Heavy quarks in Instanton Liquid Model with perturbative QCD corrections

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Outline

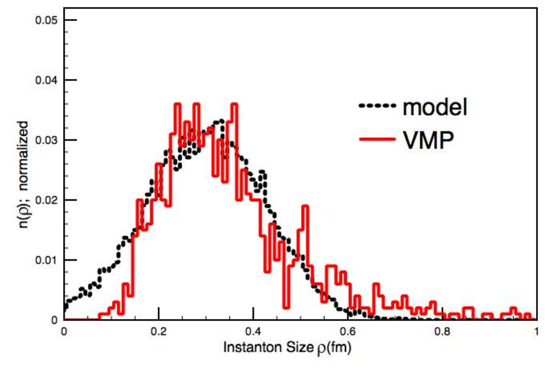
based on:
MM, N. Rakhimov, e-Print: 2111.07519 [hep-ph];
MM, U. Yakhshiev, Int.J.Mod.Phys.E30(2021)2141005;
MM, N. Rakhimov, U. Yakhshiev, Phys.Rev.D102(2020)076022;
MM, O. Egamberdiev, Phys.Lett.B 779 (2018) 206.

Instanton Liquid Model for QCD vacuum

- Instanton is tunneling path between the C-S states, instanton collective coordinates ξ:
  $4\ (position) + 1\ (size) + (4N_c - 5)\ (orientations)\ =\ 4N_c$;
- Instanton Liquid Model $\sim$ instanton sum ansatz $A = \sum_{\pm} A_{\pm}$, $N_+ = N_- = N/2$, $DA \to D\xi$ (Shuryak1981, Diakonov-Petrov1983);
- Two main parameters: average instanton size $\bar{\rho}$ and inter-instanton distance $R = (V/N)^{1/4}$;
- Estimates:
  - $R \approx 0.89\ fm$, $\bar{\rho} \approx 0.36\ fm$ – lattice;
  - $R \approx 1.00\ fm$, $\bar{\rho} \approx 0.33\ fm$ – phenomenological;
  - $R \approx 0.76\ fm$, $\bar{\rho} \approx 0.32\ fm$ – ILM $\sim$ ChPT.

Here $\sim$ 20% uncertainty. Packing parameter $\lambda = \bar{\rho}^4/R^4 \sim 0.01 - 0.03 \Rightarrow$ Independent averaging over instanton positions and orientations.
At the typical values of the parameters ILM vacuum energy density $\epsilon \approx -500\,\mathrm{MeV/fm^3}$.

Instanton size distribution



Instanton size $n(\rho)$ – lattice vs ILM (Millo,Faccioli2011).

Instantons vs heavy quarkonium sizes

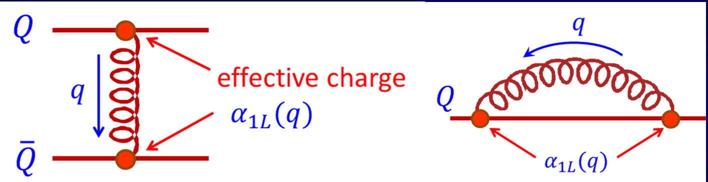
Quarkonium states and its sizes in non-relativistic potential model (see Satz2012).

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
mass [Gev]	3.07	3.53	3.68	9.46	9.99	10.02	10.26	10.36
size r [fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

We assume, for heavy quarkonium lowest states we may safely apply ILM.

Heavy quarkonium $Q\bar{Q}$ in perturbative QCD

Is the perturbative QCD able to describe heavy quarkonium?



Essential ingredient of such calculations are the QCD $Q\bar{Q}$ potential $V_{QCD}(r)$ and pole heavy quark mass m_{pole}.

Heavy quarkonium $Q\bar{Q}$ in perturbative QCD

In the colorless state and LL approximation (Sumino2014)

$$V_{QCD}(r) \approx -\frac{4}{3}\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\vec{r}} \frac{4\pi\alpha_s(q)}{q^2},$$

$$m_{pole} \approx m_{\overline{MS}}(\mu) + \frac{2}{3}\int_{q<\mu} \frac{d^3q}{(2\pi)^3} \frac{4\pi\alpha_s(q)}{q^2},$$

$$\alpha_s(q) \approx \alpha_{1L}(q) = \frac{2\pi}{\beta_0 \ln \frac{q}{\Lambda_{QCD}}} = \frac{\alpha_s(\mu)}{1 + \frac{\beta_0\alpha_s(\mu)}{2\pi}\ln\frac{q}{\mu}}$$

Both integrals here are ill-defined at $q < \mu$ but in the

$$E_{\rm pert}(r) = 2m_{pole} + V_{QCD}(r)$$

the dangerous parts are canceled.!! Perturbative QCD succeeded to describe bottomonium (and partly charmonium) spectra.

ILM nonperturbative contributions to the heavy quarkonium spectra

Non-perturbative contributions are still an open question and usual assumption, that they are of the order of 100 MeV. This type of estimations request a heavy quarkonium wave functions. On the other hand, including higher-order terms of perturbative QCD, $E_{\rm pert}(r)$ becomes steeper at larger r $(r\Lambda_{QCD} < 1) \sim$ a “Cornell” shape $\sim$ Cornell potential

$$V_{\rm C}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$$

Our aim to get nonperturbative contributions in ILM and estimate their effects in the $Q\bar{Q}$ spectra using "Cornell"-type WF.

Perturbative corrections in ILM

ILM partition function $Z[j]$ ($Z[0]=1$) with account of perturbative gluons a_μ and their sources j_μ is approximated by

$$Z[j]=\int D\xi Da e^{-[S_{eff}[a,A(\xi)]+(ja)]}\approx\int D\xi e^{-\frac{1}{2}(j_\mu S_{\mu\nu}(\xi)j_\nu)},$$

Here $\mathcal{O}(a^3,a^4)$ are neglected, $(ja)=\int d^4x j^a_\mu(x)a^a_\mu(x)$,
$(j_\mu S_{\mu\nu}(\xi)j_\nu)=\int d^4x d^4y j^a_\mu(x)S^{ab}_{\mu\nu}(x,y,\xi)j^b_\nu(y)$,
$S^{ab}_{\mu\nu}(x,y,\xi)$ – a gluon propagator in the ILM background $A(\xi)$.
To account perturbative and nonperturbative effects both $\Rightarrow$

double expansion series in terms of α_s and λ.

Scalar "gluons" in ILM.

The propagators $\Delta = (p^2 + \sum_i(\{p, A_i\} + A_i^2) + \sum_{i\neq j} A_i A_j)^{-1}$,
$\Delta_i = (p^2 + \{p, A_i\} + A_i^2)^{-1}, \;\; \Delta_0 = p^{-2}$.
Define: $\tilde{\Delta} = (p^2 + \sum_i(\{p, A_i\} + A_i^2))^{-1}$.
ILM propagators $< \Delta >= \int D\xi\, \Delta, \; < \tilde{\Delta} >= \int D\xi\, \tilde{\Delta}$.
Multi-scattering expansion

$$\tilde{\Delta} - p^{-2} = \sum_I (\Delta_I - p^{-2}) + \sum_{I\neq J} (\Delta_I - p^{-2}) p^2 (\Delta_J - p^{-2})$$

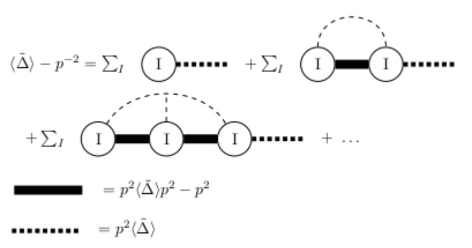
$$+ \sum_{I\neq J, J\neq K} (\Delta_I - p^{-2}) p^2 (\Delta_J - p^{-2}) p^2 (\Delta_K - p^{-2}) + ...$$

Averaging of this one – essential point here: instantons I, J, K are different or whether some of them coincide.

Main tool – the extension of Pobylitsa equation (Pobylitsa89).

The extension of Pobylitsa Eq. for scalar "gluons"

Follow Pobylitsa89, Diakonov,Petrov,Pobylitsa89. N_c counting – density of instantons $N/V \sim N_c$, averaging over instanton color orientation $\sim 1/N_c$.



A circle with I inside is $(\Delta_{\rm I} - p^{-2})$. Dashed lines connect the circles with the same instanton. At a large N_c the planar graphs dominate: neglect by crossed dashed lines. Summed-up planar graphs given as a skeleton expansion for the operator $\tilde\Delta - p^{-2}$, $\Rightarrow$ the extension of Pobylitsa Eq. for scalar "gluons" in ILM as

$$< \tilde\Delta >^{-1} - p^2 = \sum_i < \left[< \tilde\Delta > + (\Delta_i^{-1} - p^2)^{-1} \right]^{-1} > .$$

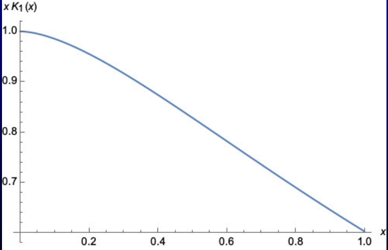
Scalar "gluon" dynamical mass in ILM.

The solution at $O(\lambda)$: $<\Delta>=<\tilde{\Delta}>$ and

$$M_s^2 =<\Delta>^{-1} - p^2 = N\, p^2(<\Delta_{\rm I}> - p^{-2})p^2,$$

Well-known result for the $\Delta_{\rm I}$ (Brown et al 1978) $\Rightarrow$

$$M_s(q) = \left[\frac{12\pi^2\lambda}{(N_c^2-1)}\right]^{1/2} \rho^{-1} F(q), \;\; F(q) = q\rho K_1(q\rho)$$



At $\rho = 0.33\,\mathrm{fm}$, $R = 1\,\mathrm{fm}$ $M_s(0) = 256\,MeV \sim \lambda^{1/2}\rho^{-1}$ *is a strength of scalar "gluon"-instanton interaction.*

Gluons in ILM. Zero-modes problem.

Gluons effective action in single instanton field is $(a_\mu M^{\rm I}_{\mu\nu} a_\nu) + O(a^3, a^4)$, $4N_c$ zero-modes $M^{\rm I}_{\mu\nu}\phi^i_\nu = 0$
The single instanton gluon propagator $S^{\rm I}_{\mu\nu}$ is
$M^{\rm I}_{\mu\nu} S^{\rm I}_{\nu\rho} = \delta_{\mu\nu} - P^{\rm I}_{\mu\nu}$, $P^{\rm I}_{\mu\nu}$ – zero-modes projection operator.
The explicit solution $S^{\rm I}_{\mu\nu}$ was given by (Brown et al 1978).
Zero-modes problem was solved by (Brown1978) introducing artificial gluon mass m_g.
Repeating the way to Pobylitsa Eq. for ILM "scalar" gluon propagator $<\Delta>$ we have at $m_g \to 0$ limit

$$M_g^2 \delta_{\rho\nu} = N {S^0}^{-1}_{\rho\sigma} (< S^{\rm I}_{\sigma\mu} > - {S^0}_{\sigma\mu}) {S^0}^{-1}_{\mu\nu}$$

Dynamical gluon mass in ILM.

Compare $S^{\mathrm{I}}_{\sigma\mu}$ and $\Delta^{\mathrm{I}} \Rightarrow$

$$M_g^2(q) = 2M_s^2(q).$$

At $\rho = 0.33\,\mathrm{fm}, R = 1\,\mathrm{fm}$
$M_g(0) = 362\,MeV \approx M(0) \sim \lambda^{1/2}\rho^{-1}$

M_g – strength of gluon-instanton interaction
$\approx M$ – strength of light quark-instanton interaction.

Essential modification of $Q\bar{Q}$ one-gluon exchange potential at the distances $r \sim M_g^{-1} \sim 0.5\,fm$.

Heavy quarks in ILM

Heavy quark Q and antiquark $\bar{Q}$ Lagrangians are

$$L_Q = Q^+(\theta^{-1} - ga - gA + ...)Q, \; L_{\bar{Q}} = \bar{Q}^+(\theta^{-1} - g\bar{a} - g\bar{A} + ...)\bar{Q},$$

Here $< t_1|\theta|t_2 >= \theta(t_1 - t_2)$, $a_4 \equiv a = a_a\lambda_a/2$,
$\bar{a}_4 \equiv \bar{a} = a_a\bar{\lambda}_a/2$, $\bar{\lambda}_a = -\lambda_a^{\mathrm{T}}$. The same for A fields.
Now ILM heavy quark propagator

$$w = \int D\xi \left[\int \left(\theta^{-1} - g\frac{\delta}{\delta j} - g\sum_i A_i \right)^{-1} \exp\left\{ \frac{1}{2}(jS(\xi)j) \right\} \right]_{j=0}.$$

Here $j \equiv j_4$, $S \equiv S_{44}$. It is easy to prove that

$$\left[\frac{1}{\theta^{-1} - g\frac{\delta}{\delta j} - gA(\xi)} \exp\left(\frac{1}{2} jS(\xi)j \right) \right]_{j=0}$$
$$= \left[\exp\left(\frac{1}{2}\frac{\delta}{\delta a_a} S_{ab}(\xi) \frac{\delta}{\delta a_b} \right) \frac{1}{\theta^{-1} - ga - gA(\xi)} \right]_{a=0}$$

This equation can be extended to any heavy quark correlator.

Heavy quark Q propagator in ILM with perturbative corrections

is:

$$w = \int D\xi \left[\theta^{-1} - \sum_i \left(gA_i(\xi_i) - g^2\left(\Delta S^i(\xi_i)\theta\right)\right)\right]^{-1},$$

where $\Delta S^i(\xi_i) = S^i(\xi_i) - S^0$ is single instanton contribution to Q propagator and last term is $\mathcal{O}(\alpha_s\lambda^{1/2})$ ILM perturbative mass operator.

w and its $g \to 0$ limit are similar on their ξ_i dependencies and we may easily extend Pobylitsa equations (DPP1989). The solution in $\mathcal{O}(\lambda, \alpha_s\lambda^{1/2})$ approximation is

$$w^{-1} = \theta^{-1} - \sum_i \int d\xi_i \theta^{-1}\left[(\theta^{-1} - gA_i(\xi_i))^{-1} - \theta\right]\theta^{-1}$$

$$-g^2\left[(\bar{S} - S^0)\theta\right].$$

Second term $\to \mathcal{O}(\lambda)\ \Delta M_Q^{\rm dir}$, third one $\to \mathcal{O}(\alpha_s\lambda^{1/2})\ \Delta M_Q^{\rm pert}$.

$O(\lambda,\, \lambda^{1/2}\alpha_s)$ ILM contribution to the heavy quark mass.

Direct instanton ILM contribution to the heavy quark mass

$$\Delta M_Q^{\rm dir} = 16\pi i_0(0)\lambda\rho^{-1}/N_c, \quad i_0(0) = 0.55.$$

At $\rho = 0.33\,{\rm fm}, R = 1\,{\rm fm}$:

$\Delta M_Q^{\rm dir} \approx 70\,{\rm MeV} \sim \lambda\rho^{-1} \sim$ *strength of a heavy quark-instanton interaction!!*

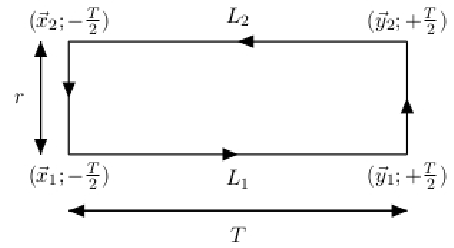
perturbative ILM contribution to the heavy quark mass:

$$-\Delta M_Q^{\rm pert} \approx \frac{2}{3}\,\alpha_s M_g(0) \sim \alpha_s \lambda^{1/2}\rho^{-1},$$

at $\alpha_s = 0.3$: $\Delta M_Q^{\rm pert} \approx -70\,{\rm MeV}$

$\Delta M_Q^{\rm dir} + \Delta M_Q^{\rm pert} \approx 0 \sim$ Perturbative gluon-instanton interaction $\Rightarrow$ cancellation of total ILM contributions to heavy quark mass.

$Q\bar{Q}$ correlator in ILM with perturbative gluons



$$W = \int D\xi \left[\exp \frac{1}{2} \sum_{i,j=1}^{2} \left(\frac{\delta}{\delta a_a^{(i)}} S_{ab}^{(ij)}(\xi) \frac{\delta}{\delta a_b^{(j)}} \right) \frac{1}{D_i^{(1)} - g a^{(1)}} \frac{1}{\bar{D}_i^{(2)} - g \bar{a}^{(2)}} \right] \Bigg|_{a=0},$$

where $D_i^{(1)} = \theta^{-1} - g A_i^{(1)}(\xi_i)$, $\bar{D}_i^{(2)} = \theta^{-1} - g \bar{A}_i^{(2)}(\xi_i)$, $a^{(1)}, A^{(1)}$ $(\bar{a}^{(2)}, \bar{A}^{(2)})$ are fields' projections to the line L_1 (L_2).

$Q\bar{Q}$ potential $V_{\rm ILM}(r)$

Solution of Pobylitsa's equation extension in the $\mathcal{O}(\lambda, \alpha_s\lambda^{1/2})$ approximation is

$$W^{-1} = w^{(1)-1}\bar{w}^{(2)-1} - \sum_i \int d\xi_i$$

$$\times\theta^{(1)-1}\left(\frac{1}{D_i^{(1)}} - \theta^{(1)}\right)\theta^{(1)-1}\theta^{(2)-1}\left(\frac{1}{\bar{D}_i^{(2)}} - \theta^{(2)}\right)\theta^{(2)-1}$$

$$-g^2\frac{\lambda_a}{2}\frac{\bar{\lambda}_b}{2}\int D\xi\, S_{ab}^{(12)},$$

right side first and second terms lead to direct instanton potential $V_{\rm dir}(r)$ derived at DPP1989, while third term gives ILM modified one-gluon exchange potential $V_{\rm pert}(r)$.

$$V_{\rm ILM}(r) = V_{\rm dir}(r) + V_{\rm pert}(r)$$

Direct instanton ($Q\bar{Q}$) potential in ILM

at colorless state is a smooth positive function

$$V_{\rm dir}(r) = \frac{N}{2VN_c}\sum_{\pm}\int d^3z_\pm \mathrm{tr}_c\,[1-$$

$$P\exp\left(i\int_{L_1} dtA_{\pm,4}\right)P\exp\left(-i\int_{L_2} dtA_{\pm,4}\right)\Big],$$

$$V_{\rm dir}(0) = 0 \text{ and } V_{\rm dir}(r\to\infty)\to 2\Delta M_Q^{\rm dir}.$$

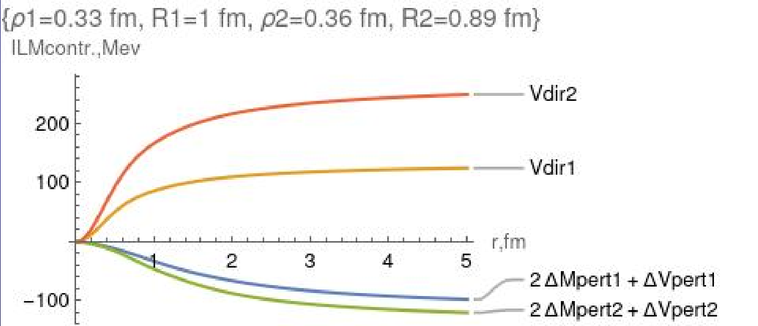
At $r << \rho$

$$V_{\rm dir}(r) \approx \frac{4\pi\rho^3}{R^4N_c}1.345\left(\frac{r^2}{\rho^2} - 0.372\frac{r^4}{\rho^4}\right).$$

Average size of charmonium $r_c \sim \rho$, while for bottomonium $r_b < \rho$.

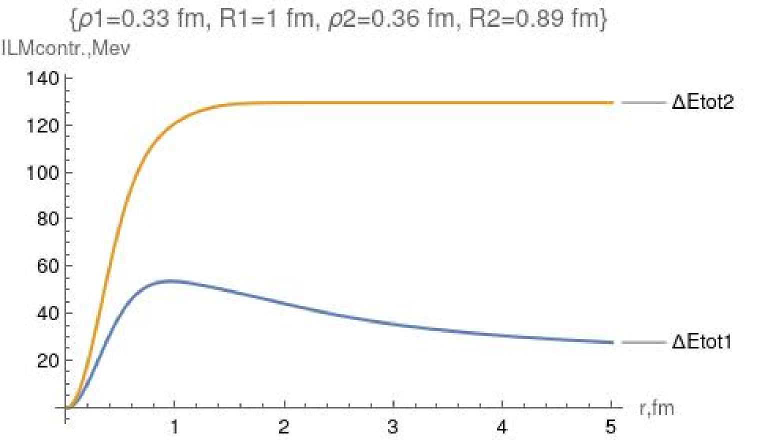
Perturbative one-gluon exchange $(Q\bar{Q})$ potential in ILM

$$V_{\rm pert}(r) = -\frac{4}{3}\,4\pi\alpha_s \int \frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2+M_g^2(q)}$$



Direct instanton $V_{dir}(r)$ and perturbative ILM $2\Delta M_{pert} + \Delta V_{pert}(r)$ contributions at $\alpha_s = 0.4$ and different sets of ILM parameters: $\rho_1 = 0.33$ fm, $R_1 = 1$ fm and $\rho_2 = 0.36$ fm, $R_2 = 0.89$ fm.

Total ILM contribution $O(\lambda, \lambda^{1/2}\alpha_s)$



Total ILM contribution
$\Delta E(r) = V_{dir}(r) + 2\Delta M_{pert} + \Delta V_{pert}(r)$ at $\alpha_s = 0.4$ and at different sets of ILM parameters: $\rho_1 = 0.33$ fm, $R_1 = 1$ fm and $\rho_2 = 0.36$ fm, $R_2 = 0.89$ fm.

Charmonium states

Total potential= "Cornell" central potential + spin one

$$V_{\rm C}(r) = -(4/3)\frac{\alpha_s}{r} + \kappa r,$$

$$V_{\rm SS}(r) = \frac{2}{3m_Q^2}\vec{\nabla}^2 V_{\rm C}(r) \to \frac{32\alpha_s\sigma^3}{9m_Q^2 r^2}\exp(-\sigma^2 r^2)$$

$$E(n^{2S+1}L_J) = 2m_Q + < V_{\rm C} > + < V_{\rm SS} >$$

$< V_{\rm SS} >= -3/4v$ for $S = 0$ and $< V_{\rm SS} >= 1/4v$ for $S = 1$.
v - radial matrix element of $V_{\rm SS}$. Fitting of $m_Q, \alpha_s, \kappa, \sigma$:
lowest $n = 1$ states $J/\psi\{1^3S_1(3096.9)\}$ and $\eta_c\{1^1S_0(2983.9)\}$
next $n = 2$ states $\psi'\{2^3S_1(3686.1)\}$ and $\eta_c'\{2^1S_0(3637.6)\} \Rightarrow$
$m_Q = 1371$ MeV, $\kappa = 0.173$ GeV^2, $\alpha_s = 0.4$, $\sigma = 1.2$ GeV.

ILM contribution to the charmonium states

$O(\lambda, \lambda^{1/2}\alpha_s)$ ILM contribution $< \Delta E >$ to charmonium S-wave n-states in MeV.

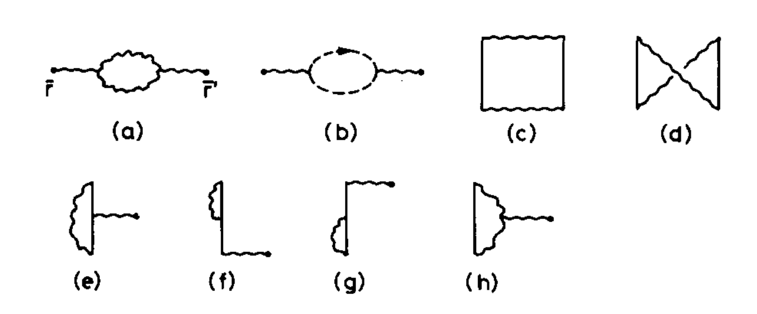
ILM parameters	$n = 1$	$n = 2$	$n = 3$	$n = 4$
$\rho = 0.33$ fm, $R = 1.00$ fm	28.4	44.7	45.4	44.3
$\rho = 0.36$ fm, $R = 0.89$ fm	56.2	98.4	109.5	114.2

$m_Q = 1371$ MeV, $\kappa = 0.173$ GeV^2, $\alpha_s = 0.4$, $\sigma = 1.2$ GeV

$O(\lambda^{1/2}\alpha_s^2)$ ILM contributions

On the ILM scale $\rho = 0.33$ fm $\alpha_s \sim 0.4$, while $\lambda \sim 0.01$. Numerically $\lambda \sim \alpha_s^4$. We have to take also terms $O(\lambda^{1/2}\alpha_s^2, \alpha_s^4)$.

$O(\alpha_s^2)$ calculations of $Q\bar{Q}$ (Fischler1977):



(h) vanishes in the Feynman gauge; No IR divergences in $(c + d + e + f + g)$; $(a), (b) \sim$ gluon self-energy.

Two scalar "gluons" in ILM

Our aim is to average the product of the propagators over instantons collective coordinates

$$\overline{\Delta(\xi)\Delta(\xi)} \equiv \int D\xi\, \Delta(\xi)\Delta(\xi).$$

In lowest order on the density we have

$$\overline{\Delta\Delta} = \Delta_0\Delta_0 + N\left(\{\Delta_0, \overline{(\Delta_I - \Delta_0)}\} + \overline{(\Delta_I - \Delta_0)^2}\right)$$

We know

$$\bar{\Delta} = \Delta_0 + N\overline{(\Delta_I - \Delta_0)} = (p^2 + M_s^2(p))^{-1}.$$

$\bar{\Delta} \Rightarrow \Delta M_{\rm pert} \sim O(\lambda^{1/2}\alpha_s)$.
We expect $N\{\Delta_0, \overline{(\Delta_I - \Delta_0)}\} \Rightarrow O(\lambda^{1/2}\alpha_s^2)$ contributions.

Instanton generated scalar "gluon"-"gluon" interaction

$N\overline{(\Delta_I-\Delta_0)^2}$ contribution to the two-gluon exchange diagrams.

$\int D\xi \Rightarrow$ scalar "gluon"-"gluon" interaction. In the momentum space we have

$$N\overline{(\Delta_I^{a_1a_2}(p_1,p_2)-\Delta_0^{a_1a_2}(p_1,p_2))(\Delta_I^{a_3a_4}(p_3,p_4)-\Delta_0^{a_3a_4}(p_3,p_4)}$$
$$=\delta_{a_1a_2}\delta_{a_3a_4}\left(\frac{3}{4\pi^2(N_c^2-1)}\right)^2\lambda\,(2\pi)^4\delta^4(p_1+p_2+p_3+p_4)$$
$$\times f_1(p_1)f_1(p_2)f_1(p_3)f_1(p_4),\quad f_1(p)=\frac{F(p)}{p^2},$$

Here four-legs "gluon"-"gluon" $O(\lambda)$ interaction vertex. Each leg/free propagator accompanied by the form-factor $F(p)$. So, in general, it leads to $O(\lambda\alpha_s^2)$ contributions in the heavy quarks correlators, which might be neglected within our accuracy.

Conclusion

- ILM effects are controlled by $\lambda = \rho^4/R^4 \sim 0.01 - 0.03$. Dynamical light quark and gluon masses $M(p)$ and $M_g(p)$ $\sim O(\lambda^{1/2}) \sim 400$ MeV, while $\Delta M_{\rm dir} \sim O(\lambda) \sim 70$ MeV.
- Perturbative QCD effects in ILM $\Rightarrow$ double expansion on $\alpha_s(\rho) \sim 0.4$ and λ. We assume $\alpha_s^4 \sim \lambda$.
- We found the mutual cancellation of the $O(\lambda)$ and $O(\alpha_s\lambda^{1/2})$ contributions in a quarkonium spectra. As an example, we calculated $< \Delta E >$ $(\sim O(\lambda) + O(\alpha_s\lambda^{1/2}))$ to the charmonium spectra and found for the ground state $< \Delta E >= 28.4$ MeV at $\rho = 1/3$ fm, $R = 1$ fm, while $< \Delta E >= 56.2$ MeV at $\rho = 0.36$ fm, $R = 0.89$ fm.
- We assume that the mentioned cancellation request th calculations of $O(\alpha_s^2\lambda^{1/2})$.
- The account of ILM effects in addition to the perturbative QCD calculations may improve QCD predictions for heavy quarks.