



# Non-conformal holographic fluid relaxation

Maximilian Attems

[arXiv:1603.01254](https://arxiv.org/abs/1603.01254)

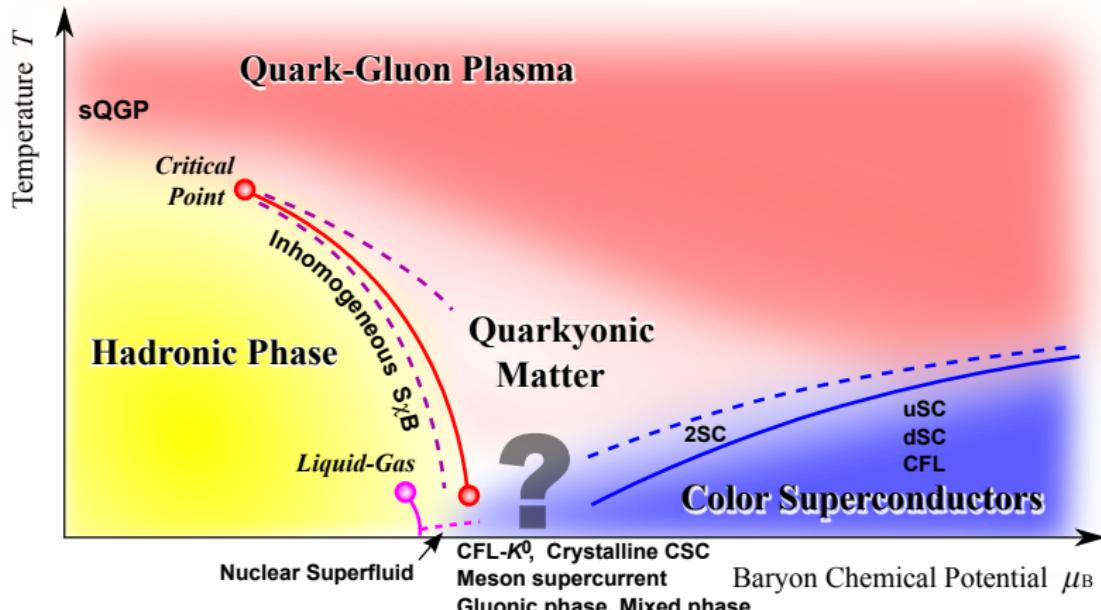
[arXiv:1703.09681](https://arxiv.org/abs/1703.09681)

[arXiv:{1805,1806}.XXXXXX](#)

Collaborators:

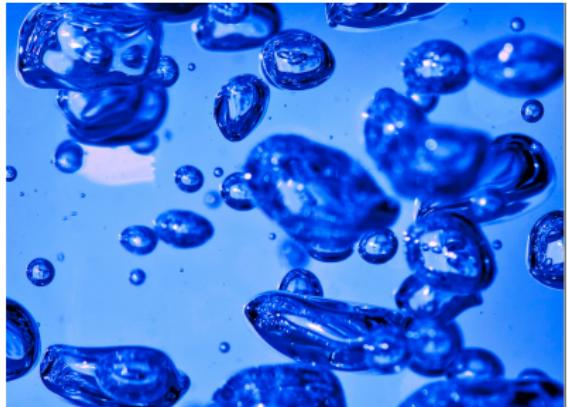
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David Mateos (UB), Miguel Zilhao (CENTRA)

Foundational aspects of relativistic hydrodynamics  
ECT\*, 7th May 2018



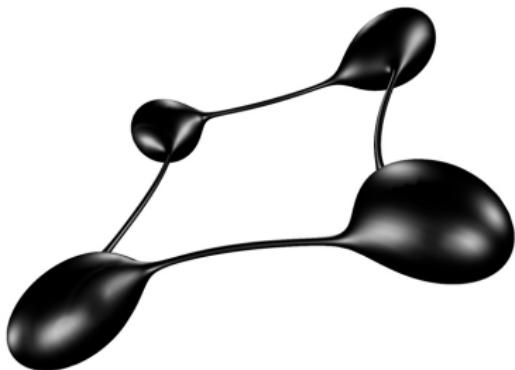
QCD phase diagram [Fukushima, Hatsudo 2010]

Phase transition:



Vaporization

Gregory-Laflamme Instability:



Black ring pinching off

gauge/gravity correspondence:

bridge between physical phenomena in gauge theories and gravity.

## Excitation and saturation of the spinodal instability

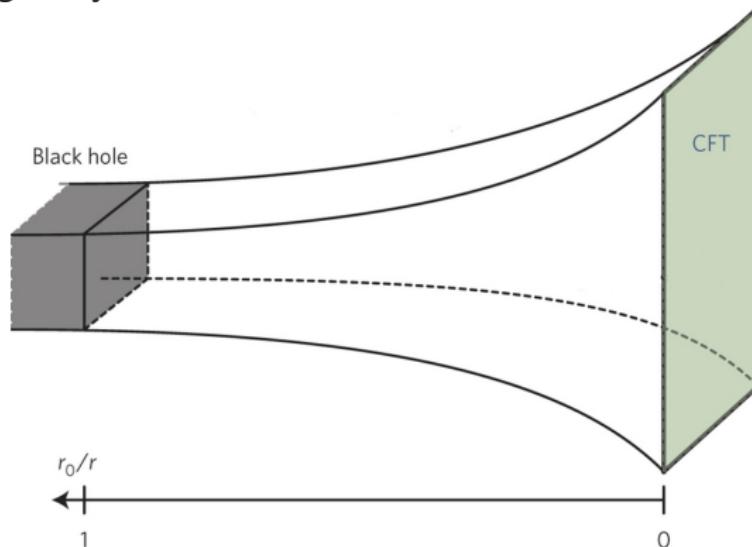
- Spinodal instability
- Inhomogeneous Horizon
- Hydrostatic + Hydrodynamic evolution
- Phase separation
- Phase mergers

## Shockwave collisions near Critical Point

- Introduction Hydrodynamics and viscosities
- Introduction shockwave collisions
- Hydrodynamization with bulk viscosity
- Collisions across a 1st-order phase transition
- From 1st-order to 2nd-order to crossover

# Introduction gauge gravity duality

Quantum gravity in  $d + 1$  dimension AdS  $\leftrightarrow$  QFT in  $d$  dimension



IIB string theory on  $\text{AdS}_5 \times \text{S}_5 \leftrightarrow \mathcal{N} = 4$  Super-Yang-Mills  
[Maldacena 1998, Witten 1998]

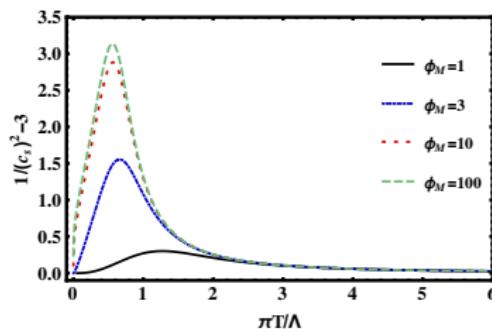
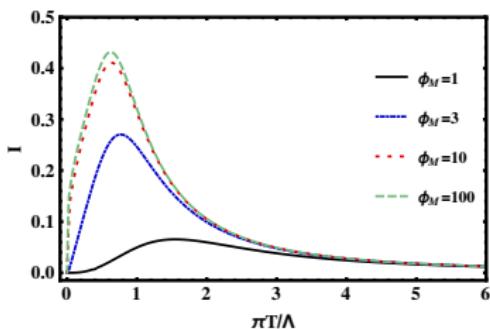
shear viscosity over entropy density ratio  $\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$   
[Policastro, Son, Starinets 2001; Kovtun, Son, Starinets 2005]

# Non-conformal General Relativity setup

Einstein-Hilbert action coupled to a scalar with non-trivial potential in five-dimensional bottom-up model:

$$S = \frac{2}{\kappa_5^2} \int d^5x \sqrt{-g} \left[ \frac{1}{4} \mathcal{R} - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right]$$

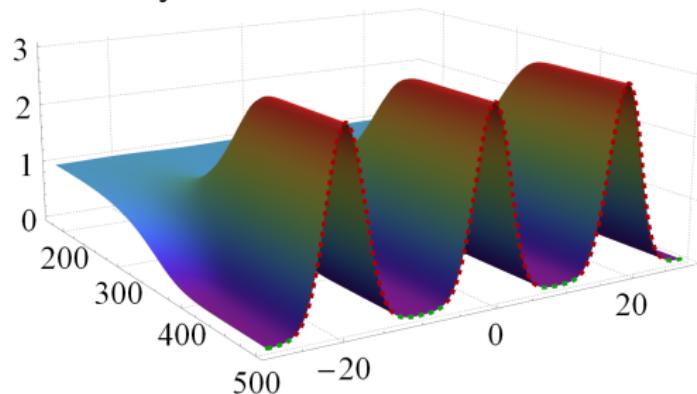
$$V(\phi) = -\frac{1}{12\phi_M^4}\phi^8 + \left( \frac{1}{2\phi_M^4} \pm \frac{1}{3\phi_M^2} \right) \phi^6 - \frac{1}{3}\phi^3 - \frac{3}{2}\phi^2 - 3$$



**NON**-conformal at intermediate  $T$ , conformal at  $IR$  and  $UV$

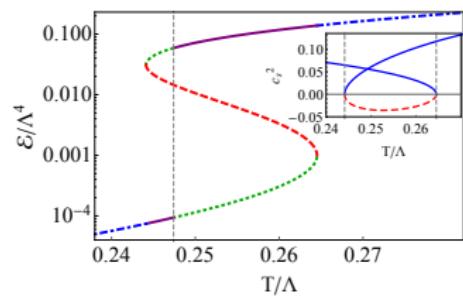
# Spinodal instability

Energy density evolution of black branes afflicted by the Gregory-Laflamme instability:



excited unstable mode growth until non-linear saturation

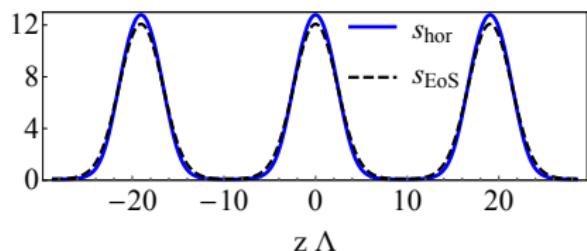
Energy density versus temperature for the gauge theory:



The dashed red curve is locally unstable, the dotted green curve metastable.

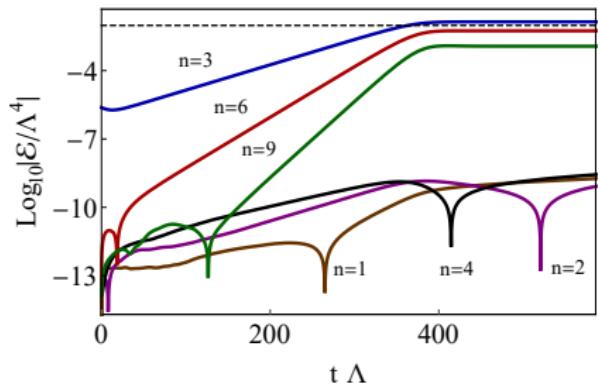
# Inhomogeneous Horizon

Final entropy density



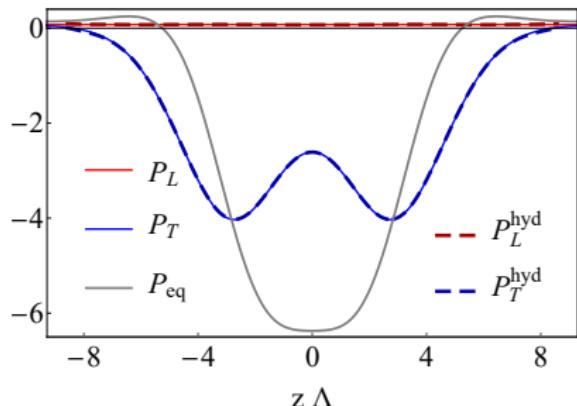
showing inhomogenous horizon  
(not just the boundary energy  
density)

Evolution of Fourier modes of the local energy density



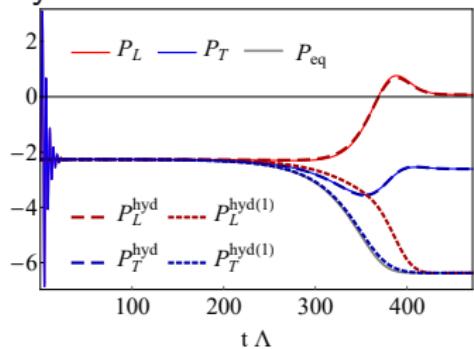
# Hydrostatic + Hydrodynamic evolution

Hydro description  $P_{L/T}^{\text{hyd}} = P_{\text{eq}}(\mathcal{E}) + c_{L/T}(\mathcal{E})(\partial_z \mathcal{E})^2 + f_{L/T}(\mathcal{E})(\partial_z^2 \mathcal{E})$



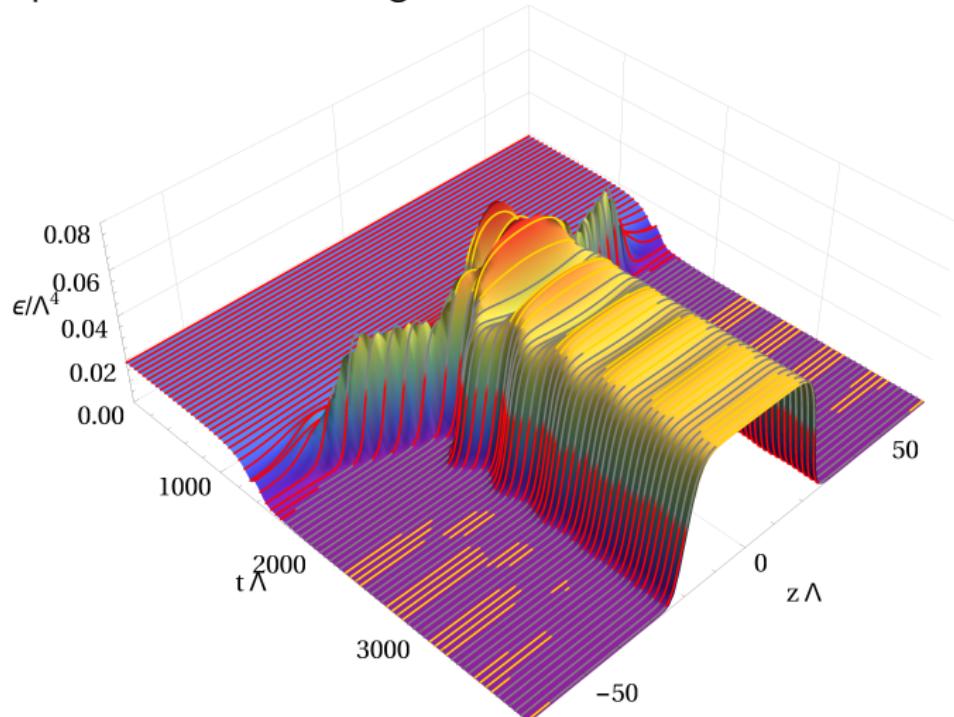
Pressures agree with hydrodynamic prediction for a different state

Pressures predicted by hydro match:



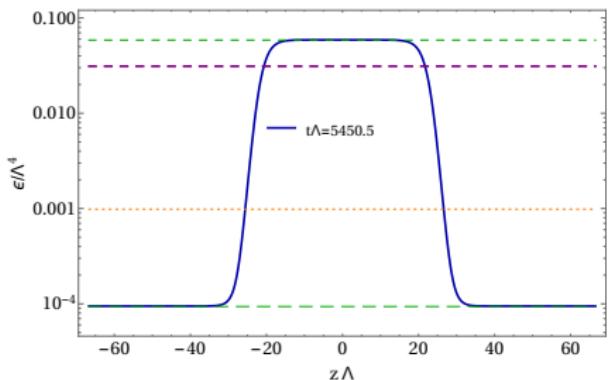
Early time behaviour with exponential decay of quasi-normal modes

Bigger periodic box with single excited mode:

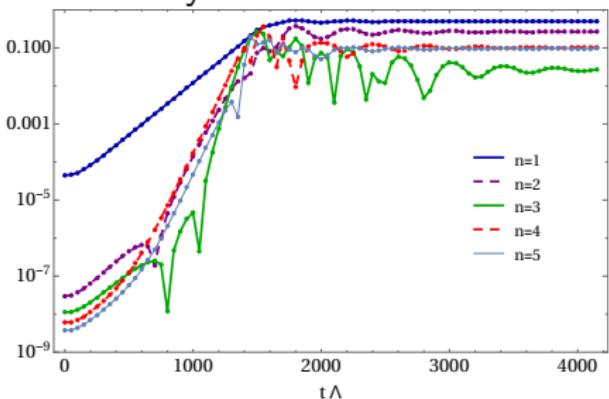


Cooled and hot regions are on the respective stable phase

Final static state:



mode analysis

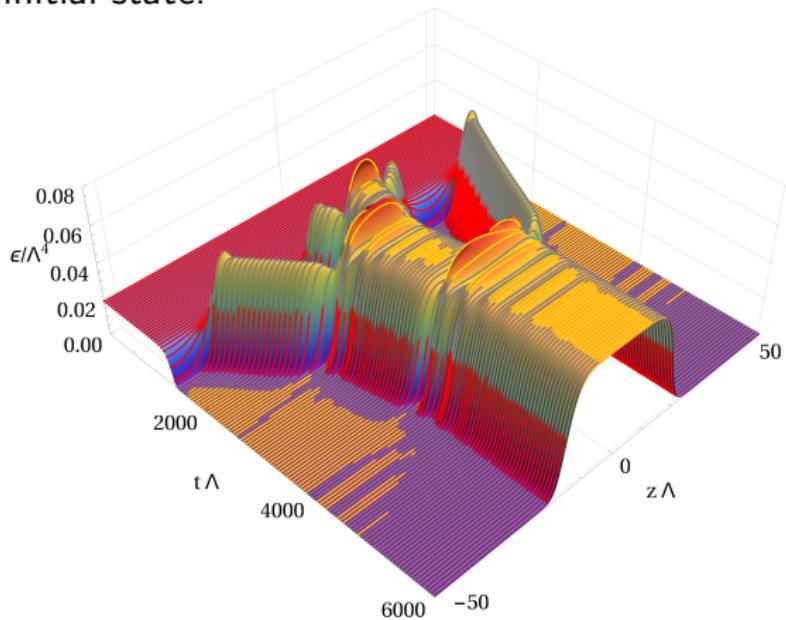


Initial  $n = 1$  excited

Phase separated final state

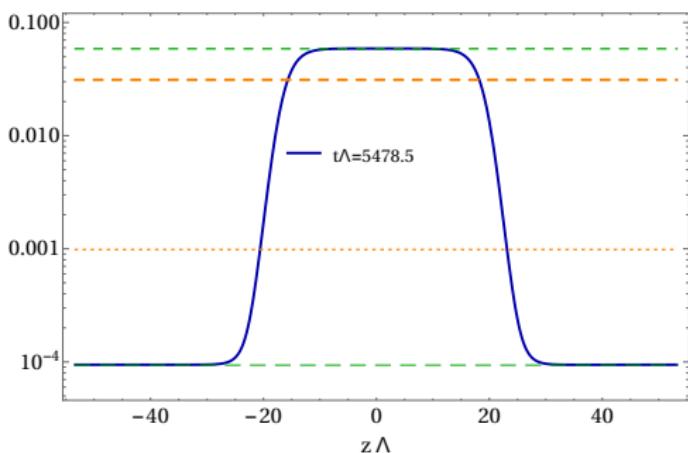
# Spinodal instability triggered by numerical noise I

Unexcited initial state:



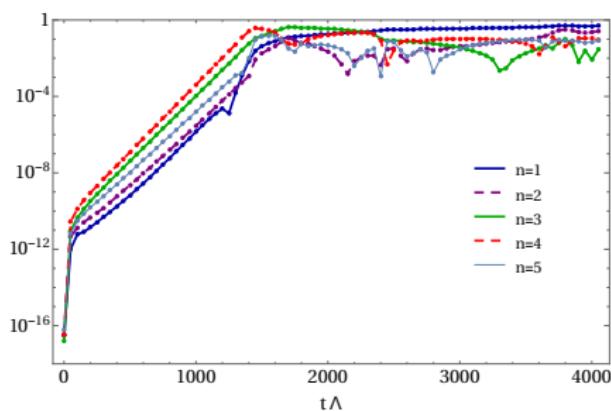
Minimizing free energy to have only a single blob

Final static state:



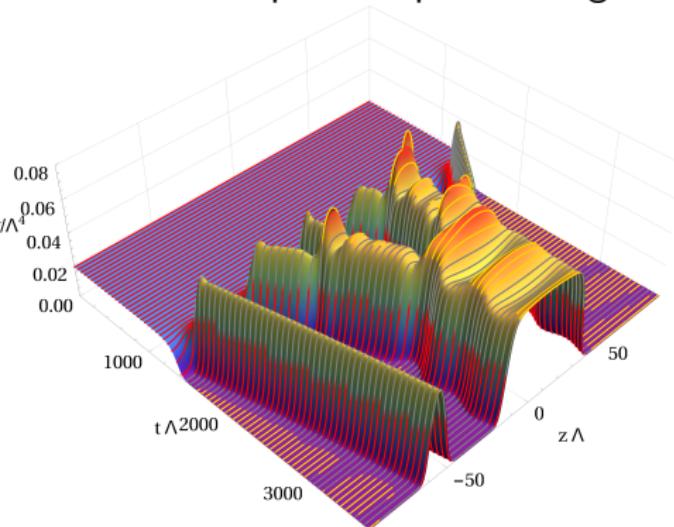
Phase separated final state

Unstable mode growth



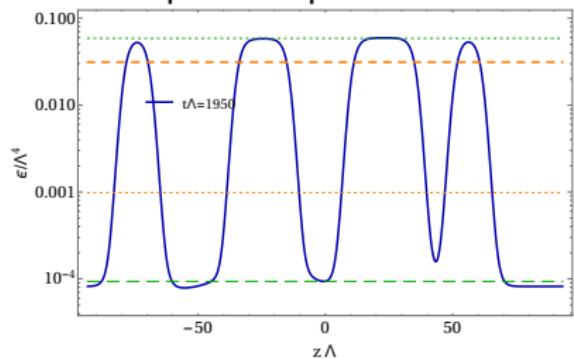
$n = 4$  pushing four peaks

Different phase separated region merge:



On the way to settle.

Phase separated peaks:



## Excitation and saturation of the spinodal instability

### Shockwave collisions near Critical Point

- Introduction Hydrodynamics and viscosities
- Introduction shockwave collisions
- Hydrodynamization with bulk viscosity
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# Introduction Hydrodynamics and viscosities

Hydrodynamics assumes mean free path goes to zero and conservation of energy and momentum

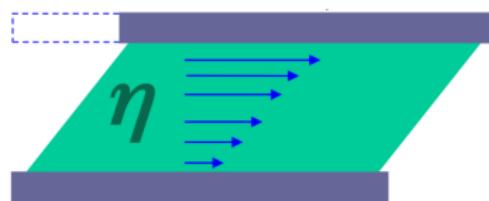
$$\partial_\mu T^{\mu\nu} = 0$$

expansion around isotropic equilibrium distribution:

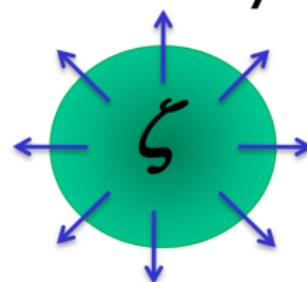
$$T_{\mu\nu}^{\text{hyd}} = T_{\mu\nu}^{\text{ideal}} - \eta \sigma_{\mu\nu} - \zeta \Pi \Delta_{\mu\nu} + \Pi_{\mu\nu}^{(2)}$$

Together with the equation of state of the fluid, which is defined as a functional relation between conserved quantities  $\mathcal{E}$ ,  $P$ , they form a closed system of equations.

## Shear viscosity



## Bulk viscosity



# Introduction shockwave collisions

Heavy-Ion collision:  
QGP formation



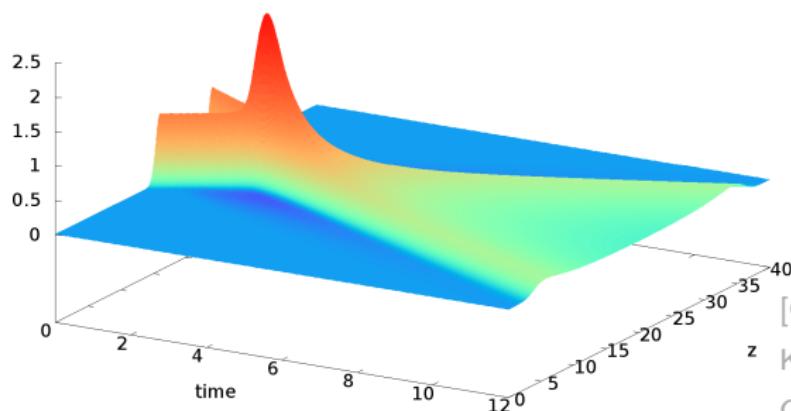
shock wave collision:  
black hole formation

$$(\mathcal{E}, J_{\mathcal{E}}, P_{x^i}, \mathcal{V})$$



$$\frac{\kappa_5^2}{2L^3} \left( -T_t^t, T_t^z, T_{x^i}^{x^i}, \mathcal{O} \right)$$

Holography allows to explore far from equilibrium dynamics:



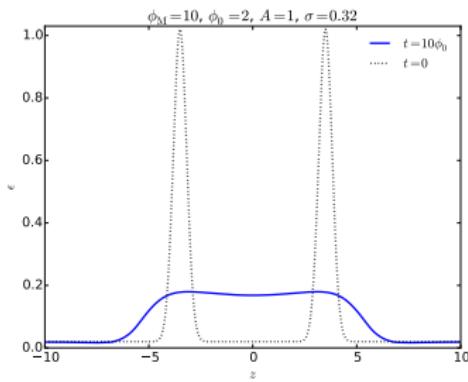
Energy density evolution of a typical scalar shock wave collision

at strong coupling  
non-perturbatively  
with fast  
hydrodynamization  
time

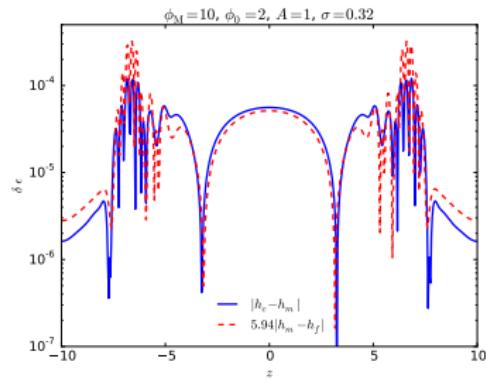
[Chesler, Yaffe 11; Albacete,  
Kovchegov, Taliotis 08;  
Grumiller, Romatschke 08; Beuf,  
Heller, Janik, Peschanski 09;  
Casalderrey-Solana, Heller,  
Mateos, v.d.Schee 13 ]

# Numerical convergence analysis

Local energy densities at simulation start and at late times:

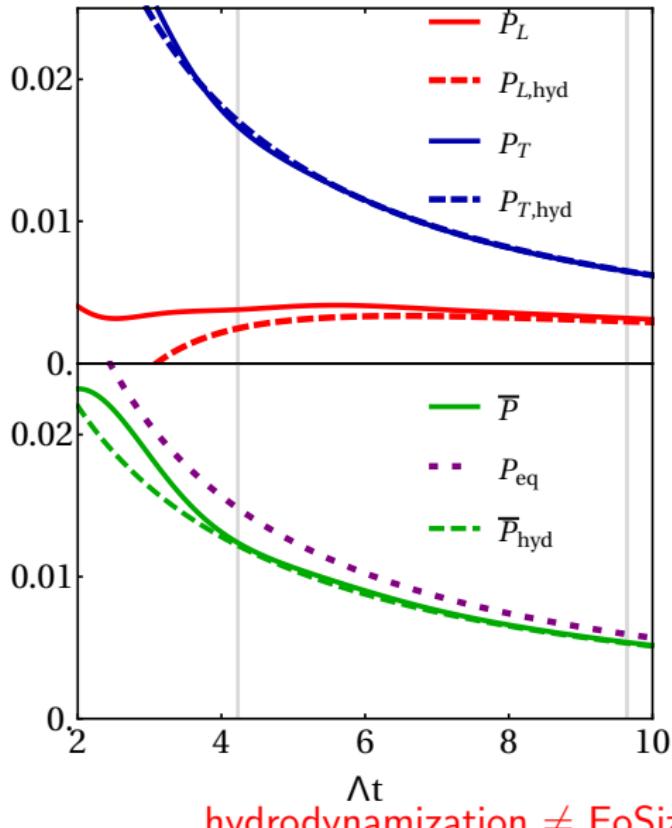


Differences between the coarse and medium (blue solid line) and the medium and fine (red dashed line) resolution run:



The results show fourth-order convergence at late times and for the relevant regions.

# Non-conformal shock collision



hydrodynamization  $\neq$  EoSization  $\neq$  isotropization

Hydrodynamics expansion:

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} + \eta\Pi^{\mu\nu} + \zeta\Pi(g^{\mu\nu} + u^\mu u^\nu)$$

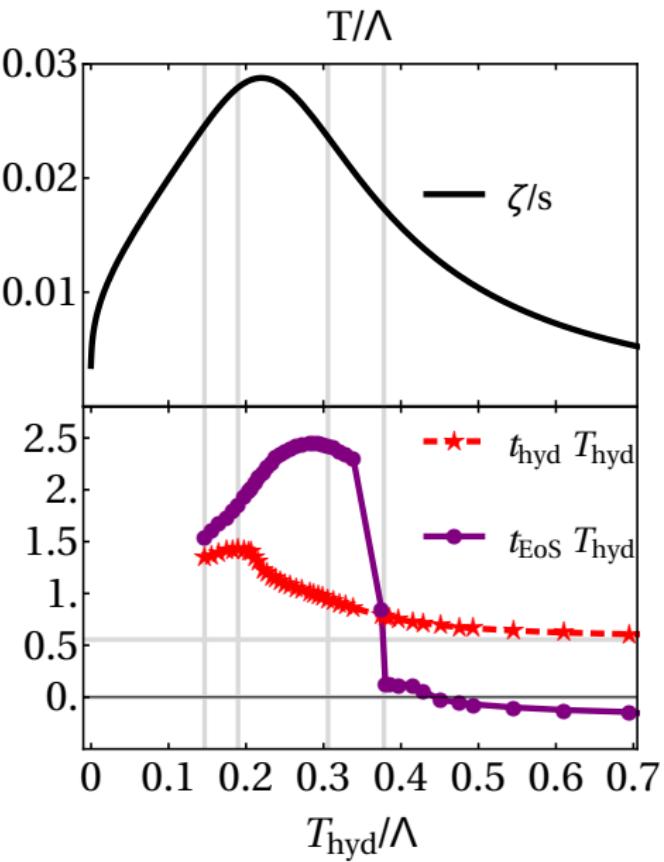
Hydrodynamization:

$$\left| P_{L,T} - P_{L,T}^{\text{hyd}} \right| / \bar{P} < 0.1$$

EoSization:

$$\left| \bar{P} - P_{eq} \right| / \bar{P} < 0.1$$

# Hydrodynamization with bulk viscosity



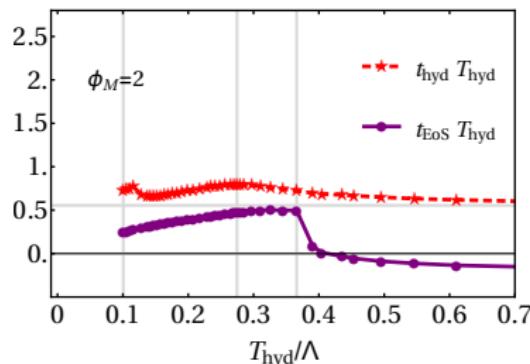
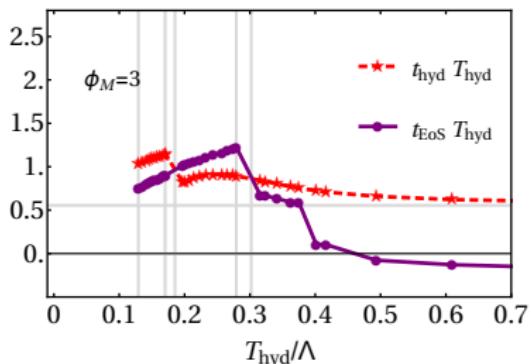
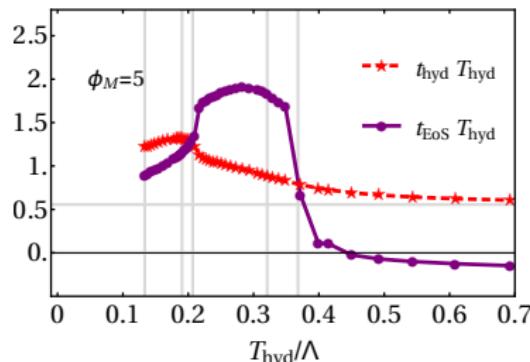
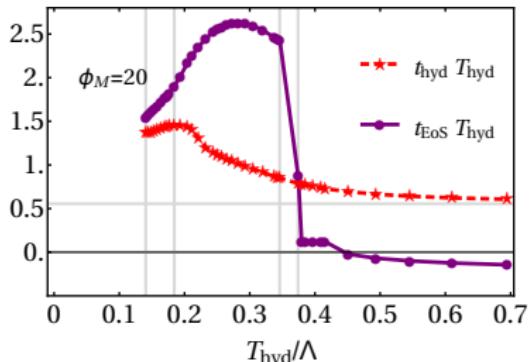
Non-conformal  $T$  scan:

- $t_{\text{hyd}}$  slow down, still very fast
- new relaxation channel: **EoSization** (= time when ideal equ. of state applies)
- $\zeta/s > 0.025$  for  $t_{\text{EoS}}$  and  $t_{\text{hyd}}$

[A., Casalderrey-Solana,  
Mateos, Santos,  
Sopuerta, Triana, Zilhao  
2017]

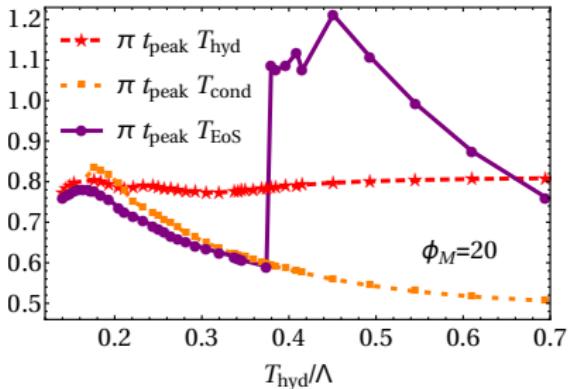
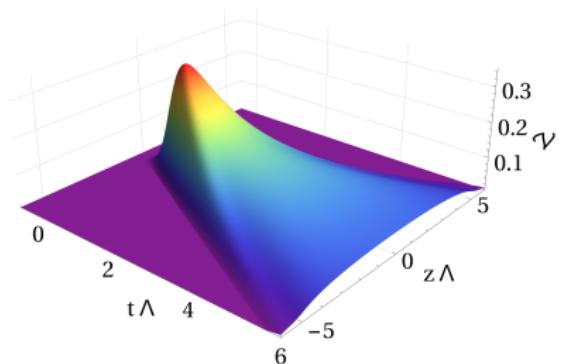
# Temperature scan in different theories

Comparing varying non-conformality  $\phi_M$ :



conservative estimate  $\zeta/s > 0.025$  needed for  $t_{\text{EoS}} > t_{\text{hyd}}$

Universal response:

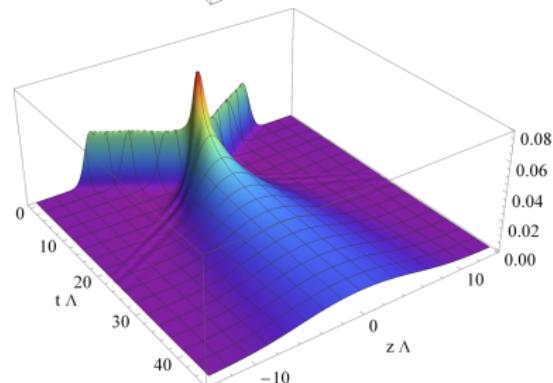
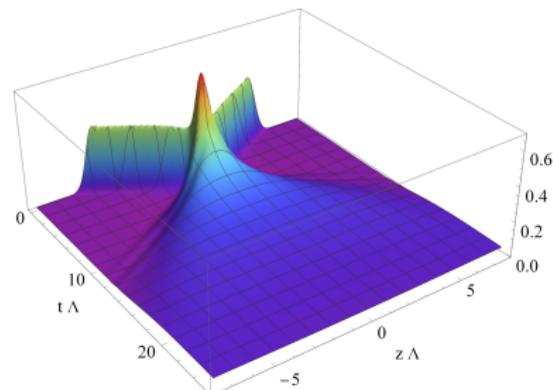
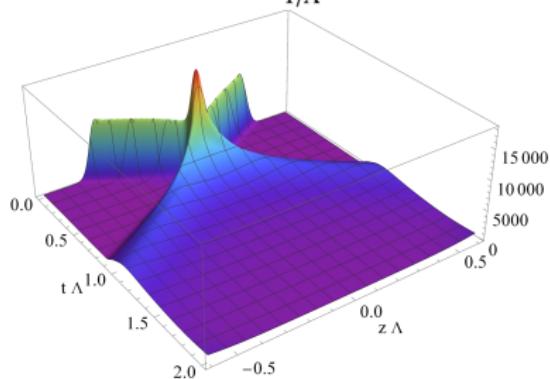
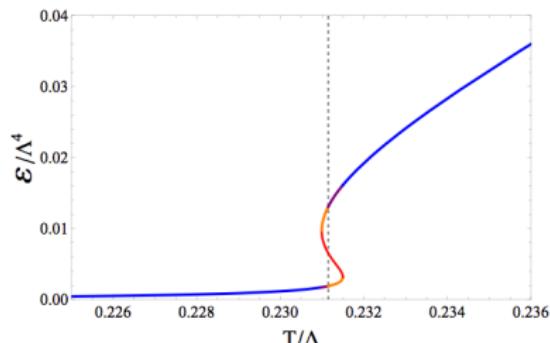


$$\pi t_{\text{peak}} T_{\text{hyd}} \approx 0.8$$

time for the effects of the dynamics near the horizon that forms deep in the bulk when the shocks collide to reach the boundary.

# Collisions across a 1st-order phase transition

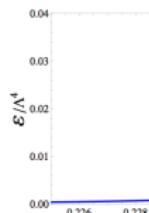
CFT shockwaves explode



Stuck in a blob

# From 1st-order to 2nd-order to crossover

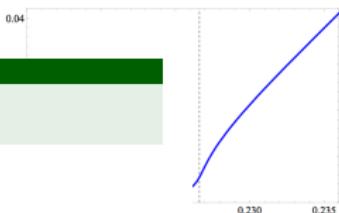
1st order:



2nd order:



crossover:



Quark Matter 2018



Contribution ID : 347

Type : Poster

## Collision Dynamics near the Critical Point at Strong Coupling

Tuesday, 15 May 2018 19:20 (20)

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**Co-author(s) :** ATTEMES, maximilian (University of Barcelona); MATEOS, David (ICREA & U. Barcelona); ZILHÃO, Miguel; Dr BEA, Yago (University of Barcelona)

**Presenter(s) :** CASALDERREY SOLANA, Jorge (University of Oxford)

**Session Classification :** Poster Session

**Track Classification :** Phase diagram and search for the critical point

- Spinodal instabilities lead to phase separation and mergers
  - The system settles (!?) into an inhomogenous state (!)
  - The final state is also described by hydrodynamics!
- First simulation of a holographic non-conformal model for heavy ion collisions:
  - Fast hydrodynamization at early time despite non-trivial equation of state despite sizeable  $\zeta/s$  bulk viscosity over entropy
  - New relaxation channel from bulk viscosity: *EoSization*
  - Conservative estimate  $\zeta/s \approx 0.025$  for non-conformal effects
  - Universal response of the scalar condensate
- First simulation of a holographic non-conformal model near a critical point:
  - New qualitative feature of most of the energy stuck in a blob (dynamics well described by hydrodynamics)