Onset of final-state-collectivity signals in nuclear collisions

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Strong-interaction matter under extreme conditions

Onset of final-state-collectivity signals in nuclear collisions

- Introduction and motivation
- Model
  - One approach, many possible setups
- A few generic results
  - Scaling behaviors

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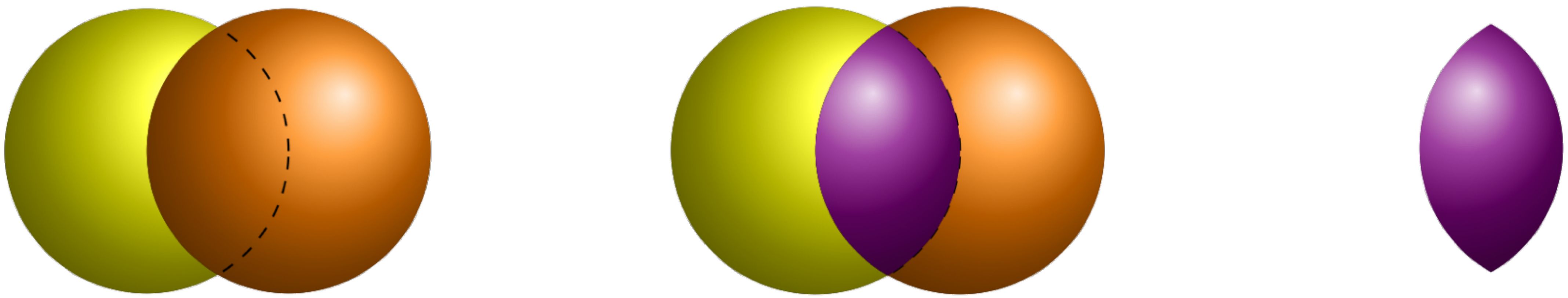
Preamble: Anisotropic collective flow

Classical idea (before ca.2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)



$t < 0$ $t = 0$ $t = 0^+$

pictures shamelessly stolen from Matt Luzum

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Classical picture (before ca.2010) in collisions of "large" systems:

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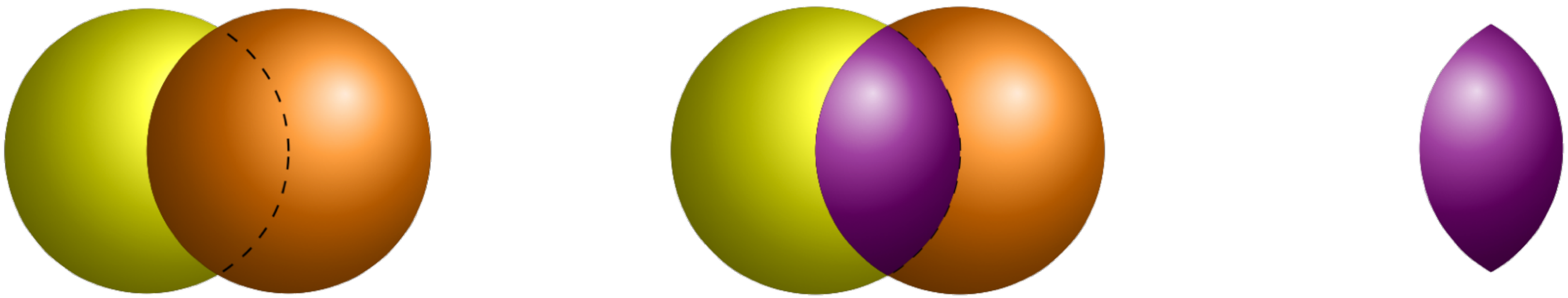
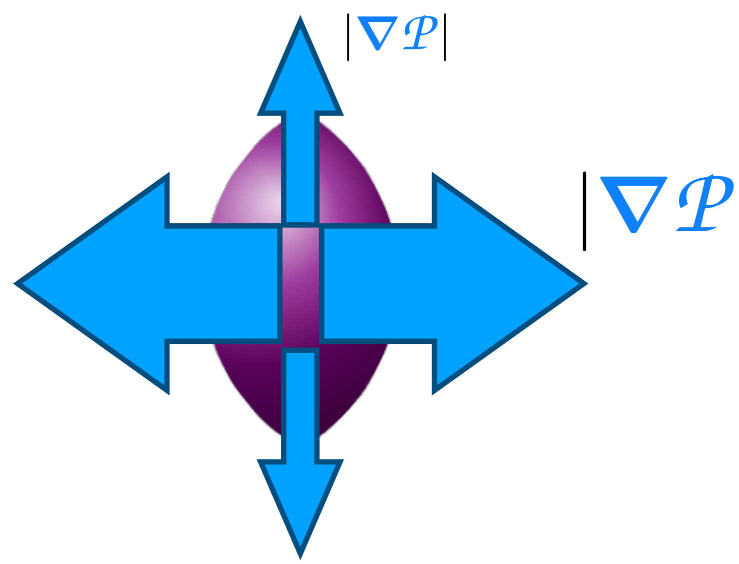


$t < 0$ $t = 0$ $t = 0^+$

$\Rightarrow$ larger pressure gradient along the impact parameter direction



$\Rightarrow$ larger fluid acceleration along the impact parameter direction

pictures shamelessly stolen from Matt Luzum

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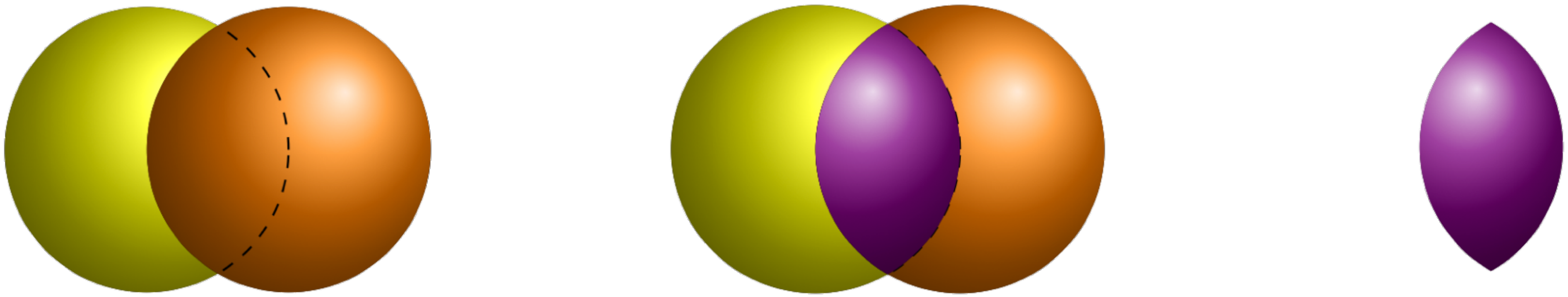
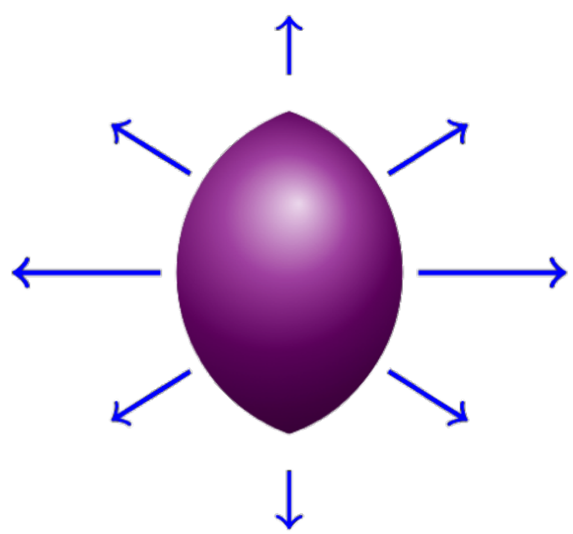
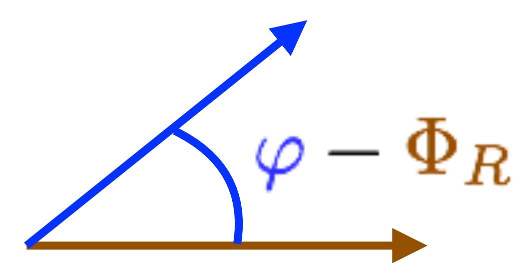


$t < 0$ $\quad$ $t = 0$ $\quad$ $t = 0^+$

$\Rightarrow$ anisotropic emission of particles





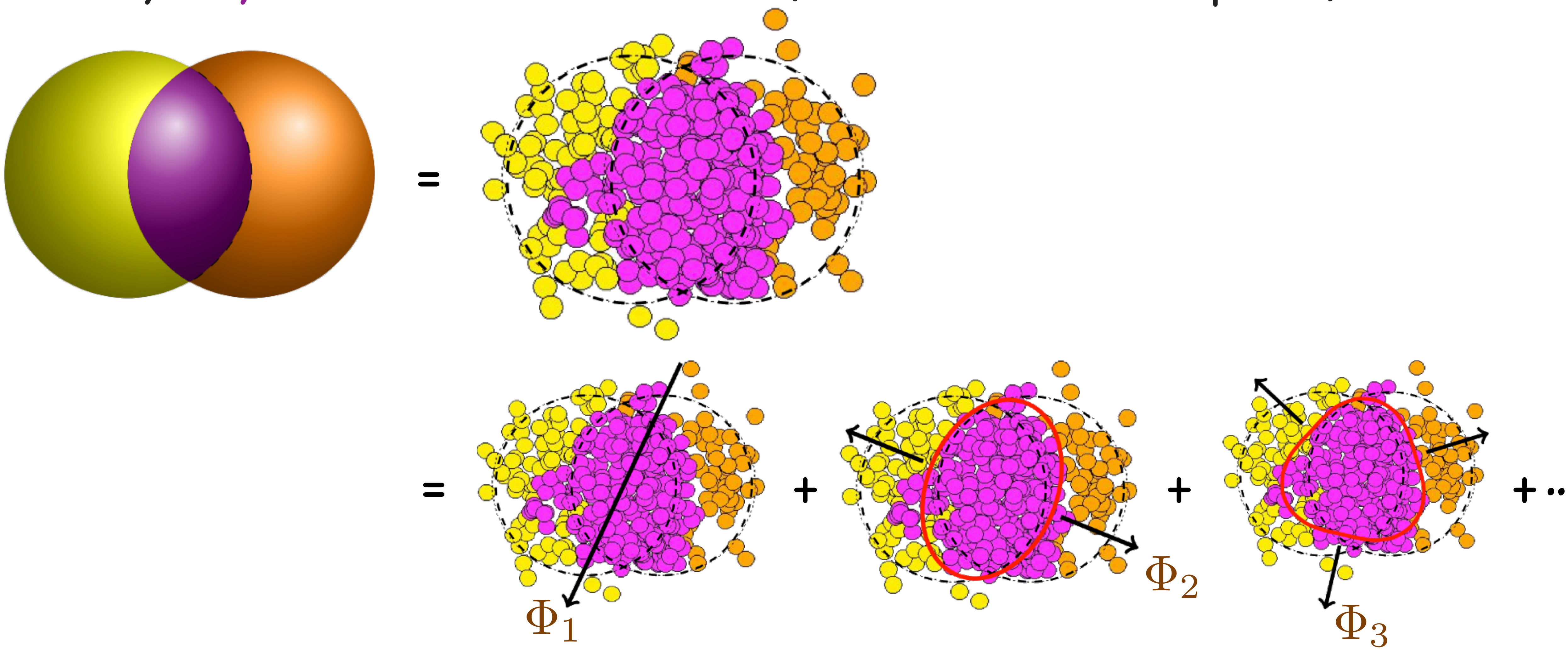
$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T\,\mathrm{d}y}\left[1 + 2v_1\cos(\varphi - \Phi_R) + 2v_2\cos 2(\varphi - \Phi_R) + \cdots\right]$$

pictures shamelessly stolen from Matt Luzum

Preamble: Anisotropic collective flow

Newer paradigm (since 2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)



$\Rightarrow$ anisotropic emission of particles

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T \mathrm{d}p_T \mathrm{d}y}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos n(\varphi - \Phi_n)\right]$$

pictures shamelessly stolen from Matt Luzum

Preamble: Anisotropic collective flow

The asymmetric initial-state (transverse) geometry, characterized by "eccentricities" ϵ_n:*

$$\epsilon_n\, \mathrm{e}^{\mathrm{i}n\Phi_n} \equiv -\frac{\langle r^n\, \mathrm{e}^{\mathrm{i}n\theta}\rangle_{\boldsymbol{r}}}{\langle r^n\rangle_{\boldsymbol{r}}}$$

is converted by the system evolution into final-state anisotropies in (transverse) momentum space, characterized by Fourier coefficients v_n.

$$v_n = f_n\big(\{\epsilon_m\}\big)$$

* and by other similar quantities

Anisotropic flow in fluid dynamics

$$v_n = f_n\big(\{\epsilon_m\}\big)$$

The "response functions" f_n still depend on the system size, but also on more intrinsic properties of the expanding fireball.

☞ in a (classical*) fluid-dynamical description of the fireball as a continuous medium, dependence on the equation of state and on the transport coefficients (η, ζ ...).

* i.e. relying on local thermodynamic equilibrium

Anisotropic flow in fluid dynamics

$$v_n = f_n\big(\{\epsilon_m\}\big)$$

Summary of findings within numerical fluid dynamical studies:*

- elliptic flow: $v_2 \simeq \mathcal{K}^{\rm hydro}_{2,2}\,\epsilon_2$
- triangular flow: $v_3 \simeq \mathcal{K}^{\rm hydro}_{3,3}\,\epsilon_3$
- quadrangular flow: $v_4 \simeq \mathcal{K}^{\rm hydro}_{4,22}\,\epsilon_2^2 + \mathcal{K}^{\rm hydro}_{4,4}\,\epsilon_4$
- pentagonal flow: $v_5 \simeq \mathcal{K}^{\rm hydro}_{5,23}\,\epsilon_2\epsilon_3 + \mathcal{K}^{\rm hydro}_{5,5}\,\epsilon_5$
- hexagonal flow: $v_6 \simeq \mathcal{K}^{\rm hydro}_{6,222}\,\epsilon_2^3 + \mathcal{K}^{\rm hydro}_{6,33}\,\epsilon_3^2 + \mathcal{K}^{\rm hydro}_{6,42}\,\epsilon_2\epsilon_4 + \mathcal{K}^{\rm hydro}_{6,6}\,\epsilon_6$

* discarding ϵ_1 and considering only the leading contributions

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* discarding $\epsilon_{1,4,5,6}$ and considering only the leading contributions in ϵ_2 and ϵ_3.

Anisotropic flow in fluid dynamics

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nonlinear flow!

* discarding $\epsilon_{1,4,5,6}$ and considering only the leading contributions in ϵ_2 and ϵ_3.

Anisotropic flow in fluid dynamics

$$v_n = f_n\big(\{\epsilon_m\}\big)$$

In a fluid dynamical description of the heavy-ion fireball evolution, one finds both linear and nonlinear responses to the initial eccentricities.

Intuition / prejudice: the nonlinear response terms should emerge more slowly(?), and thus possibly probe a different stage of the evolution.

(N.B. & J.-Y.Ollitrault 2005: nonlinearity as a freeze-out effect?)

☞ How does it look like in other approaches to the fireball dynamics?

$\Rightarrow$ in a kinetic description à la Boltzmann?

Further motivation for such models: data from "small systems"

An apology…

Onset of final-state-collectivity signals
in nuclear collisions

actually means

Anisotropic flow far from equilibrium

N.B., Steffen Feld & Nina Kersting, arXiv:1804.05729
N.Kersting & N.B., work (= more explicit calculations) in progress

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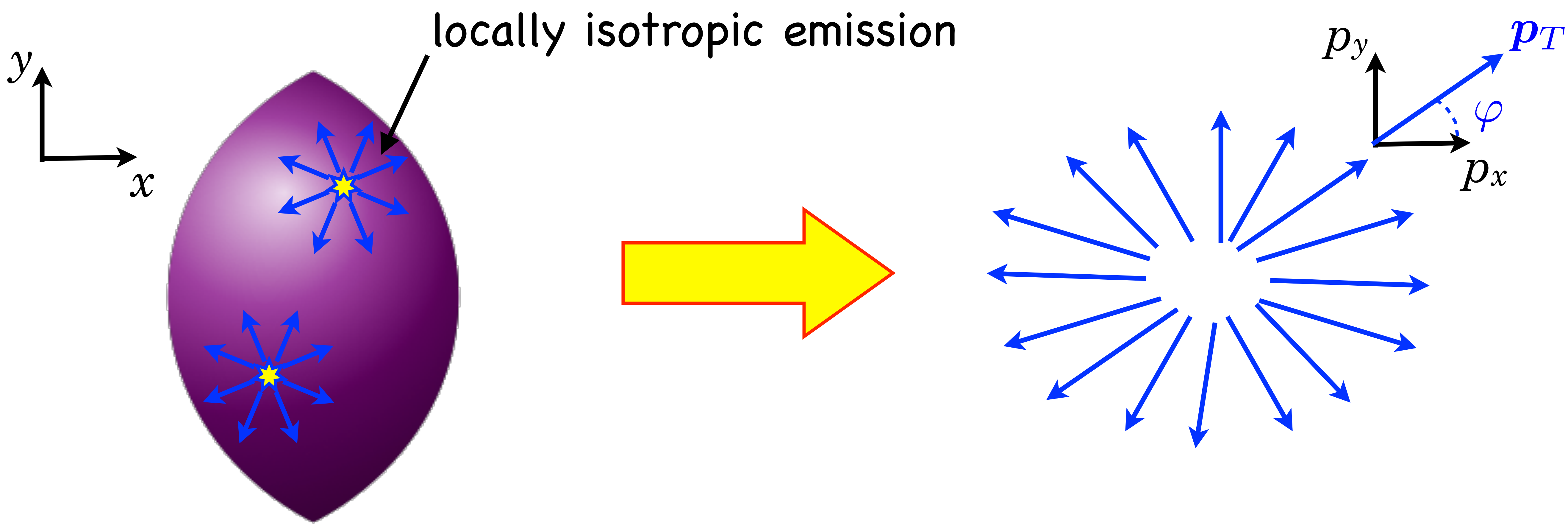
Anisotropic flow: the particle picture



In non-central nucleus-nucleus collisions, the initial spatial asymmetry of the overlap region in the transverse plane is converted by particle rescatterings into an anisotropic transverse-momentum distribution of the outgoing particles: anisotropic flow.

Kinetic theoretical approach

Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t,\mathbf{x},\mathbf{p})$
- total particle number $$\int_{\mathbf{x},\mathbf{p}} f(t,\mathbf{x},\mathbf{p}) = N(t)$$
- momentum distribution $$\int_{\mathbf{x}} f(t,\mathbf{x},\mathbf{p}) = E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}}(t,\mathbf{p})$$

$\Rightarrow$ anisotropic flow coefficients $v_n(t)$

final-state observables: let $t\to\infty$

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

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$\Rightarrow$ anisotropic flow coefficients $v_n(t)$

final-state observables: let $t\to\infty$

Focus on transverse anisotropic flow: consider 2-dimensional system

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

Kinetic theoretical approach

Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t, \mathbf{x}, \mathbf{p})$
- initial state: at some time t_0
  - eccentricities: "ellipticity" ϵ_2 , "triangularity" ϵ_3 ...
  - typical (transverse) size R

  $\Rightarrow$ typical particle number density $n \sim \dfrac{N}{R^2}$ (remember: 2D)

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

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Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t, \mathbf{x}, \mathbf{p})$

- initial state: at some time t_0

  - eccentricities: "ellipticity" ϵ_2 , "triangularity" ϵ_3 ...

  - typical size R ⇒ a possible setup (easy to work with analytically):

$$f(t_0, \mathbf{x}, \mathbf{p}) = F(\mathbf{p})\, \mathrm{e}^{-r^2/2R^2}\left[1 + \bar{\epsilon}_2\left(\frac{r}{R}\right)^2 \cos[2(\theta - \Phi_2)] + \bar{\epsilon}_3\left(\frac{r}{R}\right)^3 \cos[3(\theta - \Phi_3)]\right]$$

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

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proportional to ϵ_2 , ϵ_3

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require a cutoff

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

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assumed to be isotropic: focus on generated anisotropies

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

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assumed to be isotropic: focus on generated anisotropies

+ independent of position

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

Kinetic theoretical approach

Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t,\mathbf{x},\mathbf{p})$
- initial state: at some time t_0
- evolution equation, of the kinetic type

$$p_\mu \partial^\mu f(t,\mathbf{x},\mathbf{p}) = -\mathcal{C}_{\rm coll.}[f]$$

$\Rightarrow$ various possible choices for the collision term ("setups")

- 2-to-2 elastic (Boltzmann), $2 \longleftrightarrow 3$ scatterings
- include / ignore Pauli blocking / Bose-Einstein enhancement
- rescattering cross section σ.

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Warm up: free streaming

Assume first that there are no rescatterings, $\mathcal{C}_{\text{coll.}}[f] = 0$.

$\Rightarrow$ the evolution equation becomes $p_\mu \partial^\mu f(t, \mathbf{x}, \mathbf{p}) = 0$,

with free-streaming solutions $f^{(0)}(t, \mathbf{x}, \mathbf{p}) = f^{(0)}(t_0, \mathbf{x} - \mathbf{v}(t - t_0), \mathbf{p})$.

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integration over space

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}}(t, \mathbf{p})\right] = 0$$

Multiply by $\cos(n\varphi)$, average over φ, let $t \to \infty$...

$$v_n = v_n(t) = v_n(t_0)$$

Boring!

Few-scattering approach

Let us turn on particles rescatterings: $\mathcal{C}_{\rm coll.}[f] \neq 0$.

$\Rightarrow$ need an ansatz for the collision term (next-to-next slide)

If the number of rescatterings per particle is low enough, f should not differ much from the free-streaming solution $f^{(0)}$:

$$f = f^{(0)} + f^{(1)}$$

"low density limit": Heiselberg & Levy, PRC **59** (1999) 2716
"far from equilibrium": N.B. & Gombeaud, EPJC **71** (2011) 1612
"eremitic expansion", Romatschke, arXiv:1802.06804
"one hit dynamics", Kurkela, Wiedemann & Wu, arXiv:1803.02072

Note: (small) expansion parameter needed: wait till slide 20!

Few-scattering approach

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In the absence of initial momentum-space anisotropy, the generated anisotropic flow is due to $f^{(1)}$, i.e. to rescatterings.

Few-scattering approach

If the number of rescatterings per particle is low enough, f should not differ much from the free-streaming solution $f^{(0)}$:

$$f = f^{(0)} + f^{(1)}$$

Insert into the kinetic evolution equation:

$$p_\mu \partial^\mu f^{(1)}(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}_{\rm coll.}[f^{(0)} + f^{(1)}]$$

$$\simeq -\mathcal{C}_{\rm coll.}[f^{(0)}]$$

Integrate over $\mathbf{x}$, multiply by $\cos(n\varphi)$, average over φ

$\Rightarrow$ anisotropic flow growth rate $\dfrac{\mathrm{d}}{\mathrm{d}t} v_n(t)$

Few-scattering approach

Let us turn on particles rescatterings: $\mathcal{C}_{\rm coll.}[f] \neq 0$.

$\Rightarrow$ need an ansatz for the collision term

- relaxation time approximation $\mathcal{C}_{\rm coll.}[f] = -\dfrac{p^\mu u_\mu}{\tau_{\rm rel.}}\big(f - f_{\rm eq.}[f]\big)$

Romatschke, arXiv:1802.06804
Kurkela, Wiedemann & Wu, arXiv:1803.02072

(nonlinearities hidden in u_μ and $f_{\rm eq.}[f]$)

Few-scattering approach

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Kurkela, Wiedemann & Wu, arXiv:1803.02072

- collision integral, including a rescattering cross section, as e.g.

$$\mathcal{C}_{\rm coll.}[f(\mathbf{1})] = \int_{\mathbf{p_2},\mathbf{p_3},\mathbf{p_4}} \big[f(\mathbf{3})f(\mathbf{4})w(\mathbf{3}+\mathbf{4}\to\mathbf{1}+\mathbf{2}) - f(\mathbf{1})f(\mathbf{2})w(\mathbf{1}+\mathbf{2}\to\mathbf{3}+\mathbf{4})\big]$$

Heiselberg & Levy, PRC **59** (1999) 2716

N.B. & Gombeaud, EPJC **71** (2011) 1612

N.B., Feld & Kersting, arXiv:1804.05729

Few-scattering approach

If the number of rescatterings per particle is low enough, f should not differ much from the free-streaming solution $f^{(0)}$:

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What is the small parameter here?

Goal: small number of rescatterings per particle: $(\sigma R)\dfrac{N}{R^2} \ll 1$

$$N_{\text{resc.}} \sim \frac{N\sigma}{R} \ll 1$$

For an explicit calculation, take an initial configuration, a given collision integral (with the free-streaming solution), integrate over $\mathbf{x}$, t, and the still unspecified momentum.

Scaling behaviors

$$f = f^{(0)} + f^{(1)}$$

- The free-streaming solution $f^{(0)}$ propagates the initial distribution, and thus only contains linear terms in the eccentricities ϵ_n.

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$$p_\mu \partial^\mu f^{(1)}(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}_{\rm coll.}[f^{(0)} + f^{(1)}] \simeq -\mathcal{C}_{\rm coll.}[f^{(0)}]$$

- The first-order correction $f^{(1)}$
  - is linear in the cross section / the small parameter $N_{\rm resc.}$
  - contains nonlinear terms in the eccentricities ϵ_n.

$$\mathcal{C}_{\rm coll.}[f^{(0)}] \ni \int f^{(0)}(\mathbf{1}) f^{(0)}(\mathbf{2})\, \sigma \ni \int \epsilon_2 \cos(2\theta_1)\, \epsilon_2 \cos(2\theta_2)\, \sigma$$

Scaling behaviors

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$$\mathcal{C}_{\rm coll.}[f^{(0)}] \ni \int f^{(0)}(\mathbf{1}) f^{(0)}(\mathbf{2})\,\sigma \ni \int \underbrace{\epsilon_2 \cos(2\theta_1)\,\epsilon_2 \cos(2\theta_2)}_{\ni\, \epsilon_2^2 \cos\left(4\frac{\theta_1+\theta_2}{2}\right)}\,\sigma$$

will contribute to v_4 $\longrightarrow$ $\ni \epsilon_2^2 \cos\left(4\frac{\theta_1+\theta_2}{2}\right)$

Scaling behaviors

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- The first-order correction $f^{(1)}$
  - is linear in the cross section / the small parameter $N_{\rm resc.}$
  - contains $\underbrace{\text{linear and quadratic}}_{}$ terms in the eccentricities ϵ_n.

if the collision integral is quadratic in f

(as in the Boltzmann ansatz)

Anisotropic flow far from equilibrium

Summary of findings with the Boltzmann collision kernel:

- elliptic flow: $v_2 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2$
- triangular flow: $v_3 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3$
- quadrangular flow: $v_4 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2^2$
- pentagonal flow: $v_5 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2\epsilon_3$
- hexagonal flow: $v_6 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3^2$

Anisotropic flow far from equilibrium vs. in fluid dynamics

- elliptic flow: $v_2 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2$ | $v_2 \simeq \mathcal{K}^{\rm hydro}_{2,2}\,\epsilon_2$
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$$\underbrace{\phantom{v_6 \simeq \mathcal{K}^{\rm hydro}_{6,222}\,\epsilon_2^3 + \mathcal{K}^{\rm hydro}_{6,33}\,\epsilon_3^2}}_{\text{reminder from slide 7}}$$

Anisotropic flow far from equilibrium vs. in fluid dynamics

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- pentagonal flow: $v_5 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2\epsilon_3$ $\quad\vdots\quad$ $v_5 \simeq \mathcal{K}^{\rm hydro}_{5,23}\,\epsilon_2\epsilon_3$
- hexagonal flow: $v_6 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3^2$ $\quad\vdots\quad$ $v_6 \simeq \boxed{\mathcal{K}^{\rm hydro}_{6,222}\,\epsilon_2^3} + \mathcal{K}^{\rm hydro}_{6,33}\,\epsilon_3^2$

not there at order $\mathcal{O}(N_{\rm resc.})$

☞ As anticipated(?), some of the nonlinear response terms seem to require more time (rescatterings) to be generated!

v_6 far from equilibrium

What about the contribution of order ϵ_2^3 to v_6?

Write $f = f^{(0)} + f^{(1)} + f^{(2)}$, insert into the evolution equation

$$p_\mu \partial^\mu \big[f^{(1)} + f^{(2)}\big] = -\mathcal{C}_{\rm coll.}[f^{(0)}+f^{(1)}+f^{(2)}]$$

$$\simeq -\mathcal{C}_{\rm coll.}[f^{(0)}+f^{(1)}]$$

$$= \underbrace{-\mathcal{C}_{\rm coll.}[f^{(0)}]}_{\text{yields } f^{(1)}} - \underbrace{\Big(\mathcal{C}_{\rm coll.}[f^{(0)}+f^{(1)}] - \mathcal{C}_{\rm coll.}[f^{(0)}]\Big)}_{\text{The terms } \mathcal{O}(N_{\rm resc.}^2) \text{ yield } f^{(2)}}$$

$f^{(2)}$ does contain a term in ϵ_2^3

☞ $v_6 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3^2 + \mathcal{O}(N_{\rm resc.}^2)\,\epsilon_2^3$ nontrivial?

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- include Pauli blocking / Bose–Einstein enhancement factors

$$f(\mathbf{1})f(\mathbf{2})[1 \pm f(\mathbf{3})][1 \pm f(\mathbf{4})]$$

  relevant only for a dense system

☞ Does one remain in the "small number of rescatterings" regime?

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- Are they confirmed by numerical studies?
  - Ultimately, how does the scaling with the number of rescatterings manifest themselves in the fluid-dynamical regime: dependence on viscosity of the proportionality coefficients $\mathcal{K}_{n,..}^{\rm hydro}$?
- Are they of any relevance for experimental data?
  - Small systems
  - Pre-hydrodynamization stage in heavy ion collision.

... And another thing...

An unpleasant curiosity?

A simple setup:

$$f(t_0,\mathbf{x},\mathbf{p}) = F(\mathbf{p})\,\mathrm{e}^{-r^2/2R^2}\left[1+\bar{\epsilon}_2\left(\frac{r}{R}\right)^2\cos[2(\theta-\Phi_2)]+\bar{\epsilon}_3\left(\frac{r}{R}\right)^3\cos[3(\theta-\Phi_3)]\right]$$

for massless particles with $2 \longleftrightarrow 2$ elastic scatterings, with an isotropic cross section.

Leads to finite v_2, v_4, v_6 but vanishing v_3, v_5 at order $\mathcal{O}(N_{\rm resc.})$...

… And another thing…

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The calculation involves that of the integral

$$\int f^{(0)}(t_0,\mathbf{x}-\mathbf{v}_1(t-t_0),\mathbf{p}_1)\,f^{(0)}(t_0,\mathbf{x}-\mathbf{v}_2(t-t_0),\mathbf{p}_2)\,\mathrm{d}^2\mathbf{x} = \int f^{(0)}(t_0,\mathbf{x},\mathbf{p}_1)\,f^{(0)}(t_0,\mathbf{x}-\mathbf{X},\mathbf{p}_2)\,\mathrm{d}^2\mathbf{x}$$

where $\mathbf{X} = (\mathbf{v}_2-\mathbf{v}_1)(t-t_0)$

A term linear in $\bar{\epsilon}_3$ comes with an odd contribution to the integrand, thus does not contribute to the integral… Leading to no v_3.

But do I understand the physics here?