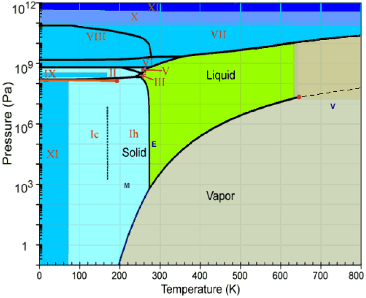
Hydrodynamics and QCD critical point

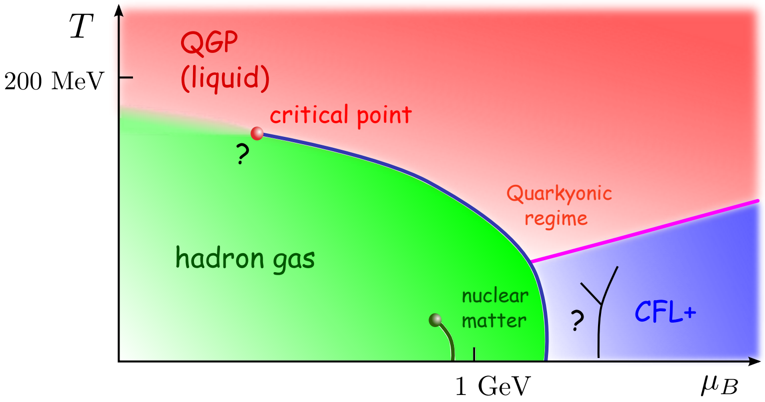
M. Stephanov

Substance[13][14]	Critical temperature	Critical pressure (absolute)
Argon	−122.4 °C (150.8 K)	48.1 atm (4,870 kPa)
Ammonia[15]	132.4 °C (405.5 K)	111.3 atm (11,280 kPa)
Bromine	310.8 °C (584.0 K)	102 atm (10,300 kPa)
Caesium	1,664.85 °C (1,938.00 K)	94 atm (9,500 kPa)
Chlorine	143.8 °C (416.9 K)	76.0 atm (7,700 kPa)
Ethanol	241 °C (514 K)	62.18 atm (6,300 kPa)
Fluorine	−128.85 °C (144.30 K)	51.5 atm (5,220 kPa)
Helium	−267.96 °C (5.19 K)	2.24 atm (227 kPa)
Hydrogen	−239.95 °C (33.20 K)	12.8 atm (1,300 kPa)
Krypton	−63.8 °C (209.3 K)	54.3 atm (5,500 kPa)
CH_4 (methane)	−82.3 °C (190.8 K)	45.79 atm (4,640 kPa)
Neon	−228.75 °C (44.40 K)	27.2 atm (2,760 kPa)
Nitrogen	−146.9 °C (126.2 K)	33.5 atm (3,390 kPa)
Oxygen	−118.6 °C (154.6 K)	49.8 atm (5,050 kPa)
CO_2	31.04 °C (304.19 K)	72.8 atm (7,380 kPa)
N_2O	36.4 °C (309.5 K)	71.5 atm (7,240 kPa)
H_2SO_4	654 °C (927 K)	45.4 atm (4,600 kPa)
Xenon	16.6 °C (289.8 K)	57.6 atm (5,840 kPa)
Lithium	2,950 °C (3,220 K)	652 atm (66,100 kPa)
Mercury	1,476.9 °C (1,750.1 K)	1,720 atm (174,000 kPa)
Sulfur	1,040.85 °C (1,314.00 K)	207 atm (21,000 kPa)
Iron	8,227 °C (8,500 K)	
Gold	6,977 °C (7,250 K)	5,000 atm (510,000 kPa)
Water[2][16]	373.946 °C (647.096 K)	217.7 atm (22.06 MPa)

Critical point is a ubiquitous phenomenon



Critical point between the QGP and hadron gas phases?



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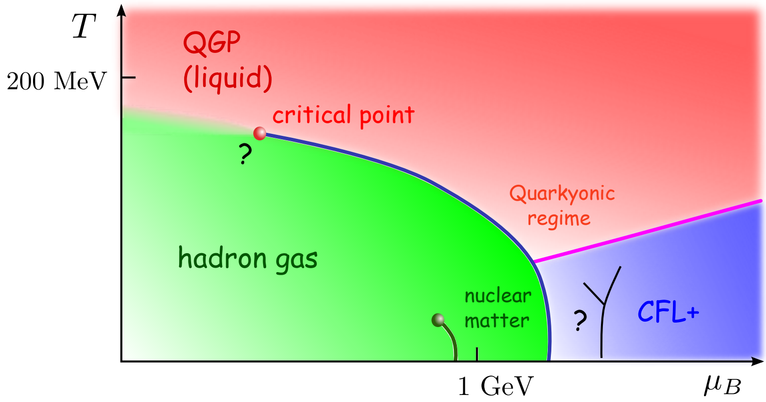


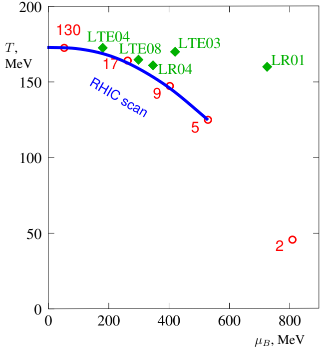
Lattice QCD at $\mu_B \lesssim 2T$ – a crossover.

C.P. is ubiquitous in models (NJL, RM, Holog., Strong coupl. LQCD, ...)

Essentially two approaches to discovering the QCD critical point.

Each with its own challenges.

- Lattice simulations. Sign problem.
- Heavy-ion collisions. *Non-equilibrium.*



Fluctuations are large and non-gaussian at a CP

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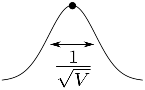

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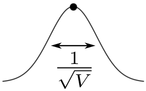
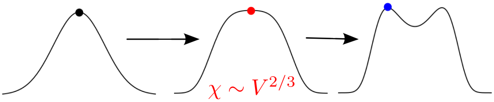


- At the critical point $S(\sigma)$ "flattens". And $\chi \equiv \langle \delta\sigma^2 \rangle V \to \infty$.



CLT?

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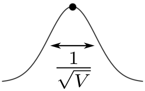

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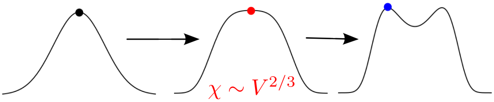
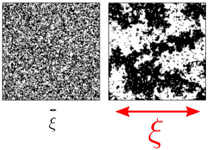




CLT?

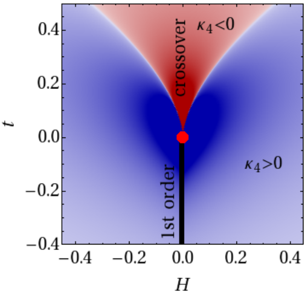
$\boxed{\delta\sigma \text{ is not an average of } \infty \text{ many } \textit{uncorrelated} \text{ contributions: } \xi \to \infty}$

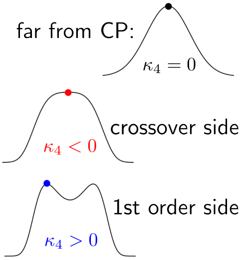
Higher order cumulants

- $n > 2$ cumulants (shape of $P(\sigma)$) depend stronger on ξ.

  E.g., $\langle\sigma^2\rangle \sim \xi^2$ while $\kappa_4 = \langle\sigma^4\rangle_c \sim \xi^7$ *[PRL102(2009)032301]*
- For $n > 2$, sign depends on which side of the CP we are.

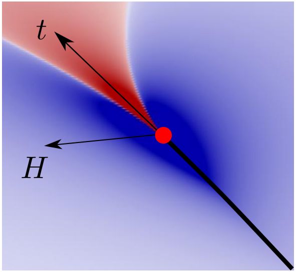
  This dependence is also universal. *[PRL107(2011)052301]*
- Using Ising model variables:





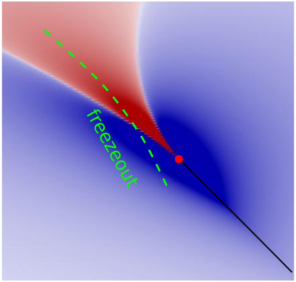
Mapping Ising to QCD phase diagram

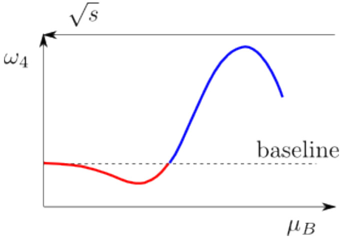
T vs μ_B:



- In QCD $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$
- $\kappa_n(N) = N + \mathcal{O}(\kappa_n(\sigma))$

Beam Energy Scan





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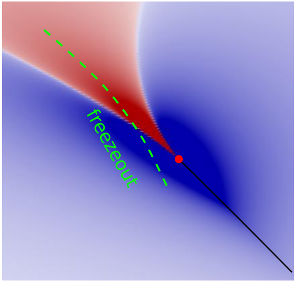
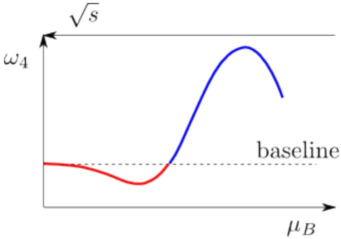
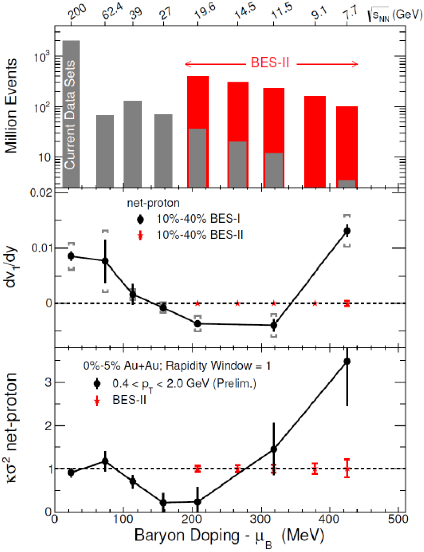
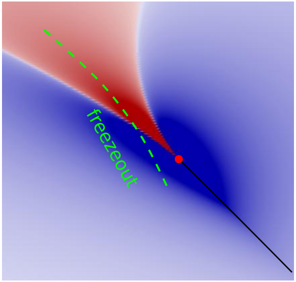
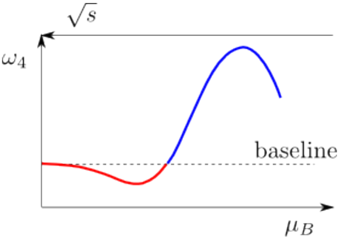
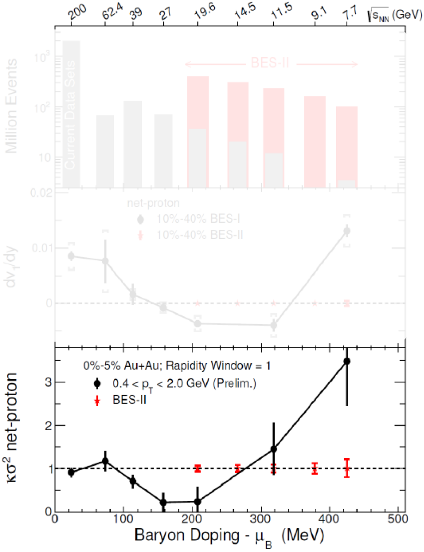







freezeout


$\sqrt{s}$
ω_4
baseline
μ_B


200
62.4
39
27
19.6
14.5
11.5
9.1
7.7
$\sqrt{s_{NN}}$ (GeV)
Million Events
Current Data Sets
BES-II
net-proton
10%-40% BES-I
10%-40% BES-II
dv_1/dy
0%-5% Au+Au; Rapidity Window = 1
0.4 < p_T < 2.0 GeV (Prelim.)
BES-II
$\kappa\sigma^2$ net-proton
Baryon Doping - μ_B (MeV)

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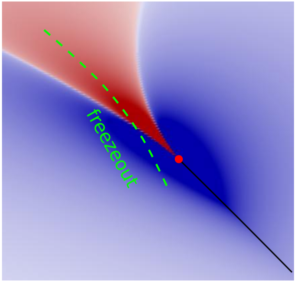

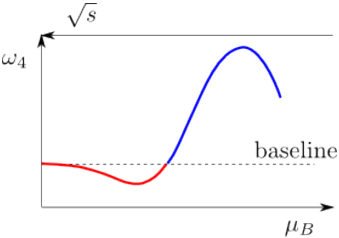

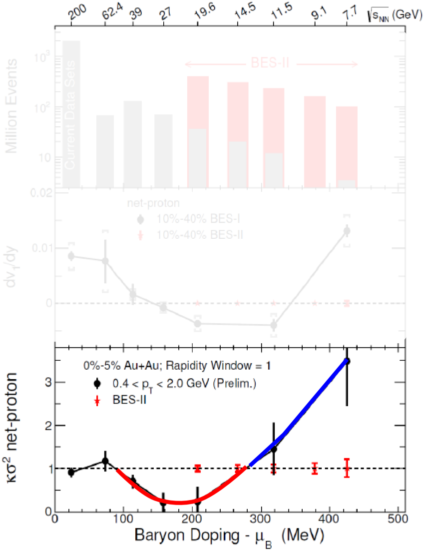

“intriguing hint” (2015 LRPNS)

Non-equilibrium physics is essential near the critical point.

The goal for BEST COLLABORATION

Why ξ is finite

$\boxed{\text{System expands and is } \textit{out of equilibrium}}$

Kibble-Zurek mechanism:

Critical slowing down means $\tau_{\text{relax}} \sim \xi^z$.
Given $\tau_{\text{relax}} \lesssim \tau$ (expansion time scale):

$$\xi \lesssim \tau^{1/z},$$

$z \approx 3$ (universal).

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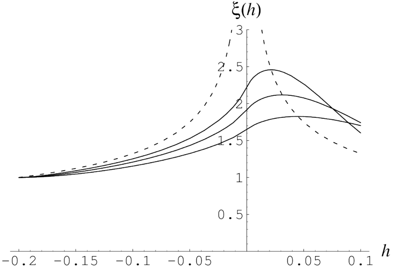
Estimates: $\xi \sim 2 - 3$ fm
(Berdnikov-Rajagopal)

KZ scaling for $\xi(t)$
and cumulants
(Mukherjee-Venugopalan-Yin)



Magnitude of observables and ξ

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- Therefore, the magnitude of fluctuation signals is determined by non-equilibrium physics.
- Can we get *critical* fluctuations from hydrodynamics *directly*?

Hydrodynamics breaks down at CP

Hydrodynamics relies on gradient expansion:

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(\epsilon u^\mu u^\nu + p\Delta^{\mu\nu} + \tilde{T}^{\mu\nu}_{\text{visc}} \right) = 0$$

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Near CP: $\zeta \sim \xi^3 \to \infty$ $(z - \alpha/\nu \approx 3)$. [Units: $T = 1$]

When $k \sim 1/\zeta \sim \xi^{-3}$ hydrodynamics breaks down.

But $k \sim \xi^{-3} \gg \mathcal{O}(1)$ or even $\mathcal{O}(\xi^{-1})$! Why?

Critical slowing down and bulk viscosity

Bulk viscosity is the effect of system taking time to adjust to local equilibrium (Mandel'shtam-Leontovich, Khalatnikov-Landau).

$$p_{\text{hydro}} = p_{\text{equilibrium}} - \zeta \, \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

$\boldsymbol{\nabla} \cdot \boldsymbol{v}$ – expansion rate

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Hydrodynamics breaks down because of *large relaxation time*.

Similar to breakdown of an effective theory due to a low-energy mode which should not have been integrated out.

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  Crossover occurs when $\tau_{\rm hydro} \sim \tau_\phi$, or $k \sim 1/\tau_\phi$.

- Regime II: $\tau_\phi \gg \tau_{\rm hydro} \gg \tau_{\rm micro}$ – "Hydro+" regime.

Advantages/motivation of Hydro+

- Extends the range of validity of hydrodynamics to shorter time ($\omega \gg 1/\tau_\phi \sim \xi^{-3}$) and length ($k \gg \xi^{-3}$) scales.

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- No large kinetic coefficients. Large ζ generated “dynamically”.

Ingredients of “Hydro+”

- As a warmup consider one extra slow mode.
- Nonequilibrium entropy, or quasistatic EOS:

$$s_{(+)}(\varepsilon, n, \phi)$$

Equilibrium entropy is the maximum of $s_{(+)}$:

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- The 6th equation (constrained by 2nd law):

$$(u \cdot \partial)\phi = -\gamma_\pi \pi - A_\phi (\partial \cdot u), \qquad \text{where } \pi = -\frac{\partial s_{(+)}}{\partial \phi}$$

  ϕ relaxes to equilibrium ($\pi = 0$) at a rate $1/\tau_\phi \equiv \Gamma = \gamma_\pi (\partial \pi / \partial \phi)$.

Linearized Hydro+

- 4 longitudinal modes (sound$\times 2$ + density + ϕ).

  In addition to c_s, D, etc. Hydro+ has two more parameters

  $\Delta c^2 = c^2_{(+)} - c^2_s$ and $\Gamma = 1/\tau_\phi$.

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$$c_s^2 = \left(\frac{\partial p}{\partial \varepsilon}\right)_{s/n,\pi=0} \text{ and } c^2_{(+)} = \left(\frac{\partial p_{(+)}}{\partial \varepsilon}\right)_{s/n,\phi} > c_s^2$$

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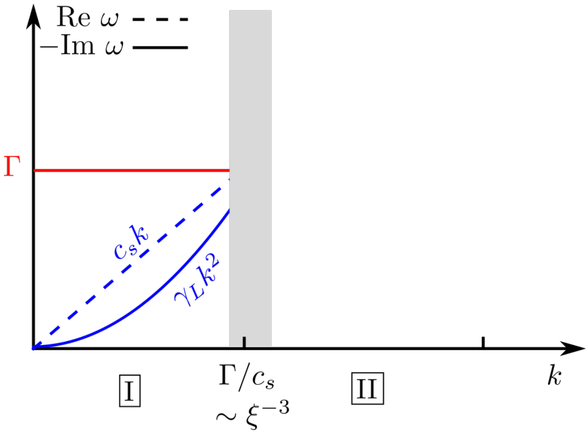
- In Regime I bulk viscosity is divergent as $\Gamma \to 0$:

$$\Delta\zeta = w \Delta c^2 / \Gamma$$

  In Regime II bulk viscosity is finite.

Modes



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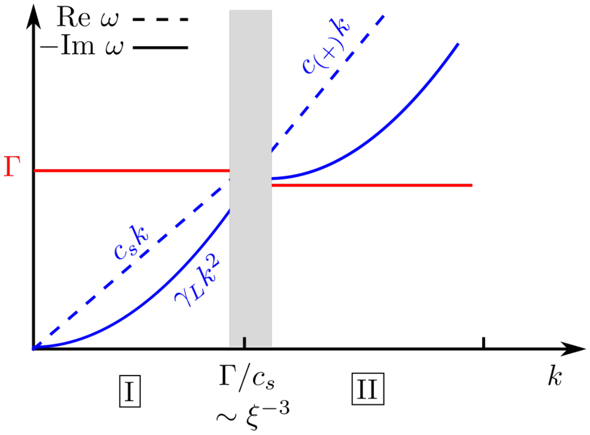

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What is the slow mode? Nonequilibrium fluctuations.

- An *equilibrium* thermodynamic state is completely characterized by $\bar{\varepsilon}$, $\bar{n}$, $\ldots$.

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- Hydrodynamics describes *partial-equilibrium states*, i.e., equilibrium is only local, because equilibration time $\sim L^2$.

  Fluctuations in such states are not necessarily in equilibrium.

- Measures of fluctuations are *additional* variables needed to characterize the partial-equilibrium state.

  2-point (and n-point) functions of fluctuating hydro variables: $\langle\delta\varepsilon\delta\varepsilon\rangle$, $\langle\delta n\delta n\rangle$, $\langle\delta\varepsilon\delta n\rangle$, ... (Or probability functional).

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- Relaxation rates of 2pt functions is of the same order as that of corresponding 1pt functions.

  But effects of fluctuations are usually suppressed due to averaging out: $\sqrt{\xi^3/V} \sim (k\xi)^{3/2}$ by CLT.

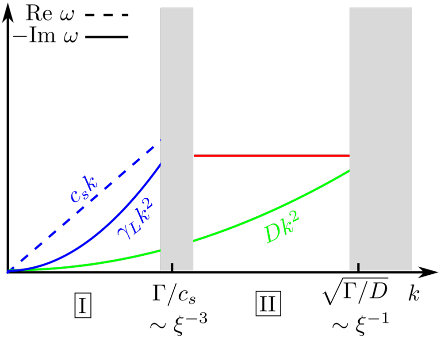
- Near CP there is *parametric* separation of relaxation time scales.

  The slowest and thus most out-of-equilibrium mode is $s/n \equiv m$.

The rate of m at scale $k \sim \xi^{-1}$,

$$\Gamma \sim D\xi^{-2} \sim \xi^{-3},$$

is of order of that for sound at much smaller $k \sim \xi^{-3}$.



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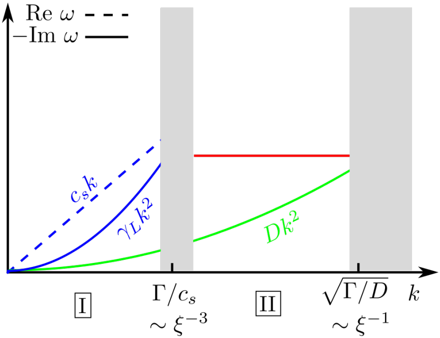

- Thus we need $\langle \delta m \delta m \rangle$ as the independent variable(s) ϕ in hydro+ equations.

Mode distribution of fluctuations

- The new variable is 2-pt function $\langle \delta m \delta m \rangle$ (Wigner transform):

$$\phi_{\boldsymbol{Q}} = \int_{\Delta \boldsymbol{x}} \langle\ \delta m(\boldsymbol{x} + \Delta \boldsymbol{x}/2)\ \delta m(\boldsymbol{x} - \Delta \boldsymbol{x}/2)\ \rangle\ e^{i\boldsymbol{Q}\cdot\Delta \boldsymbol{x}}$$

- Dependence on $\boldsymbol{x}$ ($\sim L$) is much slower than on $\Delta \boldsymbol{x}$ ($\sim \xi$).

Mode distribution of fluctuations

- The new variable is 2-pt function $\langle\delta m\delta m\rangle$ (Wigner transform):

$$\phi_{\boldsymbol{Q}} = \int_{\Delta\boldsymbol{x}} \langle\, \delta m(\boldsymbol{x}+\Delta\boldsymbol{x}/2)\, \delta m(\boldsymbol{x}-\Delta\boldsymbol{x}/2)\,\rangle\, e^{i\boldsymbol{Q}\cdot\Delta\boldsymbol{x}}$$

- Dependence on $\boldsymbol{x}$ ($\sim L$) is much slower than on $\Delta\boldsymbol{x}$ ($\sim \xi$).

- What is $s_{(+)}(\phi_{\boldsymbol{Q}})$?

  For a given ensemble: $S = \sum_i p_i \log(1/p_i)$. $\ldots$

Entropy of fluctuations

- Similar to 2-PI action: *(1712.10305)*

$$s_{(+)}(\varepsilon, n, \phi_{\boldsymbol{Q}}) = s(\varepsilon, n) + \frac{1}{2}\int_{\boldsymbol{Q}} \left(1 - \phi_{\boldsymbol{Q}}/\bar{\phi}_{\boldsymbol{Q}} + \log \phi_{\boldsymbol{Q}}/\bar{\phi}_{\boldsymbol{Q}}\right)$$

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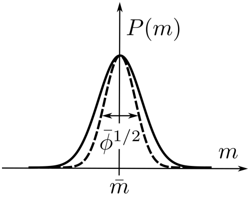
- Two competing effects: e.g., for $\phi > \bar{\phi}$

  $\log(\phi/\bar{\phi})^{1/2}$ wider distribution $P(m)$ – more microstates – larger entropy;

  vs

  $-(1/2)\phi/\bar{\phi}$ – penalty for deviating from maximum entropy (at $\delta m = 0$).

  Balance is achieved (max. $s_{(+)}$) at $\phi = \bar{\phi}$.



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  Similar separation of scales: $Q \gg k \sim 1/L$.

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- The equation for $\phi_{\boldsymbol{Q}}$ is a relaxation equation:

$$(u \cdot \partial)\phi_{\boldsymbol{Q}} = -\gamma_\pi(\boldsymbol{Q})\pi_{\boldsymbol{Q}}, \quad \pi_{\boldsymbol{Q}} = -\left(\frac{\partial s_{(+)}}{\partial \phi_{\boldsymbol{Q}}}\right)_{\varepsilon,n}$$

  $\gamma_\pi(\boldsymbol{Q})$ is known from mode-coupling calculation in model H (Kawasaki). It is universal.

  Asymptotics of γ_π: $\sim DQ^2$ for $Q\xi \ll 1$ and $\sim Q^3$ for $Q\xi \gg 1$.

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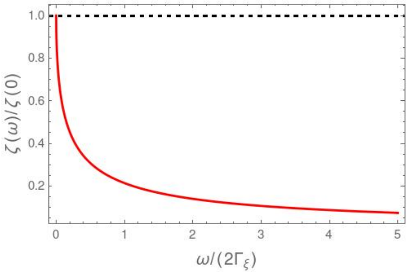
- Near the critical point the relevant scale of Q is ξ^{-1}.

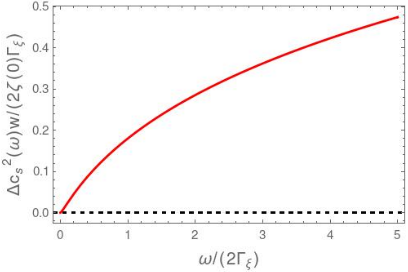
  Characteristic rate $\Gamma_\xi = D\xi^{-2} \sim \xi^{-3}$.

Hydro+ vs Hydro: real-time response

Dissipation during expansion is overestimated in hydro (dashed):



Stiffness of eos (sound speed) is underestimated.



Summary

- A fundamental question for Heavy-Ion collision experiments:

  Is there a critical point on the boundary between QGP and hadron gas phases?

- Intriguing results from experiments (BES-I).

  More to come (BES-II, FAIR/CBM, NICA, J-PARC).

  Quantitative theoretical framework is needed $\Rightarrow$ BEST COLLABORATION .

- Large (non-gaussian) fluctuations – universal signature of a critical point.

- In H.I.C., the magnitude of the signatures is controlled by dynamical *non-equilibrium effects*. The physics of the interplay of critical and dynamical phenomena can be captured in Hydro+.

More

Critical fluctuations and experimental observables

Observed fluctuations are related to fluctuations of σ.

[MS-Rajagopal-Shuryak PRD60(1999)114028; MS PRL102(2009)032301]

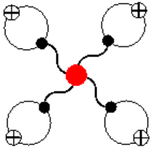
Think of a collective mode described by field σ such that $m = m(\sigma)$:

$$\delta n_{\boldsymbol{p}} = \delta n_{\boldsymbol{p}}^{\text{free}} + \frac{\partial \langle n_{\boldsymbol{p}} \rangle}{\partial \sigma} \times \delta\sigma$$

The cumulants of multiplicity $M \equiv \int_{\boldsymbol{p}} n_{\boldsymbol{p}}$:

- $$\kappa_4[M] = \underbrace{\langle M \rangle}_{\text{baseline}} + \underbrace{\kappa_4[\sigma] \times g^4 \underbrace{\left(\bullet\!\!\bigcirc\!\!\otimes\right)^4}_{\sim M^4}}_{\text{this is } \hat{\kappa}_4 (\text{a.k.a.} C_4^{\text{Bzdak-Koch}})} + \ldots,$$

$$\bullet\!\!\bigcirc\!\!\otimes = \int_{\boldsymbol{p}} \frac{n_{\boldsymbol{p}}}{\gamma_{\boldsymbol{p}}}$$ $\leftarrow$ acceptance dependent



2-PI entropy

$$S = \sum_i p_i \log \frac{1}{p_i}$$

- Microcanonical: e^{S_0} states in interval $\Delta\Psi$ – $p_i = e^{-S_0}$.
  $S = S_0(\Psi)$. $\Psi = (\varepsilon, n, \dots)$

- Canonical: $p_i = e^{J\Psi - W[J]}$.
  $S = W[J] - J\langle\Psi\rangle$. "1-PI." $J = (\beta, \beta\mu, \dots)$.

- Partial equilibrium: $p_i = e^{J\Psi + \frac{1}{2}\Psi K \Psi - W[J,K]}$.
  $S = W[J,K] - J\langle\Psi\rangle - \frac{1}{2}\langle\Psi K \Psi\rangle$. "2-PI"

Hydro+ vs one slow mode vs just hydro