Emergent strongly coupled fixed point and symmetric mass generation in 4 dimensions

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Gauge Topology Workshop
Trento
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arXiv:2204.04801

The conformal window

Take $SU(N_c)$ color with N_f fundamental flavors

$$\beta = \mu^2 \frac{dg^2}{d\mu^2} = b_0 g^4 + b_1 g^6 + \dots$$

The coefficients of $\beta(g^2)$ are known perturbatively up to 5 loops

$$b_0 = \frac{1}{16\pi^2}\left(-\frac{11}{3}N_c + \frac{2}{3}N_f\right), \qquad b_1 = \frac{1}{(16\pi^2)}\left(-\frac{34}{3}N_c^2 + N_f\left(\frac{10}{3}N_c + \frac{N_c^2-1}{N_c}\right)\right)$$

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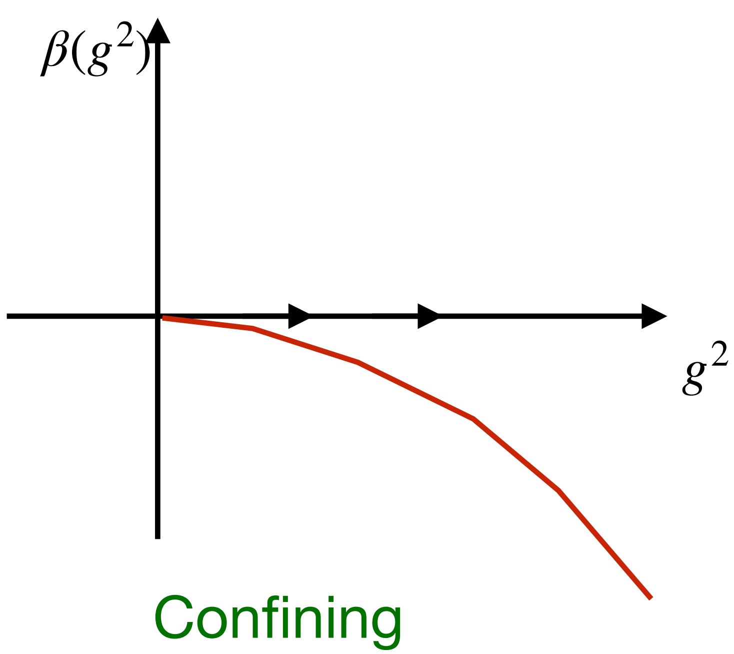
small N_f



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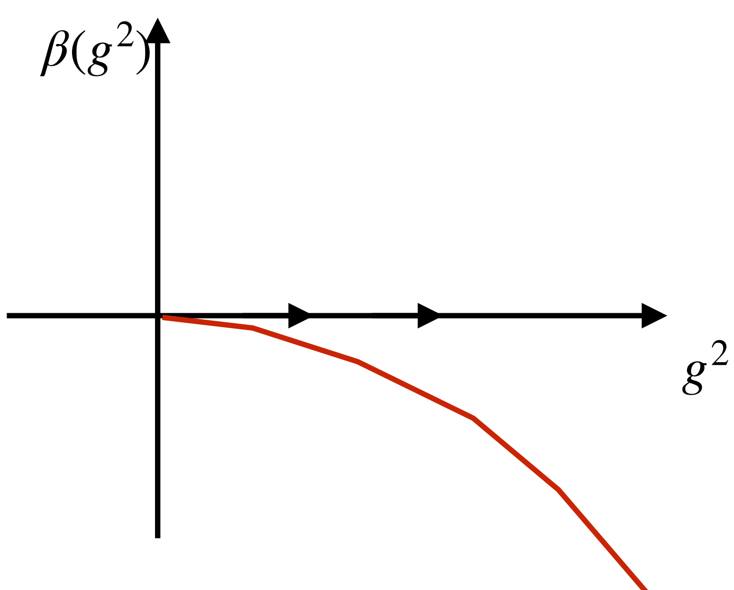
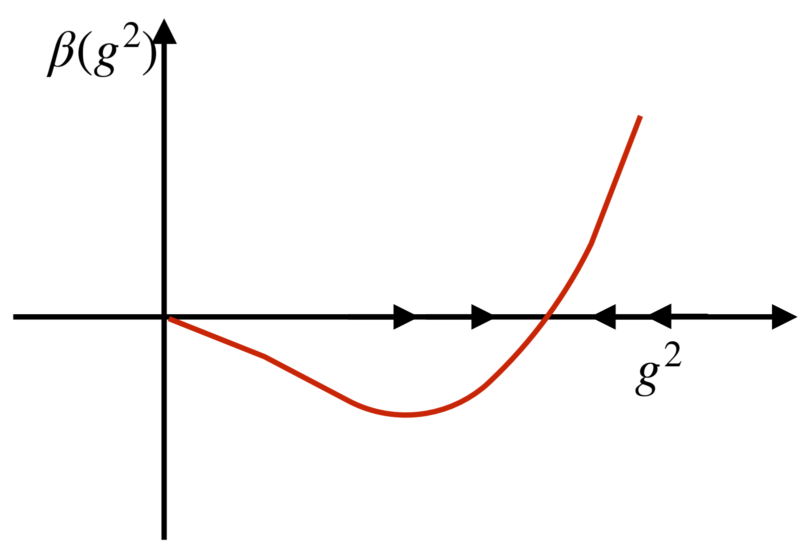


Confining

$N^* < N_f < N^{IF}$



Conformal

Perturbatively the IR fixed point emerges at $g^2 = \infty$ and moves in as $N_f \to N^{IF}$

The conformal window

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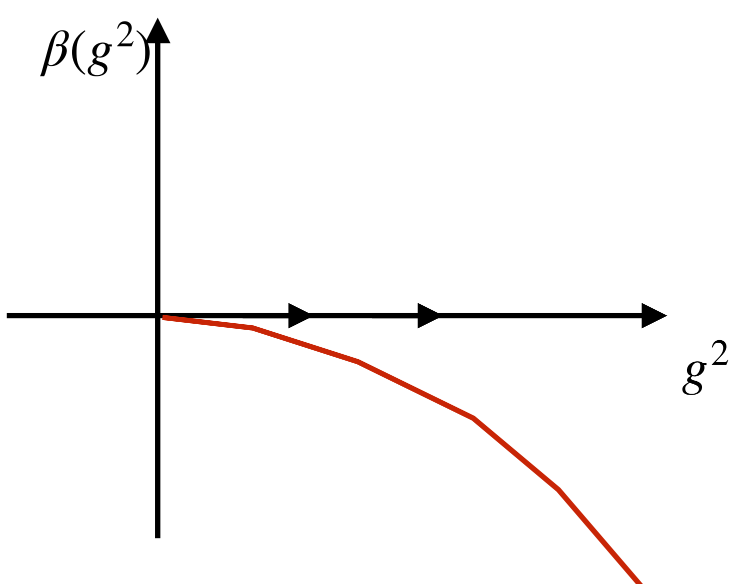

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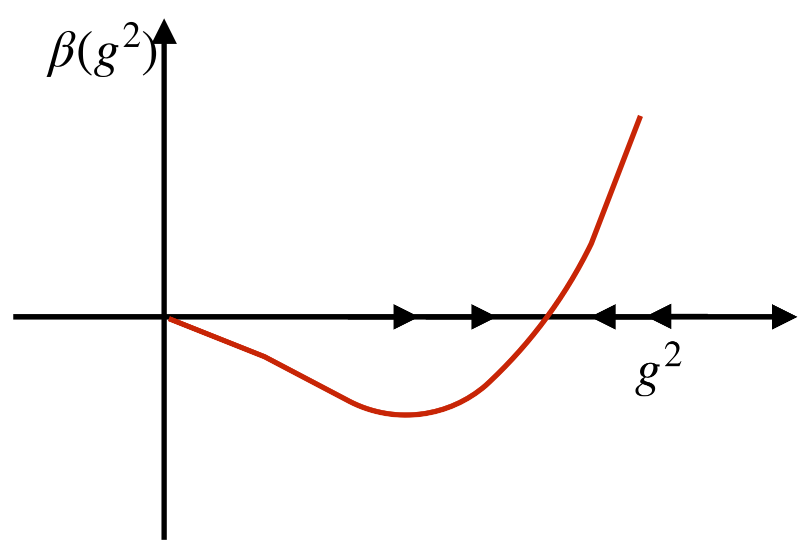
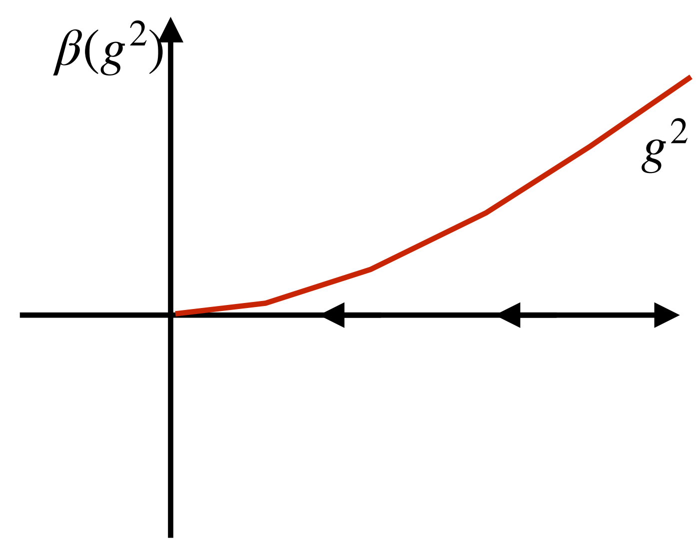


Conformal

$N_f > N^{IF}$



Infrared free

Perturbatively the IR fixed point emerges at $g^2 = \infty$ and moves in as $N_f \to N^{IF}$

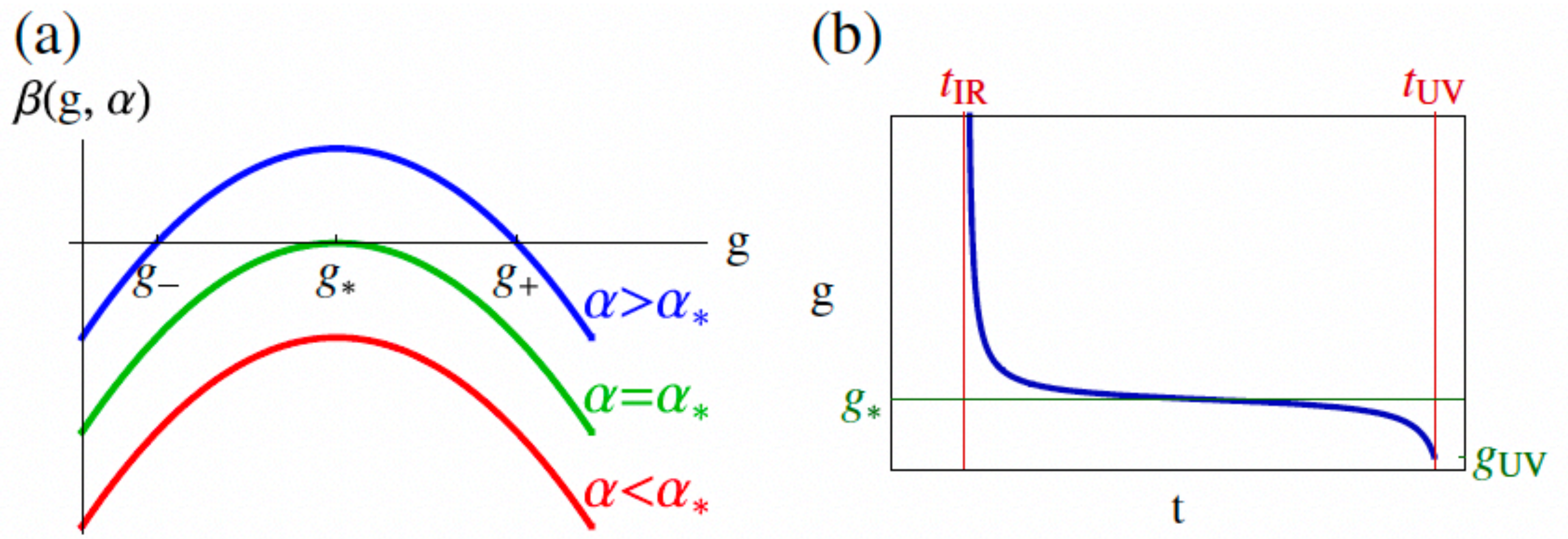
Opening the conformal window

Conformality could emerge at finite g_*^2 coupling

$$\beta(g;\alpha) = (\alpha - \alpha_*) - (g - g_*)^2 + \ldots$$

Conformality lost at $\alpha = \alpha_*$:

Kaplan et al PRD80,125005 (2009)
L. Vecchi PRD82, 045013 (2010)
Gorbenko et al JHEP10, 108 (2018)



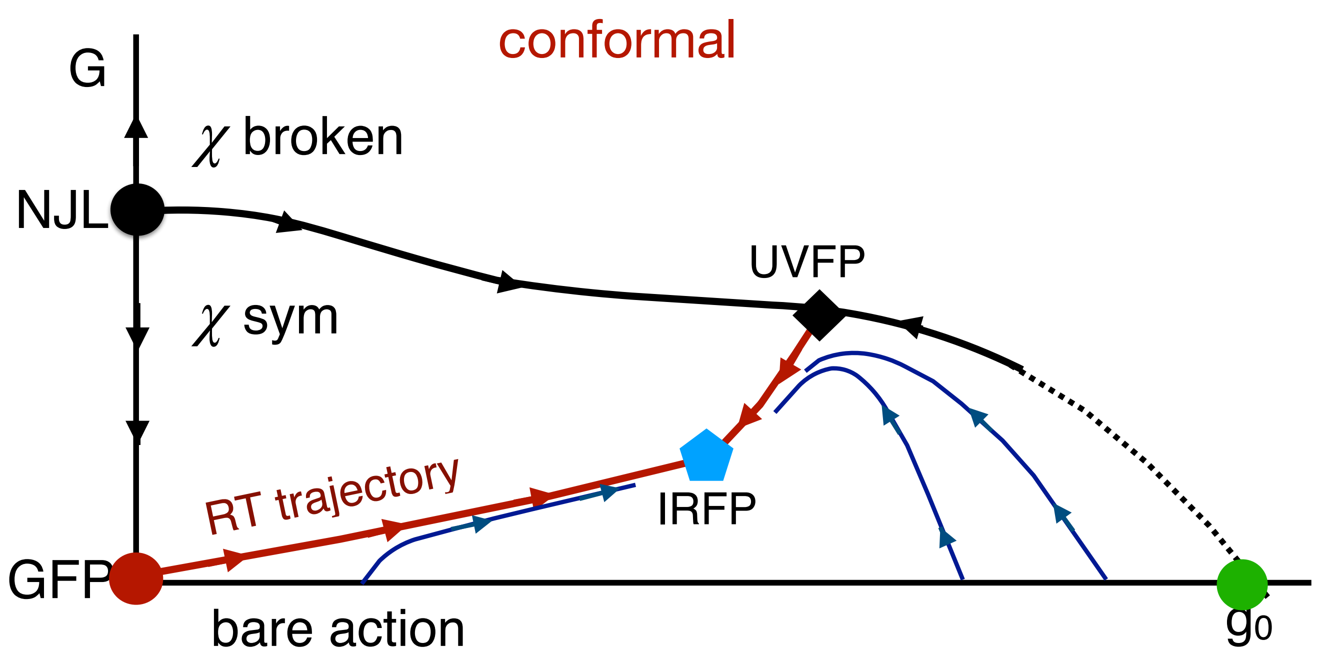
Kaplan et al PRD80,125005 (2009)

- conformity is lost due to fixed point merger
- there is a new relevant operator at g_+, g_*

Possible numerical signals:

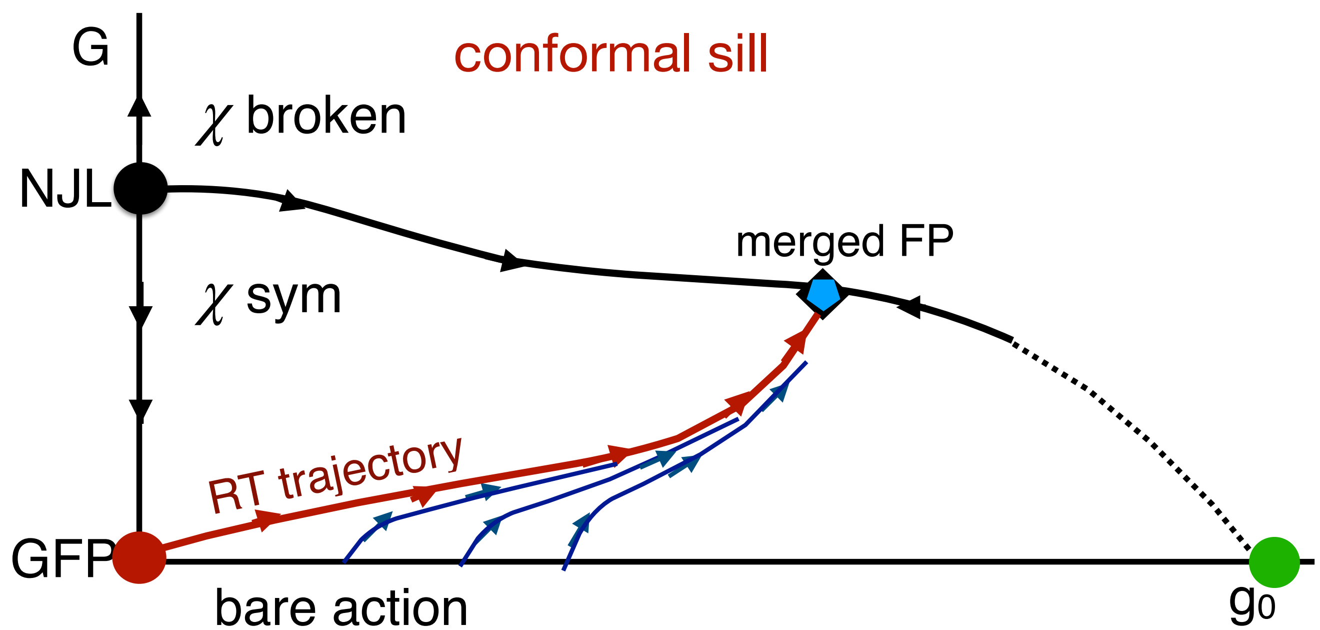
- Continuous phase transition at g_+, g_*
- BKT scaling at g_*, 2nd order scaling at g_+

Phase diagram in extended parameter space



- What is the new relevant operator?
- what is the chirally broken phase?

- Is ● a continuous phase transition?
- Can we define a continuum theory on the strong coupling side?



At the moment of FP merger, we have a BKT "walking scaling" phase transition

- What kind of scaling is in the weak/strong coupling side?
- What is the scaling in mass?

~~Outline~~ Summary of this talk

A.H. 2204.04801

I study SU(3) with $N_s = 2$ staggered flavors

- Staggered fermions are Dirac-Kaehler fermions
  - equivalent to $N_f = 8$ Dirac flavors at the GF, could be different at $g^2 \neq 0$
- In the chiral limit $N_s = 2$ is t'Hooft anomaly free, allowing symmetric mass generation (SMG) Catterall et al *Phys.Rev.D* 104 (2021)

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Prior studies of $N_f = 8$ showed strong first order bulk phase transitions

- The bulk transition appears to be a lattice artifact
- Adding heavy Pauli-Villars regulator bosons reduces cutoff effects, the bulk transition weakens and eventually disappears

AH, Shamir, Svetitsky, PRD104, 074509 (2021)

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Two distinct phases are observed: conformal and SMG

- Test the order of the phase transition using finite size scaling
  - I use the GF coupling here -this is a new application
  - The phase transition is consistent with BKT scaling, inconsistent with 1st or 2nd order phase transition
- The strong coupling phase is gapped but symmetric: SMG
  - is it a topological phase driven by instant condensation?

Staggered fermions

are Kaehler-Dirac fermions distributed in a 2^4 hypercube

Becher, Joos 1982

$$S = \frac{1}{2}\sum_{n,\mu}(\bar{\chi}_n \alpha_\mu(n) U_\mu(n) \chi_{n+\mu} + cc) + m \sum_n \bar{\chi}_n \chi_n\,, \qquad \alpha_\mu(n) = (-1)^{n_0 + \ldots n_{\mu-1}}$$

χ : 1-component fermion

1 set of staggered fermions ≡ 4 Dirac flavors in flat space, $g_0^2 = 0$

2 sets of massless staggered fermions ≡ 4 sets of reduced staggered

≡ 16 Weyl fermions

Catterall et al 2101.01026

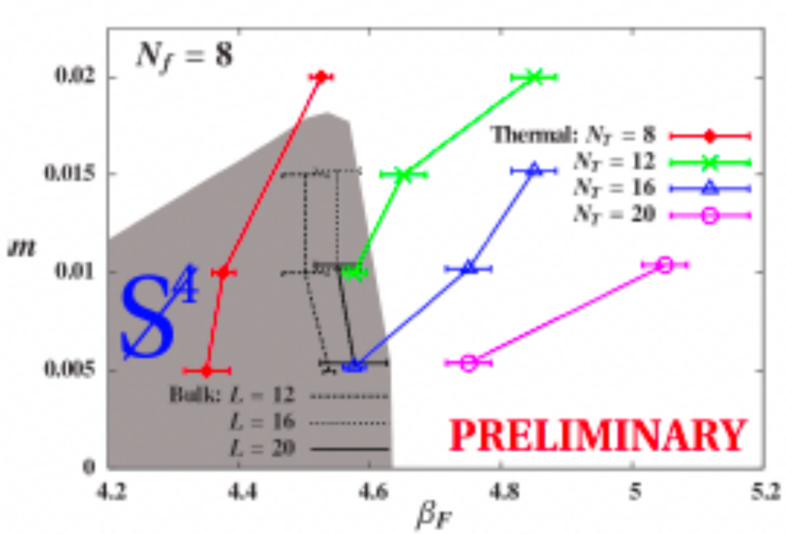
Massless staggered fermions suffer from Z_4 gauge anomaly - cancelled when 2 staggered species are present

—> 2 staggered species could exhibit symmetric mass generation : mass without spontaneous symmetry breaking

What do we know about $N_f = 8$?

Very close to the conformal sill. But no direct evidence if it is below

- Spectrum is well described by dilation ChPT : close to the sill
- All prior studies found a 1st order bulk transition preventing strong couplings
- No finite T phase transition at $am = 0$ on $N_\tau \leq 24$



1506.08791
AH,Schaich, Rinaldi

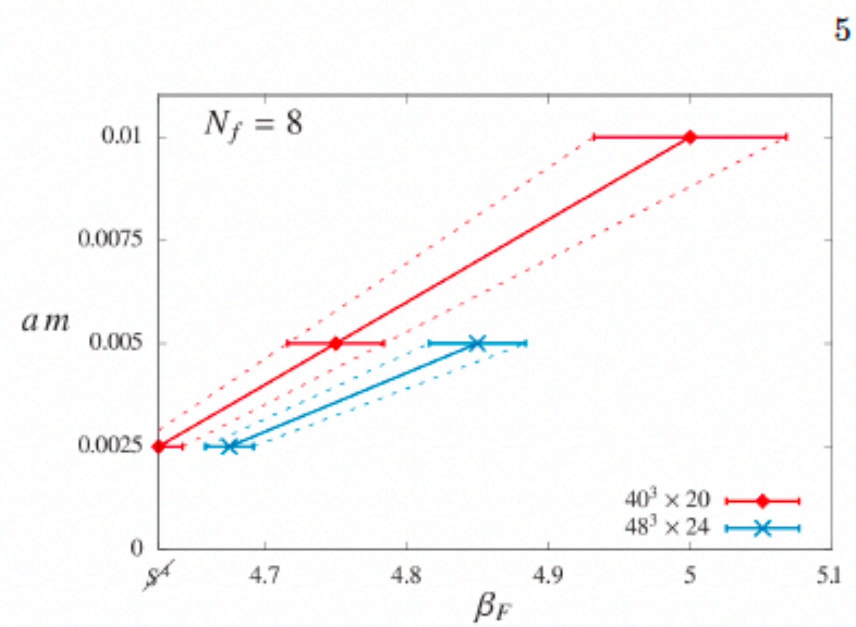


FIG. 4. Finite-temperature transitions from lattices with temporal extents $N_t = 20$ and 24, with lines connecting points to guide the eye. The region above these lines is confined and chirally broken, while the region below is deconfined and chirally symmetric. The left edge of the plot indicates the bulk transition into the $\cancel{S}^4$ lattice phase. The finite-temperature transitions merge with this bulk transition at $am > 0$, preventing a direct confirmation of spontaneous chiral symmetry breaking.

LSD PRD99,014509

PV boson improved actions

Fermions induce an effective gauge interaction

- bare gauge coupling forced to strong coupling
- UV fluctuation are large

If the 1st order bulk transition is due to UV fluctuations, an improved action could open up the parameter space:

PV boson improved actions

Fermions induce an effective gauge interaction

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If the 1st order bulk transition is due to UV fluctuations, an improved action could open up the parameter space:

Add a set of Pauli-Villars bosons with heavy mass to the action:

- heavy bosons integrate out, do not influence the IR dynamics
- the PV bosons induce an effective gauge action, countering the fermions.
- At leading order just a (smeared) plaquette

$$\beta_{ind}^{(p)} = -\frac{N_s N_{PV}}{(2am_{PV})^4}$$

- the bare gauge coupling increases to compensate for β_{ind}
- the PV action has smaller UV fluctuations

AH, Y. Shamir, B. Svetitsky PRD104, 074509 (2021) $(N_f = 12)$

Numerical detail

- SU(3) gauge with plaquette and adjoint plaquette gauge action
- nHYP smeared fermions
- PV fields :
    - 0PV : no PV bosons : $\beta_c \approx 4.6$
    - 8PV-m0.75 : 8 PV per staggered flavors, $am_{PV} = 0.75$: $\beta_c \approx 8.8$
    - 4PV-m0.5 : 4 PV per staggered flavors, $am_{PV} = 0.5$: $\beta_c \approx 8.1$

The fermions are in the chiral limit $am_f = 0$

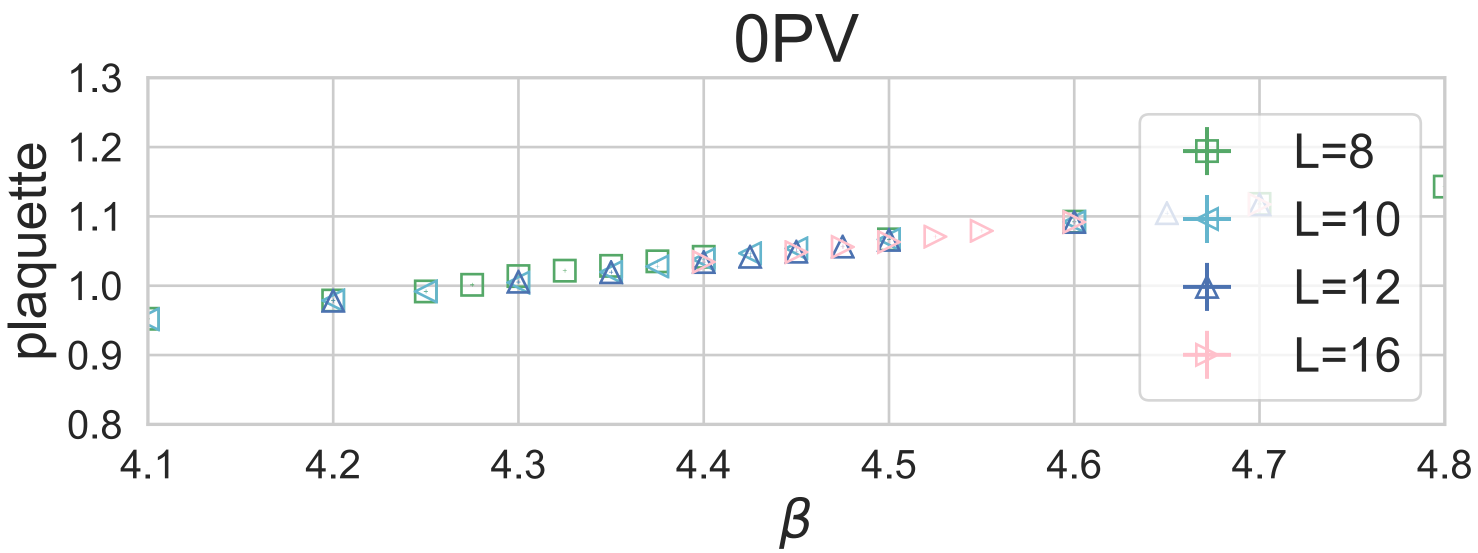
Simulations are still OK as neither phases are chirally broken

- Gradient flow observable:

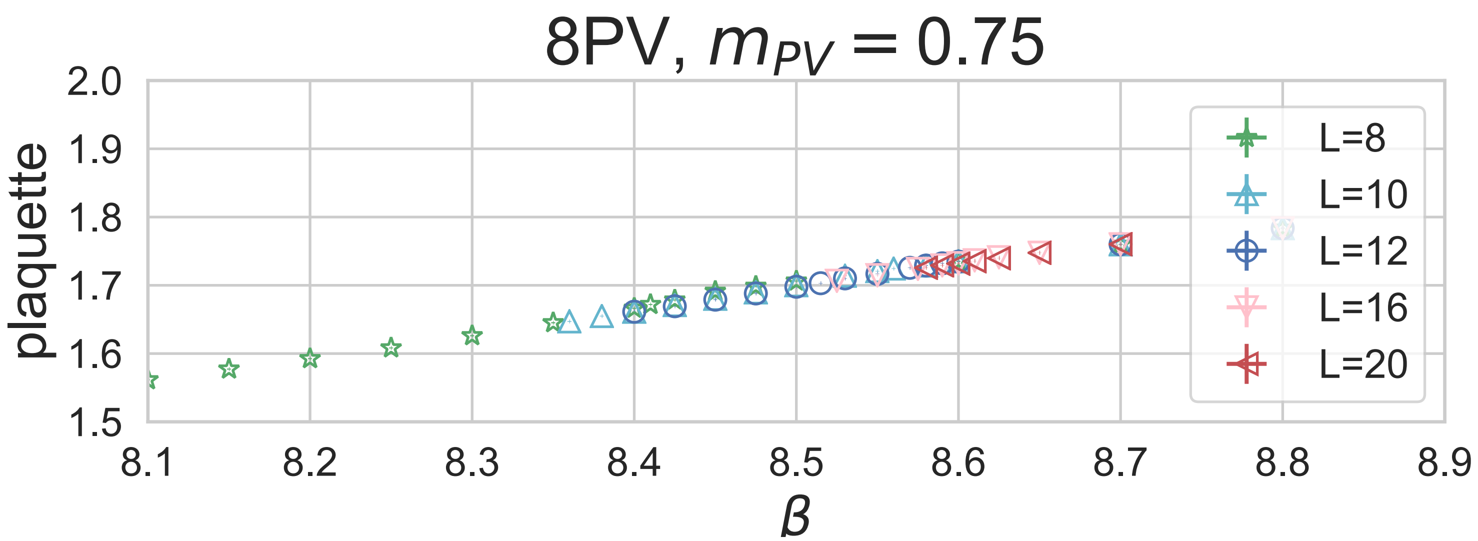
$$g^2_{GF}(\beta, L; t) = \mathcal{N} t^2 \langle E(t) \rangle_{\beta,L}$$

g^2_{GF} is dimensionless (both canonical and anomalous); It measures the RG flow along the renormalized trajectory

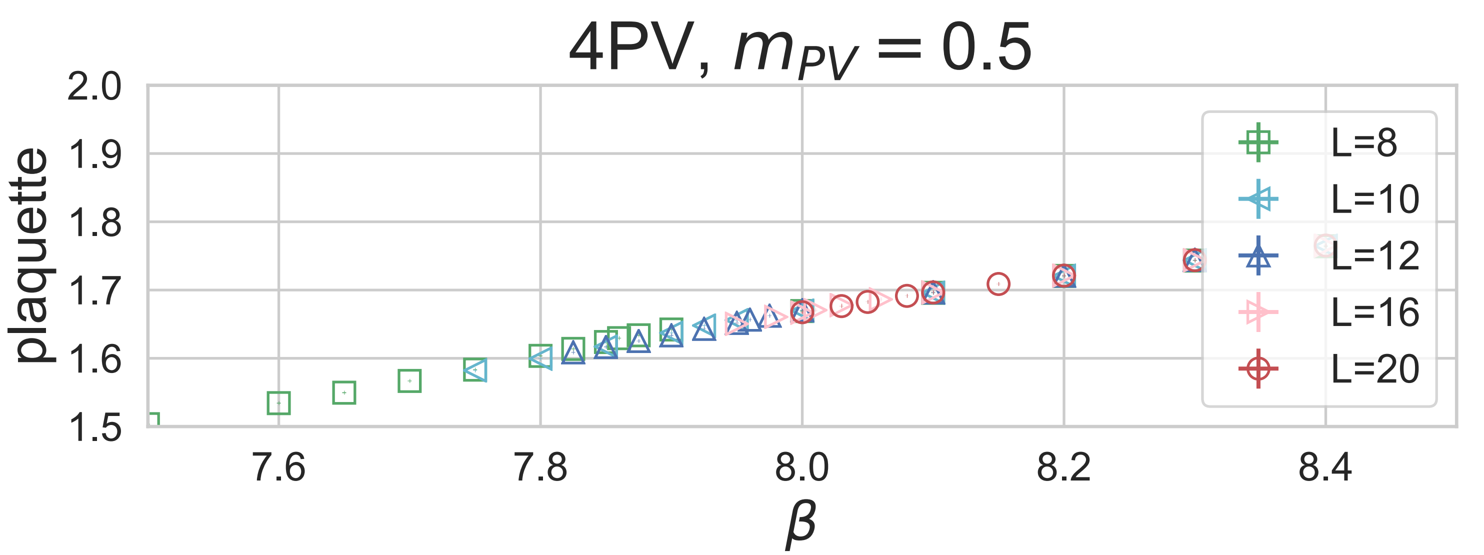
Phase structure - plaquette



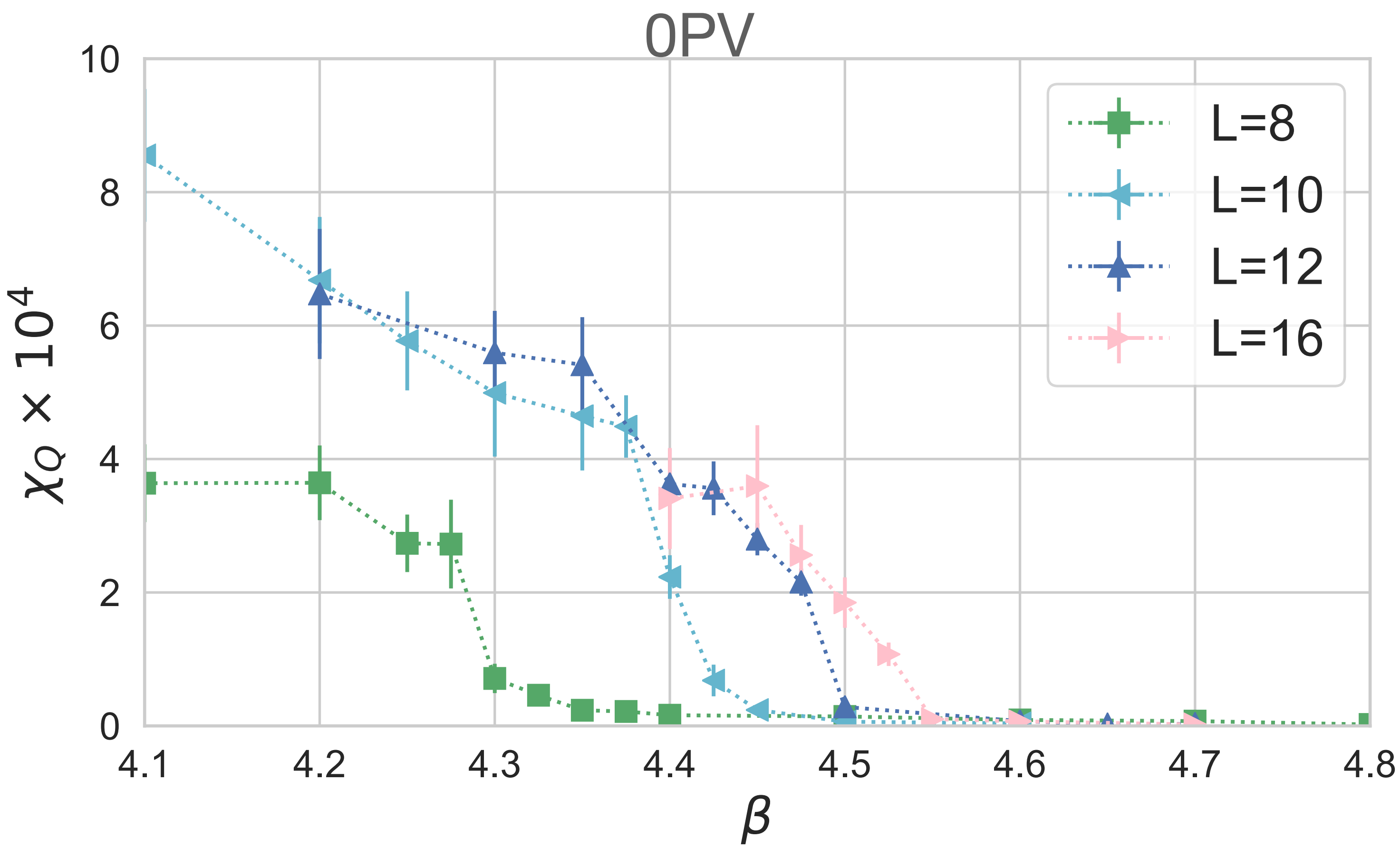
Small discontinuity with 0PV is not resolved



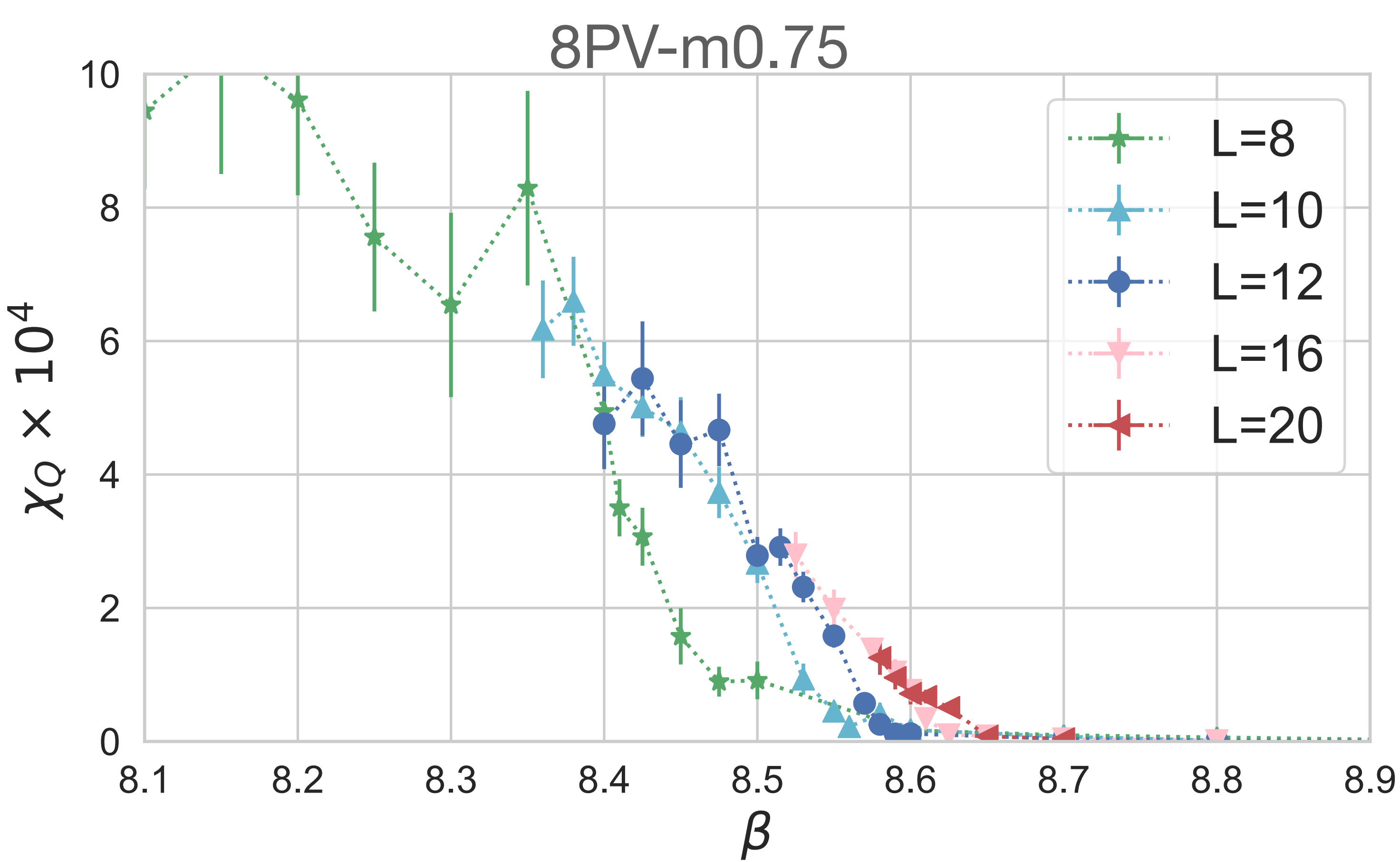
The plaquette value increases from 1.0 to 1.7 : significant reduction in UV fluctuations



Phase structure - topological susceptibility

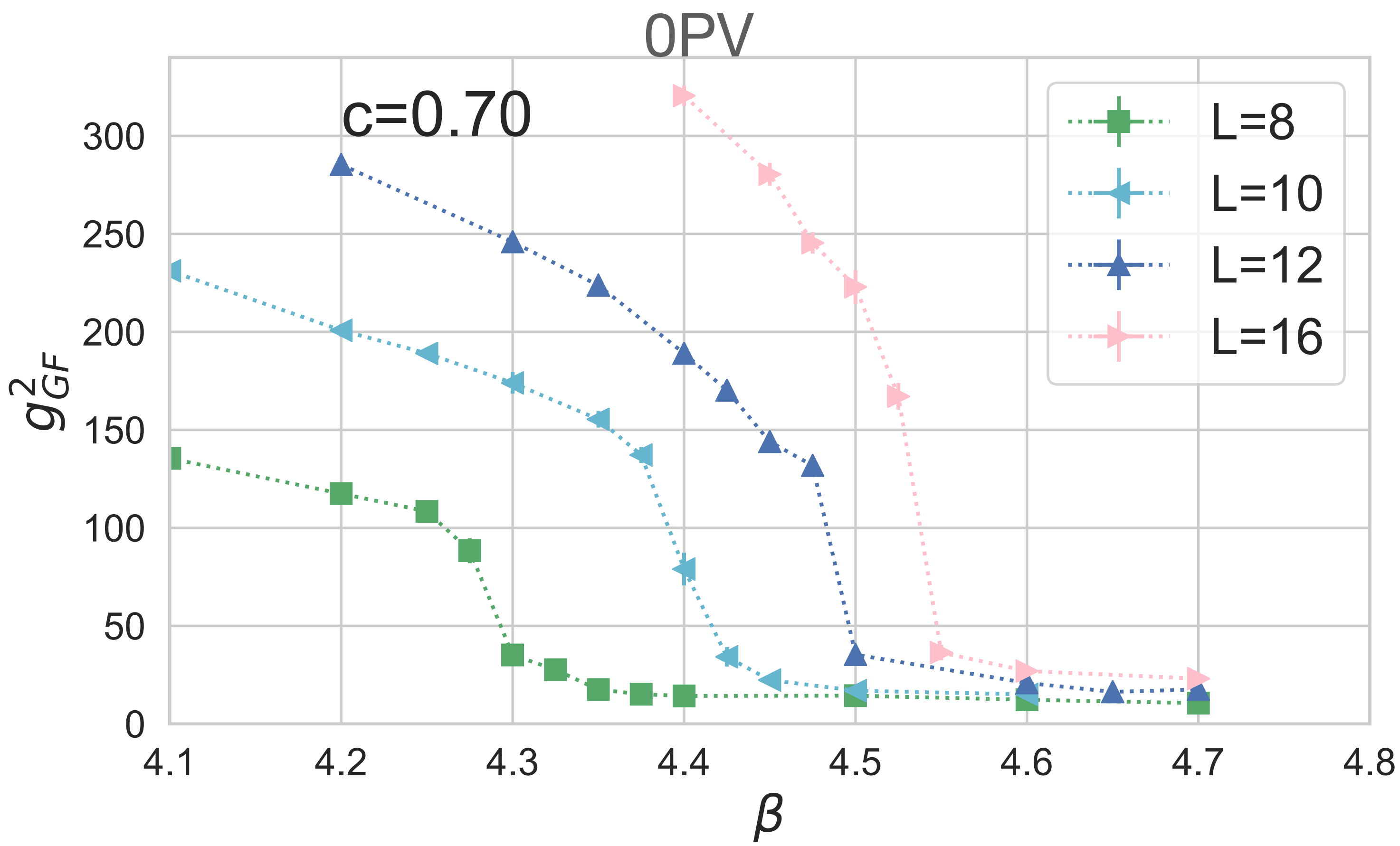


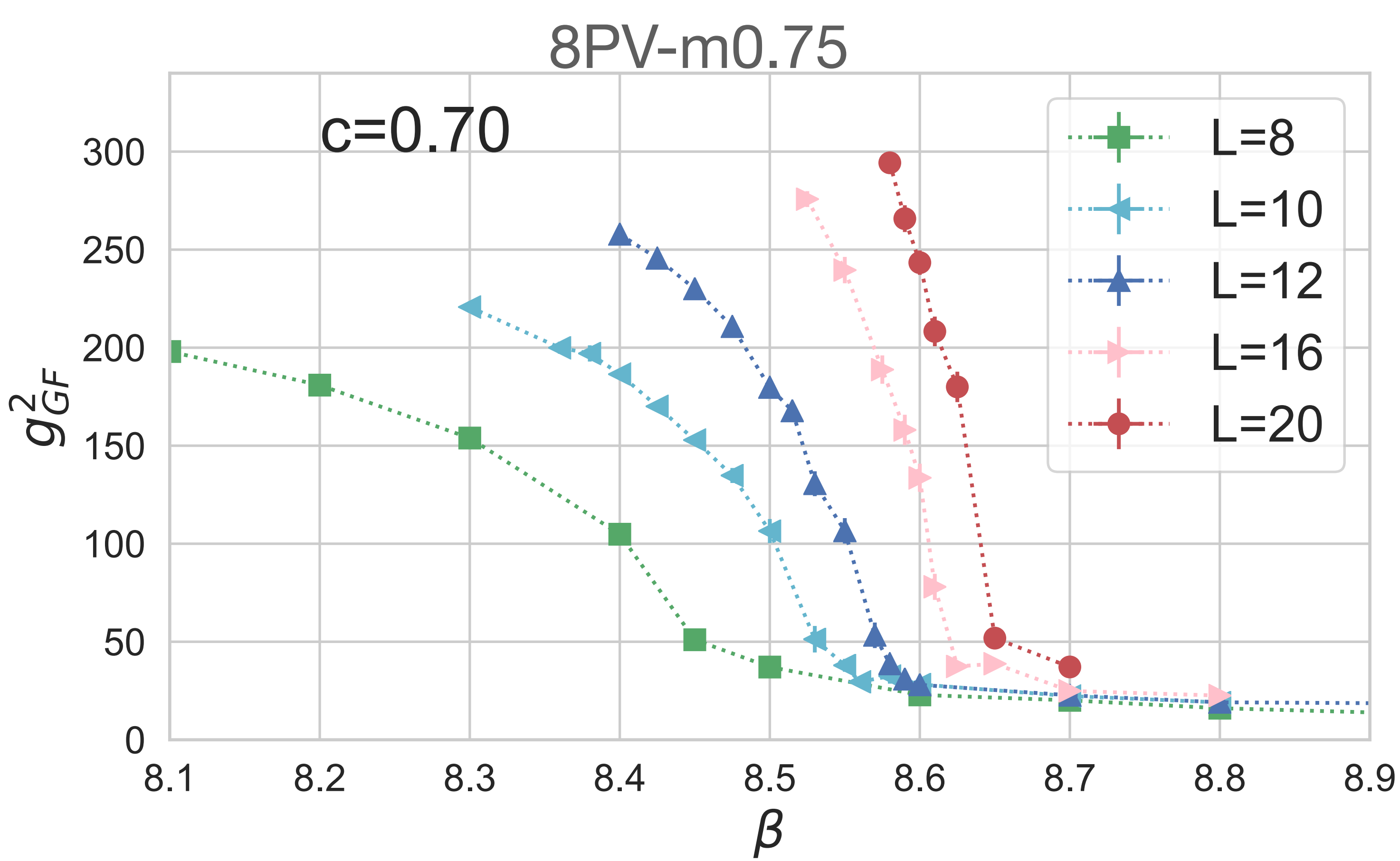
In the chiral limit topology is suppressed both in conformal and chirally broken systems
(The massless Dirac operator has a zero mode on instantons)



The new strongly coupled phase is full with unpaired instantons
- do they condense to avoid the suppression?

Phase structure - g_{GF}^2





Tie the flow time to the lattice volume $c = \sqrt{8t}/L$

$$c = 0.3 - 1.0$$

Small c: could be far from the FP and renormalized trajectory

Large c: poor statistics

Precise determination requires large c and/or large L

Finite size scaling

GF renormalized coupling $g^2_{GF}(\beta, L; t) = \mathcal{N} t^2 \langle E(t) \rangle_{\beta,L}$ has zero anomalous, zero canonical dimension; it measures the flow along the renormalized trajectory

Finite size scaling:

fix $c = \sqrt{8t}/L$ and vary the bare coupling

- 2nd order scaling: $\xi \propto |\beta/\beta_* - 1|^{-\nu}$

$$g^2_{GF}(\beta, L; c) = f^{(c)}_{2nd}\left(L\,|\beta/\beta_* - 1|^{\nu}\right)$$

- BKT scaling: $\xi \propto e^{\zeta|\beta/\beta_* - 1|^{-\nu}}$ ($\nu = 1$ is expected)

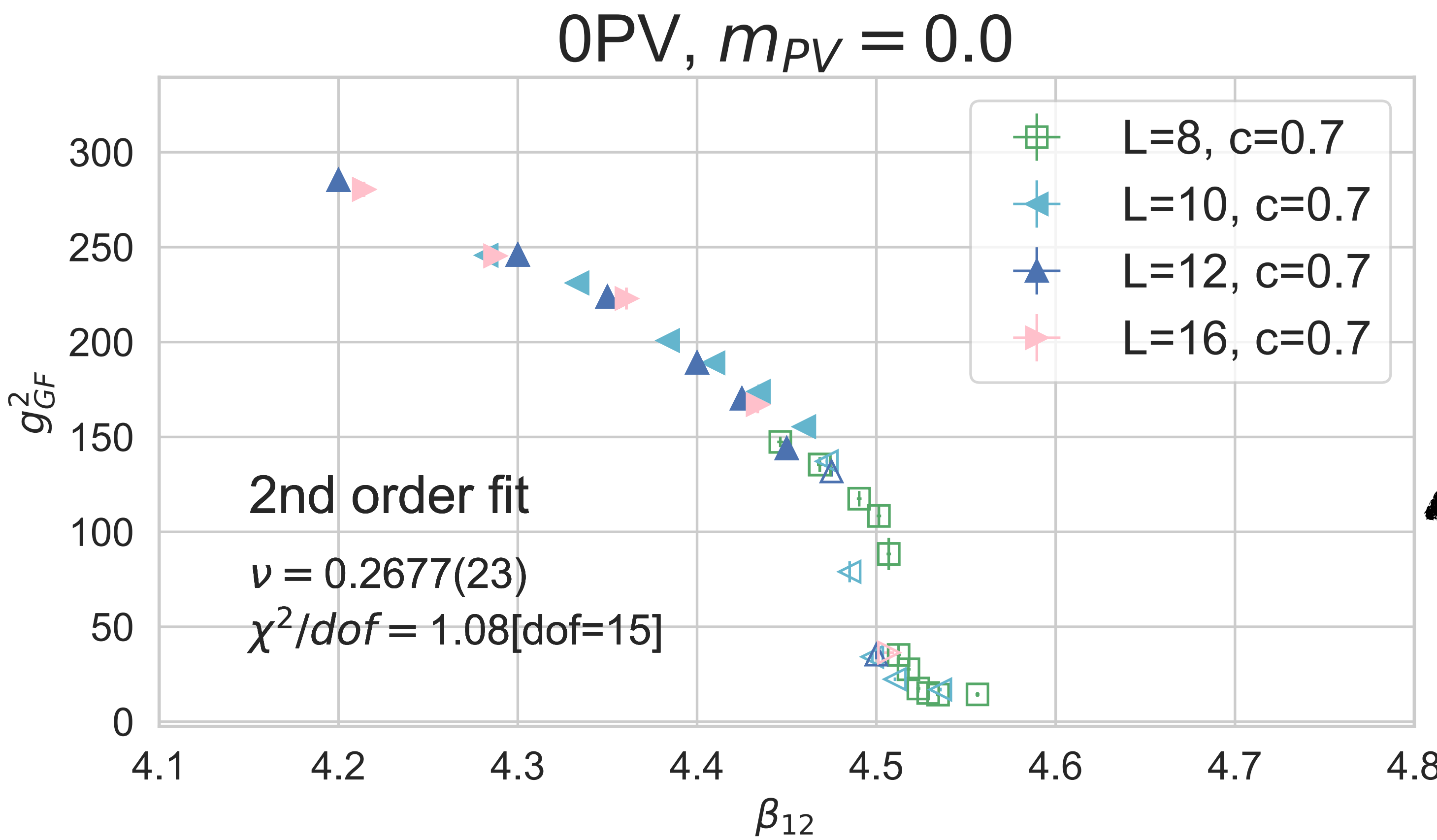
$$g^2_{GF}(\beta, L; c) = f^{(c)}_{BKT}\left(L\, e^{-\zeta|\beta/\beta_* - 1|^{-\nu}}\right)$$

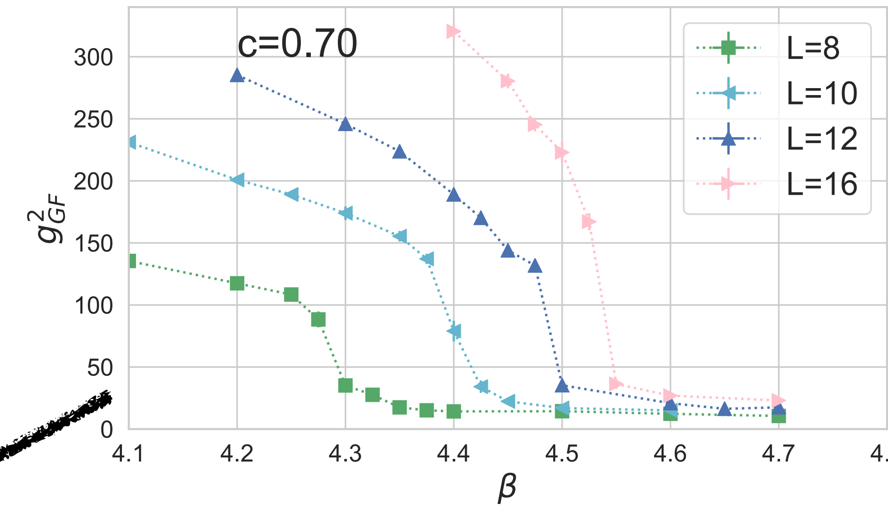
Find the exponents by standard curve-collapse analysis ;

Any $c = \sqrt{8t}/L$ can be used, the predicted β_c, ν, ζ must be independent of c

ν must be independent of the action as well

Curve collapse - 2nd order scaling, 0PV





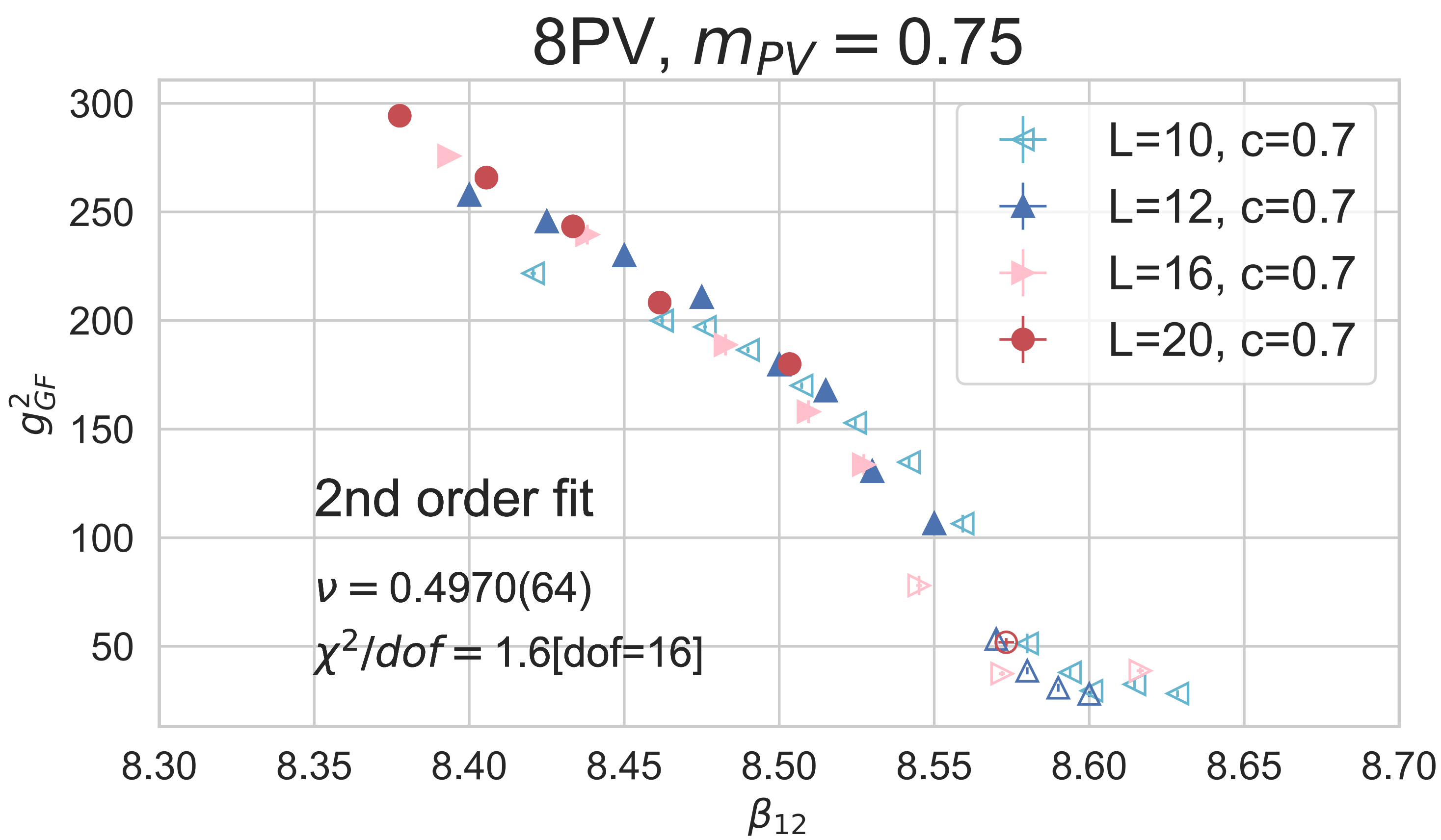
Sanity check : 0PV

- $\beta_{12}(\beta)$ is the solution of the scaling relation

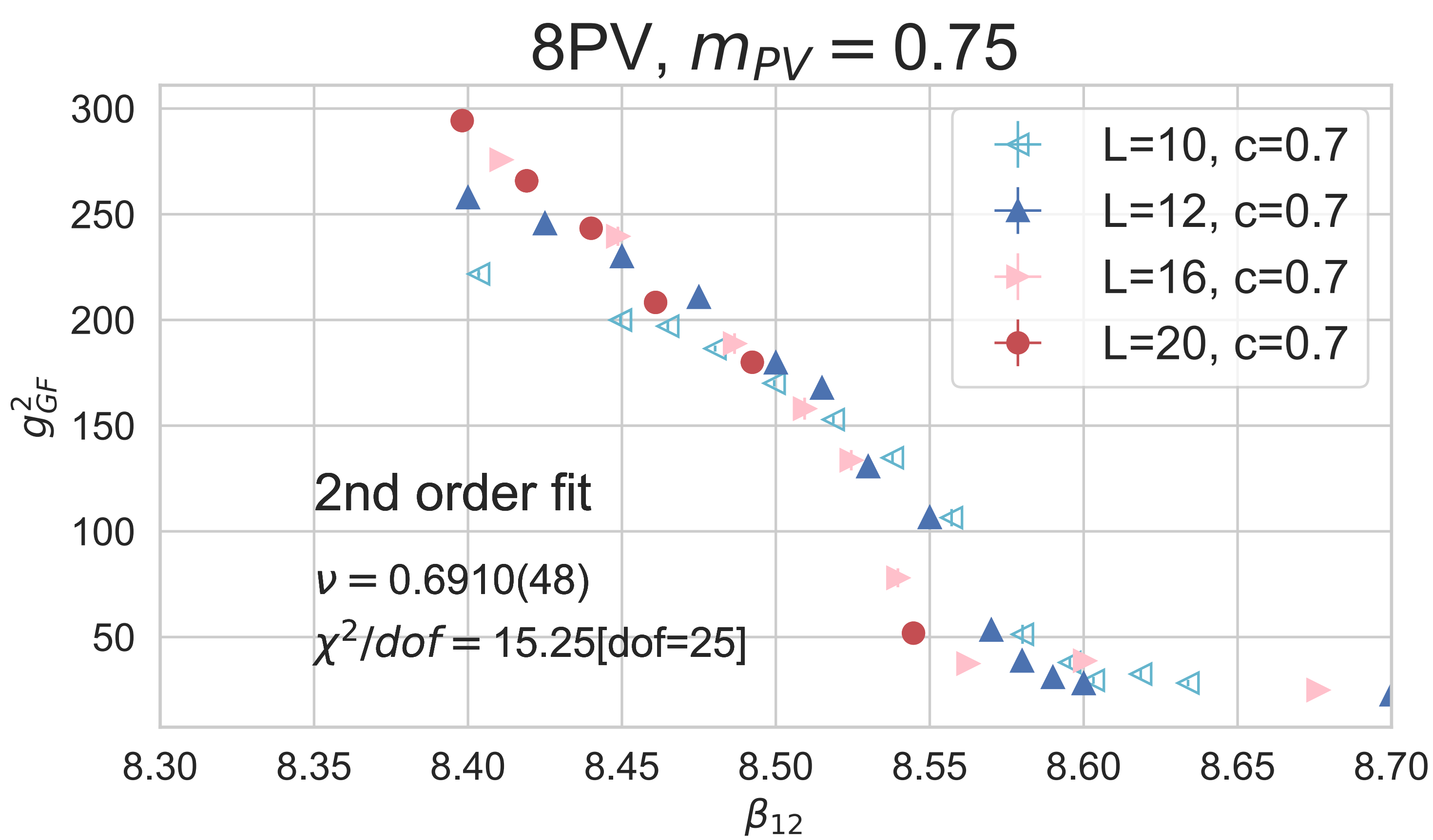
$$L^{1/\nu}(\beta/\beta_* - 1) = L_0(\beta_{12}/\beta_* - 1),\ L_0 = 12$$

- Good χ^2/dof , $\nu \approx 0.27$ — consistent with first order transition
- Only filled symbols are included in the FSS fit; no change if L=8 is added

Curve collapse - 2nd order scaling, 8PV

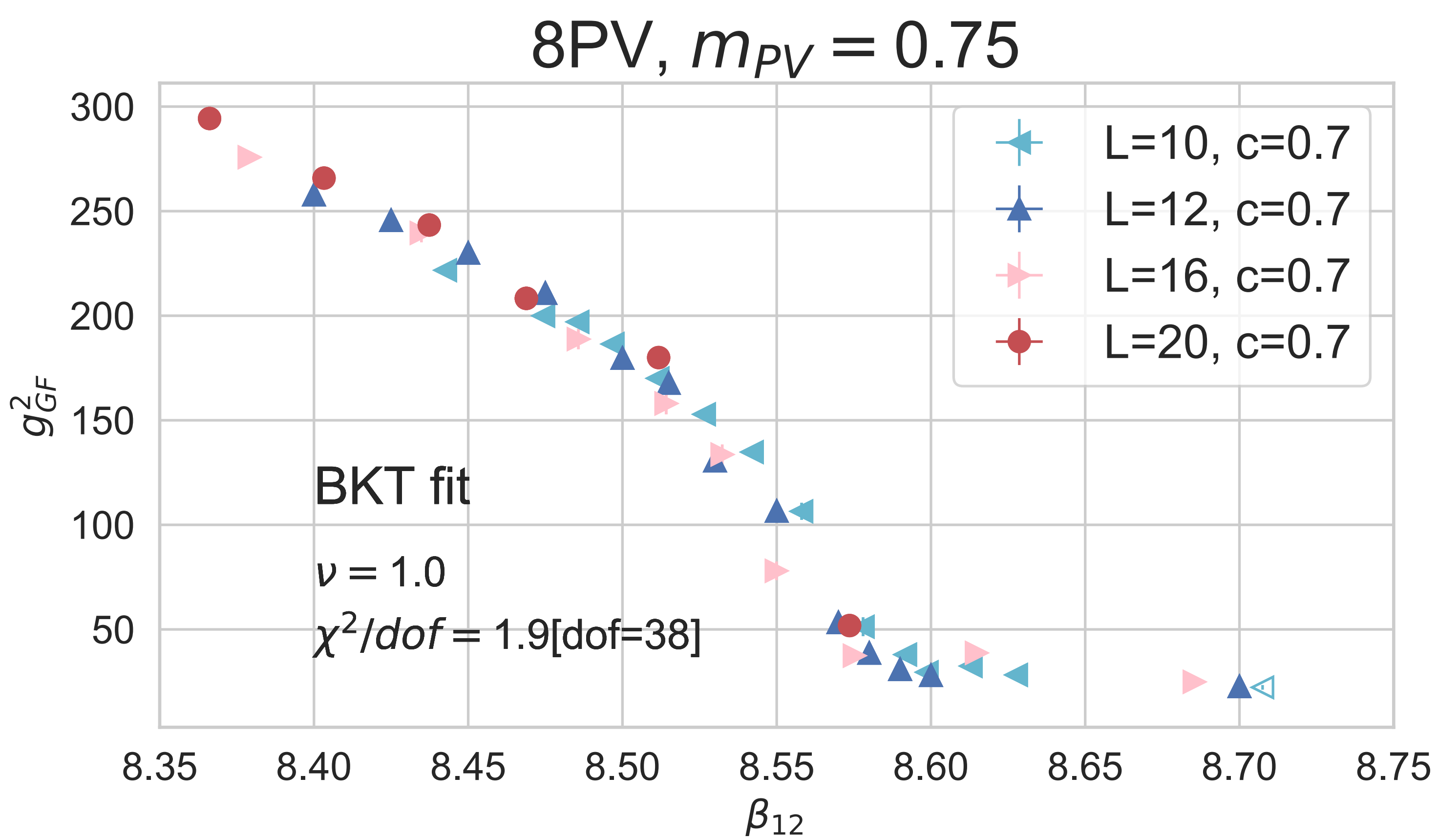


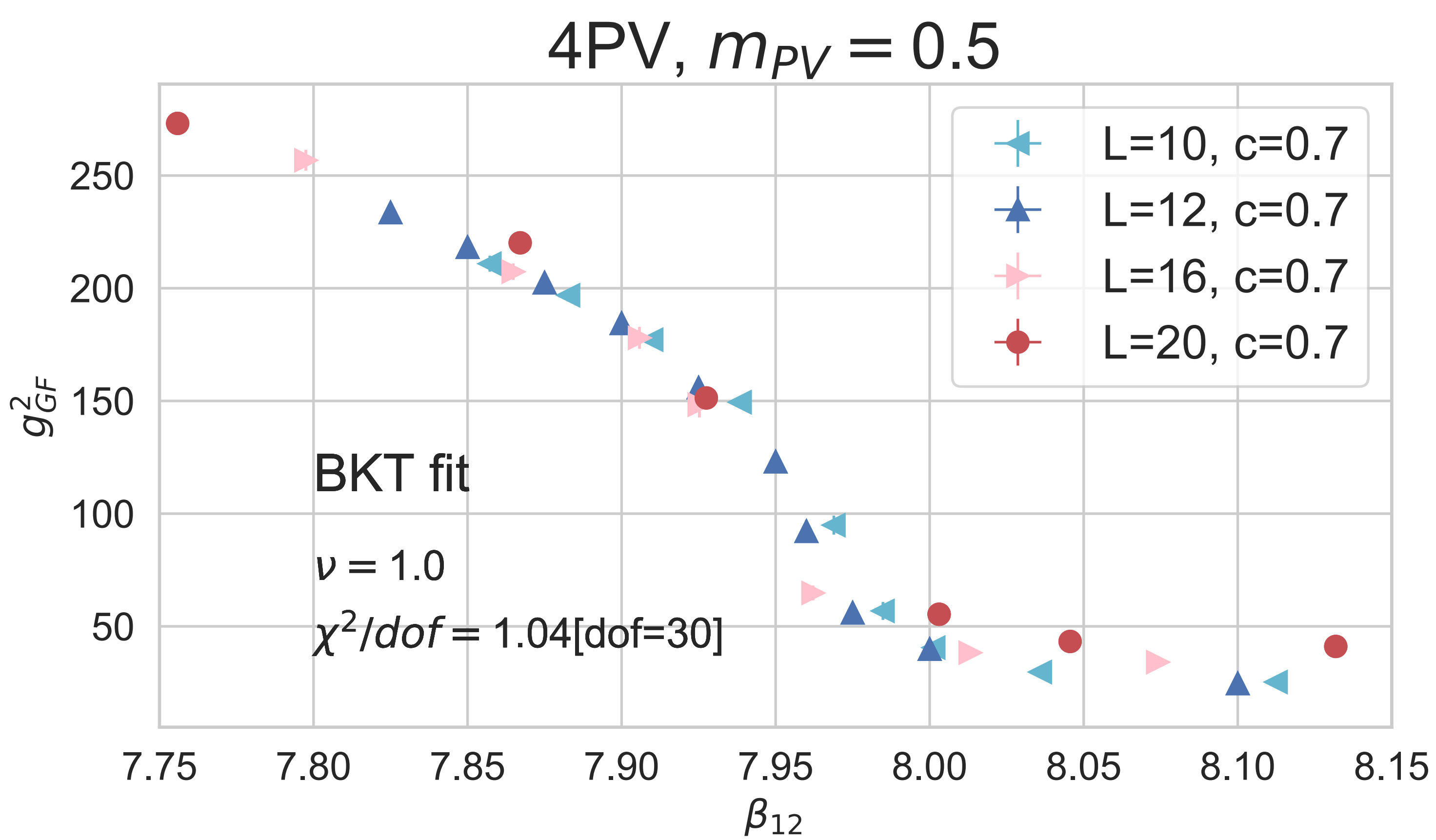
Only $L \geq 12, \nu \approx 0.50$
2nd order transition?



Include all data up to β_*
Unacceptable curve collapse
—> 2nd order transition is disfavored, 1st order inconsistent

Curve collapse - BKT scaling, $\nu = 1.0$, 8PV





$L \geq 10$

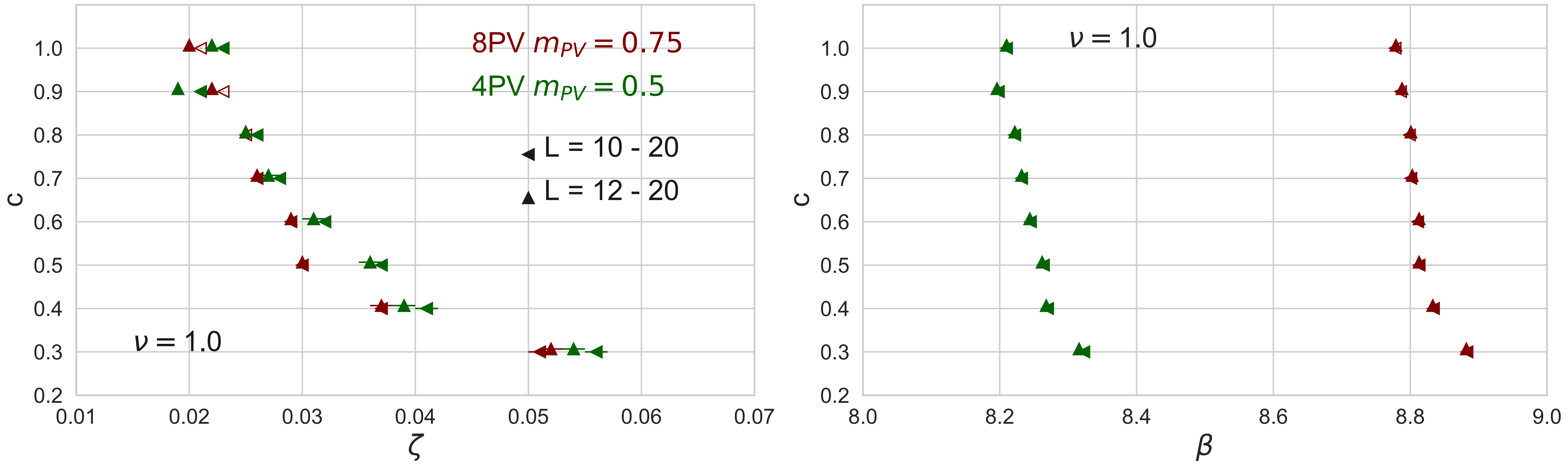
Good χ^2/dof

good curve collapse including all data up to β_*

Only mild dependence on c

BKT scaling, vary c

Repeat with different c, different volumes - consistent



The strong coupling phase. (S4)

Both PV actions show a continuous phase transition that is
- inconsistent with 1st or 2nd order scaling
- consistent with BKT or “walking” scaling

Is there indeed a phase transition?
- S4 phase, with an order parameter
- phase extends to finite mass

Cheng et al, *Phys.Rev.D* 85 (2012) 094509

Properties of S4 phase
- confining
- chirally symmetric
- gapped
- topological

symmetric mass generation

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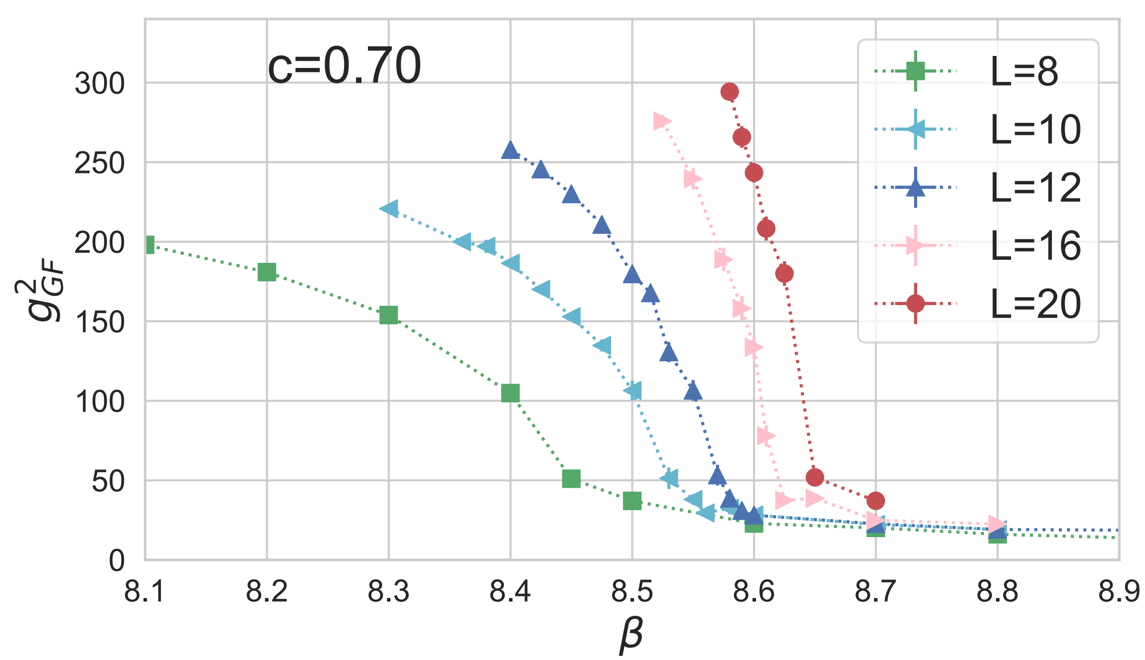
Confining



S4 phase gapped, chiral symmetric

Zero momentum correlators $C(t) = \sum_{\bar{x},\bar{y}} \langle O_S(\bar{x}, t=0) O_S(\bar{y}, t) \rangle$

"Pion states" : spin $\otimes$ taste in terms of 1-component fields

pseudoscalar : $P1 = \gamma_5 \otimes \gamma_5$: $\mathcal{O}_S = \sum_{\bar{x}} \bar{q}(\bar{x})\, q(\bar{x})\, (-1)^{x_1+x_2+x_3}$

scalar : $S1 = \gamma_0\gamma_5 \otimes \gamma_0\gamma_5$: $\mathcal{O}_S = \sum_{\bar{x}} \bar{q}(\bar{x})\, q(\bar{x})$

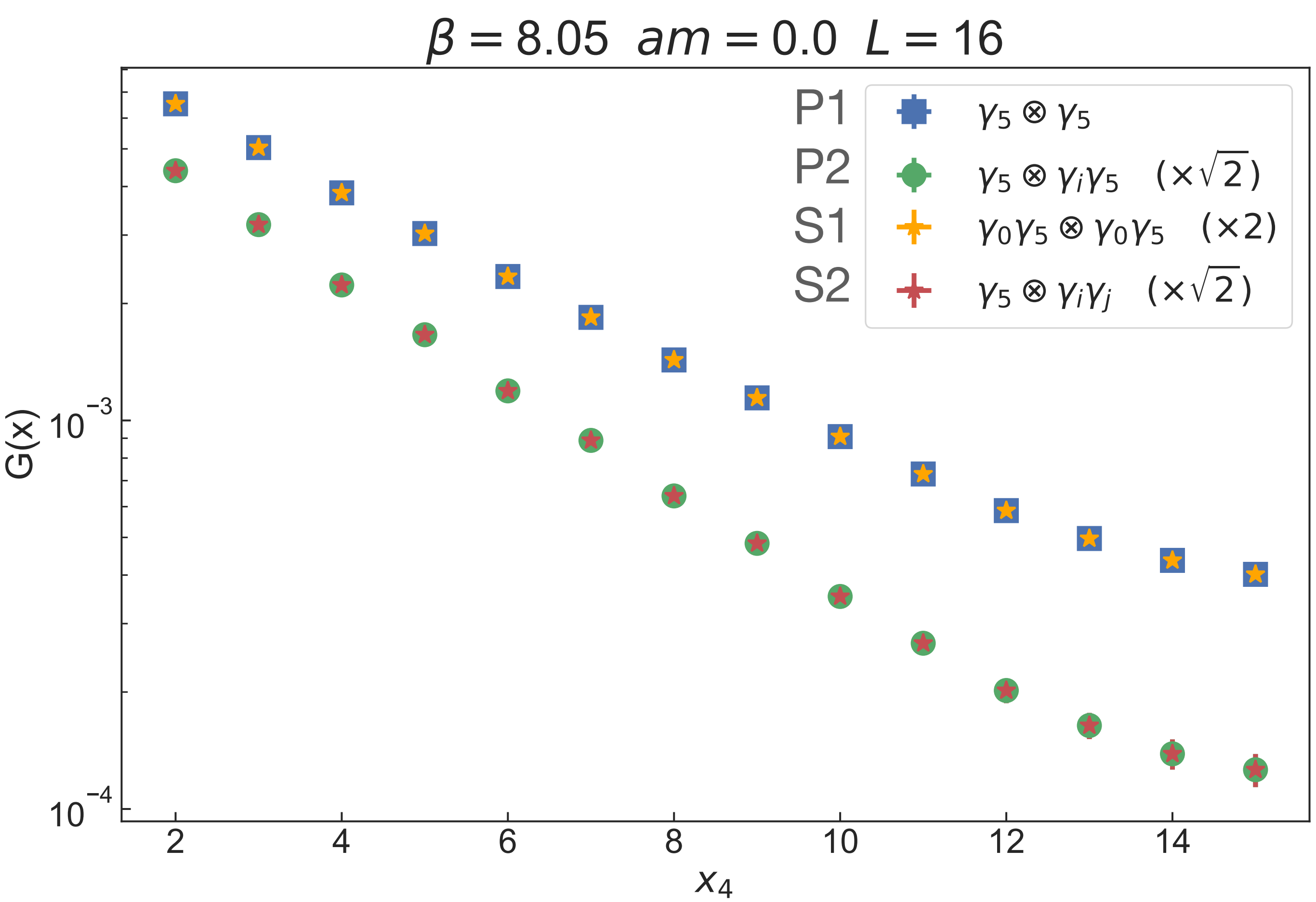
pseudoscalar : $P2 = \gamma_5 \otimes \gamma_i\gamma_5$: $\mathcal{O}_S = \sum_{\bar{x}} \bar{q}(\bar{x}) U_i(\bar{x}) q(\bar{x}+i)(-1)^{x_1+x_2+x_3}$

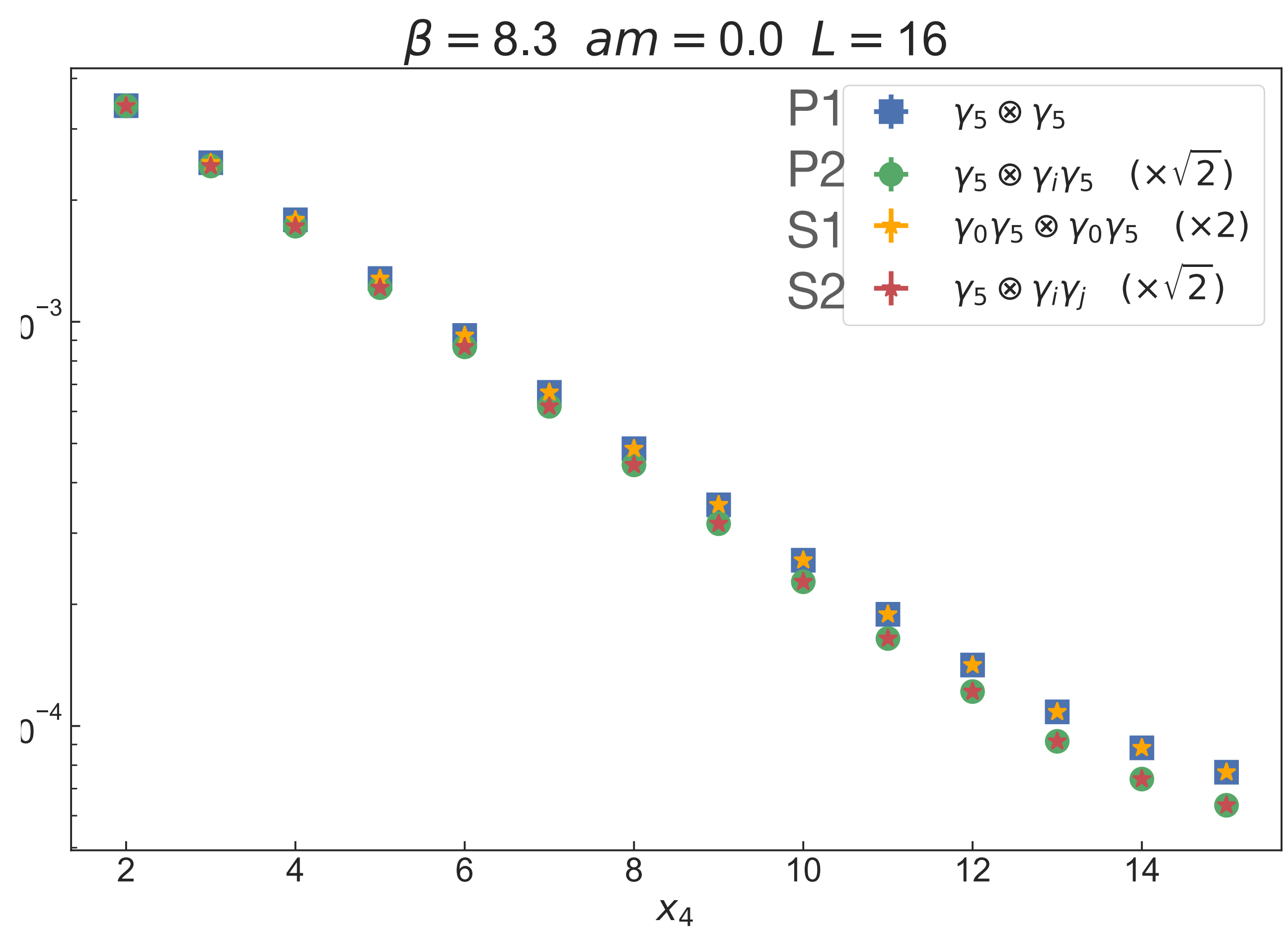
scalar : $S2 = \gamma_5 \otimes \gamma_5$: $\mathcal{O}_S = \sum_{\bar{x}} \bar{q}(\bar{x}) U_i(\bar{x}) q(\bar{x}+i)$

(all four operators couple to scalar and pseudoscalar, but mostly to one only)

S4 phase chiral symmetric

“Pion” correlators





S4 phase

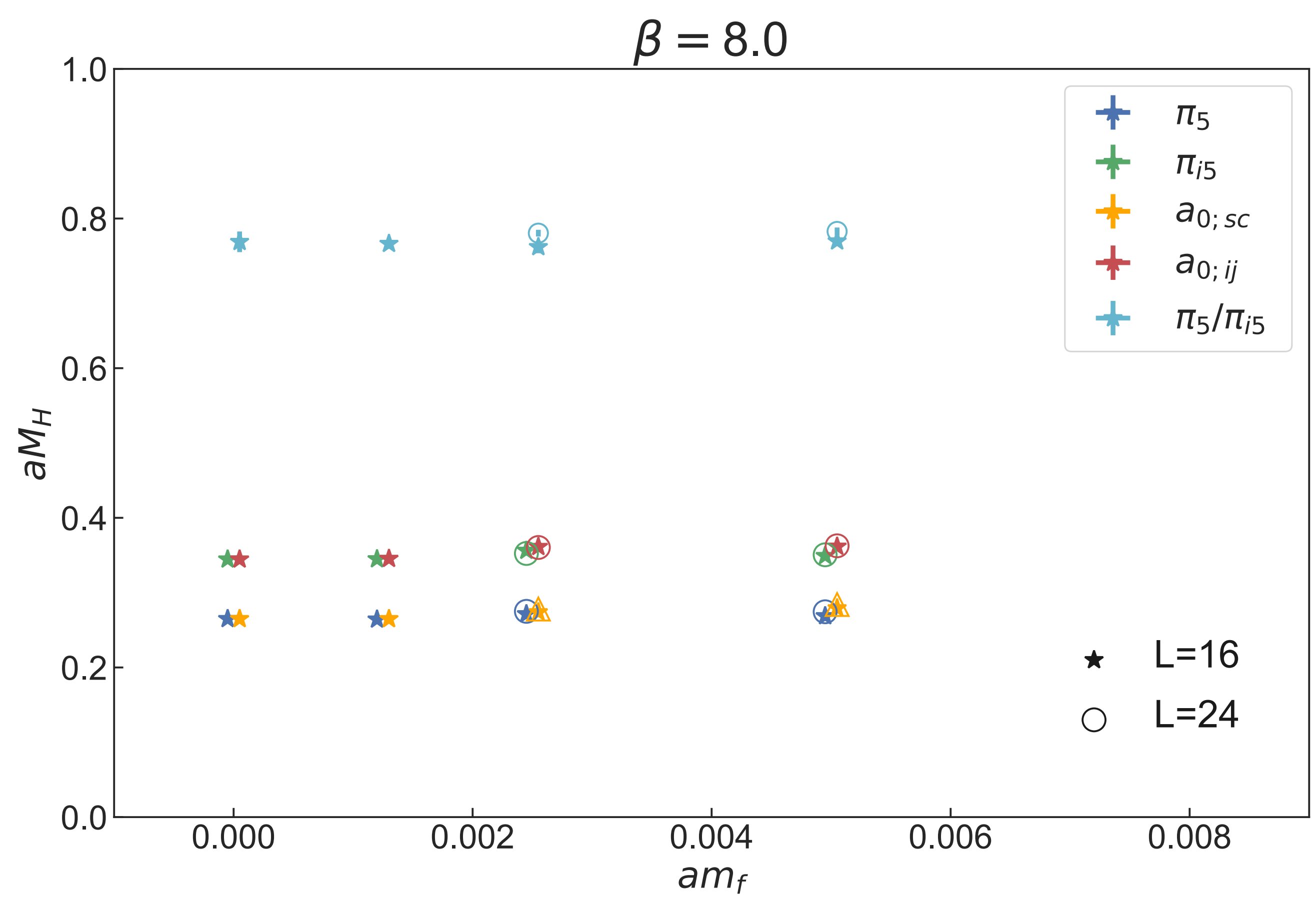
- chirally symmetric (P = S)
- P1-P2, S1-S2 are broken

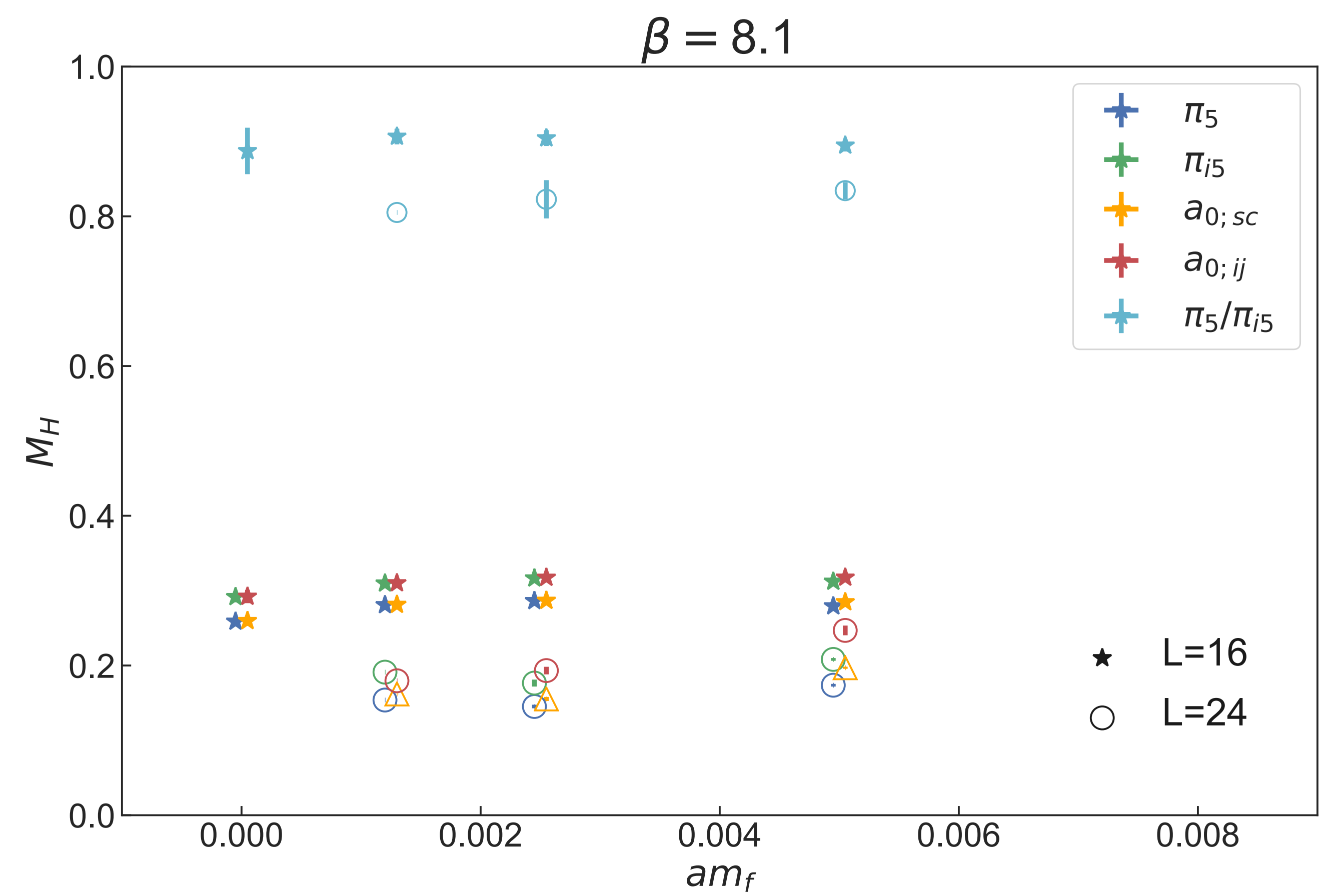
Conformal phase

- chirally symmetric (P = S)
- P1,P2, S1,S2 are nearly degenerate (good taste symmetry)

S4 phase gapped

"Pion" masses





S4 phase :mesons are massive

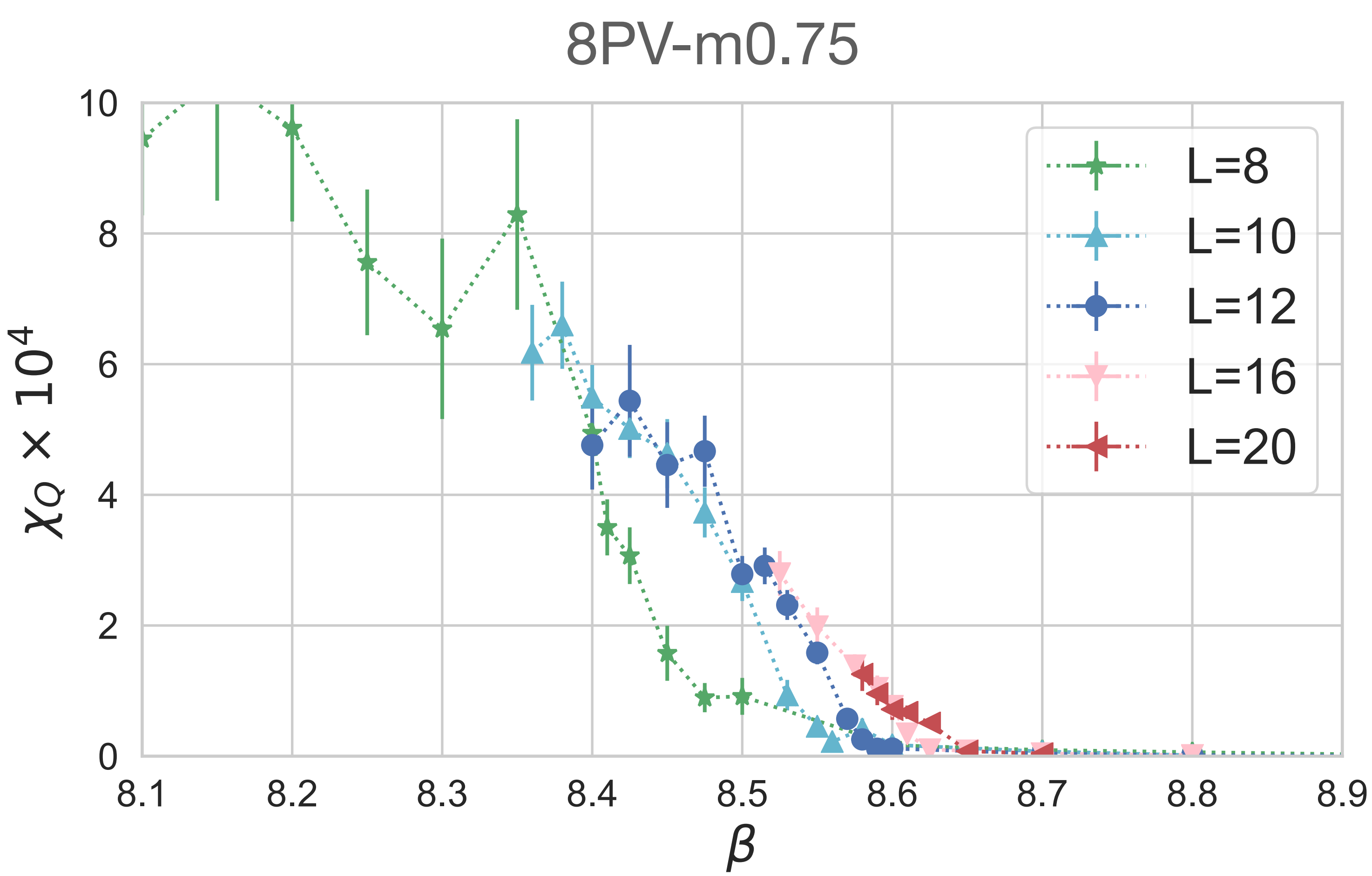
- nearly constant in fermion mass
- nearly independent of volume

Conformal phase :mesons are massive

- due to finite volume!
- all masses vanish in
  the infinite volume chiral limit

S4 phase topological

Topological susceptibility:



S4 phase :

Large topological susceptibility

- are these surface modes?

Conformal phase :

Topology is suppressed, as expected

Summary

The action with two sets of staggered fermions ($N_f = 8$) shows a continuous phase transition (when simulated with PV improved action)

The transition is at stronger couplings than what is accessible without PV improvement

Finite size scaling

- is not consistent with 1st order transition, or with 2nd order
- consistent with "walking scaling" transition (slight preference for $\nu = 1$)

The strong coupling phase (S4):

- Chirally symmetric and confining
- Strong topology
- Shows symmetric mass generation

If $N_f = 8$ is the sill of the conformal window, there has to be a symmetry driving this

- Is it specific to staggered or would it be the same with DWF?
- Could this FP introduce a new paradigm for BSM models?

This is a new work, with quite surprising, provocative results.
Independent verification would be great.
If proven correct

- $N_f = 8$ is the sill of the conformal window
- SMG phase could provide a new beyond-standard model mechanism

EXTRA SLIDES

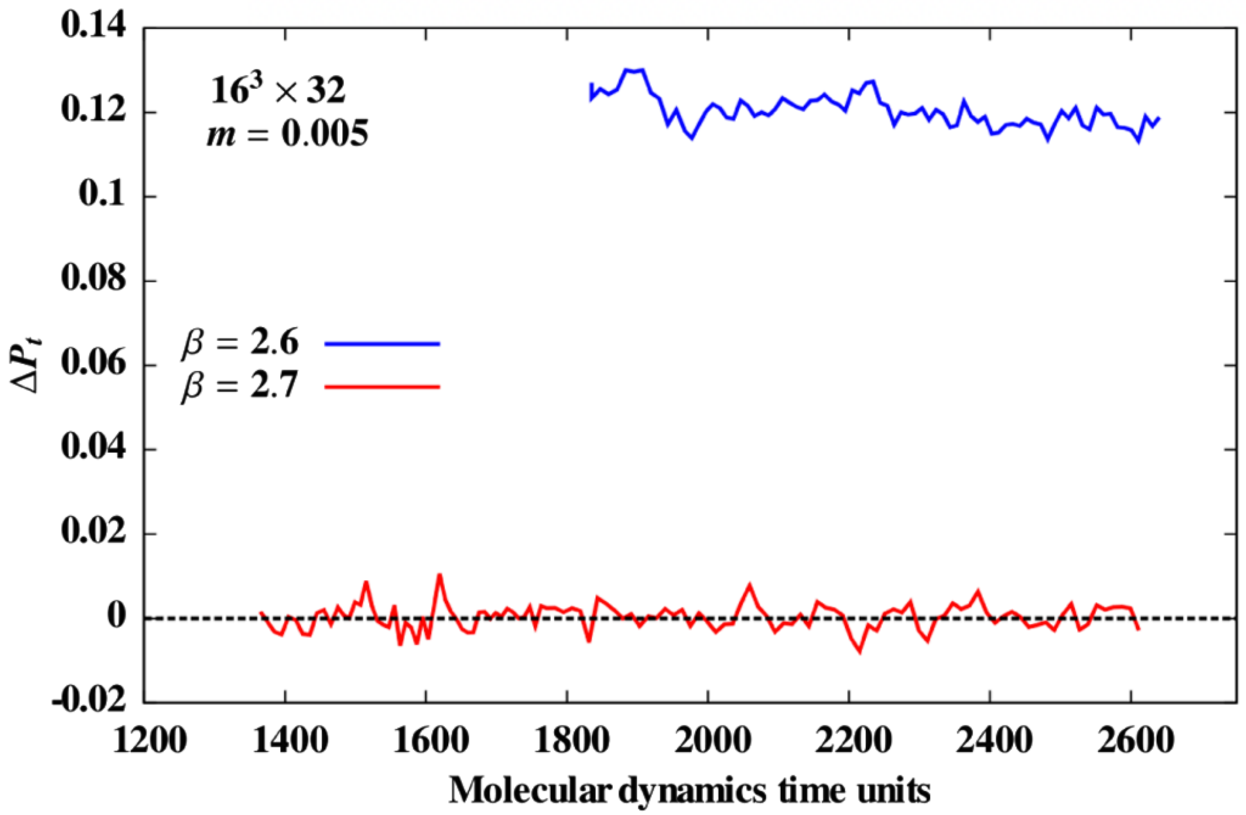
S4 phase

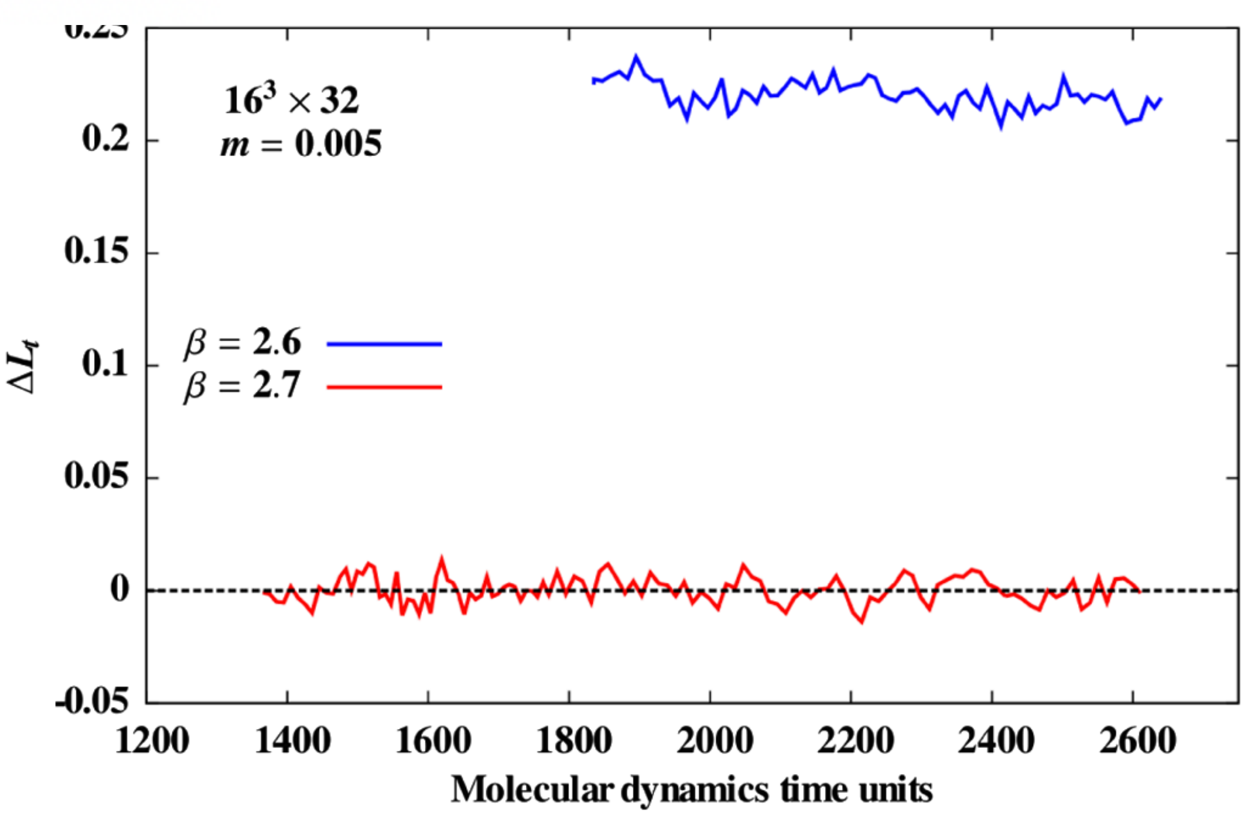
Cheng et al, PRD85, 094509

- Breaks single site translational symmetry
- Confining, all hadrons are heavy in the chiral limit
- Chirally symmetric
- Has a local order parameter that measures staggered symmetry breaking

$$\Delta P_\mu = \langle \mathrm{Re}\,\mathrm{Tr}\Box_n - \mathrm{Re}\,\mathrm{Tr}\Box_{n+\mu}\rangle_{n_\mu \mathrm{even}}, \qquad (3)$$

$$\begin{aligned}\Delta L_\mu = &\langle \alpha_\mu(n)\bar{\chi}(n)U_\mu(n)\chi(n+\mu) \\ &- \alpha_\mu(n+\mu)\bar{\chi}(n+\mu)U_\mu(n+\mu)\chi(n+2\mu)\rangle_{n_\mu \mathrm{even}},\end{aligned} \qquad (4)$$





Symmetric mass generation

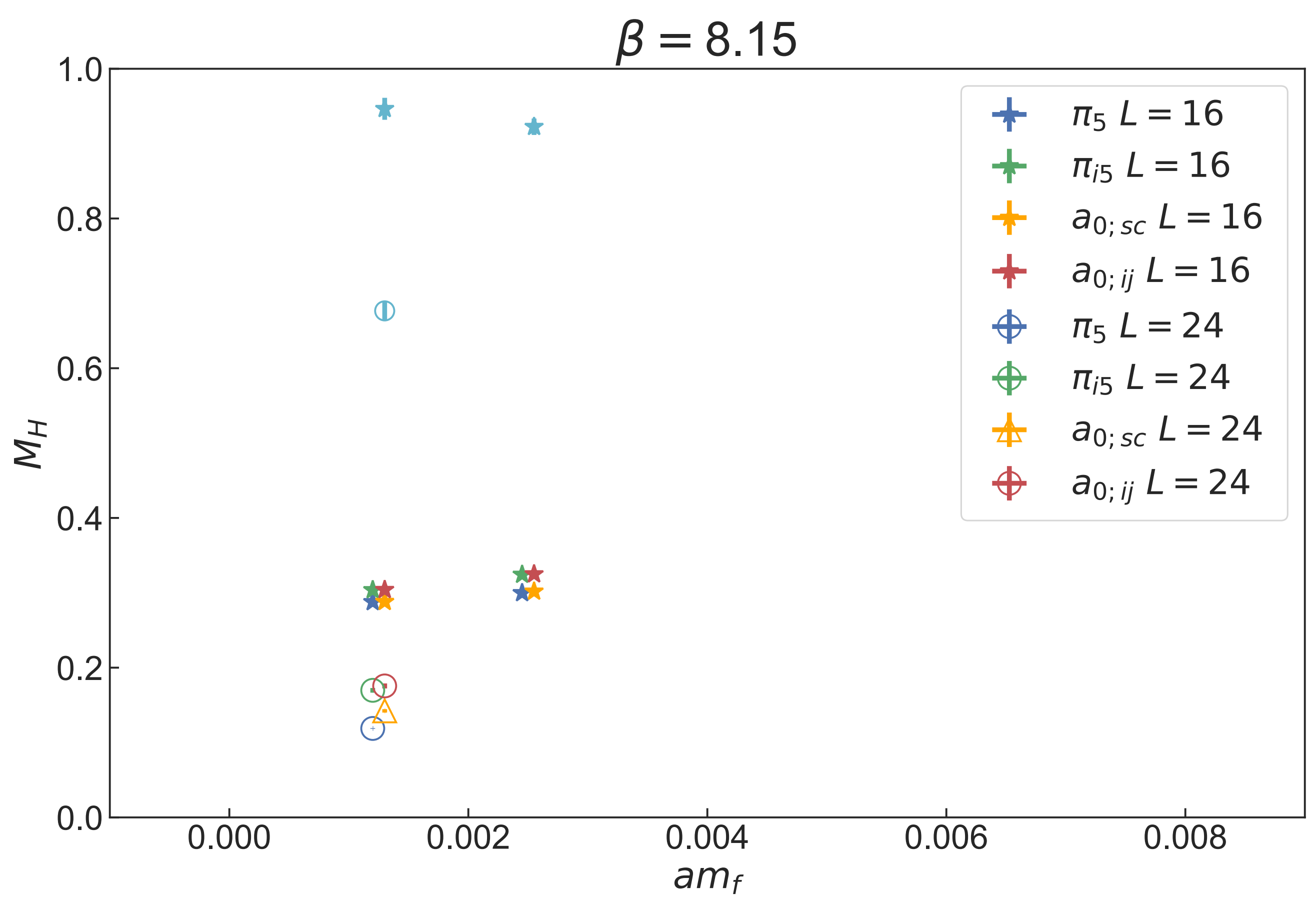
- Systems where the fermions are gapped but there is no spontaneous symmetry breaking
- Simon Catterall, David Tong, etc… : there is a U(1) symmetry that is anomalous unless the number of (Dirac) flavors equal to 8
- If there is no anomaly t'Hooft anomaly matching is not needed and confinement can occur without chiral symmetry breaking

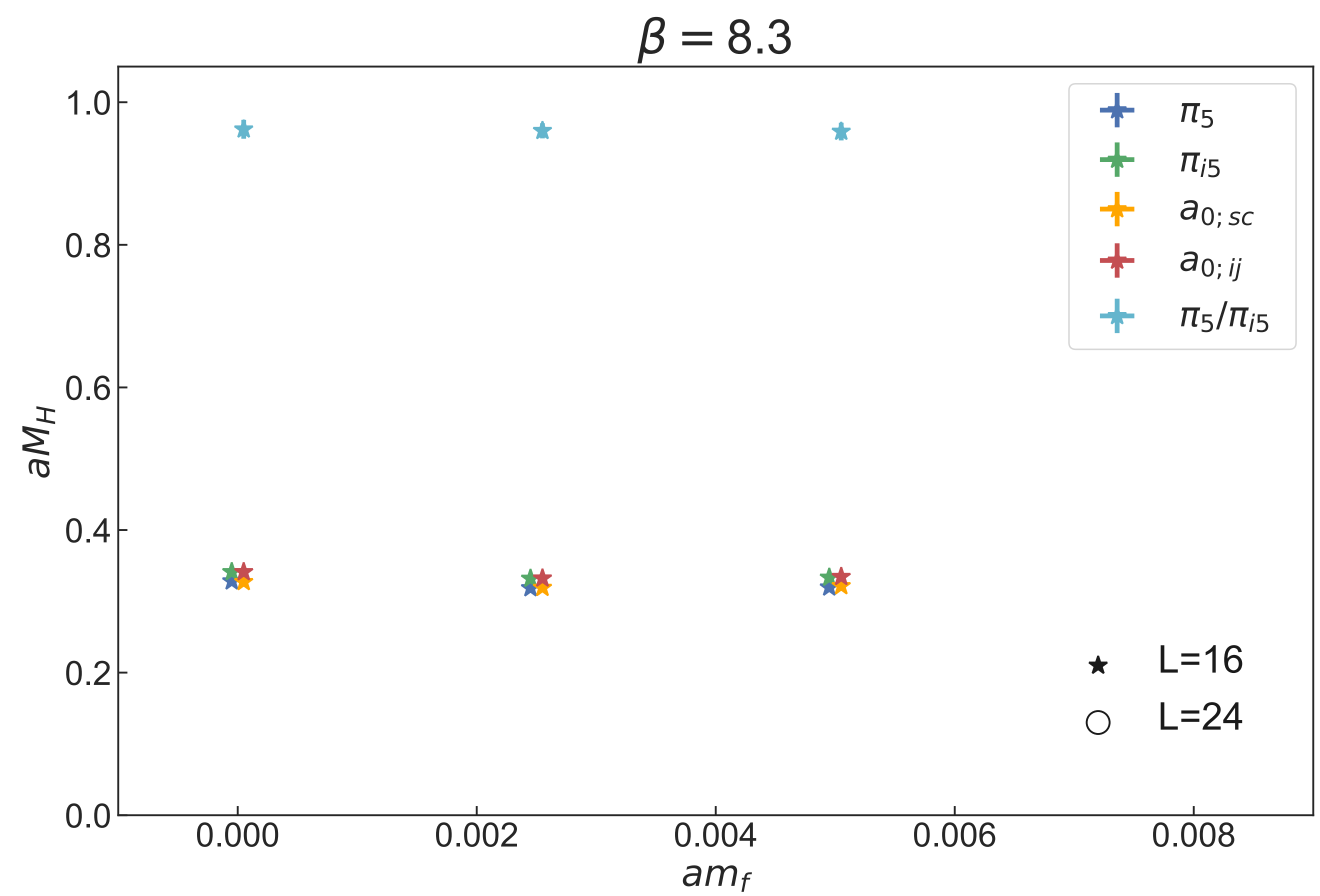
✦All examples rely on strong 4-fermion interaction generated by scalars via Yukawa coupling

✦Gauge interactions generate 4-fermion interactions as well — could gauge+fermion systems have symmetric mass phases?

S4 phase gapped

"Pion" masses





S4 phase :mesons are massive

- nearly constant in fermion mass
- nearly independent of volume

Conformal phase :mesons are massive

- due to finite volume!
- all masses vanish in
  the infinite volume chiral limit