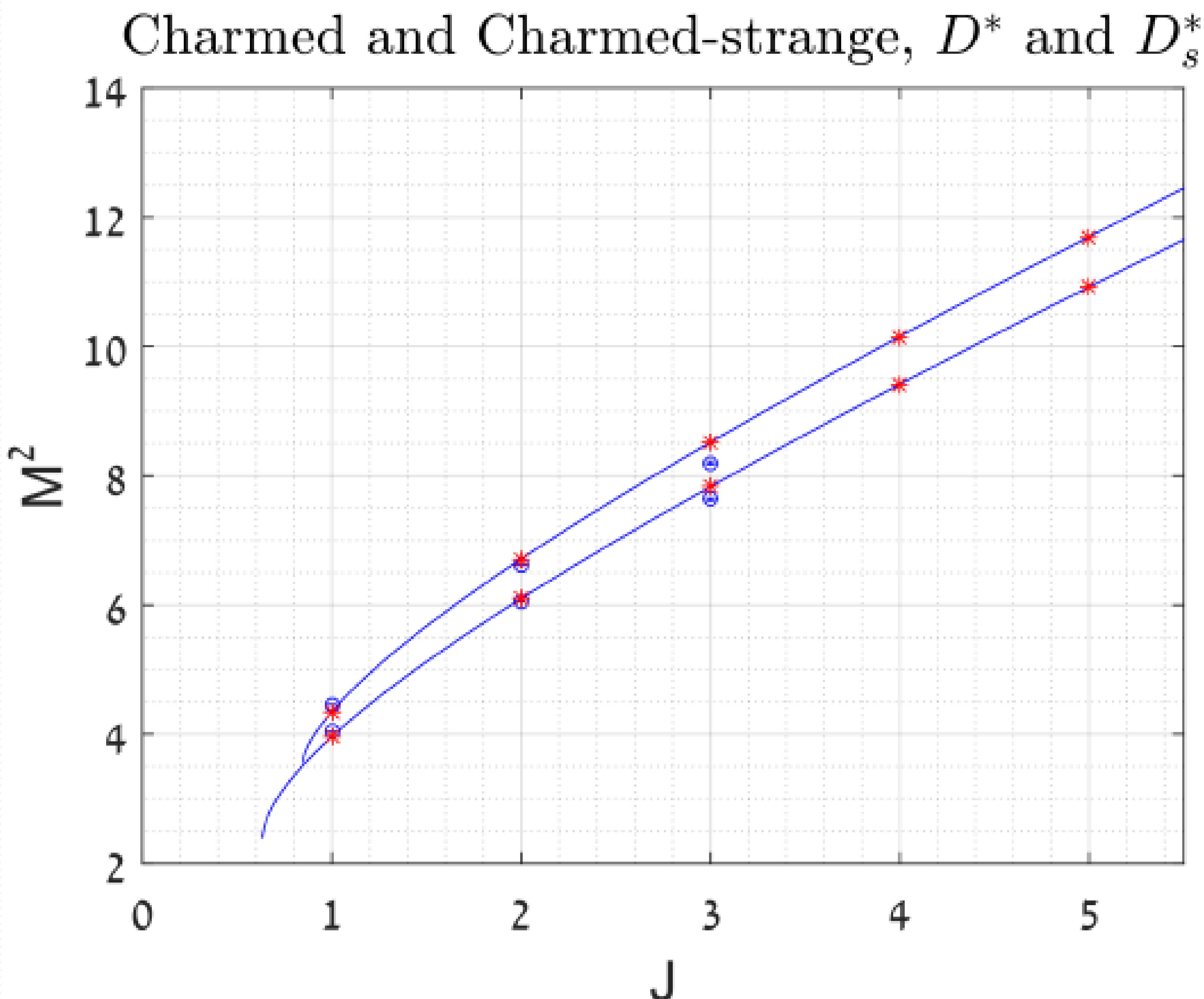

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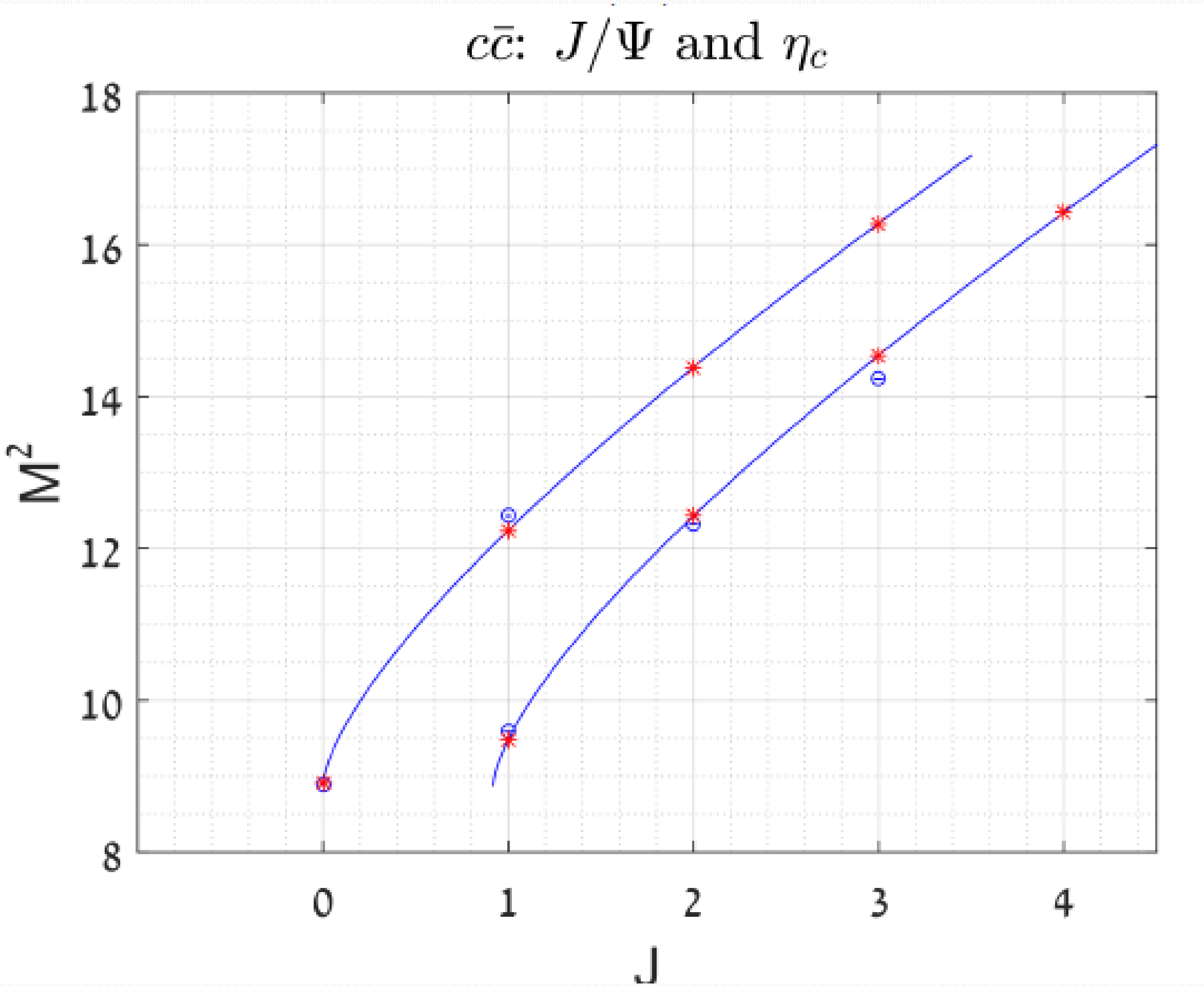

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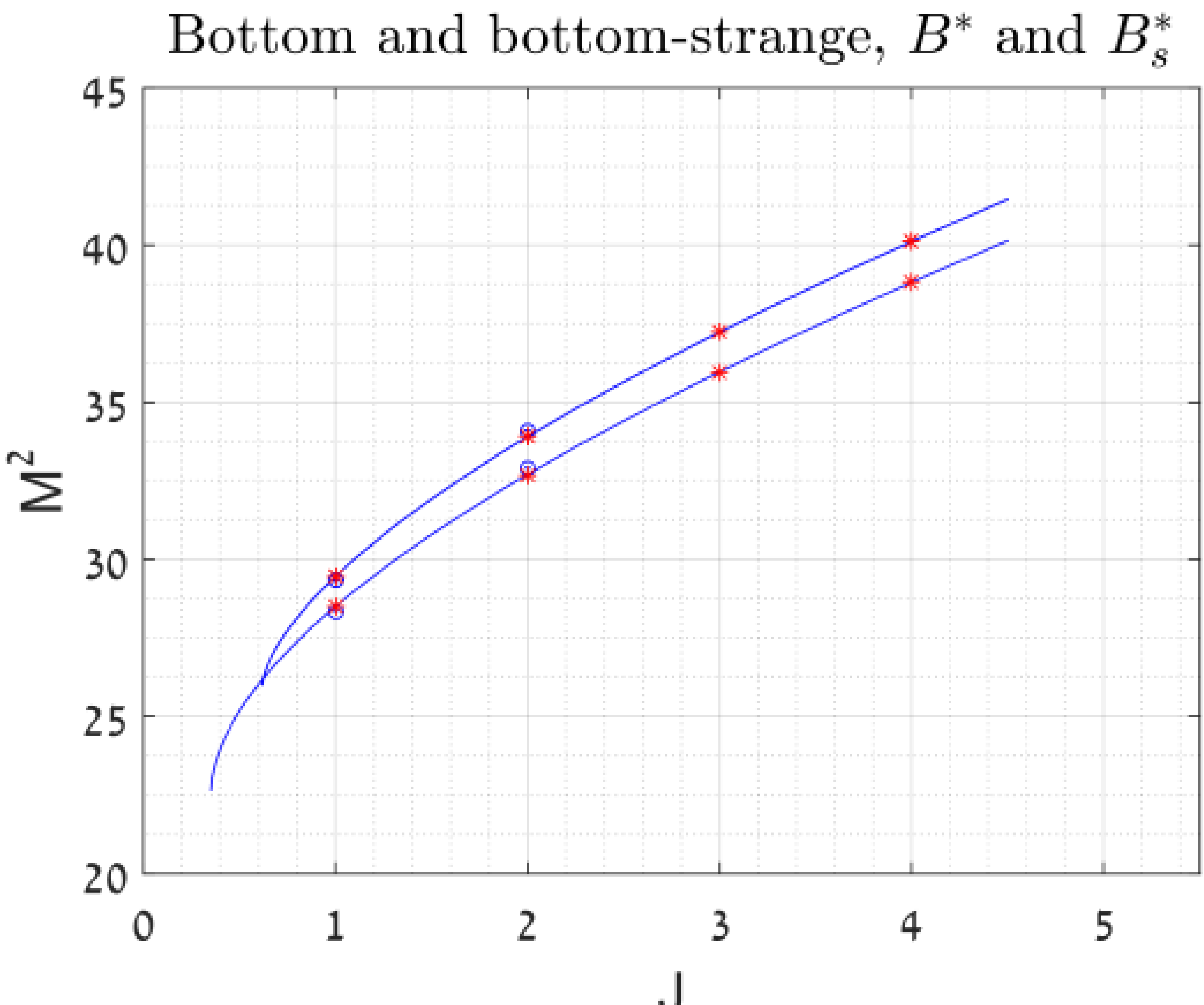
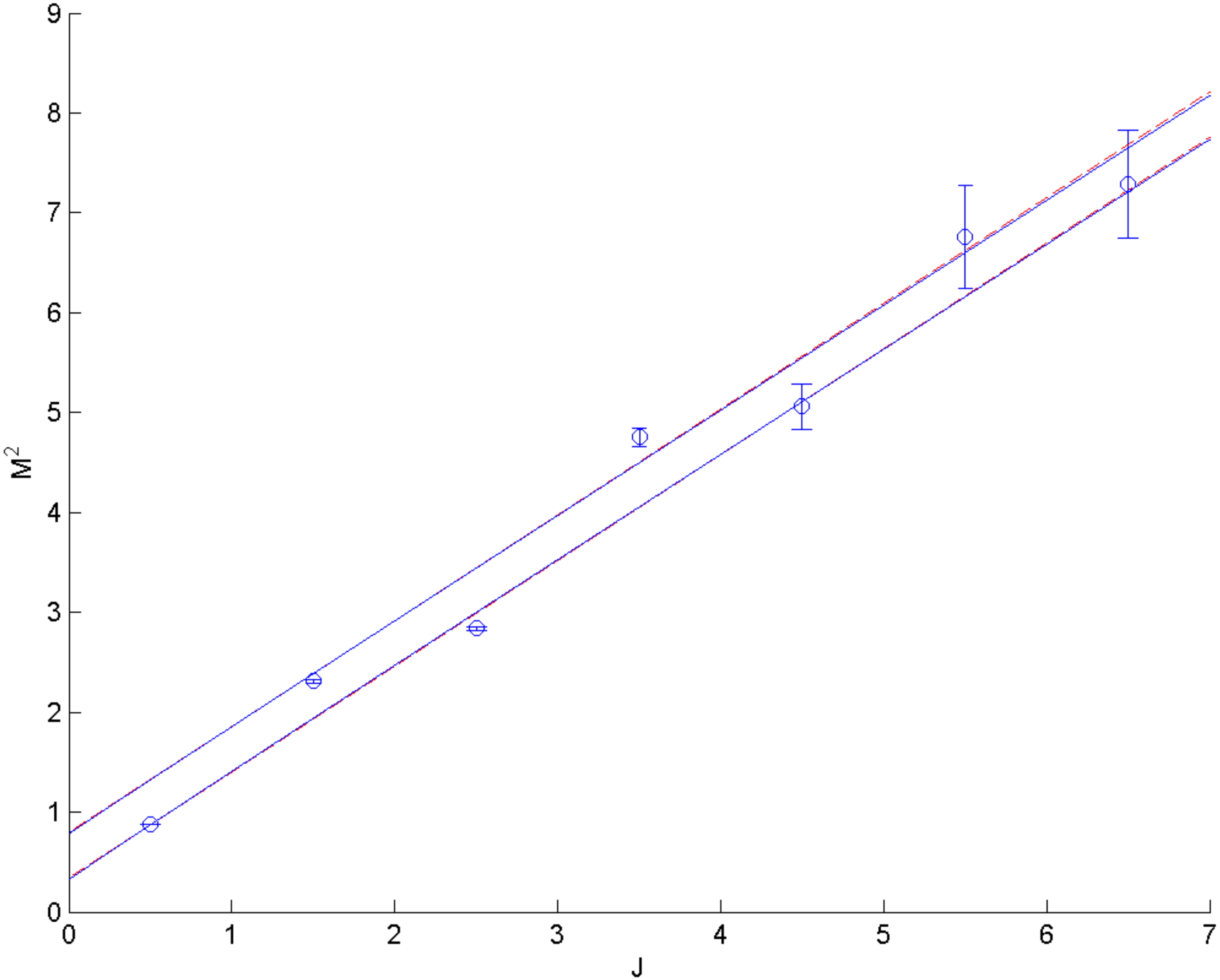
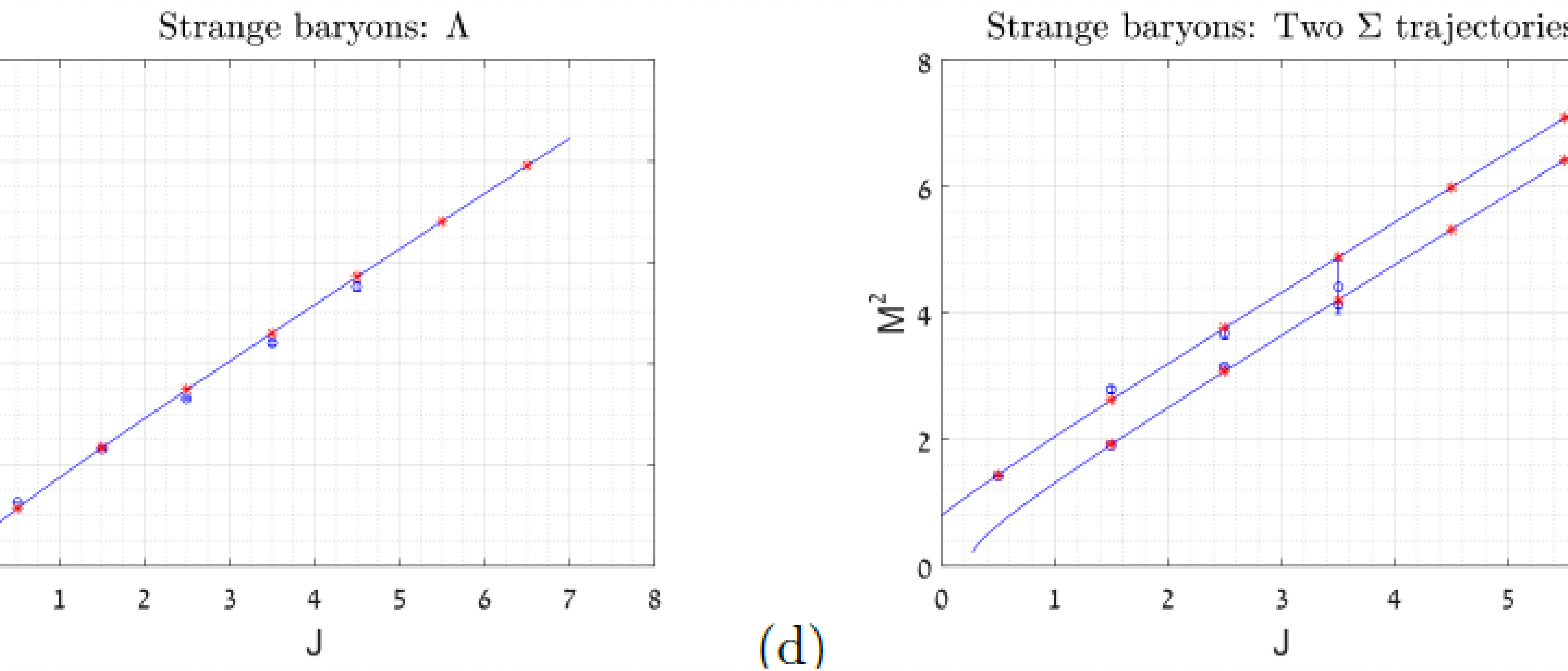


The spectra fits of Nucleons

- Trajectories for even and odd J **nucleons**

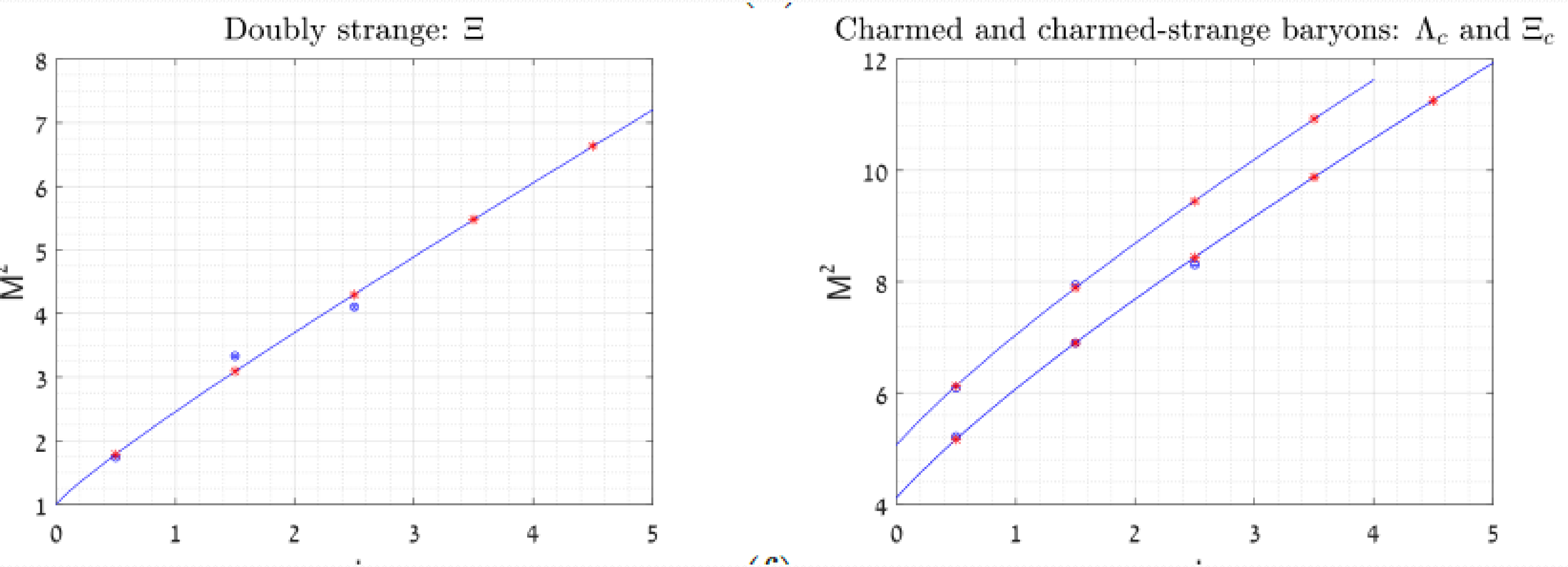


Trajectories of Λ and Σ

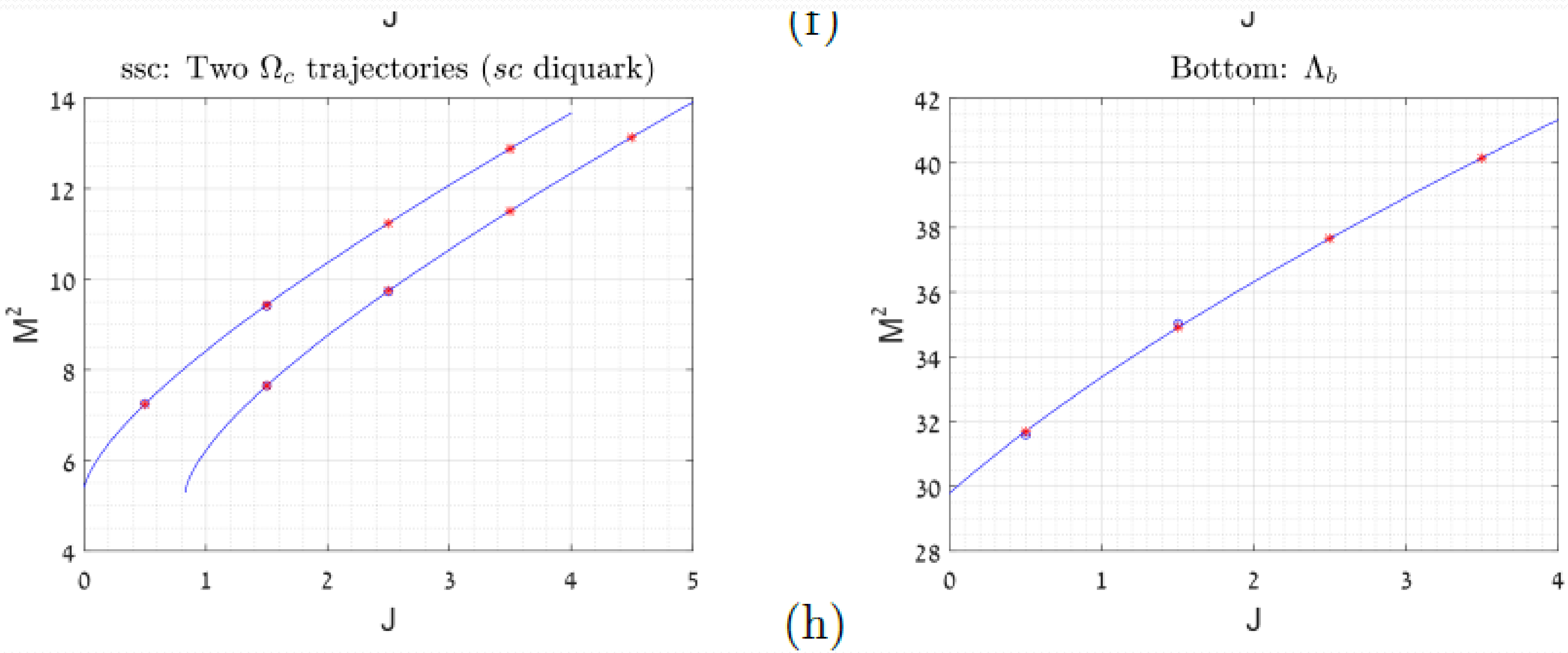


(d)

Trajectories of Ξ Λ_c and Ξ_c



Trajectories of Ω_c and Λ_b



Stringy hadrons in holography and HISH

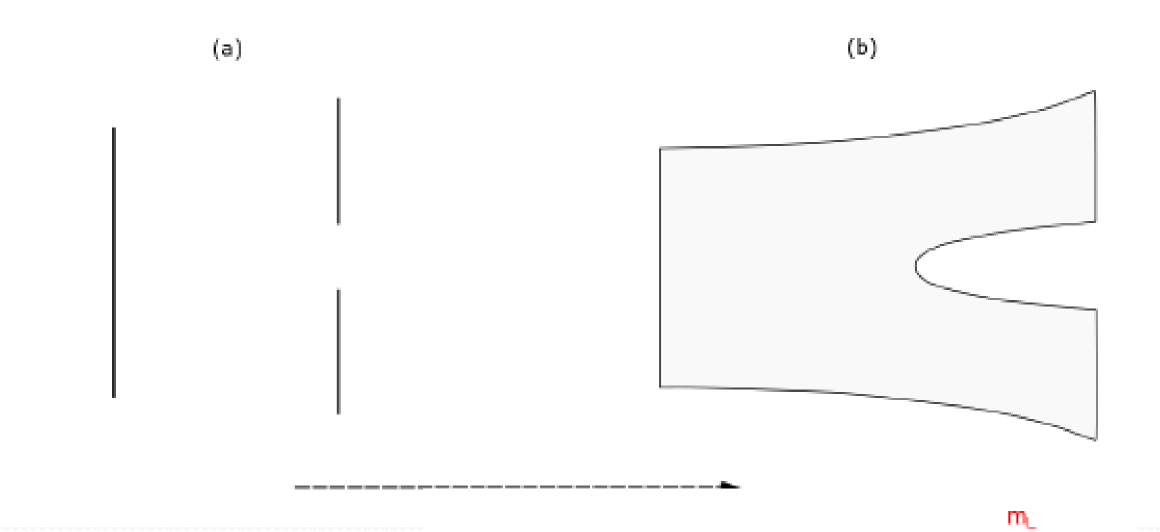
- It is important to emphasize the differences between the **hadronic strings** and the ordinary open strings.
- For the latter the spin zero state is a tachyon but for hadronic strings that have masses on their ends and also negative intercept it is a massive scalar meson
- Similarly the spin one of ordinary string is a massless gauge field and in the stringy hadron picture it is a massive vector meson.
- For the pion which is the gs on its trajectory m/TL is large so we cannot trust our intercept calculations.

$$M = 2\,m + TL \sim 2x\,60 + 20 \longrightarrow m/TL = 3$$

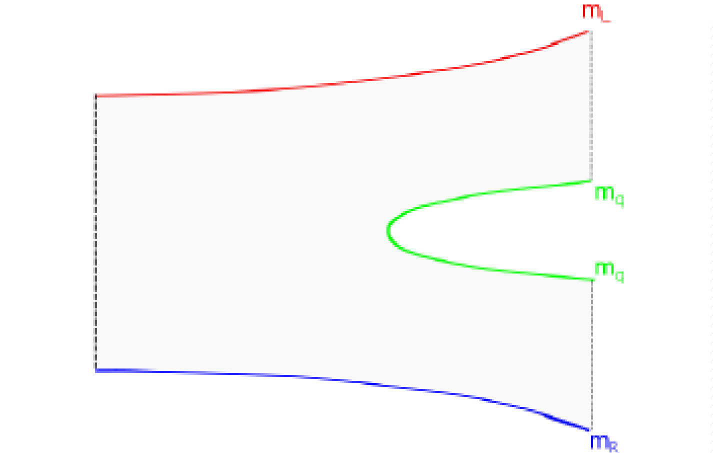
Step 7- Quantum calculations of the Decay widths and Branching ratios

The decay of a long string

- The decay of a hadron is in fact the **breaking of a string into two strings**
- A type I open string can undergo such a split

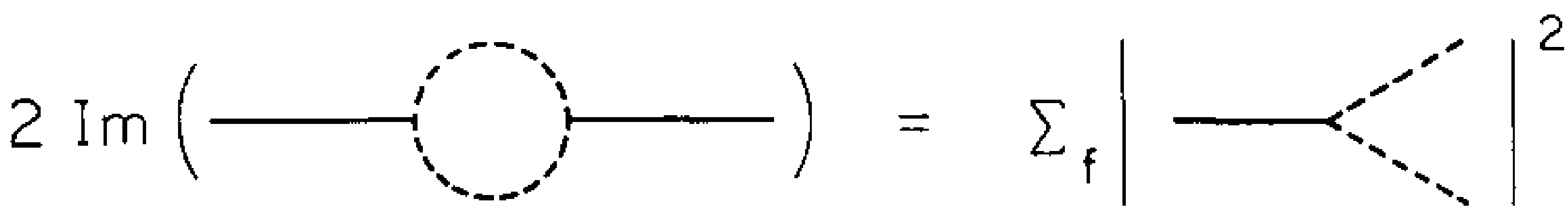


- A split of a string with
massive Endpoints



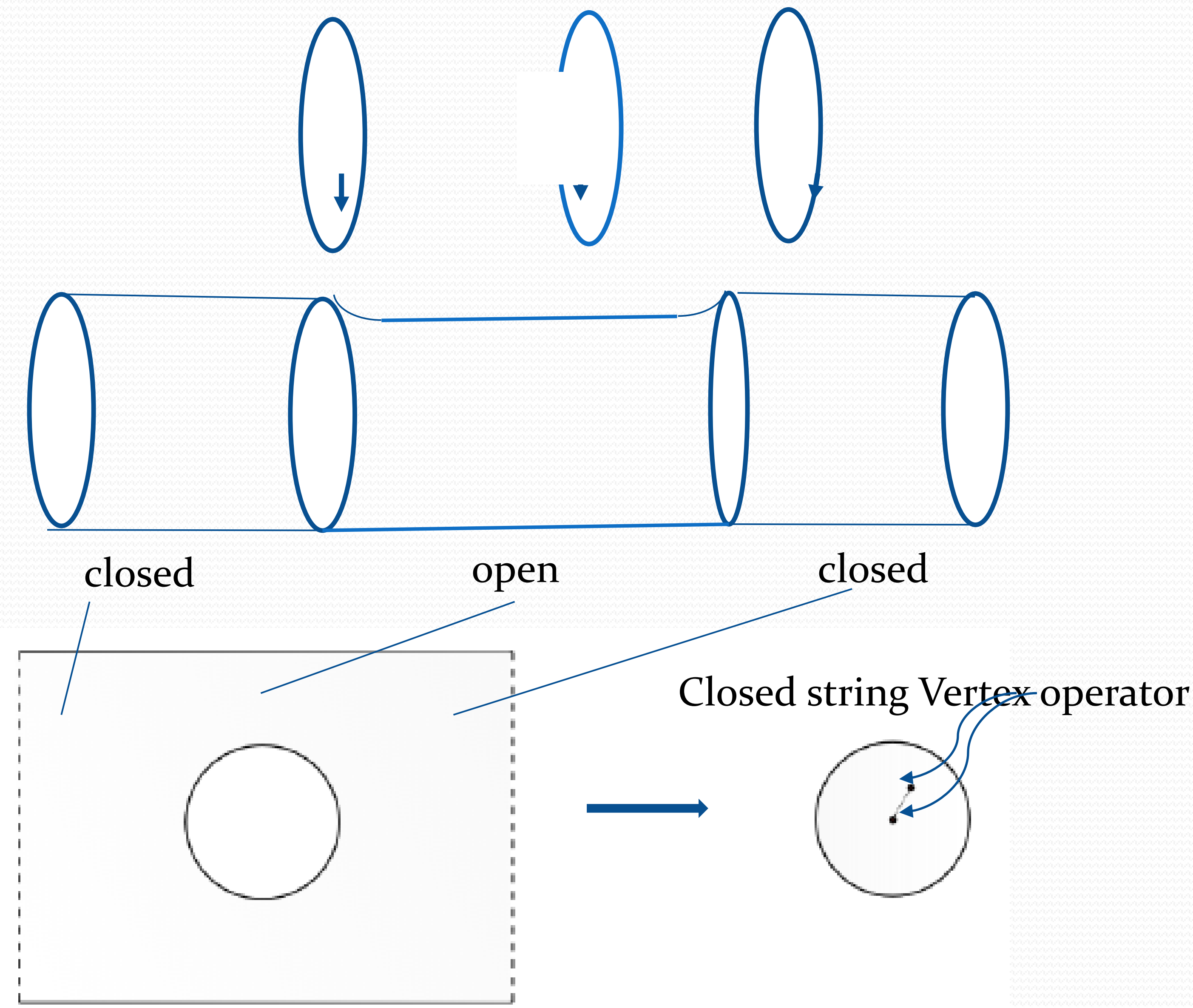
The decay of a long string in critical flat space-time

- The **total decay width** is related by the optical theorem to the imaginary part of the self-energy diagram



- A trick that Polchinski et al used is to compactify one space coordinate and consider incoming and outgoing strings that wrap this coordinate so one can avoid an annulus open string diagram and instead compute a disk diagram with simple vertex operator of a closed string

The string amplitude



The decay of a long string in critical flat space-time

- We would like to determine the dependence of the **string amplitude** on the **string length L**

$$iA_2 = \frac{iTN}{g^2} L \left[\frac{\kappa}{2\pi\sqrt{L}}\right]^2 \int_{|z|<1} \mathrm{d}^2 z \, \langle : e^{ip\cdot X(0)} :: e^{-ip\cdot X(z)} : \rangle$$

open string coupling

Zero mode

Gravitational coupling

Normalization Of the vertex

Vertex operator

$$iA_2 = \frac{iTN\kappa^2}{2\pi g^2} \lim_{t\to 0} \frac{\Gamma(t-1)\Gamma(1-\tilde{J})}{\Gamma(t-\tilde{J})} \rightarrow \Gamma = \frac{TN\kappa^2}{4g^2}\left[L_{tot} + \frac{4\pi}{T}\frac{1}{L_{to}}\right.$$

Check of the linear dependence on L

- The final result for long strings is a **linear dependence on the length L**

$$\Gamma = \frac{\pi}{2} A T L(M, m_1, m_2, T) .$$

- For short strings with important role of the massive endpoints we add a **phase space factor**

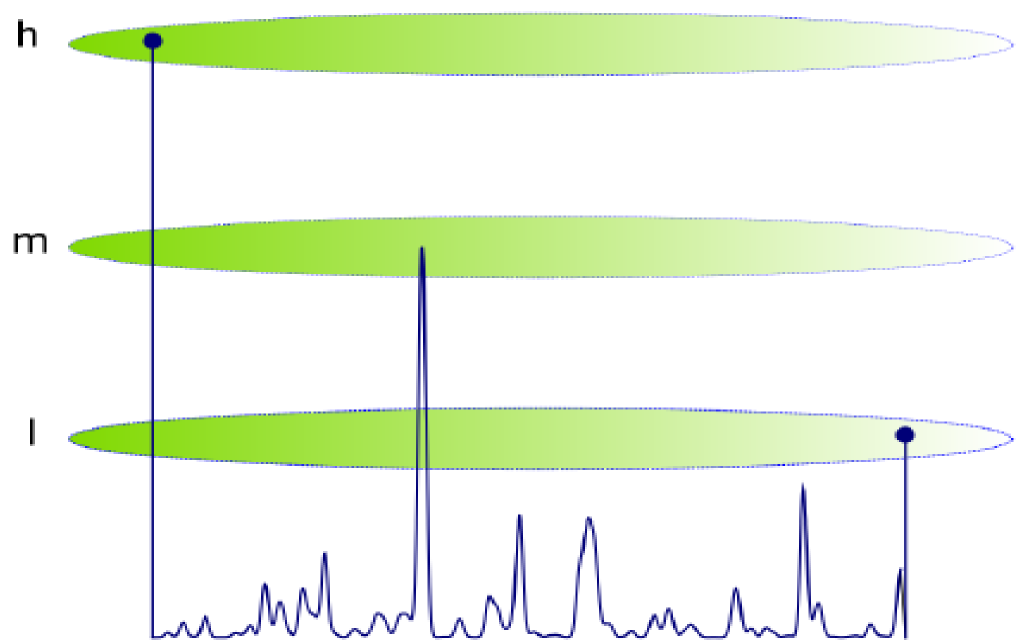
$$\Gamma = \frac{\pi}{2} A \times \Phi(M) \times T L(M, m_1, m_2, T) .$$

- The phase space factor

$$\Phi(M, M_1, M_2) \equiv 2\frac{|p_f|}{M} = \sqrt{\left(1 - (\frac{M_1 + M_2}{M})^2\right)\left(1 - (\frac{M_1 - M_2}{M})^2\right)}$$

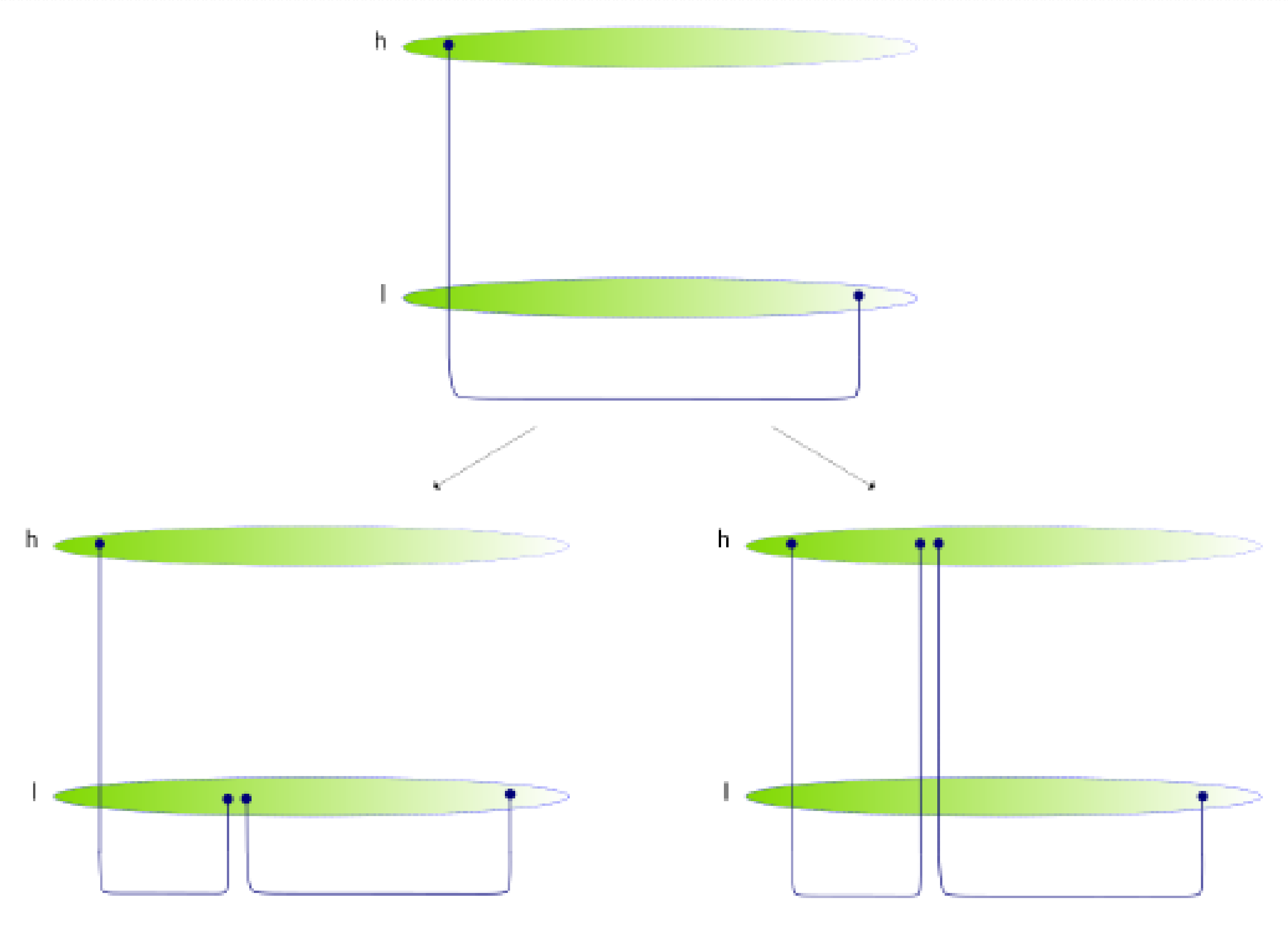
The suppression factor for stringy holographic hadrons

- The horizontal segment of the stringy hadron **fluctuates** and can reach flavor branes
- When this happens the string may **break up** , and the two new endpoints connect to a flavor brane



The suppression factor for stringy holographic hadrons

- There are in fact several possible **breakup patterns**



Determination of the suppression factor

- Assuming first that the string stretches in flat space-time we found using both a **string beads model** and a continues one that

$$\Gamma = \text{Const} \quad \exp\left(-1.0\,\frac{z_B^2}{\alpha'_{\text{eff}}}\right) = \exp\left(-2\pi\,\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

- There are further corrections due to the curvature and due to the massive endpoints. K.Peeters, M.Zamaklar JS

- $$\Gamma = \exp\left(-2\pi C(T_{\text{eff}}, M, m_i)\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

$$C(T_{\text{eff}}, M, m_i) \approx 1 + c_c\frac{M^2}{T_{\text{eff}}} + \sum_{i=1}^{2} c_{m_i}\frac{m_i}{M}.$$

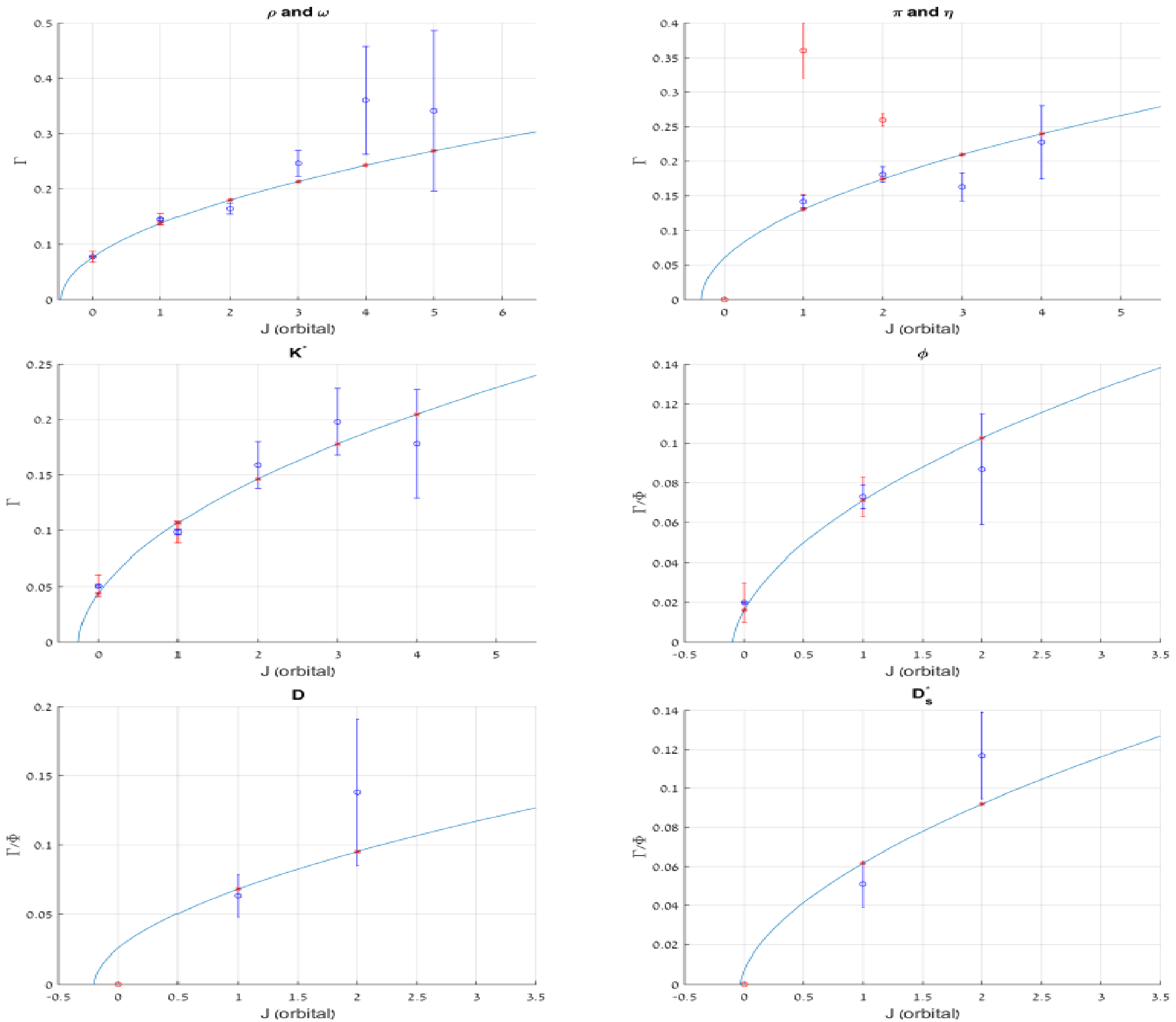
Fit results: the total decay width of mesons

- Fits of the **decay width of Mesons**

$$\Gamma = \frac{\pi}{2} A T L(M, m_1, m_2, T)\,.$$

Trajectory (No. of states)		a (from spectrum)	A (fitted value)	$\sqrt{\chi^2/DOF}$
ρ	5[a]	-0.46	0.097	1.76
ω	5[a]	-0.40	0.120	2.31
ρ and ω (avg.)	6	-0.46	0.108	1.14
π	3[a]	-0.34	0.100	1.66
η	3[a]	-0.29	0.108	1.56
π and η (avg.)	4	-0.29	0.109	1.52
K^*	5	-0.25	0.098	0.77
ϕ	3	-0.10	0.074	0.50
D	2	-0.20	0.072	0.87
D_s^*	2	-0.03	0.076	1.44

Fit results: the meson trajectories



Exponential suppression of pair creation

- The **ratio of the decay width** to a strange pair versus to a light quark pair is

$$\lambda_s = \exp\left(-2\pi C(m_s^2 - m_{u/d}^2)/T_{\text{eff}}\right) \approx 0.3$$

Hadron	J^P	Light channel		$s\bar{s}$ channel		Ratio	λ_s
$\rho_3(1690)$	3^-	$\omega\pi$	16±6%	$K\bar{K}\pi$	3.8±1.2%	0.24±0.12	0.30±0.15
$K_4^*(2045)$	4^+	$K^*\pi\pi\pi$	7±5%	$\phi\bar{K}^*$	1.4±0.7%	0.20±0.17	0.32±0.28

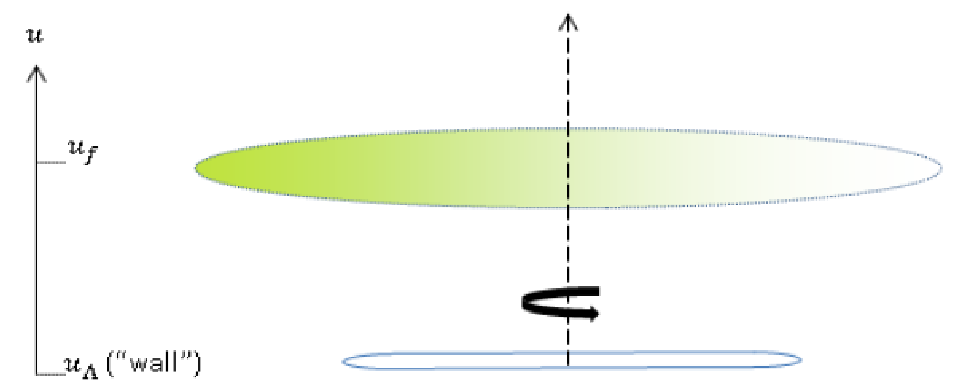
- In **radiative decays**

$$\frac{\Gamma(J/\Psi \to \gamma f_2'(1525))}{\Gamma(J/\Psi \to \gamma f_2(1270))} = 0.31 \pm 0.06\,. \quad \frac{\Gamma(\Upsilon \to \gamma f_2'(1525))}{\Gamma(\Upsilon \to \gamma f_2(1270))} = 0.38 \pm 0.10$$

Step 8-a. Glueball States as closed strings,

Glueballs as closed strings

- What are stringy glue balls in holography and HISH?
- Since they do not incorporate quarks it is natural to assume that they are rotating closed strings
- Angular momentum associates with rotation of **folded closed strings**



- The folded string is like two strings and therefore

$T_{gb}=2\,T$ - $\frac{1}{2}\alpha' = \alpha'_{gb}$ $a_{gb}=2\,a$

Fits of (potential) glueball spectra

- A rotating and exciting folded closed string admits in flat space-time a **linear Regge trajectory**

$$J + n = \alpha'_{gb} M^2 + a \qquad \alpha'_{gb} = \frac{1}{2}\alpha' \quad a = 2a_0$$

- The basic candidates of glueballs are **flavorless** hadrons f_0 of 0++ and f_2 of 2++. There are 9 (+3) f_0 and 12 (+5) f_2.
- The question is whether one can fit all of them into meson and separately some glueball trajectories.
- We found various different possibilities of fits.

Glueball 0++ fits of experimental data

- Assignment with $f_0(1380)$ as the glueball ground-state

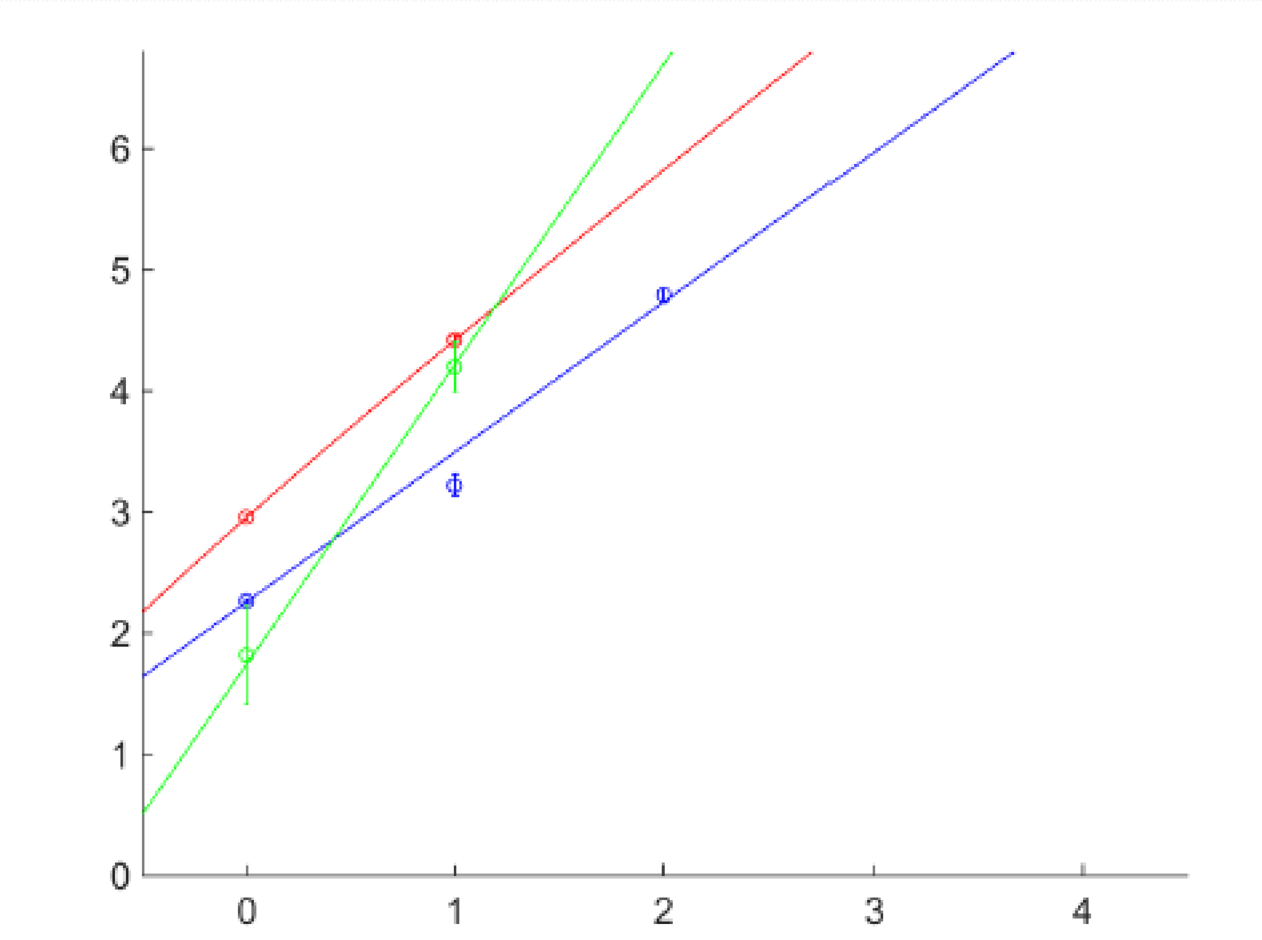
Light :	1500, *1800, 2200
$s\bar{s}$:	1710, 2100
Glue :	1370, *2060

Light		$s\bar{s}$		Glueball	
Exp.	Thry.	Exp.	Thry.	Exp.	Thry.
1505±6	1503	1720±6	1720	1350±150	1321
1795±25	1870	2103±8	2103	2050±50	2055
2189±13	2176				

Table 4. The results of the fit to the assignment with $f_0(1370)$ as the glueball ground state. The slope is $\alpha' = 0.808$ GeV^{-2} and the mass of the s quark $m_s = 439$ MeV. This fit has $\chi^2 = 1.76$. The intercepts obtained are (-1.81) for light mesons, (-1.17) for $s\bar{s}$, and (-0.71) for glueballs.

Glueball 0++ fits of experimental data

- The meson and glueball trajectories based on $f_0(1380)$ as a glueball lowest state.



On the identification of glueball trajectory

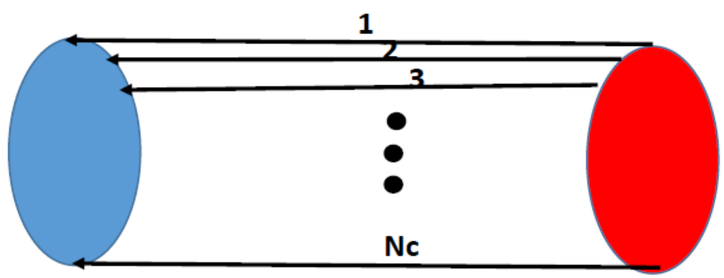
- Unfortunately there exists no unambiguous way to assign the known flavorless hadrons into **trajectories of mesons and glueballs**,
- But it is clear that **one cannot sort** all the known resonances into **meson trajectories alone**.
- One of the main problems in identifying glueball trajectories is simply the lack of experimental data, particularly in the mass region between 2.4 GeV and the cc threshold, where we expect the first excited states of the glueballs to be found.
- It is because of this that we cannot find a glueball trajectory in the angular momentum plane.

Glueballs made out of baryonic vertices

- In addition to ordinary closed string there is a zoo of stringy configurations without quarks built from **BVs and anti-BVs.**
- In general glueballs must have

  # BVs= # anti-Bvs

- These configurations look differently for **different Nc**
- The simplest configuration is



- The **mas**s of such a glueball is

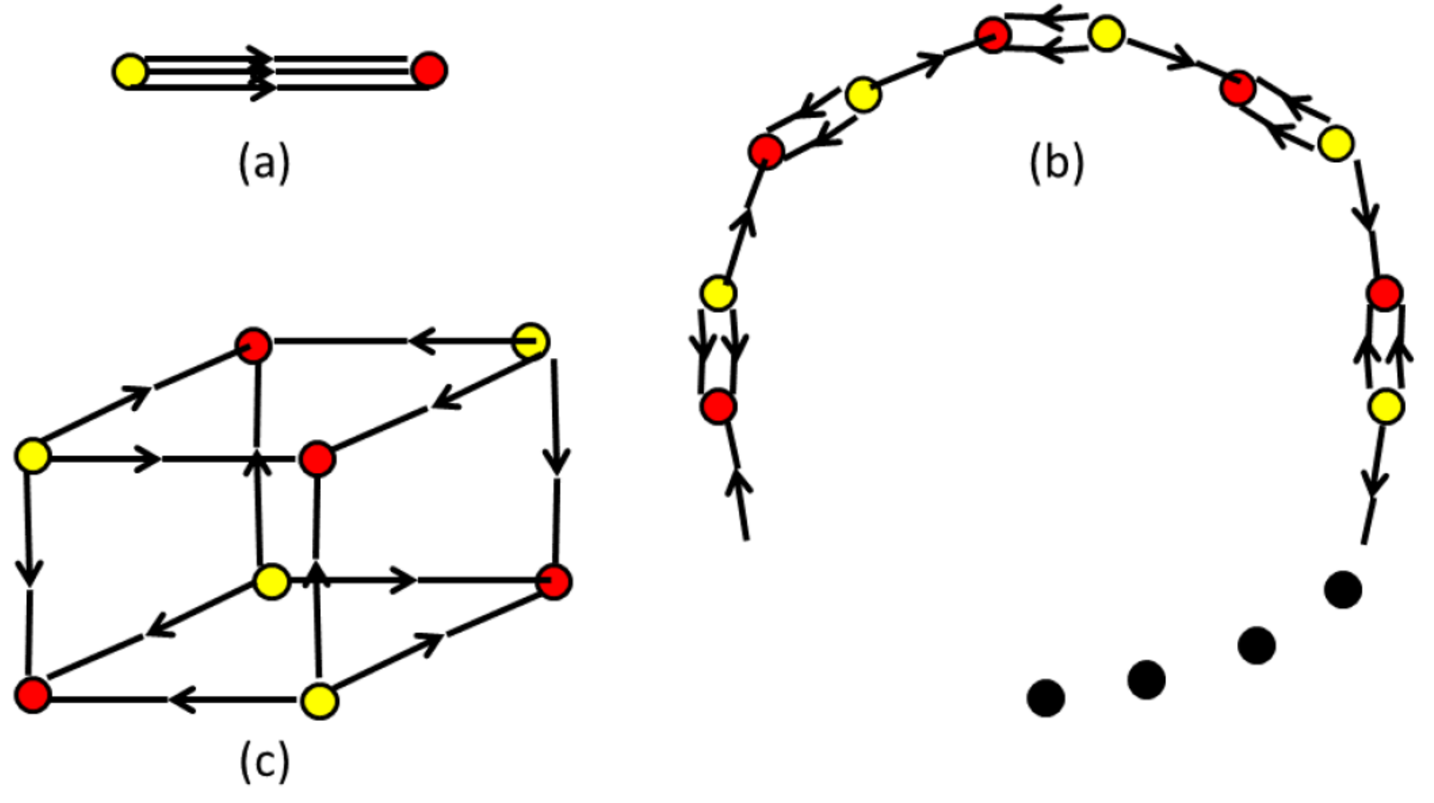
$$M_{gb} = N_c T L \qquad T = \frac{2\pi a}{L^2} \qquad \rightarrow \alpha' M_{gb}^2 = N_c |a|$$

- The corresponding **slope**

$$T_{gb} = N_c T \rightarrow \alpha'_{gb} = \frac{1}{N_c}\alpha'$$

Glueballs made out of baryonic vertices

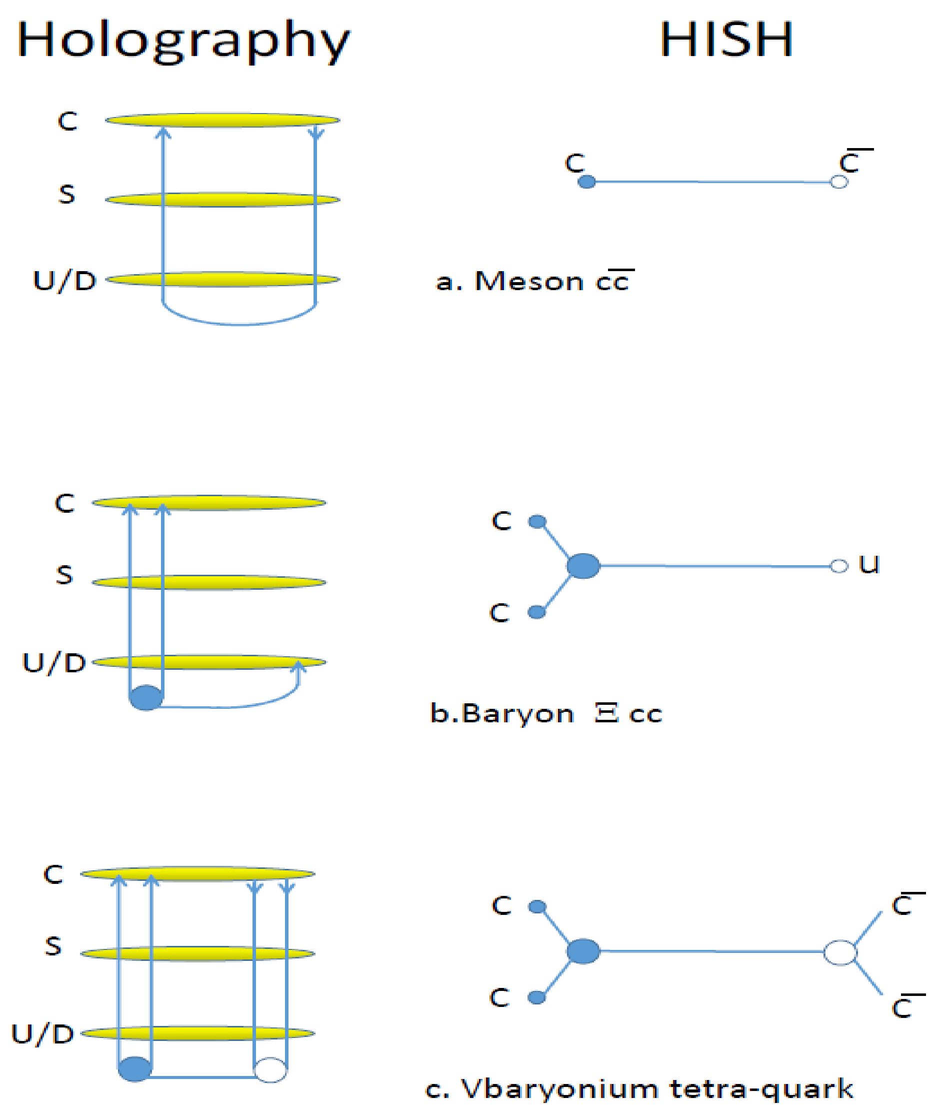
- In a similar way we can have a closed loop with n BVs and n anti-BVs with k and Nc-k strings from each BV.
- There also 3d configuraions depending on Nc



Step 8-b. Exotic Hadrons

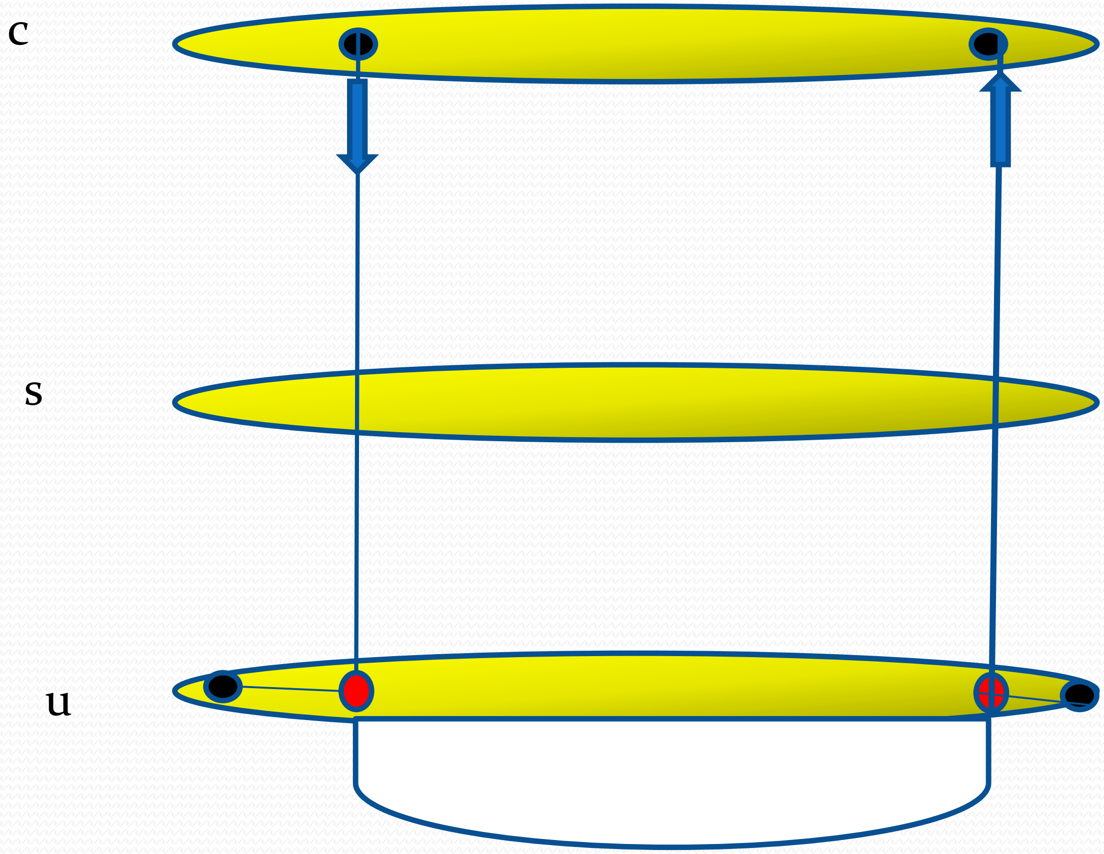
Mesons, bayons and tetra-quarks in holography andHISH

- We demonstrate the structures for **charmed hadrons**



An example of a Holographic tetra quark

- A configuration of a bryonic vertex connected to a u c di-quark and connected to an anti-baryonic vertex which is connected to anti- u and anti- c



Types of tetra-quarks

- In the construction of a tetra quark as a string with a di-quark on one end and an anti-diquark on the other end, there are three types of tetra quarks. Altogether there **225** possibilities
- **Symmetric** – the anti di-quark is made out of the anti-quarks that make up the di-quark There are obviously **15** of this type like
- **Semi-symmetric**- one pair of quark and anti-quark of the same flavor and one with different flavors. There are 100 such tetra quark for i $(cu)(\bar{c}\bar{s})$
- **Asymmetric** – both pairs are of different flavor. There are **110** of this kind like $(cs)(\bar{u}\bar{d})$

Regge-like trajectories of tetra-quarks.

- Since the structure of the tetra quark is of a **single string** with a BV+ a di-quark on one side and an anti-BV and an anti-di-quark on the other side, it has to admit a **Regge like trajectories** like mesons and baryons in J and n.
- We computed the spectra along these trajectories. **Discovering a trajectory** is a clear indication that the exotic object is a **genuine tetra quark** and not a **molecule**.
- A particular trajectory includes the Y_c (4630) and its Yb analog

Predictions of trajectory of charmed tetra quarks

- Based on the Y(4630) that was observed to decay predominantly to $\Lambda_c^+\Lambda_c^-$. If we assume that it is on a Regge-like trajectory and we borrow the slop and the endpoint masses from the J/Ψ trajectory we get

n	Mass	Width
0	4634^{+9}_{-11}	92^{+41}_{-32}
1	4902±95	103±46
2	5148±99	114±51
3	5378±104	124±55
4	5594±109	134±60

J^{PC}	Mass	Width
1^{--}	4634^{+9}_{-11}	92^{+41}_{-32}
2^{++}	4791±64	98±44
3^{--}	4939±66	105±47
4^{++}	5080±67	111±49
5^{--}	5215±69	117±52

- The gs is 1-- thus easy to create in e+ e- collisions

Predictions for the trajectory of bottom tetra quarks

- In a similar manner we predict a trajectory of Yb tetra quark that decays predominantly to $\Lambda_b \bar{\Lambda}_b$

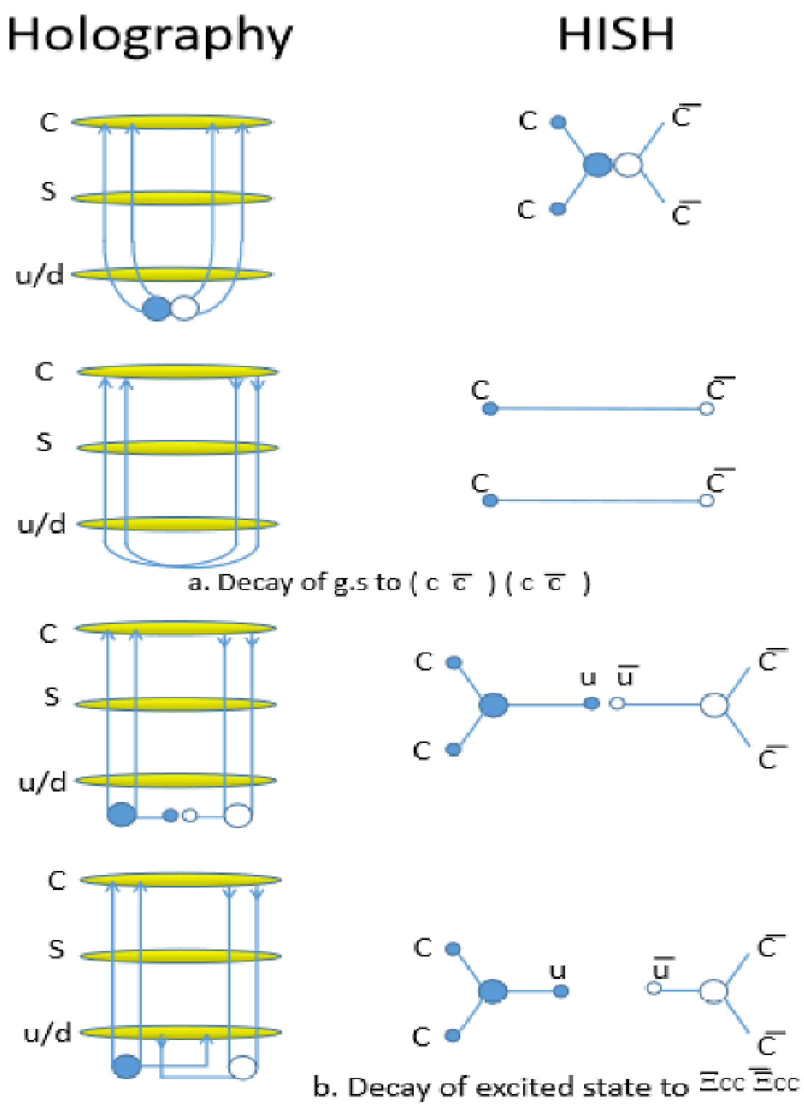
n	Mass
"-2"	10870±50
"-1"	11080±50
0	11280±40
1	11460±40
2	11640±40
3	11810±40
4	11980±40

J^{PC}	Mass
1^{--}	11280±40
2^{++}	11410±40
3^{--}	11550±40
4^{++}	11670±40
5^{--}	11800±40

Possible decays of a stringy tetra quark

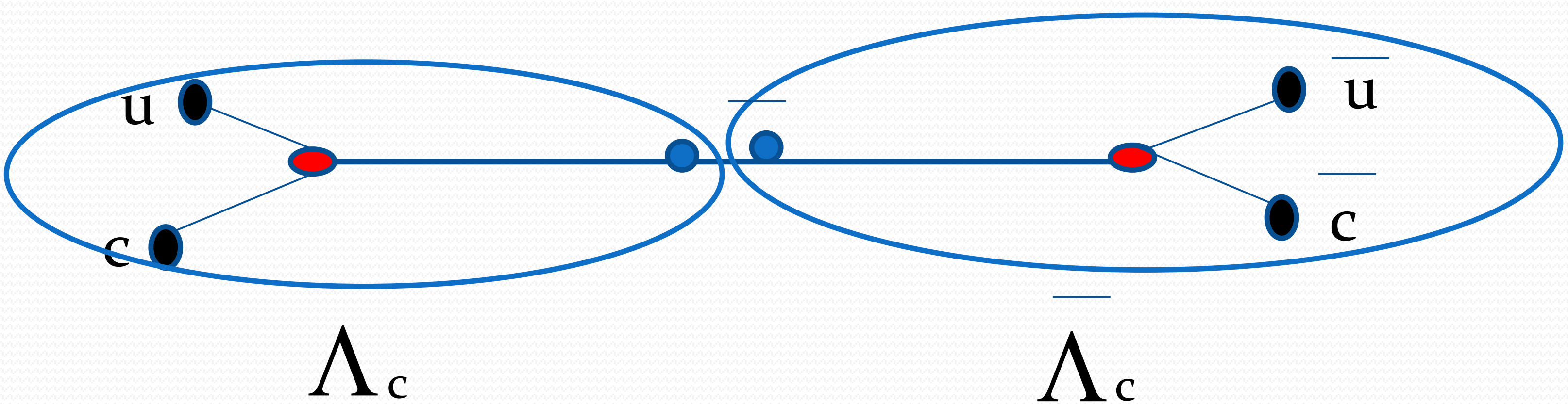
- If the mass of the tetra quark is above the threshold of the mass of a baryon and anti-baryon, it will decay via the standard **breaking** of a string.
- If the mass is below this threshold but above the threshold of a pair of mesons it will decay via an **annihilation** process of the BV and anti-BV
- If it is below this threshold it will be strong interaction **stable**.
- Using the stringy structure one can determine the conditions for these 3 possibilities based on properties of the spectra of mesons and baryons.

Possible decays of the tetra-quarks



Decays of the tetra quarks

- The tetra quark can naturally decay into a by baryon anti-baryon tearing apart the string that connects them and creating a quark anti quark pair
- For instance a creation of a d anti-d pair at the endpoints of the torn apart string between a baryonic vertex that connects to a uc di-quark and a similar anti- baryonic vertex we get a pair of Λc and anti Λc



A test case : $cc\bar{c}\bar{c}$ tetra-quarks

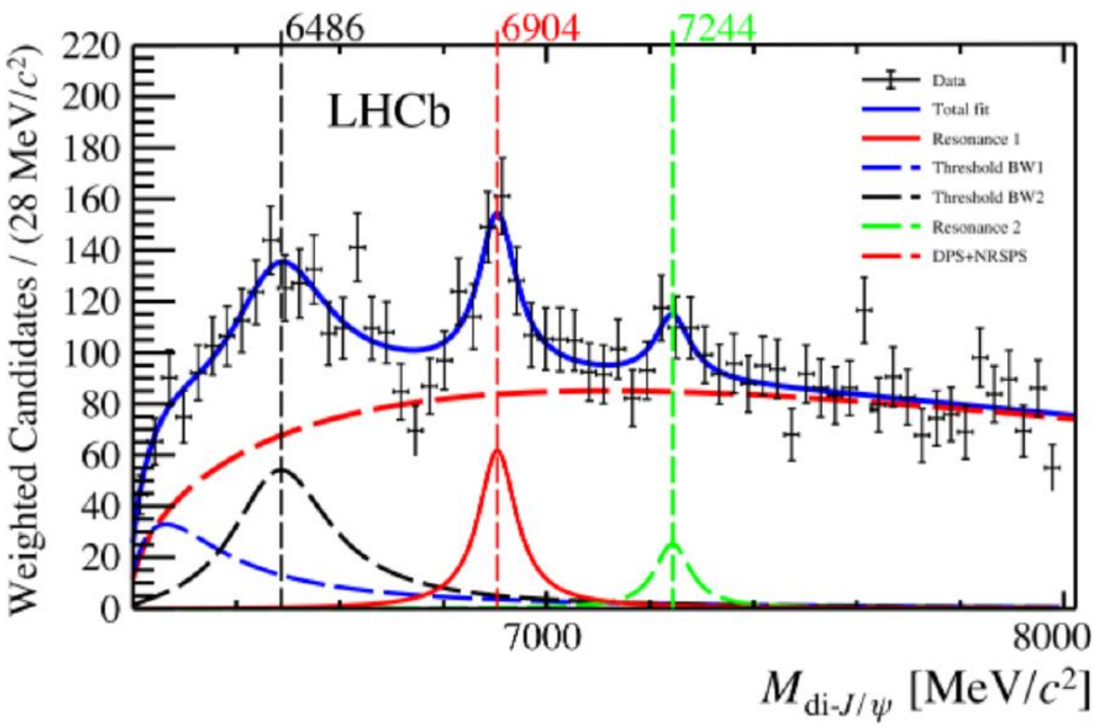


Figure 1: Location of peaks in the LHCb data. Adapted from figure 7 in [1]. The 7.2 GeV state appears to be almost exactly on the $\Xi_{cc}\bar{\Xi}_{cc}$ threshold, which is at 7242 MeV.

The $cc\bar{c}\bar{c}$ tetra-quarks

- Two states have been identified one at 6.9 GeV which is below the threshold to decay to $\Xi_{cc}\bar{\Xi}_{cc}$
- Another state was discovered at 7.2 Gev above this threshold and hence we predict that a channel of decay to $\Xi_{cc}\bar{\Xi}_{cc}$ should be discovered.

Step 9- From stringy to partonic scattering amplitudes

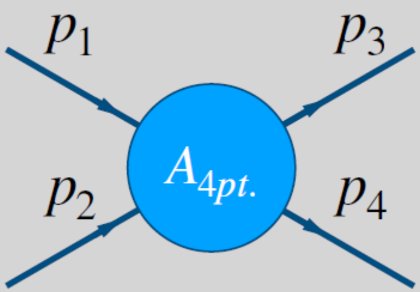
Scattering amplitude

- We consider here 2-> 2 scattering

$$s = -(p_1+p_2)^2, \quad t = -(p_1-p_3)^2, \quad u = -(p_1-p_4)^2$$

It always holds that $s+t+u = \sum_{i=1}^{4} m_i^2 \ (=0)$.

Physical regime: $s > 0, t < 0$. Scattering angle $\cos\theta_s = 1 + \frac{2t}{s}$



- There are two interesting **high energy limits**

- Fixed angle: $s \to \infty$, with fixed t/s
- Regge limit: $s \to \infty$ with fixed t (small angle)

Stringy scattering amplitude

- Veneziano string amplitude

$$A_{Ven.}(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))},$$

- For large s and fixed angle θ the string amplitude is **soft**

$$\mathcal{A}(s,t) \sim [F(\theta)]^{-\alpha(s)}$$

- **SLAC deep inelastic scattering** of electrons from fixed target found a power law falloff.
- This was the historical **mismatch** between **string theory** and **hadron physics**.
- On the other hand a QCD parton description was in accordance with the experimental results.
- That was one of the main reasons for the demise of strong interactions in terms of string theory

Partonic scattering

- An old QFT argument of Brodsky-Farrar based on dimensional argument

$$A \sim s^{2-N_p/2}$$

total number of constituent partons

- For example in a **p π** scattering Np=10 and

$$A \sim s^{-3}, \qquad \frac{d\sigma}{dt} \sim \frac{1}{s^2}|A|^2 \sim s^{-8}$$

- Polchinski Strassler proposed a holographic prescription that **bridges** between the **soft** and **hard** amplitude

The Polchinski Strassler prescription

- Consider scattering of **closed string** in a confining background like the Ads **hard wall** model.
- A key property is the **wrapping**

$$\sqrt{\alpha'}\tilde{p} = \frac{r_0}{r}\sqrt{\hat{\alpha}'}p \quad \text{and} \quad r_0 = \Lambda L^2$$

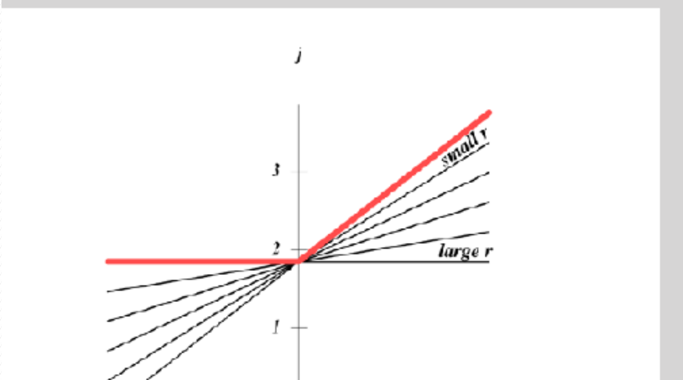
10 d inertial momentum

4d momentum

holographic coordinate

- This implies a **scale dependent string tension**

- This yields the **bending** of the **trajecory**



The Polchinski Strassler prescription

- The closed string **scattering amplitude** is

$$\mathcal{A}_{QCD}(s,t) = \int_{r_0}^{\infty} dr d^5\Omega \sqrt{|G|} \prod_{i=1}^{4} \psi_i(r,\Omega) \mathcal{A}_{string}(\tilde{s},\tilde{t})$$

holographic geometry

The Sugra wavefunction

The Virasoro-Shapiro amplitude

$$\mathcal{A}_{string}(\tilde{s},\tilde{t}) = K_8(\tilde{p},h) \frac{\Gamma(1-\frac{\alpha'}{4}\tilde{s})\Gamma(1-\frac{\alpha'}{4}\tilde{t})\Gamma(1-\frac{\alpha'}{4}\tilde{u})}{\Gamma(1+\frac{\alpha'}{4}\tilde{s})\Gamma(1+\frac{\alpha'}{4}\tilde{t})\Gamma(1+\frac{\alpha'}{4}\tilde{u})}$$

Scalled kinematic variable

$$s \to \tilde{s} = g^{xx} s$$

The Polchinski Strassler prescription

- A simple calculation shows that indeed the PS prescription yields a **passage from soft to hard** behavior.

$\Psi_i \sim (r_0/r)^{\Delta_i}$ with $\Sigma_i \Delta_i \equiv \Delta$

$$\mathcal{A}_{QCD}(s,t) \approx \frac{g^2 \alpha'^3}{L^6 r_0^4} \int_{r_0}^{\infty} dr r^3 \prod_{i=1}^{4} \frac{r_0^{\Delta_i}}{r^{\Delta_i}} A_{string}(\tilde{s}, \tilde{t})$$

$$A_4 \sim \int_1^{\infty} dr r^{3-\Delta} e^{-f(\theta)s/r^2} \sim s^{2-\frac{\Delta}{2}}$$

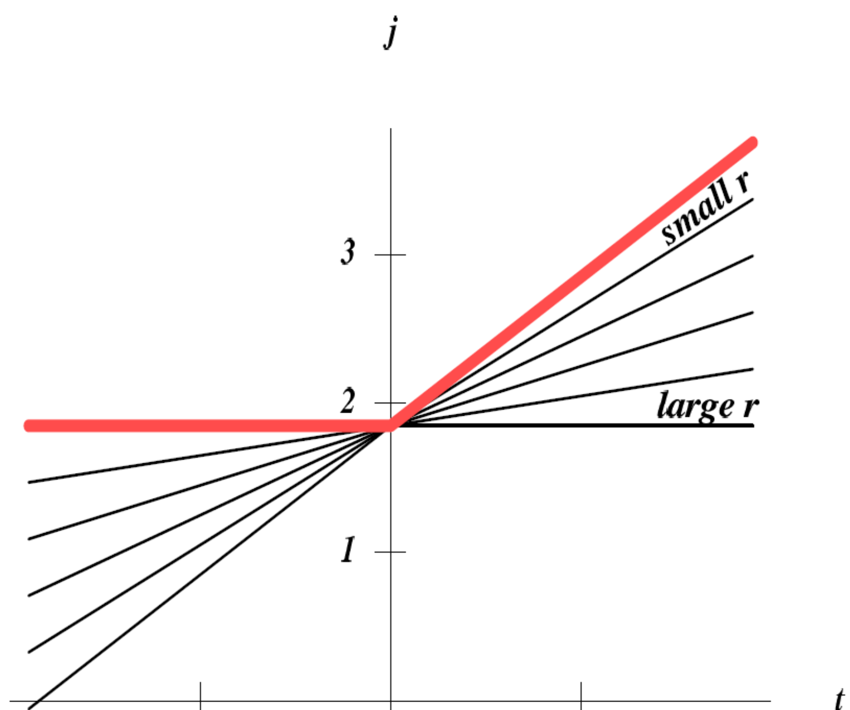
$$A_{10} \sim e^{-f(\theta)s}$$

- In QCD terms

$$\approx \frac{(gN)^{\frac{\Delta-2}{4}}}{N^2} \frac{\Lambda^{\Delta-4}}{p^{\Delta-4}}$$

From soft to hard scattering-general idea

- The bending implies moving from linear trajectory at large positive t to zero slope at large negative t.
- The zero slope region corresponds to the asymptotically free region of QCD.
- This implies a transition from a soft scattering to a hard one



Types of computations

- The **scattering amplitude** is determined in:

(i) The **fixed angle** limit

$$s \to \infty \text{ with fixed } s/t$$

(ii) The **Regge** limit

$$s \to \infty \text{ while keeping } t \text{ fixed.}$$

(iii) Expanding around the poles

Expansion around the s-channel poles, perform the integral and then resum.

i. The fixed angle approximation

- We use $$\Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin(\pi z)}\,, \qquad \Gamma(z) \approx \sqrt{\frac{2\pi}{z}}\left(\frac{z}{e}\right)^z$$

To make the arguments positive and take the limit of large s and –t and –u.

$$t = -\frac{s}{2}(1-\cos\theta)\,, \qquad u = -\frac{s}{2}(1+\cos\theta)$$

- So we can write the amplitude as a function of s and the scattering angle

$$\mathcal{A}_{10}^{\rm FA} \approx \frac{\sin(\frac{\pi}{4}\alpha' s c_+)\sin(\frac{\pi}{4}\alpha' s c_-)}{\sin(\frac{\pi}{4}\alpha' s)} \cdot \frac{1+c_+^4+c_-^4}{c_+c_-} \cdot s e^{-2\beta_{stu}}$$

$$\beta_{stu} = -\frac{\alpha' s}{4}\left(i\pi + c_+\log c_+ + c_-\log c_-\right)$$

$$c_\pm = \frac{1}{2}(1\pm\cos\theta)$$

i. The fixed angle approximation

- The pre-factor ratio of sine functions is rapidly varying and contains the zeros and poles of the amplitude.
- The last part $se^{-2\beta_{stu}}$ gives the average of the amplitude at high energies.
- We verified that omitting the rapidly oscillating term does not alter the result
- Thus the **amplitude** is given by

$$\mathcal{A}_4^{\mathrm{FA}}(s,\theta) \approx \frac{1+c_+^4+c_-^4}{c_+c_-}\int_{r_0}^{\infty} dr\sqrt{-g}\,\psi(r)^4\,\tilde{s}(r)\,e^{-2\tilde{\beta}_{stu}(r)}$$

ii. The Regge regime

- In the **large s and fixed t** limit

$$\mathcal{A}_{10}^{\mathrm{R}}(s,t) = \frac{\sin[\frac{1}{4}\pi\alpha'(s+t)]}{\sin(\frac{1}{4}\pi\alpha' s)} \frac{\Gamma(-\alpha' t/4)}{\Gamma(1+\alpha' t/4)} \left(\frac{\alpha' s}{4}\right)^{\frac{\alpha' t}{2}+2}$$

- Again we can omit the oscillating pre-factor so

$$\mathcal{A}_{10}^{\mathrm{R}}(s,t) \simeq \frac{1}{\alpha' t} \left(\frac{\alpha' s}{4}\right)^{\frac{\alpha' t}{2}+2}$$

- The **4d amplitude** is given by

$$\mathcal{A}_{4}^{\mathrm{R}}(s,t) = \int_{r_0}^{\infty} dr \sqrt{-g}\,\psi(r)^4 \mathcal{A}_{10}^{\mathrm{R}}(\tilde{s},\tilde{t})$$

ii. The Regge regime

- We use the **saddle point approximation**. For that we write

$$\mathcal{A}_4^{\mathrm{R}}(s,t) = \int_{r_0}^{\infty} dr\, e^{F_{st}(r)}$$

$$F_{st}(r) = \log(\sqrt{-g}\psi(r)^4) - \log[\alpha'\tilde{t}(r)] + \left(2 + \frac{\alpha'\tilde{t}(r)}{2}\right)\log\frac{\alpha'\tilde{s}(r)}{4}$$

- We solve the saddle point equation

$$F'_{s,t}(r) = 0,$$

- The **amplitude**

$$\mathcal{A}_4^{\mathrm{R}}(s,t) \simeq \frac{e^{F_{st}(r^*)}}{\sqrt{F''_{st}(r^*)}}$$

r* the location of the saddle

iii. Expansion around the poles

- We determine the **s channel poles** (apart from s=0)
- Then the amplitude

$$\mathcal{A}_{10} = -4(\alpha' s)^3 (1 + c_+^4 + c_-^4) \frac{\Gamma(1 - \alpha' s/4)\Gamma(\alpha' s\, c_+/4)\Gamma(\alpha' s\, c_-/4)}{\Gamma(1 + \alpha' s/4)\Gamma(1 - \alpha' s\, c_+/4)\Gamma(1 - \alpha' s\, c_-/4)}$$

- The poles are at $\alpha' s = 4(n+1)$ and the **amplitude**

$$\mathcal{A}_{10} = 4^4 \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)^2}{(n!)^2} \frac{(1 + c_+^4 + c_-^4)}{\alpha' s/4 - (n+1)} \frac{\Gamma\big((n+1)c_+\big)\Gamma\big((n+1)c_-\big)}{\Gamma\big(1-(n+1)c_+\big)\Gamma\big(1-(n+1)c_-\big)}$$

$$\equiv \sum_{n=0}^{\infty} \frac{\mathcal{R}_n(\theta)}{\alpha' s/4 - (n+1)}$$

iii. Expansion around the poles

- The residue at each pole $\mathcal{R}_n(\theta)$ lynomial of $\cos(\theta)$
- When we rescale the Mandelstam variables their ratio is fixed
- Therefore the angle and the residue do not depend on the holographic coordinate in the integral
- Thus the final form of the amplitude is

$$\mathcal{A}_4(s,t,u) = \sum_{n=0}^{\infty} \mathcal{R}_n(\theta) \int_{r_0}^{\infty} dr \sqrt{-g} \frac{\psi(r)^4}{\alpha' \tilde{s}(r)/4 - (n+1)}$$

The hard wall model

- The bulk is that of the $AdS_5 \times S_5$
- It stretches from the Wall at r=ro to the boundary at r= infinity
- The corresponding metric

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

- The warp factor

$$p_\mu \to \tilde{p}_\mu = \frac{R}{r}p_\mu \qquad \Rightarrow \qquad s \to \tilde{s} = \frac{R^2}{r^2}s$$

- The wave function can be approximated as

$$\psi_i(r) \simeq \left(\frac{r_0}{r}\right)^{\Delta_i}$$

The hard wall: model expanding around the poles

- The amplitude is

$$\mathcal{A}_4(s,t,u) = \frac{r_0^{\Delta}}{R^4}\int_{r_0}^{\infty} dr\, r^{3-\Delta}\, \mathcal{A}_{10}\left(\frac{R^2}{r^2}\alpha' s, \frac{R^2}{r^2}\alpha' t, \frac{R^2}{r^2}\alpha' u\right)$$

$$\Delta \equiv \sum_{i=1}^{4} \Delta_i$$

- One expects that Δ_i is replaced by the twist $\tau = \Delta - s$
- The outcome of the integral is

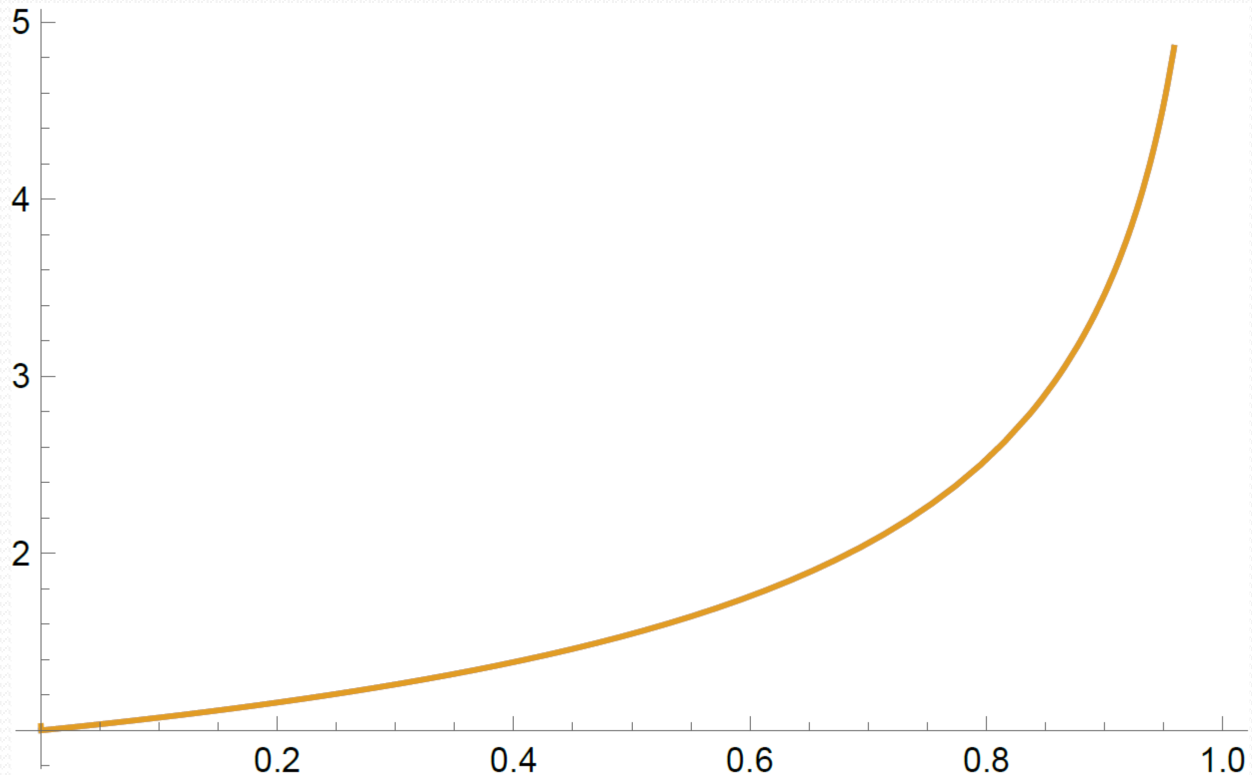
$$\mathcal{A}_4(s,t,u) =$$

$$\frac{r_0^4}{R^4}\frac{1}{4-\Delta}\sum_{n=0}^{\infty}\frac{\mathcal{R}_n(\theta)}{n+1}\; {}_2F_1\left(1; \frac{\Delta}{2}-2; \frac{\Delta}{2}-1; \frac{\alpha_0' s}{4(n+1)}\right)$$

The hard wall model

- The hypergeometric function has a simple form for integer, even Δ. for Δ=8

$$ {}_2F_1(1\,;k\,;k+1\,;z) = -\frac{k\log(1-z)}{z^k} - \sum_{l=1}^{k-1}\frac{k}{(k-l)z^l} $$



The hard wall model

- For any Δ we find a **logarithmic singularity** (**branch cut** starting $\alpha'_0 s > 4(n+1)$
- This contradict expectations that there are no branch cuts in large Nc QCD.
- The imaginary part of the amplitude comes from the log term

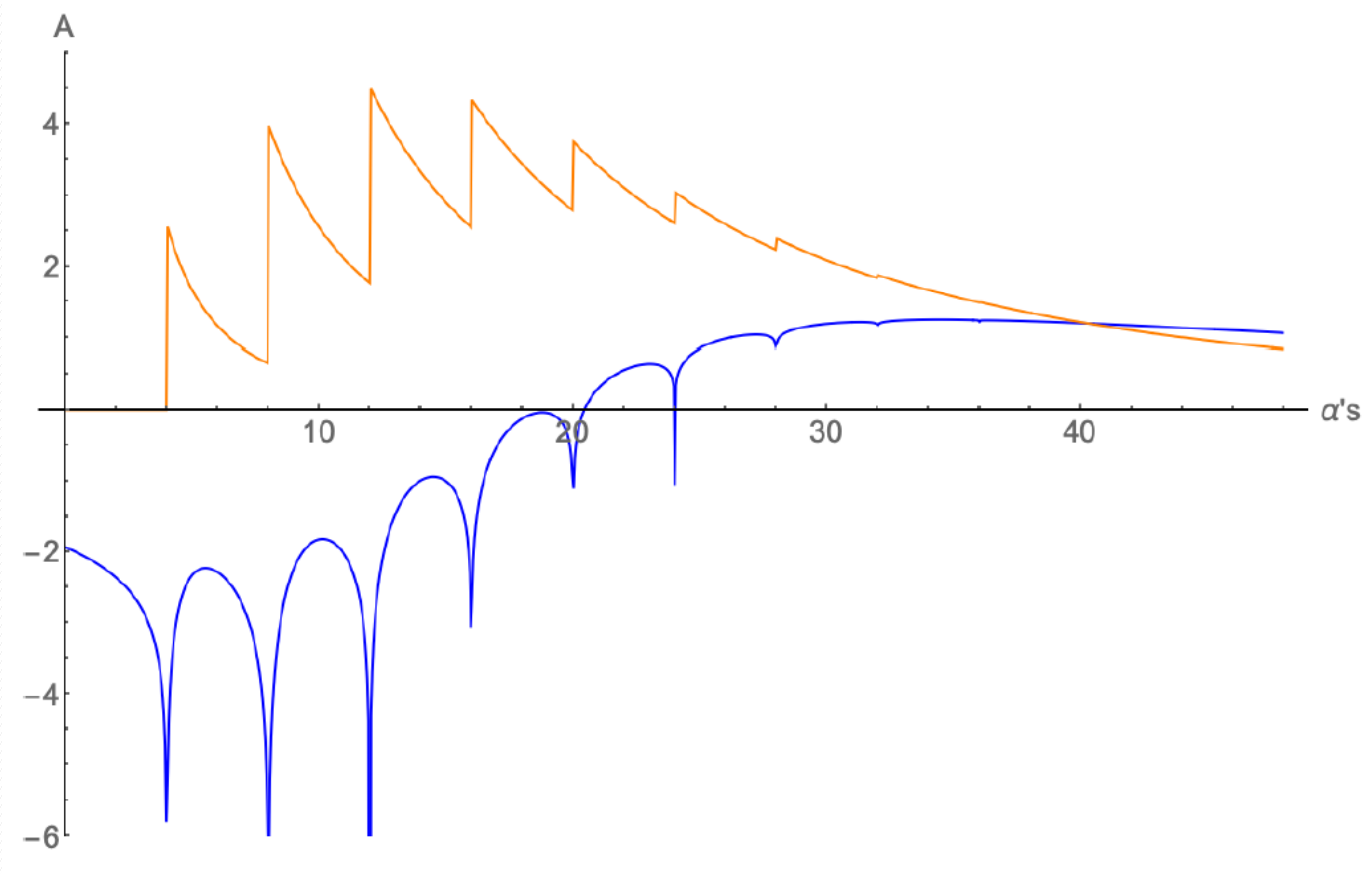
$$\mathrm{Im}\left[\mathcal{A}_4\right] = s^{2-\frac{\Delta}{2}} \sum_{n=0}^{N_M} (n+1)^{\frac{\Delta}{2}-3} \mathcal{R}_n(\theta)$$

- Altogether the asymptotic behavior of the amplitude

$$\mathrm{Re}\left[\mathcal{A}_4\right] \sim s^{-1}, \qquad \mathrm{Im}\left[\mathcal{A}_4\right] \sim s^{2-\Delta/2}$$

The hard wall model

- The real part in blue and imaginary part in yellow



The hard wall model

- The real and imaginary parts of the amplitude at intermediate energies
- For fixed angle the log singularities fade away for high energies
- We would like to interpret this as a consequence of asymptotic freedom

The hard wall model: The fixed angle

- Using

$$\mathcal{A}_4^{\mathrm{FA}}(s,\theta) \approx \frac{1+c_+^4+c_-^4}{c_+c_-}\int_{r_0}^{\infty} dr\sqrt{-g}\,\psi(r)^4\,\tilde{s}(r)\,e^{-2\tilde{\beta}_{stu}(r)}$$

- We get

$$\mathcal{A}_4^{\mathrm{FA}} \simeq \frac{\alpha' s}{2}\,\beta_{stu}^{1-\frac{\Delta}{2}}\left[\Gamma\left(\frac{\Delta}{2}-1\right)-\tilde{\Gamma}\left(\frac{\Delta}{2}-1;\beta_{stu}\right)\right]$$

- The incomplete Γ is negligible so

$$\mathcal{A}_4^{\mathrm{FA}} \simeq \quad \sim s\beta_{stu}^{1-\frac{\Delta}{2}} \sim s^{2-\frac{\Delta}{2}}$$

The hard wall model: Regge regime

- We compute the amplitude

$$\mathcal{A}_4^{\text{Regge}}(s,t) = \int_{r_0}^{\infty} dr r^{3-\Delta} \mathcal{A}_{10}^{\text{Regge}}(\tilde{s},\tilde{t})$$

- Using a saddle point method which is at

$$r^* \approx R\sqrt{\frac{\alpha'|t|}{\Delta-1}\log\left(\frac{s}{|t|}\right)}$$

- The final result is

$$\mathcal{A}_4^{\text{Regge}}(s,t) \sim (\alpha' s)^2 (\alpha'|t|)^{-\frac{\Delta}{2}} (\log \frac{s}{|t|})^{1-\frac{\Delta}{2}}$$

Witten's models

- There are in fact two models:
- Compactified one space dimension in $AdS_7 \times S_4$.

$$ds^2 = \frac{r^2}{R^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(r)dx_4^2 + dx_{11}^2\right) + \frac{R^2}{r^2}\frac{dr^2}{f(r)} + \frac{R^2}{4}d\Omega_4^2,$$

with $f(r) = 1 - \frac{r_0^6}{r^6}$.

- Compactified one space dimension in D4 background

$$ds^2 = \left(\frac{U}{R_4}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)dx_4^2\right) + \left(\frac{R_4}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

with $f(U) = 1 - \frac{U_0^3}{U^3}$ and a non-trivial dilaton $e^\Phi = \left(\frac{U}{R_4}\right)^{3/4}$.

Witten's model: the 11d model

- The same spectra of glueballs for the two models
- Is the scattering amplitude the same?
- The scaling of s is different

$$\tilde{s}(r) = s r^{-2} \qquad \tilde{s}(U) = s U^{-3/2}$$

11 d — 10 d

- The pole expansion in the 11 d case is

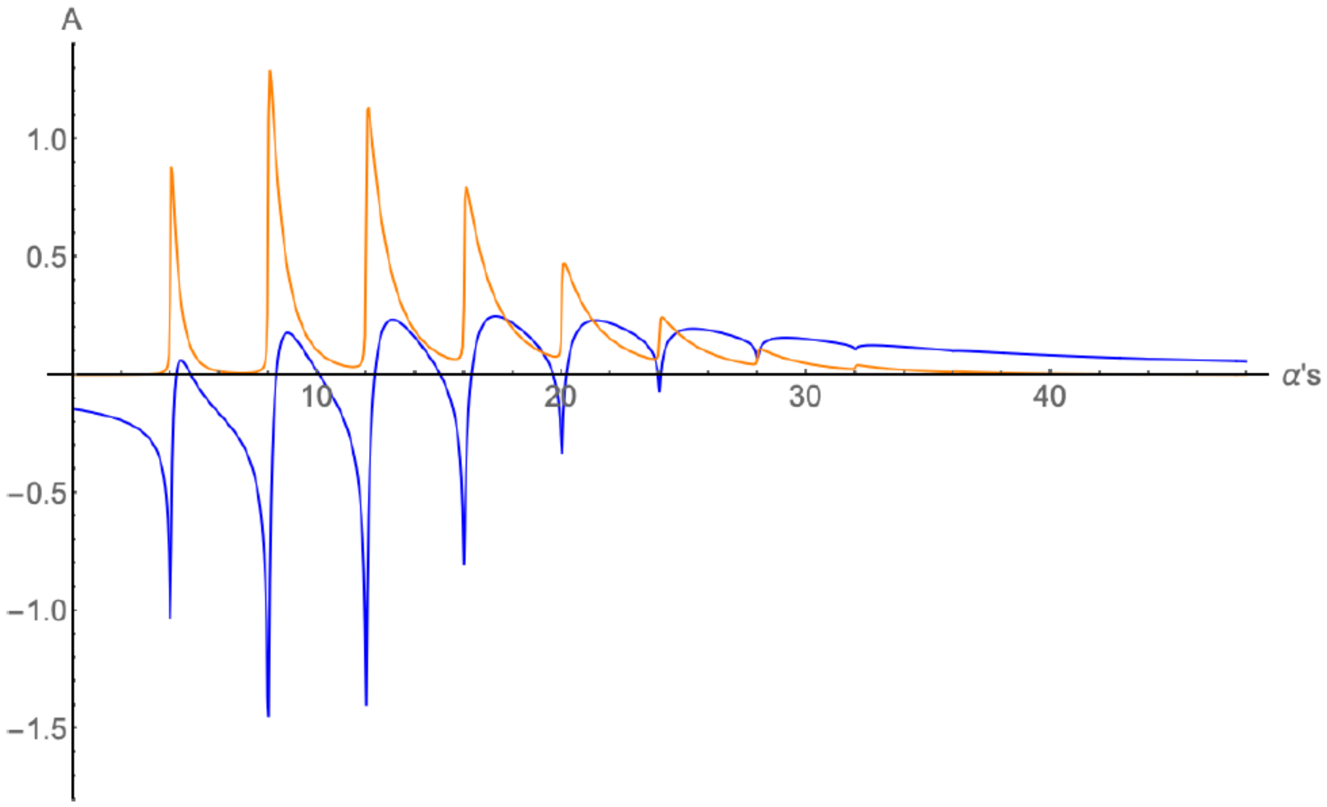
$$\mathcal{A}_4(s,t,u) = \sum_{n=0}^{\infty} \mathcal{R}_n(\theta) \int_{r_0}^{\infty} dr \, \frac{r^{5-\Delta}}{r^{-2}\alpha' s - 4(n+1)}$$

$$\mathcal{A}_4(s,t,u) = \frac{1}{4-\Delta} \sum_{n=0}^{\infty} \frac{\mathcal{R}_n(\theta)}{n+1} \, {}_2F_1\left(1\,;\frac{\Delta}{2}-3\,;\frac{\Delta}{2}-2\,;\frac{\alpha_0' s}{4(n+1)}\right)$$

$$\mathrm{Re}\,\mathcal{A}_4 \sim s^{-1}\,, \qquad \mathrm{Im}\,\mathcal{A}_4 \sim s^{3-\frac{\Delta}{2}}$$

Witten's model: the 11d model

- The amplitudes again admit logarithmic singularities that fade away for high energy



Witten's model: the 11d model

- The fixed angle amplitude

$$\mathcal{A}_4^{\rm FA} \simeq \int_1^\infty dr\, r^{5-\Delta} \frac{\alpha' s}{r^2} e^{-2\beta_{stu}/r^2} \approx \frac{\alpha' s}{2}\, \beta_{stu}^{2-\frac{\Delta}{2}} \Gamma(\frac{\Delta}{2} - 3)$$

- Which again scales like $s^{3-\frac{\Delta}{2}}$.
- In the Regge regime

$$\mathcal{A}_4^R \sim (\alpha' s)^2 (\alpha' |t|)^{1-\frac{\Delta}{2}} (\log \frac{s}{|t|})^{2-\frac{\Delta}{2}}$$

General Confining Background

- The amplitude has a real part s^{-1}, and imaginary part s^{α}.

$$\alpha = \frac{1+b-\Delta}{a}$$

- For the fixed angle we get

$$s\int_{U_\Lambda}^{\infty} dU U^{b-\Delta-a} e^{-\beta_{stu} U^{-a}} \approx \frac{1}{a} s \beta^{\frac{1+b-\Delta}{a}-1} \Gamma\left(1-\frac{1+b-\Delta}{a}\right) \sim s^{(1+b-\Delta)/a}$$

- The dependence s^{α} is not only for the imaginary part
- For the Regge limit we get

$$\mathcal{A}_4^R \sim (\alpha' s)^2 (\alpha' |t|)^{-2+\alpha} (\log \frac{s}{|t|})^{-1+\alpha}$$

Witten's model: Compactified D4 brane

- In the 10 d the results are
- Summation over poles

$$\int_{U_\Lambda}^{\infty} dU \frac{U^{5/2-\Delta}}{U^{-3/2}\alpha' s - 4(n+1)} = \frac{1}{4(n+1)(\Delta - \frac{11}{2})} {}_2F_1\Big(1\,;\frac{2}{3}(\Delta-\frac{7}{2})\,;\frac{2}{3}(\Delta-\frac{7}{2})+1\,;\frac{\alpha_0' s}{4(n+1)}\Big)$$

Which asymptotes to

$$\mathrm{Re}\,[\mathcal{A}] \sim s^{-1}\,,\ \mathrm{Im}\,[\mathcal{A}] \sim s^{\frac{7}{3}-\frac{2}{3}\Delta}$$

With $\Delta = 12$

$$\mathrm{Im}\,[\mathcal{A}] \sim s^{-\frac{17}{3}}$$

- The fixed angle

$$\mathcal{A}^{FA} \simeq \int_{U_\Lambda}^{\infty} dU U^{\frac{5}{2}-\Delta} \frac{\alpha' s}{U^{3/2}} e^{-\beta_{stu} U^{-3/2}} \simeq \frac{2}{3}\Gamma\big(\frac{2}{3}\Delta - \frac{4}{3}\big)\, s\, \beta_{stu}^{-\frac{2}{3}(\Delta-2)}$$

- The Regge behavior

$$\mathcal{A}^R = \int_{U_\Lambda}^{\infty} dU U^{\frac{5}{2}-\Delta} \mathcal{A}^R(\tilde{s},\tilde{t},\tilde{u}) \sim (\alpha' s)^2 (\alpha'|t|)^{\frac{1}{3}-\frac{2}{3}\Delta} (\log \frac{s}{|t|})^{\frac{4}{3}-\frac{2}{3}\Delta}$$

The scattering amplitude

- We derived the amplitude in the hard-wall , soft-wall and Witten's model in 10d and 11d at fixed angle, Regge limit and expanding around the poles

1) Fixed angle approximation ($|t| \sim s$) before radial/holographic integration:

- Hard wall (polynomial ψ): $\mathcal{A} \sim s^{2-\Delta/2}$. We can take $\Delta = 8$, (replacing dimension with twist or number of constituents $\Delta_i \to 2$) so that $A \sim s^{-2}$.
- Hard wall (Bessel ψ): $\mathcal{A} \sim s^{2-\Delta/2}$. *Caveat*: for a massless scalar in AdS_5 $\Delta_i = 4$, so $\Delta = 16$. Using 'exact' ψ corresponds to taking Δ_i, not the twist or the number of constituents (gluons) as desired, and therefore $\mathcal{A} \sim s^{-6}$.
- Soft wall: $\mathcal{A} \sim s^{2-\Delta/2}$. We can take $\Delta = 8$ (replacing dimension with twist or number of constituents) so that $A \sim s^{-2}$.
- Witten's model (11D): $\mathcal{A} \sim s^{3-\Delta/2}$. Here the dependence on Δ is altered because asymptotically the background is a higher dimensional AdS_7. Then the results do not match with the expectation in four dimensional QCD. In addition, from the scalar wave function we read $\Delta = 24$ for a final result of $\mathcal{A} \sim s^{-9}$.
- Witten's model (10D): $\mathcal{A} \sim s^{7/3-2\Delta/3}$. The warp factor of $U^{3/2}$ in this background results in fractional powers rather than integer ones, and a discrepancy with the 11D formulation. With $\Delta = 12$ we find $\mathcal{A} \sim s^{-17/3}$

The scattering amplitude

2) Pole expansion, integration, summation:

- All models: $\mathrm{Re}\,[\mathcal{A}] \sim s^{-1}$, $\mathrm{Im}\,[\mathcal{A}] \sim s^{\alpha}$. Where α for each model is the same power that one finds in the fixed angle calculation detailed above ($\alpha = 2 - \Delta/2$ for hard wall, etc.). With the exception of:
- Soft wall model: $\mathrm{Re}\,[\mathcal{A}] \sim s^{3-\Delta/2}$, $\mathrm{Im}\,[\mathcal{A}] \sim s^{2-\Delta/2}$.

3) Regge regime, large s, fixed t (small angle):

- All models: $\mathrm{Re}\,\mathcal{A} \sim s^2\, t^{-2+\alpha}\, (\log \frac{s}{t})^{-1+\alpha}$; $\mathrm{Im}\,\mathcal{A} \sim s^2\, t^{-2+\alpha}\, (\log \frac{s}{t})^{-2+\alpha}$ with the same α as above.

From soft open string Scattering to partonic scattering

Meson and baryon scattering as open string scattering

- Next we would like to apply the Polchinski Starssler prescription to the scattering on open strings that describe the scattering of mesons and baryons.
- The endpoints of holographic open strings reside on probe flavor branes. They can be added to the models discussed for closed strings. In particular adding them to Witten's model is the Sakai-Sugimoto model.
- Like for the closed string, for simplicity, we will use the wavefunction associated with the supergravity modes (not strings). These are the fluctuations of the embeddings ((pseudo)scalars) and of the flavor gauge fields (vectors)

Meson and baryon scattering as open string scattering

- We used the following open string amplitudes:
- The **Lovelace-Shapiro** describing pion scattering
- The **Veneziano** originally suggested to describe the decay of a vector w meson to 3 pions
- Vector superstring
- These amplitudes have the same high energy by different low energy behviour
- We will use wave functions of the form $\psi(r)_i \sim r^{\Delta_i}$
- For these wave functions the analysis is very similar to the closed string one.

Superstring amplitude

- The 10d amplitude for four massless vectors reads

$$\mathcal{A}_{10}(s,t) = -\frac{1}{4\alpha'^2}\frac{\mathcal{F}^4}{st}\frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}$$

$$\mathcal{F}^4 = 2\,\mathrm{Tr}\,(f_1 f_2 f_3 f_4) - \frac{1}{2}\,\mathrm{Tr}\,(f_1 f_2)\,\mathrm{Tr}\,(f_3 f_4) + \text{cyclic in } (234)$$

- where $f_j^{\mu\nu} = k_j^{[\mu} a_j^{\nu]}$ are the usual field strength
- The supersymmetric multiplet includes fermionic and pseudo scalar partners.
- For the scalars

$${}_{4p} = \left(\frac{u}{s}(\phi_1\cdot\phi_2)(\phi_3\cdot\phi_4) + (\phi_1\cdot\phi_3)(\phi_2\cdot\phi_4) + \frac{u}{t}(\phi_1\cdot\phi_4)(\phi_2\cdot\phi_3)\right)\frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}$$

Superstring amplitude

- At large s the dominant term is

$$-\frac{s}{t}(\phi_1\cdot\phi_4)(\phi_2\cdot\phi_3)\frac{\Gamma(1-\alpha's)\Gamma(1-\alpha't)}{\Gamma(1-\alpha's-\alpha't)}\bigg|_{s\gg 1}$$

- In the fixed angle regime

$$\mathcal{A}_{4pt} \simeq -\frac{s}{t}e^{-\beta_{st}}$$

$$\beta_{st} = \alpha's\Big(-c_+\log(-c_+) - c_-\log(-c_-)\Big)$$

- This is the same as the closed string up to a rescaling α' by a factor of 4.

Superstring amplitude

- In the hard wall the holographic amplitude is

$$\mathcal{A}_4 = \frac{1}{(4-\Delta)} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(n+1)!} \frac{\Gamma\big(c_+(n+1)+1\big)}{\Gamma\big(c_+(n+1)-n\big)} \, {}_2F_1\Big(1\,;\frac{\Delta}{2}-2\,;\frac{\Delta}{2}-1\,;\frac{\alpha_0' s}{n+}$$

- The integral over a single pole gives the same result as for the closed string apart from the residue and the locations of the singularities are now at $\alpha' s = (n+1)$ rather than $4(n+1)$
- The behavior at high energies is the same

$$\mathrm{Re}\,[\mathcal{A}] \sim s^{-1}\,, \qquad \mathrm{Im}\,[\mathcal{A}] \sim s^{2-\frac{\Delta}{2}}$$

Superstring amplitude

- The amplitude for Δ=8

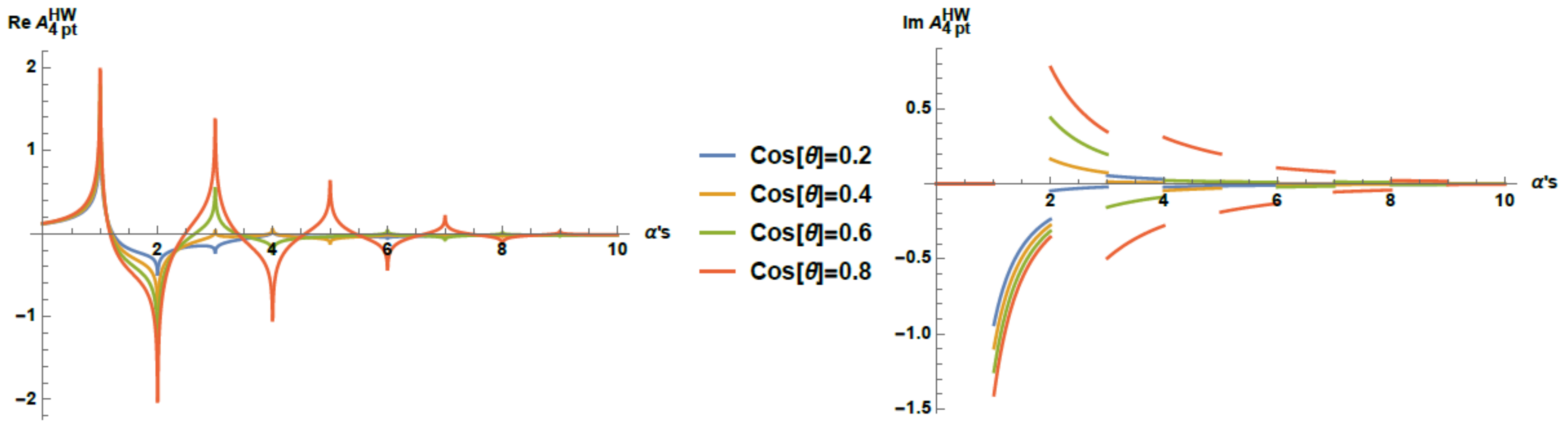


Figure 7. Real and imaginary part of the open superstring amplitude in the hard wall background at different values of the angle.

- For the fixed angle

$$\mathcal{A}_4^{\rm FA} \simeq -\frac{s}{t}\int_1^\infty dr\, r^{3-\Delta} e^{-\frac{\beta}{r^2}} = \frac{1}{2c_+}\beta_{st}^{2-\frac{\Delta}{2}}\left[\Gamma\left(\frac{\Delta}{2}-2\right)-\widetilde{\Gamma}\left(\frac{\Delta}{2}-2;\beta_{st}\right)\right]$$

- Which yields the same $\mathcal{A} \sim s^{2-\Delta/2}$

Witten Sakai Sugimoto model

- For the WSS model we get the integral

$$\mathcal{A}_4(s,t,u) = \int_{U_\Lambda}^{\infty} du \sqrt{-g} e^{-2\phi} U^{-\Delta} \mathcal{A}_{10}(\tilde{s},\tilde{t},\tilde{u})$$

- Now the scaling is $\tilde{s} = sU^{-3/2}$
- For the fixed angle we find

$$\mathcal{A}_4^{\text{FA}} \sim s^{\frac{7}{3}-\frac{2}{3}\Delta}$$

$$\mathcal{A}_{\text{pseudo}}^{\text{FA}} \sim s^{-\frac{17}{3}}$$

$$\mathcal{A}_{\text{vector}}^{\text{FA}} \sim s^{-\frac{11}{3}}$$

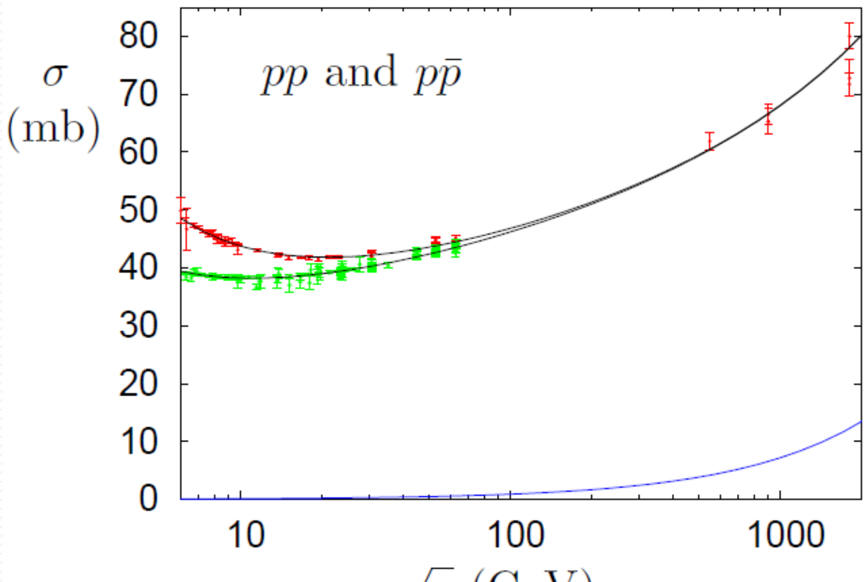
- From fluctuation of the embedding $\mathcal{A}_{\text{scalar}}^{\text{FA}} \sim s^{-\frac{29}{3}}$

Witten Sakai Sugimoto model

- All these result are in accordance with the partonic picture in general but the specific power s^{-2}. is not. We would expect to get for mesons with the total number of partons is 8

Proton proton stirngy total cross section

- We showed how stringy amplitude can yield a partonic one. This was needed to account for the deep inelastic scattering.
- On the other hand the total cross section of **pp scattering** , as determined by the Totem experiment from the imaginary part of the forward scattering amplitude, admits a stringy behavior that QCD has to account for



Proton proton stirngy total cross section

- The total cross section can parametrized as

$$\sigma_{tot}(s) \sim As^{-0.5} + Bs^{0.08}$$

- This has a simple interpretation as an exchange of open string (Reggeon) and a closed string (Pomeron).
- The string result in the Regge region takes the form

$$\sigma_{tot}(s) \sim As^{\alpha'_o t + a - 1} + Bs^{\alpha'_c t + a - 1}$$

- The intercept of a typical open string the rho meson a= 0.5 and for the closed string $a_c = 2\, a_o$

So for small it admits the experimental result

Open questions

- In a similar way to identifying a confining background using the stringy Wilson line one may wonder whether one can characterize backgrounds that ensure the transition from soft stringy to hard partonic scattering amplitudes

- In the present work we looked at backgrounds where the scaling of the Mandelstam variables is with a simple power in the radial variable. In some holographic models, e.g. Klebanov-Strassler, this is not the case, so one can extend our analysis to a more general case.

Open questions

- The various different models studied in this work yielded **different results** for the amplitudes in particular at **fixed angle.** Understanding these differences and evaluating the quality of the various results is still an open question.
- The prescription used in this paper, following the PS seminal paper of is a **hybrid formulation**. Deriving a procedure that will be fully stringy and not a hybrid one would be an important challenge.
- Perhaps the most interesting process that one can imagine getting from the stringy holographic picture is proton-proton scattering. For that purpose one has to use in the hybrid procedure the wave functions associated with the baryons which takes the form of **flavor instantons in the SS model**. The instanton is a soliton of the five dimensional theory on the flavor brane after integrating over the four sphere.

Step 10- Predictions

Predictions of the HISH model: Mesons

Trajectory	Quarks	$J^{P[C]}$	Mass	Width	$J^{P[C]}$	Mass	Width
π/b	$I=1$	5^{+-}	2480	240	6^{-+}	2700	270
η/h	$I=0$	5^{+-}	2470	260	6^{-+}	2690	290
ρ/a	$I=1$	7^{--}	2720	260	8^{++}	2920	280
ω/f	$I=0$	7^{--}	2710	320	8^{++}	2910	350
K	$s\bar{q}$	3^{+}	2050	220	4^{-}	2330	250
K^*	$s\bar{q}$	6^{+}	2620	230	7^{-}	2840	250
ϕ	$s\bar{s}$	4^{++}	2260	130	5^{--}	2520	150
D	$c\bar{q}$	3^{+}	3030	70	4^{-}	3270	90
D^*	$c\bar{q}$	4^{+}	3070	100	5^{-}	3310	120
D_s	$c\bar{s}$	2^{-}	2890	-	3^{+}	3160	-
D_s^*	$c\bar{s}$	4^{+}	3160	120	5^{-}	3400	140
Ψ	$c\bar{c}$	4^{++}	4020	90	5^{--}	4230	130
η_c	$c\bar{c}$	2^{-+}	3790	-	3^{+-}	4030	-
B	$b\bar{q}$	2^{-}	5980	-	3^{+}	6210	-
B^*	$b\bar{q}$	3^{-}	6000	-	4^{+}	6230	-
B_s	$b\bar{s}$	2^{-}	6080	-	3^{+}	6320	-
B_s^*	$b\bar{s}$	3^{-}	6100	-	4^{+}	6330	-
Υ	$b\bar{b}$	4^{++}	10420	Narrow	5^{--}	10630	-
η_b	$b\bar{b}$	2^{-+}	10180	Narrow	3^{+-}	10410	Narrow

Predictions of the HISH model: Mesons

- Predictions of ``radial" excited **mesonic** states

Traj.	Quarks	J^{PC}	n	Mass	Width	n	Mass	Width
π	$I=1$	0^{-+}	5	2610	300	6	2830	330
π_2	$I=1$	2^{-+}	3	2520	300	4	2740	350
a_1	$I=1$	1^{++}	2	1990	350	4	2520	390
h_1	$I=0$	1^{--}	4	2470	400	5	2700	450
ω	$I=0$	1^{--}	5	2560	360	6	2780	390
ω_3	$I=0$	3^{--}	3	2510	230	4	2740	250
ϕ	$s\bar{s}$	1^{--}	2	2000	100	4	2570	120
η_c	$c\bar{c}$	0^{-+}	2	4020	-	3	4330	-
Ψ	$c\bar{c}$	1^{--}	4	4620	110	5	4860	120
χ_{c1}	$c\bar{c}$	1^{++}	1	3920	-	2	4240	-
Υ	$b\bar{b}$	1^{--}	6	11310	90	7	11510	100
χ_{b1}	$b\bar{b}$	1^{++}	3	10800	-	4	11040	-

Predictions of the HISH model: Baryons

- Prediction of **higher J baryonic** states

Traj.	Quarks	J^P	Mass	Width	J^P	Mass	Width
N	qqq	$15/2^-$	2950	690	$17/2^+$	3050	580
Δ	qqq	$17/2^-$	3180	450	$19/2^+$	3160	490
Λ	qqs	$11/2^-$	2610	120	$13/2^+$	2810	140
Σ	qqs	$9/2^+$	2450	160	$11/2^-$	2660	180
Σ	qqs	$9/2^-$	2310	200	$11/2^+$	2530	230
Ξ	qss	$7/2^-$	2340	-	$9/2^+$	2570	-
Ω	sss	$5/2^-$	2070	-	$7/2^+$	2370	-
Λ_c	qqc	$7/2^-$	3140	-	$9/2^+$	3350	-
Σ_c	qqc	$3/2^-$	2760	-	$5/2^+$	3020	-
Σ_c	qqc	$5/2^-$	2820	-	$7/2^+$	3060	-
Ξ_c	qsc	$5/2^+$	3070	-	$7/2^-$	3300	-
Ω_c	$(ss)c$	$5/2^+$	3310	-	$7/2^-$	3540	-
Ω_c	$s(sc)$	$5/2^+$	3350	-	$7/2^-$	3590	-
Ω_c	$(ss)c$	$7/2^+$	3360	-	$9/2^-$	3580	-
Ω_c	$s(sc)$	$7/2^+$	3390	-	$9/2^-$	3620	-
Ξ_{cc}	$(qc)c$	$3/2^-$	3870	Narrow?	$5/2^+$	4090	-
Ξ_{cc}	$q(cc)$	$3/2^-$	4000	-	$5/2^+$	4270	-
Λ_b	qqb	$5/2^+$	6140	-	$7/2^-$	6340	-
Σ_b	qqb	$3/2^-$	6060	-	$5/2^+$	6260	-
Σ_b^*	qqb	$5/2^-$	6070	-	$7/2^+$	6280	-
Ξ_b	qsb	$3/2^-$	6060		$5/2^+$	6280	-

Predictions of the HISH model: Baryons

- Predictions of **higher ``radial" baryonic** states

Traj.	Quarks	J^P	n	Mass	n	Mass
N	qqq	$1/2^+$	4	2330	5	2560
N	qqq	$3/2^-$	3	2380	4	2610
N	qqq	$5/2^+$	2	2260	3	2490
N	qqq	$1/2^-$	2	2150	3	2400
N	qqq	$3/2^+$	2	2290	3	2520
N	qqq	$5/2^-$	2	2270	3	2510
Δ	qqq	$3/2^+$	3	2210	4	2450
Λ_b	qqb	$1/2^+$	1	6070	2	6420
Λ_b	qqb	$3/2^-$	1	6290	2	6600
Σ_b	qqb	$1/2^+$	1	6210	2	6530
Σ_b^*	qqb	$3/2^+$	1	6230	2	6540
Ξ_b	qsb	$1/2^+$	2	6560	3	6840
Ω_b	$(ss)b$	$1/2^+$	1	6470	2	6790
Ω_b	$s(sb)$	$1/2^+$	1	6520	2	6870

Open questions

- Determine a string model with **negative** values of **a-s**. This may require incorporating fermionic fluctuations of the string.
- Quantizing the string with **charges** on its endpoint that do not sum to zero.
- Quantizing the string with **spins** on its ends.
- Accounting for the **total and partial cross sections** of hadrons scattering like the pp collisions in LHC (Totem experiment).

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Additional transperacies:

HISH- Holography Inspired Stringy Hadron

- The construction of the HISH model is based on the following steps.
- (i) Analyzing classical string configurations in confining holographic string models that correspond to hadrons.
- (ii) Performing a transition from the holographic regime (for fields) of large Nc and large λ to the real world that bypasses expansions in $\frac{1}{N_c}$ and $\frac{1}{\lambda}$
- (iii) Proposing a model of **stringy hadrons** in **flat four dimensions with massive endpoint particles** that is **inspired** by the corresponding holographic model
- (iv)Dressing the endpoint particles with structure like **baryonic vertex**, **charge, spin** etc
- (v) Confronting the outcome of the models with **experimental data** .

Action and equations of motion

- The action describing a stringy hadron

$$S = S_{st} + (S_{pm} + S_{pq})\,|_{\sigma=0} + (S_{pm} + S_{pq})\,|_{\sigma=\ell}$$

- The string action

$$S_{st} = -T\int d\tau d\sigma\sqrt{-h} = -T\int d\tau d\sigma\sqrt{\dot{X}^2X'^2 - (\dot{X}\cdot X')^2}.$$

Where $\mu,\nu = 0,\ldots D-1.$ $-\infty < \tau < \infty$ and $0 \le \sigma \le \ell.$

- The endpoint actions

$$S_{pm} = m_i\int d\tau\sqrt{-\dot{X}^2} \qquad S_{pq} = Tq_i\int d\tau A_\mu(X)\dot{X}^\mu$$

Action and equations of motion

- One can consider the interaction between the charges by turning on $$S \to S - \frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}$$
- We consider here only the interaction with a background electromagnetic field.
- For the neutral case $q_1 = -q_2 = q$ S_{pq} can be written as a bulk action

$$S_{sq} = -\frac{T}{2}\int d\tau d\sigma \Big(qF_{\mu\nu}\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu) \Big)$$

- The bulk equation of motion

$$\partial_\alpha(\sqrt{-h}h^{\alpha\beta}\partial_\beta X^\mu) = 0 \qquad X''^\mu - \ddot{X}^\mu = 0$$

Action and equations of motion

- The boundary conditions read

$$T X'^{\mu} + m_1 \partial_\tau \frac{\dot{X}^{\mu}}{\sqrt{-\dot{X}^2}} + T q_1 F^{\mu}{}_{\nu} \dot{X}^{\nu} = 0 \qquad \sigma = 0$$

$$T X'^{\mu} - m_2 \partial_\tau \frac{\dot{X}^{\mu}}{\sqrt{-\dot{X}^2}} - T q_2 F^{\mu}{}_{\nu} \dot{X}^{\nu} = 0 \qquad \sigma = \ell$$

- For the neutral case and with no masses

$$\boxed{X'^{\mu} + q F^{\mu}{}_{\nu} \dot{X}^{\nu} = 0 \qquad \sigma = 0,\ \ell}$$

The scattering amplitude

- Now we are ready to compute the scattering amplitude of 2->2 strings with opposite charges in their ground state