Holographic heavy-ion collisions: new analytic solutions

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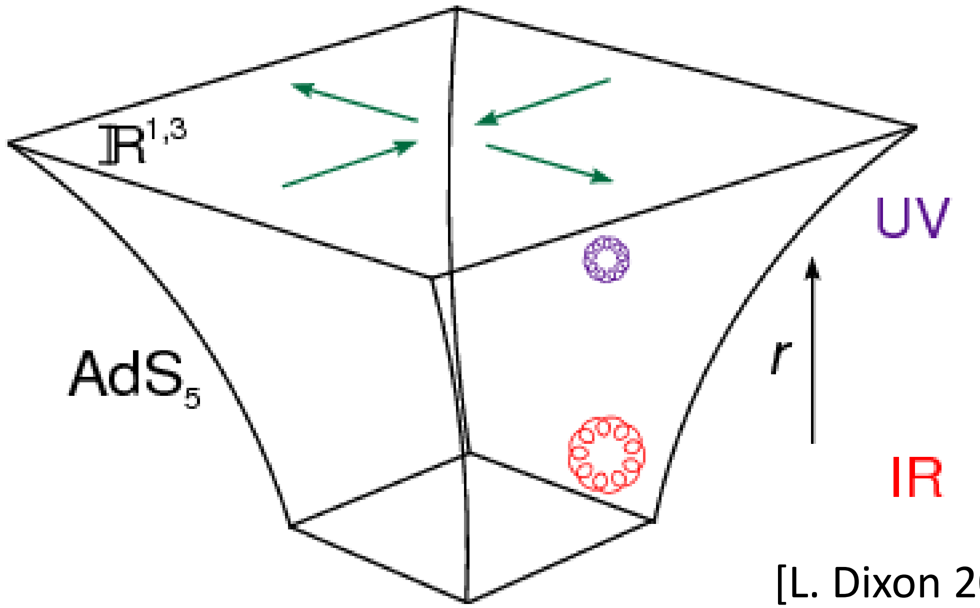
based on

Bantilan, Ishii, PR, 1803.10774

What is Holography?

- Conjectured Duality between gauge theory and gravity in one higher dimension [Maldacena, 1997]
- Best-studied example: N=4 SYM for large N_c and large coupling dual to classical Einstein Gravity in Asymptotic AdS_5



[L. Dixon 2008]

Matter in Holography

- Field theory matter in equilibrium at temperature T
- Static Black Hole with Hawking temperature T

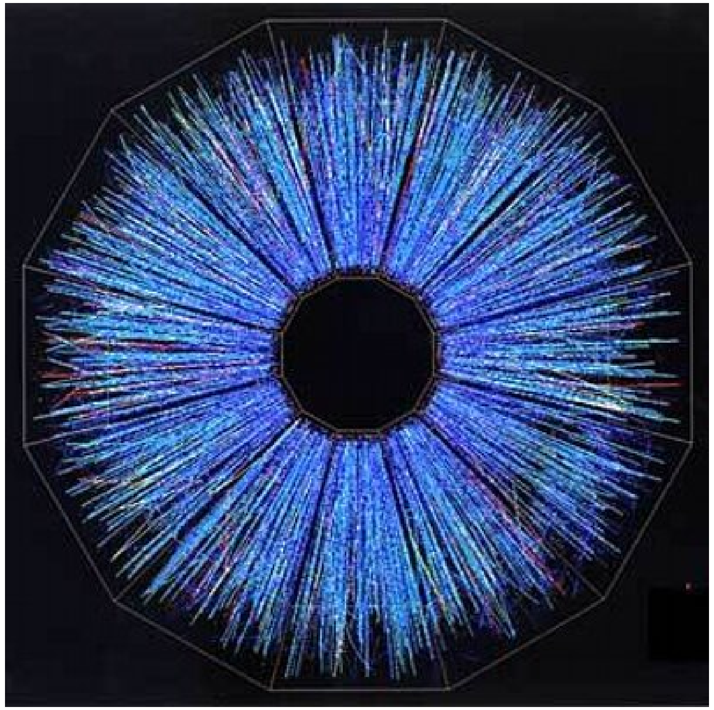
} Related by Holographic Duality

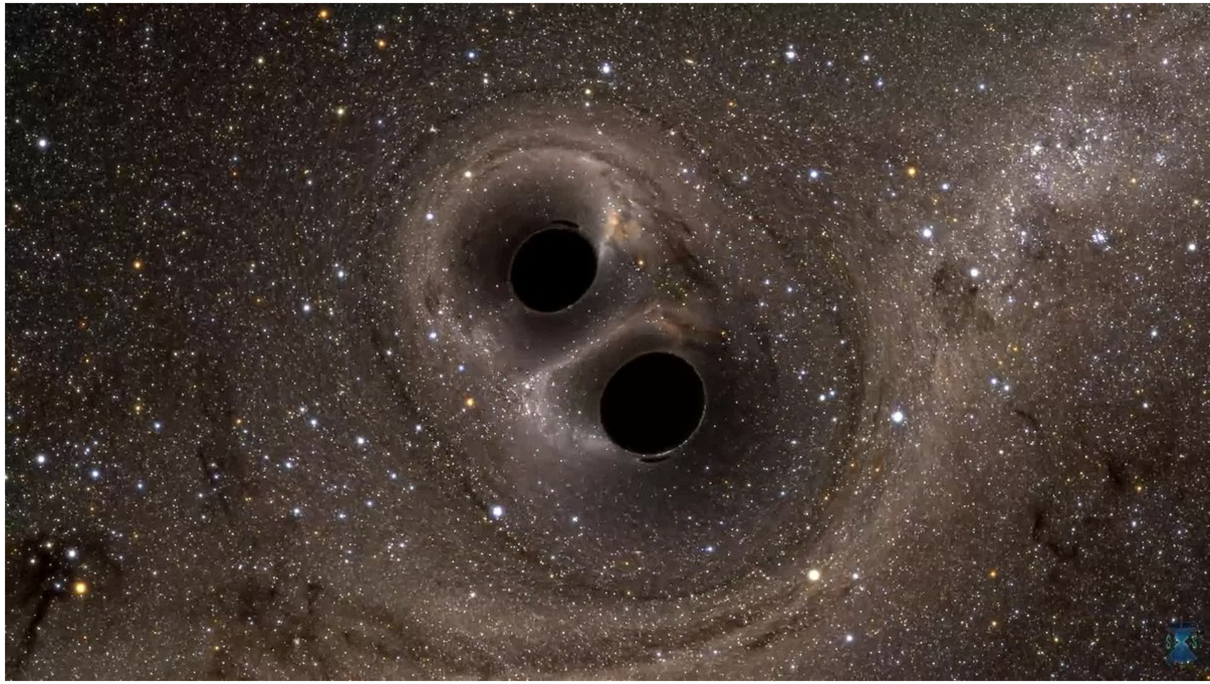
Matter Dynamics in Holography

Out of equilibrium dynamics of strongly coupled QFT matter

$\leftrightarrow$

Out of equilibrium dynamics black holes





Holographic Heavy-Ion Collisions

- Holographic Dual of Heavy-Ion Collision is Collision of localized “Stuff” in AdS_5
- Easy to handle “localized stuff”: Black Holes

  Heavy Ion Collisions <-> Black Hole Collisions in AdS_5

Collisions in AdS_5

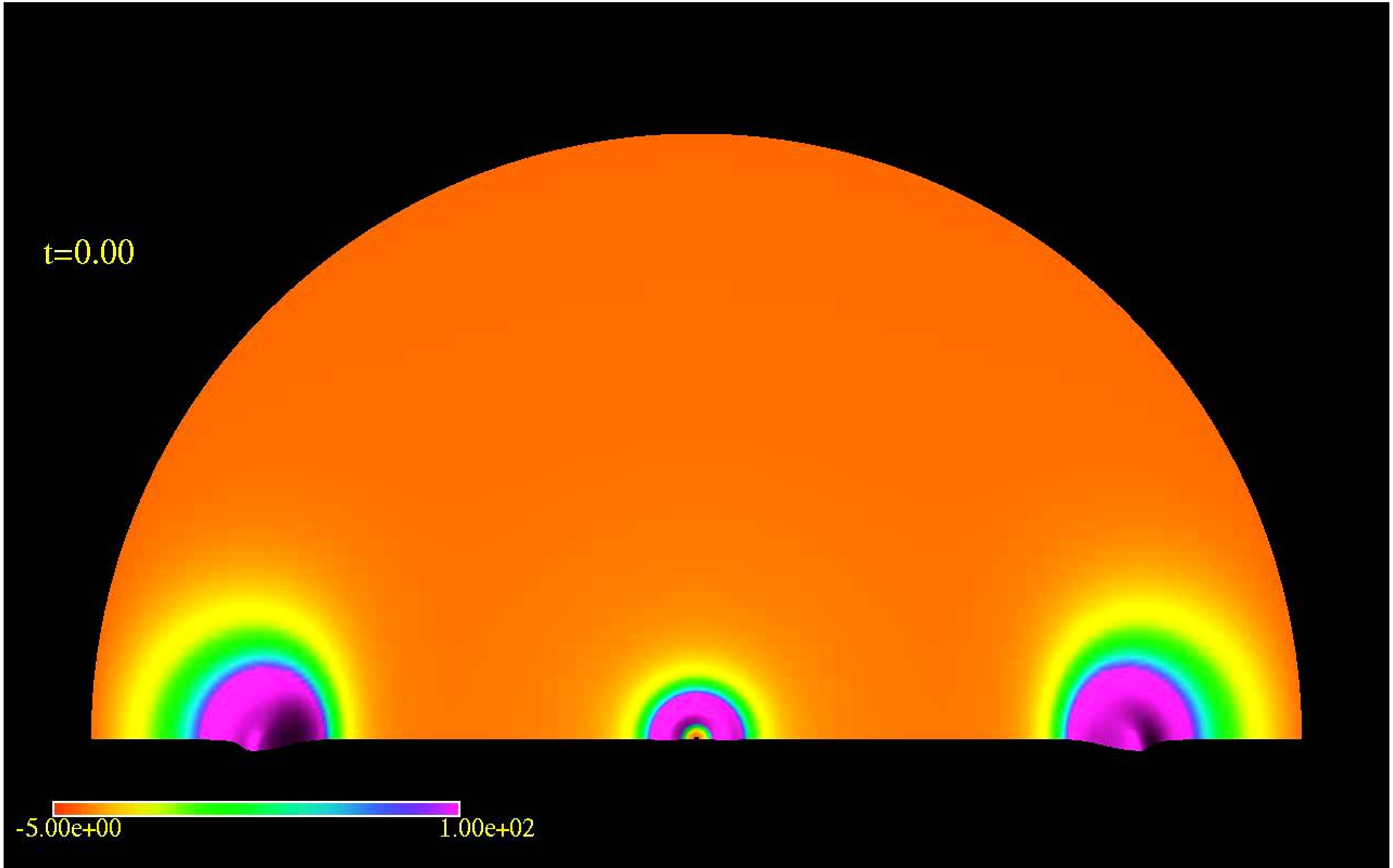
- Collisions of non-compact objects (shock-waves)

  [analytic: Albacete, Kovchegov, Taolitis; Grumiller & PR;
  numerical: Chesler & Yaffe, van der Schee, Attems et al, ...]

- Collisions of compact objects (BHs)

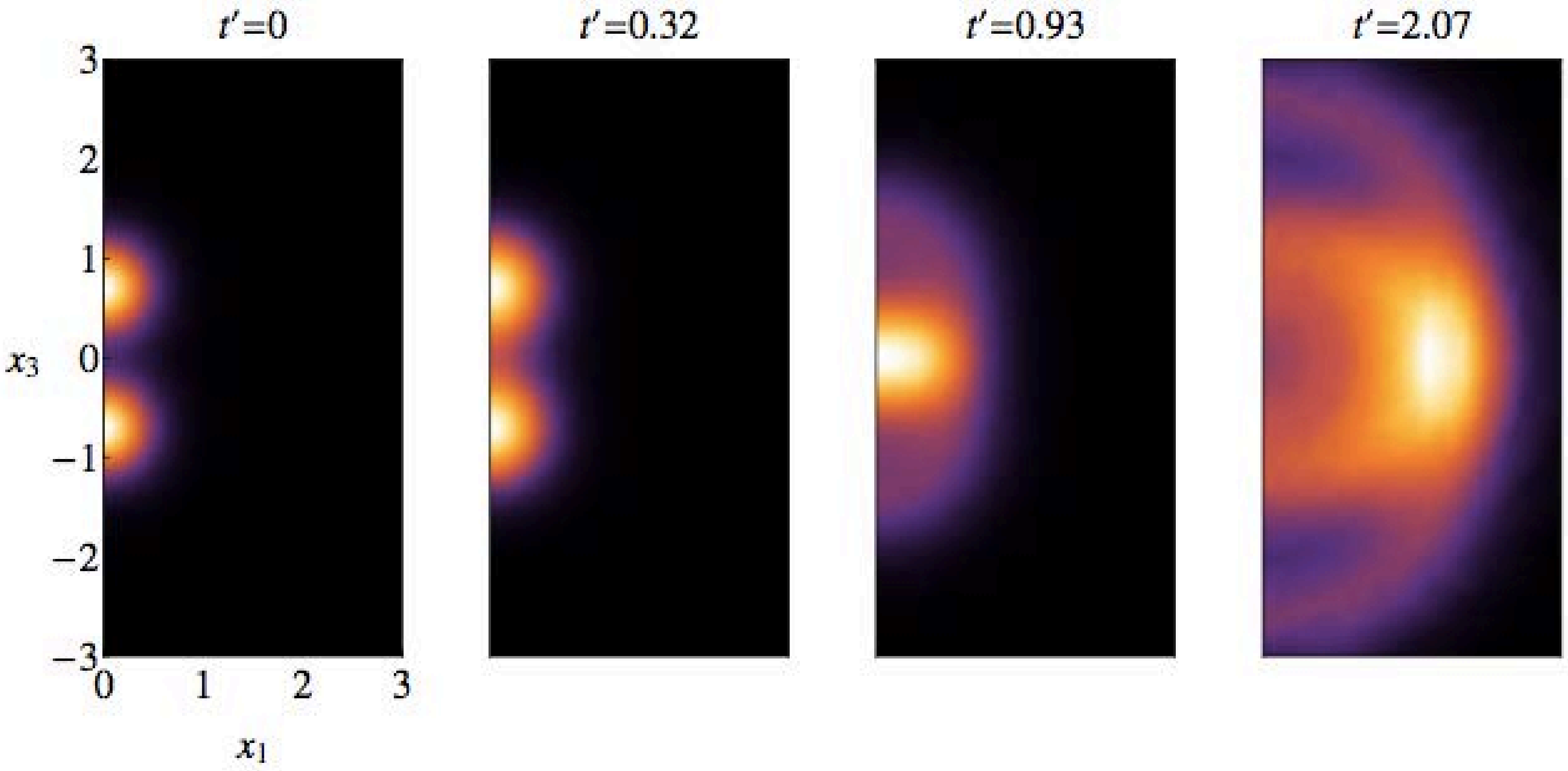
  [numerical: Bantilan & PR, 2014]

Simulations of Black Hole Collisions in AdS_5



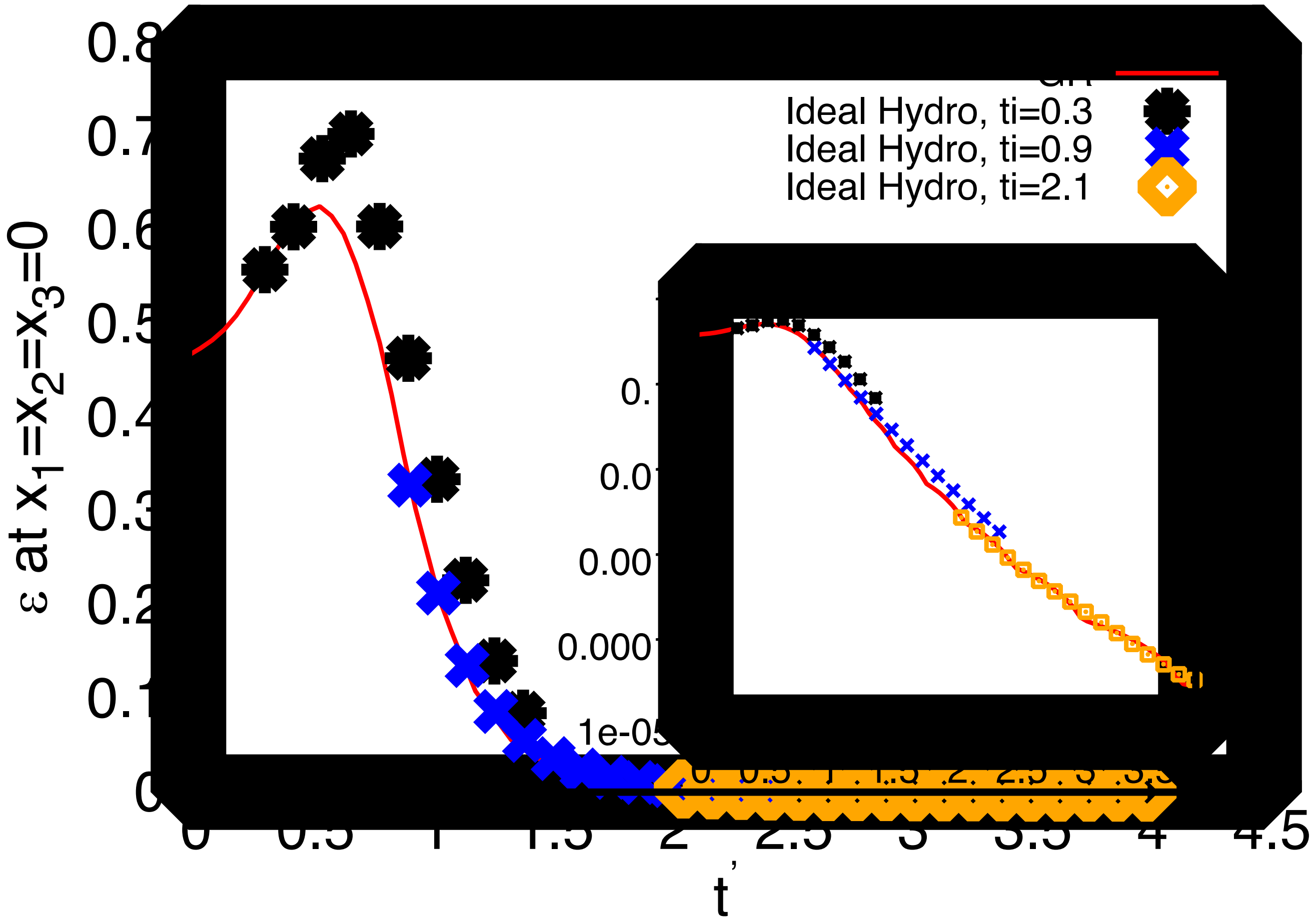
[Bantilan & PR, 2014]

Head-On Black Hole Collisions in AdS$_5$



[Bantilan & PR, 2014]

Head-On Black Hole Collisions in AdS_5



- Collision leads to deformed AdS-Schwarzschild BH
- BH is ringing down
- Becomes stationary AdS-Schwarzschild BH
- Trafo to Minkowski boundary: dilution of energy density

[Bantilan & PR, 2014]

How to generalize to off-center BH collisions?

- I) Direct Numerical Solution of Einstein Equations
- II) Think/Cheat: Head-on Collision led to stationary Schwarzschild in global AdS after ring-down, what would off-center collision look like?

Myers-Perry Black hole

(generalization of Kerr to AdS_5)

Holographic Off-Center BH Collisions

- Myers-Perry black hole in global AdS_5 parametrized by 2 rotation parameters

$$|w_{1,2}|<1$$

- Line element in Boyer-Lindquist coordinates known [Hawking, Hunter, Taylor 1998]
- Stationary BH, no dissipative effects
- Stress tensor on global AdS R^1xS^3 known [Bhattacharyya, Lahiri, Loganayagam, Minwalla, 2007]
- Coordinate/Conformal transformation to obtain Stress Tensor on $R^{1,3}$
- This is ideal hydrodynamic stress tensor since dissipative effects are absent

Holographic Off-Center BH Collisions

After QNM Ring-down, holographic solution for energy density ε and four-velocity $u^\mu = \gamma\,(1, v^x, v^y, v^\xi/\tau)$

$$\epsilon = 16L^8T_0^4\left[(L^4 + 2L^2\mathbf{x}_\perp^2 + (\tau^2 - \mathbf{x}_\perp^2)^2)(1-\omega_2^2) + 2L^2(\tau^2 - 2y^2)(\omega_1^2 - \omega_2^2) + 2L^2\tau^2(1-\omega_1^2)\cosh 2\xi\right]^{-2}$$

$$\gamma = \frac{[(L^2 + \tau^2 + \mathbf{x}_\perp^2)\cosh\xi + 2(\tau\omega_2 x - L\omega_1 y \sinh\xi)]}{(16L^8T_0^4/\epsilon)^{1/4}}$$

$$v^x = \frac{2\tau x\cosh\xi + \omega_2(L^2 + \tau^2 + x^2 - y^2)}{(L^2 + \tau^2 + \mathbf{x}_\perp^2)\cosh\xi + 2(\tau\omega_2 x - L\omega_1 y\sinh\xi)},$$

$$v^y = \frac{2\tau y\cosh\xi + 2\omega_2 xy - 2L\tau\omega_1\sinh\xi}{(L^2 + \tau^2 + \mathbf{x}_\perp^2)\cosh\xi + 2(\tau\omega_2 x - L\omega_1 y\sinh\xi)},$$

$$v^\xi = -\frac{(L^2 - \tau^2 + \mathbf{x}_\perp^2)\sinh\xi - 2L\omega_1 y\cosh\xi}{(L^2 + \tau^2 + \mathbf{x}_\perp^2)\cosh\xi + 2(\tau\omega_2 x - L\omega_1 y\sinh\xi)},$$

[Bantilan, Ishii, PR, 2018]

- 2-parameter analytic solution to conformal relativistic ideal hydrodynamics

$$u^\mu \nabla_\mu \epsilon = -(\epsilon + P)\nabla_\mu u^\mu; \quad (\epsilon + P)u^\mu \nabla_\mu u^\alpha = -\nabla_\perp^\alpha P$$

Holographic Off-Center BH Collisions

- New 2-parameter analytic solution to conformal relativistic ideal hydrodynamics
- Reduces to Gubser's solution for $w_1=w_2=0$ [Gubser, 2010]
- Reduces to other known hydrodynamic solutions for $w_2/w_1=0$

[Nagy, 2009; Hatta, Noronha, Xiao 2014]

What does it look like on the boundary?

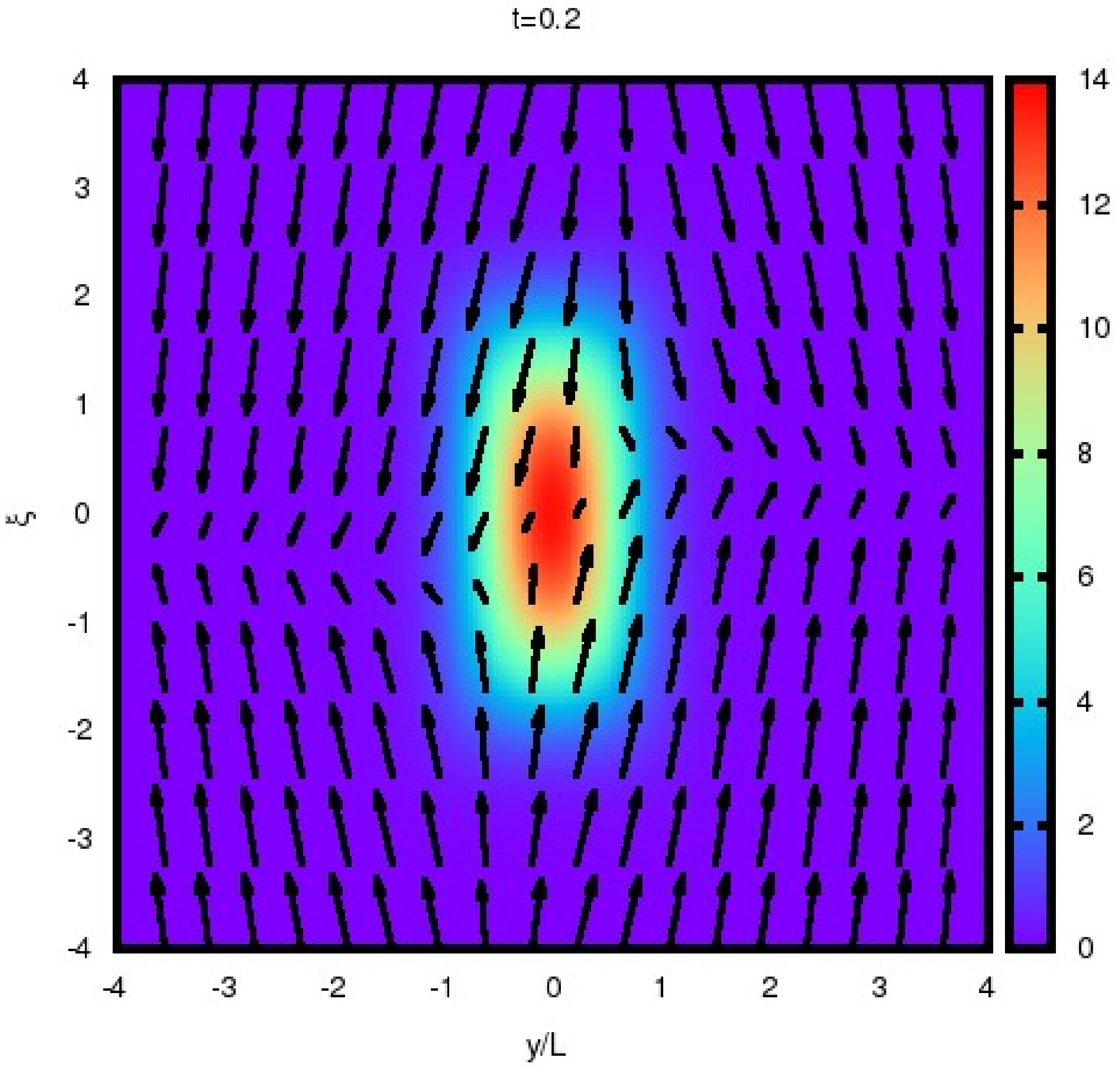
Qualitative features of analytic solution: 1/3

- Negative Initial Longitudinal Flow (Inward)
- Turns positive at late time
- Generic feature of holographic collisions

[Grumiller & PR, 2008; Bantilan & PR 2014]

- May have been seen in experiment

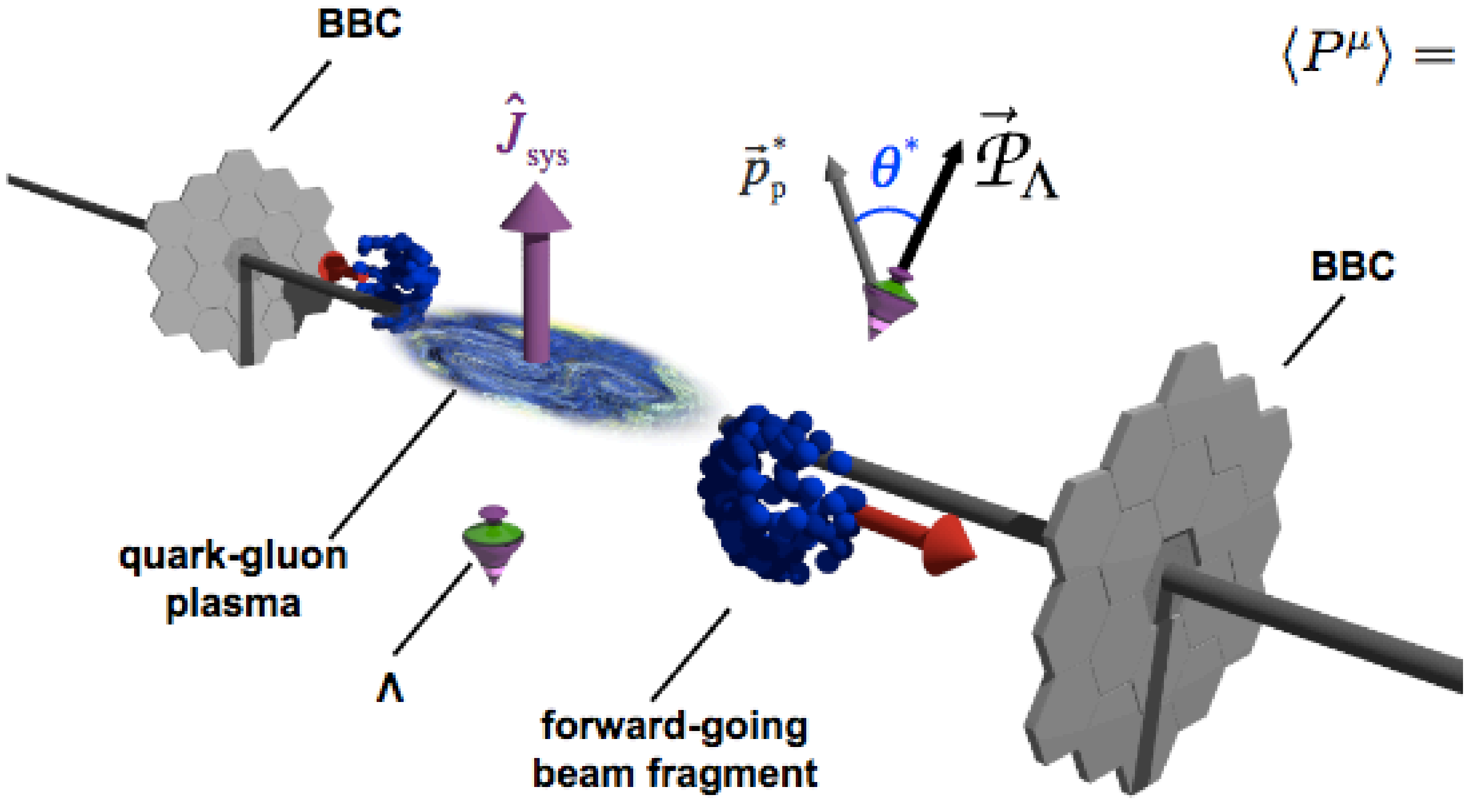
[Stephanov & Yin, 2014]



Recent excitement in HIC: observing vorticity

Polarization of Lambda Hyperons couples to fluid vorticity Ω

$$\langle P^\mu \rangle = \frac{1}{4m_\Lambda} \frac{C^{\mu\nu\alpha\beta} \int d\Sigma^\lambda T_{\lambda\nu} \Omega_{\rho\sigma}/T}{N_\Lambda},$$



[STAR, Nature 2017]

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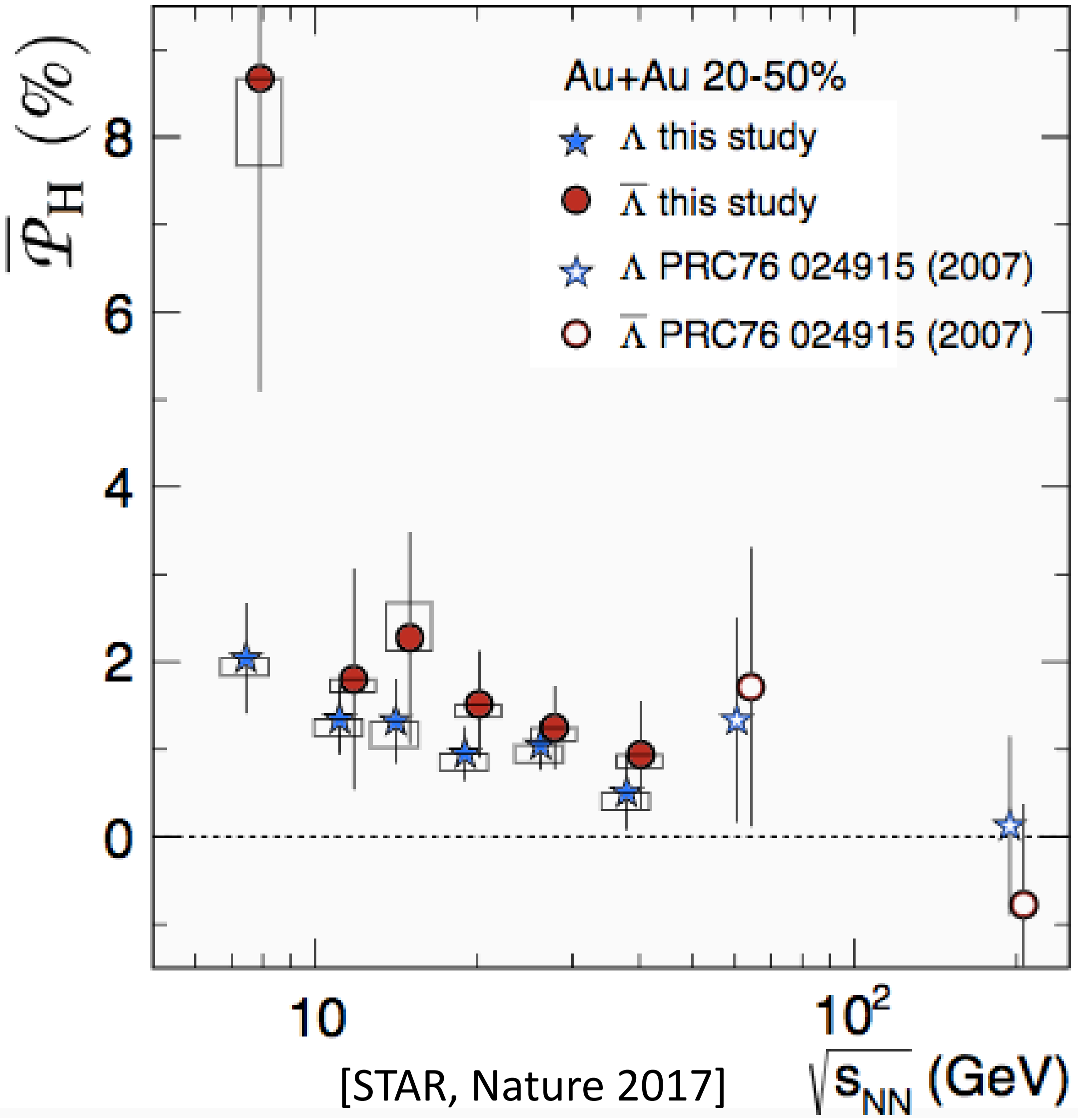


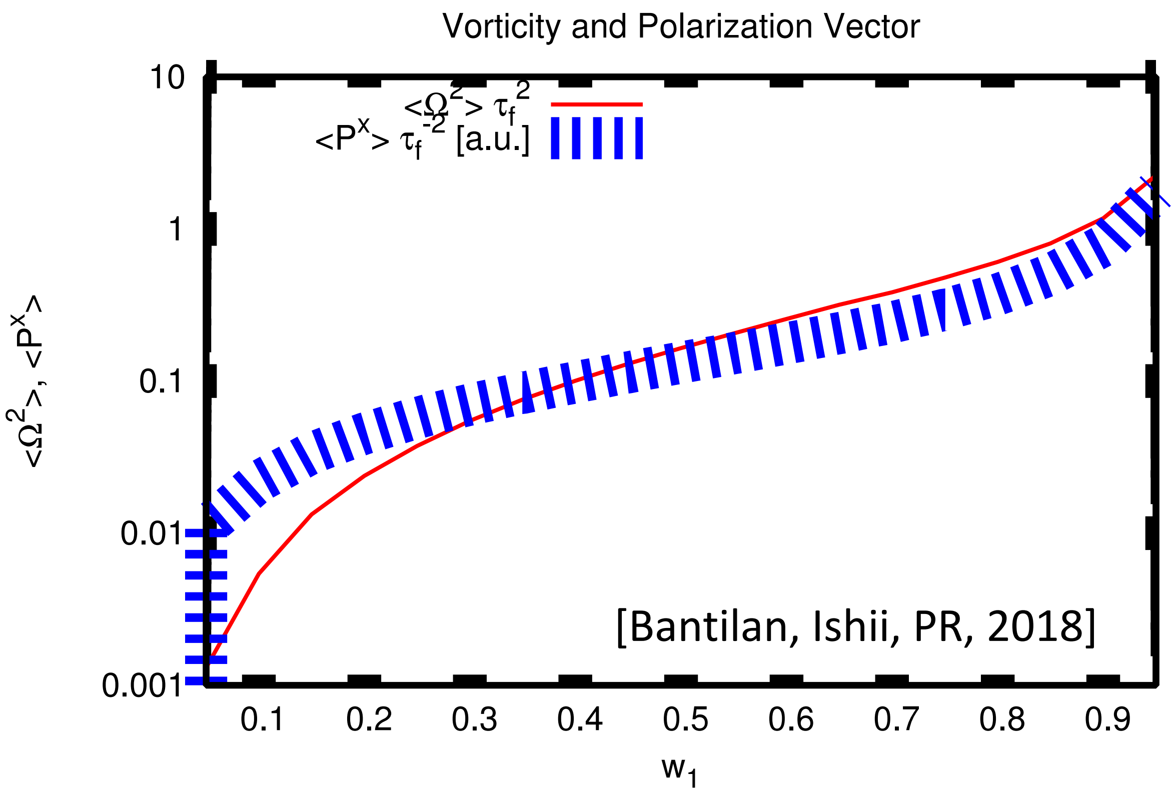
[STAR, Nature 2017]

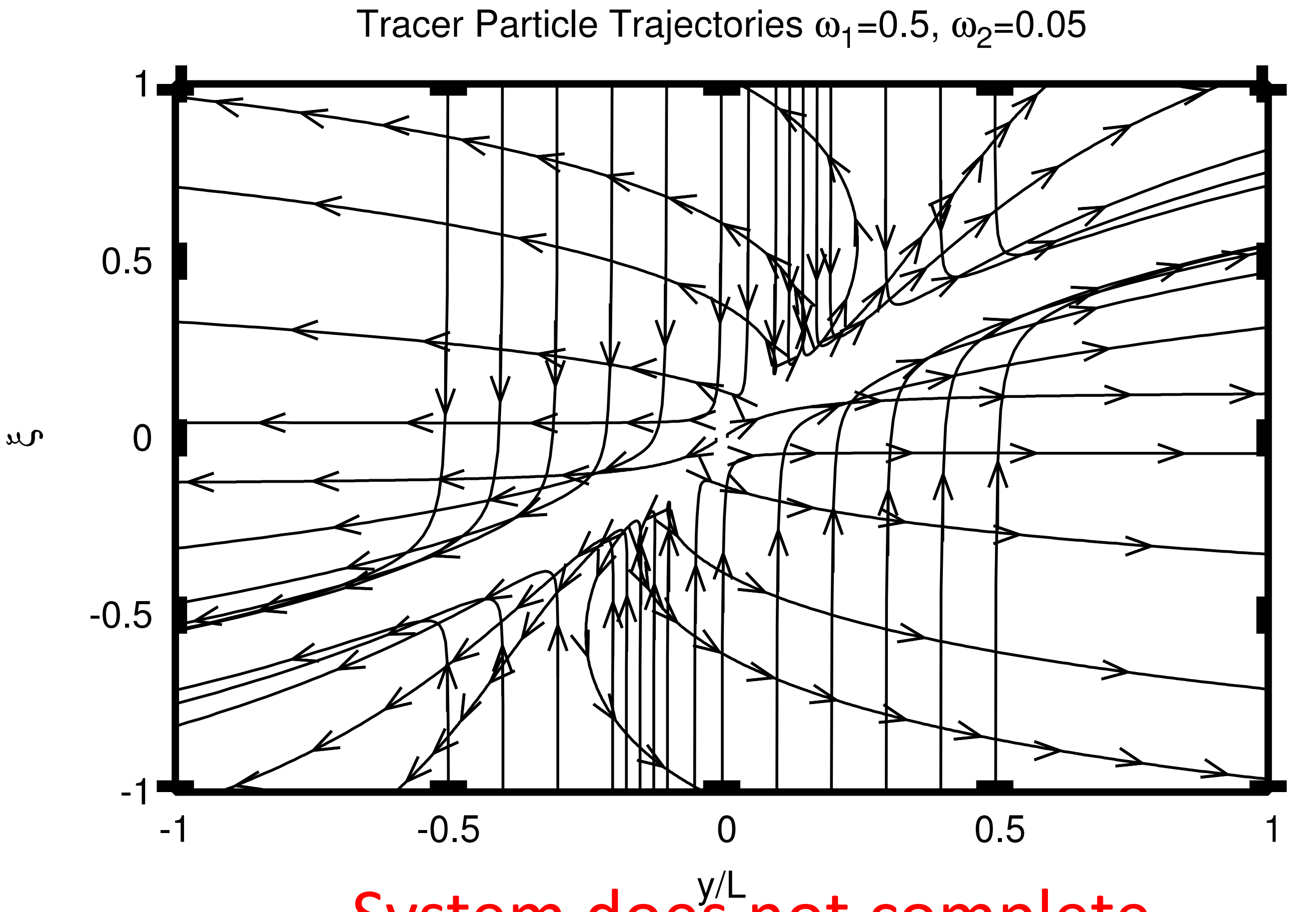
Qualitative Questions

- For off-central heavy-ion collision, non-vanishing angular momentum
- Natural to expect non-vanishing vorticity
- But does the system actually rotate?

Qualitative features of analytic solution: 2/3

- Vorticity controlled by w_1
- Non-vanishing polarization

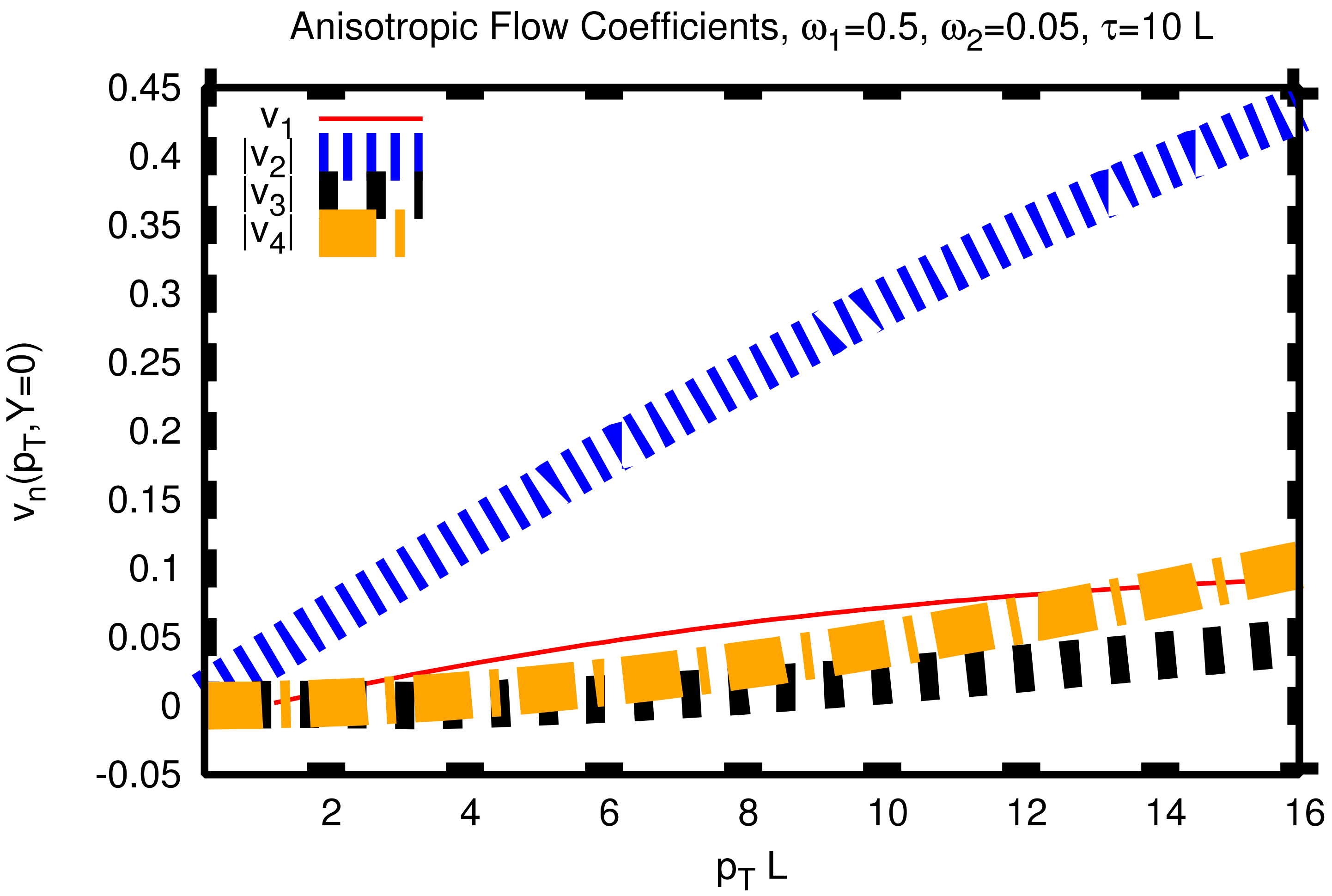




System does not complete single rotation!

Qualitative features of analytic solution: 3/3

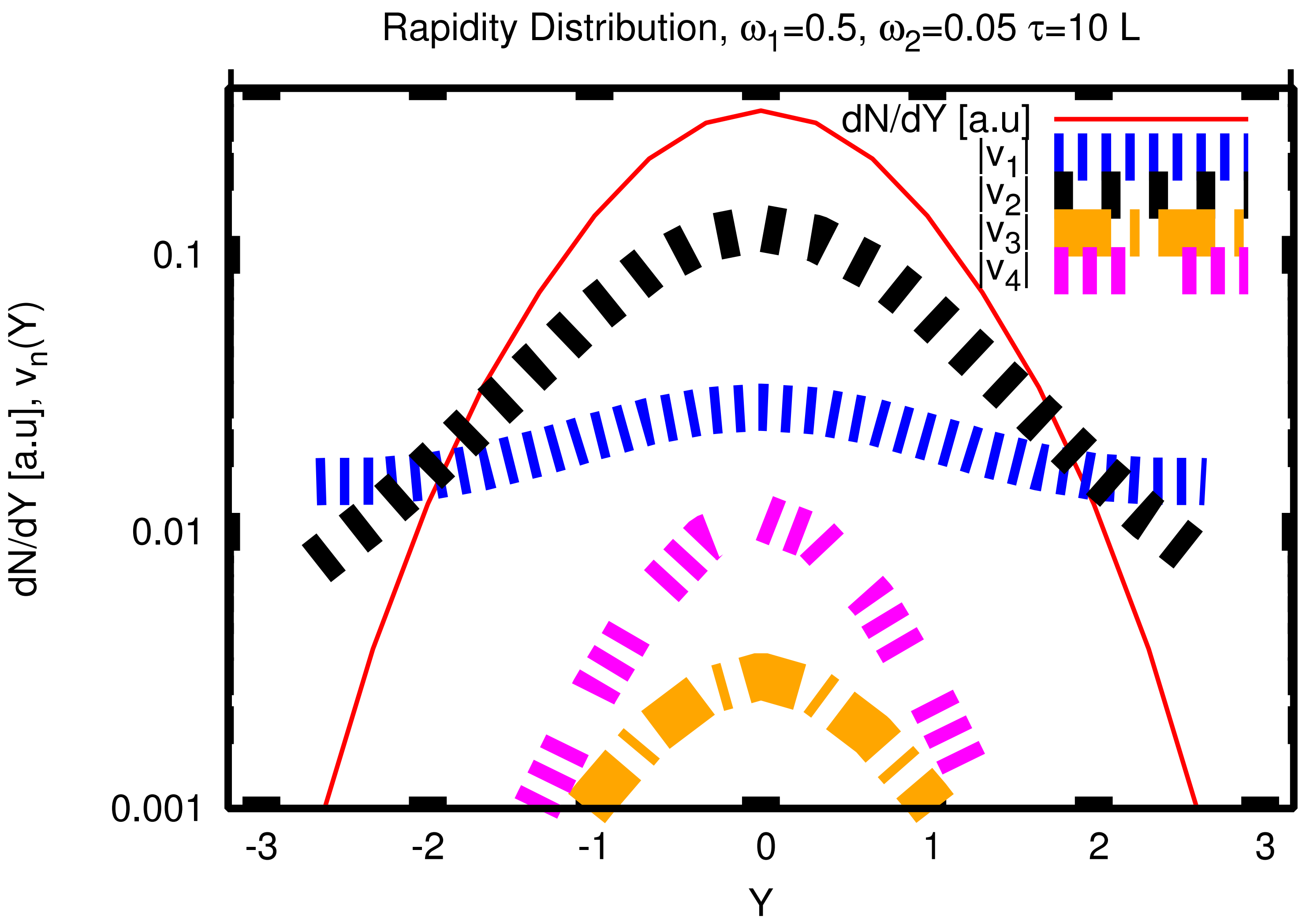
- Non-vanishing $v_1, v_2, v_3, v_4, \ldots$



Qualitative features of analytic solution: 3/3

- Non-vanishing $v_1, v_2, v_3, v_4, \ldots$
- Qualitatively reasonable rapidity-profile
- Quantitatively too narrow
- Generic feature of holographic collisions

  [Casalderrey-Solana, Heller, Mateos, van der Schee, 2013]



Summary and Outlook

- Analytic solutions for off-center holographic collisions
- Solutions qualitatively reasonable: longitudinal, direct, elliptic, triangular flow and vorticity
- Solutions indicate that while vorticity is non-vanishing, heavy-ion systems do not rotate

- Outlook: solutions have known gravity dual; allows calculation of entanglement entropy in heavy-ion collision via Ryu-Takayangi

Bonus Material

Myers-Perry Metric

$$\frac{ds_5^2}{L^2} = -\frac{\Delta(1+r^2)}{\Sigma_1\Sigma_2}dt'^2 + \frac{\rho^2}{V-2M}d\tilde{r}^2 + \frac{2M}{\rho^2}\left(\frac{\Delta}{\Sigma_1\Sigma_2}dt' - \frac{\omega_1 \sin^2\tilde{\theta}}{\Sigma_1}d\phi' - \frac{\omega_2\cos^2\tilde{\theta}}{\Sigma_2}d\chi'\right)^2$$
$$+\frac{\rho^2}{\Delta}d\tilde{\theta}^2 + \frac{r^2+\omega_1^2}{\Sigma_1}\sin^2\tilde{\theta}d\phi'^2 + \frac{r^2+\omega_2^2}{\Sigma_2}\cos^2\tilde{\theta}d\chi'^2\,,$$