

Thermal Fluctuations in Relativistic Hydrodynamics

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Outline

- ▶ Hydrodynamic Fluctuations
- ▶ Correlations induced in a static fluid
- ▶ Effects on an expanding medium
- ▶ Thermal noise in higher-order theories
- ▶ Numerical simulations

Stochastic Hydrodynamics



$$T^{\mu\nu} = T_{id}^{\mu\nu} + \pi^{\mu\nu} + \Xi^{\mu\nu}$$

$\Xi^{\mu\nu}(x)$ is a stochastic field in space-time

- ▶ Addition of $\Xi^{\mu\nu}(x)$ makes each hydro variable i.e, ϵ , u^μ and $\pi^{\mu\nu}$ stochastic, instead of being deterministic
- ▶ To determine correlation function : $\langle \Xi^{\mu\nu}(x_1) \Xi^{\alpha\beta}(x_2) \rangle$
- ▶ $\langle \Xi^{\mu\nu} \rangle = 0 \implies$ averages of all quantities that depend linearly on $\Xi^{\mu\nu}$ are 0

- ▶ Consider the set of equations

$$\dot{x}_a = - \sum_a \gamma_{ab} X_b + y_a$$

X_b are “driving” forces, y_a are random fluctuations

- ▶ Rate of change of entropy

$$\dot{S} = - \sum_a \dot{x}_a X_a$$

- ▶ As the probability of fluctuating variables are e^S

$$\langle y_a(t_1) y_b(t_2) \rangle = (\gamma_{ab} + \gamma_{ba}) \delta(t_1 - t_2)$$

- ▶ For a fluid, entropy flux:

$$S^\mu = s u^\mu$$

- ▶ This implies

$$\frac{dS}{dt} = \int d^3x \quad \pi^{\mu\nu} \left[\frac{1}{2T} (\nabla_\mu u_\nu + \nabla_\nu u_\mu) \right]$$

- ▶ Identify

$$\dot{x}_1 \longrightarrow \pi_{vis}^{\mu\nu}, \quad X_1 \longrightarrow -\frac{1}{2T} (\nabla_\mu u_\nu + \nabla_\nu u_\mu) \Delta V$$

- ▶

$$\begin{aligned} \langle \Xi^{\mu\nu}(x_1) \Xi^{\alpha\beta}(x_2) \rangle &= 2\eta T \left[(\Delta^{\mu\alpha} \Delta^{\nu\beta} + \Delta^{\mu\beta} \Delta^{\nu\alpha}) \right. \\ &\quad \left. - \frac{2}{3} \Delta^{\mu\nu} \Delta^{\alpha\beta} \right] \delta^4(x_1 - x_2) \end{aligned}$$

Fluctuations in a static fluid

- ▶ For a fluid with one spatial dimension

$$\pi^{\mu\nu} = \nu s \Delta^{\mu\nu} \theta, \quad \Xi^{\mu\nu} = -wf \Delta^{\mu\nu}, \quad (\nu \equiv \frac{4\eta}{3s})$$

- ▶ Auto-correlations of fluctuations :

$$\langle f(t_1, z_1) f(t_2, z_2) \rangle = \frac{2\nu}{w} \delta(t_1 - t_2) \delta(z_1 - z_2)$$

- ▶ Express hydro variables as:

$$\epsilon = \epsilon_0 + \delta\epsilon, \quad u^\mu = u_0^\mu + \delta u^\mu \implies T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$$

- ▶ The two equations for $\delta\epsilon$ and δu^z are :

$$\frac{\partial \delta\epsilon}{\partial t} + w_0 \frac{\partial \delta u^z}{\partial z} = 0$$

$$w_0 \frac{\partial \delta u^z}{\partial t} + \frac{\partial \delta p}{\partial z} - \nu s_0 \frac{\partial^2 \delta u^z}{\partial z^2} + w_0 \frac{\partial f}{\partial z} = 0$$

Solution for the fluctuations

- ▶ Define Fourier transformed variables:

$$\delta\epsilon(t, z) = \int \frac{d\omega}{2\pi} \frac{dk}{2\pi} e^{-i\omega t} e^{-ikz} \tilde{\delta\epsilon}(\omega, k)$$

$$\delta u^z(t, z) = \int \frac{d\omega}{2\pi} \frac{dk}{2\pi} e^{-i\omega t} e^{-ikz} \tilde{\delta u^z}(\omega, k)$$

- ▶ The solution of Fourier transformed variables are

$$\tilde{\delta\epsilon} = \frac{w_0 k^2 \tilde{f}}{\omega^2 - c_s^2 k^2 + i\alpha k^2 \omega}$$

$$\tilde{\delta u^z} = \frac{\omega k \tilde{f}}{c_s^2 k^2 - \omega^2 - i\alpha k^2 \omega},$$

where $\alpha = \frac{\nu s_0}{w_0}$

Equal time correlations

- ▶ Two-point correlators of energy fluctuations,

$$\langle \delta\epsilon(t_1, z_1) \delta\epsilon(t_2, z_2) \rangle = \frac{2 T_0 \nu s_0}{(2\pi)^2} \int d\omega dk \quad e^{i\omega(t_1-t_2)} e^{-ik(z_1-z_2)} \\ \frac{k^4}{(\omega^2 - cs^2 k^2)^2 + \alpha^2 k^4 \omega^2}$$

- ▶ Equal time correlations ,

$$\int_{-\infty}^{\infty} \frac{d\omega}{(\omega^2 - cs^2 k^2)^2 + \alpha^2 k^4 \omega^2} = \frac{\pi}{\alpha c_s^2 k^4}$$

- ▶

$$\langle \delta\epsilon(t, z_1) \delta\epsilon(t, z_2) \rangle = \frac{w_0 T_0}{c_s^2} \delta(z_1 - z_2)$$

- ▶ At equal times, there are no long range correlations.

Solution: Temporal Correlations

- ▶ for unequal times :

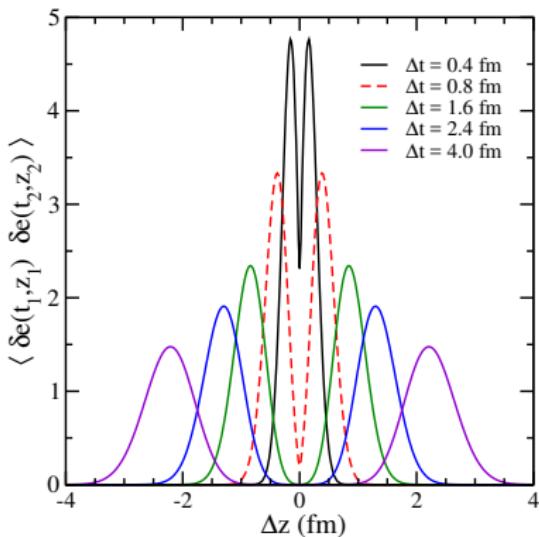
$$\langle \delta\epsilon(t_1, z_1) \delta\epsilon(t_2, z_2) \rangle = \frac{T_0 w_0}{c_s^2} \int \frac{dk}{2\pi} e^{-ik\Delta z} e^{-\frac{\alpha k^2 \Delta t}{2}} \left[\cos\left(\frac{\Delta t \zeta(k)}{2}\right) + \frac{\alpha k^2}{\zeta(k)} \sin\left(\frac{\Delta t \zeta(k)}{2}\right) \right]$$

where $\zeta(k) = \sqrt{4c_s^2 k^2 - \alpha^2 k^4}$

- ▶ At large k , the integrand $\rightarrow e^{-ik\Delta z} e^{-\frac{c_s^2 \Delta t}{\alpha}}$ \implies δ fn at $\Delta z = 0$.
- ▶ In the limit that $\nu \rightarrow 0$,

$$\langle \delta\epsilon(t_1, z_1) \delta\epsilon(t_2, z_2) \rangle \rightarrow \frac{T_0 w_0}{2c_s^2} \left[\delta(\Delta z - c_s \Delta t) + \delta(\Delta z + c_s \Delta t) \right]$$

Correlations in static fluid



- viscosity 'broadens' and dampens the correlations over space and time.

Correlations in Bjorken expansion

[Kapusta, Muller, Stephanov, PRC 85,054906(2012)]

- ▶ After adding fluctuations,

$$\epsilon = \epsilon_0(\tau) + \delta\epsilon(\tau, \eta), \quad u^\mu = (1, 0) + (0, \delta u^\eta)$$

- ▶

$$\frac{d\epsilon_0}{d\tau} = -\frac{1}{\tau} (\epsilon_0 + p_0 - \pi_0), \quad \pi_0 = \frac{4}{3} \frac{\eta_\nu}{\tau}$$

- ▶ Linearised hydrodynamic evolution,

$$\frac{\partial}{\partial \tau}(\tau \delta\epsilon) + \frac{\partial}{\partial \eta}(\tau \mathcal{U}_0 \delta u^\eta) = -\delta\mathcal{V},$$

$$\frac{\partial}{\partial \tau}(\tau \mathcal{U}_0 \delta u^\eta) + \frac{\partial}{\partial \eta} \left(\frac{\delta\mathcal{V}}{\tau} \right) = -2\mathcal{U}_0 \delta u^\eta,$$

where $\mathcal{U}_0(\tau) \equiv w_0 - \pi_0$, $\delta\mathcal{V}(\eta, \tau) \equiv \delta p + \tau^2 \delta\pi'^{\eta\eta}$

Singular part of the correlators

- ▶ Work in Fourier coordinates,

$$\tilde{X}(\tau, k) = - \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'} \tilde{G}_X(\tau', \tau, k) f(\tau', k)$$

- ▶

$$\langle X(\eta, \tau) Y(\eta', \tau') \rangle = \frac{2}{A_{\perp}} \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'^3} \frac{4\eta_v}{3sw_0(\tau')} G_{XY}(\eta - \eta'; \tau, \tau').$$

- ▶

$$G_{XY}(\eta - \eta'; \tau, \tau') = \int_{-\infty}^{\infty} \frac{d k}{2\pi} e^{ik(\eta - \eta')} \tilde{G}_{XY}(k; \tau, \tau'),$$

$$\tilde{G}_{XY}(k; \tau, \tau') \equiv \tilde{G}_X(k; \tau, \tau') \tilde{G}_Y(-k; \tau, \tau')$$

Singular part

- ▶ Denoting $\rho \equiv 3\delta e/(4e_0)$ and $\omega \equiv \tau\delta u^\eta$



$$\begin{aligned}\tilde{G}_{\rho\rho}^{\text{sing}}(k; \tau, \tau') &= (a_1 k^2 + b_1) + (a_2 k^2 + b_2) \cos(2c_s \gamma k) \\ &\quad + \frac{a_3 k^2 + b_3}{k} \sin(2c_s \gamma k).\end{aligned}$$

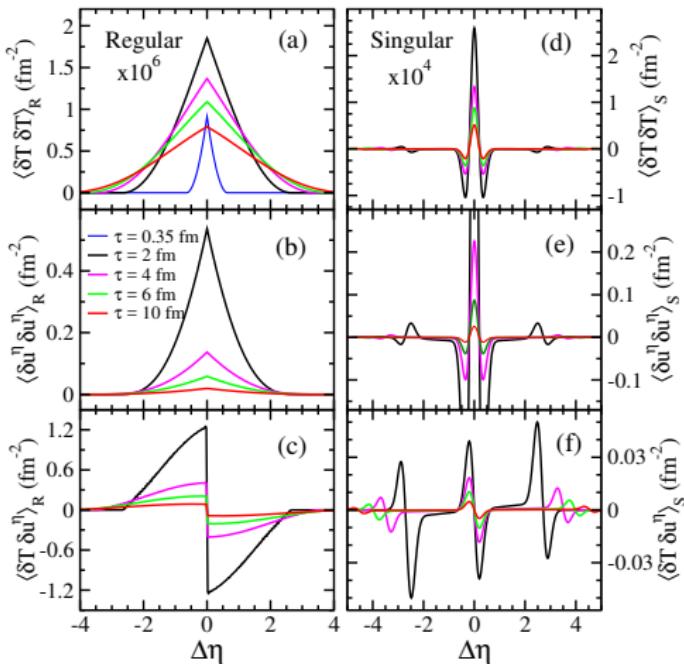


$$\tilde{G}_{\rho\omega}^{\text{sing}}(k; \tau, \tau') = d_1 k + d_2 k \cos(2c_s \gamma k) + (d_3 k^2 + d_4) \sin(2c_s \gamma k),$$



$$\begin{aligned}\tilde{G}_{\omega\omega}^{\text{sing}}(k; \tau, \tau') &= (w_1 k^2 + w_2) + (w_3 k^2 + w_4) \cos(2c_s \gamma k) \\ &\quad + \left(\frac{w_5 k^2 + w_6}{k} \right) \sin(2c_s \gamma k),\end{aligned}$$

Singular and regular part of correlators



Using $\tau_0 = .15$ fm, $T_0 = 600$ MeV, $\tau_f = 10$ fm

- ▶ Hydrodynamics continued till $T(\tau)$ falls below a certain value.
- ▶ The number of particles formed from each fluid cell are:

$$\frac{dN_s}{d^2 p_\perp dy} = \int_{\Sigma_f} d^3 \Sigma_\mu p^\mu \theta(\hat{\sigma} \cdot p) d_s f_s(x, p),$$

where $f_s(x, p) = \text{Exp}[-\frac{(u \cdot p)}{T}]$ and $y = \frac{1}{2} \log(\frac{E+p^z}{E-p^z})$.

- ▶ In Bjorken,

$$\frac{dN_s}{dy} = \frac{d_s A \tau_f}{(2\pi)^3} \int d\eta \cosh(y-\eta) \times \int d^2 p_\perp m_\perp e^{-\cosh(y-\eta-\omega)m_\perp/T},$$

Rapidity correlation

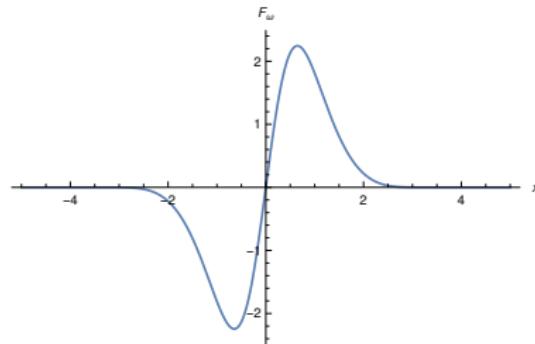
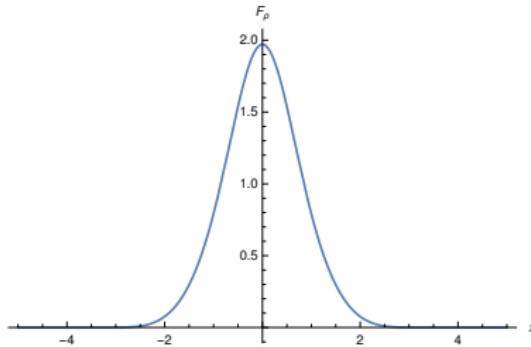
- ▶ The correlator may be separated as:

$$\frac{dN_s}{dy} = \left\langle \frac{dN_s}{dy} \right\rangle_0 + \delta \left(\frac{dN_s}{dy} \right)$$

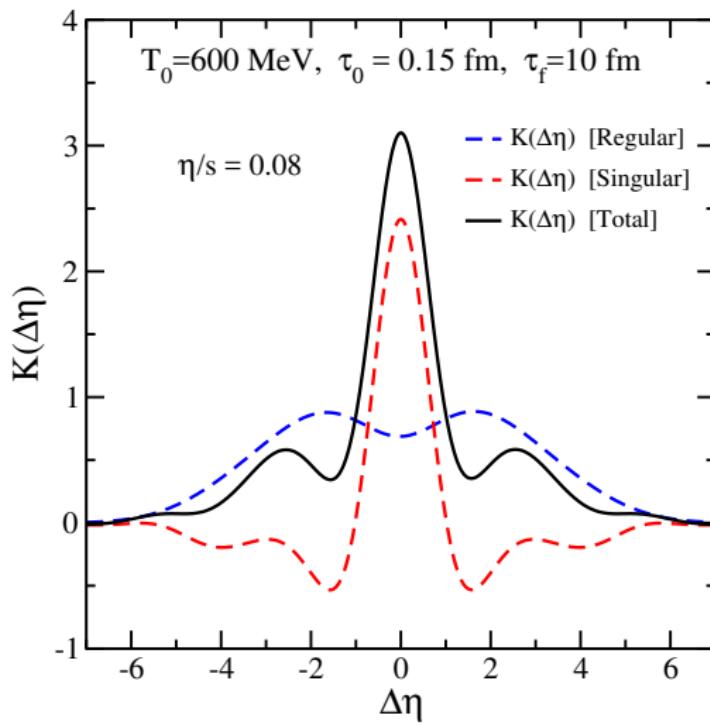
- ▶ The rapidity correlator:

$$\left\langle \delta \frac{dN_s}{dy_1} \delta \frac{dN_s}{dy_2} \right\rangle = \sum_{X,Y=\rho,\omega} \int \int d\eta dz F_X(y) F_Y(\Delta y + \eta + z) C_{XY}(z)$$

where C_{XY} are the correlators $\langle X(\tau_f, \eta_1) Y(\tau_f, \eta_2) \rangle$



Correlator in Bjorken



Chapman-Enskog hydrodynamics

- ▶ Boltzmann Eq. in relaxation-time approximation,

$$p^\mu \partial_\mu f = -(u \cdot p) \frac{\delta f}{\tau_R},$$

τ_R is the relaxation time.

- ▶ Solve perturbatively assuming small relaxation time τ_R .
- ▶ To first- and second-order in derivatives,

$$\delta f^{(1)} = -\frac{\tau_R}{u \cdot p} p^\mu \partial_\mu f_{eq},$$

$$\delta f^{(2)} = \frac{\tau_R}{u \cdot p} p^\mu p^\nu \partial_\mu \left(\frac{\tau_R}{u \cdot p} \partial_\nu f_{eq} \right).$$

Equations of shear stress tensor

- Relaxation type equation:

$$\pi^{\mu\nu} = \Delta_{\alpha\beta}^{\mu\nu} \int dp p^\alpha p^\beta \delta f$$
$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_R} = - \int dp p^\mu p^\nu p^\gamma \nabla_\gamma f$$

- Second-order equation for $\pi^{\mu\nu}$,

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu} \omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} - \frac{4}{3}\pi^{\mu\nu}\theta,$$

where $\omega^{\mu\nu} \equiv (\nabla^\mu u^\nu - \nabla^\nu u^\mu)/2$ and $\beta_\pi = 4P/5$.

Second-order entropy four-current

- ▶ Boltzmann's H-theorem,

$$S^\mu = \int dp p^\mu f (\log f - 1)$$

- ▶ Use perturbative solution of $f(x, p)$
- ▶ Second-order entropy,

$$S^\mu = s u^\mu - \frac{\beta_2}{2T} u^\mu \pi^{\alpha\beta} \pi_{\alpha\beta}$$

where $\beta_2 = 1/2\beta_\pi$.

- ▶ Using $\partial_\mu S^\mu$,

$$\frac{dS}{dt} = \int d^3x \frac{\pi^{\mu\nu}}{T} [\nabla_\mu u_\nu - \beta_2 \dot{\pi}_{\mu\nu} - \beta_2 \lambda_\pi \theta \pi_{\mu\nu}].$$

Fluctuation-dissipation relation

- ▶ In analogy to

$$\dot{S} = - \sum_a \dot{x}_a X_a, \quad \dot{x}_a = -\gamma_{ab} X_b + y_a$$

- ▶ Identify

$$\dot{x}_a \rightarrow \pi^{\mu\nu}$$

$$X_a \rightarrow -\frac{1}{T} [\nabla_\mu u_\nu - \beta_2 \dot{\pi}_{\mu\nu} - \beta_2 \lambda_\pi \theta \pi_{\mu\nu}] \quad \Delta V \equiv X_{\mu\nu}$$

- ▶

$$\pi^{\mu\nu} = -\gamma^{\mu\nu\alpha\beta} X_{\alpha\beta} + \xi^{\mu\nu}$$

$$\begin{aligned} \pi^{\mu\nu} &= 2\eta_\nu \left[-\beta_2 \dot{\pi}^{\langle\mu\nu\rangle} + \sigma^{\mu\nu} + 2\beta_2 \pi_\gamma^{\langle\mu} \omega^{\nu\rangle\gamma} - \beta_2 \frac{10}{7} \pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} \right. \\ &\quad \left. - \frac{4}{3} \beta_2 \pi^{\mu\nu} \theta \right] \end{aligned}$$

Constraints on $\gamma^{\mu\nu\alpha\beta}$

- ▶ Symmetries of $\pi^{\mu\nu}$: $\gamma^{\mu\nu\alpha\beta} = \gamma^{\nu\mu\alpha\beta}$, $\gamma_\mu^{\mu\alpha\beta} = 0$, and $\gamma^{\mu\nu\alpha\beta} u_\mu = 0$.
- ▶ Identification of $X_{\mu\nu}$ is not unique: $X_{\mu\nu} \rightarrow X_{\mu\nu} + H_{\mu\nu}$, keeps dS/dt invariant, if $H_{\mu\nu}$ is orthogonal to $\pi^{\mu\nu}$.
- ▶ To obtain an autocorrelation which is insensitive to such transformations, $\gamma^{\mu\nu\alpha\beta} = \gamma^{\mu\nu\beta\alpha}$, $\gamma_\alpha^{\mu\nu\alpha} = 0$, and $\gamma^{\mu\nu\alpha\beta} u_\alpha = 0$.

- ▶ $\gamma^{\mu\nu\alpha\beta}$ consistent with the constraints:

$$\begin{aligned}\gamma^{\mu\nu\alpha\beta} = & 2\eta_v T \left(\Delta^{\mu\nu\alpha\beta} - \frac{10}{7} \beta_2 \Delta_{\zeta\kappa}^{\mu\nu} \pi_\gamma^\zeta \Delta^{\kappa\gamma\alpha\beta} \right. \\ & \left. + 2\tau_\pi \Delta_{\zeta\kappa}^{\mu\nu} \omega_\gamma^\zeta \Delta^{\kappa\gamma\alpha\beta} \right)\end{aligned}$$

- ▶ Correspondingly one obtains the noise autocorrelation:

$$\begin{aligned}\langle \xi^{\mu\nu}(x) \xi^{\alpha\beta}(x') \rangle = & 4\eta_v T \left(\Delta^{\mu\nu\alpha\beta} - \frac{5}{7} \beta_2 \Delta_{\zeta\kappa}^{\mu\nu} \pi_\gamma^\zeta \Delta^{\kappa\gamma\alpha\beta} \right. \\ & - \frac{5}{7} \beta_2 \Delta_{\zeta\kappa}^{\alpha\beta} \pi_\gamma^\zeta \Delta^{\kappa\gamma\mu\nu} \\ & \left. + \omega - \text{terms} \right) \delta^4(x - x')\end{aligned}$$

Noise in MIS theory: A simpler case

- ▶ Entropy four-current is same as before,

$$\frac{dS}{dt} = \int d^3x \frac{\pi^{\mu\nu}}{T} [\nabla_\mu u_\nu - \beta_2 \dot{\pi}_{\mu\nu} - \beta_2 \lambda_\pi \theta \pi_{\mu\nu}].$$

- ▶ Evolution equation for shear tensor ($\partial_\mu S^\mu \geq 0$)

$$\pi^{\mu\nu} = 2\eta_\nu [\sigma^{\mu\nu} - \beta_2 \dot{\pi}^{\mu\nu} - \beta_2 \lambda_\pi \theta \pi^{\mu\nu}]$$

- ▶

$$\gamma^{\mu\nu\alpha\beta} = 2\eta_\nu T \Delta^{\mu\nu\alpha\beta}$$
$$\langle \xi^{\mu\nu}(x) \xi^{\alpha\beta}(x') \rangle = 4\eta_\nu T \Delta^{\mu\nu\alpha\beta} \delta^4(x - x').$$

White noise vs Colored noise

- ▶ In the Navier-Stokes theory,

$$T^{\mu\nu} = T_{id}^{\mu\nu} + \pi^{\mu\nu} + \Xi^{\mu\nu}, \quad \pi^{\mu\nu} = 2\eta_v \sigma^{\mu\nu}$$

- ▶ Local auto-correlation function for $\Xi^{\mu\nu}(x_1)$
- ▶ In higher order theories,

$$T^{\mu\nu} = T_{id}^{\mu\nu} + \pi^{\mu\nu} + \Xi^{\mu\nu}, \quad \pi^{\mu\nu} = \tau_\pi \dot{\pi}^{\mu\nu} + 2\eta_v \sigma^{\mu\nu} + \dots$$

- ▶ In the CE equation

$$\dot{\Xi}^{\langle\mu\nu\rangle} = -\frac{1}{\tau_\pi} (\Xi^{\mu\nu} - \xi^{\mu\nu}) - \frac{10}{7} \Xi^{\langle\mu}_{\gamma} \sigma^{\nu\rangle\gamma} - \lambda_\pi \Xi^{\mu\nu} \theta.$$

For MIS,

$$\dot{\Xi}^{\langle\mu\nu\rangle} = -\frac{1}{\tau_\pi} (\Xi^{\mu\nu} - \xi^{\mu\nu}) - \lambda_\pi \Xi^{\mu\nu} \theta$$

Numerical Application: Back to Bjorken!

- ▶ Linearization of the problem independent fluctuating component, $\delta\pi$, reads

$$\begin{aligned}\frac{\partial\delta\pi}{\partial\tau} + \frac{\delta\pi}{\tau_\pi} = & \frac{1}{\tau_\pi} \left[\tau^2 \xi^{\eta\eta} + \frac{4\eta_v}{3s} (s_0\delta\theta + \delta s\theta_0) \right] \\ & - \lambda_\pi (\theta_0\delta\pi + \delta\theta\pi_0) \\ & - \frac{\delta\tau_\pi}{\tau_\pi} \left(\lambda_\pi\theta_0\pi_0 + \frac{d\pi_0}{d\tau} \right)\end{aligned}$$

- ▶ Autocorrelation function

$$\begin{aligned}\langle \xi^{\eta\eta}(\eta, \tau) \xi^{\eta\eta}(\eta', \tau') \rangle = & \frac{8\eta_v(\tau) T_0(\tau)}{3A_\perp \tau^5} [1 - \mathcal{A}\beta_2\pi_0] \\ & \times \delta(\tau - \tau') \delta(\eta - \eta').\end{aligned}$$

$\mathcal{A} = 0$ in MIS and $5/7$ in CE theory.

Cooper-Frye freezeout



$$E \frac{dN}{d^3 p} = \frac{g}{(2\pi)^3} \int_{\Sigma} d\Sigma_{\mu} p^{\mu} f(x, p),$$

- ▶ Include viscous corrections in $f(x, p) = f_{\text{eq}}(x, p) + f_{\text{vis}}(x, p)$,

$$f_{\text{vis}} = f_{\text{eq}} \frac{p^{\mu} p^{\nu} \pi_{\mu\nu}}{2(\epsilon + p) T^2}.$$

- ▶ Linearized fluctuations:

$$\begin{aligned} \delta f = & \delta f_{\text{eq}} + K_{0\mu\nu} \left[\delta f_{\text{eq}} \pi_0^{\mu\nu} + (f_{\text{eq}})_0 \delta \pi^{\mu\nu} \right. \\ & \left. - (f_{\text{eq}})_0 \pi_0^{\mu\nu} \left(2 \frac{\delta T}{T_0} + \frac{\delta \epsilon + \delta p}{\epsilon_0 + p_0} \right) \right] \end{aligned}$$

Rapidity Correlators

- ▶ The fluctuating parts can be expressed as

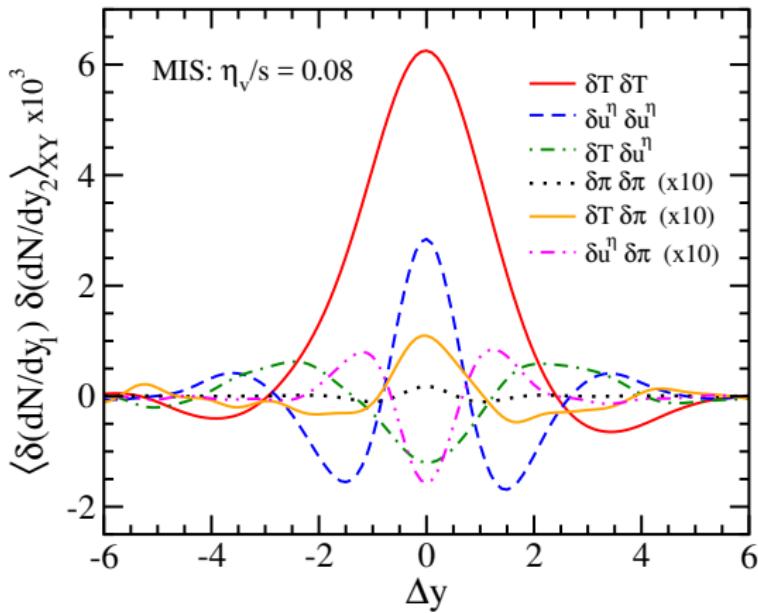
$$\delta \frac{dN}{dy} = \frac{g\tau_f T_0^3 A_\perp}{(2\pi)^2} \int d\eta \left[\mathcal{F}_T(y - \eta) \frac{\delta T(\eta)}{T_0} + \mathcal{F}_u(y - \eta) \tau_f \delta u^\eta(\eta) + \mathcal{F}_\pi(y - \eta) \frac{\delta \pi(\eta)}{w_0} \right]$$

- ▶ Two particle-rapidity correlators:

$$\begin{aligned} \left\langle \delta \frac{dN}{dy_1} \delta \frac{dN}{dy_2} \right\rangle &= \left[\frac{g\tau_f T_0^3 A_\perp}{(2\pi)^2} \right]^2 \int d\eta_1 \int d\eta_2 \\ &\times \sum_{X,Y} \mathcal{F}_X(y_1 - \eta_1) \mathcal{F}_Y(y_2 - \eta_2) \\ &\times \langle X(\eta_1) Y(\eta_2) \rangle \end{aligned}$$

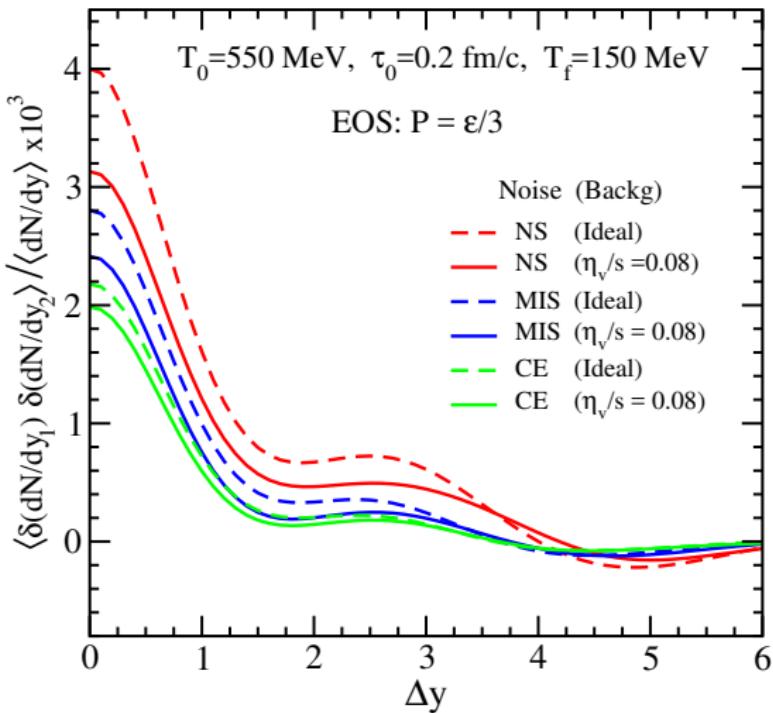
Here $(X, Y) \equiv (\delta T, \delta u^\eta, \delta \pi)$

Results: Correlators at Freeze-out

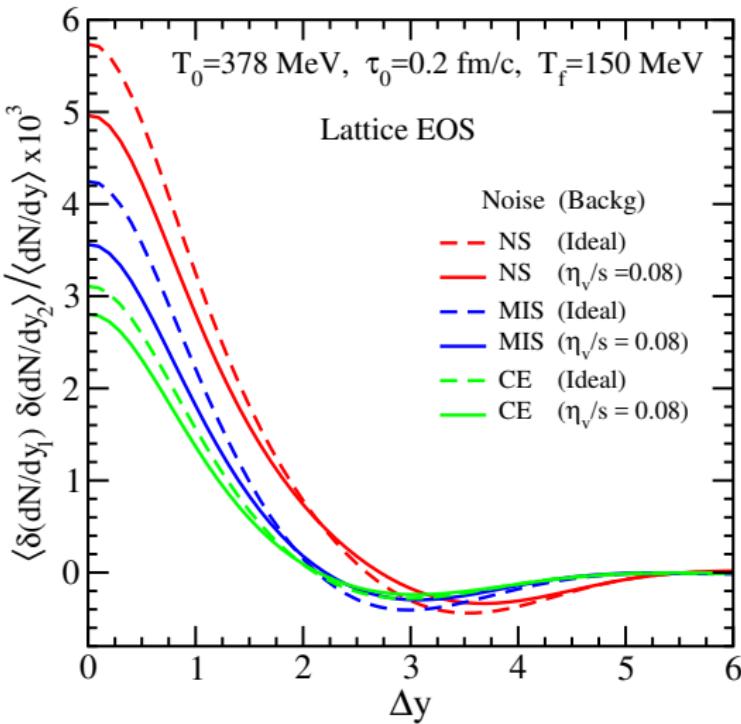


- ▶ $T_0 = 550 MeV$, $\tau_0 = 0.2 fm/c$, $T_f = 150 MeV$.

Results: Two particle rapidity correlations



Results: Using Lattice EOS



Summary and Future Work

- ▶ Obtained analytical solutions for thermal correlations in static fluid.
- ▶ Analyzed the effects of the thermal correlations in expanding medium on particle spectra.
- ▶ Derived fluctuation-dissipation relation in higher-order theories.
- ▶ Ongoing work:
 - ▶ Developed (1+1)D linearised hydro code and studied thermal fluctuations in this setup.
 - ▶ Study the effects of bulk viscosity on fluctuations.