# Dynamics, phase transitions and holography

Jakub Jankowski

with R. A. Janik, H. Soltanpanahi

Phys. Rev. Lett. **119**, no. 26, 261601 (2017)

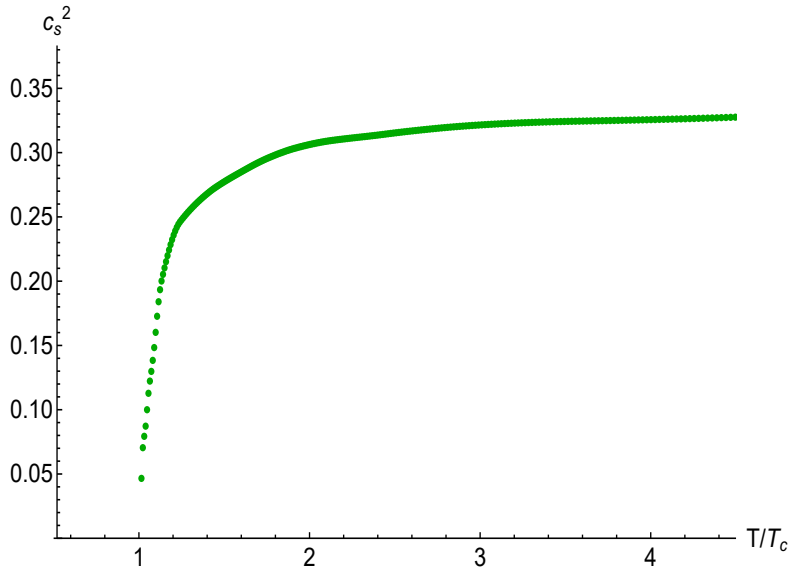
Faculty of Physics, University of Warsaw

NARODOWE CENTRUM NAUKI

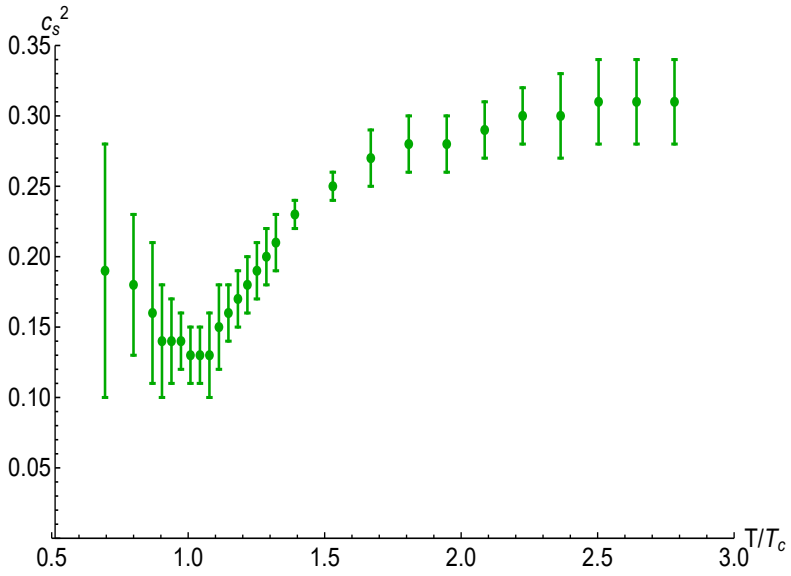
UNIVERSITY OF WARSAW

# Phase structure at strong coupling

- Systems at strong coupling exhibit various phase structures
- Pure gluon system $\longrightarrow$ $1^{\mathrm{st}}$ order phase transition (left)
- Gluons + quarks $\longrightarrow$ smooth crossover (right)




G. Boyd *et al.* Nucl. Phys. B **469**, 419 (1996)




S. Borsanyi *et al.* JHEP **1009**, 073 (2010)

# Phase structure at strong coupling

- Lattice methods do not reach real time dynamics easily
- Use other methods to model strongly coupled phase transitions
- Compute the spectrum of linearized perturbations
- Compute transport coefficients and non-hydrodynamic modes
- Compute the non-linear time evolution
- Investigate the dynamical appearance of diverse phases
- Check linear and non-linear stability

**Method:**

Use a string theory based approach to formulate models at strong coupling.

# Questions

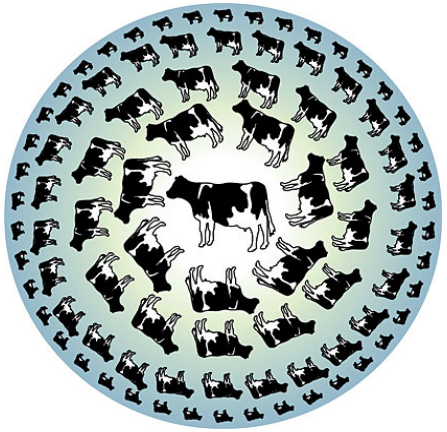
- Does spinodal instability appear for a holographic system with a $1^{\mathrm{st}}$ order phase transition?
- Does the phase separation effect appear dynamically?
- Are there black hole solution with inhomogeneous horizons?
- How do non-hydrodynamic degrees of freedom behave in the critical region?
- Do diffusive modes appear?

**Method:**

Use a string theory based approach to formulate models at strong coupling.

# Holography and Quantum Field Theory

- Holographic principle
  Quantum gravity in $d$ dimensions must have a number of DOF which scales like that of QFT in $d-1$ dimensions
  't Hooft and Susskind '93



- String Theory realization: *AdS/CFT correspondence*
  Theory is *conformal* and *supersymmetric* Maldacena '97
- Extensions to *non-supersymmetric* and *non-conformal* field theories are possible
- Applications: elementary particle physics and condensed matter physics

# Non-conformal holographic plasma

- Top-down construction

  $\mathcal{N} = 4$ broken to $\mathcal{N} = 2^*$ SUSY theory. Known, but complicated dual gravity description

  A. Buchel, S. Deakin, P. Kerner, J. T. Liu, Nucl. Phys. B **784**, 72 (2007)

- Bottom-up construction

  Assuming AdS/CFT dictionary, try to model gravity+matter background to approach as closely as possible to your favourite physics

  U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
  S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

# Holographic bottom-up construction

- **Boundary:** add a source for an operator $O_\phi$ in a $\mathrm{CFT_d}$

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + \Lambda^{d-\Delta} O_\phi$$

- **Bulk:** a gravity-scalar system in $D = d + 1$

$$S = \frac{1}{2\kappa_D^2} \int_{\mathcal{M}} d^D x \sqrt{-g} \left[ R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] + S_{\mathrm{GH}} + S_{\mathrm{ct}}$$

with the potential

$$V(\phi) = 2\Lambda_C \left(1 + a\,\phi^2\right)^{1/4} \cosh(\gamma\,\phi) + b_2\,\phi^2 + b_4\,\phi^4 + b_6\,\phi^6$$

- $\Lambda_C = -d(d-1)/2$ is the cosmological constant

U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

# Phase transitions in holography

- Finite $T$ sates $\longleftrightarrow$ black hole solutions in the dual spacetime
- Phase structure is determined by the choice of $a, \gamma$ and $b_2,\ b_4,\ b_6$, coefficients of $V(\phi)$
- With $a \neq 0$ confinig models (IHQCD)
- It is possible to tune parameters to mimic

  $\rightarrow$ crossover e.g. QCD

  $\rightarrow$ $1^{\mathrm{st}}$ order phase transition e.g. pure gluon systems

  $\rightarrow$ $2^{\mathrm{nd}}$ order phase transition

U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

# Equilibrium configurations

- Metric ansatz for a homogeneous configuration

$$ds^2 = e^{2A(r)}\left(-h(r)dt^2 + d\vec{x}^2\right) - 2e^{A(r)+B(r)}drdt$$

  with $\phi(r) = r$ the holographic coordinate
- Solve Einstein + matter equations
- The event horizon: $h(r_H) = 0$
- Entropy and Hawking temperature

$$s = \frac{2\pi}{\kappa_D^2} e^{(d-1)A(r_H)} \qquad T = \frac{e^{A(r_H)+B(r_H)}|V'(r_H)|}{4\pi}$$

U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

# Equation of State (EoS)

- The free energy is defined by the action

$$F = TS_{\mathrm{on-shell}}$$

- Energy density is defined by holographic renormalization

H. Elvang, M. Hadjiantonis, JHEP **1606**, 046 (2016)

- Configurations characterized by **the horizon radius**
- Condition for the $1^{\mathrm{st}}$ order phase transition

$$F_{\mathrm{BH}_1} = F_{\mathrm{BH}_2}$$

- Similar to Hawking-Page transition of pure *AdS*

S. W. Hawking, D. N. Page, Commun. Math. Phys. **87**, 577 (1983)

E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998)

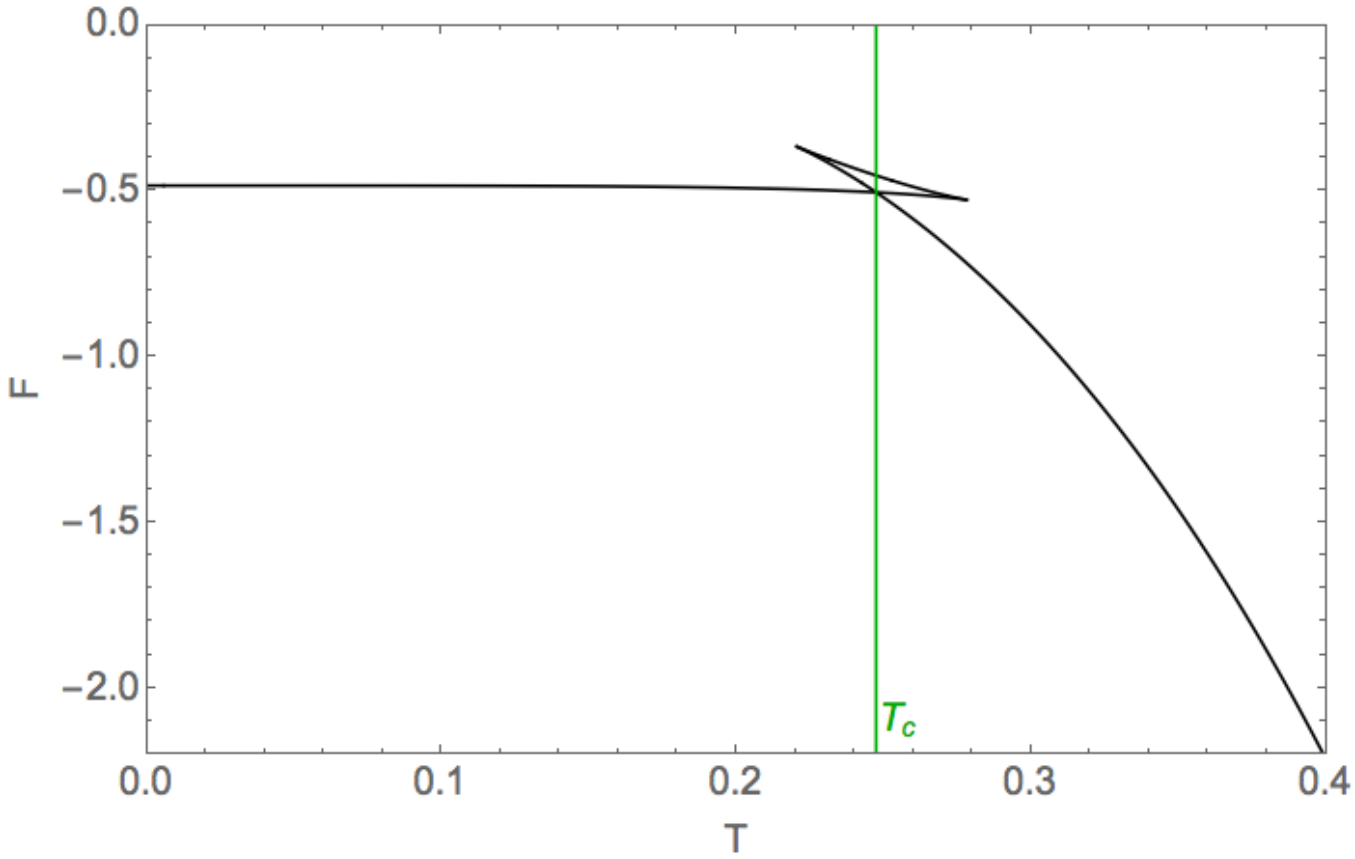
# Example: Transition in $2+1$ dimensions

- In $d = 2+1$ we choose

$$V(\phi) = -6\,\cosh\left(\phi/\sqrt{3}\right) - 0.2\,\phi^4$$

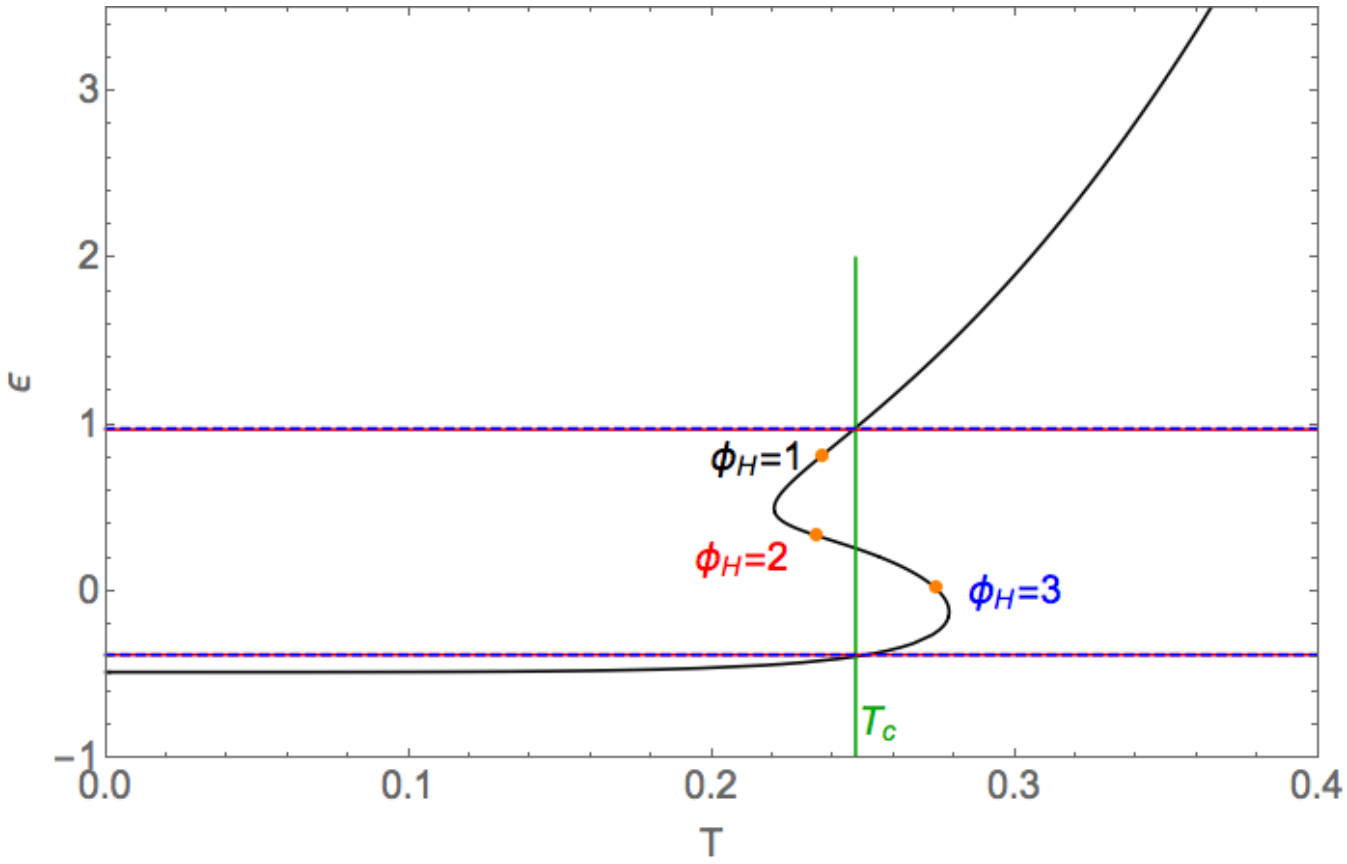
- Configurations characteristics $\phi(z=1) = \phi_H$
- Conformal dimension of the scalar operator is $\Delta = 2$
- Transition between two different black hole solutions
- Transition condition $F_{\mathrm{BH}_1} = F_{\mathrm{BH}_2}$
- $T_c = 0.246$ in $\Lambda = 1$ units

# Example: Free Energy




$T_c = 0.246$ in $\Lambda = 1$ units

# Example: Energy Density




$\phi_H = 1 \rightarrow$ stable configuration

$\phi_H = 2$ and $\phi_H = 3 \rightarrow$ unstable configurations

# Spinodal instability

- When $c_s^2 < 0$ we have purely damped hydro-modes

$$\omega \approx \pm i|c_s|\,k - \frac{i}{2\,T}\left(\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s}\right)k^2 = \pm i|c_s|\,k - i\Gamma_s k^2$$

  so for small enough $k$ we have $\mathrm{Im}\ \omega > 0$

- This mode is present for a finite range of $0 < k < k_{\max}$
- The maximum momentum for the unstable mode is $k_{\max} = |c_s|/\Gamma_s$
- This appears for systems with a $1^{\mathrm{st}}$ order phase transition; *spinodal* instability

P. Chomaz, M. Colonna, J. Randrup, Phys. Rept. **389**, 263 (2004)

# Examples of spinodal instabilities

- Water: superheated liquid and supercooled vapour


- Spinodal instability in nuclear matter liquid-gas transition
  Nuclear multifragmentation: Xe+Sn @ 32 MeV/A

  B. Borderie *et al.* Phys. Rev. Lett. **86**, 3252 (2001)

  P. Chomaz, M. Colonna, J. Randrup, Phys. Rept. **389**, 263 (2004)

# Time dependent set-up

- The metric ansatz Eddington-Finkelstein coordinates

$$ds^2 = -A\,dt^2 - \frac{2\,dt\,dz}{z^2} - 2\,B\,dt\,dx + S^2\,\left(G\,dx^2 + G^{-1}\,dy^2\right)$$

with $0 \leq z \leq 1$ and $x$ periodic

- Initial state in the *spinodal* region with x-dependent perturbations

$\rightarrow$ single mode

$$\delta S(t,x,z) = S_0\,z^2\,(1-z)^3\,\cos(kx)$$

$\rightarrow$ Gaussian

$$\delta S(t,x,z) = S_0\,z^2\,(1-z)^3\,\exp\left(-w_0\cos^2\left(\tilde{k}x\right)\right)$$

# Technicalities

- Characteristic formulation of Einstein's equations

  P. M. Chesler, L. G. Yaffe, JHEP **1407**, 086 (2014)

- Chebyshev and Fourier spectral methods; ABM4 time evolution

  P. Grandclement, J. Novak, Living Rev. Rel. **12**, 1 (2009)

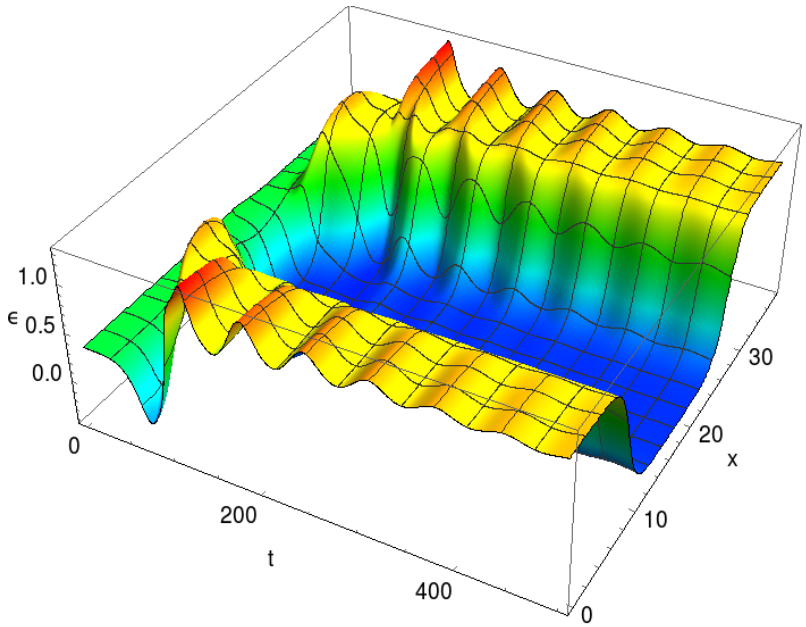
- We use $k = 1/6$ and $\tilde{k} = 1/12$ and $S_0 = 0.1 - 0.5$, $w_0 = 10$

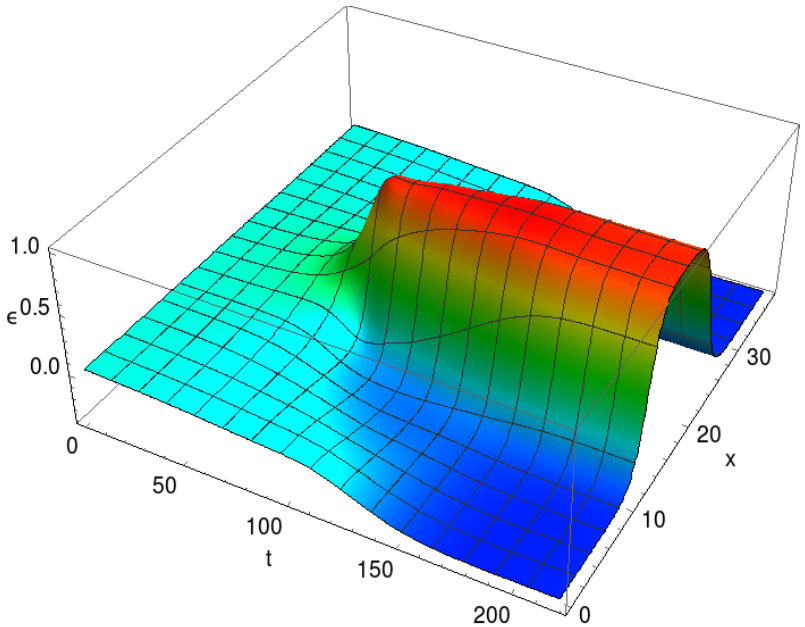
- $\epsilon$ and $\langle O_\phi \rangle$ are defined by holographic renormalization

  H. Elvang, M. Hadjiantonis, JHEP **1606**, 046 (2016)

- Results in this talk are with $x$ period $12\pi$

# Time dependence of the energy density




$\phi_H = 2$ Gaussian perturbation $\phi_H = 3$ cosine perturbation

- Flat domains of specific thermodynamic phase
- Narrow domain walls
- Stabilization time $t \sim 300$ simulation units

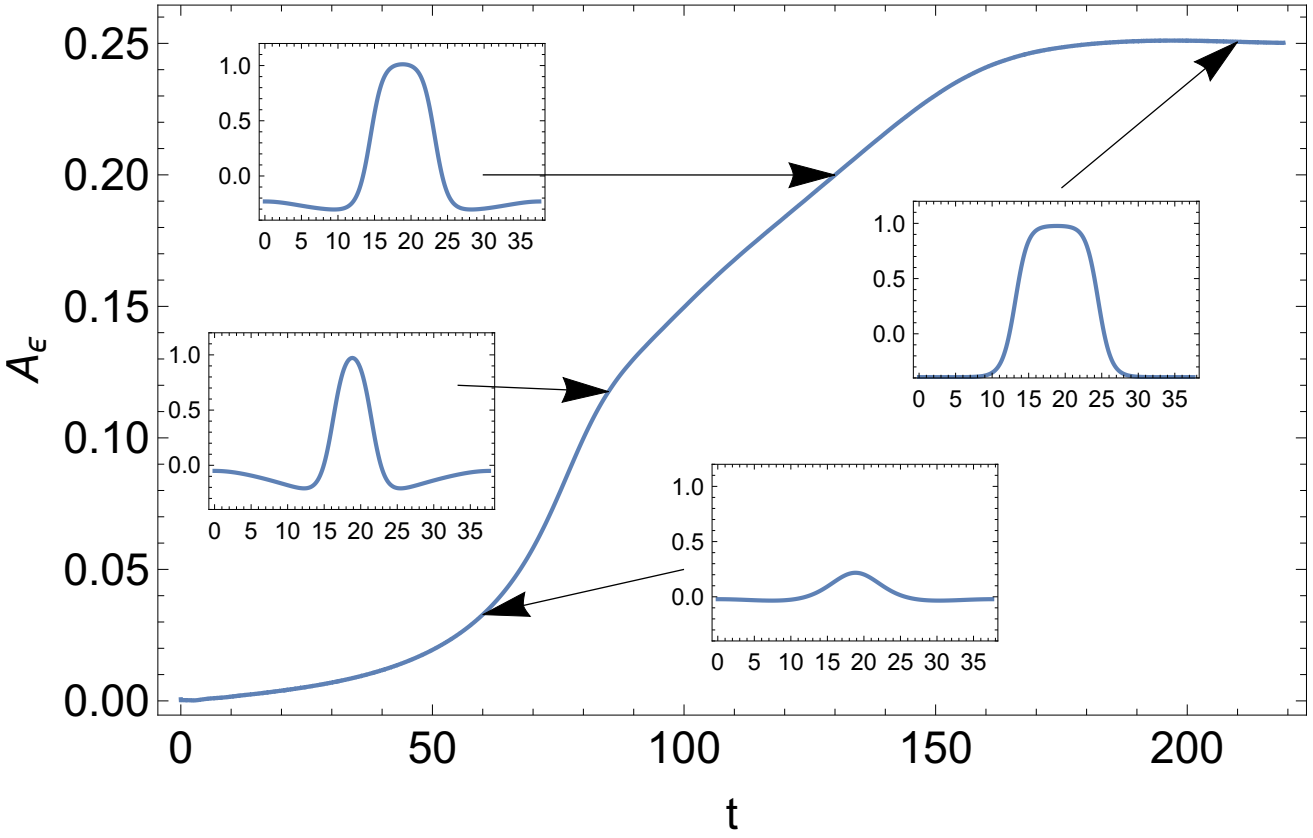
# Generic aspects of results

- Energy conservation (microcanonical ensemble)
- $\phi_H = 1$ is a stable configuration
- Inhomogeneous final state $\rightarrow$ domains of different phases
- To quantify the evolution we use

$$A_\epsilon(t) = \frac{1}{12\pi} \int_{\epsilon > \epsilon_0} \epsilon(t, x) dx$$

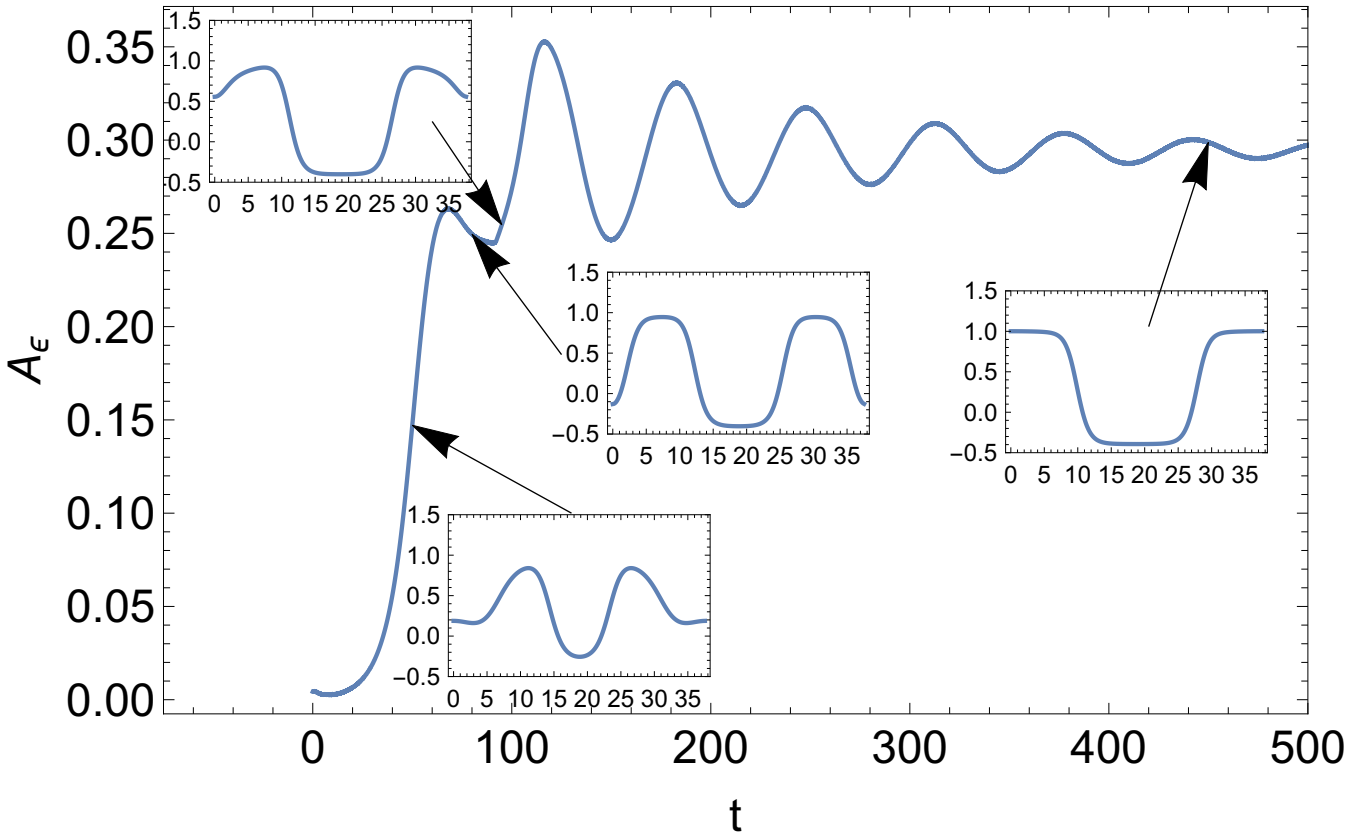
where $\epsilon_0$ - mean energy of the system at $t = t_0$.

# Energy density $\phi_H = 3$ case



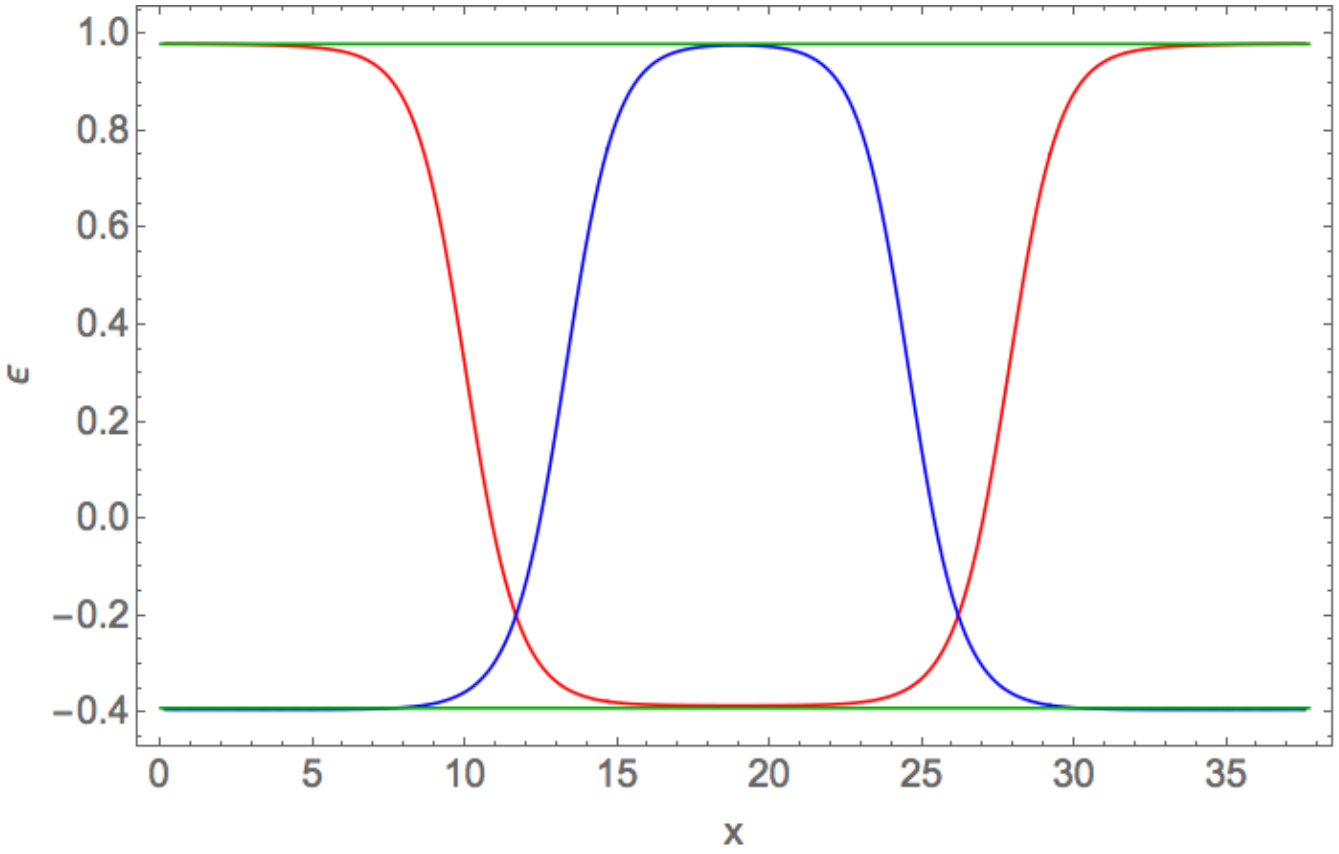

- Single mode perturbation
- Three regions of evolution (exponential, linear, saturation)

# Energy density $\phi_H = 2$ case




- Gaussian perturbation
- Formation and subsequent collision of two bubbles

# Final state energy density




- $\phi_H = 2$ (red) and $\phi_H = 3$ (blue)
- Uniform Hawking temperature $T = T_c$
- Universal shape of the domain walls

# Summary

- Non-trivial phase structure limits the applicability of hydrodynamics
- Phase separation effect in the context of holographic models
- Large black holes with $T < T_c$ are stable against perturbations
- A large class of inhomogeneous black hole solutions
- Spontaneous breakdown of translational invariance
  $\rightarrow$ holographic model for a lattice

A. Donos, J. P. Gauntlett, JHEP **1404**, 040 (2014)

G. T. Horowitz, J. E. Santos, D. Tong, JHEP **1207**, 168 (2012)

# Some open directions

- Extensions to higher dimensions
- Relaxation of symmetry assumptions
- Detailed study of various temporal regimes
- Conserved charges fluctuations
- Applicability of hydrodynamics at late times

M. Attems, *et.al* JHEP **1706**, 129 (2017)