Conservation Laws on the Cooper-Frye Surface and Hadronic Rescattering

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Motivation and Outline

- **Hybrid** transport+hydrodynamics approaches are successfully applied for the description of the dynamics of heavy ion collisions

- There are **2 ad hoc** transitions
  - Initial assumption on local equilibration
    - Coarse-grained transport approach
  - Final *Cooper-Frye* sampling/Particlization
    - Negative contributions: How large?
    - Global and local conservation laws
    - Broad spectral functions for resonances

- Hadronic rescattering within SMASH
  - High viscosity in the hadron gas phase
Evolution of Heavy Ion Reactions

- Initial and final state require non-equilibrium treatment
- Nearly ideal hydrodynamics provides framework for the hot and dense stage of the evolution including a phase transition
- There are 2 crucial interfaces: Initial state for hydrodynamic evolution and Cooper-Frye particlization
Local Equilibration
What is Usually Done?

- To calculate the energy-momentum tensor and four-current from particles a **smearing kernel** (Gaussian) is used:

\[
T_{\mu\nu}^{\text{init}}(r) = \sum_i \frac{p_i^\mu \cdot p_i^\nu}{p_i^0} K(r - r_i, p)
\]

\[
j_{\mu}^{\text{init}}(r) = \sum_i \frac{p_i^\mu}{p_i^0} K(r - r_i, p)
\]

- **Assuming** that the resulting tensor has the form for relativistic ideal fluid dynamics, the following equations are solved iteratively

\[
\begin{align*}
T^{00} &= (\epsilon + p)\gamma^2 - p \\
T^{0i} &= (\epsilon + p)\gamma^2 \nu \\
j_B^0 &= n\gamma \\
p &= p_{\text{EoS}}(n, \epsilon)
\end{align*}
\]

- The other option: Solve the eigenvalue problem and decompose the tensor in the **Landau frame**
Coarse-Grained UrQMD

1. Several thousands Au+Au collisions at $E_{\text{lab}} = 5\text{-}160$ AGeV beam energy and different centralities

2. Calculate $T^{\mu\nu}$ on a space-time grid

3. Transform to the Landau rest frame
   - Investigate **locally** two measures of isotropization:
     - Pressure anisotropy:
       \[
       X \equiv \frac{|T_{L}^{11} - T_{L}^{22}| + |T_{L}^{22} - T_{L}^{33}| + |T_{L}^{33} - T_{L}^{11}|}{T_{L}^{11} + T_{L}^{22} + T_{L}^{33}} \ll 1
       \]
     - Off-diagonality:
       \[
       Y \equiv \frac{3(|T_{L}^{12}| + |T_{L}^{23}| + |T_{L}^{13}|)}{T_{L}^{11} + T_{L}^{22} + T_{L}^{33}} \ll 1
       \]
   - $X,Y \leq 0.3 \to$ viscous hydrodynamics applicable
Time Evolution

- $E_{lab} = 80A$ GeV, $b=6$ fm, pressure anisotropy
- After initial collisions anisotropy develops minimum over a large region in space
- Later stages: Rise due to resonance decays

D. Oliinychenko and HP, PRC93(2016)
Initial Switching Time

- Isotropization time deviates from geometrical overlap criterion for higher beam energies

\[ t_{iso} = 2R \left( \frac{E_{lab}}{2m_N} \right)^{-1/2} \]

- Centrality dependence is weaker than expected from geometry

\[ t_0(b) = t_0(b = 0) + \frac{R}{\gamma_v} \left( \hat{1} - \sqrt{1 - \left(\frac{b}{2R_R}\right)^2} \right) \]
Particlization
Freeze-out Procedure

- **Deconfinement/Confinement** transition happens through equation of state in hydrodynamics.

- **Transition** from hydro to transport when temperature/energy density is smaller than **critical value**.

- Particle distributions are generated according to the **Cooper-Frye** formula:
  \[
  E \frac{dN}{d^3p} = \int f(x, p)p^\mu d\sigma_\mu
  \]

- Same EoS on both sides of the transition hypersurface.

- Rescatterings and final decays calculated via **hadronic cascade**.

- Separation of **chemical** and **kinetic** freeze-out is taken into account.

- **Large viscosity** in hadron gas stage!
Hypersurface Finding

- **Cornelius**: 3D hypersurface in 4 dimensions
- Constant energy density
- Avoiding holes and double-counting
- Applicable as a subroutine
  - Input: 16-tuples of spatio-temporal information
  - Output: Hypersurface vectors and interpolated thermodynamic quantities

P. Huovinen, HP, EPJA 48, 2012
Fortran and C++ subroutines, cornelius, implementations of this algorithm in 3D and 4D, are available at https://karman.physics.purdue.edu/OSCAR
Negative Contributions

- **Definition:**
  - Particles outward: \( p^\mu d\sigma_\mu > 0 \)
  - Particles inward: \( p^\mu d\sigma_\mu < 0 \)

- **Different options:**
  - Account for feedback in hydro
  - Account effectively by weights in transport
  - Neglect them and violate conservation laws

- **Systematic study** of the size of negative contributions by comparison to actual transport

\[ d\sigma_\mu - \text{normal 4-vector} \]
\[ u_\mu = (\gamma, \gamma \vec{V}) - \text{4-velocity} \]
\[ T - \text{temperature} \]
\[ \mu - \text{chemical potential} \]

S. Pratt, Phys.Rev. C89 (2014) 2, 024910
Energy Dependence

- Iso-energy density hypersurfaces ($\varepsilon_c = 0.3$ GeV/fm$^3$)

- Maximum at $E_{\text{lab}} \sim 25$ AGeV, decreasing at higher energies
- Actual particles are always less likely to fly inward

D. Oliinychenko and HP, PRC91(2015)
Global Conservation Laws
Why Conservation Laws?

- Event-by-event hybrid approach
  - One initial state (with fluctuations), one hydro run, one (or multiple) sample(s) of particles in the final state
  - Finite net baryon number $B$, net strangeness $S$, electric charge $Z$ and energy $E$ in initial state

- In nature quantum numbers are conserved

- Hydrodynamic evolution conserves energy and net baryon number explicitly

- Apply global conservation laws to sampling
  - No effect on single particle observables expected
  - Important for correlation and fluctuation observables, e.g. higher moments at low beam energies
Lower Beam Energies

- Loop through all cells (allcells):
  - Total quantum numbers fluctuate
- Randomly choose cells, until total energy is conserved (mode), reproduce finite S, B, Q

P. Huovinen, HP, EPJA 48, 2012
Steps for the Particle Production

1) Numbers of each particle species in the element
   \[ N_i = j^\mu d\sigma_\mu = n_i u^\mu d\sigma_\mu \quad n_i = \frac{4\pi g_i m_i^2 T}{(2\pi^3)} \exp\left(\frac{\mu}{T}\right) K_2\left(\frac{m_i}{T}\right) \]

2) Sum to get the total particle number

3) Particle production according to Poisson distribution

4) Particle type chosen according to respective probabilities

5) Isospin randomly assigned

6) Generate four-momenta (rejection method)
   \[ \frac{dN(x)}{d^3 p} = \frac{1}{E} f(x, p) p^\mu d\sigma_\mu \quad \text{with} \quad f(x, p) p^\mu d\sigma_\mu > 0 \]

7) Particle vector information is transferred to hadronic transport approach
Mode Sampling

- Seven loops over the hypersurface elements:
  - 1) Strange particles
  - 2) Anti-strange particles
  - 3) Non-strange anti-baryons
  - 4) Non-strange baryons
  - 5) Negatively charged non-strange mesons
  - 6) Positively charged non-strange mesons
  - 7) Neutral non-strange mesons
- Loops 1, 3, 5 and 7 are cut by energy conservation
- Other loops by respective conservation laws
- Has been successfully applied within UrQMD hybrid approach at finite net baryon densities

HP et al., PRC 78, 2008
P. Huovinen, HP, EPJA 48, 2012
SPREW Sampling

- **Single Particle Rejection with Exponential Weights**
- If a certain particle is chosen to be produced, $\Delta X$ is calculated for each quantum number, where
  \[ \Delta X = X_{particles} - X_{hypersurface} \]
- If $\Delta X$ and the $X_i$ have the same sign, the particle is rejected with probability
  \[ 1 - e^{\mid\Delta X\mid} \]
- Energy and momenta are rescaled to fit the values on the hypersurface
- Test the performance of both algorithms in single cell

C. Schwarz et al., JPG 452018
Two different algorithms (mode and SPREW) are compared to conventional sampling.
Thermal Box

- Even in small system (5fm length), SPREW reproduces the proper multiplicities including fluctuations.

- Bias in mode sampling appears in small systems.

C. Schwarz et al., JPG 452018
Pions in Heavy Ion Collision

• Comparison of the full distribution for pion production in Au+Au at 200 GeV

C. Schwarz et al., JPG 452018

• SPREW sampling minimizes bias by conservation laws
Hadronic Rescattering and Broad Spectral Functions
Resonance Masses

- Typically resonances are sampled only with their pole masses at particlization.
- In hadronic transport approaches spectral functions are used, usually Breit-Wigner distributions.
- Vector meson masses as shown below:

- Influences dilepton emission from hadronic afterburner.
• Hadronic transport approach:
  – Includes all mesons and baryons up to \( \sim 2 \) GeV
  – Geometric collision criterion
  – Binary interactions: Inelastic collisions through resonance/string excitation and decay
  – Infrastructure: C++, Git, Redmine, Doxygen, (ROOT)

* Simulating Many Accelerated Strongly-Interacting Hadrons

\[ T_L^{00} \text{[GeV} \cdot \text{fm}^{-3}] \]

Au+Au at \( E_{\text{Kin}} = 0.8 \text{ AGeV}, b=3 \text{ fm} \)
Influence of Global Conservation

- Comparison between hydrodynamic calculation only with resonance decays and with full hadronic rescattering

- Global conservation laws do not affect single particle observables as expected
Comparison SMASH vs UrQMD

- SMASH rescattering yields qualitatively similar results as UrQMD afterburner

- Results are sensitive to missing baryon-antibaryon annihilation and AQM cross-sections
Shear Viscosity of the Hadron Gas
Transport Coefficients

- Within hydrodynamics/hybrid approaches the shear viscosity is an input parameter

- Application of Bayesian techniques allows extraction of temperature dependence

Existing Results - Discrepancy

Green-Kubo formalism
UrQMD

Discrepancy with hydro-inspired B3D and VISHNU

Long standing question: Why are the results so different from each other?

Shear Viscosity over Entropy Density

- Box with periodic boundary condition in chemical and thermal equilibrium
- Entropy is calculated via Gibbs formula from thermodynamic properties
- The shear viscosity is extracted following the Green-Kubo formalism:

\[ \eta = \frac{V}{T} \int_{0}^{\infty} C^{xy}(t) \, dt \]

\[ C^{xy}(t) = \frac{1}{N} \sum_{s} T^{xy}(s) T^{xy}(s + t) \]

\[ T^{\mu\nu} = \frac{1}{V} \sum_{i}^{N_{\text{part}}} \frac{p_{i}^{\mu} p_{i}^{\nu}}{p_{i}^{0}} \]

\[ C^{xy}(t) \simeq C^{xy}(0) \exp \left( -\frac{t}{\tau} \right) \]

\[ \eta = \frac{V C^{xy}(0) \tau}{T} \]
Resonance Dynamics

- Energy-dependence of cross-sections is modelled via resonances
- Point-like in analytic calculation and finite lifetime in transport approach

- Agreement recovered by decreasing $\rho$ meson lifetime

- Closest similarity to Bass/Demir result as expected
Point-like Interactions

- Adding a constant elastic cross section leads to agreement with B3D result

- Approximately linear relationship between relaxation time and mean free time is recovered

Summary

- **Hybrid approaches** based on relativistic hydrodynamics and hadron transport provide realistic dynamical description.
- Two transitions have been studied systematically using coarse-grained UrQMD calculations.
- Different algorithms to conserve quantum numbers globally at the partilization transition have been proposed.
- **SPREW sampling** is computationally efficient and reproduces the mean values and fluctuations properly.
- Broad spectral functions are employed in the sampling process.
- Hadronic rescattering within SMASH yields similar results as within UrQMD.
- Hadron gas viscosity is sensitive to the lifetimes of the resonances.
- Outlook: Electromagnetic emission from non-equilibrium hadronic stage.
Backup
Influence of Statistics

- From $N$ random thermal pions, the effect of finite particle statistics on the deviations of the energy-momentum tensor from equilibrium can be estimated.
Parameter Sensitivities

- Comparison of coarse-grained transport with Cooper-Frye calculation vs actual particles

- No significant dependence on cell sizes
- Saturation for large enough number of events
- Dependence on $\sigma$ due to smearing of surface velocities
Hypersurface Results

- Energy and net baryon number conservation on hypersurface

\[ E = \int \sigma T^\mu_\mu d\sigma_\mu \quad \text{and} \quad B = \int \sigma n_B u^\mu d\sigma_\mu. \]

- where \( d\sigma_\mu T^\mu^0 \geq 0 \) and \( d\sigma_\mu n_B u^\mu \geq 0 \) specify the positive and negative contributions

- Results at RHIC for central and mid-central collisions

<table>
<thead>
<tr>
<th></th>
<th>E [GeV]</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>total</td>
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<tr>
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<tr>
<td>final</td>
<td>2336</td>
<td>2455</td>
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</tbody>
</table>

Dashed lines indicate possible positions of elements with negative contributions \( d\sigma_\mu u^\mu < 0 \)

see also Oliinychenko et al arXiv:1411.3912
Negative Contributions

- The elements with particle flow inwards are located at high fluid velocities
- Coincides with peak in space-like surface elements with outward particle flow
- At midrapidity positive particle flux is dominant
Pion Spectra

- Largest effect on low $p_T$ pions
## Different Approaches

<table>
<thead>
<tr>
<th>Model</th>
<th>Initial condition</th>
<th>Hydro</th>
<th>Switching criterion</th>
<th>Smearing kernel</th>
<th>Getting $T_{\text{ideal}}^{\mu\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>UrQMD hybrid [12]</td>
<td>UrQMD cascade</td>
<td>ideal 3+1D, SHASTA</td>
<td>$t_{CM}[\text{fm/c}] = \max(2R\sqrt{\frac{E_{lab}}{2m_N}}, 1.0)$</td>
<td>Gaussian z-contracted</td>
<td>$T^{\mu0}$, $j^0$</td>
</tr>
<tr>
<td>Skokov-Toneev hybrid [13]</td>
<td>Quark-Gluon-String-Model</td>
<td>ideal 3+1D, SHASTA</td>
<td>$t_{CM}$ such that $S/Q_B = \text{const}$</td>
<td>not mentioned</td>
<td>$T^{\mu0}$, $j^0$</td>
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<tr>
<td>EPOS [15]</td>
<td>Strings (Regge-Gribov model)</td>
<td>ideal 3+1D</td>
<td>$\tau$</td>
<td>Gaussian z-contracted</td>
<td>Landau frame</td>
</tr>
<tr>
<td>NeXSPhERIO hybrid [16, 17]</td>
<td>Strings (Regge-Gribov model)</td>
<td>ideal 3+1D, SPH</td>
<td>$\tau = 1 \text{ fm}[18]$</td>
<td>Gaussian in $x, y, \tau \eta$</td>
<td>Landau frame</td>
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<tr>
<td>Gale et al [19]</td>
<td>IP-glasma</td>
<td>viscous 3+1D, MUSIC</td>
<td>$\tau = 0.2 \text{ fm/c}$ \ \ ($\sqrt{s_{NN}} = 2.76 \text{ TeV}$)</td>
<td>not mentioned</td>
<td>Landau frame</td>
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<td>Karpenko hybrid [20]</td>
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<td>viscous 3+1D</td>
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<td>Gaussian with $\sigma_{\perp}$ and $\sigma_{\eta}$</td>
<td>$T^{\mu0}$, $j^0$</td>
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<td>$\tau$</td>
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