

Flow fluctuations in small systems

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Workshop on
Foundational aspects of relativistic hydrodynamics
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Outline

- What do we mean by collectivity ?
- Specificities of small systems
- A few recent predictions

Giacalone, Noronha-Hostler, JYO 1702.01730

Particle emission in hydrodynamics

- Particles emitted **independently** (no correlations) on the freeze-out surface
- The anisotropy of the **single-particle momentum distribution** (v_n) is driven by the **initial density profile**.
- There is certainly more, but we can use these two properties as a first definition of *collectivity*.

Property #1: The flow paradigm

- Particles are emitted **independently** in every event with momentum distribution $f(p)$
- $f(p)$ **fluctuates** event to event
 - azimuthal angle of impact parameter fluctuates
 - more generally: fluctuations in density profile, hot spots..
- **Averaging over events** generates non-trivial **correlations to all orders**, e.g., the pair distribution is $\langle f(p_1) f(p_2) \rangle$

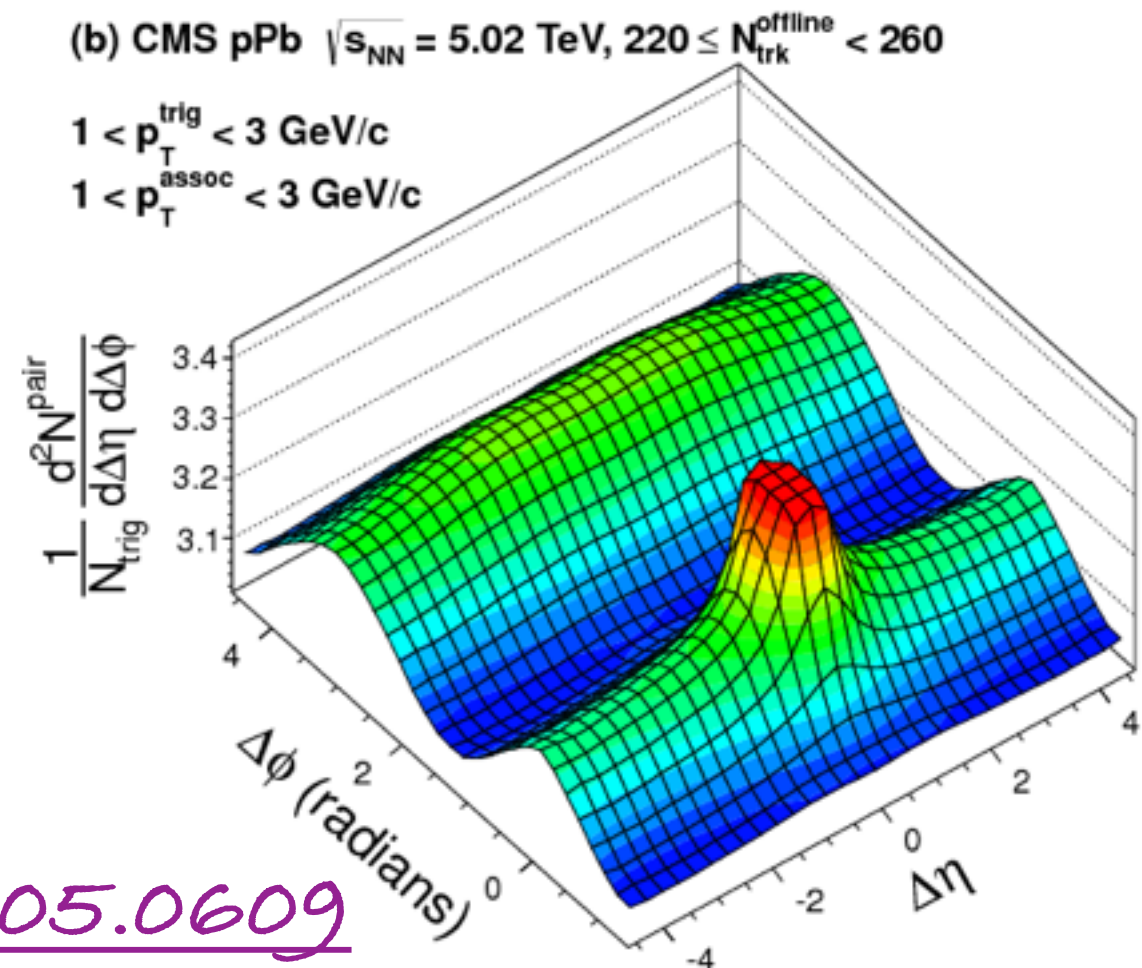
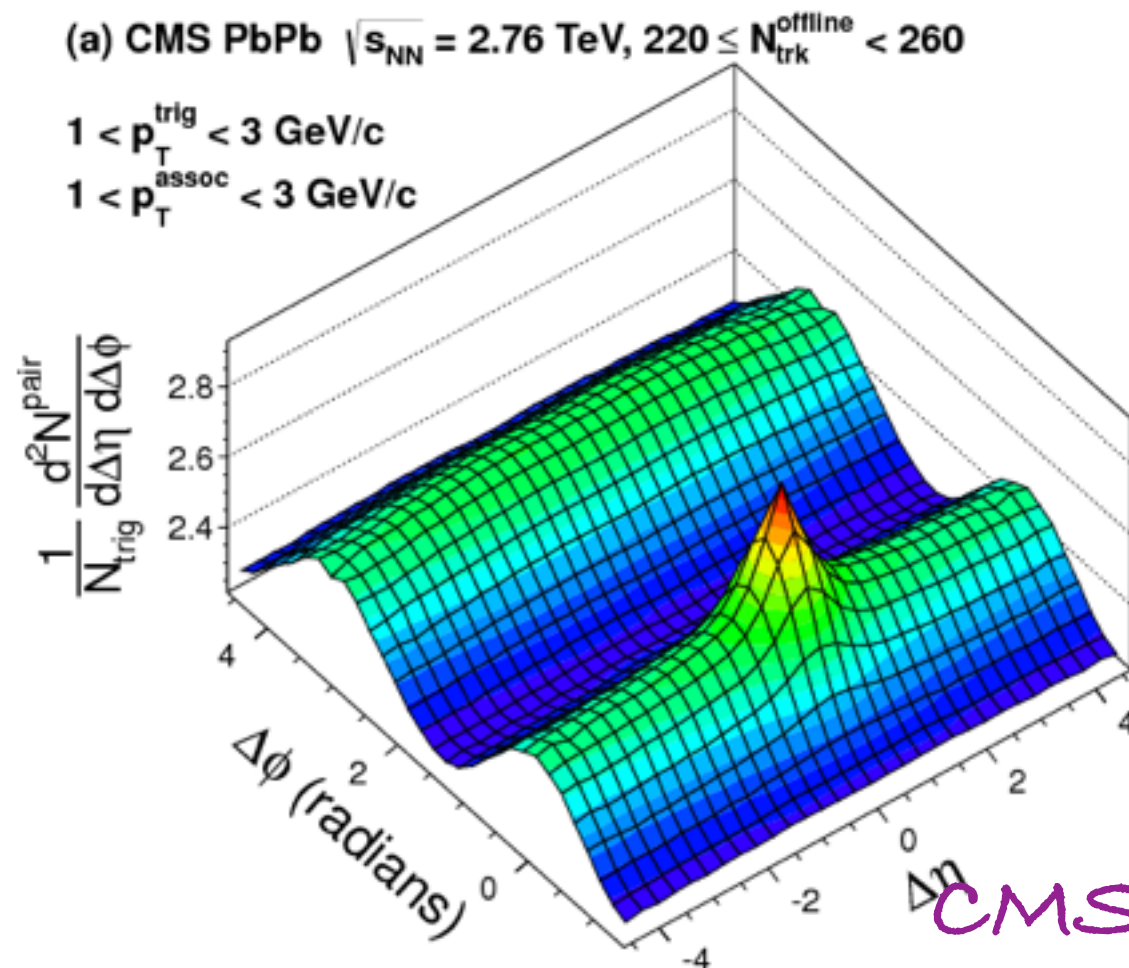
Alver & Roland 1003.0194

Flow paradigm naturally explains the ridge

- At the expense of an additional symmetry assumption:
 $f(p)$ is essentially independent of rapidity in every event

Pb+Pb

p+Pb



CMS 1305.0609

Hydro versus CGC

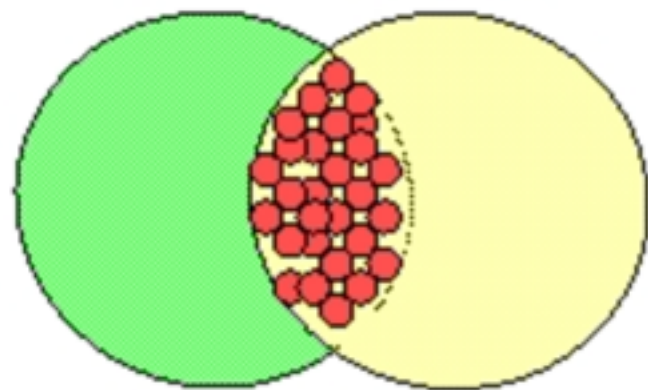
- Note that some alternatives to hydro referred to as CGC strictly comply with this *flow paradigm*.

Dusling Mace Venugopalan 1705.00745

$$\frac{d^m N}{d^2 \mathbf{p}_{i\perp} \cdots d^2 \mathbf{p}_{m\perp}} = \prod_{i=1}^m \int d^2 \mathbf{b}_i \int \frac{d^2 \mathbf{k}_i}{(2\pi)^2} W_q(\mathbf{b}_i, \mathbf{k}_{i\perp})$$
$$\cdot \int d^2 \mathbf{r}_i e^{i(\mathbf{p}_{i\perp} - \mathbf{k}_{i\perp}) \cdot \mathbf{r}_i} \left\langle \prod_{j=1}^m D \left(\mathbf{b}_j + \frac{\mathbf{r}_j}{2}, \mathbf{b}_j - \frac{\mathbf{r}_j}{2} \right) \right\rangle. \quad (1)$$

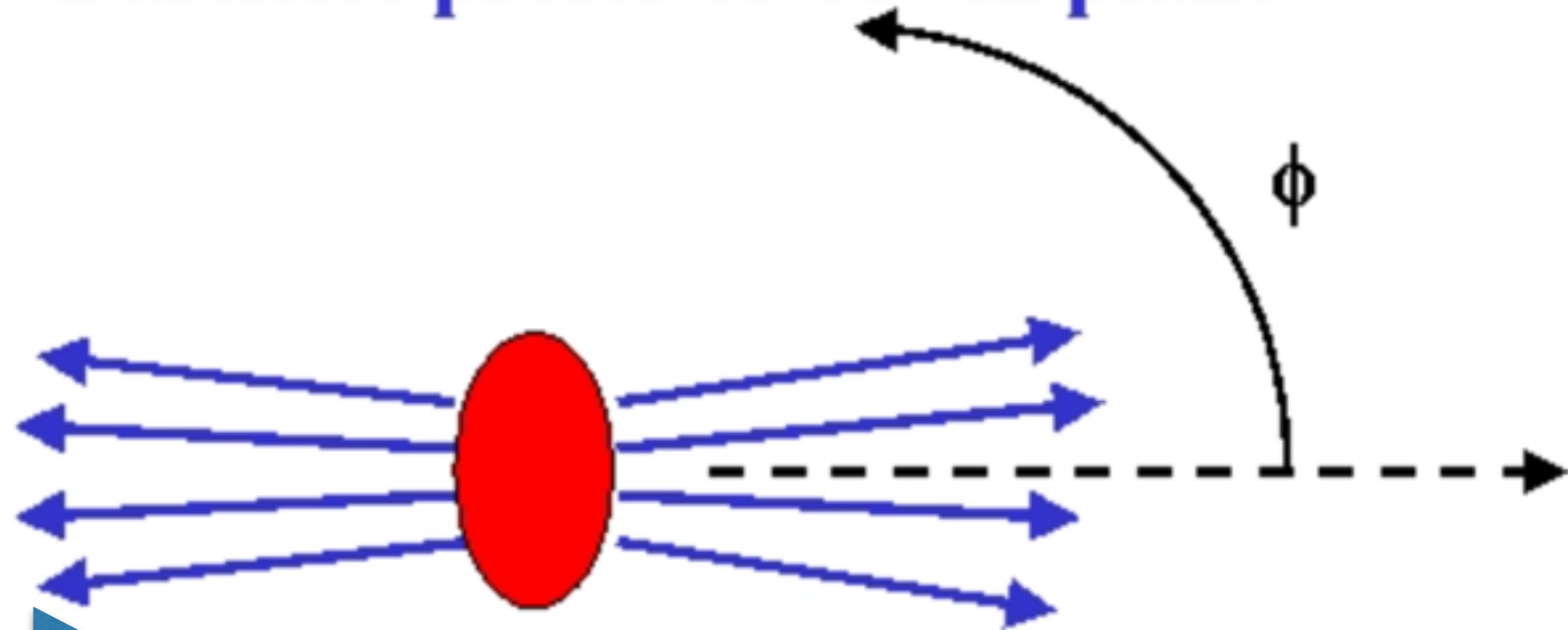
Property #2: Initial spatial anisotropy as the seed of anisotropic flow

Beam's eye view of a non-central collision:



Pressure gradient : flow

Particles prefer to be “in plane”:



(courtesy Mark Baker)

An elliptic density profile produces elliptic flow
A triangular density profile produces triangular flow...

Specificities of small systems

- *Nonflow* correlations, breaking the flow paradigm, are **larger**.
- **Initial anisotropies** are solely produced by fluctuations (in p+p and p+Pb) and these fluctuations are **larger** in small systems.

Nonflow

- Particles sometimes come in clusters (jets, resonance decays).
- The probability that two arbitrary particles come from the same cluster scales like $1/(\# \text{ of clusters})$.
- Hence, nonflow contribution to a 2-particle correlation typically scales like $1/N_{\text{ch}}$ and is larger for small systems.
- Nonflow both present at short $\Delta\eta$ (short range) and large $\Delta\eta$ (away-side)

Methods to suppress nonflow

- Rapidity gap: eliminates short-range nonflow.
Usually implemented in 2-particle correlations

$v_n\{2\}$

PHENIX [nucl-ex/0305013](#)

- Cumulants: higher-order correlations: $v_n\{4\}$, $v_n\{6\}$.

Borghini et al [nucl-th/0007063](#) [nucl-th/0105040](#)

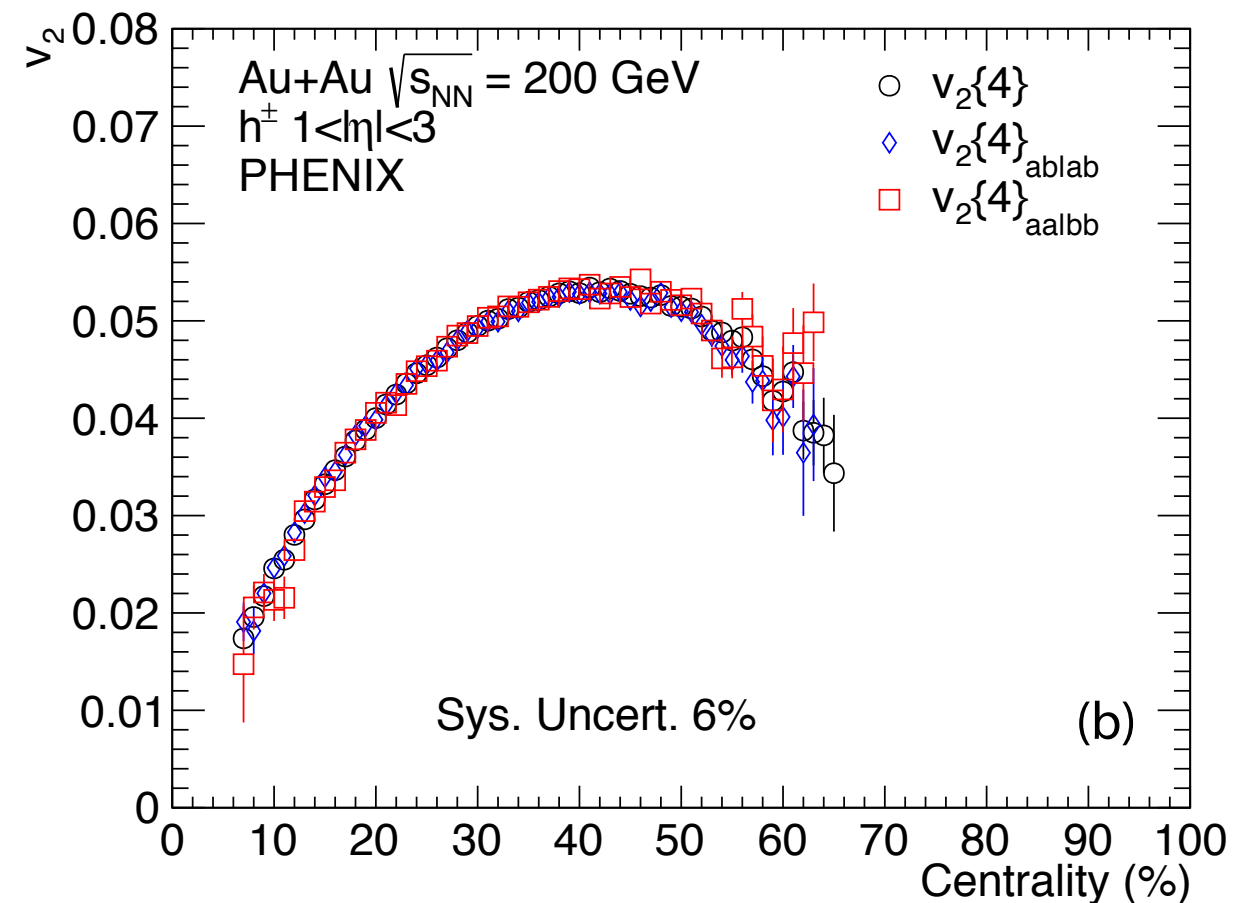
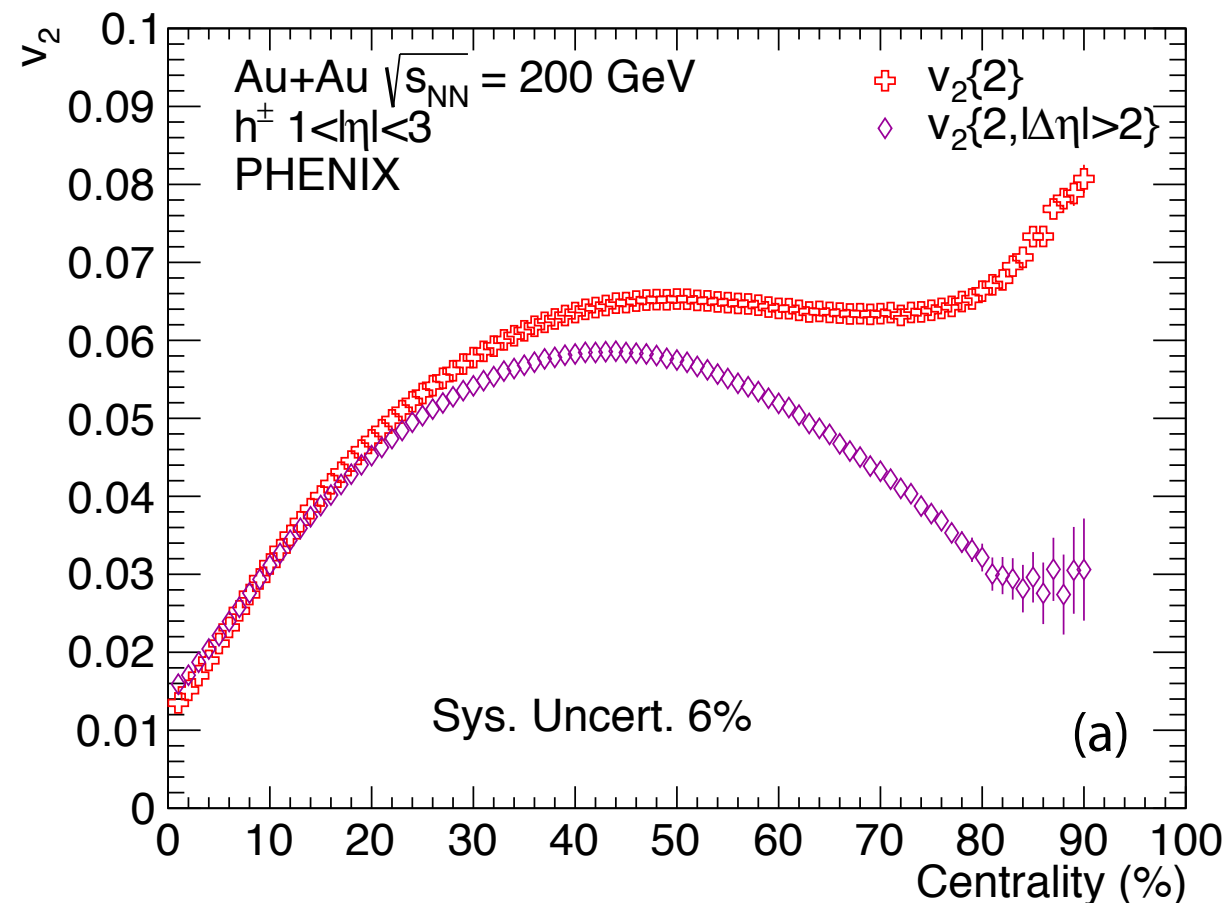
Bilandzić et al [1010.0233](#)

- Note that flow depends on rapidity and comparing $v_n\{2\}$ with gap and $v_n\{4\}$ without gap is not apples-to-apples.

Recent progress: cumulants with gaps

Jia Zhou Trzuppek [1701.03830](#)

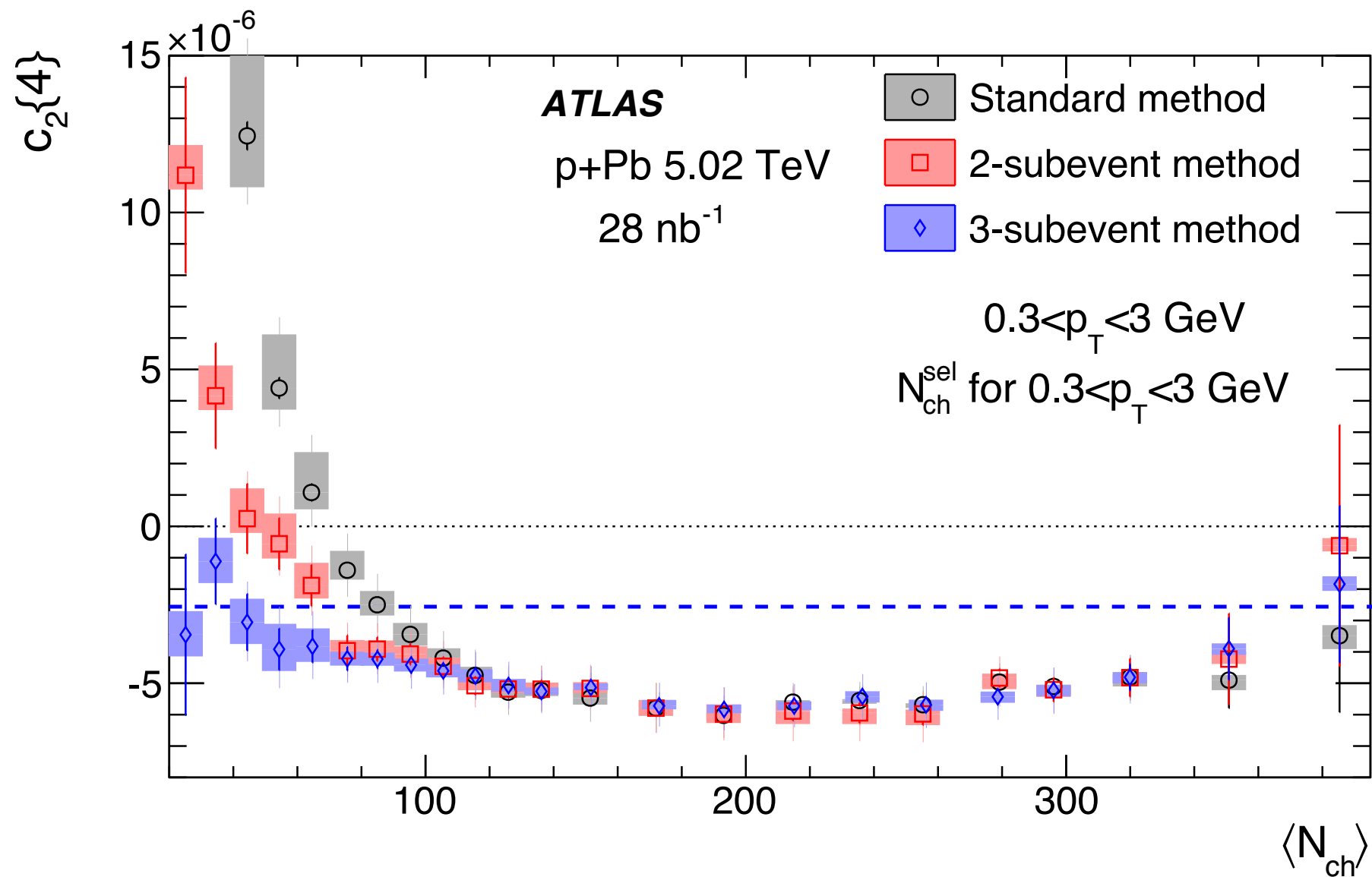
Dí Francesco et al [1612.05634](#)



PHENIX [1804.10024](#)

In Au+Au, rapidity gaps matter for $v_2\{2\}$, not $v_2\{4\}$.

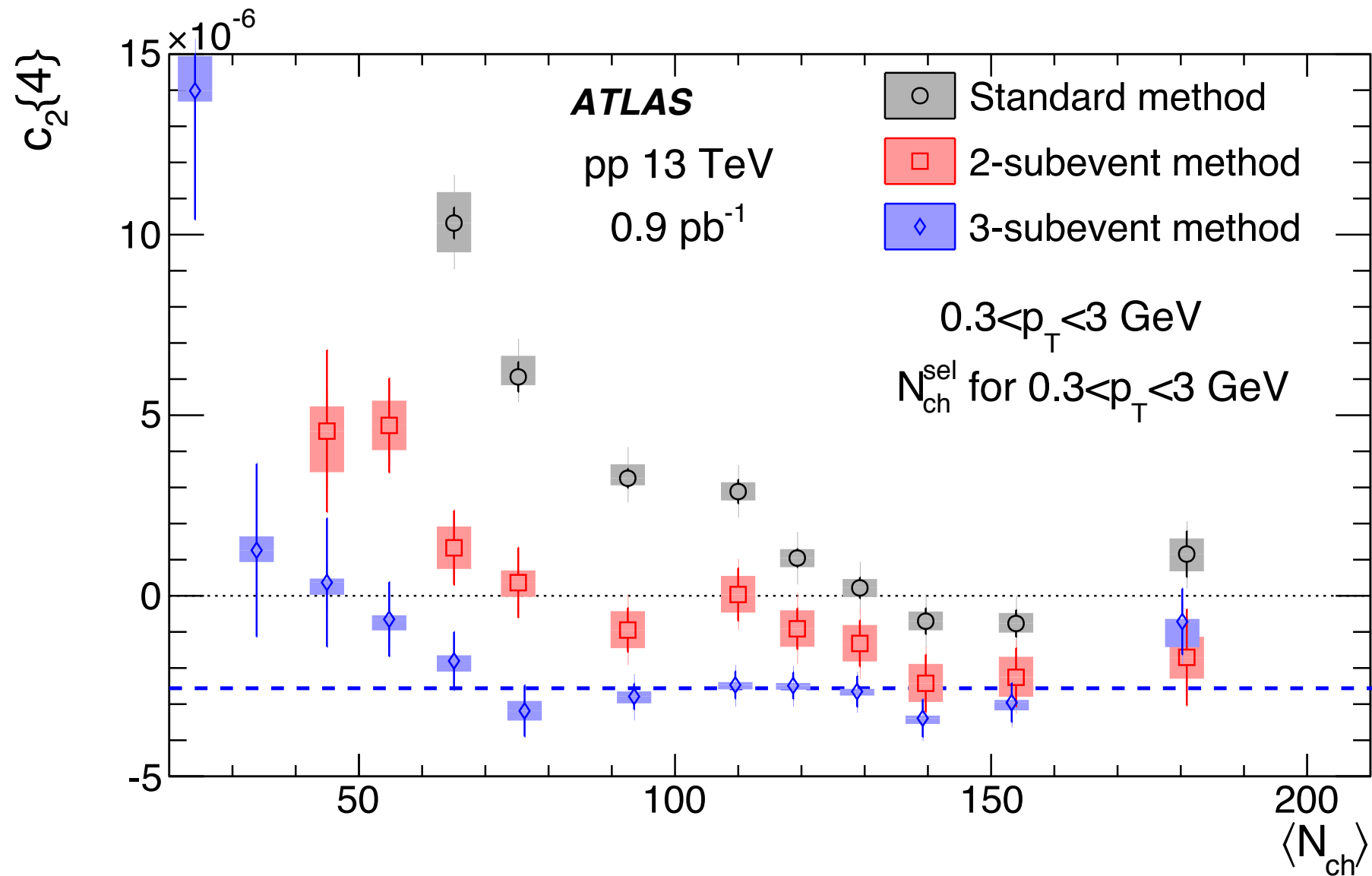
Recent progress: cumulants with gaps



ATLAS 1708.03559

In p+Pb, $v_2\{4\}$ depends somewhat on the gap

Recent progress: cumulants with gaps



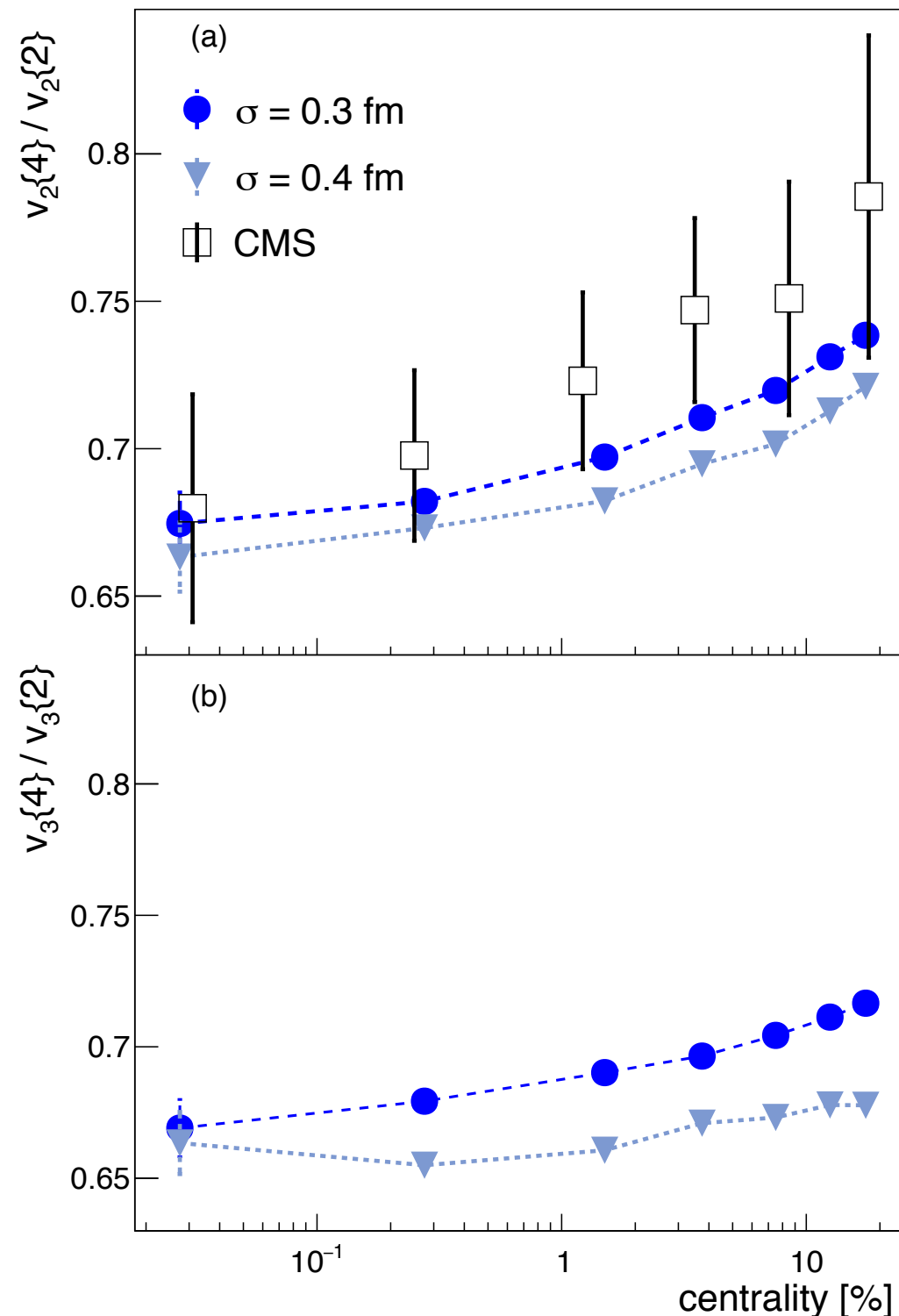
ATLAS 1708.03559

In p+p, a gap changes everything

Anisotropy fluctuations

- **Small fluctuations** are often **Gaussian** (central limit theorem).
- This also applies to ε_n fluctuations
Voloshin, Poskanzer, Aihong Tang, Gang Wang 0708.0800
- =Transverse, 2-dimensional Gaussian fluctuations usually dubbed **Bessel-Gaussian**
- Such fluctuations imply $v_n\{4\}=v_n\{6\}=\dots=0$ in small systems (no mean v_2 in reaction plane)
- **Small systems** have **large fluctuations**, and this results in **non-Gaussianities**.

Non-Gaussianities in p+Pb



Giacalone et al [1702.01730](#)

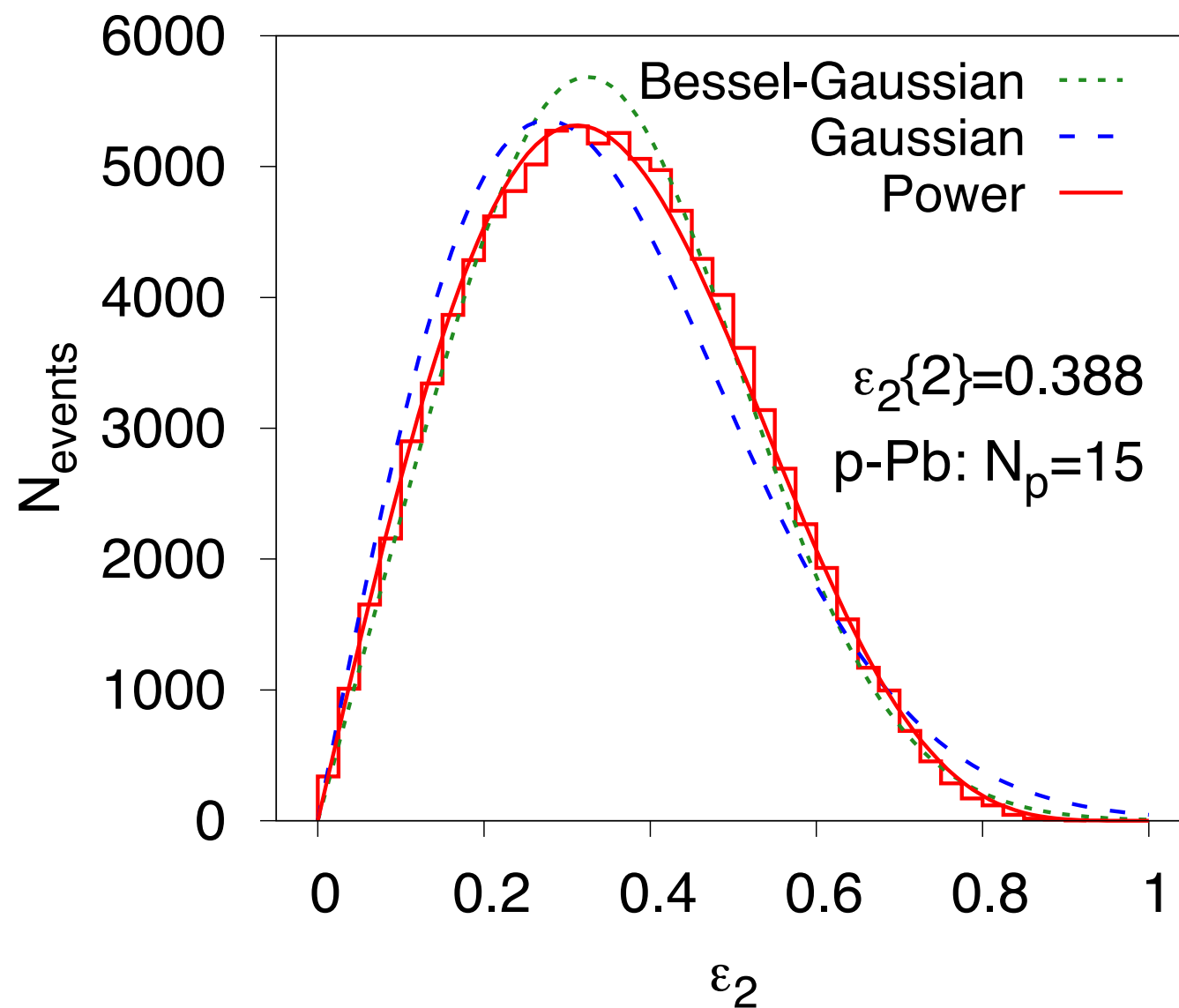
Monte Carlo simulations based on the Trento model of initial conditions (essentially a Monte Carlo Glauber)

Moreland Bernhard Bass [1412.4708](#)

No hydro here: We assume that v_n is proportional to ϵ_n in every event.

A generic prediction is that non Gaussianities increase as a function of centrality %: smaller systems are less Gaussian. Seen by CMS.

Anisotropy fluctuations



Li Yan JY0 1312.6555

New *Power*

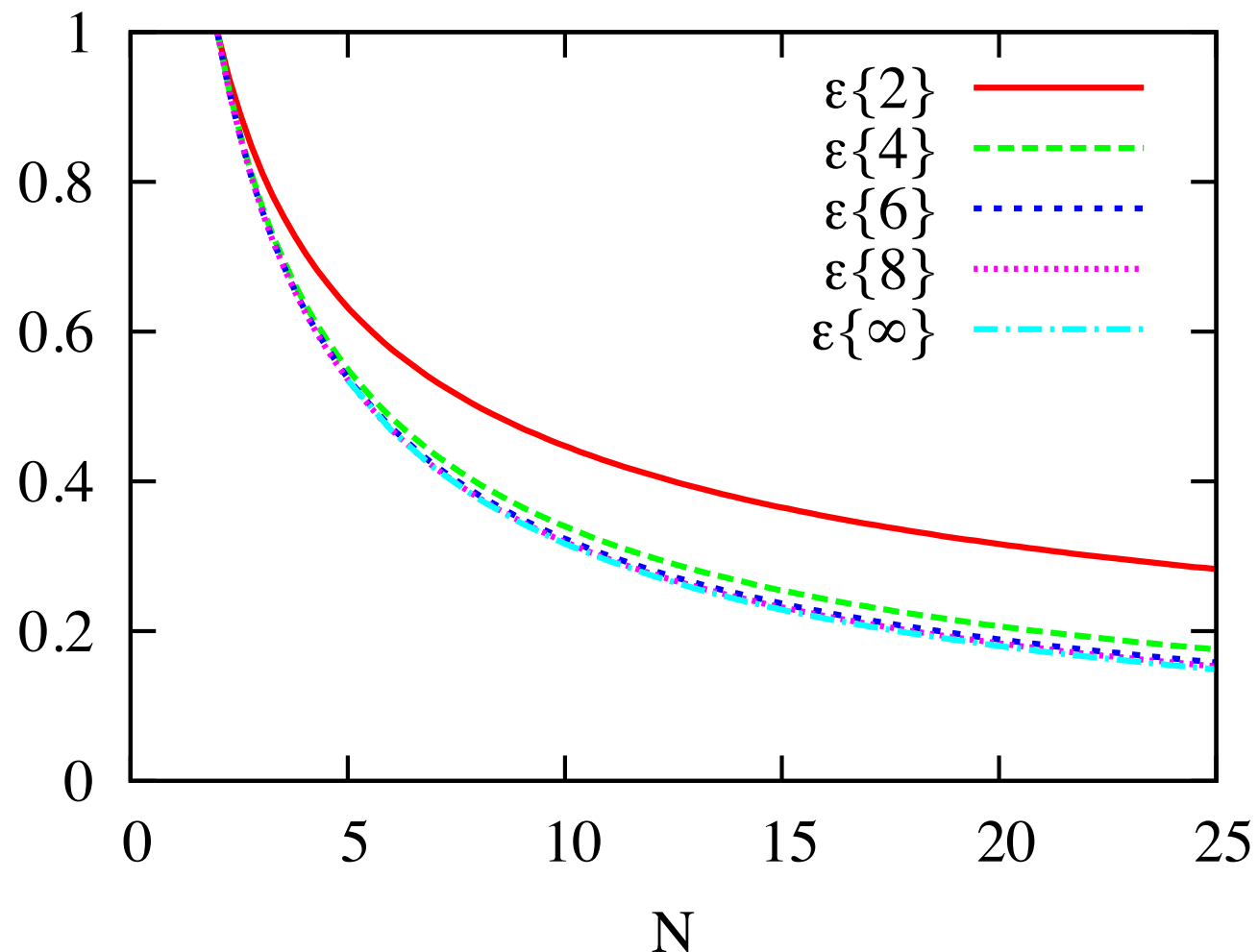
Parametrization of the distribution of ϵ_n in the form

$$P(\epsilon_n) = \epsilon_n (1 - \epsilon_n^2)^\alpha$$

Takes into account the bound $\epsilon_n < 1$

A single parameter α fixes the width.

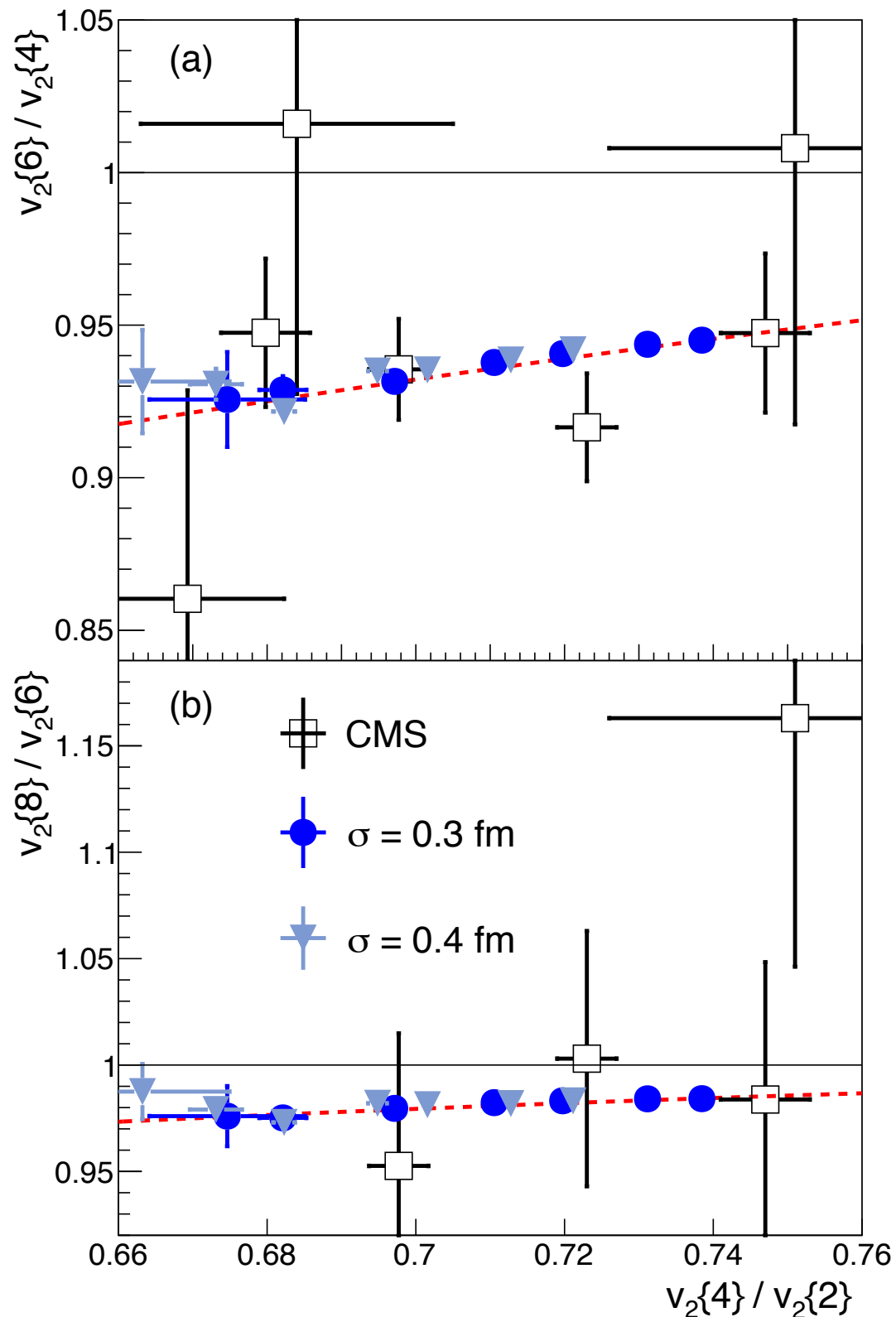
Higher-order Cumulants



(number of pointlike sources)

- The Power distribution predicts a near degeneracy of higher-order cumulants in small systems, even though anisotropy is solely due to fluctuations
- Once $v\{4\}/v\{2\}$ is known, the small lifting of degeneracy between higher-order cumulants is precisely predicted

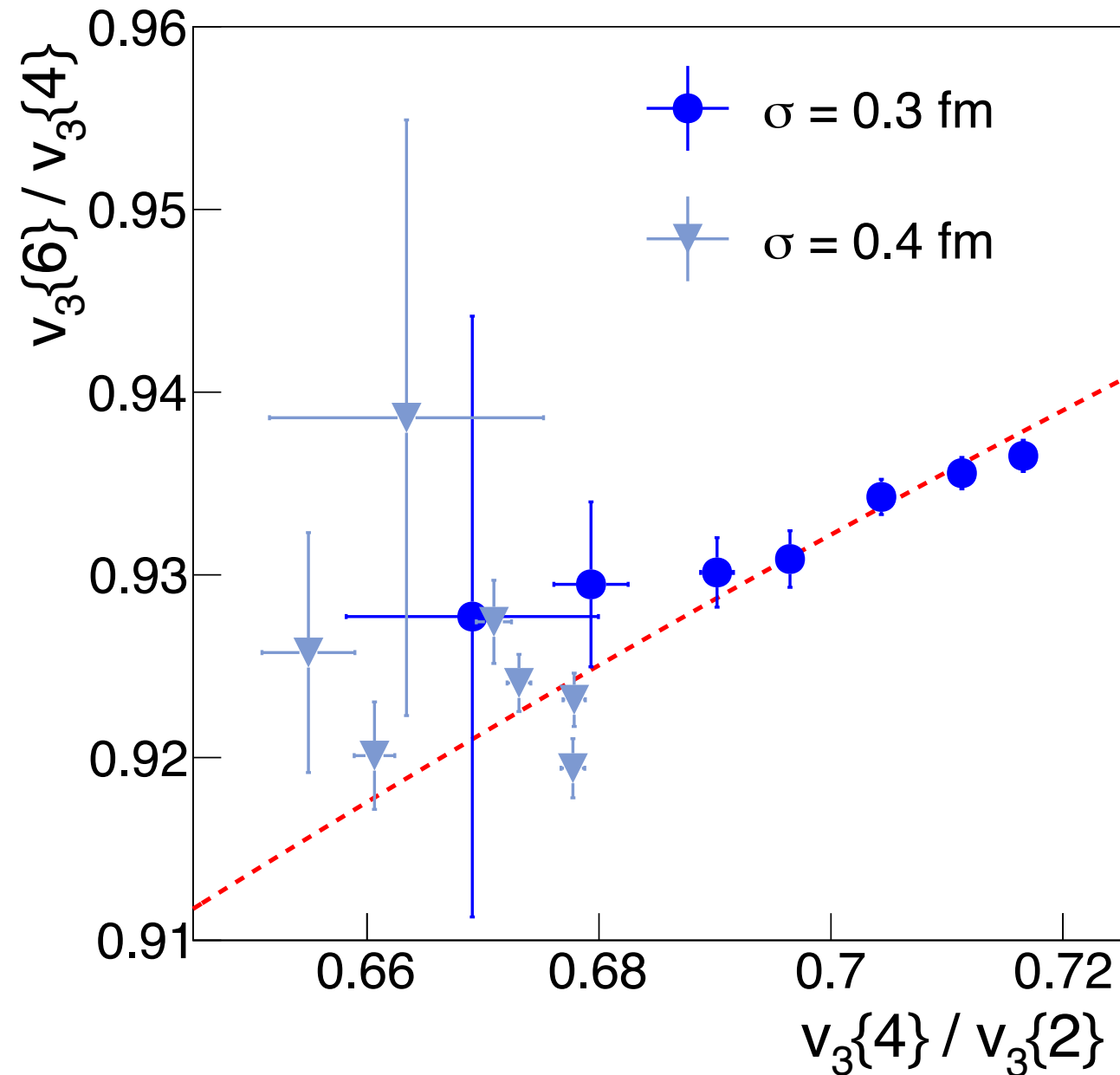
Higher-order Cumulants



- Monte Carlo simulations of the initial state (Trento model) agree with the analytic prediction from the Power distribution
- CMS preliminary data also in good agreement

Giacalone et al [1702.01730](#)

Higher-order Cumulants



- Also works for triangular flow (higher order cumulants not yet measured in p+Pb)

Giacalone et al [1702.01730](#)

Conclusions

- Long range correlations naturally explained by independent particle emission from a fluctuating source (flow paradigm). This also applies to the CGC framework.
- Assuming that anisotropic flow is a linear response to the initial anisotropy allows one to make accurate, quantitative predictions for higher-order cumulants, which can be tested against data.

Backup slides

The fluctuations of elliptic flow

- One can measure much more than the rms value of v_2 . Also higher order moments and cumulants

$$v_2\{2\} = (\langle v_2^2 \rangle)^{1/2}$$

$$v_2\{4\} = (2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle)^{1/4}$$

$$v_2\{6\} = ((\langle v_2^6 \rangle - 9\langle v_2^4 \rangle \langle v_2^2 \rangle + 12\langle v_2^2 \rangle^3)/4)^{1/6}$$

- $v_2\{4\} < v_2\{2\}$ if v_2 fluctuates
- $v_2\{4\} = v_2\{6\}$ if fluctuations are 2-dim. Gaussian.