Long time tails and quark mass dependence of the QCD bulk viscosity

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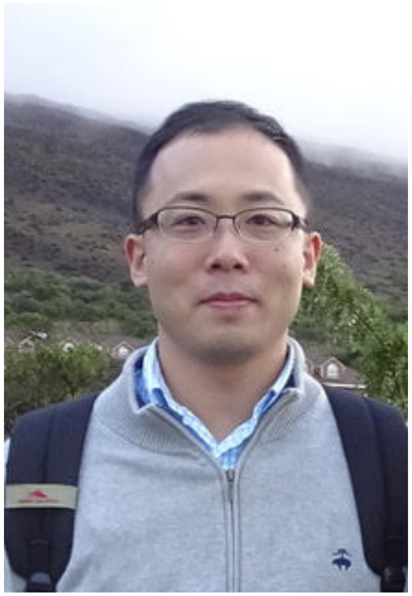
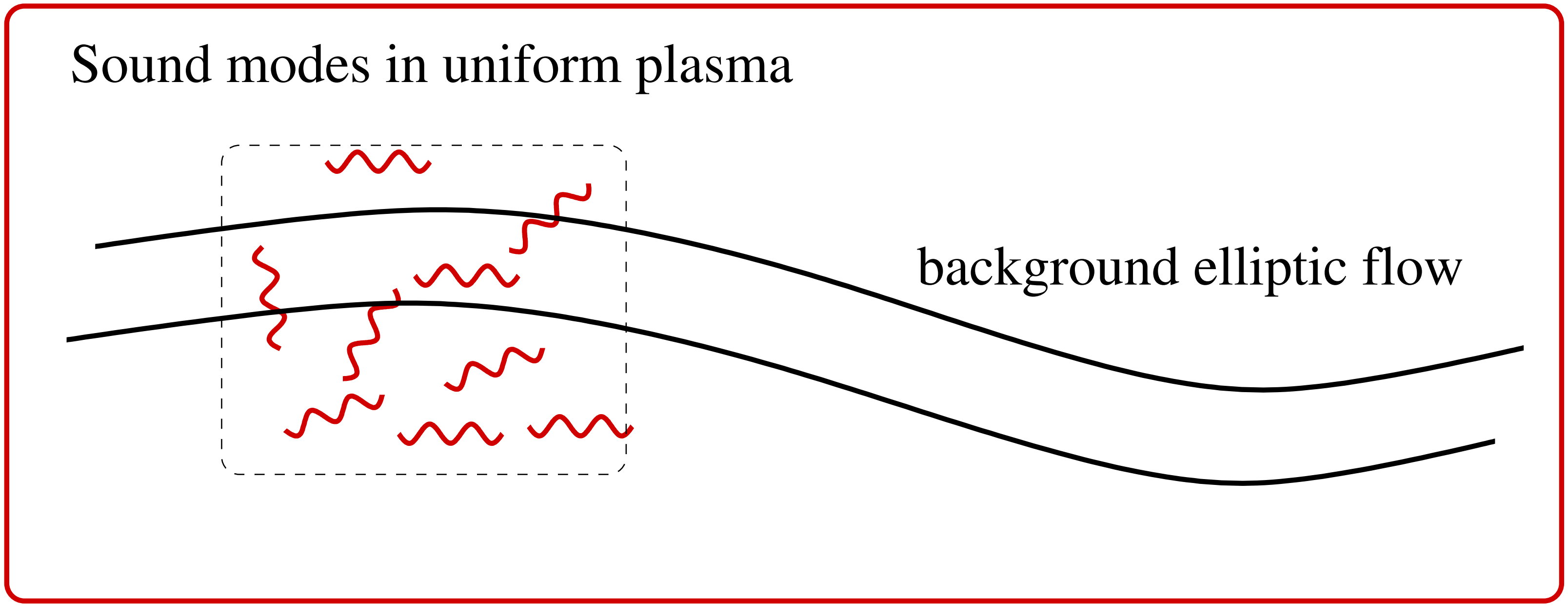
- ⋆ Yukinao Akamatsu, Aleksas Mazeliauskas, DT, PRC, arXiv:1606.07742
- ⋆ Yukinao Akamatsu, Aleksas Mazeliauskas, DT; PRC, arXiv:1708.05657
- ⋆ Y. Akamatsu, DT, Fanglida Yan, Yi Yin; almost done. See Quark Matter
- ⋆ Y. Akamatsu, DT, Fanglida Yan, Juan-Tores Ricon; in progress



Thermal fluctuations and longtime tails:

Kovtun, Yaffe; Kovtun, Moore, Romatschke



These hard sound modes are part of the bath, adding to the pressure and shear viscosity

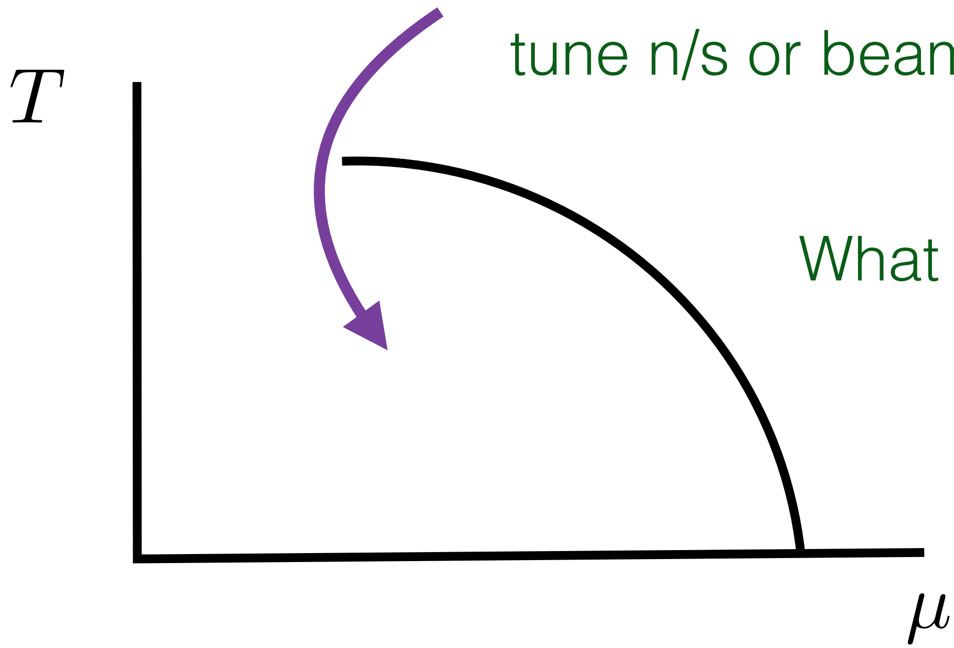
$$N^{\rm eq}_{ee}(\boldsymbol{k},t) \equiv \underbrace{\langle e^*(\boldsymbol{k},t)e(\boldsymbol{k},t)\rangle}_{\text{energy-density flucts}} = T^2 c_v$$

$$N^{\rm eq}_{gg}(\boldsymbol{k},t) \equiv \underbrace{\left\langle g^{*i}(\boldsymbol{k},t)g^{j}(\boldsymbol{k},t)\right\rangle}_{\text{momentum, } g^i \equiv T^{0i}} = (e+p)T\delta^{ij}$$

In a driven system these correlators will fall out of equilibrium.

We should calculate this.

How are thermal fluctuations distorted by the expansion near the critical point?



tune n/s or beam energy to go through CP

What if we miss it?

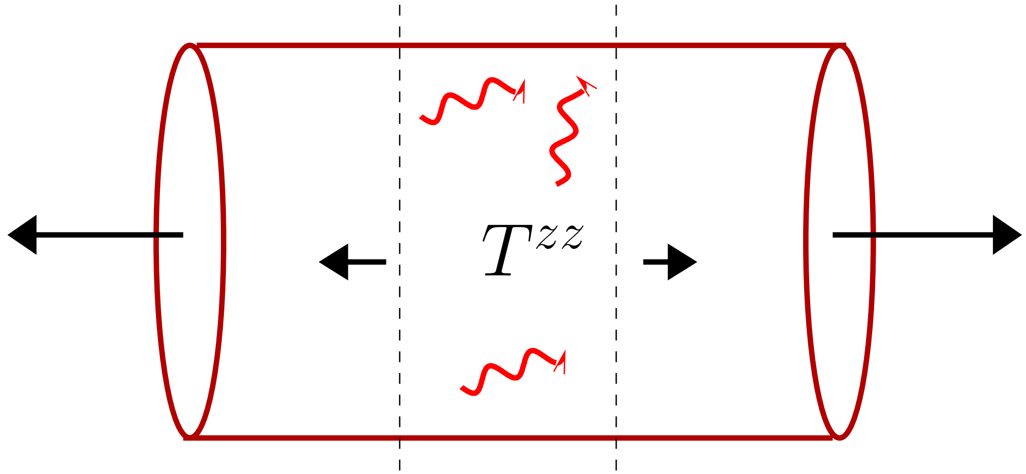
Expanding?

See a great talk at Quark Matter by Fanglida Yan (my student)

Today: chiral symmetry breaking plays no role in our hydro model.

Seems wrong to me!

A prototypical driven system: the Bjorken expansion



1. The system has an expansion rate of $\partial_\mu u^\mu = 1/\tau$

2. The hydrodynamic expansion parameter is

$$\epsilon \equiv \frac{\gamma_\eta}{\tau} \ll 1 \qquad \gamma_\eta \equiv \frac{\eta}{e+p} \sim \text{typical relaxation time}$$

   and corrections to hydrodynamics are organized in powers of ϵ

$$T^{zz} = p\Big[1 + \underbrace{\mathcal{O}(\epsilon)}_{\text{1st order}} + \underbrace{\mathcal{O}(\epsilon^2)}_{\text{2nd order}} + \ldots\Big]$$

High k modes are brought to equilibrium by the dissipation and noise

The transition regime:

⋆ There is a wave number where the damping rate competes with the expansion

$$\underbrace{\gamma_\eta k^2}_{\text{damping rate}} \sim \underbrace{\frac{1}{\tau}}_{\text{expansion rate}}$$

and thus the transition happens for: $\gamma_\eta \equiv \eta/(e+p)$

$$k \sim k_* \equiv \frac{1}{\sqrt{\gamma_\eta \tau}}$$

need $k \gg k_*$ to reach equilibrium!

⋆ This is an intermediate scale $k_* \equiv 1/(\tau\sqrt{\epsilon})$, $\epsilon \equiv \eta/(e+p)\tau$

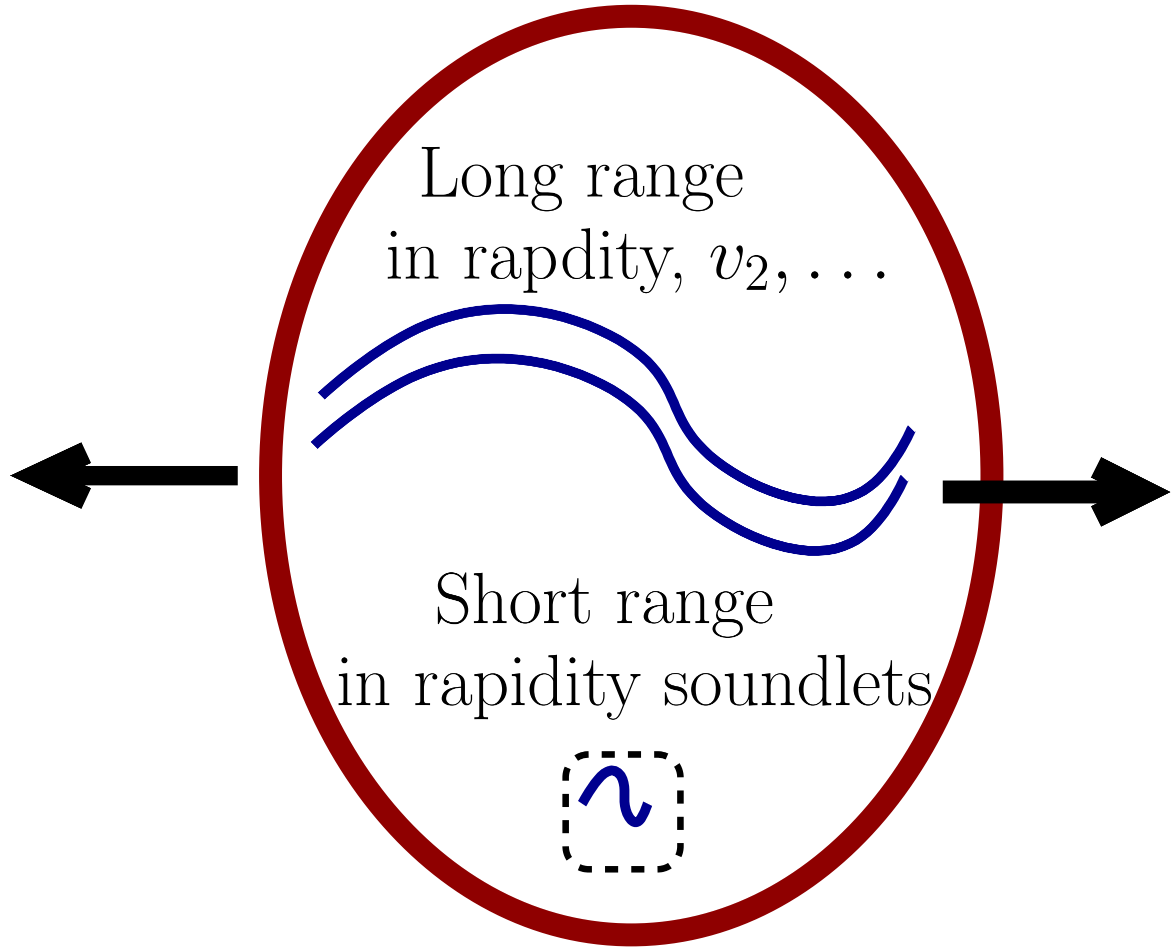
these are the same

$$\frac{1}{\tau} \ll k_* \ll \frac{1}{\ell_{\mathrm{mfp}}}$$

$$\epsilon \ll \sqrt{\epsilon} \ll 1$$

We will determine the phase-space density of sound modes with $k \sim k_*$

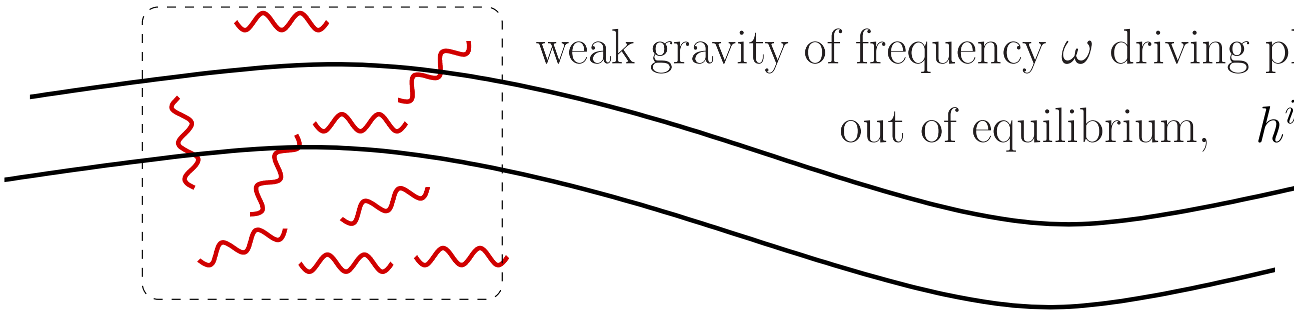
(using the scale separation $\epsilon \ll \sqrt{\epsilon} \ll 1$ to simplify the problem)



$$\underbrace{\ell_{\rm mfp}}_{\text{microscopic}} \ll \underbrace{\frac{1}{k_*}}_{\text{soundlets}} \ll \underbrace{R}_{\text{macroscopic flow } v_2,\, v_3\, \ldots}$$

Equilibrium Linear Response

Kovtun, Yaffe; Kovtun, Moore, Romatschke



Hydro prediction

$$\langle T^{ij}\rangle = -p h^{ij} - \eta\Big(\overbrace{\nabla^i u^j}^{\text{cov-deriv}} + \nabla^j u^i - \frac{2}{3}\nabla\cdot u\Big) + \text{2nd order}$$

So

$$\langle T^{xy}(\omega)\rangle = -\Big[p + \overbrace{i\omega\eta}^{\text{1st order}} + \overbrace{(\eta\tau_\pi - \tfrac{1}{2}\kappa)\omega^2}^{\text{2nd order}}\Big] h^{xy}(\omega)$$

Thermal flucts. are not included, and are driven slightly out of equilibrium for $k \sim k_*$

$$\frac{\omega}{c_s} \ll \quad k_* \sim \sqrt{\frac{\omega}{\gamma_\eta}} \ll \frac{1}{\ell_{\rm mfp}}$$

I will develop a kinetic theory for thermal fluctuations with $k_* \sim \sqrt{\omega/\gamma_\eta}$

Evolving the phase space density of sound – linearized (stochastic) hydro in a box

1. Evolve fields of linearized hydro with bare parameters $p_0(\Lambda)$, $\eta_0(\Lambda)$, $s_0(\Lambda)$ etc

$$\phi_a(\boldsymbol{k}) \equiv \Big(e(\boldsymbol{k}), g^x(\boldsymbol{k}), g^y(\boldsymbol{k}), g^z(\boldsymbol{k}) \Big)$$

2. Then the equations are schematically the same as the Brownian motion

$$\frac{d\phi_a(\boldsymbol{k})}{dt} = \underbrace{\mathcal{L}_{ab}(\boldsymbol{k})}_{\text{ideal} \sim c_s k} \phi_b(\boldsymbol{k}) + \underbrace{D_{ab}\phi_b}_{\text{visc} \sim -\eta_0 k^2} + \xi_a \qquad \langle \xi_a \xi_b \rangle = 2T\mathcal{D}_{ab}(\boldsymbol{k})\delta_{tt'}$$

3. Break up the equations into eigen modes of $\mathcal{L}_{ab}$, and analyze exactly same way:

$$\underbrace{\text{right moving sound}}_{\lambda_+ = +ic_s k} \qquad \underbrace{\text{left moving sound}}_{\lambda_- = -ic_s k} \qquad \underbrace{\text{two diffusion modes}}_{\lambda_T = 0}$$

So for k in the z direction, work with the following linear combos (eigenvects)

$$\phi_A \equiv \Big[\underbrace{e(\boldsymbol{k}) \pm \frac{1}{c_s} g^z(\boldsymbol{k})}_{\phi_+ \text{ and } \phi_-} , \underbrace{g^x(\boldsymbol{k})}_{\equiv \phi_{T_1}} , \underbrace{g^y(\boldsymbol{k})}_{\equiv \phi_{T_2}} \Big]$$

The hydro-kinetic equations without expansion

1. Compute how the phase-space density of sound (squared amplitude) evolves:

$$N_{++}(\boldsymbol{k},t) = \left\langle \phi_+^*(\boldsymbol{k},t)\phi_+(\boldsymbol{k},t)\right\rangle \qquad N_{T_1T_1} = \left\langle \phi_{T_1}^*(\boldsymbol{k},t)\phi_{T_1}(\boldsymbol{k},t)\right\rangle$$

2. The phase space distribution evolution (hydro-kinetic equation):

$$\underbrace{\frac{dN_{++}}{dt}}_{\text{phase-space dist}} = \underbrace{-\tfrac{4}{3}\gamma_\eta k^2\left[N_{++} - N_{++}^{\rm eq}\right]}_{\text{damping to equilibrium}}$$

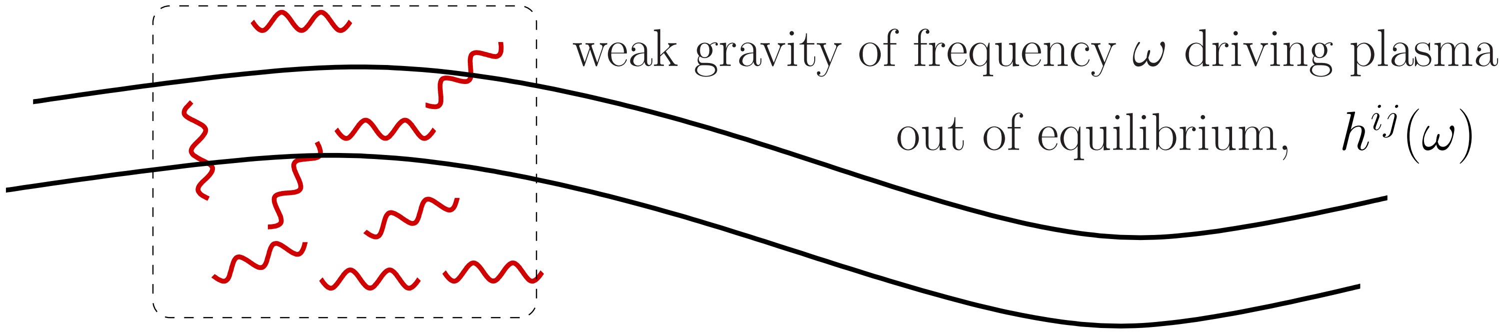
   and similar equations for N_{--}, $N_{T_1T_1}$ and $N_{T_2T_2}$. Here

$$N_{++}^{\rm eq} \equiv T^2 c_v = \text{equilibrium}$$

3. Neglect (rapidly rotating) off diagonal components of density matrix e.g. N_{+T_1}

Solving relaxation time kinetic equations is much easier than hydro!

Kinetic equations for perturbed system



Hydro equations become $\phi_a \equiv \Big(e(\boldsymbol{k}), g^x(\boldsymbol{k}), g^y(\boldsymbol{k}), g^z(\boldsymbol{k})\Big)$

$$\frac{d\phi_a(\boldsymbol{k})}{dt} = \underbrace{\mathcal{L}_{ab}(\boldsymbol{k})\phi_b(\boldsymbol{k})}_{\text{ideal}} + \underbrace{D_{ab}\phi_b}_{\text{visc}} + \underbrace{\xi_a}_{\text{noise}} + \underbrace{\mathcal{P}_{ab}\phi_b}_{\text{perturbation}}$$

with

$$\mathcal{P}_{ab} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}\partial_t h_{ij} \end{pmatrix}, \qquad h_{ij}(t) = \text{metric perturbation}$$

Kinetic equations for perturbed system:

- ⋆ Turn on a weak gravitational perturbations, $h_{ij} = h(t)\,\mathrm{diag}(1,1,-2)$

$$\partial_t N_{++}(k) = -\underbrace{\tfrac{4}{3}\gamma_\eta k^2\left[N_{++} - N_{++}^{\rm eq}\right]}_{\text{damping}} - \underbrace{\partial_t h\,(\sin^2\theta_k - 2\cos^2\theta_k)}_{\text{perturbation}\quad h_{ij}\hat{k}^i\hat{k}^j} N_{++}$$

- ⋆ Solve the equations to first order in the gravitational, $h(t) = he^{-i\omega t}$

$$\delta N_{++} = \frac{i\omega h\,(\sin^2\theta_k - 2\cos^2\theta_k) N_{++}^{\rm eq}}{-i\omega + \frac{4}{3}\gamma_\eta K^2} \qquad \Longleftarrow \qquad \text{solution}$$

- ⋆ Calculate the stress tensor

$$\delta T^{ij} = (e+p)\left\langle v^i v^j\right\rangle = \int \frac{d^3K}{(2\pi)^3}\frac{\left\langle g^i(\boldsymbol{k})g^j(-\boldsymbol{k})\right\rangle}{e+p}$$

- ⋆ Find an HTL like expression

$$\langle \delta T^{xx} + \delta T^{yy} - 2\delta T^{zz}\rangle \supset \int \frac{d^3K}{(2\pi)^3}\,\delta N_{++}\,\underbrace{(\sin^2\theta - 2\cos^2\theta)}_{\hat{k}^x\hat{k}^x + \hat{k}^y\hat{k}^y - 2\hat{k}^z\hat{k}^z}$$

Precisely reproduces Yaffe-Kovtun hydro loop calculation

1. Kovtun-Yaffe

$$\langle T^{xy}(\omega)\rangle = -\Big[p - \underbrace{i\omega\eta}_{\text{1st order}} + \underbrace{\frac{(7+(3/2)^{3/2})}{240\pi}T\left(\frac{\omega}{\gamma_\eta}\right)^{3/2}}_{\text{3/2 order}} + \underbrace{\mathcal{O}(\omega^2)}_{\text{2nd order}}\Big]h^{xy}(\omega)$$

2. For a Bjorken expansion expansion in $1/\tau$:

$$\frac{\langle T^{zz}\rangle}{e+p} = \Big[\underbrace{\frac{p}{e+p}}_{\sim 1} - \underbrace{\frac{4}{3}\frac{\gamma_\eta}{\tau}}_{\text{1st order}} + \underbrace{\frac{1.08318}{s\,(4\pi\gamma_\eta\tau)^{3/2}}}_{\text{non-universal 3/2 order}} + \underbrace{\frac{(\lambda_1-\eta\tau_\pi)}{e+p}\frac{8}{9\tau^2}}_{\text{2nd order}}\Big]$$

Non-universal power law terms

dominate beyond first order in the gradient expansion

QCD and the Chiral limit and Broken Symmetry: Son hep-ph/9912267; Son and Stephanov hep-ph/020422

1. The approximately conserved quantities

$$\hat{j}^{\mu}_{a} = \bar{\psi}\gamma^{5}\gamma^{\mu}\tau^{a}\psi$$

$$\underbrace{T^{\mu\nu}}_{\text{stress}} \qquad \underbrace{J^{\mu}_{B}}_{\text{Baryon number}} \qquad \underbrace{J^{\mu}_{a}}_{\text{isovector}} \qquad \underline{\text{and}} \qquad \underbrace{\hat{J}^{\mu}_{a}}_{\text{iso-axial vector}}$$

2. There is the phase of the chiral condensate and associated pion field $\varphi^a = \pi^a/f$

$$U = \text{Phase of } \langle \bar{q} q \rangle \equiv e^{i\tau^a \pi^a/f}$$

3. The pion field φ^a is an field like β, $\vec{u}$, μ_B, μ_I, and $\hat{\mu}$ in the constitutive relations

4. Include a mass term so the Goldstone fields decay at large distances

Need to write down a theory of superfluid hydro (Son '99)

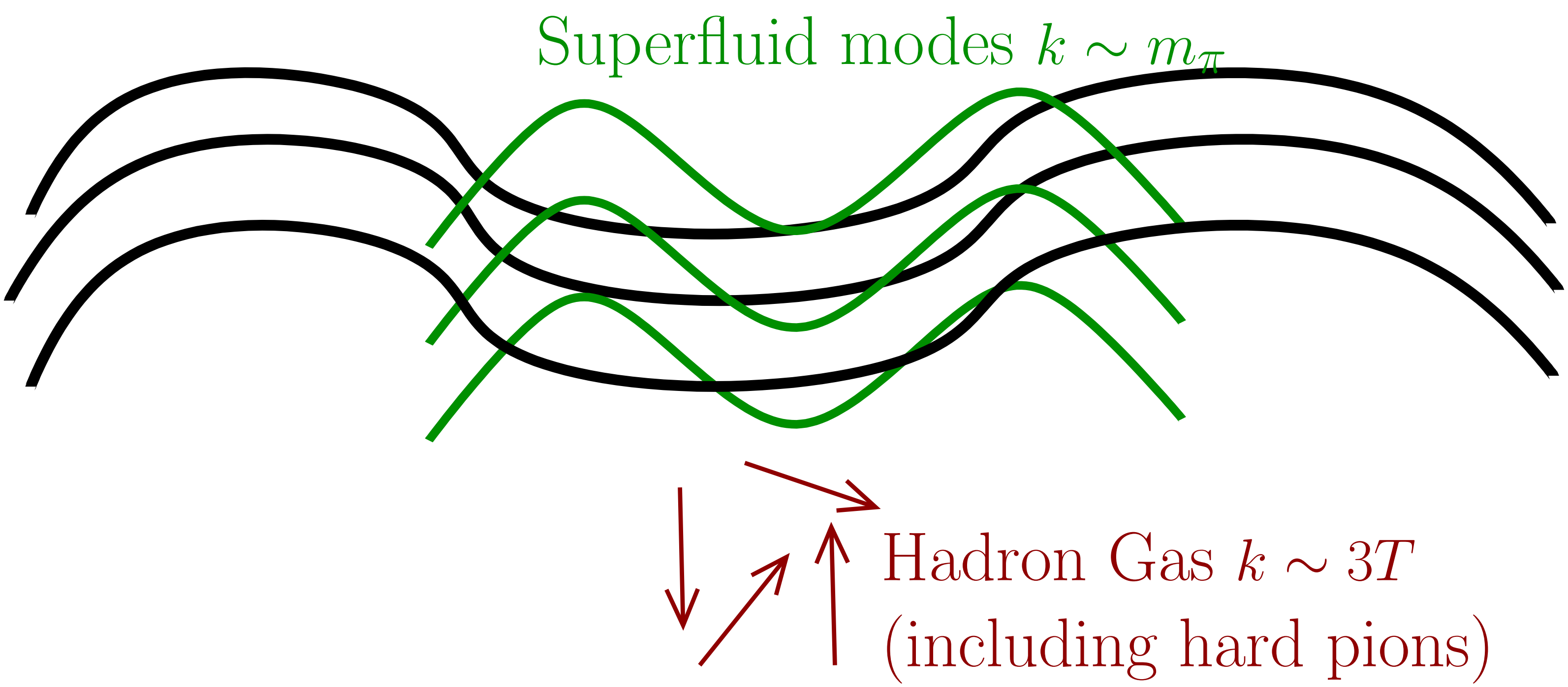
Equilibrium picture:

Son and Stephanov hep-ph020422

⋆ Work in the regime

$$k_* \ll m_\pi \ll \pi T \sim \pi \Lambda_{QCD}$$

equilibrated hydro modes $k_* \ll k \ll m_\pi$



How do these superfluid modes contribute to pressure and (bulk) viscosity?

The pressure from soft modes:

⋆ Use 3D dimensionally reduced chiral perturbation theory

$$Z_{QCD} = \underbrace{e^{\beta p_0(T,\hat{\mu})V}}_{\text{from hard modes } p \sim T} \times \underbrace{\int^{\Lambda} [D\varphi] \exp\left(-\beta \int d^3\boldsymbol{x} \mathcal{L}_{\text{eff}}\right)}_{\text{from soft modes } p \sim m_\pi T}$$

then using $U = e^{i\varphi_a \tau_a}$

$$\mathcal{L}_{\text{eff}} \simeq \frac{f^2}{4} \operatorname{Tr} \vec{\nabla} U \cdot \vec{\nabla} U^\dagger + \frac{f^2 m^2}{2} \operatorname{Re} \operatorname{Tr} U \quad \Rightarrow \quad \frac{f^2}{2} (\nabla \varphi_a)^2 + \frac{f^2 m^2}{2} \varphi_a^2$$

⋆ leading to a correction to the QCD pressure

$$p(T,\hat{\mu}) = \underbrace{p_0(T,\hat{\mu})}_{\text{analytic in } m_q} + \underbrace{\frac{T m^3}{4\pi}}_{\propto m_q^{3/2}} + \mathcal{O}(m^4)$$

The effective Lagrangian sums all non-analytic terms in the quark mass!

Stress and Current for Superfluids:

Son; Jensen et al 1203.3556, Bhattacharya et al

- The pressure in the presence of the phase is

$$p_\varphi(T,\nabla\varphi,m^2\varphi^2) = p_0(T) + \frac{1}{2}\chi\hat{\mu}^2 - \frac{f^2}{2}(\nabla\varphi)^2 + \frac{f^2m^2}{2}\varphi^2$$

- Derive the ideal stress and current from pressure

$W = \int d^4x\sqrt{g}p_\varphi$

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta W}{\delta g_{\mu\nu}} = (e_\varphi + p_\varphi)u^\mu u^\nu + \eta^{\mu\nu}p_\varphi + \underbrace{f^2\partial^\mu\varphi\partial^\nu\varphi}_{\text{super fluid stress}}$$

$$\hat{J}^\mu_a = \frac{1}{\sqrt{-g}}\frac{\partial W}{\partial A_\mu} = n_a u^\mu - \underbrace{f\partial^\mu\varphi_a}_{\text{super fluid current}}$$

- In addition find from an extension of the Jensen et al formalism

$$\underbrace{u^\mu\partial_\mu\varphi_a = \mu_a}_{\text{Josephson constraint}} \qquad \text{and} \qquad \underbrace{\partial_\mu\hat{J}^\mu_a = f^2m^2\varphi_a}_{\text{PCAC}}$$

Dissipative corrections:

Son and Stephanov hep-ph/020422 + a tiny bit by us

⋆ PCAC in the rest frame with dissipative hydro

$$\underbrace{\partial_t \hat{J}^0 + \nabla \cdot \hat{\boldsymbol{J}} = S}_{\text{not conserved}} \qquad \text{and} \qquad \underbrace{\partial_t \varphi_a = \hat{\mu}_a}_{\text{Joseph constraint}}$$

⋆ Then expand the current

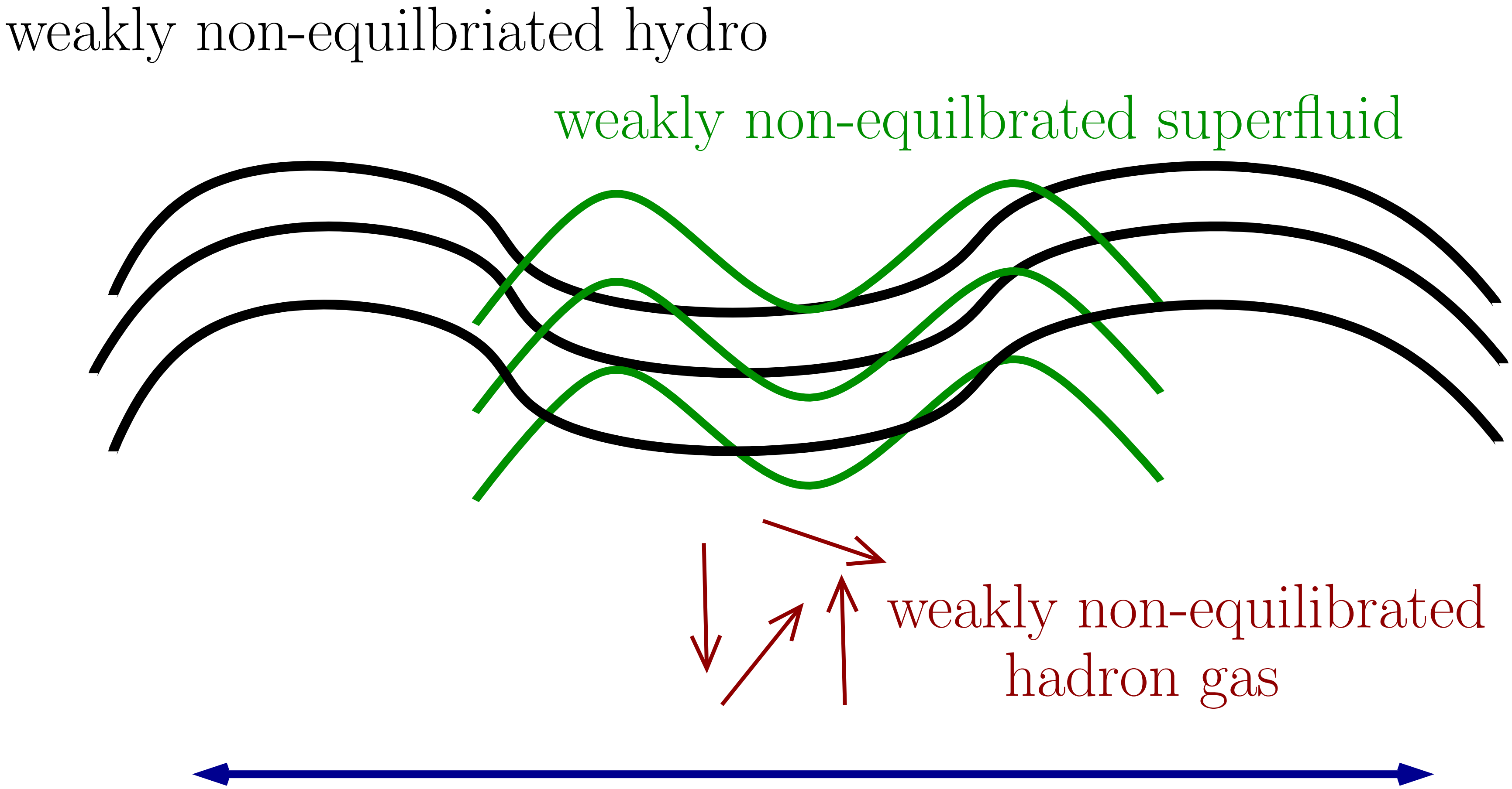
$$\hat{\boldsymbol{J}} = \underbrace{-f\nabla\varphi}_{\text{ideal current}} - \underbrace{\lambda_1 \nabla \hat{\mu}}_{\text{normal diffusivity}} + \underbrace{\vec{\xi}_J}_{\text{normal noise}}$$

and the source

$$S = \underbrace{f^2 m^2 \varphi}_{\text{ideal fluid}} - \underbrace{\lambda_2 m^2 \hat{\mu}}_{\text{correction}} + \underbrace{\xi_S}_{\text{noise}}$$

⋆ To reach the equilibrium fluctuations we must have $\langle (\delta \hat{Q})^2 \rangle \equiv \hat{\chi}$

$$\left\langle \xi_J^i \xi_J^j \right\rangle = 2T\lambda_1 \, \hat{\chi} \, \delta^{ij} \delta^4(x - x') \qquad \langle \xi_S \xi_S \rangle = 2T\lambda_2 \, \hat{\chi} \, \delta^4(x - x')$$



Drive the system with gravity $h_{ij}(\omega) = he^{-i\omega t}\delta_{ij}$

Superfluid fluctuations and shorter are <u>absorbed</u> into the transport coefficients

$$\frac{1}{3}\left\langle \delta T^{i}_{i} \right\rangle = -\zeta \nabla \cdot U = +i\omega \tfrac{3}{2} h(\omega)\zeta$$

Superfluid-hydro kinetics with gravity

⋆ There are two eigen modes known as second sound: $\omega_q^2 = f^2(q^2+m^2)/\hat{\chi}$

$$\hat{\phi}_{\pm} = \hat{J}^0 \pm i\omega_q\,\hat{\chi}\,\varphi \qquad \text{with} \qquad \hat{v}_{\pm} = \pm\sqrt{\frac{f^2}{\hat{\chi}}}$$

⋆ The correlators $\hat{N}_{++} = \left\langle \hat{\phi}_+ \hat{\phi}_+ \right\rangle$ satisfy:

$$\frac{d\hat{N}_{++}}{dt} = -\underbrace{\frac{\lambda_1 q^2 + \lambda_2 m^2}{\hat{\chi}}}_{\text{damping rate}} \left[\hat{N}_{++} - T\hat{\chi}\right] - \frac{3}{2}\partial_t h\,\hat{N}_{++}$$

⋆ Solve to find the deviation from equilibrium at small frequency

$$\delta\hat{N}_{++}(\mathbf{q},t) = \underbrace{\frac{T\chi}{\hat{D}q^2 + \kappa m^2} \times i\omega\tfrac{3}{2}h(\omega)}_{\text{this is } \delta f_{\rm bulk}!} \qquad \hat{D} \equiv \frac{\lambda_1}{\hat{\chi}} \quad \kappa \equiv \frac{\lambda_2}{\hat{\chi}}$$

Estimate for the bulk viscosity

- ⋆ We may estimate their contribution to the stress

$$\langle \delta T^i_i \rangle \sim f^2 \langle \partial^i \varphi \partial_i \varphi \rangle$$

- ⋆ With the $\delta f_{\rm bulk}$ we find:

$$\langle \delta T^i_i \rangle \sim \int \frac{d^3q}{(2\pi)^3} \underbrace{\frac{q^2}{q^2+m^2}}_{\text{kinematics}} \times \underbrace{\frac{T}{\hat{D}q^2 + \kappa m^2}(\nabla \cdot U)}_{\propto\, \delta f_{\rm bulk}}$$

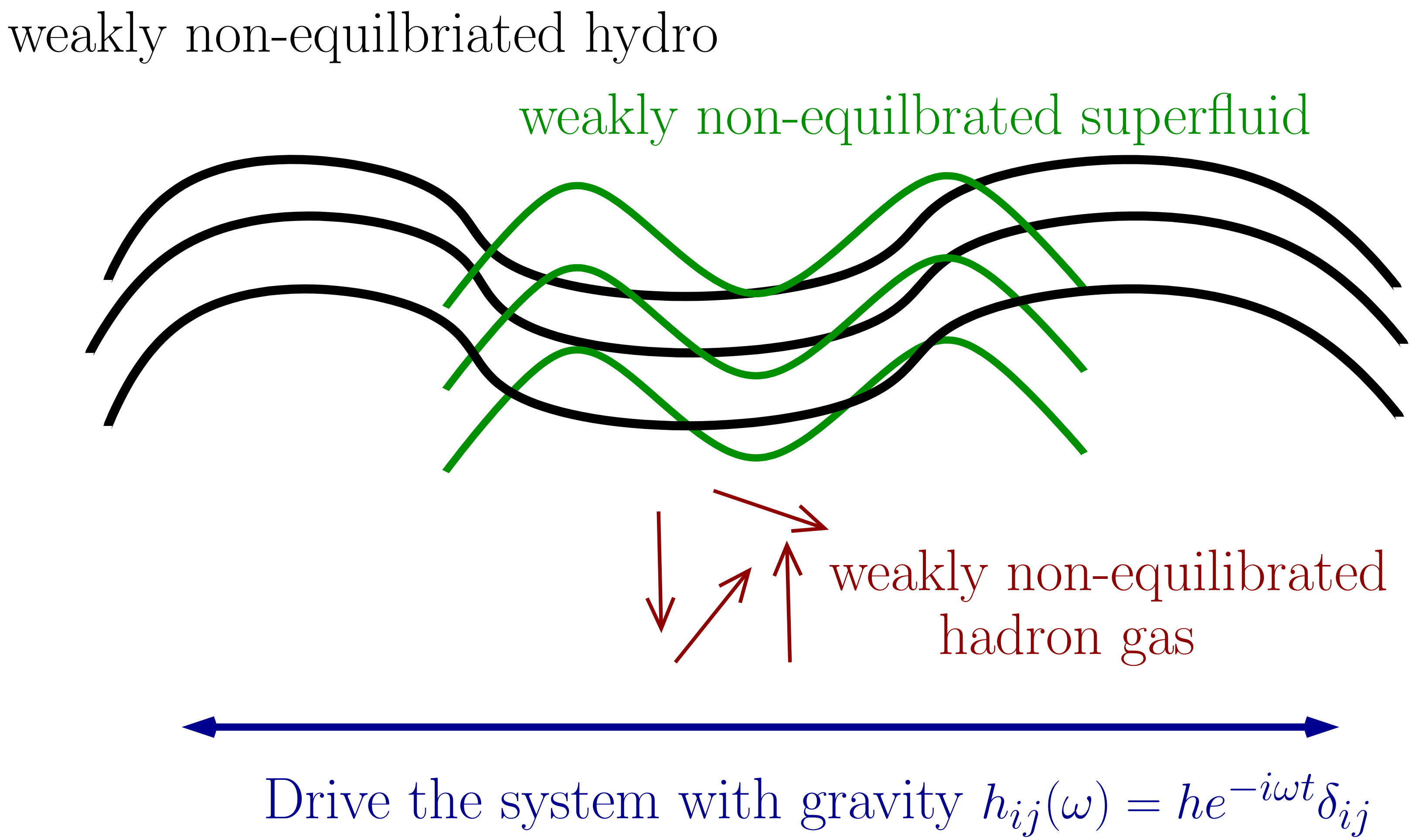
- ⋆ Find

$$\langle \delta T^i_i \rangle \sim \underbrace{\frac{Tm}{\hat{D}} \left(\frac{\kappa}{\hat{D}} \right)^{1/2}}_{\text{Bulk viscosity } \zeta} \times \nabla \cdot U$$

This is the leading quark mass dependence $\zeta \propto \sqrt{m_q}$.

A complete answer will wait for another day

A picture and questions



What happens near the $O(4)$ critical point?

Interplay of Ising-like and chiral modes?