Conservation Laws on the Cooper-Frye Surface and Hadronic Rescattering

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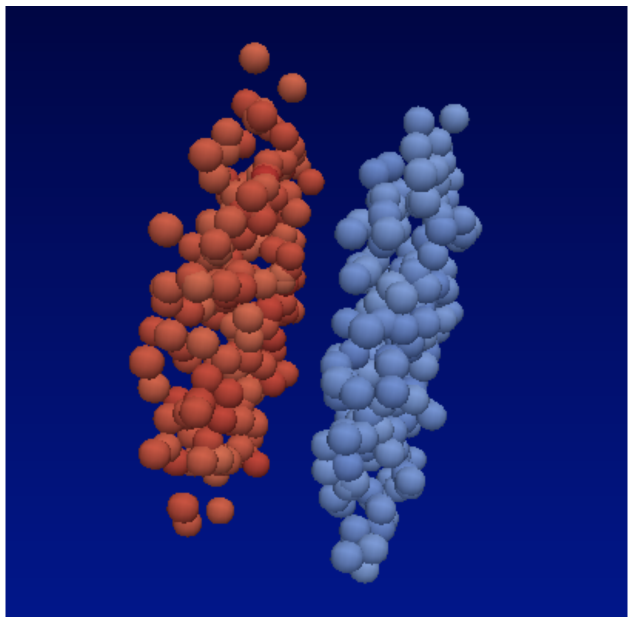
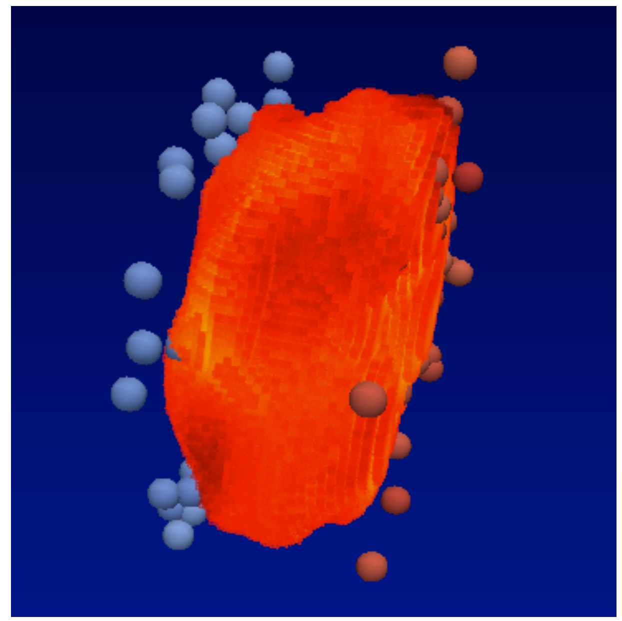
Motivation and Outline

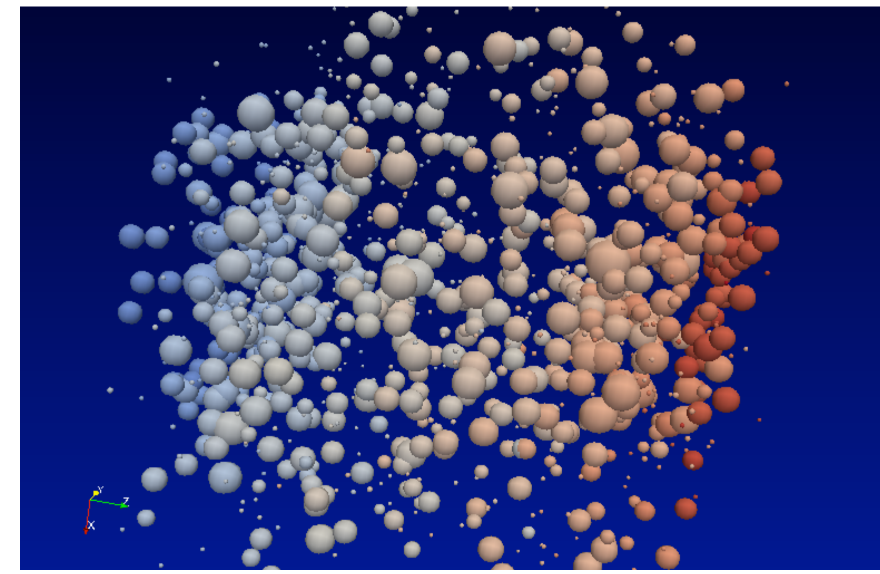
- Hybrid transport+hydrodynamics approaches are successfully applied for the description of the dynamics of heavy ion collisions
- There are 2 ad hoc transitions
  - Initial assumption on local equilibration
    - Coarse-grained transport approach
  - Final Cooper-Frye sampling/Particlization
    - Negative contributions: How large?
    - Global and local conservation laws
    - Broad spectral functions for resonances
- Hadronic rescattering within SMASH
  - High viscosity in the hadron gas phase

Evolution of Heavy Ion Reactions

Nuclei at 99 % speed of light	Quark gluon plasma	Observable fragments in the detector







<1 fm/c — 1-10 fm/c — 10-30 fm/c

- Initial and final state require non-equilibrium treatment
- Nearly ideal hydrodynamics provides framework for the hot and dense stage of the evolution including a phase transition
- There are **2 crucial interfaces**: Initial state for hydrodynamic evolution and Cooper-Frye particlization

Local Equilibration

What is Usually Done?

D. Oliinychenko and HP, PRC93(2016)

- To calculate the energy-momentum tensor and four-current from particles a **smearing kernel** (Gaussian) is used:

$$T^{\mu\nu}_{init}(r) = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\boldsymbol{r} - \boldsymbol{r_i}, \boldsymbol{p})$$

$$j^{\mu}_{init}(r) = \sum_i \frac{p_i^\mu}{p_i^0} K(\boldsymbol{r} - \boldsymbol{r_i}, \boldsymbol{p})$$

- **Assuming** that the resulting tensor has the form for relativistic ideal fluid dynamics, the following equations are solved iteratively

$$\begin{cases} T^{00} = (\epsilon + p)\gamma^2 - p \\ T^{0i} = (\epsilon + p)\gamma^2 \boldsymbol{v} \\ j^0_B = n\gamma \\ p = p_{EoS}(n, \epsilon) \end{cases}$$

- The other option: Solve the eigenvalue problem and decompose the tensor in the **Landau frame**

Coarse-Grained UrQMD

D. Oliinychenko and HP, PRC93(2016)

1. Several thousands Au+Au collisions at E_{lab} = 5-160 AGeV beam energy and different centralities
2. Calculate $T^{\mu\nu}$ on a space-time grid
3. Transform to the Landau rest frame

- Investigate **locally** two measures of isotropization:
  - Pressure anisotropy:

$$X \equiv \frac{|T_L^{11} - T_L^{22}| + |T_L^{22} - T_L^{33}| + |T_L^{33} - T_L^{11}|}{T_L^{11} + T_L^{22} + T_L^{33}} \ll 1$$

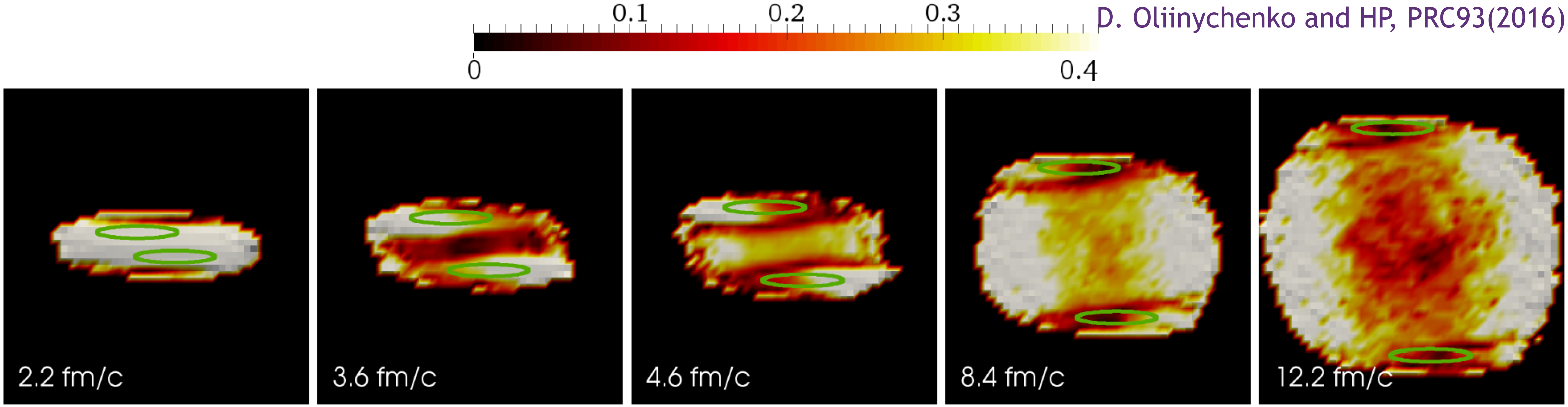
  - Off-diagonality:

$$Y \equiv \frac{3(|T_L^{12}| + |T_L^{23}| + |T_L^{13}|)}{T_L^{11} + T_L^{22} + T_L^{33}} \ll 1$$

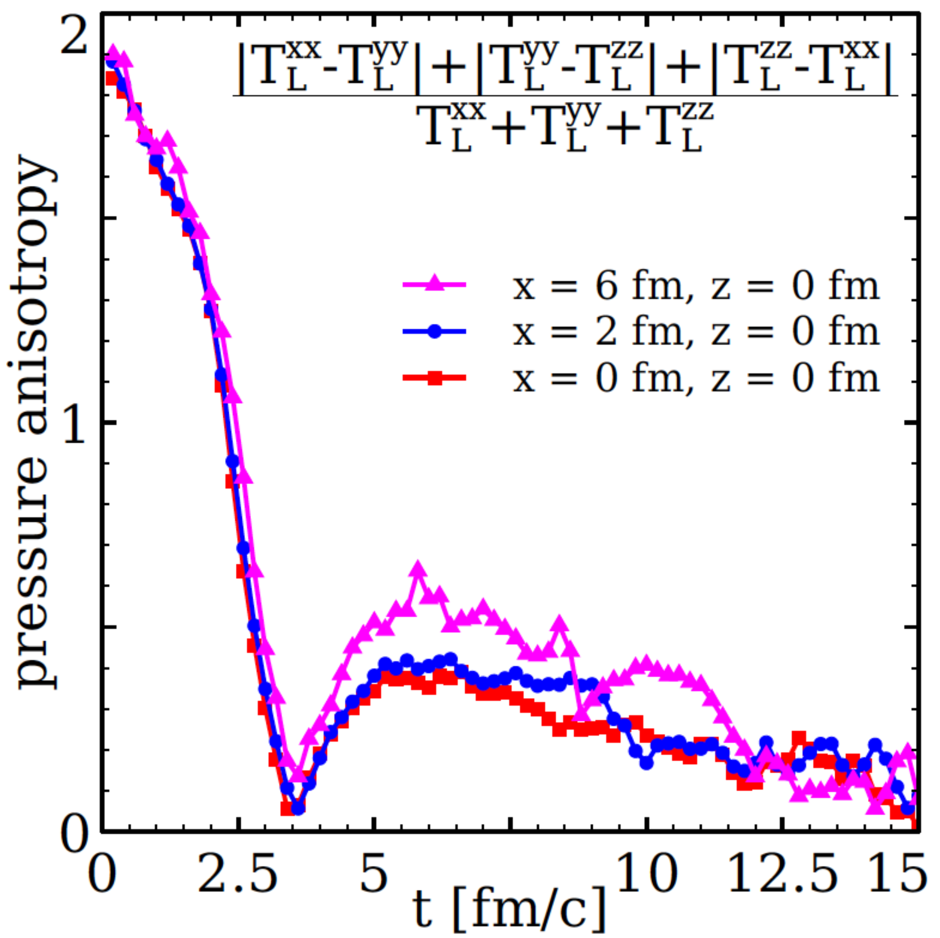
- X,Y ≤ 0.3 → viscous hydrodynamics applicable

Time Evolution

D. Oliinychenko and HP, PRC93(2016)



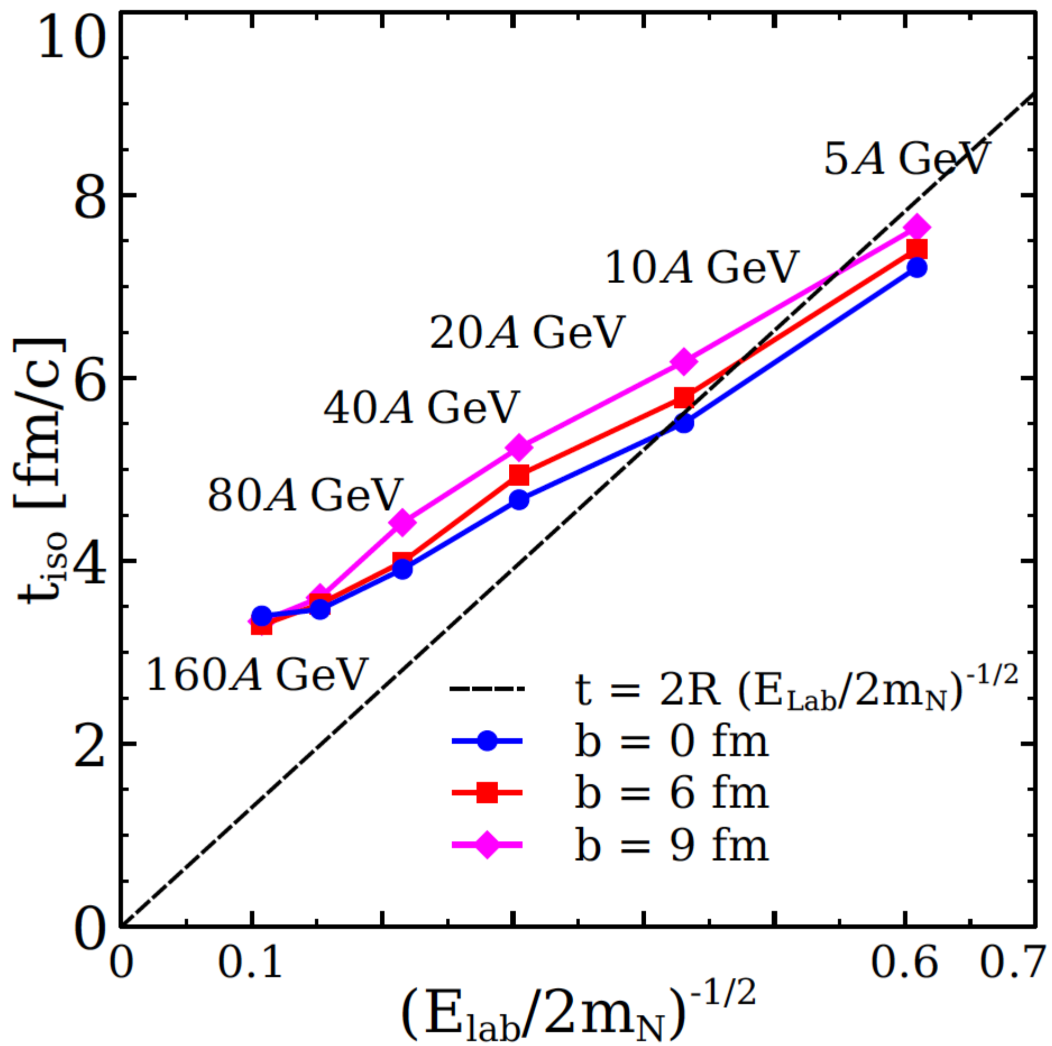
- E_{lab} = 80A GeV, b=6 fm, pressure anisotropy
- After initial collisions anisotropy develops minimum over a large region in space
- Later stages: Rise due to resonance decays

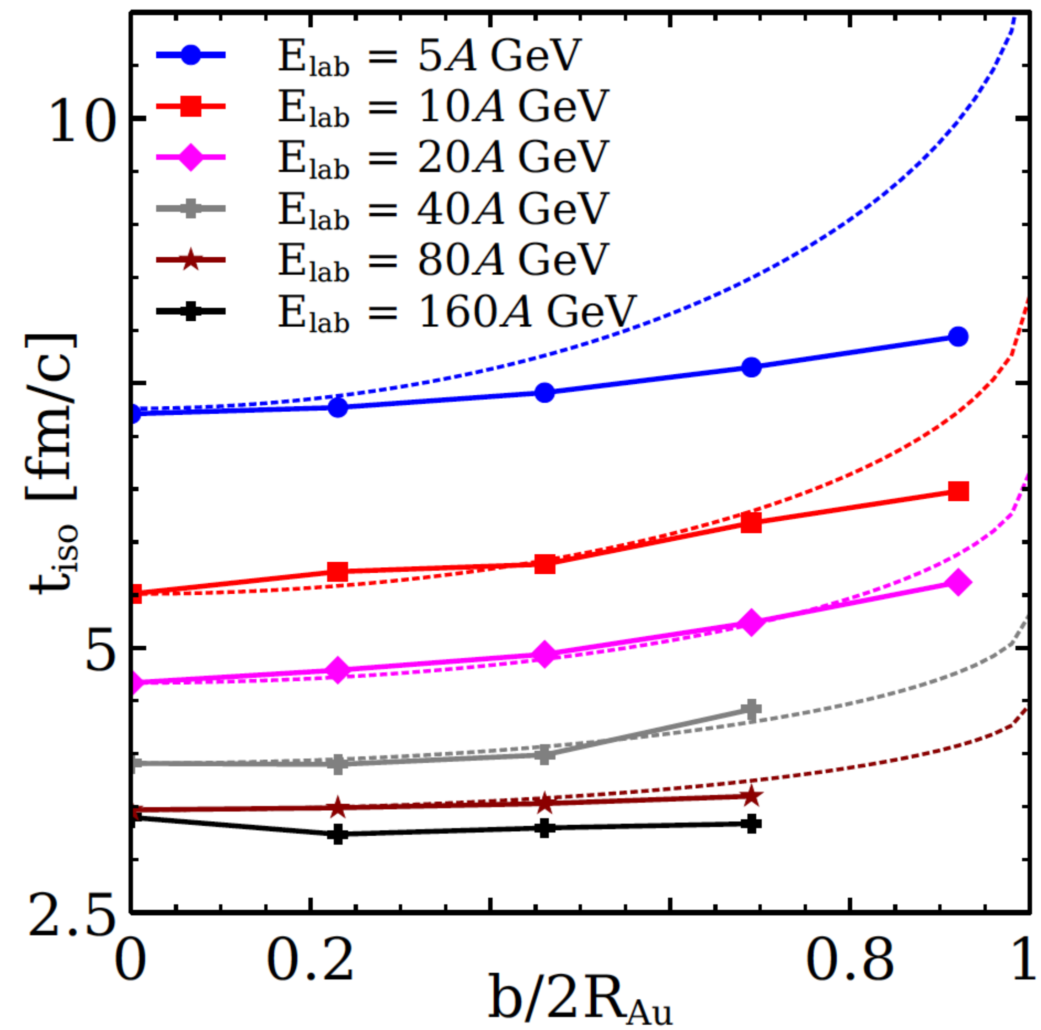


Initial Switching Time

D. Oliinychenko and HP, PRC93(2016)

- Isotropization time deviates from geometrical overlap criterion for higher beam energies





- Centrality dependence is weaker than expected from geometry

$$t_0(b) = t_0(b=0) + \frac{R}{\gamma v}\left(1 - \sqrt{1-(b/2R)^2}\right)$$

Particlization

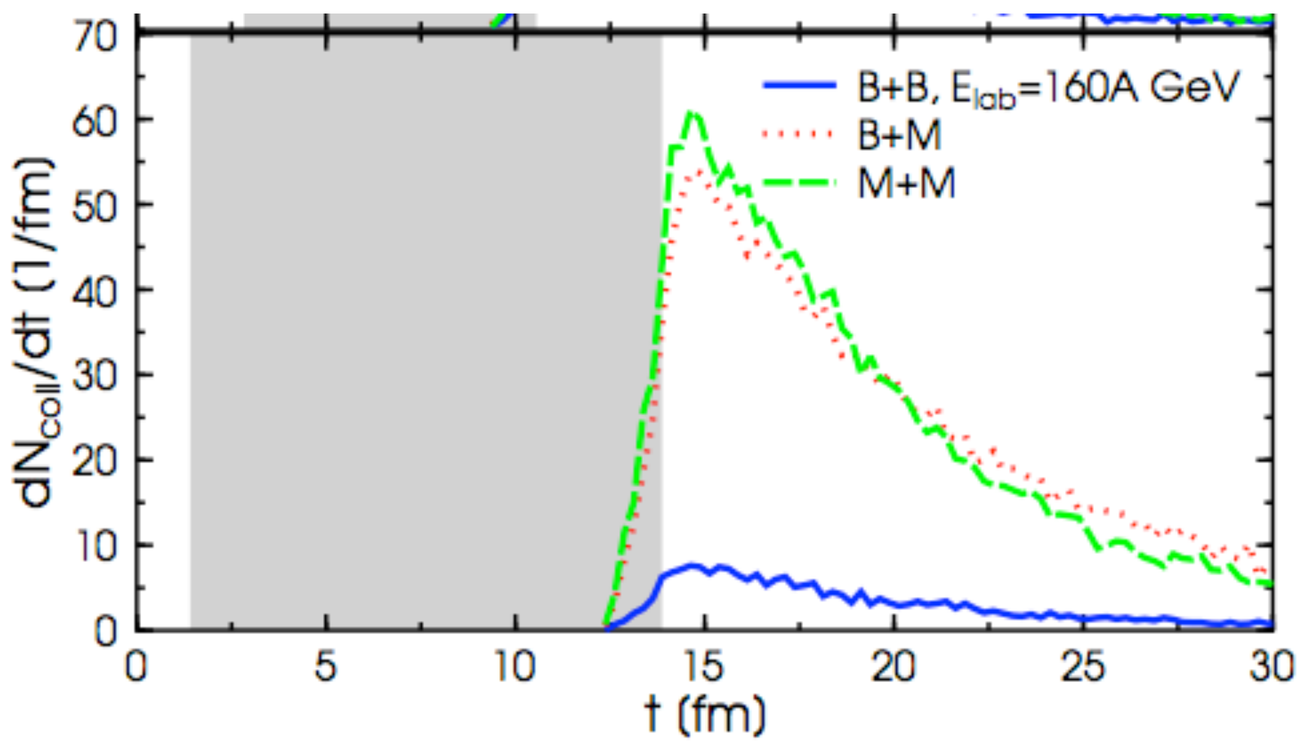
Freeze-out Procedure

- **Deconfinement/Confinement** transition happens through equation of state in hydrodynamics
- **Transition** from hydro to transport when temperature/energy density is smaller than **critical value**
- Particle distributions are generated according to the **Cooper-Frye** formula

$$E\frac{dN}{d^3p} = \int_\sigma f(x,p)p^\mu d\sigma_\mu$$

- Same EoS on both sides of the transition hypersurface
- Rescatterings and final decays calculated via **hadronic cascade**

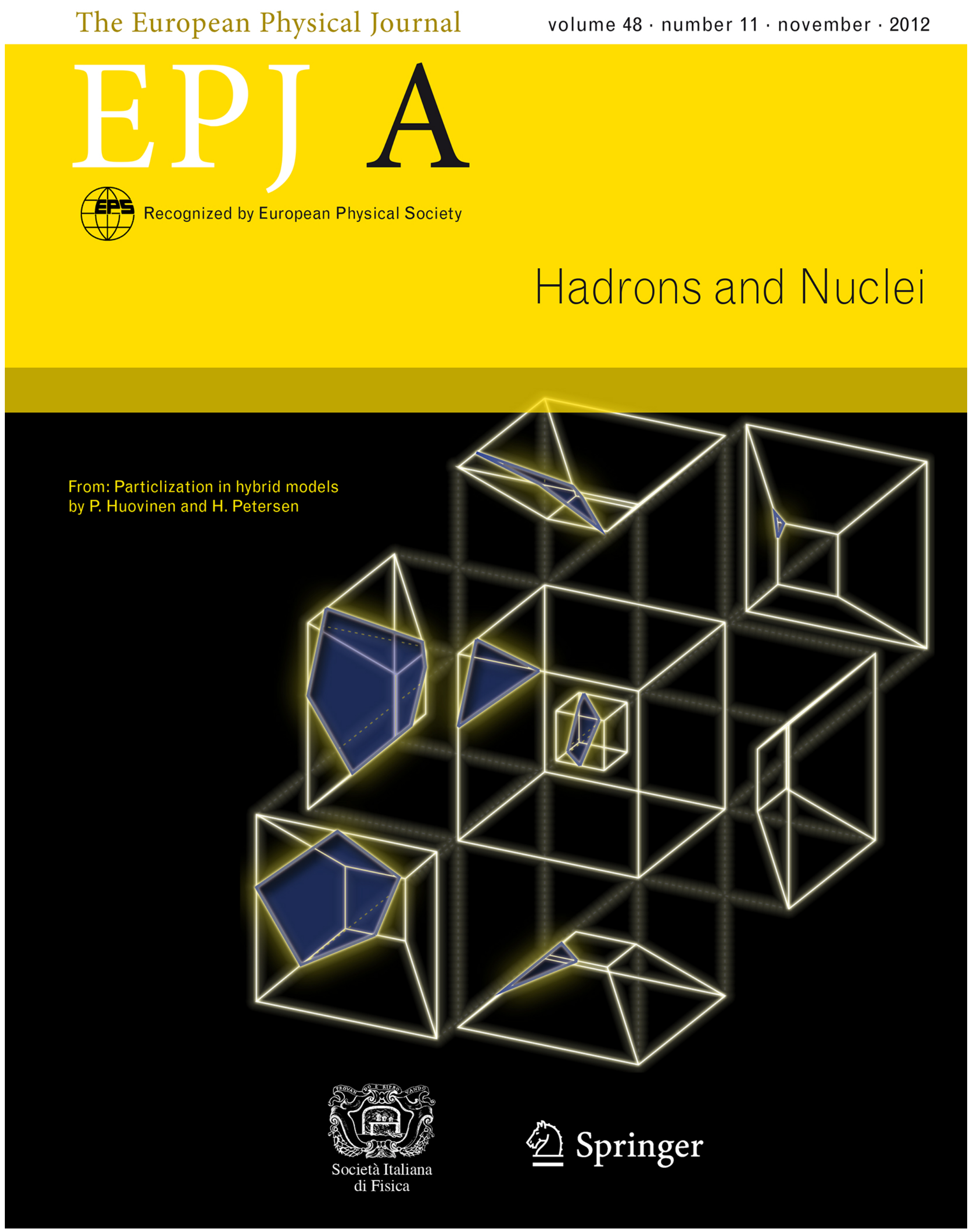
- Separation of **chemical** and **kinetic** freeze-out is taken into account
- **Large viscosity** in hadron gas stage!



H.P. et al, Phys.Rev. C78 (2008) 044901

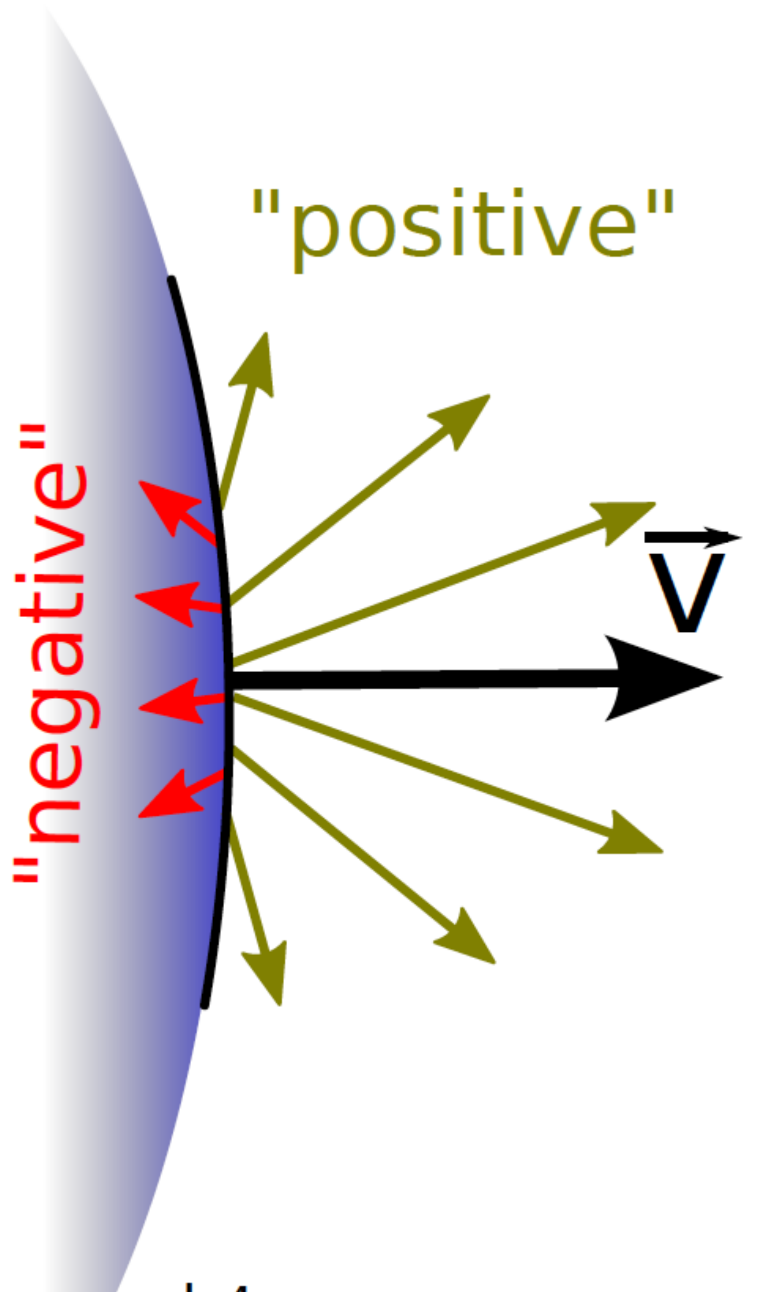
Hypersurface Finding

- **Cornelius**: 3D hypersurface in 4 dimensions
- Constant energy density
- Avoiding holes and double-counting
- Applicable as a subroutine
  - Input: 16-tuples of spatio-temporal information
  - Output: Hypersurface vectors and interpolated thermodynamic quantities



P. Huovinen, HP, EPJA 48, 2012
Fortran and C++ subroutines, cornelius, implementations of this algorithm in 3D and 4D, are available at https://karman.physics.purdue.edu/OSCAR

Negative Contributions



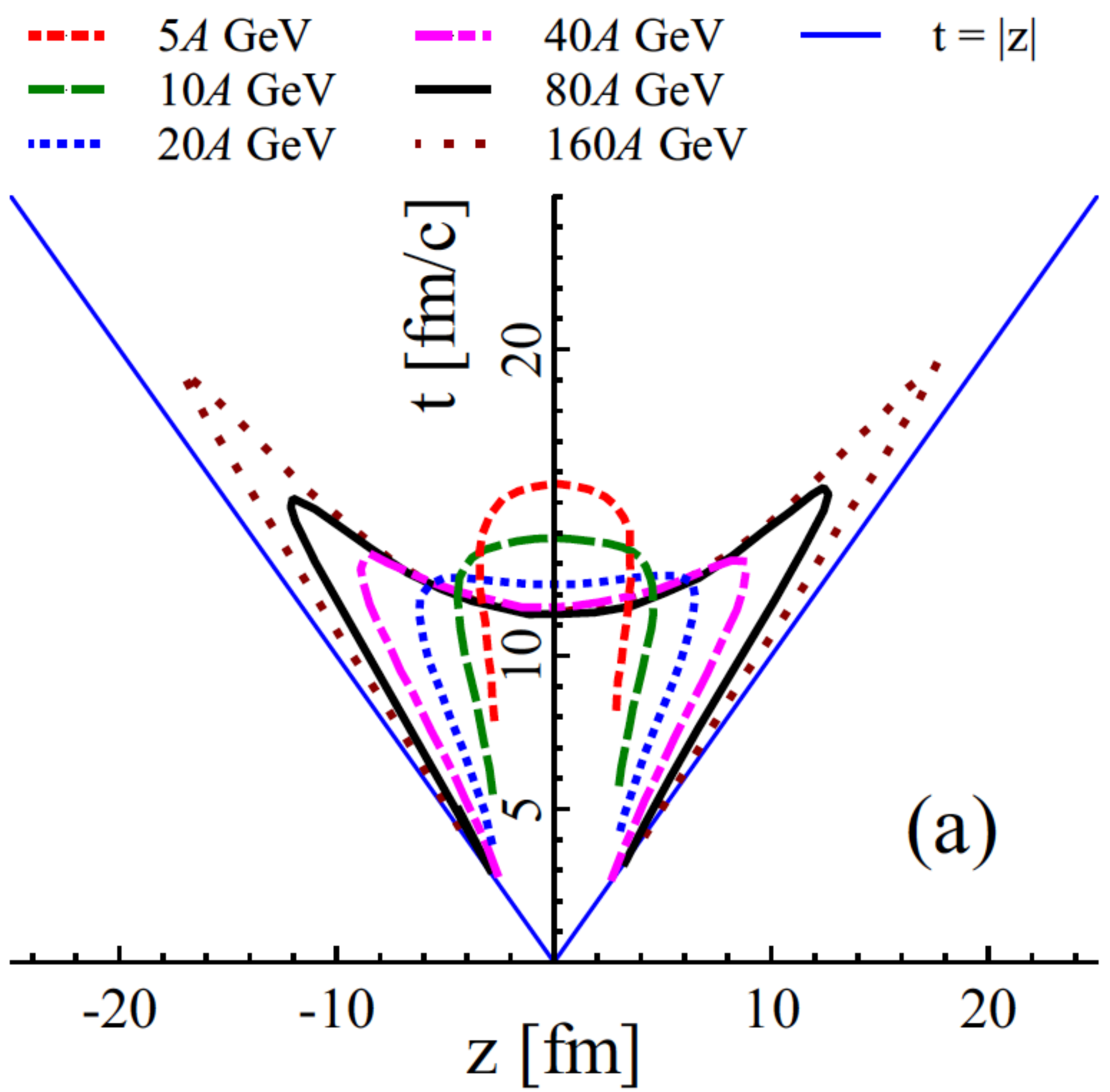
$d\sigma_\mu$ - normal 4-vector
$u_\mu = (\gamma, \gamma \vec{v})$ - 4-velocity
T - temperature
μ - chemical potential

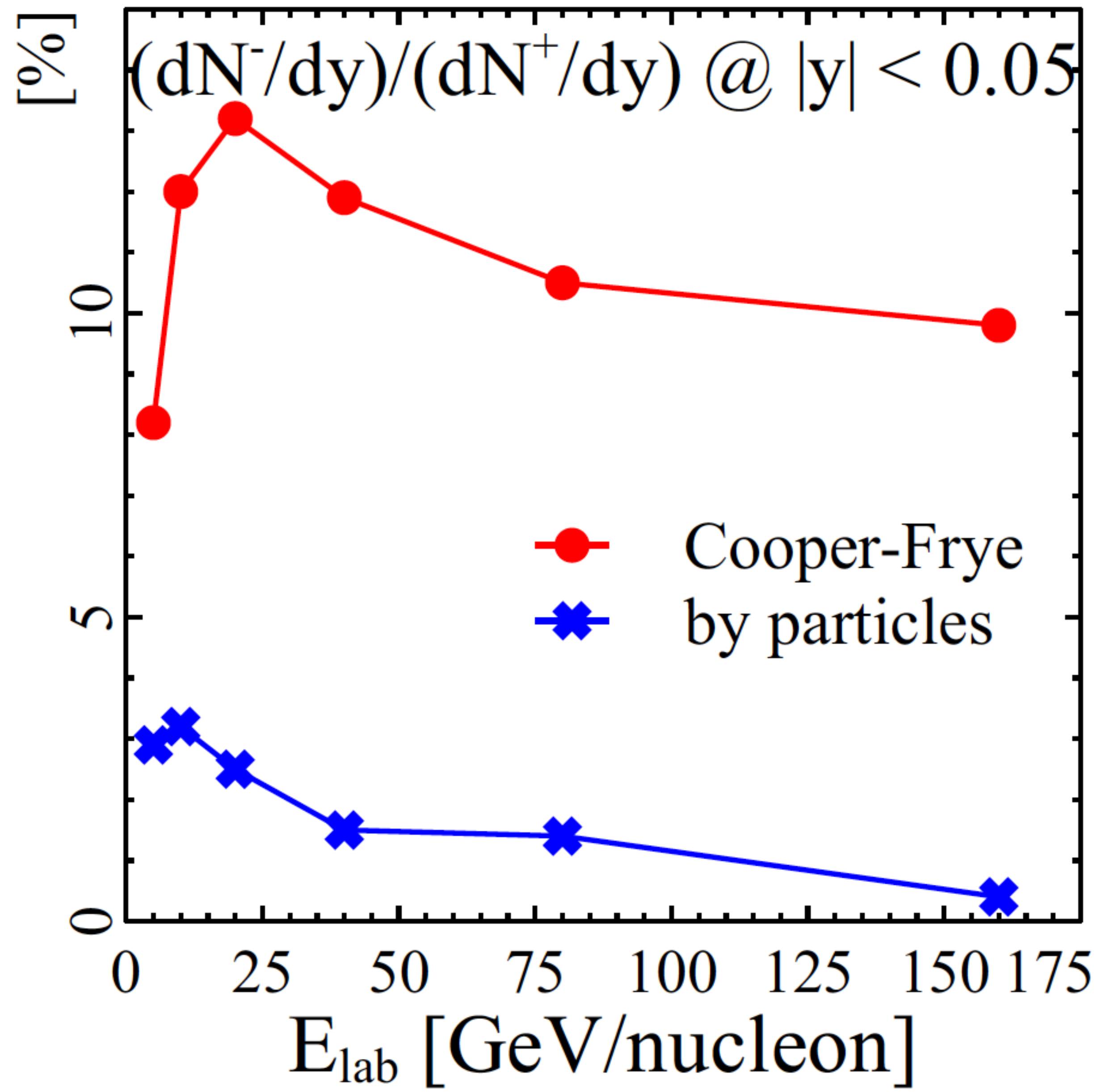
- Definition:
  - Particles outward: $p^\mu d\sigma_\mu > 0$
  - Particles inward: $p^\mu d\sigma_\mu < 0$
- Different options:
  - Account for feedback in hydro
    K. Bugaev, Phys Rev Lett. 2003; L. Czernai, Acta Phys. Hung., 2005
  - Account effectively by weights in transport
    S. Pratt, Phys.Rev. C89 (2014) 2, 024910
  - Neglect them and violate conservation laws
- **Systematic study** of the size of negative contributions by comparison to **actual transport**

Energy Dependence

- Iso-energy density hypersurfaces (ε_c = 0.3 GeV/fm^3)

D. Oliinychenko and HP, PRC91(2015)





- Maximum at E_{lab}~25 AGeV, decreasing at higher energies
- Actual particles are always less likely to fly inward

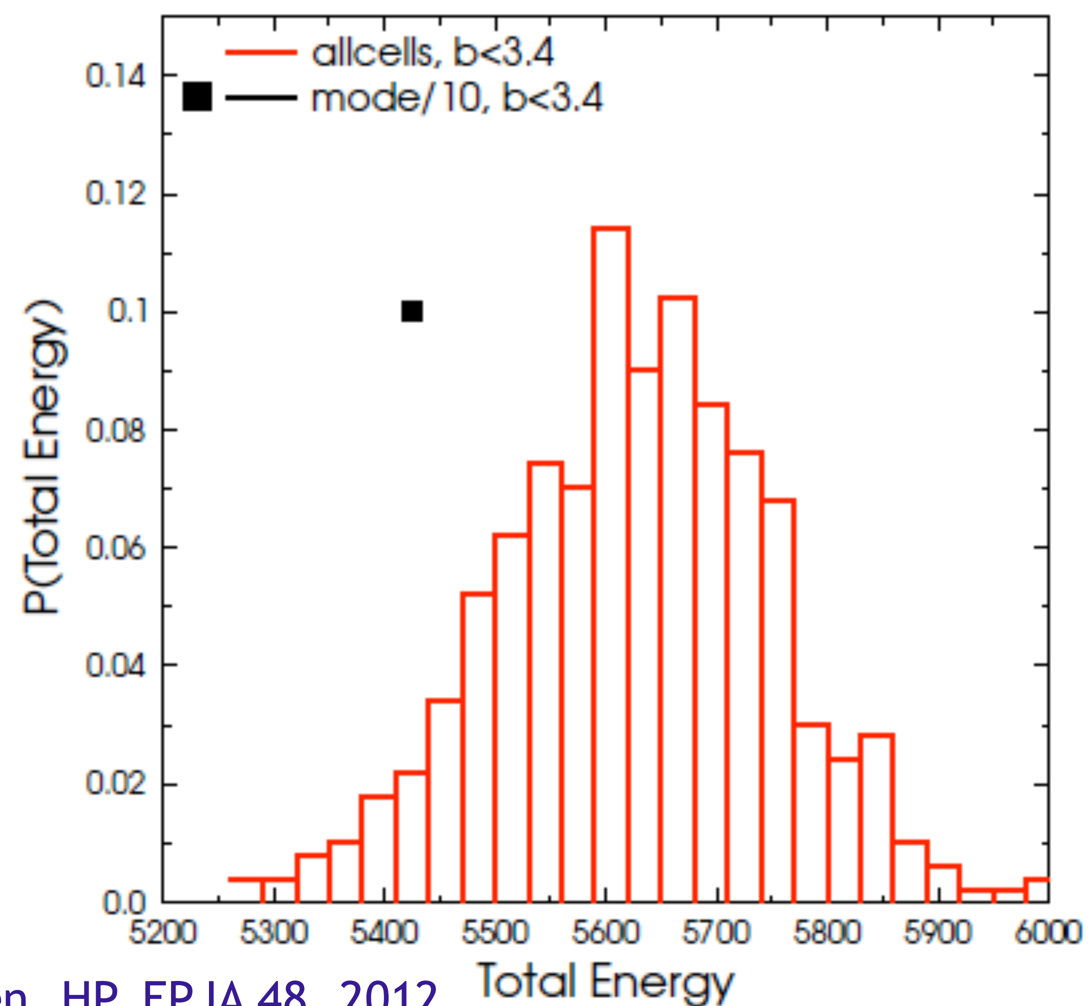
Global Conservation Laws

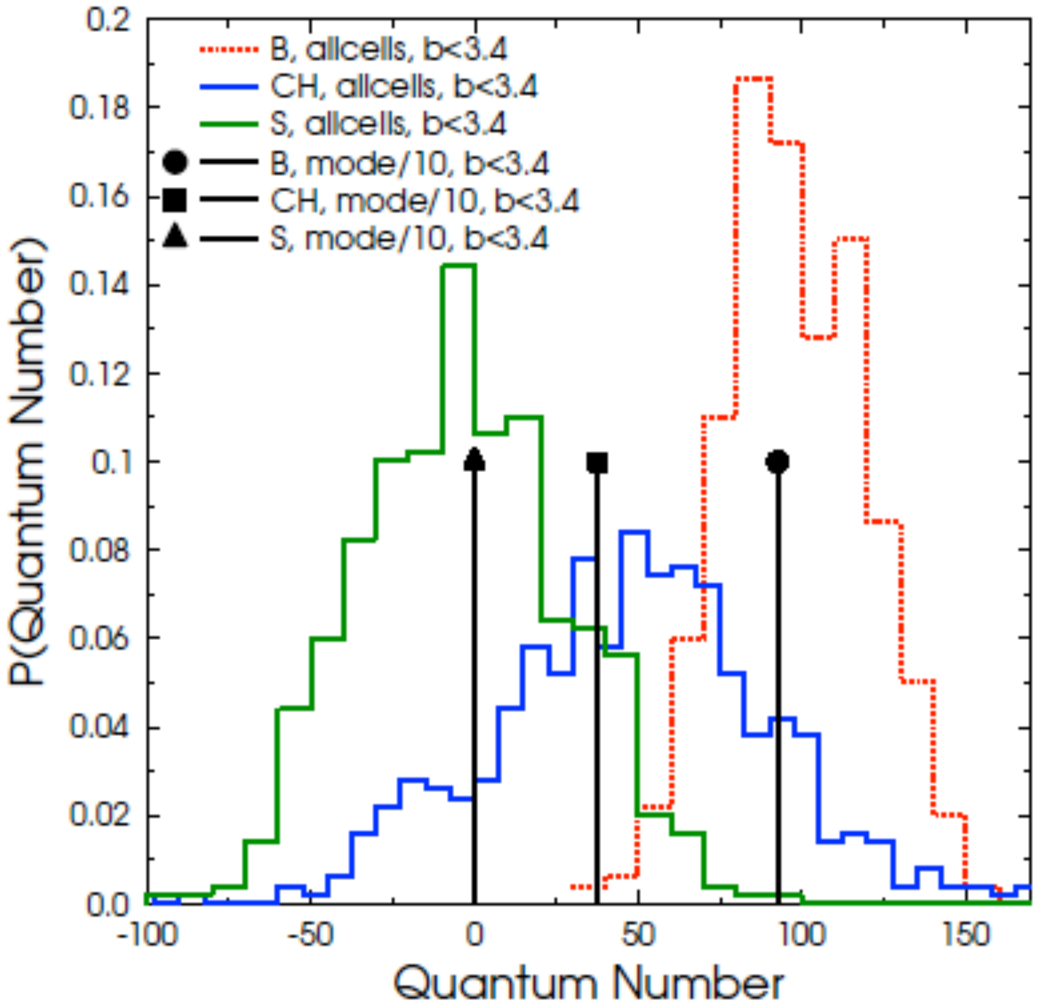
Why Conservation Laws?

- Event-by-event hybrid approach
  - One initial state (with fluctuations), one hydro run, one (or multiple) sample(s) of particles in the final state
  - Finite net baryon number B, net strangeness S, electric charge Z and energy E in initial state
- In nature quantum numbers are conserved
- Hydrodynamic evolution conserves energy and net baryon number explicitly
- Apply global conservation laws to sampling
  - No effect on single particle observables expected
  - Important for correlation and fluctuation observables, e.g. higher moments at low beam energies

Lower Beam Energies

- Loop through all cells (allcells):
  - Total quantum numbers fluctuate
- Randomly choose cells, until total energy is conserved (mode), reproduce finite S, B, Q





P. Huovinen, HP, EPJA 48, 2012

Steps for the Particle Production

1) Numbers of each particle species in the element
   - $N_i = j^\mu d\sigma_\mu = n_i u^\mu d\sigma_\mu \qquad n_i = \frac{4\pi g_i m_i^2 T}{(2\pi^3)} \exp\left(\frac{\mu}{T}\right) K_2\left(\frac{m_i}{T}\right)$
2) Sum to get the total particle number
3) Particle production according to Poisson distribution
4) Particle type chosen according to respective probabilities
5) Isospin randomly assigned
6) Generate four-momenta (rejection method)
   - $\frac{dN(x)}{d^3p} = \frac{1}{E} f(x,p) p^\mu d\sigma_\mu$ with $f(x,p) p^\mu d\sigma_\mu > 0$
7) Particle vector information is transferred to hadronic transport approach

Mode Sampling

- Seven loops over the hypersurface elements:
  - 1) Strange particles
  - 2) Anti-strange particles
  - 3) Non-strange anti-baryons
  - 4) Non-strange baryons
  - 5) Negatively charged non-strange mesons
  - 6) Positively charged non-strange mesons
  - 7) Neutral non-strange mesons

HP et al., PRC 78, 2008
P. Huovinen, HP, EPJA 48, 2012

- Loops 1,3,5 and 7 are cut by energy conservation
- Other loops by respective conservation laws
- Has been successfully applied within UrQMD hybrid approach at finite net baryon densities

SPREW Sampling

- Single Particle Rejection with Exponential Weights
- If a certain particle is chosen to be produced, ΔX is calculated for each quantum number, where

$$\Delta X = X_{particles} - X_{hypersurface}$$

- If ΔX and the X_i have the same sign, the particle is rejected with probability

$$1 - e^{|\Delta X|}$$

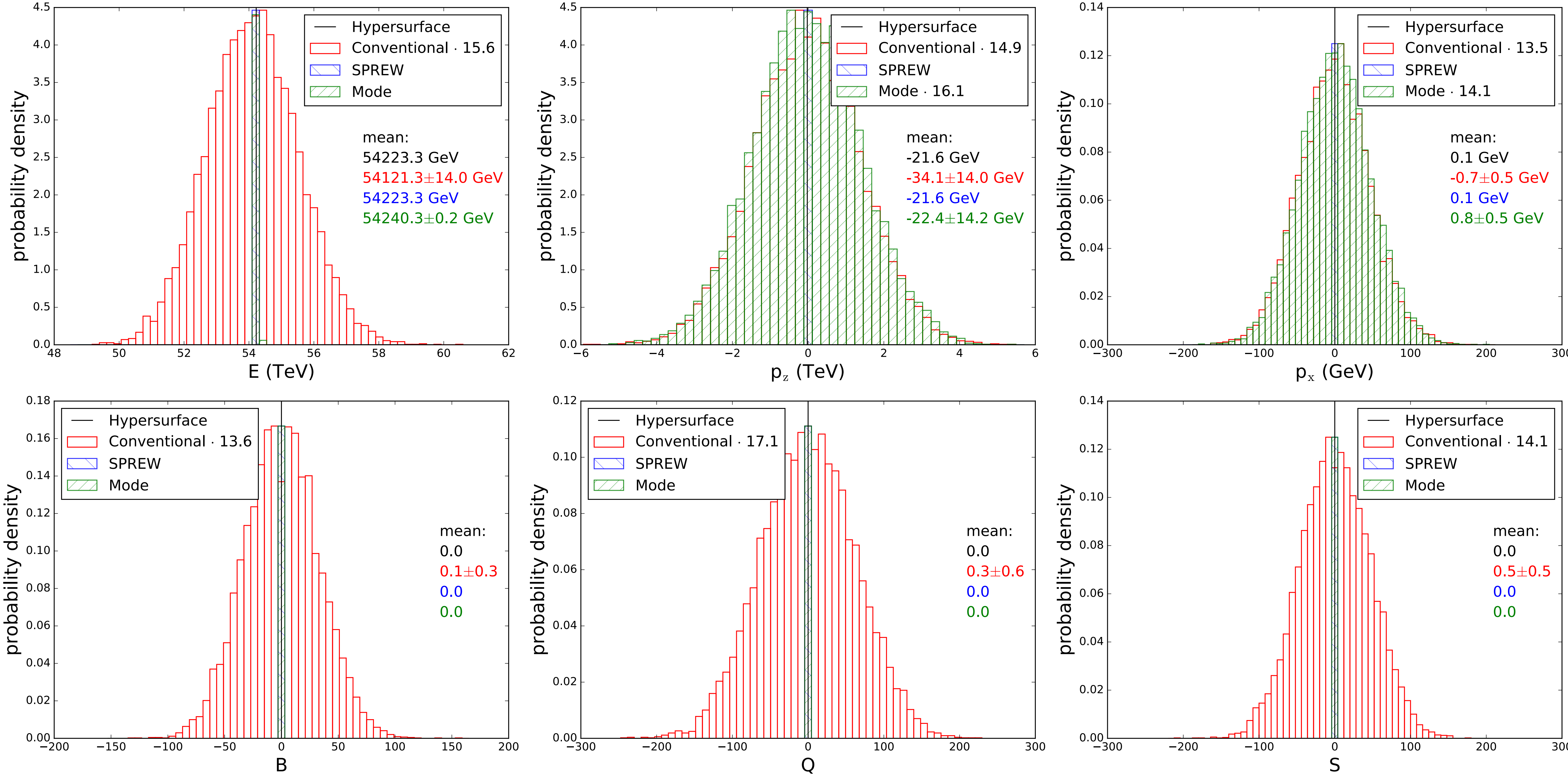
- Energy and momenta are rescaled to fit the values on the hypersurface
- Test the performance of both algorithms in single cell

C. Schwarz et al., JPG 452018

Au+Au at 200 GeV

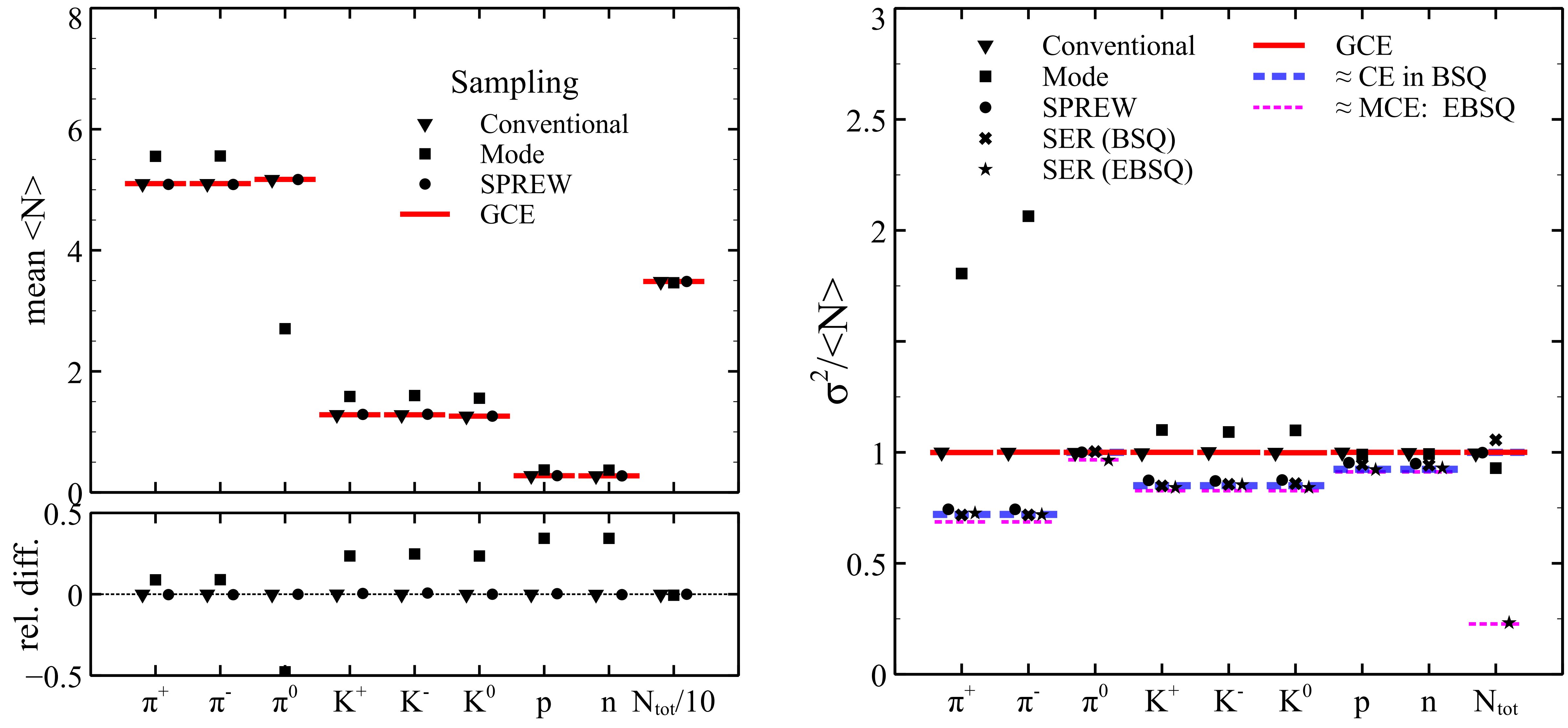
- Two different algorithms (mode and SPREW) are compared to conventional sampling

C. Schwarz et al., JPG 452018



Thermal Box

- Even in small system (5fm length), SPREW reproduces the proper multiplicities including fluctuations



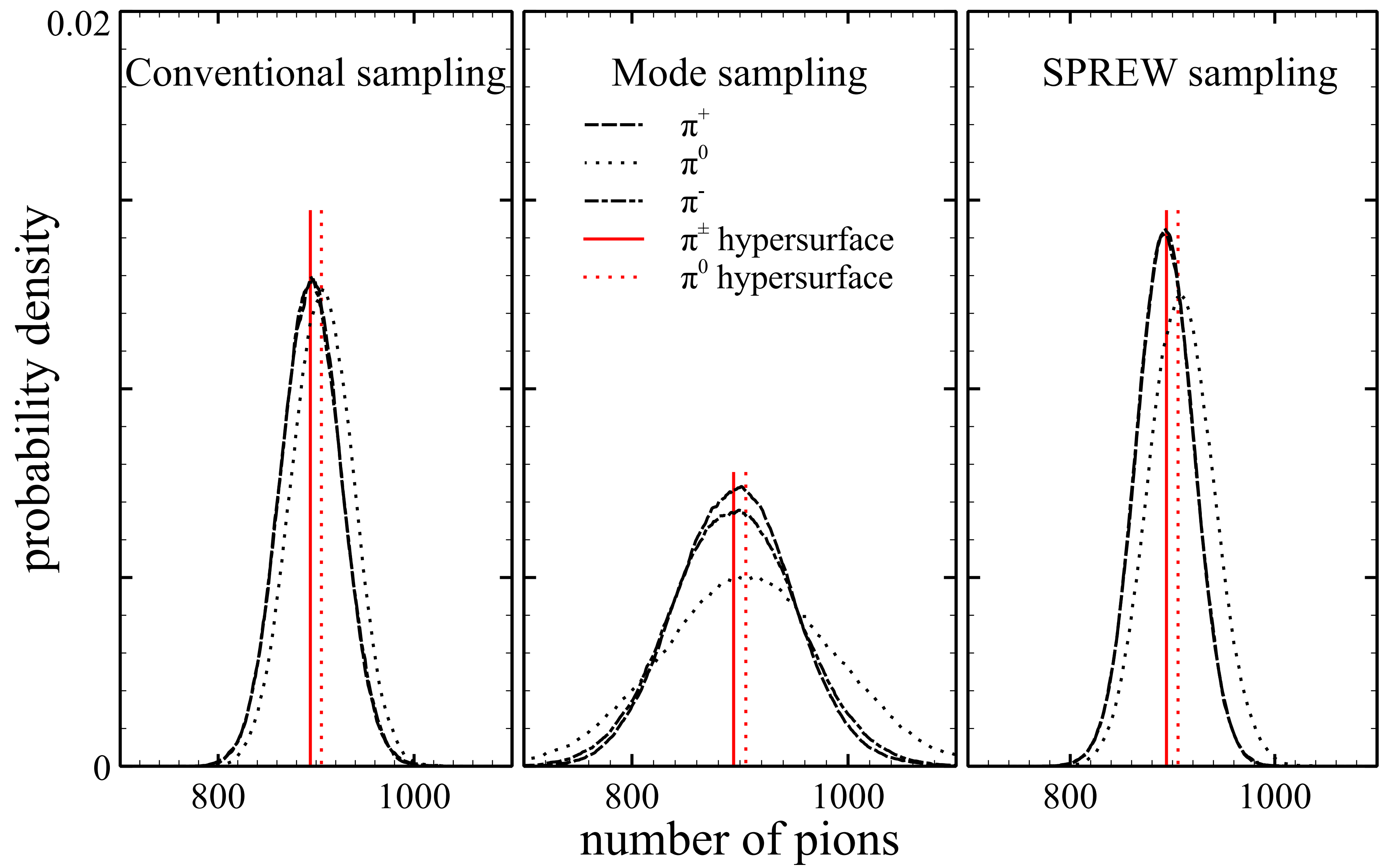
- Bias in mode sampling appears in small systems

C. Schwarz et al., JPG 452018

Pions in Heavy Ion Collision

- Comparison of the full distribution for pion production in Au+Au at 200 GeV

C. Schwarz et al., JPG 452018

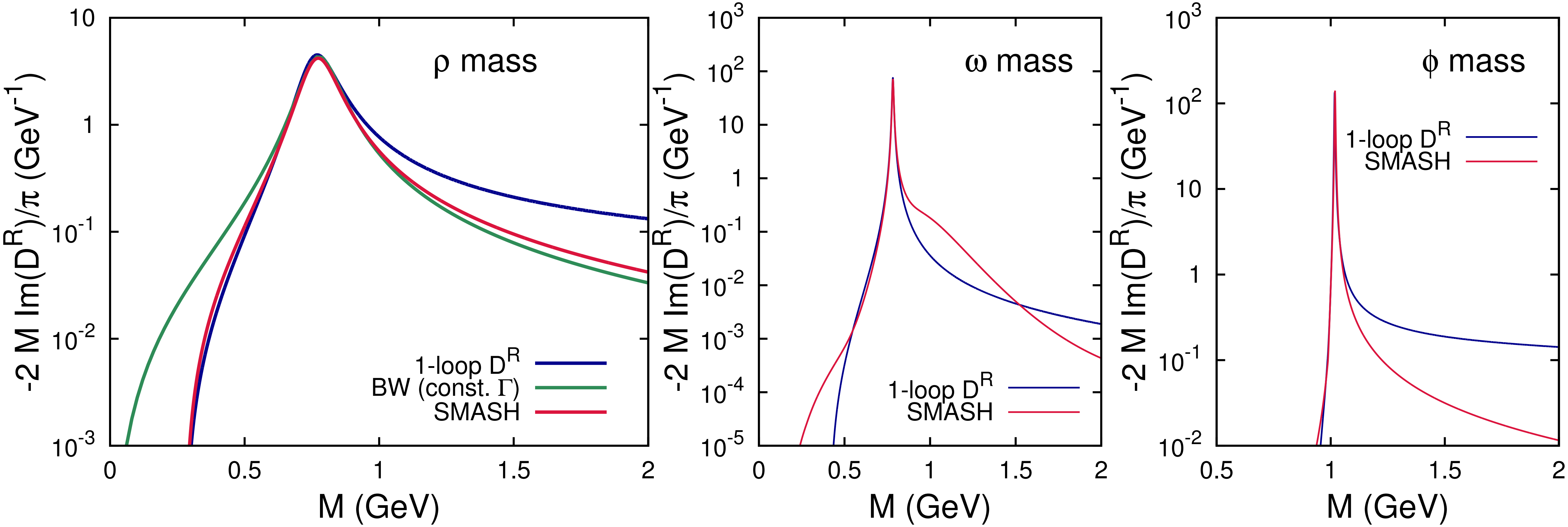


- SPREW sampling minimizes bias by conservation laws

Hadronic Rescattering and Broad Spectral Functions

Resonance Masses

- Typically resonances are sampled only with their pole masses at particlization
- In hadronic transport approaches spectral functions are used, usually Breit-Wigner distributions
- Vector meson masses as shown below:

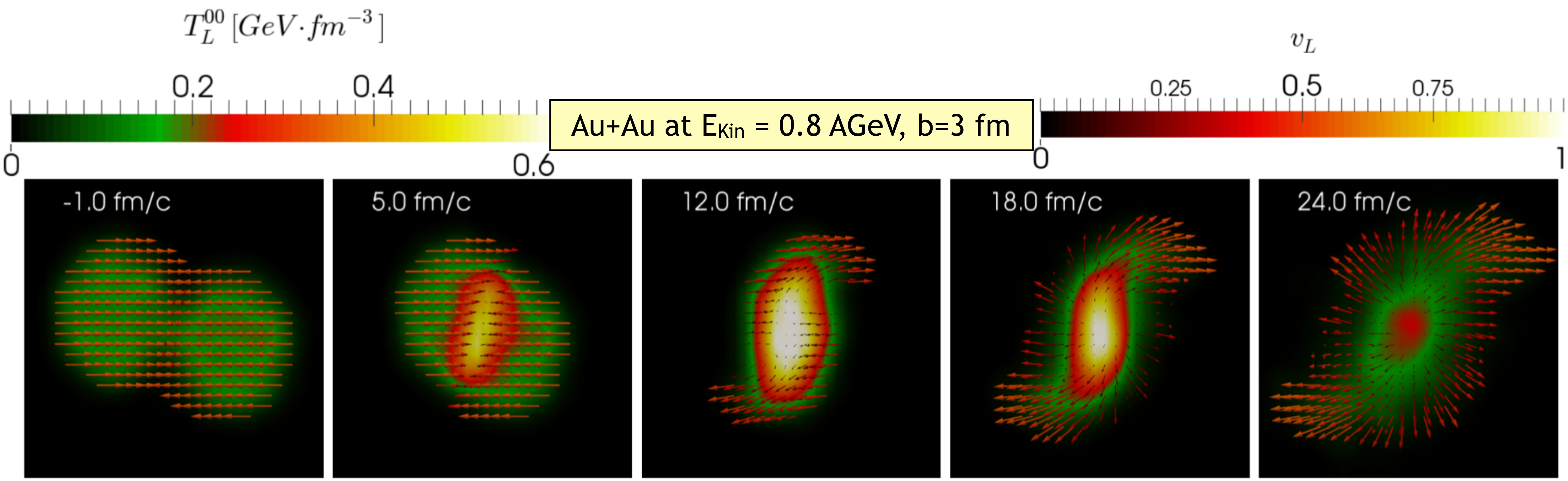


- Influences dilepton emission from hadronic afterburner

SMASH*

- Hadronic transport approach:
  - Includes all mesons and baryons up to ~2 GeV
  - Geometric collision criterion
  - Binary interactions: Inelastic collisions through resonance/string excitation and decay
  - Infrastructure: C++, Git, Redmine, Doxygen, (ROOT)

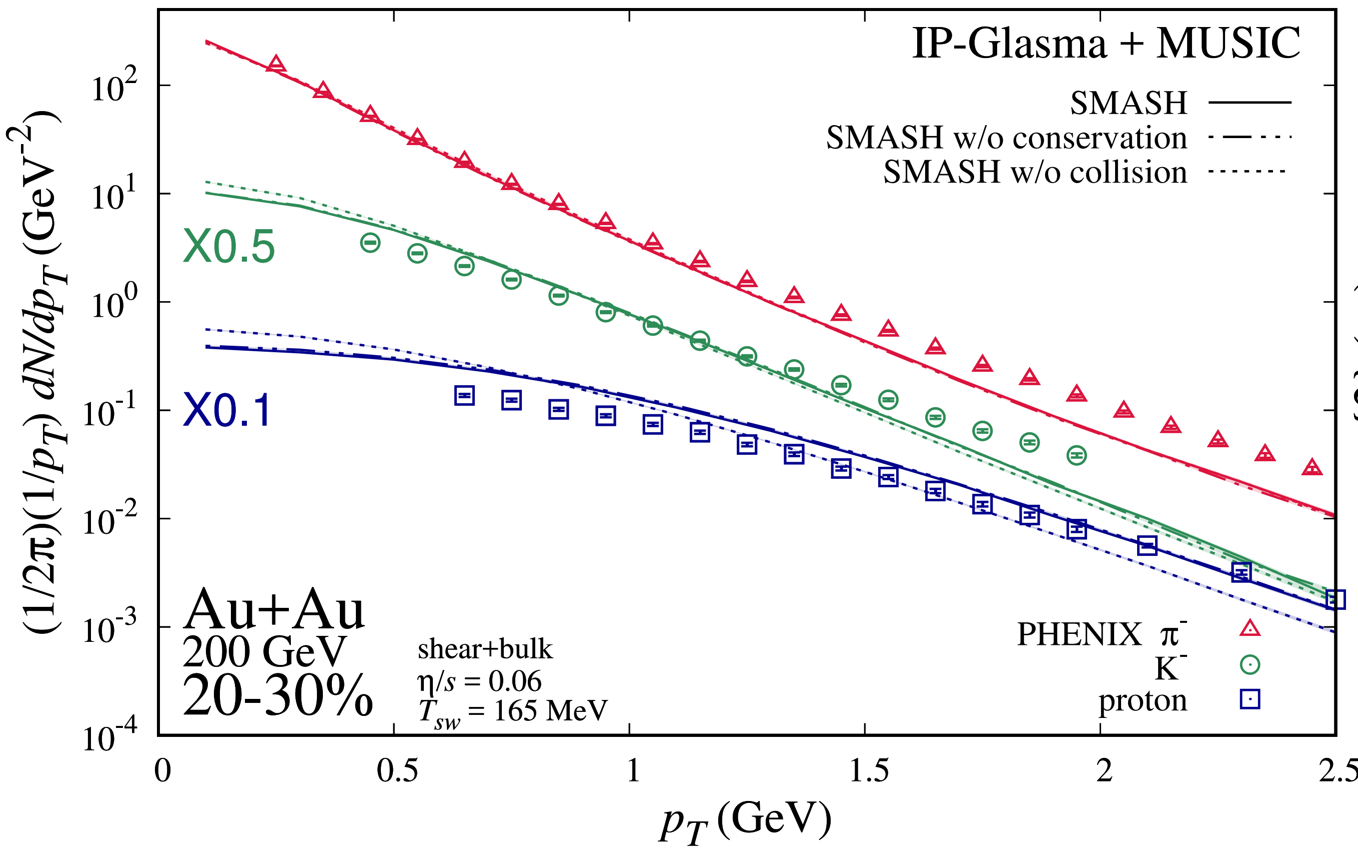
J. Weil et al, PRC 94 (2016)

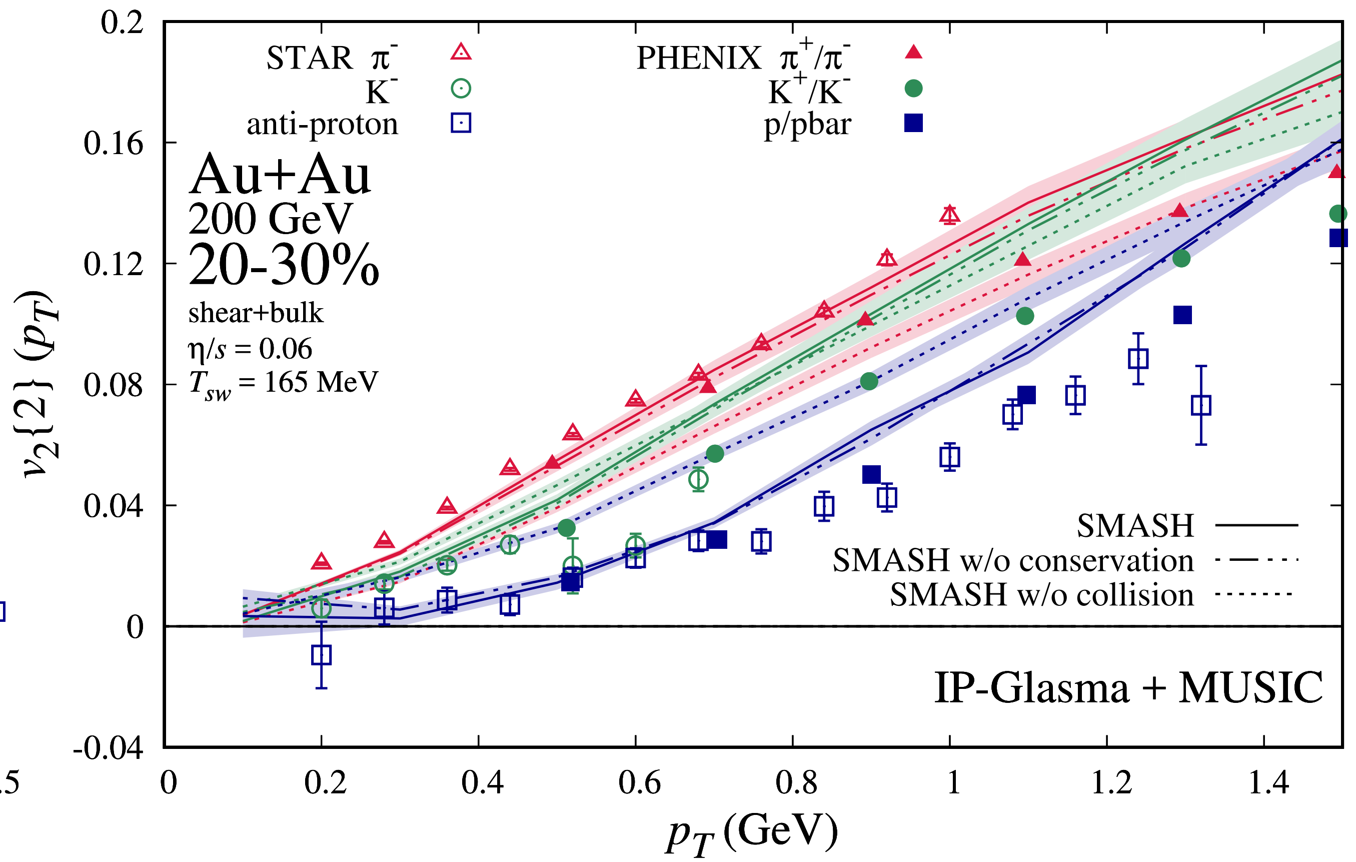


* Simulating Many Accelerated Strongly-Interacting Hadrons

Influence of Global Conservation

- Comparison between hydrodynamic calculation only with resonance decays and with full hadronic rescattering



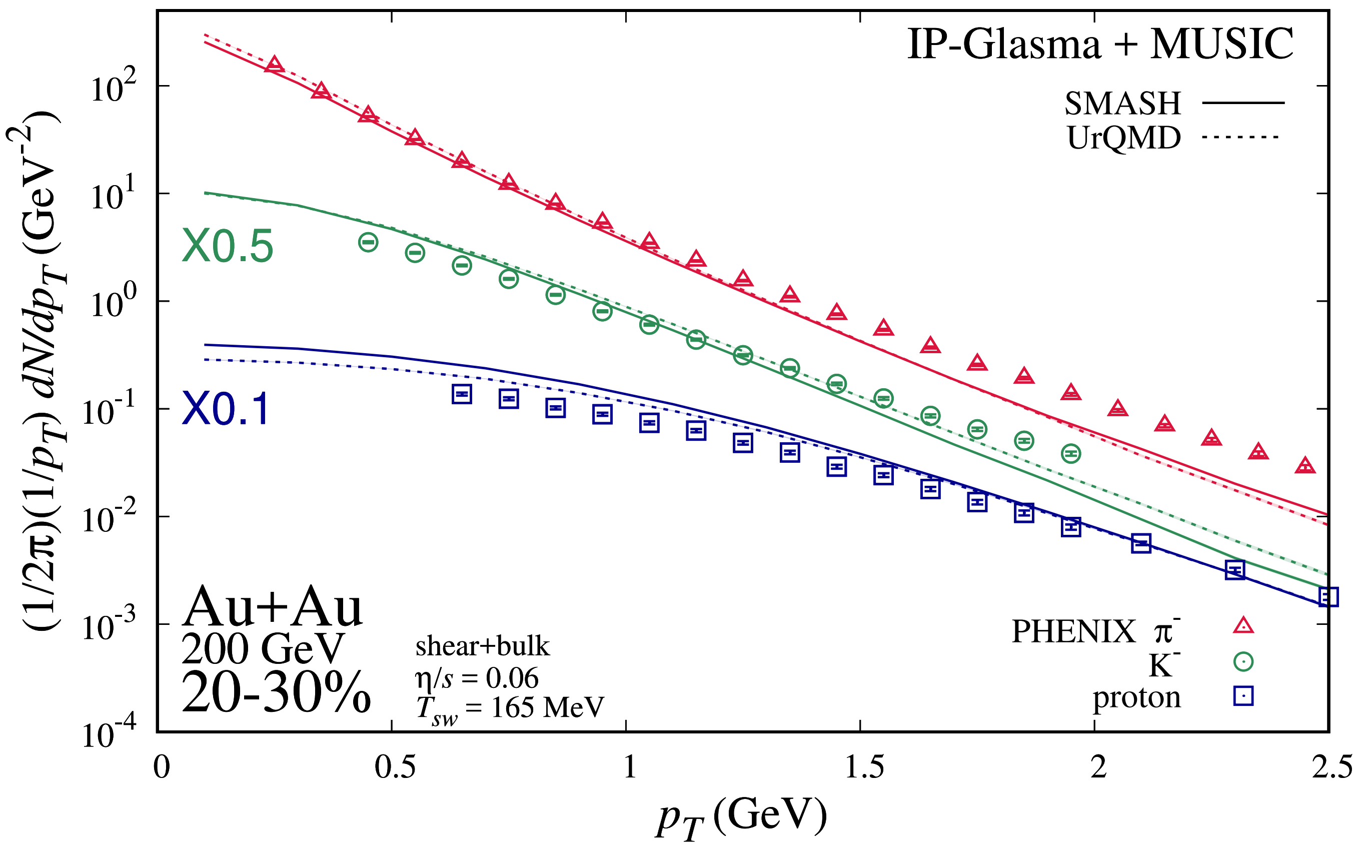


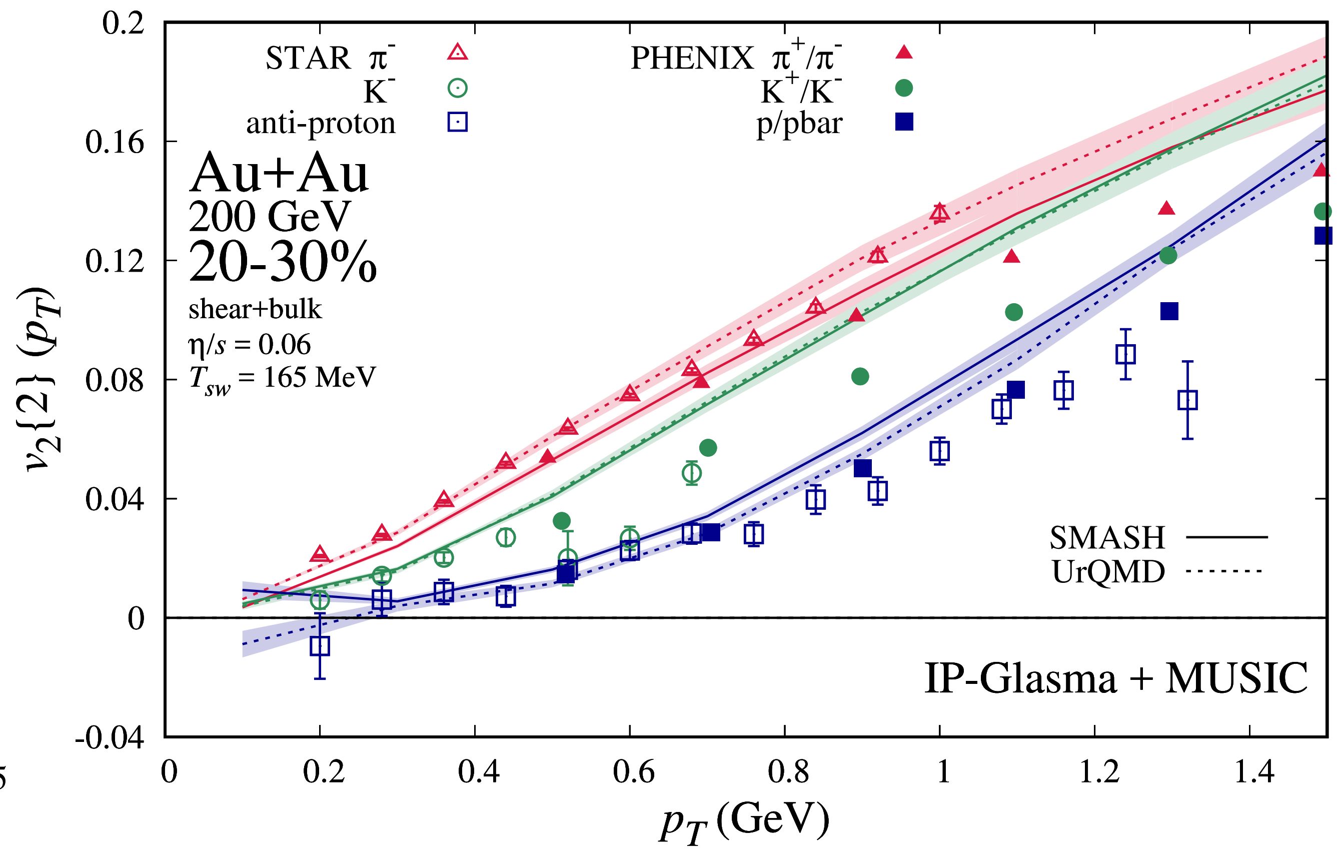
- Global conservation laws do not affect single particle observables as expected

Comparison SMASH vs UrQMD

- SMASH rescattering yields qualitatively similar results as UrQMD afterburner

MUSIC+UrQMD from S. Ryu et al, PRC 97 (2018)



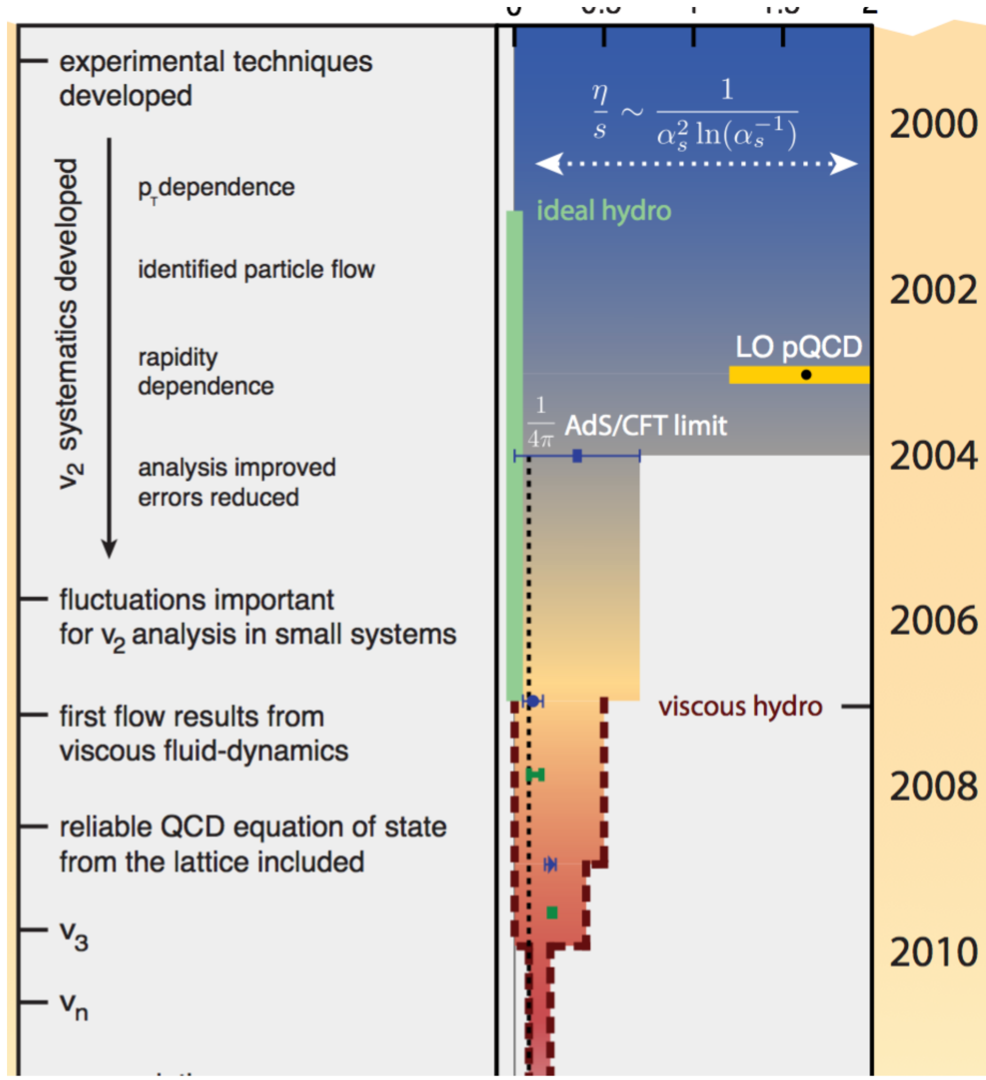


- Results are sensitive to missing baryon-antibaryon annihilation and AQM cross-sections

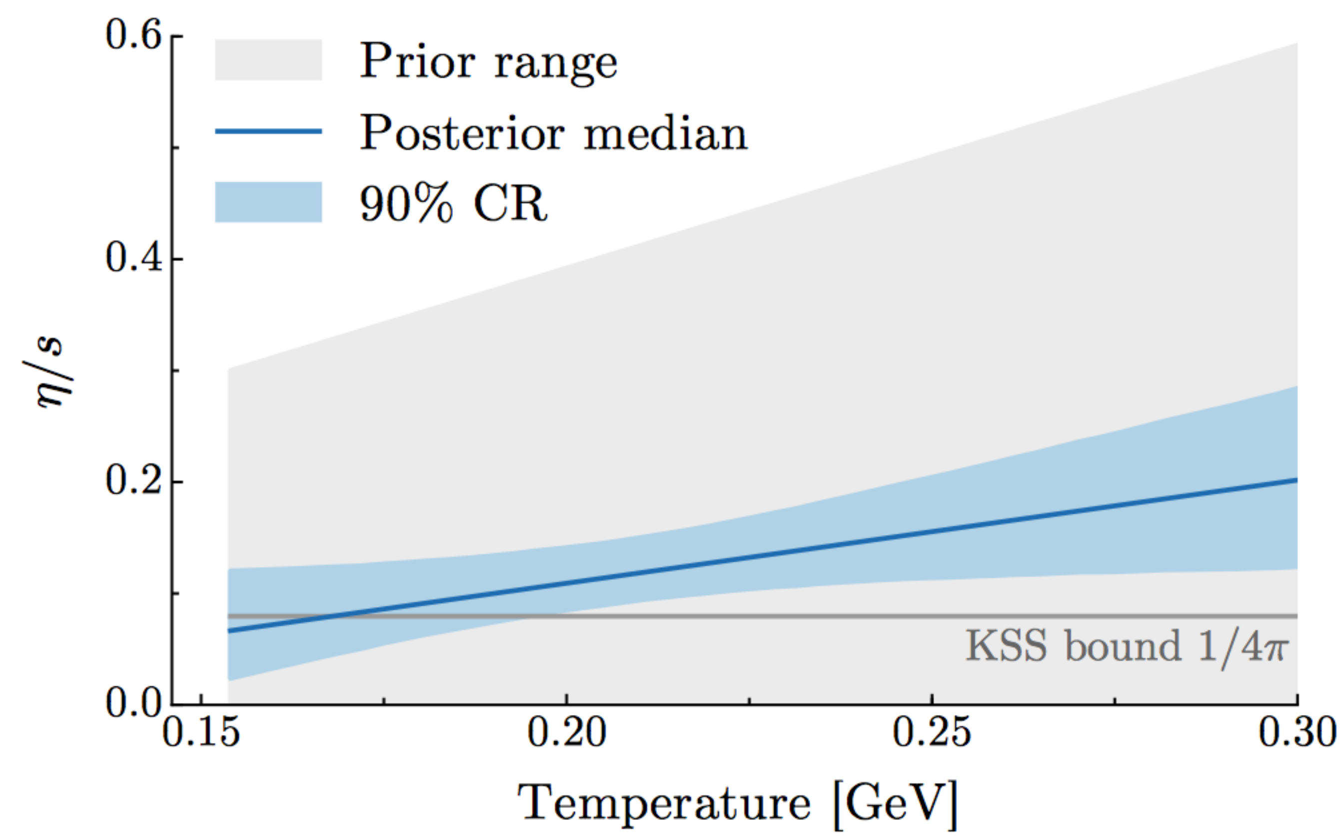
Shear Viscosity of the Hadron Gas

Transport Coefficients

- Within hydrodynamics/hybrid approaches the shear viscosity is an input parameter



RHIC White paper, 2012

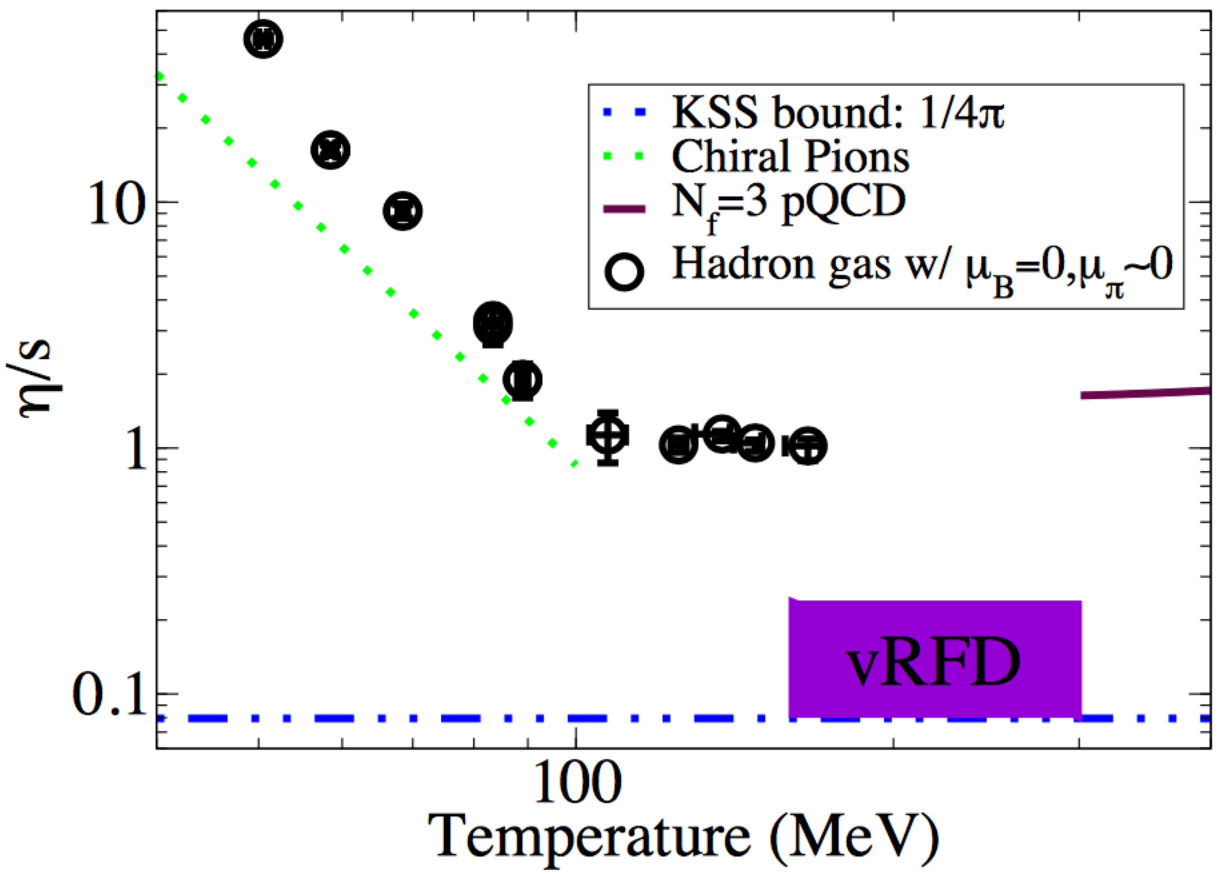


J. Bernhard et al, Phys.Rev. C94 (2016)

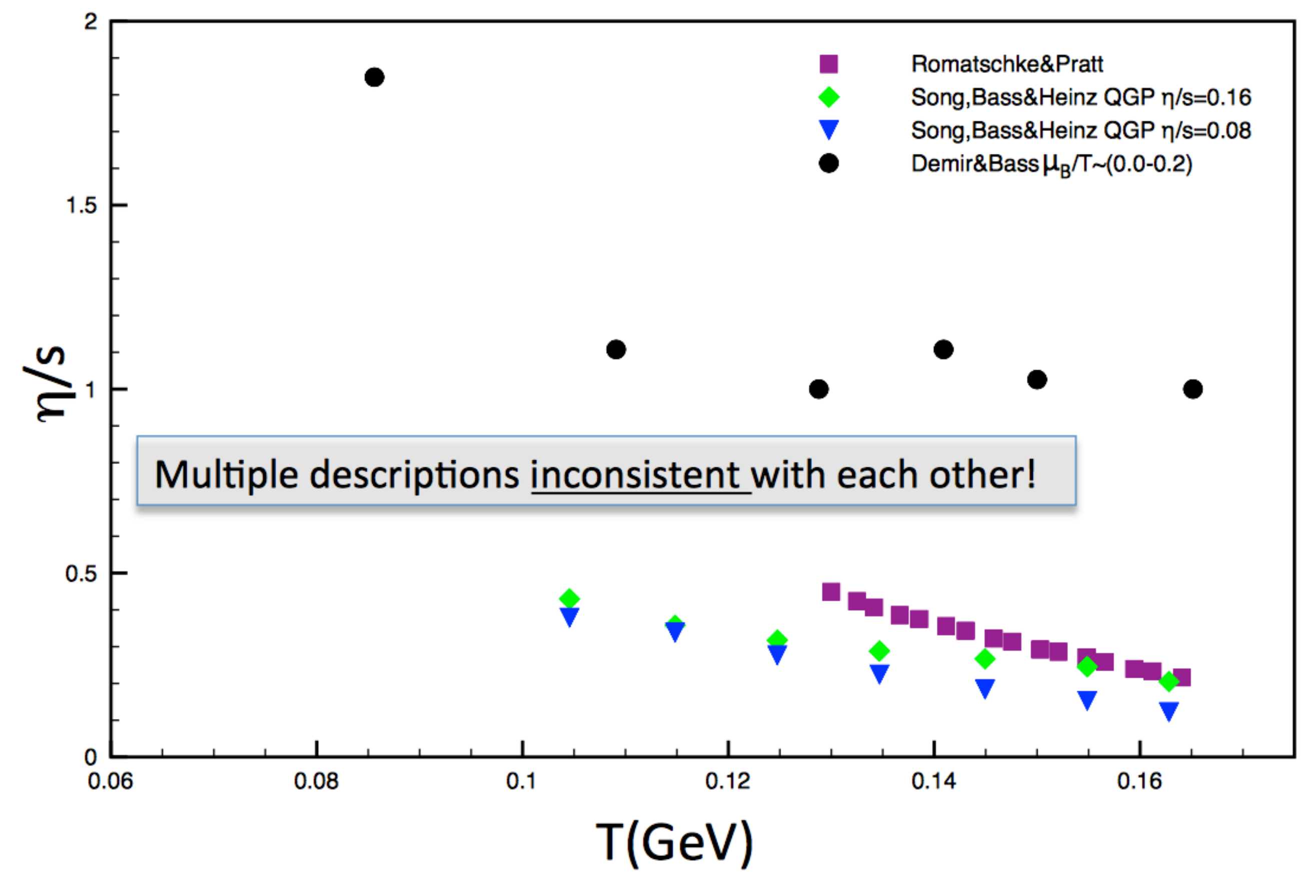
- Application of Bayesian techniques allows extraction of temperature dependence

Existing Results - Discrepancy

Green-Kubo formalism UrQMD	Discrepancy with hydro-inspired B3D and VISHNU



N. Demir and S.A. Bass, Phys.Rev.Lett. 102 (2009)



-Romatschke & Pratt, arXiv:1409.0010v1
-Song, Bass & Heinz, Phys. Rev. C83 (2011) 024912
-Demir & Bass, Phys.Rev.Lett. 102 (2009) 172302

- Long standing question: Why are the results so different from each other?

Shear Viscosity over Entropy Density

- Box with periodic boundary condition in chemical and thermal equilibrium
- Entropy is calculated via Gibbs formula from thermodynamic properties
- The shear viscosity is extracted following the Green-Kubo formalism:

$$\eta = \frac{V}{T} \int_0^\infty C^{xy}(t) dt$$

$$C^{xy}(t) = \frac{1}{N} \sum_s^N T^{xy}(s) T^{xy}(s+t)$$

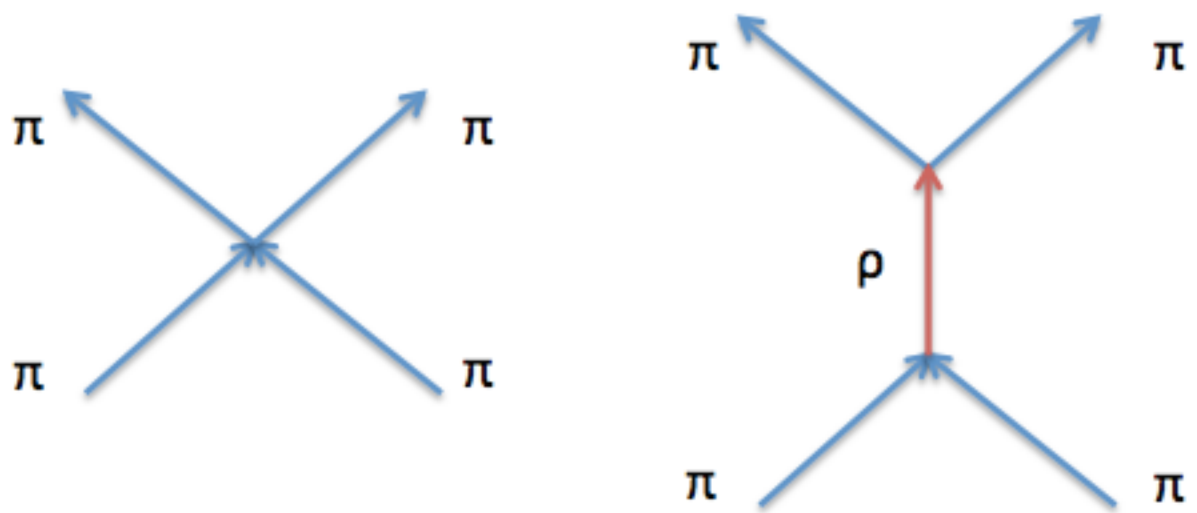
$$T^{\mu\nu} = \frac{1}{V} \sum_i^{N_{part}} \frac{p_i^\mu p_i^\nu}{p_i^0}$$

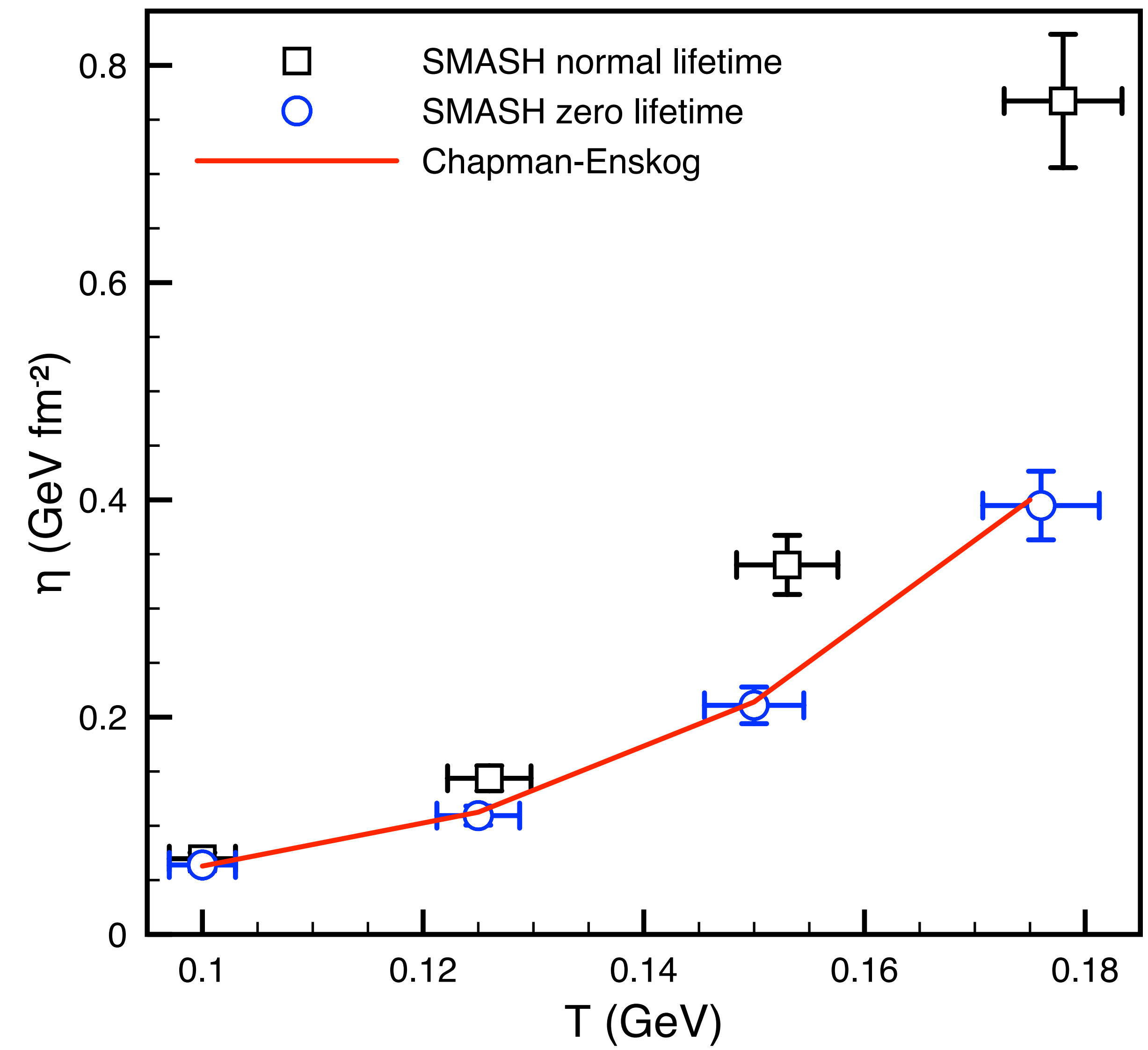
$$C^{xy}(t) \simeq C^{xy}(0) \exp\left(-\frac{t}{\tau}\right)$$

$$\eta = \frac{V C^{xy}(0) \tau}{T}$$

Resonance Dynamics

- Energy-dependence of cross-sections is modelled via resonances
- Point-like in analytic calculation and finite lifetime in transport approach



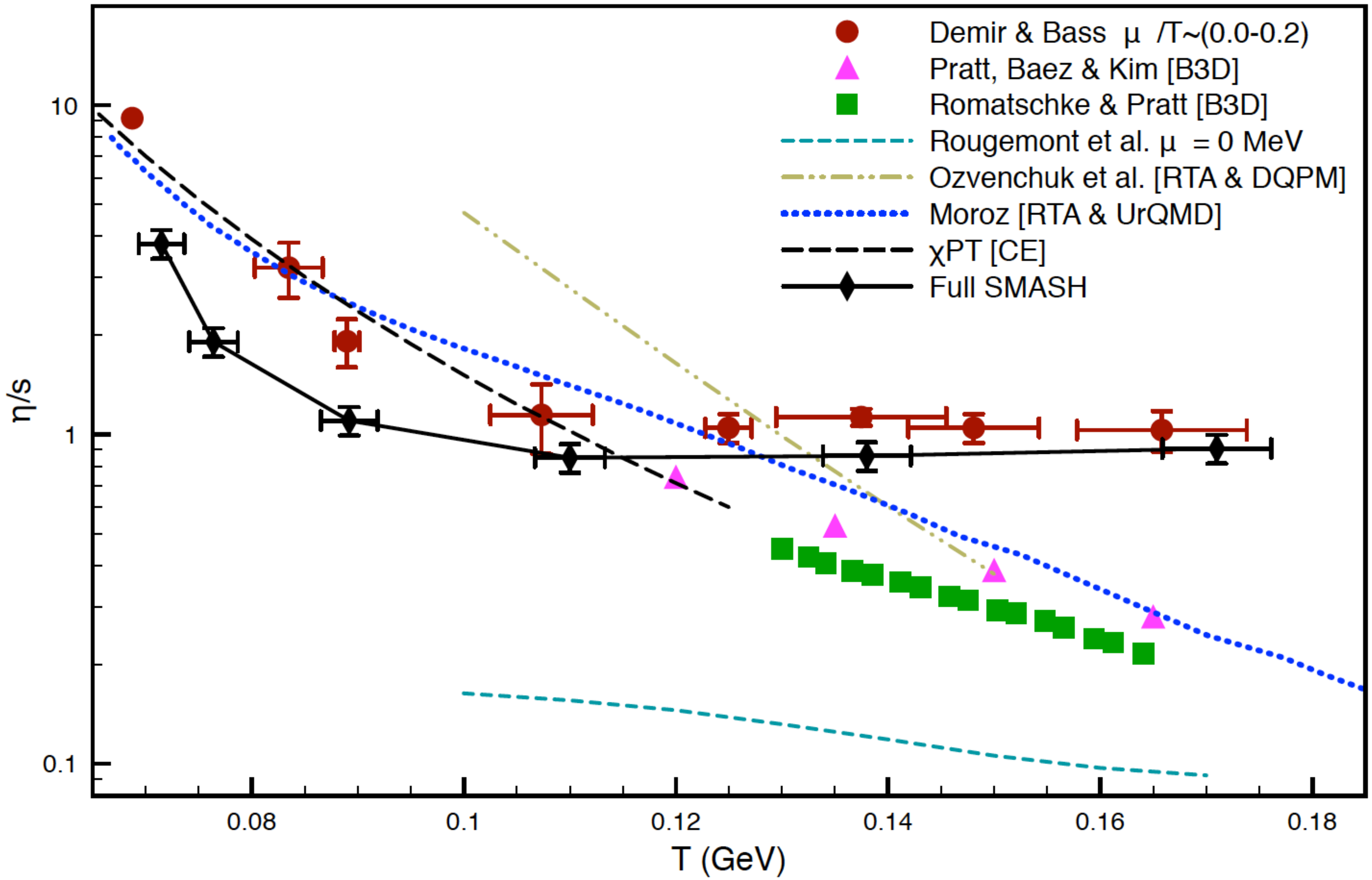


- Agreement recovered by decreasing ρ meson lifetime

J.-B. Rose et al., arXiv:1709.00369 and arXiv:1709.03826

Comparison to Literature

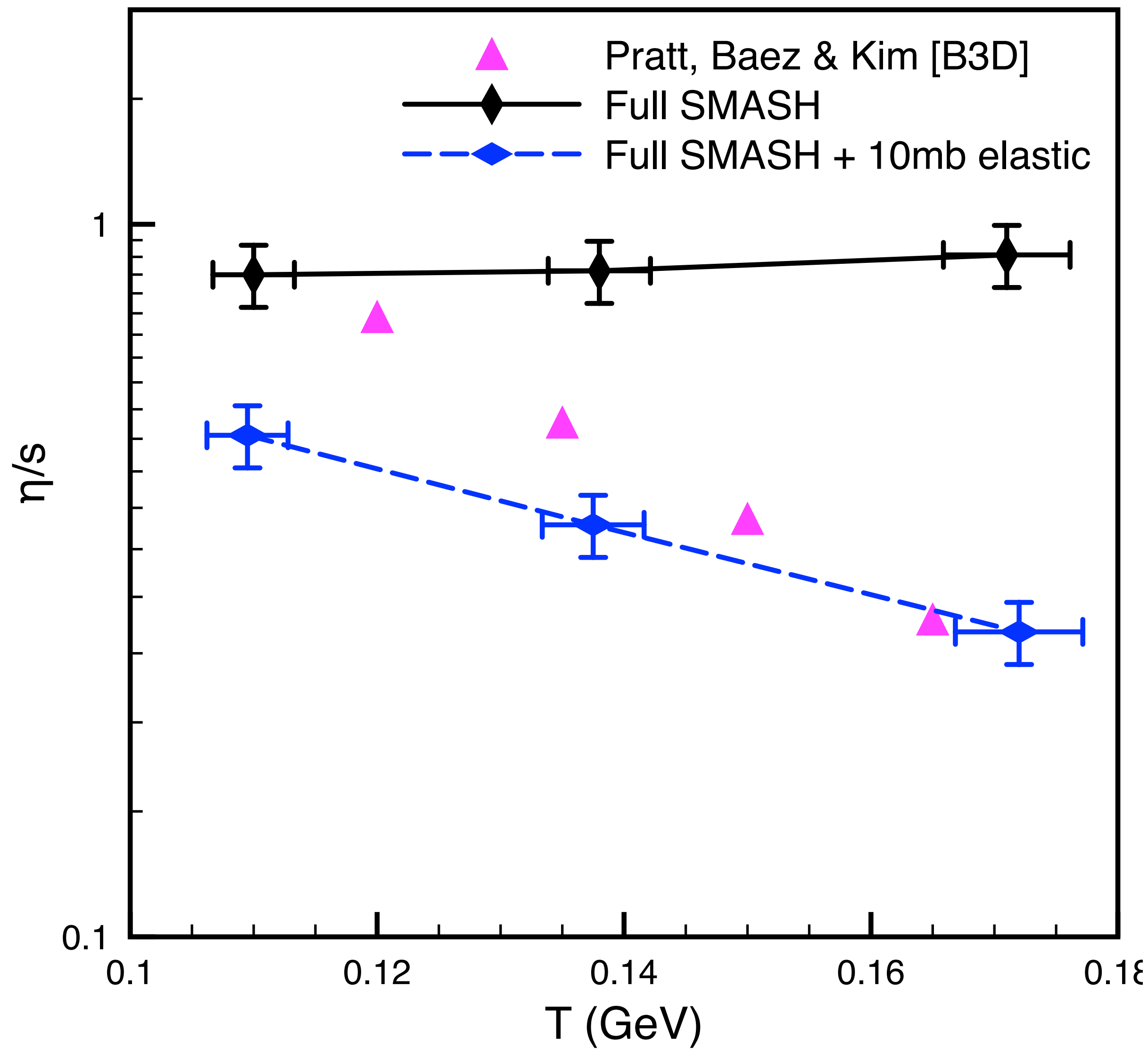
J.-B. Rose et al., arXiv:1709.00369 and arXiv:1709.03826

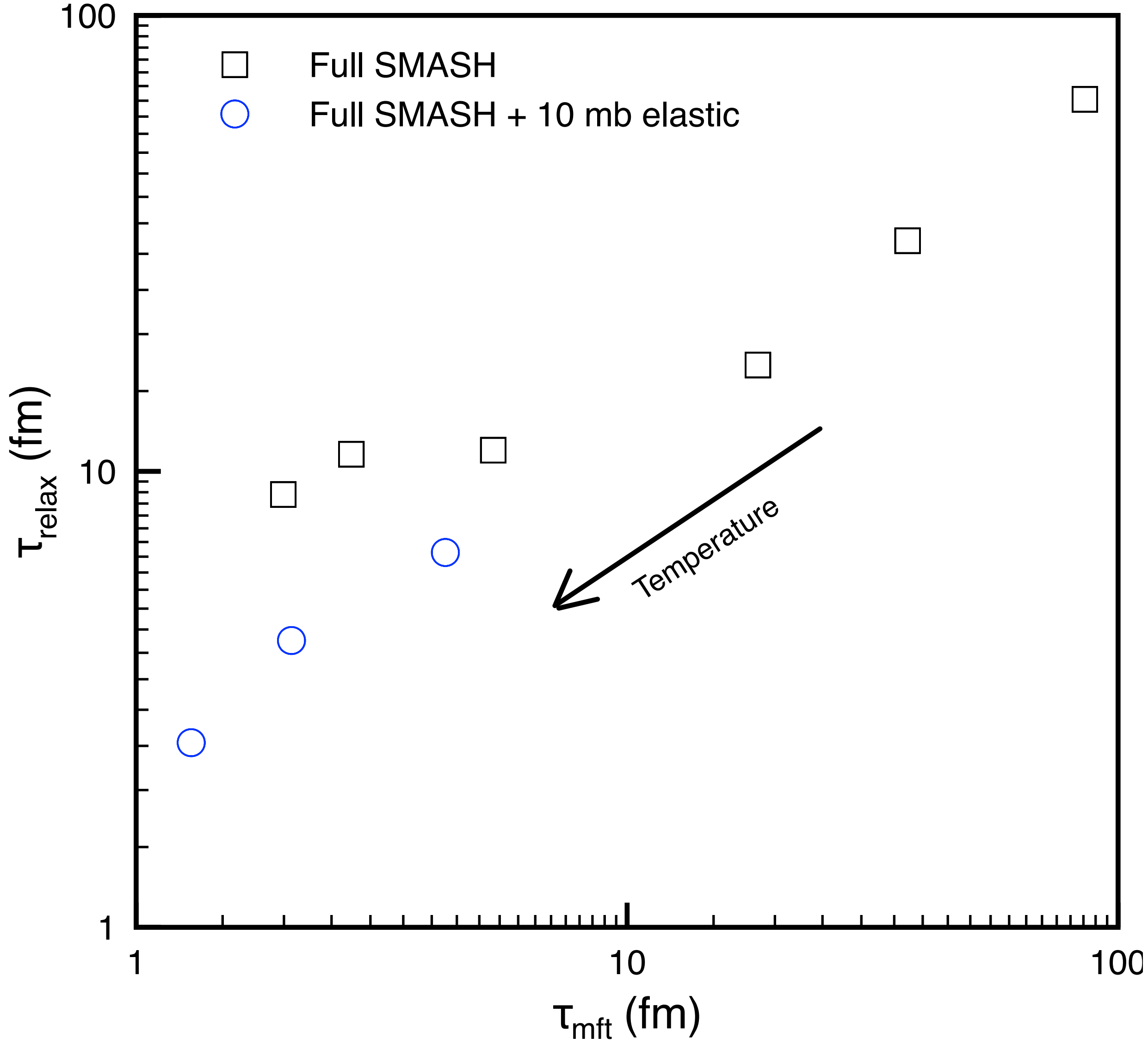


- Closest similarity to Bass/Demir result as expected

Point-like Interactions

- Adding a constant elastic cross section leads to agreement with B3D result





- Approximately linear relationship between relaxation time and mean free time is recovered

J.-B. Rose et al., arXiv:1709.00369 and arXiv:1709.03826

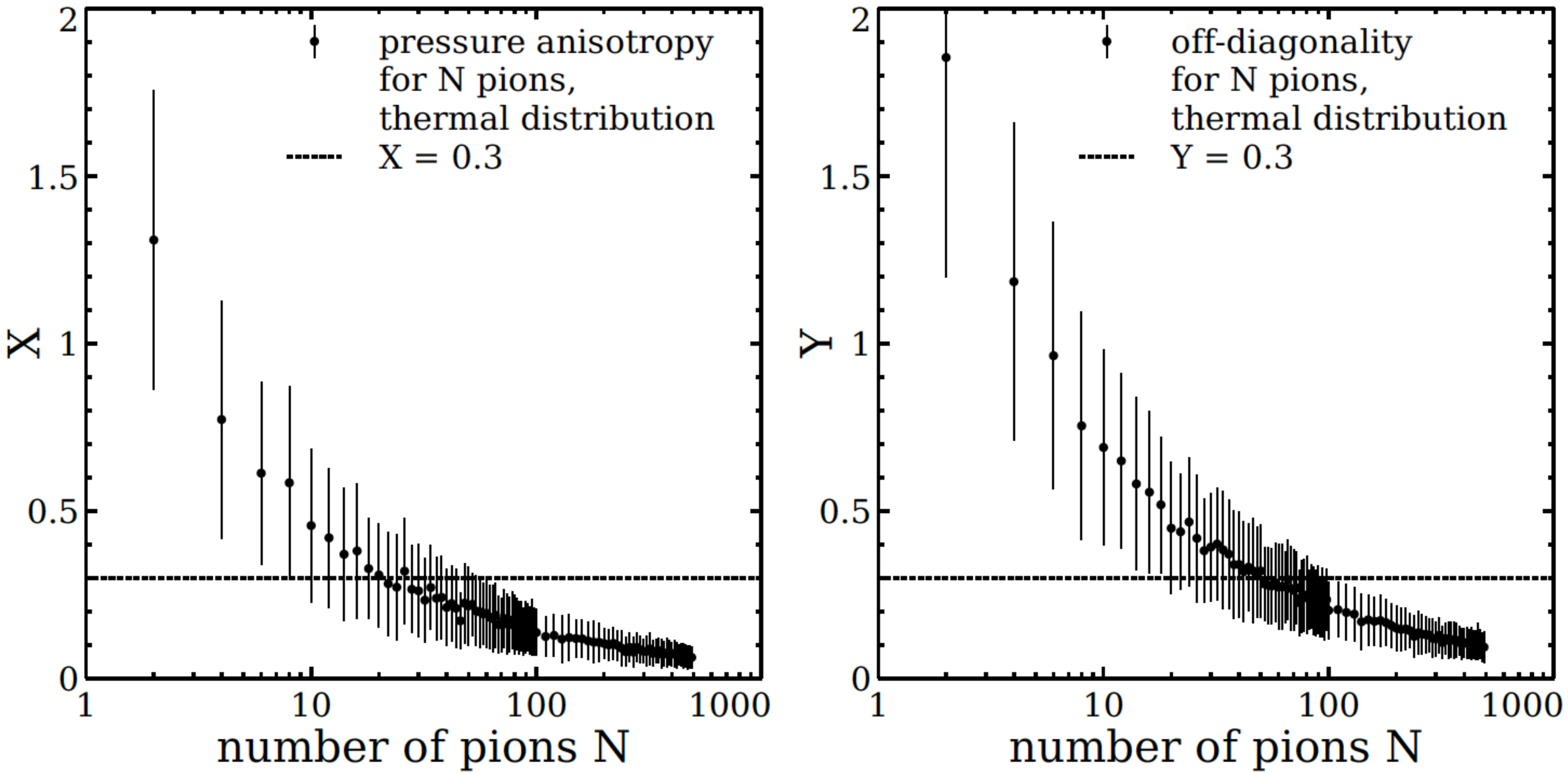
Summary

- **Hybrid approaches** based on relativistic hydrodynamics and hadron transport provide realistic dynamical description
- Two transitions have been studied systematically using **coarse-grained** UrQMD calculations
- Different algorithms to conserve **quantum numbers** globally at the particlization transition have been proposed
- **SPREW sampling** is computationally efficient and reproduces the mean values and fluctuations properly
- Broad **spectral functions** are employed in the sampling process
- Hadronic rescattering within **SMASH** yields similar results as within UrQMD
- Hadron gas viscosity is sensitive to the **lifetimes** of the resonances
- Outlook: **Electromagnetic emission** from non-equilibrium hadronic stage

Backup

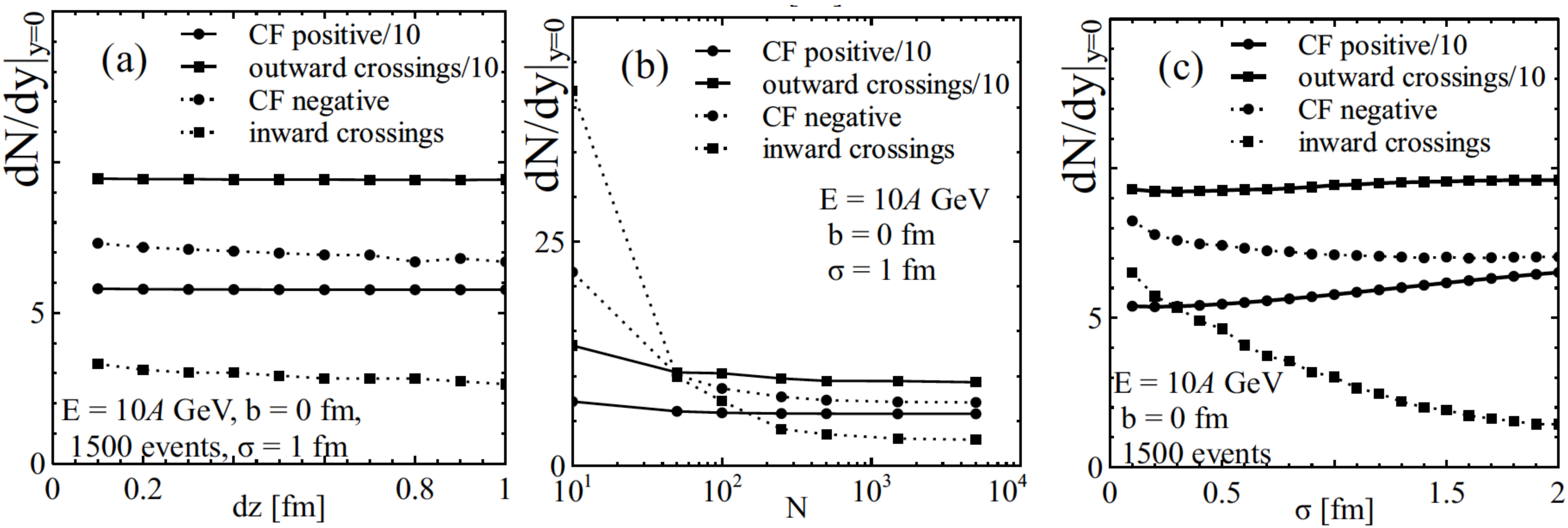
Influence of Statistics

- From N random thermal pions, the effect of finite particle statistics on the deviations of the energy-momentum tensor from equilibrium can be estimated



Parameter Sensitivities

- Comparison of coarse-grained transport with Cooper-Frye calculation vs actual particles



- No significant dependence on **cell sizes**
- **Saturation** for large enough number of events
- Dependence on σ due to **smearing** of surface velocities

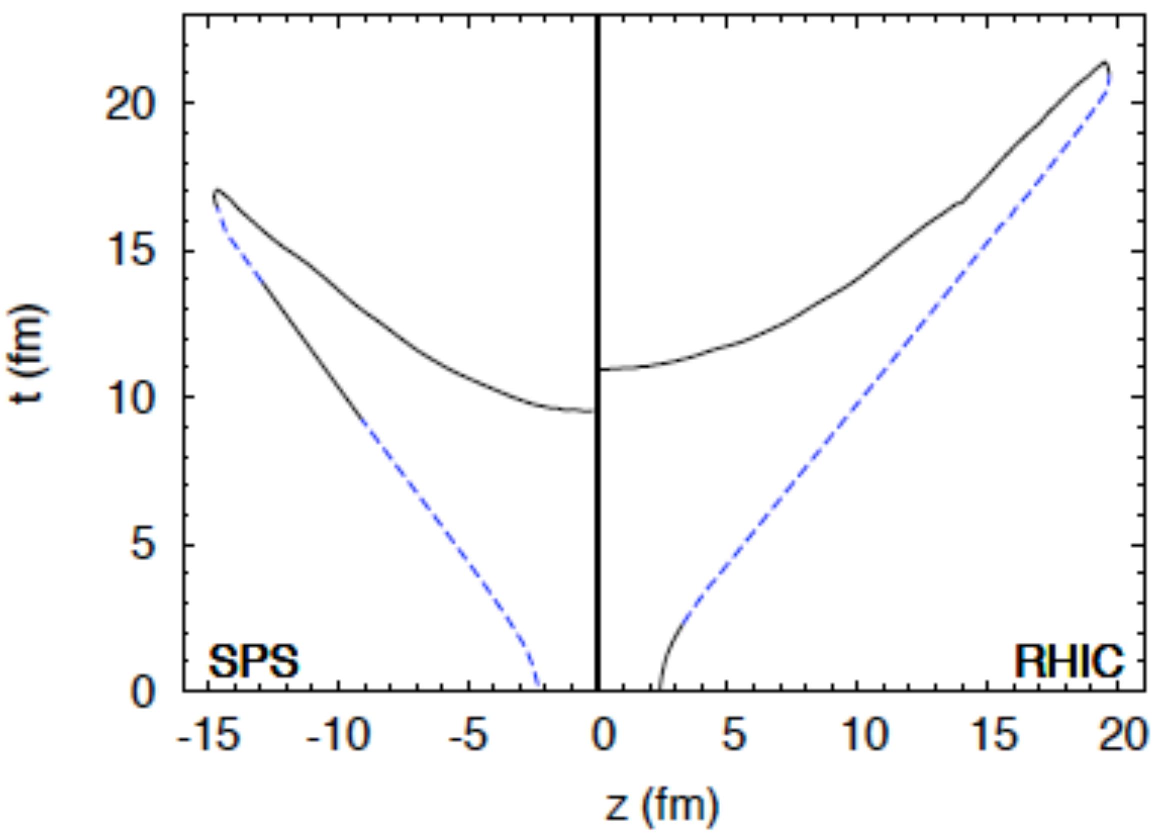
Hypersurface Results

- Energy and net baryon number conservation on hypersurface

$$E = \int_\sigma T^{0\mu} \mathrm{d}\sigma_\mu \quad \text{and} \quad B = \int_\sigma n_B u^\mu \mathrm{d}\sigma_\mu;$$

- where $d\sigma_\mu T^{\mu 0} \gtrless 0$ and $d\sigma_\mu n_B u^\mu \gtrless 0$ specify the positive and negative contributions
- Results at RHIC for central and mid-central collisions

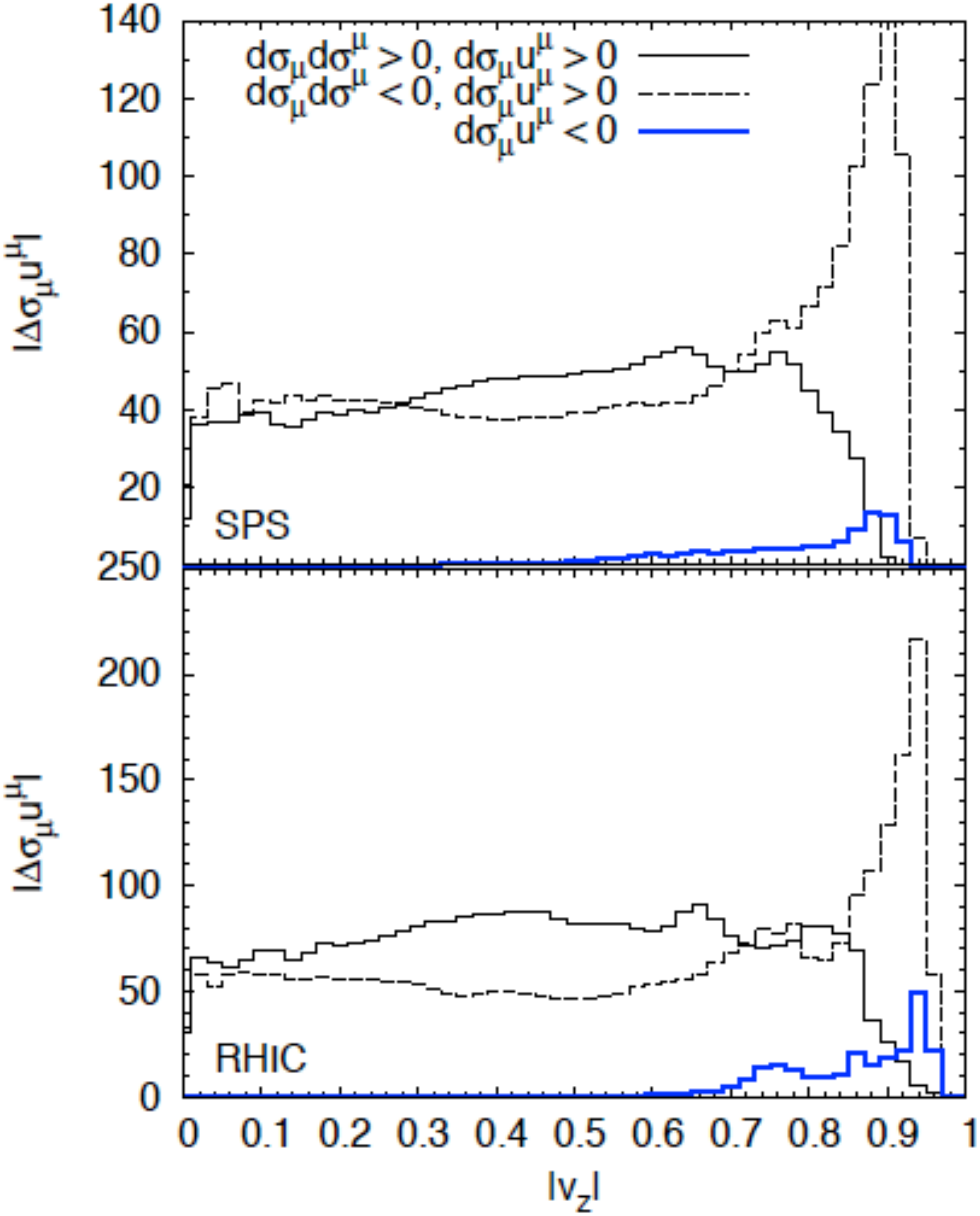
	E [GeV]			B		
	total	*pos.*	*neg.*	total	*pos.*	*neg.*
initial	5431			93.23		
final	5430	5861	-431	92.74	97.74	-5.00
initial	2327			35.84		
final	2336	2455	-119	35.80	37.10	-1.30



Dashed lines indicate possible positions of elements with negative contributions
$$d\sigma_\mu u^\mu < 0$$

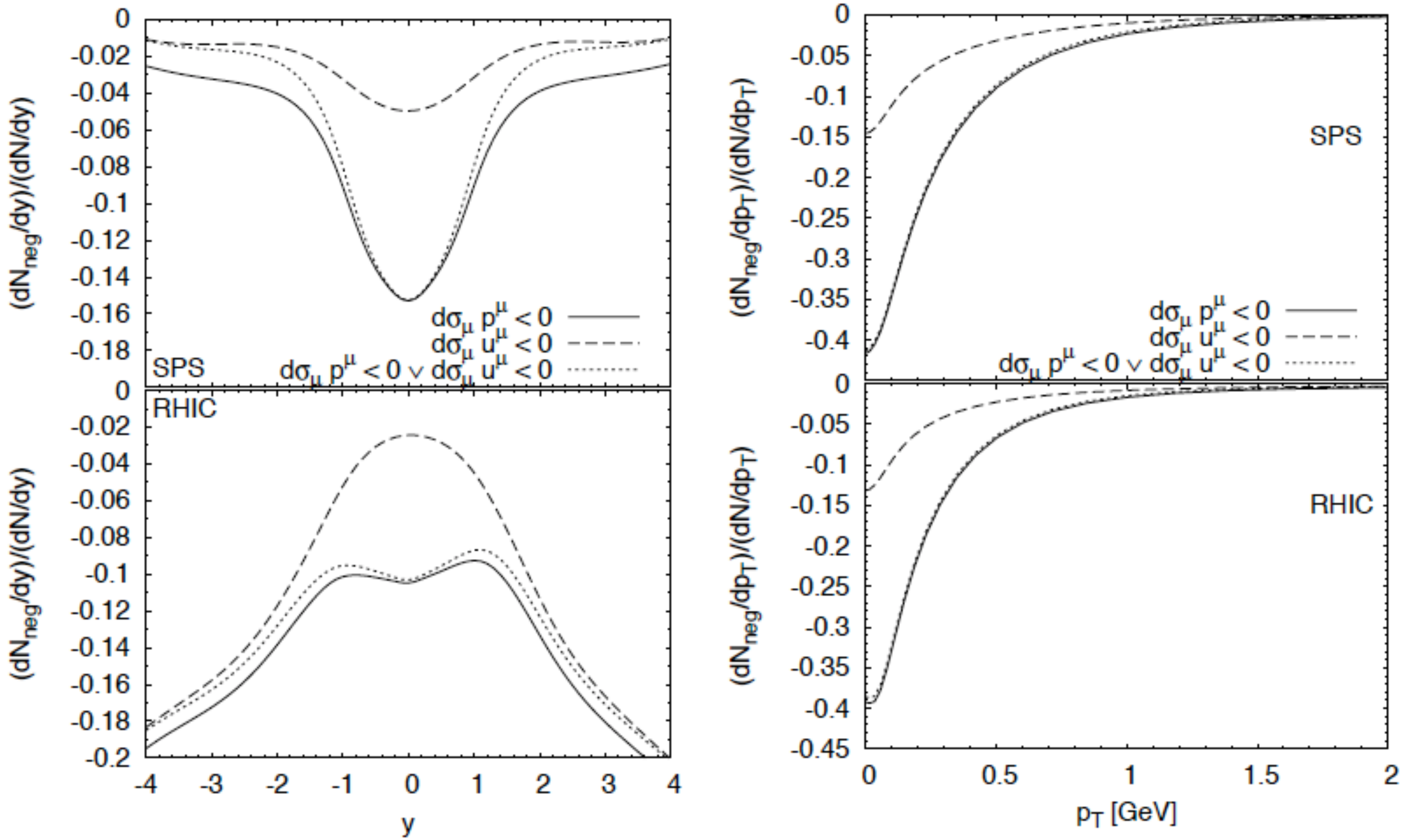
see also Oliinychenko et al arXiv:1411.3912

Negative Contributions



- The elements with particle flow inwards are located at high fluid velocities
- Coincides with peak in space-like surface elements with outward particle flow
- At midrapidity positive particle flux is dominant

Pion Spectra



- Largest effect on low p_T pions

Different Approaches

Model	Initial condition	Hydro	Switching criterion	Smearing kernel	Getting $T^{\mu\nu}_{ideal}$
UrQMD hybrid [12]	UrQMD cascade	ideal 3+1D, SHASTA	$t_{CM}[\text{fm/c}] = max(2R\sqrt{\frac{E_{lab}}{2m_N}}, 1.0)$	Gaussian z-contracted	$T^{\mu 0}$, j^0
Skokov-Toneev hybrid [13]	Quark-Gluon-String-Model	ideal 3+1D, SHASTA	t_{CM} such that $S/Q_B = \text{const}$	not mentioned	$T^{\mu 0}$, j^0
EPOS [15]	Strings (Regge-Gribov model)	ideal 3+1D	τ	Gaussian z-contracted	Landau frame
NeXSPheRIO hybrid [16, 17]	Strings (Regge-Gribov model)	ideal 3+1D, SPH	$\tau = 1$ fm [18]	Gaussian in x, y, $\tau\eta$	Landau frame
Gale et al [19]	IP-glasma	viscous 3+1D, MUSIC	$\tau = 0.2$ fm/c ($\sqrt{s_{NN}} = 2.76$ TeV)	not mentioned	Landau frame
Karpenko hybrid [20]	UrQMD cascade	viscous 3+1D	τ_{geom}	Gaussian with $\sigma_\perp$ and σ_η	$T^{\mu 0}$, j^0
Pang et al hybrid [21]	AMPT	ideal 3+1D, SHASTA	τ	Gaussian with $\sigma_\perp$ and σ_η	$T^{\mu 0}$, j^0