Transient effects in hydronization

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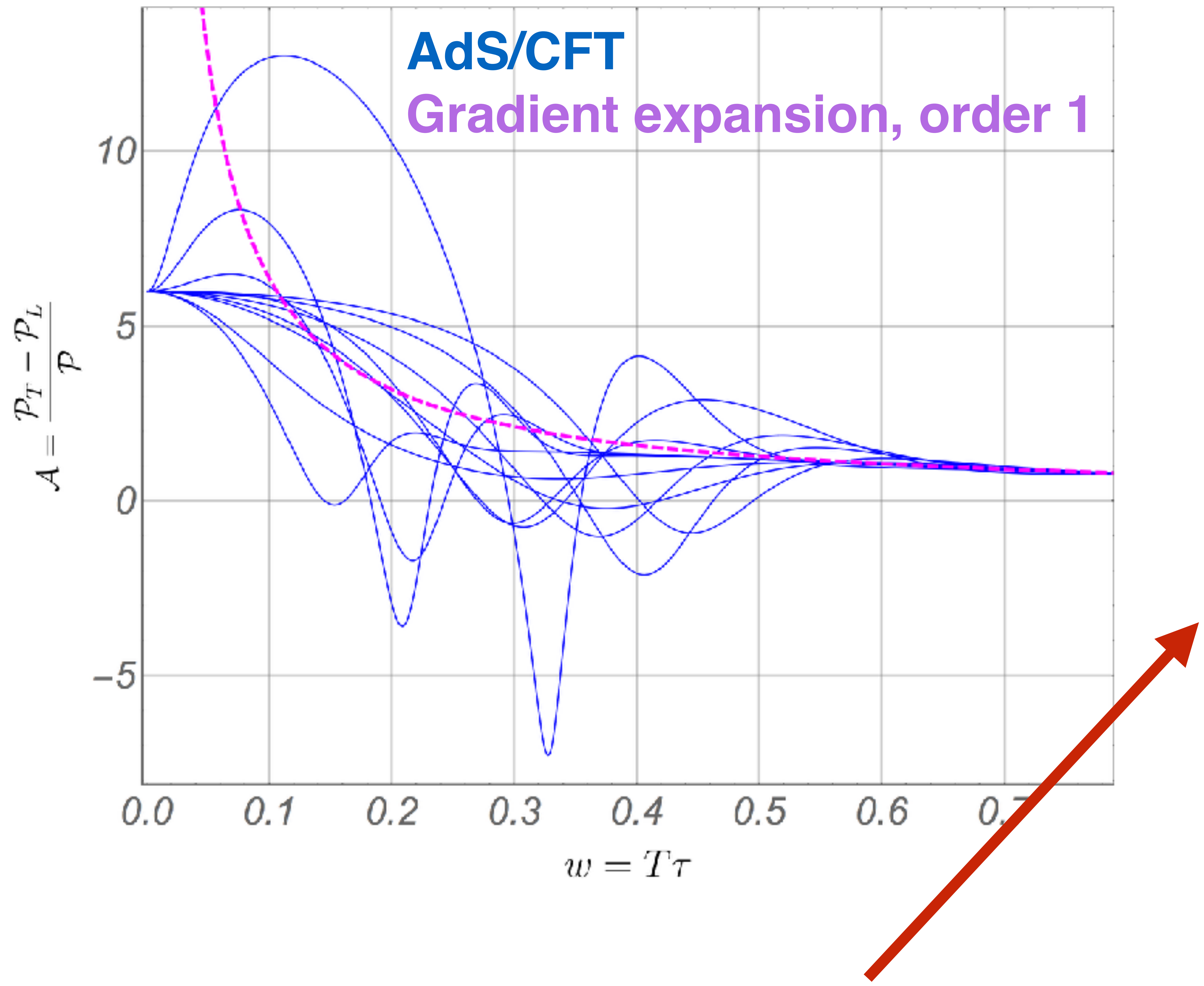
Introduction

Hydronization: the process of reaching a stage of evolution where “hydrodynamics works”.

- Proceeds by the decay of transients, leaving long-lived excitations.
- Very convenient to use special variables, which behave universally at late times, leading to attractor behaviour.

For SYM, transients exhibit damped-oscillatory behaviour, which can be seen in numerical solutions at intermediate times.



Out there, where most non-hydro modes have decayed

Role of the gradient expansion

$$\boxed{\begin{gathered} T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}(\mathcal{E})(g^{\mu\nu} + u^\mu u^\nu) + \Pi^{\mu\nu} \\ \nabla_\alpha T^{\alpha\beta} = 0 \end{gathered}}$$

Navier-Stokes theory

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$$

needs a **"UV-completion"** to cure its acausality (MIS, BRSSS, aHYDRO, …)

- This introduces non-hydrodynamic modes which act as a **regulator**
- No unique way to do this
- Domain of applicability of hydrodynamics: **regulator independence**

Standard approach: MIS, BRSSS, …

can be matched to the least damped QNM of N=4 SYM

$$(\tau_\pi \mathcal{D} + 1)\, \Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \dots$$

HJSW: an alternative designed to have oscillating transient modes

$$\left((\frac{1}{T}\mathcal{D})^2 + 2\omega_I \frac{1}{T}\mathcal{D} + |\omega|^2 \right) \Pi^{\mu\nu} = -\eta|\omega|^2\sigma^{\mu\nu} - c_\sigma \frac{1}{T}\mathcal{D}\,(\eta\sigma^{\mu\nu}) + \dots$$

The choice determines the non-hydrodynamic sector (transient behaviour)

The gradient expansion: matching hydrodynamics to a microscopic theory

$$\Pi^{\mu\nu} = \boxed{-\eta\sigma^{\mu\nu}} + \tau_\pi \mathcal{D}(\eta\sigma^{\mu\nu}) + \dots$$

Bonus: divergent gradient expansions carry information about the non-hydro sector.

Bjorken flow

- As a function of the "clock variable" $w \equiv \tau T \sim \tau/\tau_\pi$ the pressure anisotropy

$$\mathcal{A} \equiv \frac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{P}}$$

is very special: it shows **universal behaviour at late times**:

$$\mathcal{A}(w) = 8\,(\eta/s)\,\frac{1}{w} + \ldots$$

independently of initial conditions.

- Because of the simple scaling form this takes, one can further rescale

$$\tilde{w} \sim \frac{w}{\eta/s}$$

so the leading behaviour is even independent of the model.

- For this observable a special **attractor** solution exists.

The attractor in BRSSS hydrodynamics

Evolution equation

$$C_{\tau_\pi}\left(1+\frac{\mathcal{A}}{12}\right)\mathcal{A}'+\left(\frac{C_{\tau_\pi}}{3w}+\frac{C_{\lambda_1}}{8C_\eta}\right)\mathcal{A}^2=\frac{3}{2}\left(\frac{8C_\eta}{w}-\mathcal{A}\right)$$

in terms of dimensionless transport coefficients

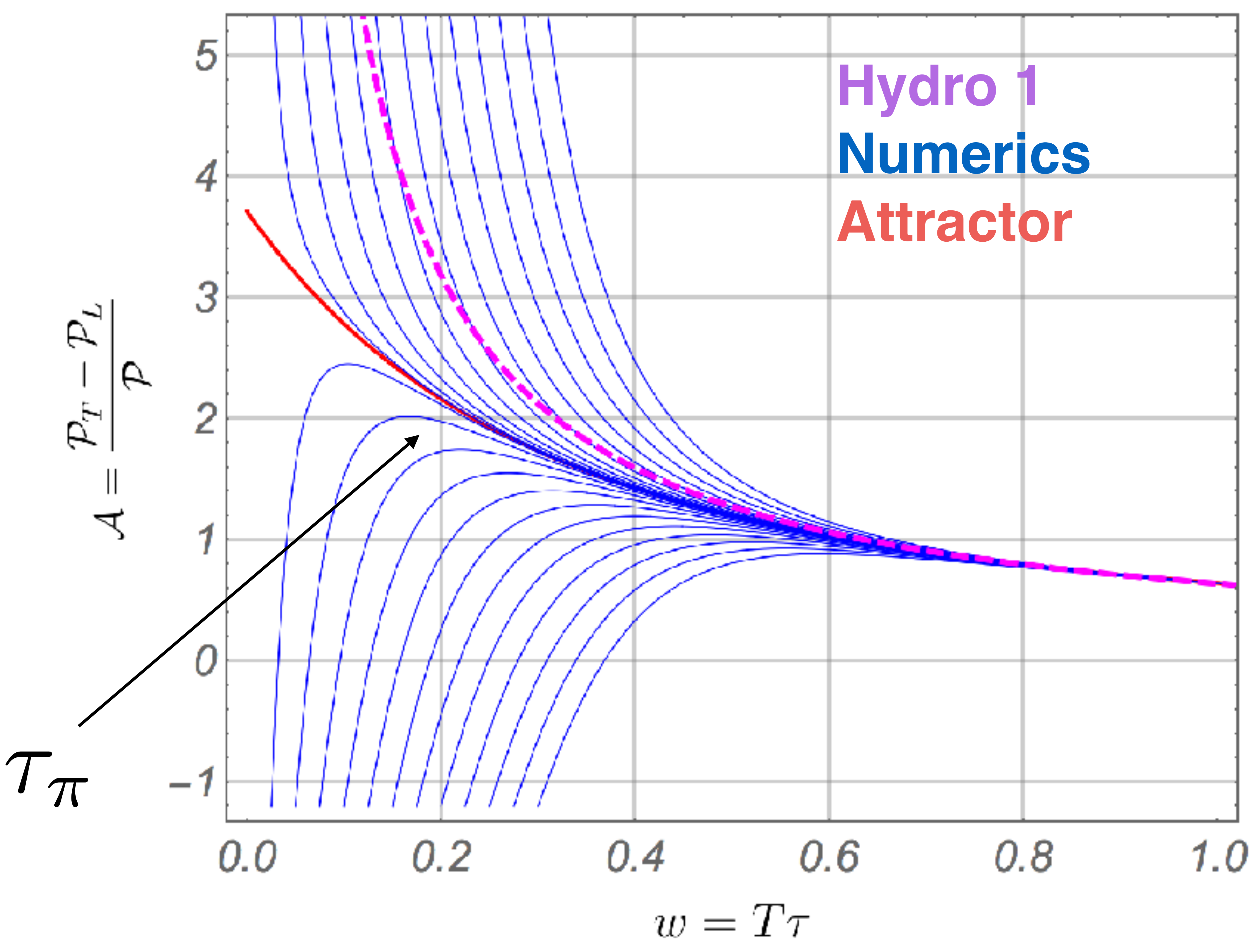
$$C_{\tau_\pi}=T\tau_\pi,\quad C_\eta=\eta/s,\quad C_{\lambda_1}=T\lambda_1/\eta$$

Gradient expansion solution

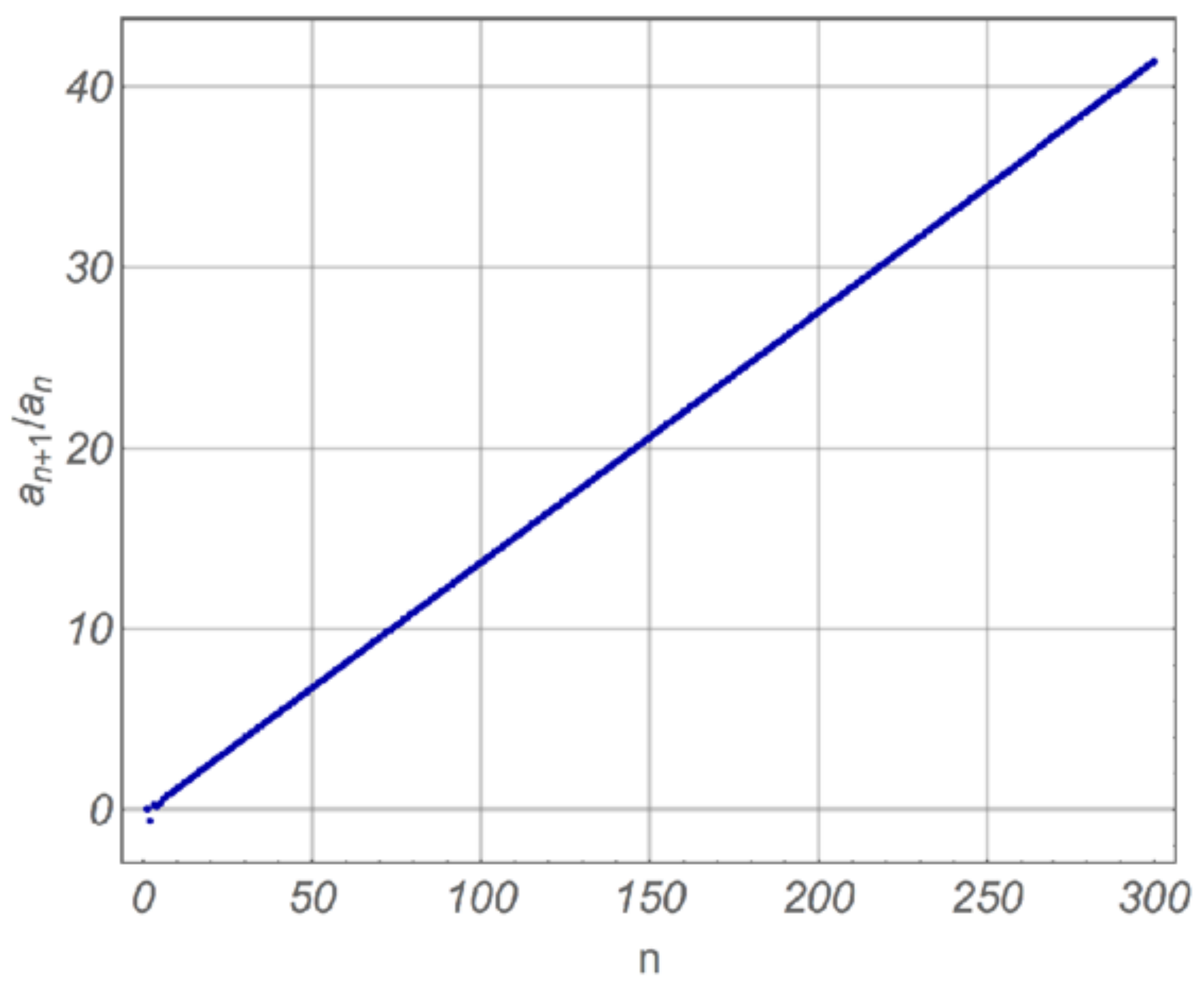
$$\mathcal{A}=\boxed{\frac{8C_\eta}{w}}+\frac{16C_\eta(C_{\tau_\pi}-C_{\lambda_1})}{3w^2}+\ldots$$

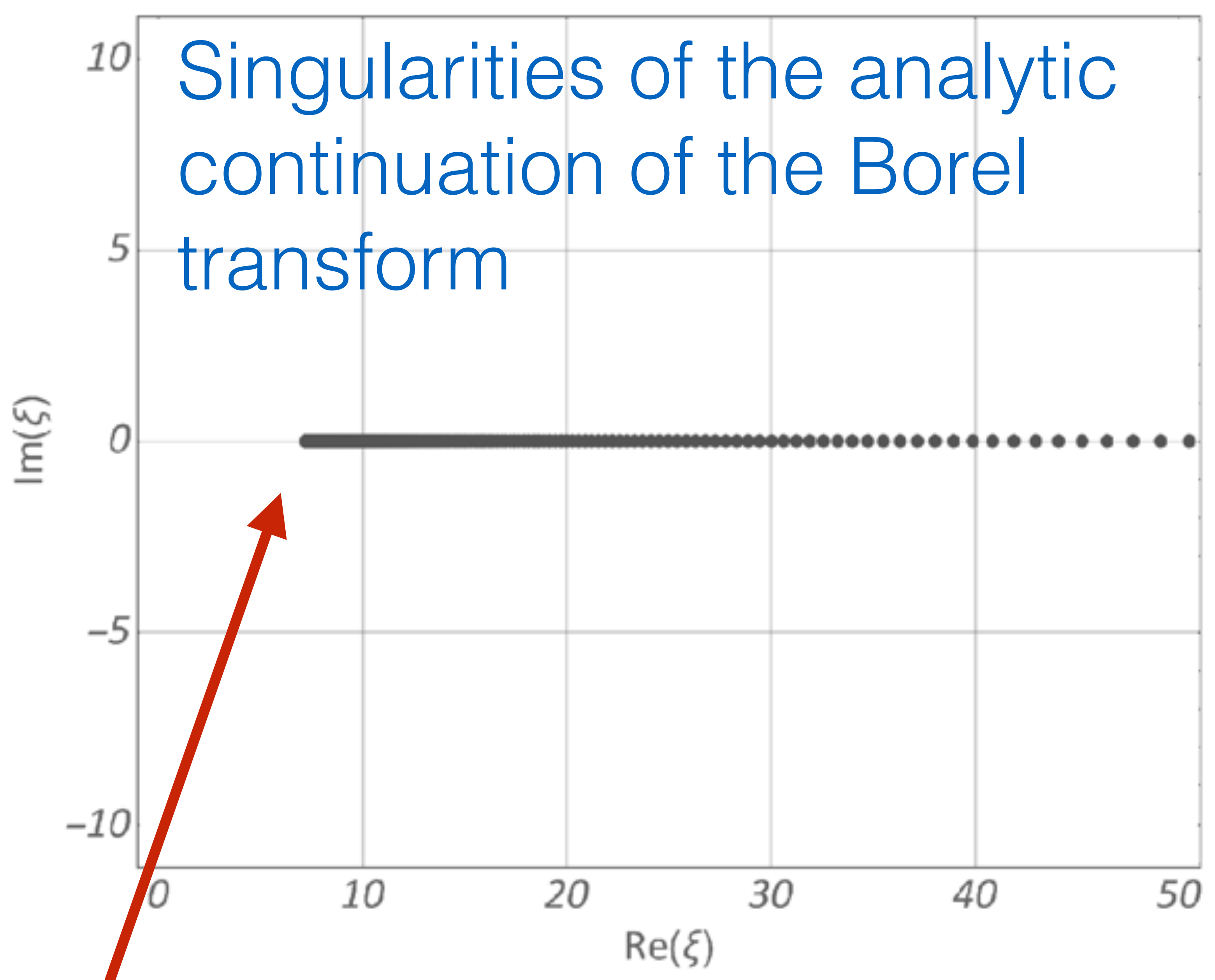
Note: the attractor depends on the values of the transport coefficients.

Question: how closely can one model the attractor of a microscopic theory?



Asymptotic behaviour in BRSSS





Borel transform method:

$$\mathcal{A}_B(\xi) = \sum_{n=1}^{\infty} \frac{a_n}{n!} \xi^n$$

$$\mathcal{A}_{\text{resummed}}(w) = w \int_C d\xi \, e^{-w\xi} \, \tilde{\mathcal{A}}_B(\xi)$$

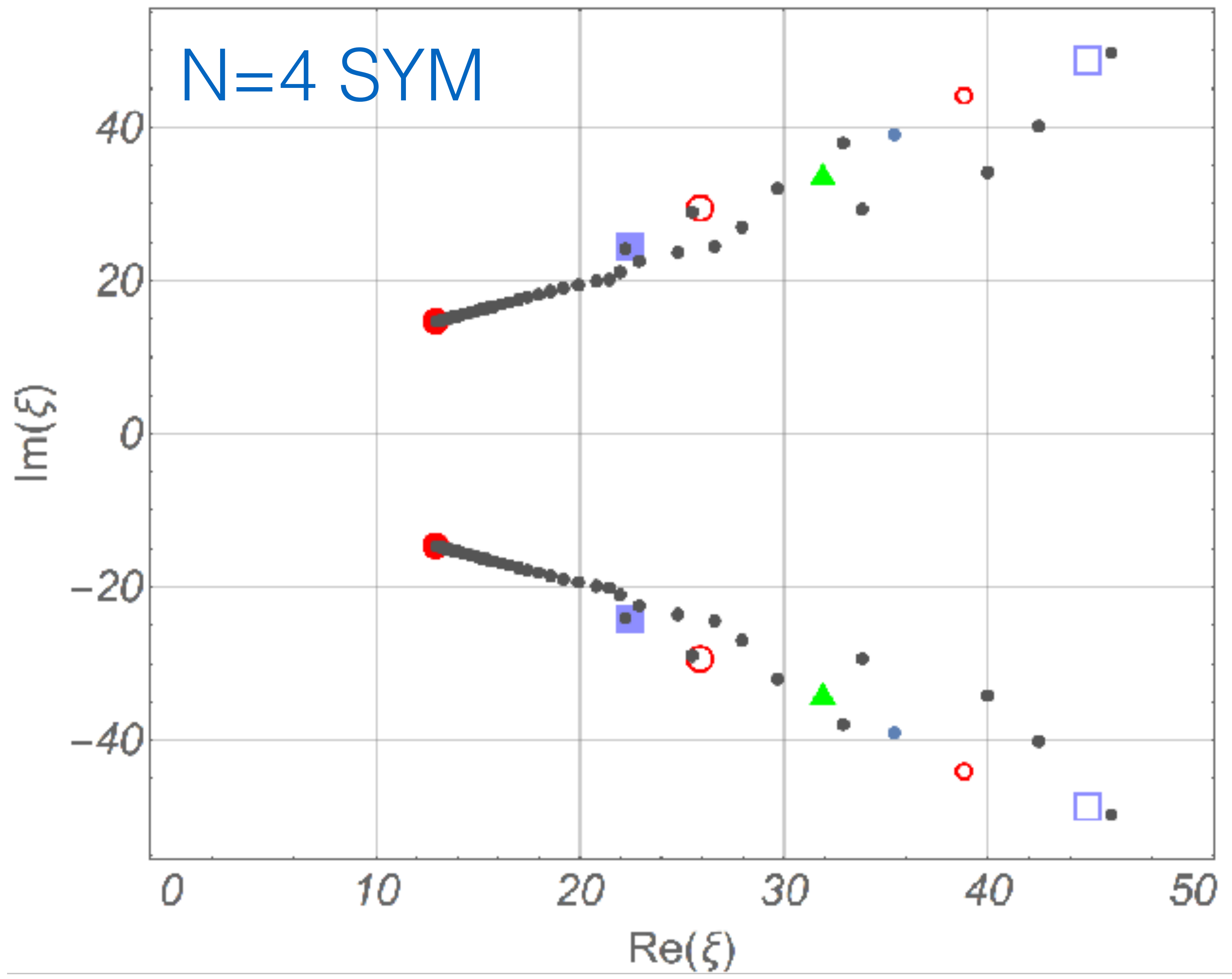
- **Branch point location determined by** τ_π
- Cannot integrate over the real line
- Complex ambiguity

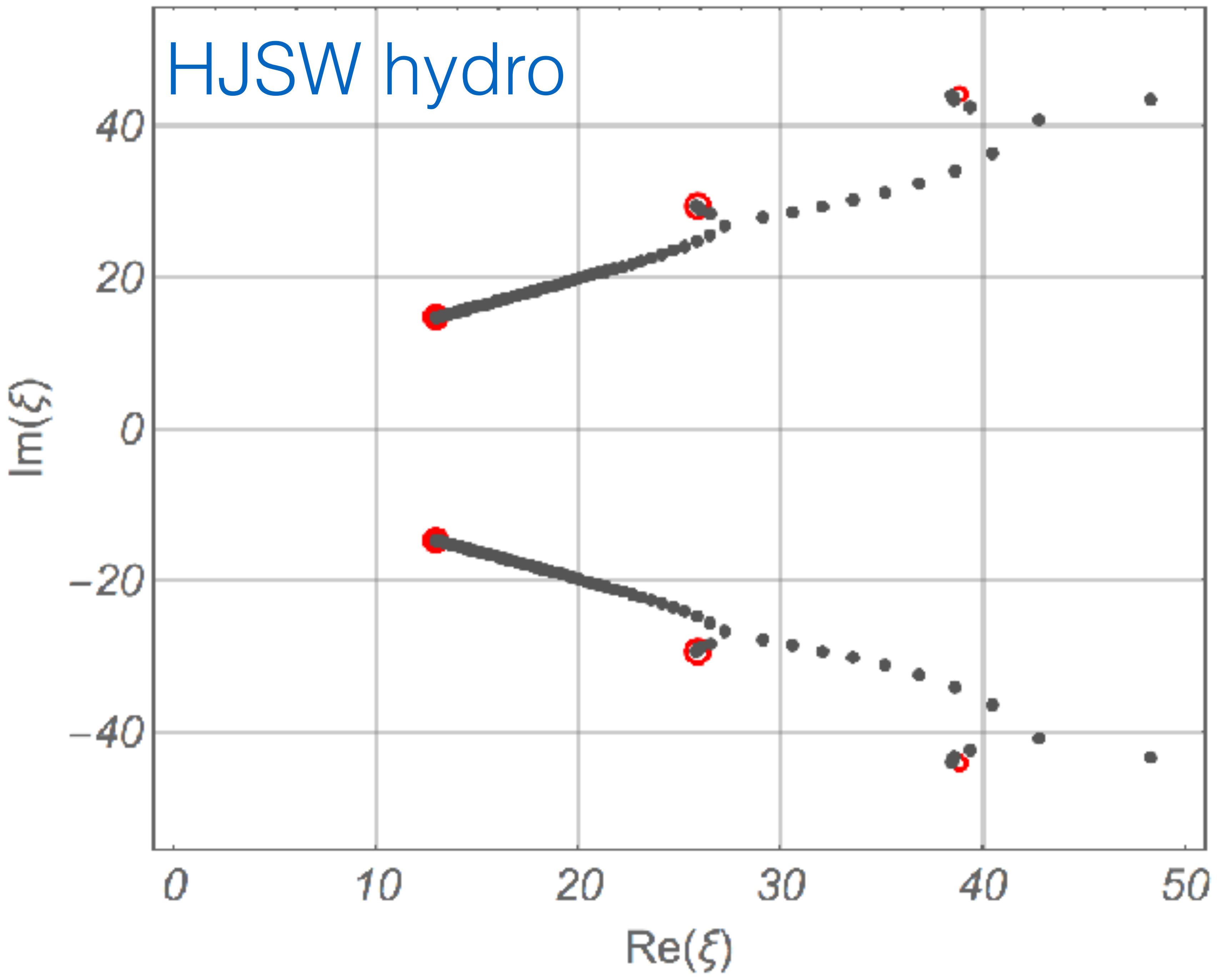
The ambiguity is resolved by incorporating “trans-series sectors”

$$\mathcal{A} = \sum_{n=0}^{\infty} a_n w^{-n} + \left(c\, e^{-\frac{3}{2C_{\tau\Pi}} w} w^{\frac{C_\eta - 2C_{\lambda_1}}{C_{\tau\Pi}}} \right) \sum_{n=0}^{\infty} a_n^{(1)} w^{-n} + \ldots$$

- The exponentially-suppressed corrections: transient, non-hydrodynamic modes.
- The coefficients in the trans-series sectors are related by resurgence relations.
- The complex trans-series parameter c is partially fixed by consistency. The remaining freedom is a single real integration constant.
- The “perturbative” sector has no memory of the initial conditions
- The exponential decay shows how the solution “forgets” the initial conditions.

Borel summation with oscillating transients



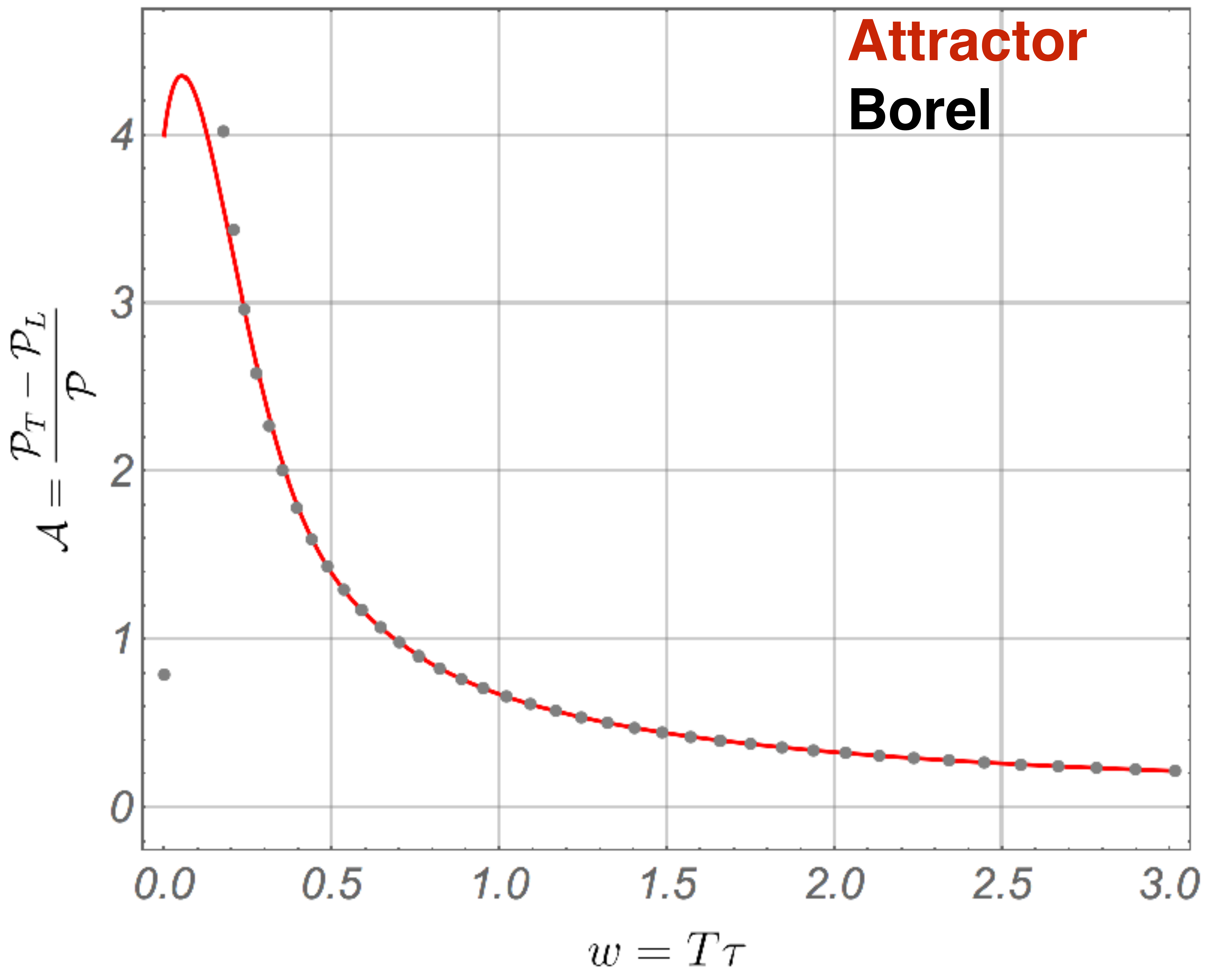


- The pattern of divergence in SYM ang HJSW hydro is almost identical
- No singularities on the real axis; ignore instanton sectors
- Idea: **use HJSW hydro as a testbed** for the Borel summation of the hydro series

Borel sum in HJSW

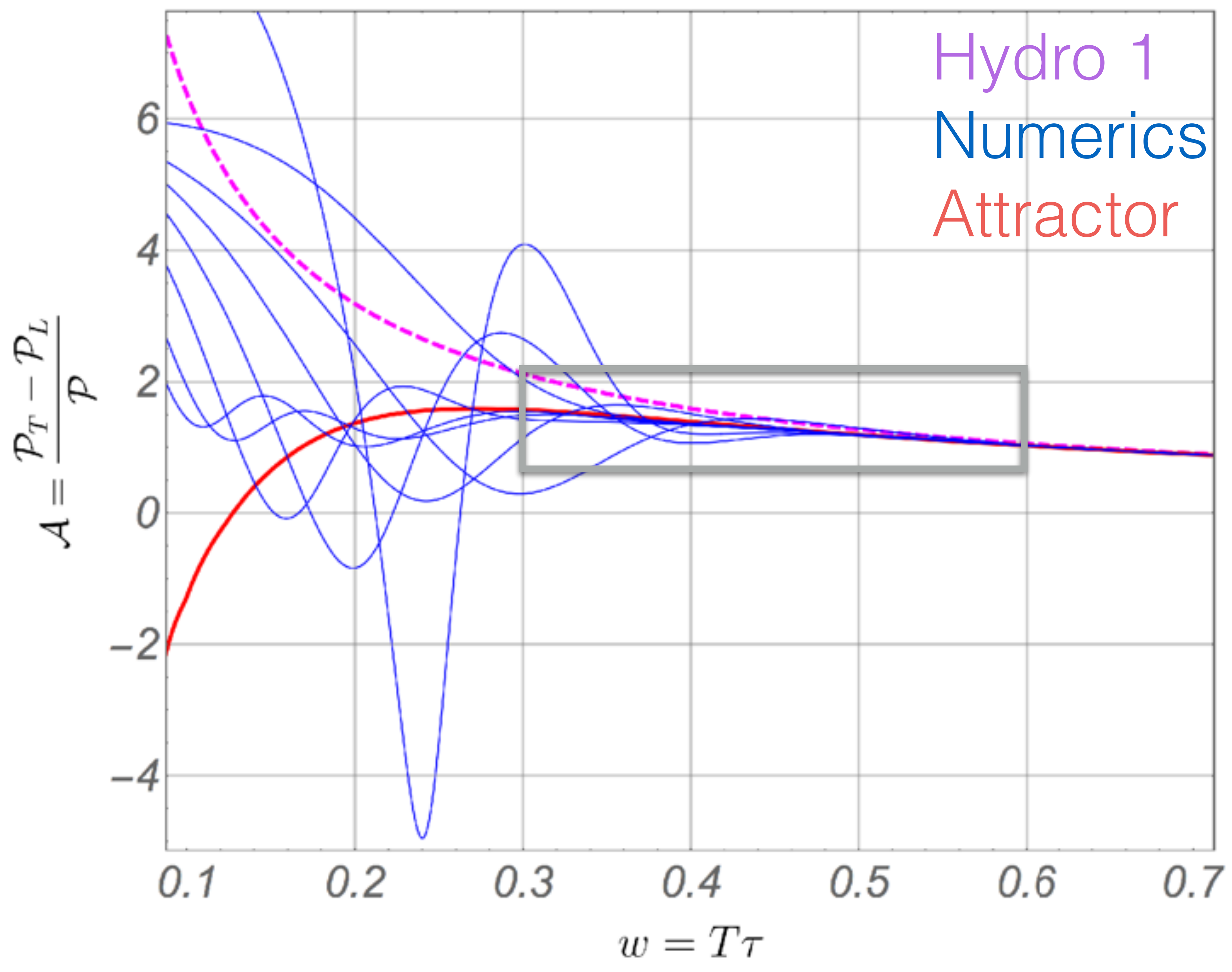
We adopt HJSW as a testing ground:

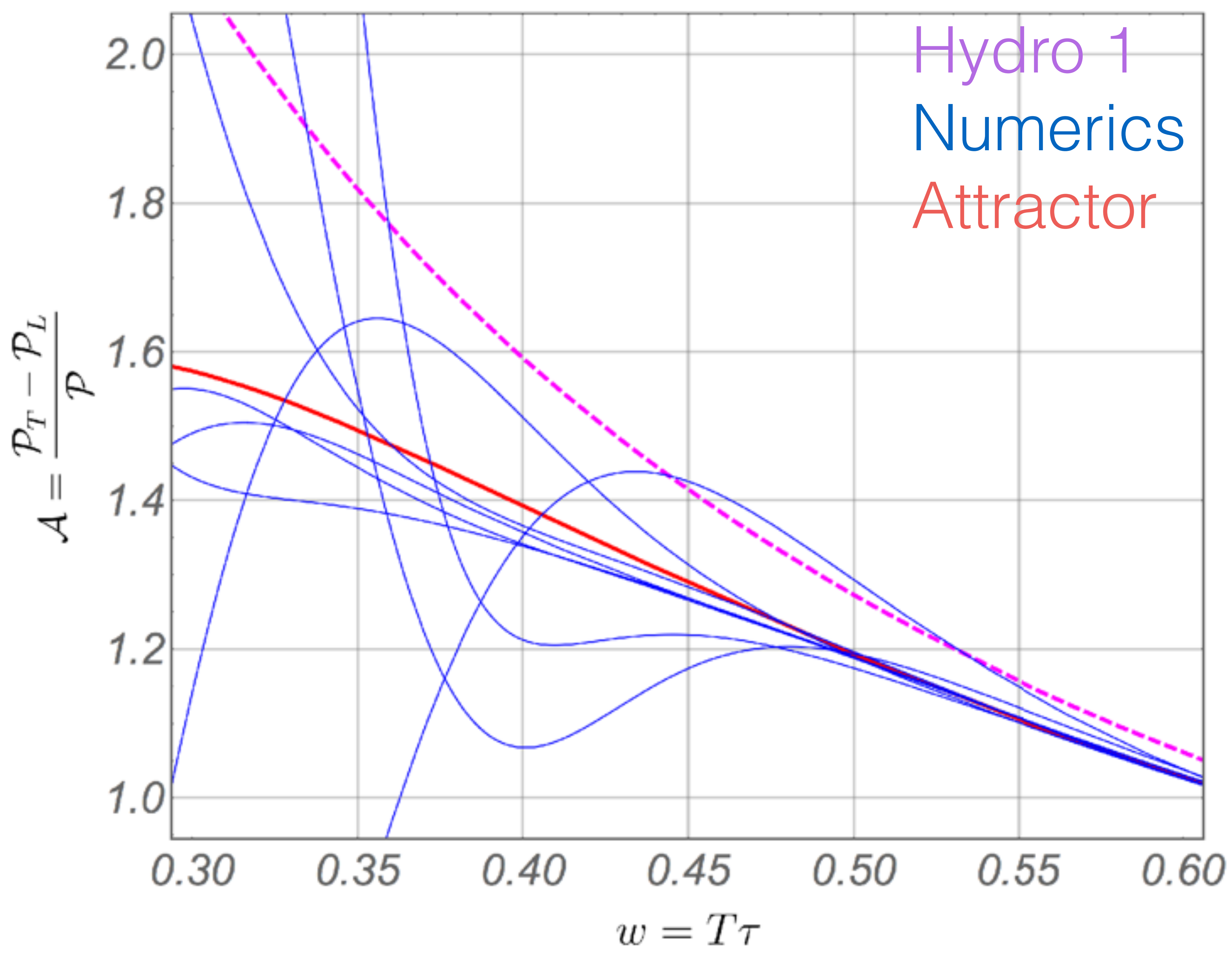
- Use 240 terms of the series
- The result **can be compared** to the numerically determined attractor
- The summation breaks down for w < 0.3, but **gets better for larger values of w.**
- Could be improved by including trans-series sectors, but this would require determining appropriate values of trans series parameters.



Next: proceed in the same way to sum the series for N=4 SYM

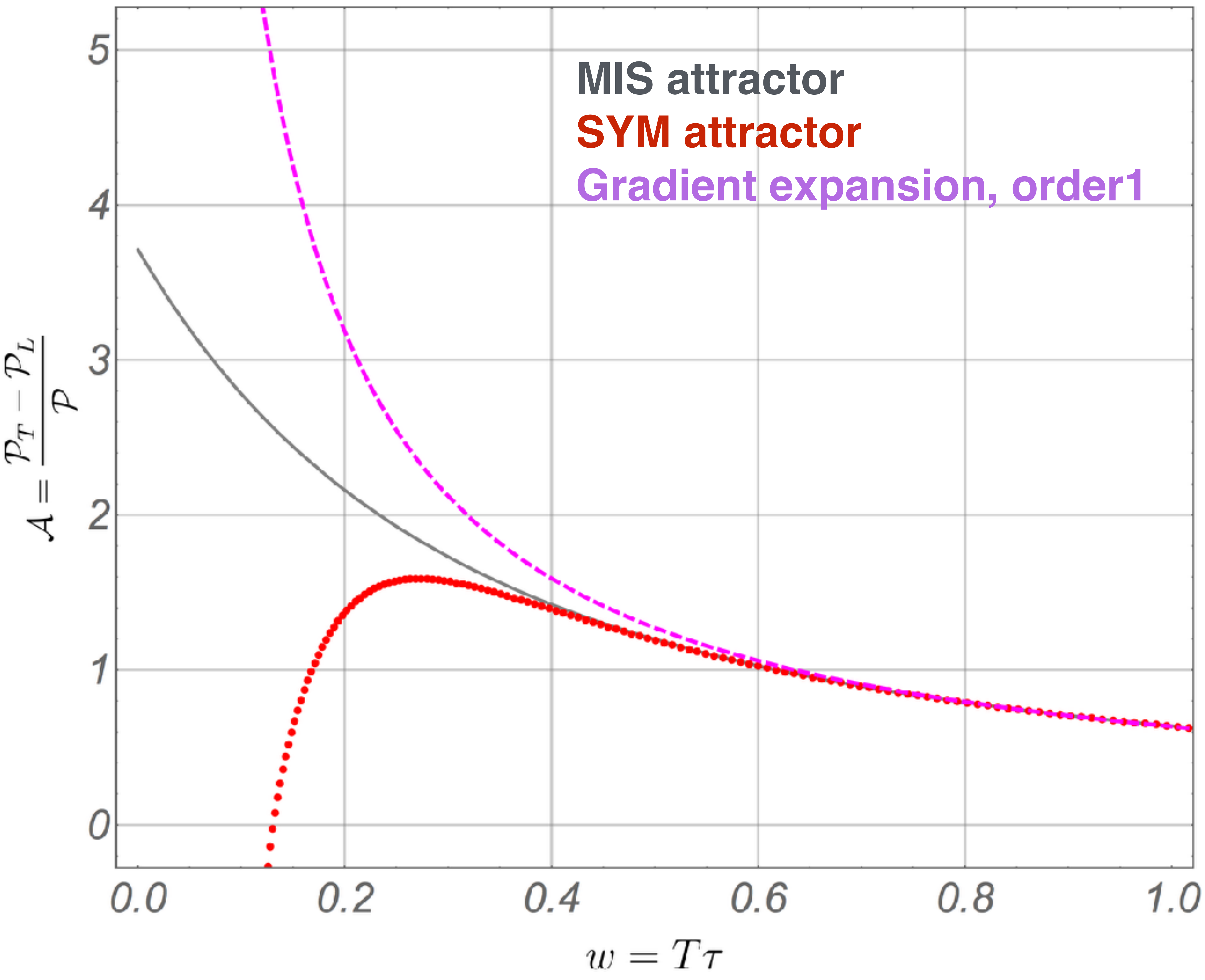
Attractor in N=4 SYM





- Very early hydronization
- Pressure anisotropy at hydrodynamization high
- The difference between the attractor and hydro-1 is parameter dependent
- Behaviour for small w hard to get this way

$$\mathcal{A}_0 = \frac{2530w - 276}{3975w^2 - 570w + 120}$$



Comparing attractors: the BRSSS attractor tracks the SYM attractor very closely: is reaches the MIS attractor as soon as the calculation makes sense.

Seeing the transients in SYM plasma

The QNM can be seen in the Borel-Pade plot. They can also be seen directly in numerical solutions of Bjorken flow obtained using AdS/CFT.

The form of the trans-series correction is

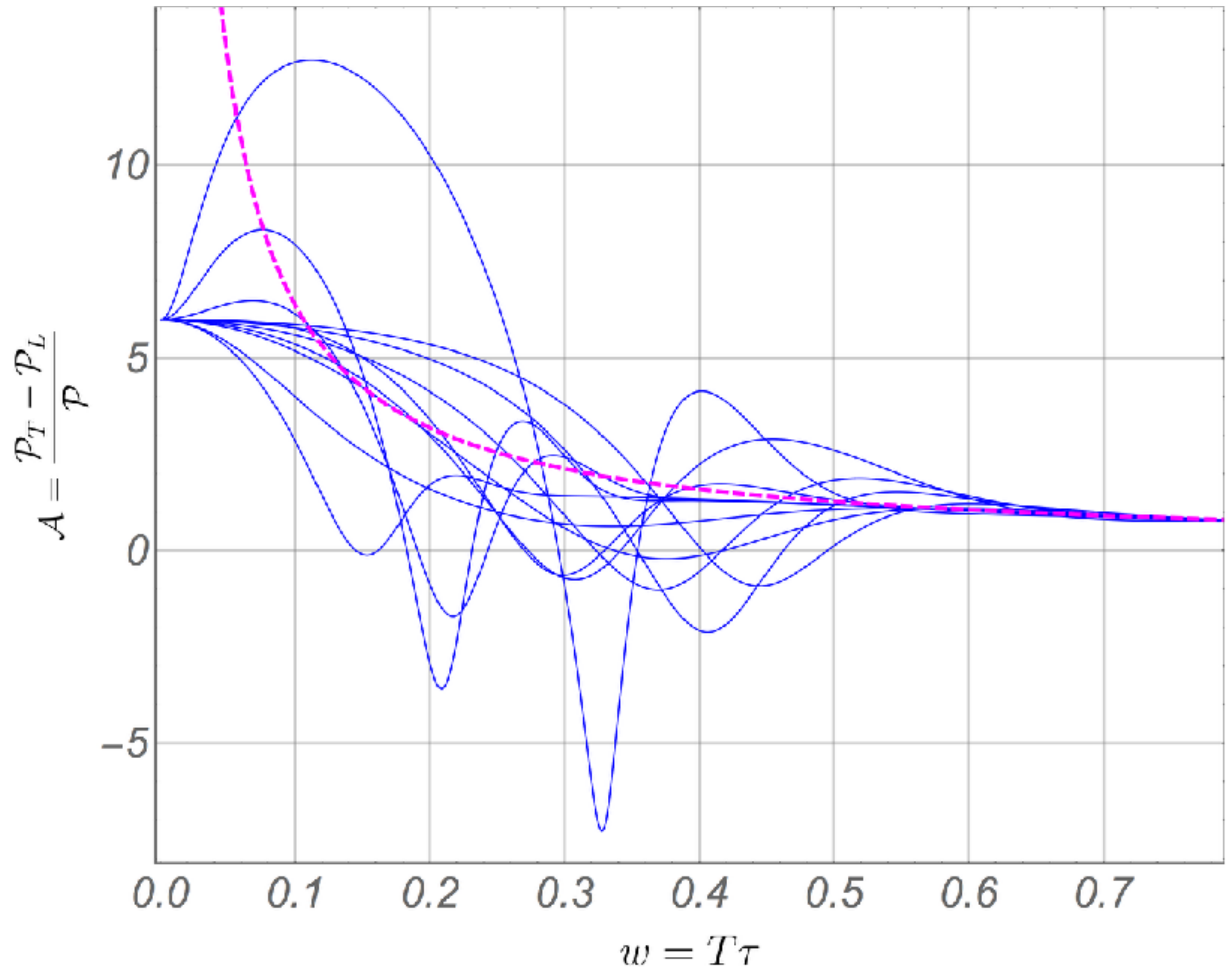
$$\mathcal{A}(w) \sim \mathcal{A}_H(w) + e^{-\frac{3}{2}\Omega_I w} w^{\beta} \left[\Phi_+(w) \cos\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) + \Phi_-(w) \sin\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) \right]$$

where

$$\Phi_\pm(w) = C_\pm \left(1 + \sum_{n>0} \frac{a_n^{(\pm)}}{w^n}\right)$$

is the "1-instanton" sector series, which we can approximate by a constant.

But we do not really know the universal part.

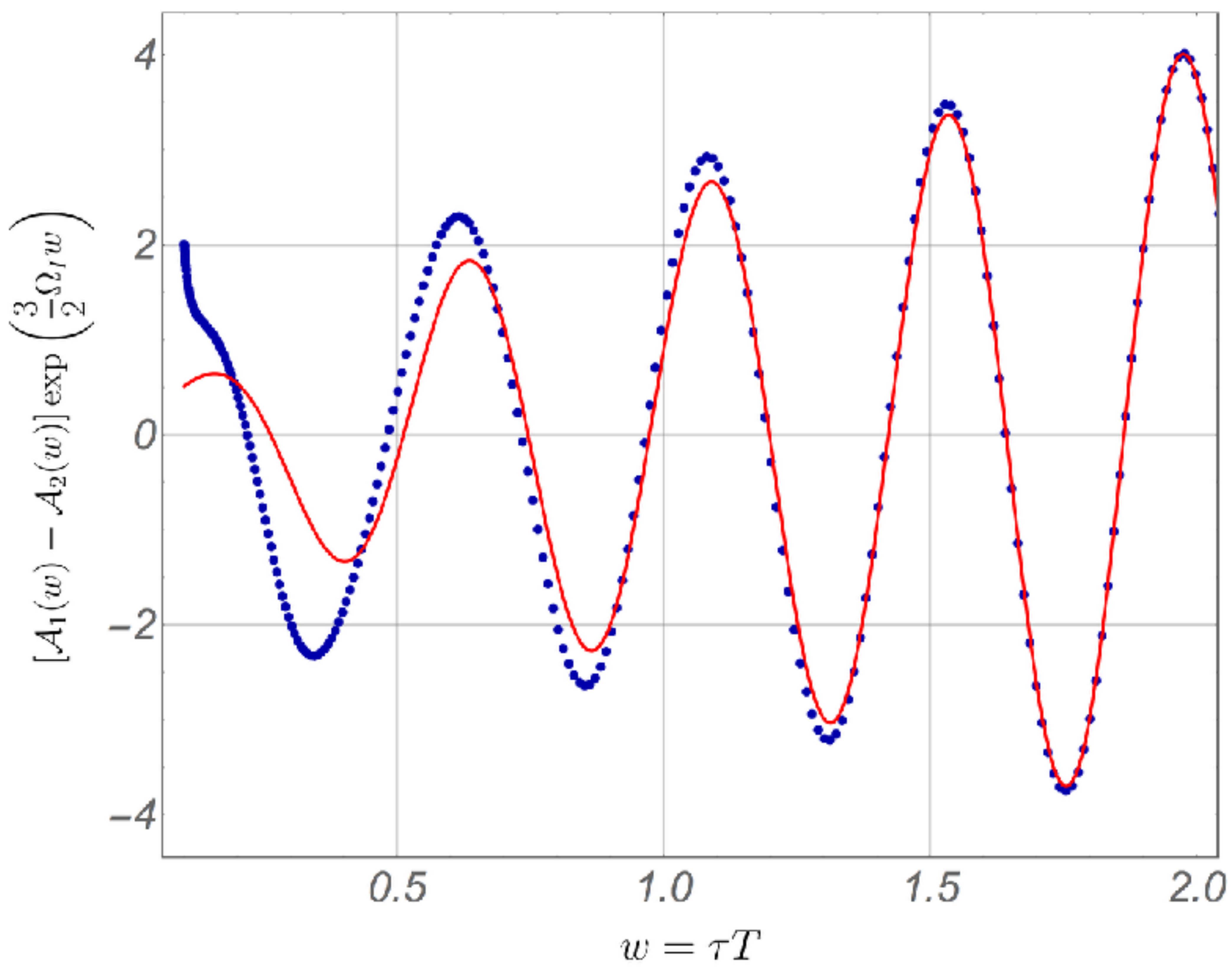


To see that the transient, damped oscillations can be resolved with the existing numerical methods we can consider **pairs of solutions**; their difference will not involve the universal, hydrodynamic part:

$$\mathcal{A}_1(w) - \mathcal{A}_2(w) \sim e^{-\frac{3}{2}\Omega_I w} w^{\beta} \left[C_{12}^{(+)} \cos\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) + C_{12}^{(-)} \sin\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right)\right]$$

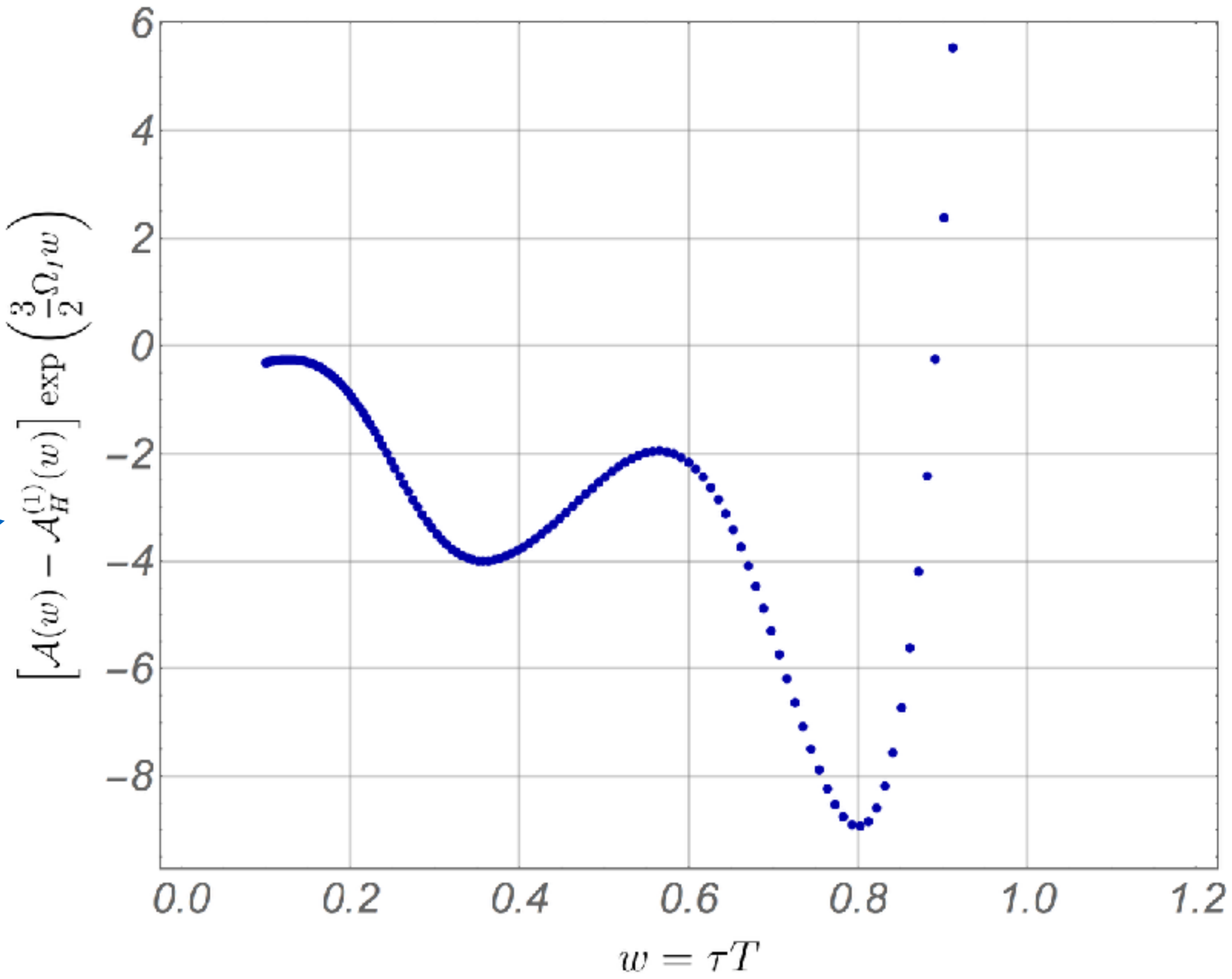
Here all the parameters are fixed apart from the **two amplitudes**, which reflect the initial conditions and differ from one pair of solution to another.

The two amplitudes appearing in the formula above can then be **fitted to the numerical solution**.



Alternatively, instead of looking at differences of solutions, we can try to subtract the universal part ("hydro to all orders") by estimating it using the gradient expansion.

Can we just remove the universal part by subtracting first or second order of the gradient expansion?

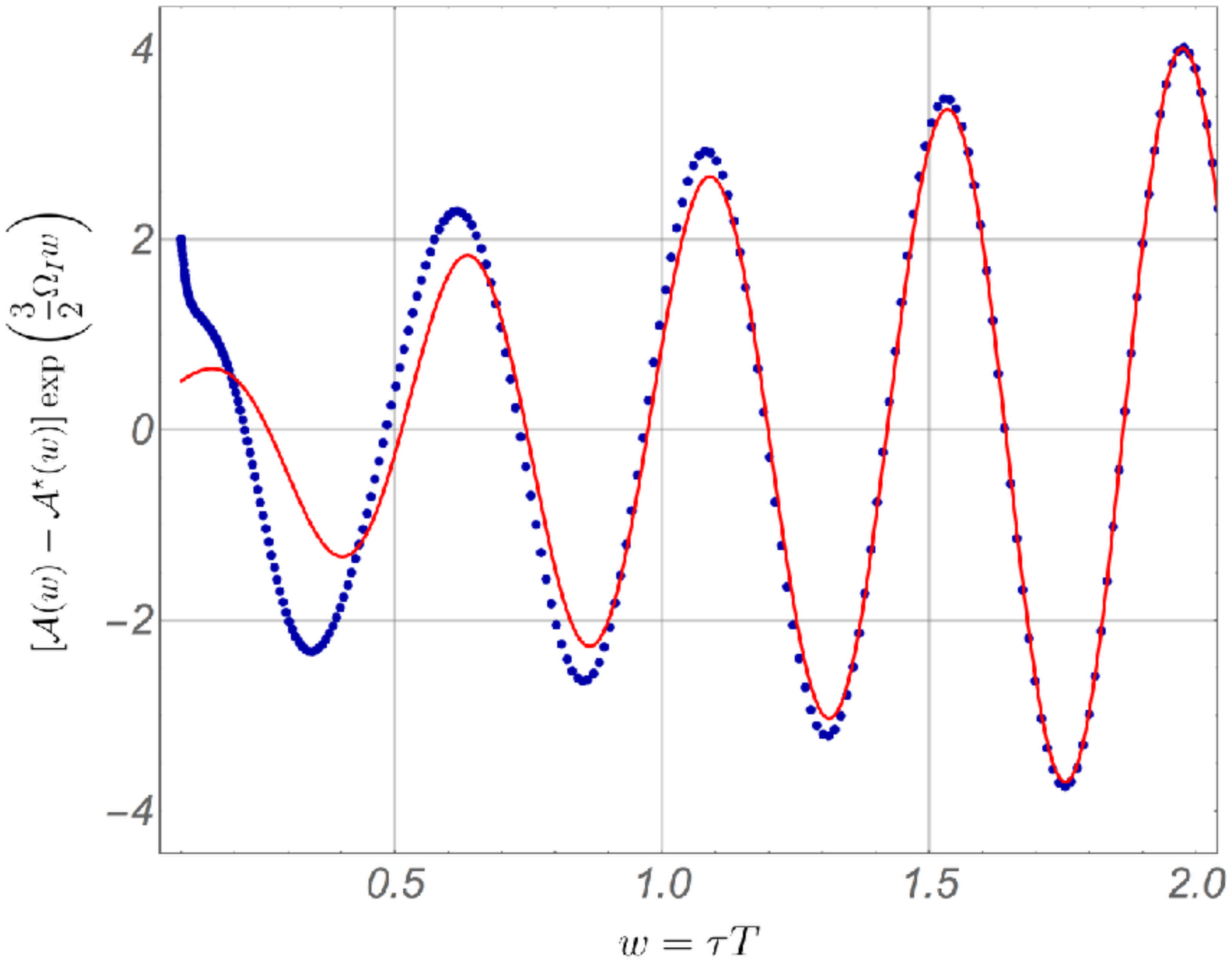


First order of the gradient expansion

$$\mathcal{A}_H^{(1)}(w) = 8\,(\eta/s)\,\frac{1}{w}$$

Instead, we can try to use the estimate of "all-order hydro" provided by the **attractor** calculated as the **Borel sum** of the truncated gradient expansion:

$$\mathcal{A}(w)-\mathcal{A}^{\star}(w) \sim e^{-\frac{3}{2}\Omega_I w} w^{\beta_R} \left[C^{(+)} \cos\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) + C^{(-)} \sin\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right)\right]$$



Summary

- Hydrodynamics can be formulated based on very general arguments and matched to microscopic theories by comparing gradient expansions.
- The emergence of hydrodynamic behaviour is governed by the decay of non-hydrodynamic modes rather than local equilibration.
- Universal observables which exhibit attractor behaviour provide a clean way to study the decay of transients.
- The naive Borel sum provides a reasonable approximation to the attractor.
- The least-damped QNM can be seen directly in the late time behaviour of numerical solutions of Bjorken flow.