

Second-order dissipative magneto-hydrodynamics

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with

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Definitions

“Dictionary”

- Non-resistive: electric conductivity $\sigma_E \rightarrow \infty \implies$ “ideal” MHD
- Resistive: electric conductivity $0 < \sigma_E < \infty \implies$ resistive MHD
- Fluid-dynamical transport coefficients: $\sim \lambda_{\text{mfp}}$ mean free path
(as opposed to “thermodynamical” transport coefficients, see P. Kovtun’s talk)
- Non-dissipative: all fluid-dynamical transport coefficients vanish
 \implies “ideal” fluid dynamics
- Dissipative: (some) fluid-dynamical transport coefficients non-zero
 \implies dissipative/viscous fluid dynamics
- Second-order dissipative: relaxation equations for dissipative currents

\implies successively reduce idealizing constraints:

Non-resistive, non-dissipative MHD

\implies Non-resistive, second-order dissipative MHD

G.S. Denicol, X.-G. Huang, E. Molnár, G.M. Monteiro, H. Niemi, J. Noronha, DHR,
Q. Wang, arXiv: 1804.05210

\implies Resistive, second-order dissipative MHD

G.S. Denicol, E. Molnár, H. Niemi, DHR, in preparation

Single-component fluid of point-like particles with spin zero and mass m

Particle current and energy-momentum tensor of fluid

$$N_f^\mu \equiv \int dK k^\mu f_k = n u^\mu + V^\mu$$

$$T_f^{\mu\nu} \equiv \int dK k^\mu k^\nu f_k = \varepsilon u^\mu u^\nu - P \Delta^{\mu\nu} + \pi^{\mu\nu}$$

- $k^\mu = (k^0, \mathbf{k})$ four-momentum of particles, $k^0 = \sqrt{\mathbf{k}^2 + m^2}$ on-shell energy
- $dK = d^3 \mathbf{k} / [(2\pi)^3 k^0]$
- f_k single-particle distribution function in momentum space
- u^μ fluid four-velocity \implies taken to be energy flow (Landau frame), $T_f^{\mu\nu} u_\nu = \varepsilon u^\mu$
- $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ 3-space projector orthogonal to u^μ
- $n \equiv N_f^\mu u_\mu$ particle density in LRF, where $E_k \equiv k^\mu u_\mu$ particle energy in LRF
- $\varepsilon \equiv T_f^{\mu\nu} u_\mu u_\nu$ energy density in LRF
- $P \equiv -\frac{1}{3} T_f^{\mu\nu} \Delta_{\mu\nu}$ isotropic pressure
- $V^\mu \equiv N_f^{\langle\mu\rangle}$ particle diffusion current, where $A^{\langle\mu\rangle} \equiv \Delta^{\mu\nu} A_\nu$
- $\pi^{\mu\nu} \equiv \Delta_{\alpha\beta}^{\mu\nu} T_f^{\alpha\beta}$ shear-stress tensor, where
 $\Delta_{\alpha\beta}^{\mu\nu} \equiv \frac{1}{2} \left(\Delta_\alpha^\mu \Delta_\beta^\nu + \Delta_\beta^\mu \Delta_\alpha^\nu \right) - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta}$ rank-4 symmetric, traceless 3-space projector orthogonal to u^μ

Conservation equations in MHD

Introduce electric charge of particles q

Charge current

$$\mathfrak{J}^\mu \equiv q N_f^\mu = n u^\mu + \mathfrak{v}^\mu$$

- $n \equiv q n$ charge density in LRF, $\mathfrak{v}^\mu \equiv q V^\mu$ charge diffusion current

Energy-momentum tensor of electromagnetic field

$$T_{em}^{\mu\nu} = -F^{\mu\lambda} F_\lambda^\nu + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

Conservation equations

$$\begin{aligned}\partial_\mu \mathfrak{J}^\mu &= 0 \\ \partial_\nu T_f^{\mu\nu} &= F^{\mu\nu} \mathfrak{J}_\nu \\ \partial_\nu T_{em}^{\mu\nu} &= -F^{\mu\nu} \mathfrak{J}_\nu \\ \Rightarrow \partial_\nu T^{\mu\nu} &= 0\end{aligned}$$

- $T^{\mu\nu} = T_f^{\mu\nu} + T_{em}^{\mu\nu}$ total energy-momentum tensor

Maxwell's equations

Maxwell's equations

$$\begin{aligned}\partial_\mu F^{\mu\nu} &= \mathcal{J}^\nu \\ \partial_\mu \tilde{F}^{\mu\nu} &= 0\end{aligned}$$

- $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ dual field-strength tensor

Electric and magnetic field strengths

$$\begin{aligned}F^{\mu\nu} &= E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta \\ \tilde{F}^{\mu\nu} &= B^\mu u^\nu - B^\nu u^\mu - \epsilon^{\mu\nu\alpha\beta} u_\alpha E_\beta\end{aligned}$$

- $E^\mu \equiv F^{\mu\nu} u_\nu$ electric field four-vector, $E^\mu u_\mu = 0$, $E_{\text{LRF}}^\mu = (0, \mathbf{E})$
- $B^\mu \equiv \tilde{F}^{\mu\nu} u_\nu$ magnetic field four-vector, $B^\mu u_\mu = 0$, $B_{\text{LRF}}^\mu = (0, \mathbf{B})$

⇒ for given charge current \mathcal{J}^μ , Maxwell's equations determine the $2 \times 3 = 6$ independent components of E^μ , B^μ

⇒ Since there are 5 conservation equations:

need 9 additional equations to determine all 14 components of N_f^μ , $T_f^{\mu\nu}$

Non-resistive MHD

Assume electric conductivity $\sigma_E \rightarrow \infty$

- ⇒ induced current $\mathfrak{J}_{ind}^\mu \equiv \sigma_E E^\mu$ needs to stay finite $\implies E^\mu \rightarrow 0$
- ⇒ only magnetic field $B^\mu \neq 0$

Introduce

- $b^\mu \equiv \frac{B^\mu}{B}$, $B \equiv \sqrt{-B^\mu B_\mu}$ $\implies b^\mu u_\mu = 0$, $b^\mu b_\mu = -1$
- $b^{\mu\nu} \equiv -\epsilon^{\mu\nu\alpha\beta} u_\alpha b_\beta \equiv -\frac{F^{\mu\nu}}{B}$ $\implies b^{\mu\nu} u_\mu = b^{\mu\nu} u_\nu = 0$
- $\Xi^{\mu\nu} \equiv \Delta^{\mu\nu} + b^\mu b^\nu$ 2-space projector orthogonal to u^μ and b^μ $\implies b^{\mu\alpha} b^\nu_\alpha = \Xi^{\mu\nu}$

Equation of motion for fluid energy

$$u_\mu \partial_\nu T_f^{\mu\nu} = -B b^{\mu\nu} \mathfrak{J}_\nu u_\mu = 0$$

- ⇒ fluid energy is conserved!

Equation of motion for fluid momentum

$$\Delta_\mu^\alpha \partial_\nu T_f^{\mu\nu} = -B b^{\alpha\nu} (\mathfrak{n} u_\nu + \mathfrak{v}_\nu) = -B b^{\alpha\nu} \mathfrak{v}_\nu$$

- ⇒ fluid momentum changes only through coupling of B with \mathfrak{v}^μ !
- ⇒ non-resistive, non-dissipative MHD: $\mathfrak{v}^\mu \equiv 0 \implies$ fluid & field evolution decouple!

Relevant scales

Boltzmann equation

$$\lambda_{\text{mfp}} \gg \ell_{\text{int}}$$

- $\lambda_{\text{mfp}} \sim (\sigma n)^{-1}$, σ cross section, n particle density
- $\ell_{\text{int}} \sim \sqrt{\sigma/\pi}$ interaction length

Since $n \sim \beta_0^{-3}$, where $\beta_0 \equiv 1/T$ thermal wavelength

$$\Rightarrow \lambda_{\text{mfp}} \sim \beta_0^3 / \ell_{\text{int}}^2 \gg \ell_{\text{int}} \Rightarrow \beta_0 \gg \ell_{\text{int}} \quad \text{dilute limit}$$

Magnetic field

$$R_T \equiv (qB\beta_0)^{-1} \gg \beta_0$$

- R_T Larmor radius for particle with electric charge q and transverse momentum $k_T \equiv \beta_0^{-1}$ in magnetic field B ("thermal Larmor radius")

$\Rightarrow \sqrt{qB} \ll T$ weak-field limit \Rightarrow allows to neglect Landau quantization

Ordering of scales

$$R_T \gg \beta_0 \gg \ell_{\text{int}}$$

Define

$$\xi_B \equiv \lambda_{\text{mfp}} / R_T \equiv qB\beta_0\lambda_{\text{mfp}}$$

$$\Rightarrow \xi_B \sim (\beta_0 / \ell_{\text{int}})^2 (\beta_0 / R_T)$$

\Rightarrow study transport coefficients as function of ξ_B

In external electromagnetic field with field-strength tensor $F^{\mu\nu}$,
single-particle distribution function $f_{\mathbf{k}}$ satisfies:

Relativistic Boltzmann equation

$$k^\mu \partial_\mu f_{\mathbf{k}} + q F^{\mu\nu} k_\nu \frac{\partial}{\partial k^\mu} f_{\mathbf{k}} = C[f_{\mathbf{k}}]$$

Collision term

$$C[f_{\mathbf{k}}] = \frac{1}{2} \int dK' dP dP' W_{\mathbf{k}\mathbf{k}' \rightarrow \mathbf{p}\mathbf{p}'} \left(f_{\mathbf{p}} f_{\mathbf{p}'} \tilde{f}_{\mathbf{k}} \tilde{f}_{\mathbf{k}'} - f_{\mathbf{k}} f_{\mathbf{k}'} \tilde{f}_{\mathbf{p}} \tilde{f}_{\mathbf{p}'} \right)$$

- $\tilde{f}_{\mathbf{k}} \equiv 1 - af_{\mathbf{k}}$, with $a = 0, \pm 1$ for Boltzmann, Fermi/Bose statistics
- Transition rate satisfies $W_{\mathbf{k}\mathbf{k}' \rightarrow \mathbf{p}\mathbf{p}'} = W_{\mathbf{k}\mathbf{k}' \rightarrow \mathbf{p}\mathbf{p}} = W_{\mathbf{p}\mathbf{p}' \rightarrow \mathbf{k}\mathbf{k}'}$

Method of moments

DNMR: G.S. Denicol, H. Niemi, E. Molnár, DHR, PRD85 (2012) 114047

Expansion around local equilibrium

$$f_{\mathbf{k}} = f_{0\mathbf{k}} + \delta f_{\mathbf{k}}, \quad f_{0\mathbf{k}} = [\exp(\beta_0 E_{\mathbf{k}} - \alpha_0) + a]^{-1}$$

⇒ write Boltzmann equation in the form

$$\delta \dot{f}_{\mathbf{k}} = -\dot{f}_{0\mathbf{k}} - E_{\mathbf{k}}^{-1} k_{\nu} \nabla^{\nu} (f_{0\mathbf{k}} + \delta f_{\mathbf{k}}) - E_{\mathbf{k}}^{-1} q B b^{\mu\nu} k_{\nu} \frac{\partial \delta f_{\mathbf{k}}}{\partial k^{\mu}} + E_{\mathbf{k}}^{-1} C [f_{0\mathbf{k}} + \delta f_{\mathbf{k}}]$$

- $\dot{A} \equiv u^{\mu} \partial_{\mu} A$, $\nabla_{\mu} \equiv \Delta_{\mu}^{\nu} \partial_{\nu}$

Irreducible moments of $\delta f_{\mathbf{k}}$

$$\rho_r^{\mu_1 \dots \mu_{\ell}} \equiv \int dK E_{\mathbf{k}}^r k^{\langle \mu_1} \dots k^{\mu_{\ell} \rangle} \delta f_{\mathbf{k}}$$

- $A^{\langle \mu_1 \dots \mu_{\ell} \rangle} \equiv \Delta_{\nu_1 \dots \nu_{\ell}}^{\mu_1 \dots \mu_{\ell}} A^{\nu_1 \dots \nu_{\ell}}$, $\Delta_{\nu_1 \dots \nu_{\ell}}^{\mu_1 \dots \mu_{\ell}}$ rank- 2ℓ generalization of $\Delta_{\alpha\beta}^{\mu\nu}$

Equations of motion for irreducible moments

$$\dot{\rho}_r^{\langle \mu_1 \dots \mu_{\ell} \rangle} \equiv \Delta_{\nu_1 \dots \nu_{\ell}}^{\mu_1 \dots \mu_{\ell}} u^{\mu} \partial_{\mu} \int dK E_{\mathbf{k}}^r k^{\langle \nu_1} \dots k^{\nu_{\ell} \rangle} \delta f_{\mathbf{k}}$$

Landau matching conditions

- $n \equiv n_0 = \int dK E_{\mathbf{k}} f_{0\mathbf{k}} \implies \rho_1 = 0$
- $\varepsilon = \varepsilon_0 = \int dK E_{\mathbf{k}}^2 f_{0\mathbf{k}} \implies \rho_2 = 0$
- $T_f^{\mu\nu} u_\nu = \varepsilon u^\mu \implies \rho_1^\mu = 0$
- $P_0 = \frac{1}{3} \int dK (E_{\mathbf{k}}^2 - m^2) f_{0\mathbf{k}}$ thermodynamic pressure in local equilibrium

Dissipative currents

$$V^\mu \equiv \rho_0^\mu, \pi^{\mu\nu} \equiv \rho_0^{\mu\nu},$$

and bulk viscous pressure

$$\Pi \equiv -\frac{m^2}{3} \rho_0 \equiv P - P_0$$

Thermodynamic integrals

$$J_{nq} \equiv \frac{1}{(2q+1)!!} \int dK E_{\mathbf{k}}^{n-2q} (E_{\mathbf{k}}^2 - m^2)^q f_{0\mathbf{k}} \tilde{f}_{0\mathbf{k}}$$

$$D_{nm} = J_{n+1,m} J_{n-1,m} - J_{nm}^2$$

$$G_{nm} = J_{n0} J_{m0} - J_{n-1,0} J_{m+1,0}$$

Truncation: 14-moment approximation

- $\rho_r^{\mu_1 \dots \mu_\ell} \equiv 0$ for $\ell \geq 3$
- $\rho_r \rightarrow -\frac{3}{m^2} \frac{J_{r0} D_{30} + J_{r+1,0} G_{23} + J_{r+2,0} D_{20}}{J_{00} D_{20} + J_{30} G_{23} + J_{40} D_{10}} \Pi$
- $\rho_r^\mu \rightarrow \frac{J_{r+2,1} J_{41} - J_{r+3,1} J_{31}}{D_{31}} V^\mu$
- $\rho_r^{\mu\nu} \rightarrow \frac{J_{r+2,2}}{J_{42}} \pi^{\mu\nu}$

Bulk viscous pressure

$$\tau_{\Pi} \dot{\Pi} + \Pi = -\zeta \theta - \ell_{\Pi V} \nabla^{\mu} V_{\mu} - \tau_{\Pi V} V_{\mu} \dot{u}^{\mu} - \delta_{\Pi \Pi} \Pi \theta - \lambda_{\Pi V} V_{\mu} \nabla^{\mu} \alpha_0 + \lambda_{\Pi \pi} \pi^{\mu\nu} \sigma_{\mu\nu}$$

formally the same as in DNMR, note, however,

$$\dot{u}^{\mu} = \frac{1}{\varepsilon_0 + P_0} [\nabla^{\mu} P_0 - \Delta_{\nu}^{\mu} \partial_{\kappa} \pi^{\kappa\nu} - \Pi \dot{u}^{\mu} + \nabla^{\mu} \Pi - \mathbf{qB} \mathbf{b}^{\mu\nu} V_{\nu}]$$

Particle diffusion current

$$\begin{aligned} \tau_V \dot{V}^{(\mu)} + V^{\mu} &= \kappa \nabla^{\mu} \alpha_0 - V_{\nu} \omega^{\nu\mu} - \delta_{VV} V^{\mu} \theta - \ell_{V\Pi} \nabla^{\mu} \Pi + \ell_{V\pi} \Delta^{\mu\nu} \nabla_{\lambda} \pi_{\nu}^{\lambda} + \tau_{V\Pi} \Pi \dot{u}^{\mu} \\ &\quad - \tau_{V\pi} \pi^{\mu\nu} \dot{u}_{\nu} - \lambda_{VV} V_{\nu} \sigma^{\mu\nu} + \lambda_{V\Pi} \Pi \nabla^{\mu} \alpha_0 - \lambda_{V\pi} \pi^{\mu\nu} \nabla_{\nu} \alpha_0 \\ &\quad - \delta_{VB} \mathbf{qB} \mathbf{b}^{\mu\nu} V_{\nu} \end{aligned}$$

Shear-stress tensor

$$\begin{aligned} \tau_{\pi} \dot{\pi}^{(\mu\nu)} + \pi^{\mu\nu} &= 2\eta \sigma^{\mu\nu} + 2\pi_{\lambda}^{(\mu} \omega^{\nu)\lambda} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi^{\lambda(\mu} \sigma_{\lambda}^{\nu)} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} \\ &\quad - \tau_{\pi V} V^{(\mu} \dot{u}^{\nu)} + \ell_{\pi V} \nabla^{(\mu} V^{\nu)} + \lambda_{\pi V} V^{(\mu} \nabla^{\nu)} \alpha_0 \\ &\quad - 2\delta_{\pi B} \mathbf{qB} \mathbf{b}^{\alpha\beta} \Delta_{\alpha\kappa}^{\mu\nu} \pi_{\beta}^{\kappa} \end{aligned}$$

keep only 1st order terms X.-G. Huang, A. Sedrakian, DHR, Annals Phys. 326 (2011) 3075

$$\begin{aligned}\Pi &= -\zeta^{\mu\nu} \partial_\mu u_\nu \\ V^\mu &= \kappa^{\mu\nu} \nabla_\nu \alpha_0 \\ \pi^{\mu\nu} &= \eta^{\mu\nu\alpha\beta} \sigma_{\alpha\beta}\end{aligned}$$

- $\zeta^{\mu\nu} = \zeta_\perp \Xi^{\mu\nu} - \zeta_\parallel b^\mu b^\nu - \zeta_\times b^{\mu\nu}$
- $\kappa^{\mu\nu} = \kappa_\perp \Xi^{\mu\nu} - \kappa_\parallel b^\mu b^\nu - \kappa_\times b^{\mu\nu}$
- $\eta^{\mu\nu\alpha\beta} = 2\eta_0 \Delta^{\mu\nu\alpha\beta} + \eta_1 (\Delta^{\mu\nu} - \frac{3}{2} \Xi^{\mu\nu}) (\Delta^{\alpha\beta} - \frac{3}{2} \Xi^{\alpha\beta}) - 2\eta_2 (\Xi^{\mu\alpha} b^\nu b^\beta + \Xi^{\nu\alpha} b^\mu b^\beta) - 2\eta_3 (\Xi^{\mu\alpha} b^{\nu\beta} + \Xi^{\nu\alpha} b^{\mu\beta}) + 2\eta_4 (b^{\mu\alpha} b^\nu b^\beta + b^{\nu\alpha} b^\mu b^\beta)$

for an alternative decomposition, see J. Hernandez, P. Kovtun, JHEP 1705 (2017) 001

Bulk viscosities

$$\zeta_x = 0$$

$$\zeta_{\perp} = \zeta_{\parallel} \equiv \zeta$$

⇒ $\Pi = -\zeta\theta$ as without magnetic field

⇒ consequence of weak-field limit

For bulk viscosities in strong fields, see

K. Hattori, X. G. Huang, DHR, D. Satow, PRD 96 (2017) 094009

⇒ in lowest-Landau-level approximation:

$$\zeta_{\perp} \ll \zeta_{\parallel} \sim qB T \left(\frac{m_q}{T} \right)^2 \frac{1}{g^2 \ln(T/m_q)}$$

Particle-diffusion coefficients

$$\kappa_{\parallel} \equiv \kappa$$

$$\kappa_{\perp} = \kappa \left[1 + (\mathbf{qB}\delta_{VB})^2 \right]^{-1}$$

$$\kappa_{\times} = \kappa_{\perp} \mathbf{qB}\delta_{VB}$$

$$\kappa_{\parallel} = \frac{3\lambda_{\text{mfp}} n_0}{16}$$

$$\kappa_{\perp} = \frac{48\lambda_{\text{mfp}} n_0}{256 + 225\xi_B^2}$$

$$\kappa_{\times} = \frac{45\xi_B \lambda_{\text{mfp}} n_0}{256 + 225\xi_B^2}$$



$\xi_B \rightarrow \infty$: Hall diffusion coefficient

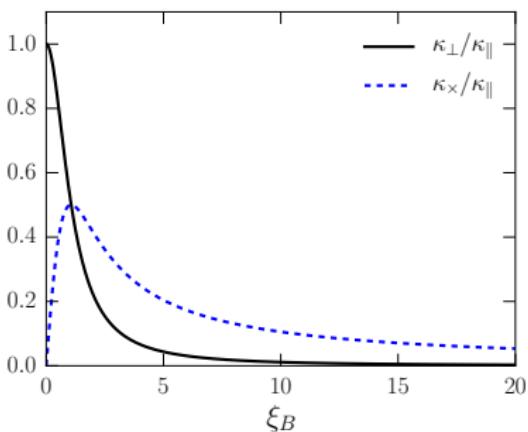
$$\kappa_{\times} \rightarrow \frac{\lambda_{\text{mfp}} n_0}{5\xi_B} \equiv \frac{n_0 R_T}{5}$$

becomes dissipationless!

For massless Boltzmann gas
and constant cross section:

$$\kappa = \frac{3\lambda_{\text{mfp}} n_0}{16}$$

$$\delta_{VB} = \frac{15\beta_0 \lambda_{\text{mfp}}}{16}$$



Shear viscosities

$$\eta_0 = \eta \left[1 + 4 (\mathbf{q}B \delta_{\pi B})^2 \right]^{-1}$$

$$\eta_1 = \frac{16}{3} \eta_0 (\mathbf{q}B \delta_{\pi B})^2$$

$$\eta_2 = 3\eta_0 (\mathbf{q}B \delta_{\pi B})^2 \left[1 + (\mathbf{q}B \delta_{\pi B})^2 \right]^{-1}$$

$$\eta_3 = \eta_0 \mathbf{q}B \delta_{\pi B}$$

$$\eta_4 = \eta \mathbf{q}B \delta_{\pi B} \left[1 + (\mathbf{q}B \delta_{\pi B})^2 \right]^{-1}$$

$$\Rightarrow \eta_0 = \frac{12 \lambda_{\text{mfp}} P_0}{9 + 4 \xi_B^2}, \quad \eta_1 = \frac{64}{9} \frac{\xi_B^2 \lambda_{\text{mfp}} P_0}{9 + 4 \xi_B^2}$$

$$\eta_2 = \frac{36 \xi_B^2 \lambda_{\text{mfp}} P_0}{[9 + 4 \xi_B^2][9 + \xi_B^2]}$$

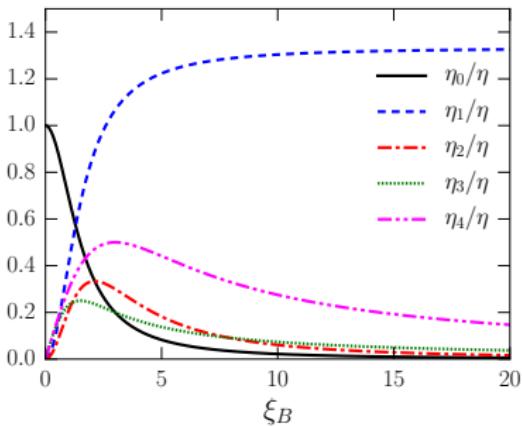
$$\eta_3 = \frac{4 \xi_B \lambda_{\text{mfp}} P_0}{9 + 4 \xi_B^2}, \quad \eta_4 = \frac{4 \xi_B \lambda_{\text{mfp}} P_0}{9 + \xi_B^2}$$

$\xi_B \rightarrow \infty : \eta_3 = \eta_4/4 \rightarrow \lambda_{\text{mfp}} P_0 / \xi_B \equiv P_0 R_T$
become dissipationless!

For massless Boltzmann gas
and constant cross section:

$$\eta = \frac{4 \lambda_{\text{mfp}} P_0}{3}$$

$$\delta_{\pi B} = \frac{\beta_0 \lambda_{\text{mfp}}}{3}$$



Equation of motion for fluid energy

$$u_\mu \partial_\nu T_f^{\mu\nu} = -E^\mu \mathfrak{v}_\mu$$

- ⇒ fluid energy is gained due to acceleration of charges along electric field E^μ
- ⇒ resistive, non-dissipative MHD: $\mathfrak{v}^\mu \equiv 0$ ⇒ fluid energy is conserved!

Equation of motion for fluid momentum

$$\Delta_\mu^\alpha \partial_\nu T_f^{\mu\nu} = n E^\alpha - B b^{\alpha\nu} \mathfrak{v}_\nu$$

- ⇒ fluid momentum is gained due to acceleration of charges in direction of E^α and due to cyclotron motion!
- ⇒ resistive, non-dissipative MHD: $\mathfrak{v}^\mu \equiv 0$ ⇒ fluid gains momentum!

Bulk viscous pressure

$$\tau_{\Pi} \dot{\Pi} + \Pi = -\zeta \theta - \ell_{\Pi V} \nabla^{\mu} V_{\mu} - \tau_{\Pi V} V_{\mu} \dot{u}^{\mu} - \delta_{\Pi \Pi} \Pi \theta - \lambda_{\Pi V} V_{\mu} \nabla^{\mu} \alpha_0 + \lambda_{\Pi \pi} \pi^{\mu \nu} \sigma_{\mu \nu} - \delta_{\Pi V E} q E^{\mu} V_{\mu}$$

Particle diffusion current

$$\begin{aligned} \tau_V \dot{V}^{(\mu)} + V^{\mu} &= \kappa \nabla^{\mu} \alpha_0 - V_{\nu} \omega^{\nu \mu} - \delta_{VV} V^{\mu} \theta - \ell_{V\Pi} \nabla^{\mu} \Pi + \ell_{V\pi} \Delta^{\mu \nu} \nabla_{\lambda} \pi_{\nu}^{\lambda} + \tau_{V\Pi} \Pi \dot{u}^{\mu} \\ &\quad - \tau_{V\pi} \pi^{\mu \nu} \dot{u}_{\nu} - \lambda_{VV} V_{\nu} \sigma^{\mu \nu} + \lambda_{V\Pi} \Pi \nabla^{\mu} \alpha_0 - \lambda_{V\pi} \pi^{\mu \nu} \nabla_{\nu} \alpha_0 \\ &\quad - \delta_{VB} q B b^{\mu \nu} V_{\nu} + \delta_{VE} q E^{\mu} + \delta_{V\Pi E} q E^{\mu} \Pi + \delta_{V\pi E} q E_{\nu} \pi^{\mu \nu} \end{aligned}$$

Note that $\kappa \beta_0 = \delta_{VE}$ (\Rightarrow see J. Liao's talk)

Induced current: $\mathfrak{J}_{\text{ind}}^{\mu} \equiv q V_{\text{ind}}^{\mu} \equiv \delta_{VE} q^2 E^{\mu} \equiv \sigma_E E^{\mu}$

\Rightarrow Electrical conductivity: $\sigma_E \equiv q^2 \delta_{VE} \equiv q^2 \kappa \beta_0$ (Wiedemann-Franz law)

Shear-stress tensor

$$\begin{aligned} \tau_{\pi} \dot{\pi}^{(\mu \nu)} + \pi^{\mu \nu} &= 2\eta \sigma^{\mu \nu} + 2\pi_{\lambda}^{(\mu} \omega^{\nu)\lambda} - \delta_{\pi \pi} \pi^{\mu \nu} \theta - \tau_{\pi \pi} \pi^{\lambda(\mu} \sigma_{\lambda}^{\nu)} + \lambda_{\pi \Pi} \Pi \sigma^{\mu \nu} \\ &\quad - \tau_{\pi V} V^{(\mu} \dot{u}^{\nu)} + \ell_{\pi V} \nabla^{(\mu} V^{\nu)} + \lambda_{\pi V} V^{(\mu} \nabla^{\nu)} \alpha_0 \\ &\quad - 2\delta_{\pi B} q B b^{\alpha \beta} \Delta_{\alpha \kappa}^{\mu \nu} \pi_{\beta}^{\kappa} + \delta_{\pi V E} q E^{(\mu} V^{\nu)} \end{aligned}$$

- considered single species of electrically charged, point-like particles with spin zero
- derived equations of motion for relativistic, non-resistive and resistive, second-order dissipative MHD from the Boltzmann equation, using method of moments in 14-moment approximation
- identified new transport coefficients due to electromagnetic fields
- computed first-order transport coefficients in constant magnetic field for massless Boltzmann gas with constant cross section
- confirmed (kinetic-theory version of) Wiedemann-Franz law for electric conductivity and particle-diffusion coefficient
- generalize beyond 14-moment approximation via resumming moments
- consider particles and antiparticles (positively and negatively charged particles)
- consider non-zero spin
 - ⇒ MHD with non-vanishing polarization, magnetization
 - ⇒ see P. Kovtun's talk