In-medium heavy quarkonia

(lattice QCD perspective)

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Quark-Gluon Plasma Characterisation with Heavy Flavour Probes (HYBRID)

ECT*, Trento, Italy, 11/17/2021

Bottomonium melting from screening correlators at high temperature,
P. Petreczky, S. Sharma, JHW, Phys.Rev.D 104 (2021) 5, 054511

Bottomonia via lattice NRQCD,
R. Larsen, S. Meinel, S. Mukherjee, P. Petreczky, Phys.Rev.D 100 (2019) 7, 074506; Phys.Lett.B 800 (2020) 135119; Phys.Rev.D 102 (2020) 114508

Heavy Quark Potential in QGP: DNN meets LQCD,
S. Shi, K. Zhou, J. Zhao, S. Mukherjee, P. Zhuang, arXiv:2105.07862[hep-ph]

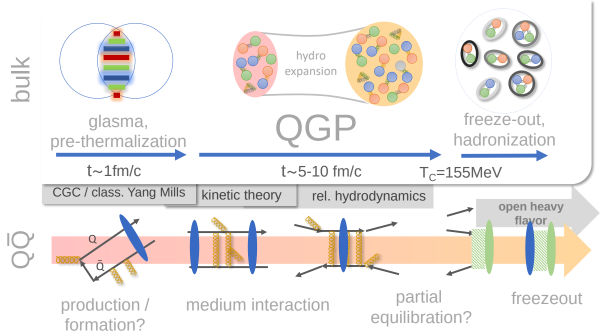
Static quark anti-quark interactions at non-zero temperature from lattice QCD,
D. Bala, O. Kaczmarek, R. Larsen, S.Mukherjee, G. Parkar,
P. Petreczky, A. Rothkopf, JHW, arXiv:2110.11659[hep-lat]

Static Potential At Non-zero Temperatures From Fine Lattices,
+A. Bazavov, D. Hoying, arXiv:2110.00565[hep-lat]

Outline

1. Appetizer: link to heavy-ion phenomenology
   - Motivation
2. Main course: modern perspective on "Heavy quarkonium at $T > 0$"
   - Historical perspective on "Heavy quarkonium at $T > 0$"
   - In-medium quarkonium at weak coupling
   - Lattice QCD
   - Relativistic bottomonium on the lattice
   - Nonrelativistic bottomonium on the lattice
   - In-medium static quarkonium
3. Dessert: bringing "Heavy quarkonium at $T > 0$" full circle

Why focus on hard probes in heavy-ion collisions?

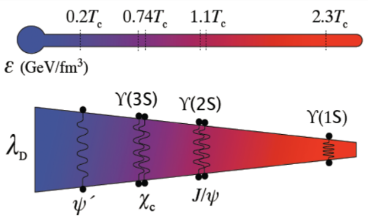


source: *Rothkopf, Phys.Rept. 858 (2020) 1-117*

- Hard probes are produced in a few **hard processes** in initial collision, neither created nor destroyed afterwards, but can alter their nature
- Most important probes: *jets*, open heavy flavor & **heavy quarkonia**
- **What happens to quarkonium if we increase the temperature?**

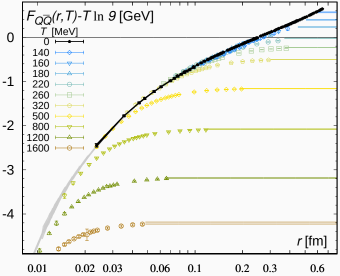
Heavy quarkonium in the hot medium as a local thermometer

- Idea to look at **quarkonium** in the QGP is old and famous
  Matsui, Satz, PLB 178 (1986)
- **Debye screening** of electric gluons (A_0) dictates a limit of the radius of hadronic bound states
- Consequence: QGP formation $\Leftrightarrow$ **quarkonium suppression**



source: *USQCD whitepaper 2018, EPJ A 55 (2019)*

- **Color screening** usually studied via Polyakov loop correlator
  $$C_P(r,T) = \langle P(0) P^\dagger(\boldsymbol{r}) \rangle_T^{\mathrm{ren}} = e^{-F_{Q\bar{Q}}(r,T)/T}$$
- $rT \ll 1$: **singlet**/**octet** decomposition
  $$C_P(r,T) = {}^1/_9 e^{-F_S(r,T)/T} + {}^8/_9 e^{-F_O(r,T)/T}$$
- $rm_D \gtrsim 1$: screening regime; decompose
  $$C_P(r,T) = \langle \mathrm{Re}\, P(0) \mathrm{Re}\, P^\dagger(\boldsymbol{r}) \rangle_T^{\mathrm{ren}} + \langle \mathrm{Im}\, P(0) \mathrm{Im}\, P^\dagger(\boldsymbol{r}) \rangle_T^{\mathrm{ren}}$$
  into $\mathcal{C}$ **even** or **odd** contributions
- QCD: $F_{Q\bar{Q},S,O}(r,T)$ **screened** @ $T = 0$



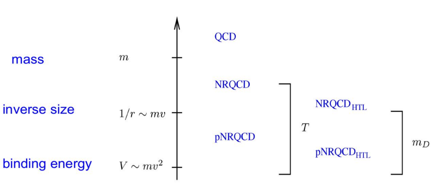
source: *Brambilla, et al., PRD 98 (2018)*

Screening is not the whole story. . . (at weak coupling)

Matsui & Satz's idea of the **quarkonium suppression mechanism** was turned inside out by **weak-coupling EFT results** emerging 15 years ago

NR hierarchy:

$v \sim \alpha_s$



Thermal hierarchy:

$m_D \sim gT$

- For $1/r \sim m_D \ll T$: $\mathrm{Re}\,[V_s] = F_S + \mathcal{O}(g^4)$ and $\mathrm{Im}\,[V_s] \sim \mathcal{O}(g^2 T)$

$$V_s(T,r) = -C_F\alpha_s\left\{\frac{e^{-rm_D}}{r} + m_D + \mathrm{i}T\phi(rm_D)\right\},\ \phi(x) = 2\int_0^\infty \frac{dz\ z}{(z^2+1)^2}\left\{1 - \frac{\sin(zx)}{zx}\right\}$$

Laine, et al., JHEP 03 (2007)

- For $\Delta V \ll 1/r \ll m_D \ll T$: $\mathrm{Re}\,[V_s] = V_s + \mathcal{O}(g^4)$ and $\mathrm{Im}\,[V_s] \sim \mathcal{O}(g^4 r^2 T^3, g^6 T)$

$$V_s(T,r) = \frac{-C_F\alpha_s}{r} + r^2 T^3\left\{\mathcal{O}(g^4) + \mathrm{i}\mathcal{O}\left(g^4, \frac{g^6}{(rT)^2}\right)\right\}$$

Brambilla, et al., PRD 78 (2008)

But an imaginary part leads to **dissociation** – is **screening** even relevant?

QCD on a lattice

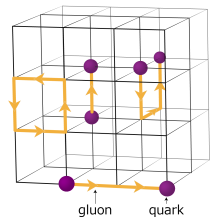
$$S_{QCD}[U,\bar{\psi},\psi] = a^4 \sum_x \sum_{f=1}^{N_f} \bar{\psi}^f(x) \left(\not{D}[U(x)] + m_f \right) \psi^f(x)$$

$$- a^4 \sum_x \sum_{\mu<\nu} \frac{2}{g_0^2} \mathrm{Re\ tr} \left\{ 1 - U_{\mu,\nu}(x) + \mathcal{O}(a^2) \right\}$$

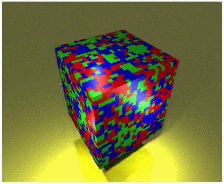
$$D_\mu[U_\mu(x)]\psi^f(x) = \frac{U_\mu(x)\psi^f(x+a\hat{\mu}) - U_\mu^\dagger(x-a\hat{\mu})\psi^f(x-a\hat{\mu})}{2a} + \mathcal{O}(a^2)$$

$$U_\mu(x) = \exp[\mathrm{i}ag_0A_\mu(x)] \qquad \text{gauge link}$$

$$U_{\mu,\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x) \qquad \text{plaquette}$$



$\overset{\text{HPC}}{\Longrightarrow}$



Lattice QCD simulations in a box on a computer

Stochastically sample the (Euclidean) QCD path integral

$$\langle O\rangle_{\mathrm{QCD}} = \frac{1}{Z}\frac{1}{N}\sum_{\{U\}}^{N} O[U]\prod_{f=1}^{N_f}\det\left(\not{D}[U]+m_f\right)\exp\left(-S_g[U]\right)+\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

using MCMC algorithm with importance sampling

QCD on a lattice with spacing a in a box of $N_\sigma^3 \times N_\tau$ points

- scale setting: lattice spacing a is determined a posteriori
  control the approach to the continuum limit $a \to 0$
- time (Euclidean): periodic for gluons, antiperiodic for quarks
- space: periodic for gluons and quarks
  always at finite temperature and in finite volume
  $aN_\tau = 1/T$ **(volumes only must be large enough)**
- quark masses: light quarks at the physical point are expensive
  control the quark mass dependence through χPT
- quark flavors: usually $N_f = 2+1$ or $N_f = 2+1+1$, or $N_f = 0$

Real-time dynamics from Lattice QCD

- Importance sampling requires an imaginary-time formalism

⇒ **Dissociation** due to real-time dynamics not directly accessible

- **Spectral functions** encode the entire dynamics
  - Stable bound states ⇒ Delta functions
  - Unstable quasiparticles ⇒ regularized peaks, locally Breit-Wigner
  - On top of a UV continuum due to scattering or merged excited states
  - At $T > 0$ potentially a substantial IR tail below the "ground state"
- Same **spectral functions** yield real- or imaginary-time correlators via different, analytically known integral kernels

$$G_T \begin{pmatrix} t \\ \tau \end{pmatrix} = \int d\omega \begin{pmatrix} K^M(T,\omega;t) \\ K^E(T,\omega;\tau) \end{pmatrix} \rho_T(\omega)$$

⇒ Strategy for lattice QCD:

1. Compute imaginary-time correlators on the lattice
2. Reconstruct **spectral functions** by inverting spectral representation
3. Directly read off some state's properties from $\rho_T(\omega)$

- Spectral reconstruction is challenging: at best N_τ resp. $N_\tau/2$ data

At which T are there either bound states or melted $q\bar{q}$ pairs?

- **Euclidean Correlators** are towers of exponential decays $G(\tau) = \sum_i A_i e^{-E_i \cdot \tau}$

  For **mesons**: same E_0 in temporal or spatial directions

- **Spatial $q\bar{q}$ pair correlators** are a **model-independent** analysis tool

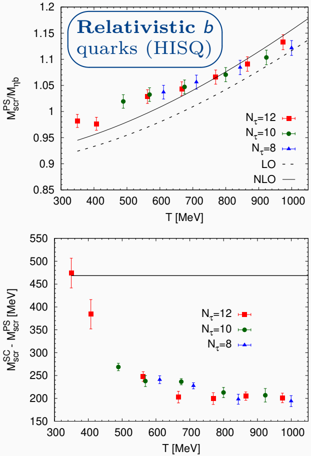
  Bazavov, et al., PRD 91 (2015)

$$G(z, T) = \int_0^{1/T} d\tau \int d^2 x_\perp \left\langle \mathcal{J}(\tau, \boldsymbol{x}_\perp, z) \mathcal{J}^\dagger(0) \right\rangle$$

$$= \int_0^\infty \frac{2d\omega}{\omega} \int_{-\infty}^{\infty} dp_z e^{ip_z z} \rho(\omega, p_z, T)$$

with **spectral function** $\rho(\omega, p_z, T)$

$$\sim \begin{cases} \delta\left[\omega^2 - p_z^2 - M_0^2\right] & \text{mesons} \\ \delta\left[\omega - \sum_{q_i} \sqrt{m_{q_i}^2 + [\pi T]^2}\right] & \text{free quarks} \end{cases}$$



source: *Petreczky, et al., PRD 104 (2021)*

- Lowest pseudoscalar & vector are hardly modified at $T \lesssim 450\,\mathrm{MeV}$
- Lowest scalar & axialvector are hardly modified at $T \lesssim 350\,\mathrm{MeV}$
- How can we understand the **melting mechanism** at work?

Nonrelativistic bottomonium with extended sources (HotQCD)

- Lattice NRQCD: no continuum limit $\Rightarrow$ upper limit on T for fixed N_τ
- NRQCD correlator study on $T = 0$ or $N_\tau = 12$ lattices: **extended sources**

  Larsen, et al., ...

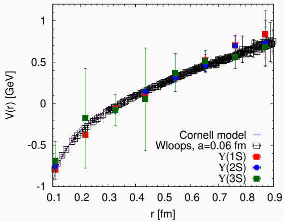
  1. Point source vs Gaussian smearing, new scheme for removing UV part: **thermal width**, small mass shift *PRD 100 (2019)*
  2. Cornell pot. eigenstates $\to$ GEVP $\Rightarrow$ obtain lattice **BS amplitudes**
  3. BS amp. allow recovering potential

$$\left(\frac{-\Delta}{m_b} + V(r)\right)\phi_\alpha = E_\alpha \phi_\alpha$$

PLB 800 (2020) + PRD 102 (2020)

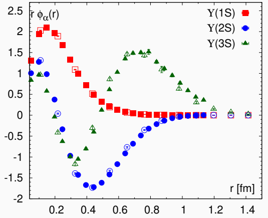
  4. **Small** τ: BS amplitudes at $T = 0$ and $T > 0$ almost ***T*-independent**
  5. **Large** τ: $3S$ BS amp. at $T > 0$ visibly **modified** $\Leftrightarrow$ thermal width

$T = 0$, $a = 0.06$ fm: NRQCD vs static $q\bar{q}$



source: *Larsen, et al., PRD 102, (2020)*

$N_\tau = 12$, $\tau \sim 0.4$ fm
$T = 334$ MeV vs 151 MeV

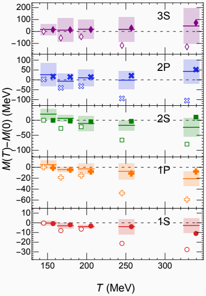


Machine learning the potential from NRQCD amplitudes

Is there room for another interpretation? Let an algorithm figure it out. . .

Machine learning (DNN) applied to lattice BS amp.: $T > 0$ potential

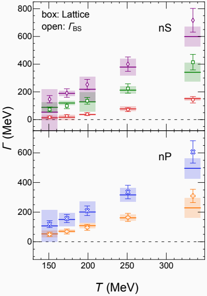
Shi, et al., arXiv:2105.07862



Lattice BS amplitudes fed into DNN ⇒ can reconstruct a nonperturbative potential V^{ML}

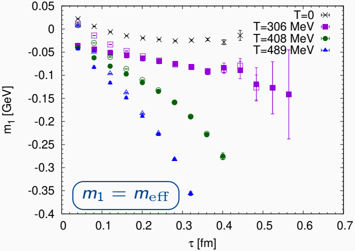
Clearly smaller thermal mass shift and larger width than in **Hard Thermal Loop** (HTL) perturbation theory ($\mathrm{Re}\,(V_s^{\mathrm{HTL}}) \sim F_S$):

$$\mathrm{Re}\,[V^{\mathrm{ML}}] \sim V_s(T=0)$$
$$\mathrm{Im}\,[V^{\mathrm{ML}}] \gg \mathrm{Im}\,[V_s^{\mathrm{HTL}}]$$



Static $q\bar{q}$ pair at $T > 0$ on the lattice

$a \approx 0.04$ fm, $r/a = 6$, $N_\tau = 64, 16, 12, 10$



source: *Bala, et al., arXiv:2110.11659*

- **Static $q\bar{q}$** interaction is encoded in (real-time) **Wilson loops**[a]

$$W_{[r,T]}(t) = \left\langle e^{ig \oint_{r\times t} dz^\mu A_\mu} \right\rangle_{\mathrm{QCD},T}$$

- Stable (ground) state Ω_r exists if

$$\Omega_{[r,T]} \equiv -i \lim_{t\to\infty} \partial_t W_{[r,T]}(t)$$

[a] We use Wilson line correlators in Coulomb gauge.

- Same **spectral functions** yield real- or imaginary-time correlators

$$W_{[r,T]} \begin{pmatrix} t \\ \tau \end{pmatrix} = \int d\omega \begin{pmatrix} e^{+i\omega t} \\ e^{-\omega\tau} \end{pmatrix} \rho_{[r,T]}(\omega)$$

- Motivates **generic decomposition**

$$\rho_{[r,T]}(\omega) = \rho^{\{\Omega;\mathcal{O}(T)\}}_{[r,T]}(\omega) + \rho^{\mathrm{tail}}_{[r,T]}(\omega) + \rho^{\mathrm{UV}}_{[r,T]}(\omega)$$

- UV continuum $\rho^{\mathrm{UV}}_{[r,T]}(\omega)$ is far above **lowest feature Ω + effects of $\mathcal{O}(T)$**

$\Rightarrow$ Guess $\rho^{\mathrm{UV}}_{[r,T]}(\omega)$ via $\rho^{\mathrm{UV}}_{[r,0]}(\omega)$ $\Rightarrow$ subtract

Note: "tail" due to backward propagating UV physics (vacuum excited states) at $\tau \lesssim 1/T$.

Cumulants of spectral functions – what can we expect?

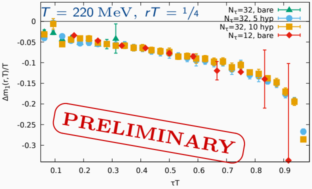
- Access **cumulants of** $\rho_{[r,T]}(\omega)e^{-\omega\tau}$ via τ **(log) derivatives** of $W_{[r,T]}(\tau)$

$$m_1^{[r,T]}(\tau) = -\partial_\tau \ln W_{[r,T]}(\tau) \quad [\equiv m_{\mathrm{eff}}^{[r,T]}(\tau)],$$

$$m_n^{[r,T]}(\tau) = -\partial_\tau m_{n-1}^{[r,T]}(\tau), \quad n > 1$$

- For $N_\tau \leq 16$ obtain up to $m_3^{[r,T]}(\tau)$: supports ≤ 5 parameters for $\rho_{[r,T]}(\omega)$
- Higher cumulants at small τ need at least $N_\tau > 16$: bad signal-to-noise

Fully vacuum-subtracted result



see: *Hoying, et al., arXiv:2110.00565 [hep-lat]*

Feasibility study with $N_\tau = 32$: $m_n^{[r,T]}$, $n > 2$?

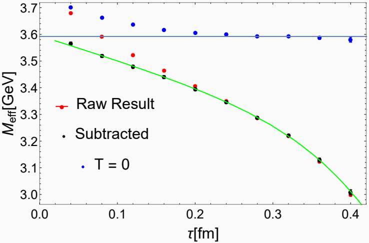
- Fine lattices: $a^{-1} \approx 7\,\mathrm{GeV}$ $m_\pi \approx 0.3\,\mathrm{GeV}$
- UV filtering (HYP) for **noise reduction**

→ distortions cancel in vacuum subtraction

- Definitely still work in progress

Lowest spectral feature from fits using Gaussian approximation

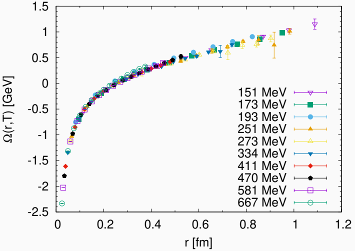
$N_\tau = 12,\ T = 408$ MeV, $r/a = 9$

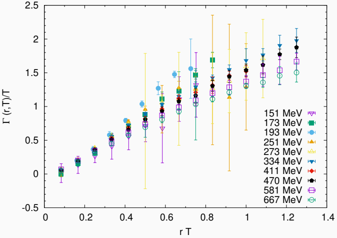


- **Quasiparticles** are represented as **Breit-Wigner** in $\rho_{[r,T]}(\omega)$
- Ansatz: **approximate BW of** $\rho_r^{\{\Omega;\mathcal{O}(T)\}}(\omega)$ locally as **Gaussian**, include delta function for $\rho_r^{\rm tail}(\omega)$

$$W_{[r,T]}(\tau) = A_{[r,T]}^{\{\Omega;\mathcal{O}(T)\}} e^{-\Omega_{[r,T]}\tau + (\Gamma^G_{[r,T]})^2\tau^2/2} + A^{\rm tail}_{[r,T]} e^{-\omega^{\rm tail}_{[r,T]}\tau}, \quad \omega^{\rm tail}_{[r,T]} \ll \Omega_{[r,T]}$$

$N_\tau = 12, \quad \Omega(r,T) \equiv \Omega_{[r,T]}, \quad \Gamma(r,T) \equiv \sqrt{2\ln 2}\,\Gamma^G_{[r,T]},$ subtracted correlators



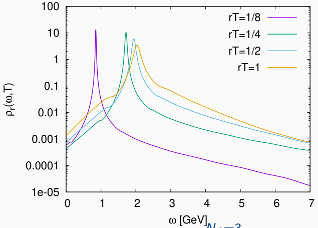


source: *Bala, et al., arXiv:2110.11659*

- Almost no T dependence in $\Omega_{[r,T]}$ (naive correspondence: Re $V_s(r,T)$)
- Naively expected scaling of $\Gamma(r,T)/T \approx \Gamma(rT)/T$ down to $T \approx T_{pc}$

Comparison: lattice QCD vs HTL

HTL spectral function for $T = 667$ MeV



[NLO, 2-loop $\alpha_s(2\pi T)$, $\Lambda_{\overline{MS}}^{N_f=3} = 332$ MeV]

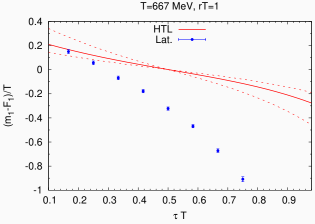
source: *Bala, et al., arXiv:2110.11659*

- **HTL** is an attractive proposition: **motivated & regularized BW**
- **HTL** result is **antisymmetric** around the midpoint $\tau = {}^1/_{2T}$:

$$\log W_{[r,T]}(\tau) = -\mathrm{Re}\, V_s(r,T) \times \tau + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ e^{-\omega\tau} + e^{-\omega(1/T-\tau)} \right\} \times \{1 + n_B(\omega)\}\, \sigma_{[r,T]}(\omega)$$

- Leading **singularity** of $\sigma_{[r,T]}(\omega)$ (transv. gluon spec. fun.) fixes $\mathrm{Im}\, V_s(r,T)$

$N_\tau = 12$, $r/a = 12$, subtracted correlator



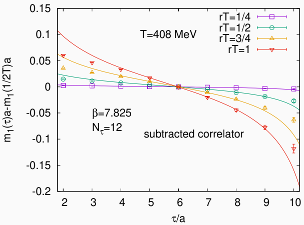
source: *Bala, et al., arXiv:2110.11659*

- **HTL** should work at $r \sim 1/m_D$
- *Subtleties* due to renormalons and regulators: consider $(m_1 - F_S)/T$

  Reminder: $\mathrm{Re}\,[V_s] = F_S + \mathcal{O}(g^4)$ in **HTL**
- No large UV component in HTL, compare UV-subtracted result
- m_1 at midpoint lower than **HTL**, and m_2 is much more negative

Lowest spectral feature from fits using HTL-motivated Ansatz

$N_\tau = 12,\ T = 408\ \mathrm{MeV},\ r/a = 9$

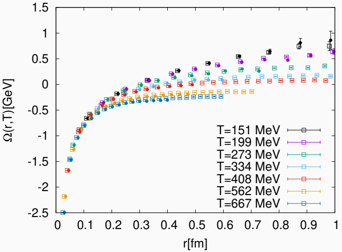


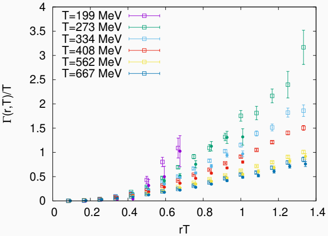
- Fit via HTL-motivated Ansatz
  Bala, Datta, PRD 101 (2020)

$$W_{[r,T]}(\tau) = A^{BD}_{[r,T]} e^{-\Omega^{BD}_{[r,T]}\tau - \mathrm{i}\frac{\Gamma^{BD}_{[r,T]}}{\pi}\log\sin(\pi\tau T)}$$

- Note: similar result via Gaussian around midpoint $\tau = 1/2T$

$N_\tau = 12,\quad \Omega(r,T) \equiv \Omega^{BD}_{[r,T]},\quad \Gamma^{BD}(r,T) \equiv \Gamma^{BD}_{[r,T]},\quad$ (un-)subtracted correlators

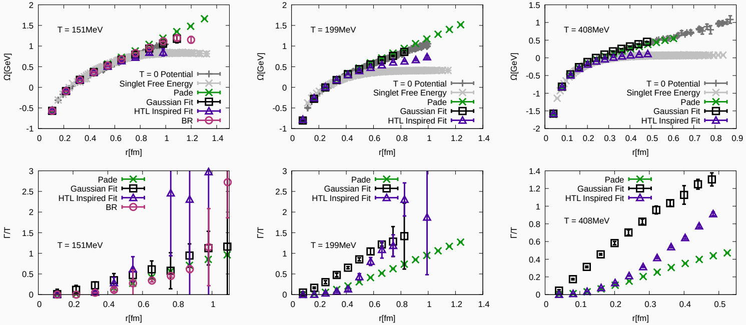




source: *Bala, et al., arXiv:2110.11659*

- Significant T dependence in $\Omega_{[r,T]}$ (naive correspondence: $\mathrm{Re}\, V_s(r,T)$)
- Weaker than naive scaling of $\Gamma(r,T)/T \approx \Gamma(rT)/T$

Comparison: lowest spectral feature from four different methods



source: *Bala, et al., arXiv:2110.11659*

- Applied two further, independent methods (Padé rational approximation, Bayesian reconstruction) not discussed in detail
- $T \approx 150$ MeV conclusive: $\Omega_{[r,T]} \approx F_S(r,T) \approx V_s(r)$ for $r \lesssim 0.8$ fm
- $T \lesssim 250$ MeV: all three methods yield $\Omega_{[r,T]} \gg F_S(r,T)$
- $T \approx 400$ MeV inconclusive: $\Omega^{BD}_{[r,T]} \approx F_S(r,T)$ vs $\Omega^{G}_{[r,T]} \approx \Omega^{P}_{[r,T]} \approx V_s(r)$
- All methods find for all T nontrivial $\Gamma_{[r,T]}$ that increases with r or T

Heavy quarkonium at finite temperature

Modern picture: quarkonium suppression due to dissociation…

- Spatial correlation functions of **relativistic bottomonium**
  - Model-independent study of quarkonium melting in LQCD
  - η_b or $\Upsilon(1S)$ largely unmodified at $T \approx 400\,\text{MeV}$; χ_{b0} or h_b already gone
- **Nonrelativistic bottomonium**
  - Extended sources or BS wave functions boost resolving power of LQCD
  - Spectral features are fully **consistent with static $q\bar{q}$ pair**
- **Static quarkonium ($q\bar{q}$ pair)**
  - Lowest spectral feature $\{\Omega; \mathcal{O}(T)\}$ + tail + UV continuum
  - Model-independent cumulant analysis → clear evidence for a large **thermal width being the main cause of quarkonium melting**
  - Consistent with minimal (Gaussian fit) or major (HTL-motivated fit) medium modification of real part → insufficient resolution with $N_\tau \leq 16$

The lattice + EFT is in good shape to deliver more accurate and more realistic results needed for HIC phenomenology in the coming years.

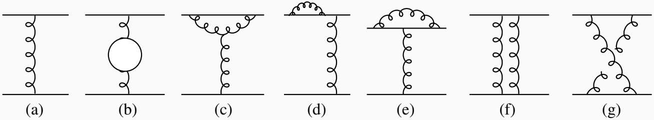
Thank you for your attention!

Jet transport coefficient $\hat{q}$ and collision kernel in weak coupling (I)

- Jet transport coefficient $\hat{q}$ is truncated integral of collision kernel $C(k_\perp)$

$$\hat{q}(k_\perp^{\max}) = \hat{q}_{\rm soft}(k_\perp^*) + \hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\max}) \equiv \int_0^{k_\perp^*} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \boldsymbol{k}_\perp^2 C_{\rm soft}(k_\perp) + \int_{k_\perp^*}^{k_\perp^{\max}} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \boldsymbol{k}_\perp^2 C_{\rm hard}(k_\perp)$$

- Soft contribution $(k_\perp \ll T)$ to collision kernel known up to $\mathcal{O}(g^6)$ in HTL



$$C_{\rm soft}(k_\perp) = g^2 T C_R \left\{ \frac{m_D^2}{k_\perp^2 \left(k_\perp^2 + m_D^2\right)} \right\} + g^4 T^2 C_R N_c \times \left\{ \frac{7}{32 k_\perp^3} + \frac{m_D}{4\pi \left(k_\perp^2 + m_D^2\right)} \left[\frac{3}{k_\perp^2 + 4m_D^2} - \frac{2}{k_\perp^2 + m_D^2} - \frac{1}{k_\perp^2} \right] \right.$$

$$\left. - \frac{k_\perp m_D + 2\left(k_\perp^2 - m_D^2\right)\arctan(\frac{k_\perp}{m_D})}{4\pi k_\perp \left(k_\perp^2 + m_D^2\right)^2} + \frac{m_{\rm E}^2 \arctan(\frac{k_\perp}{2m_D})}{2\pi k_\perp^3 \left(k_\perp^2 + m_D^2\right)} + \frac{2k_\perp m_D - \left(k_\perp^2 + 4m_D^2\right)\arctan(\frac{k_\perp}{2m_D})}{16\pi k_\perp^5} \right\} + \mathcal{O}(g^6)$$

Caron-Huot, PRD 79 (2009)

$$\hat{q}_{\rm soft}(k_\perp^*) = \frac{g^2 T m_D^2 C_R}{2\pi} \ln \frac{k_\perp^*}{m_D} + \frac{g^4 T^2 m_D C_R N_c}{2\pi} \left\{ -\frac{k_\perp^*}{16 m_D} + \frac{3\pi^2 + 10 - 4\ln 2}{16\pi} + \mathcal{O}\left(\frac{m_D}{k_\perp^*}\right) \right\} + \mathcal{O}(g^6)$$

Jet transport coefficient $\hat{q}$ and collision kernel in weak coupling (II)

$$\hat{q}(k_\perp^{\max}) = \hat{q}_{\rm soft}(k_\perp^*) + \hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\max}) \equiv \int_0^{k_\perp^*} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}\boldsymbol{k}_\perp^2\, C_{\rm soft}(k_\perp) + \int_{k_\perp^*}^{k_\perp^{\max}} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}\boldsymbol{k}_\perp^2\, C_{\rm hard}(k_\perp)$$

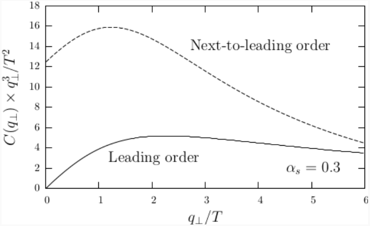
- Hard contribution ($k_\perp \gtrsim T$) to collision kernel known up to $\mathcal{O}(g^6)$ as well

$$\hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\max}) = g^4 T^3 C_R \left\{ \frac{N_c}{6\pi}\left[\log\left(\frac{T}{k_\perp^*}\right) + \frac{\zeta(3)}{\zeta(2)}\log\left(\frac{k_\perp^{\max}}{T}\right) - 0.06885\ldots + \frac{3}{16}\frac{k_\perp^*}{T} + \ldots\right]\right.$$
$$\left. + \frac{N_f T_f}{6\pi}\left[\log\left(\frac{T}{k_\perp^*}\right) + \frac{3}{2}\frac{\zeta(3)}{\zeta(2)}\log\left(\frac{k_\perp^{\max}}{T}\right) - 0.07286\ldots + \ldots\right]\right\}$$

Arnold, Xiao, PRD 78 (2008)

- $\hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\max} \to \infty)$ is finite
- Cancellation of $k_\perp^*$ dependence between $\hat{q}_{\rm soft}(k_\perp^*)$, $\hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\max})$
- Soft $\mathcal{O}(g^5)$ exceeds $\mathcal{O}(g^4)$ term $\Rightarrow$ expansion in g converges poorly
- Contributions at $\mathcal{O}(g^6)$ from magnetic scale ($g^2 T$) are small

Laine, EPJC 72 (2012)

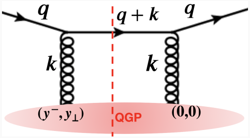


source: *Caron-Huot, PRD 79 (2009)*

LO is accidentally small, NLO is regularly large, and the non-perturbative magnetic contribution is hardly relevant – is this the **end of the story**???

Hard jet scattering a QGP brick

- Uniform **QGP brick** of length L, temperature T, Debye mass m_D
- A **jet** with energy E, virtuality Q traverses this QGP: $E \gg Q \gg T, m_D$
- On-medium scattering is dominated by **one-gluon exchange** (OGE)



source: *arXiv:2010.14463*

- Regularize integrals in finite box $V = L^3$, interaction time $T_I = L/c$
- The average momentum broadening among N_e OGE scattering events is

$$\hat{q} = \sum_i^{N_e} \frac{[k_\perp^i]^2}{N_e T_I}$$

$\hat{q}$ w. tree-level collision kernel as near light-cone **field-strength correlator**

$$\hat{q} = c_0 \int \frac{dy^- d^2y_\perp}{(2\pi)^3} d^2k_\perp \exp\left[-i\frac{\boldsymbol{k}_\perp^2}{2q^-}y^- + i\boldsymbol{k}_\perp.\boldsymbol{y}_\perp\right] \times \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \operatorname{tr}\left[F^{+j}(0)[\ldots]F_j^+(y^-, y_\perp)\right]|n\rangle ,$$

- $c_0 \propto g^2 C_R$ known perturbatively
- Events $\xrightarrow{\text{ergodicity}}$ inital states
- $[\ldots]$ = omitted Wilson lines
- $j = \{1, 2\}$ transverse directions

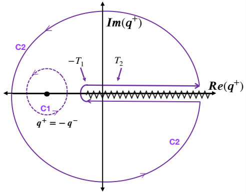
Enable lattice calculation via OPE of a generalized coefficient $\hat{Q}(q^+)$

$$\hat{Q}(q^+) = c_0 \int \frac{d^4y d^4k}{(2\pi)^4} \frac{2q^- e^{ik\cdot y}}{(q+k)^2 + i\epsilon} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \operatorname{tr}\left[F^{+j}(0)F_j^+(y)\right]|n\rangle$$

OPE in deep space-like region & integration via contour deformation

$$\hat{Q}(q^+) = c_0 \int \frac{d^4y d^4k}{(2\pi)^4} \frac{2q^- e^{ik\cdot y}}{(q+k)^2 + i\epsilon} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n | \operatorname{tr}\left[F^{+j}(0) F_j^+(y)\right] | n \rangle$$

- $-T_1$, T_2 bound the **thermal discontinuity** of $q^+ = k^+ + k_\perp^2/2(q^- + k^-)$
- **vacuum discontinuity** (real hard gluon radition): subtract vacuum



source: dissertation A. Kumar

$$I = \oint_{C_1} \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{q^+ + q^-} = \hat{Q}(q^+ = -q^-)$$

$$= \int_{-T_1}^{T_2} \frac{dq^+}{2\pi i} \frac{Disc\left[\hat{Q}(q^+)\right]}{q^+ + q^-}$$

$$+ \int_0^\infty \frac{dq^+}{2\pi i} \frac{Disc\left[\hat{Q}(q^+)\right]}{q^+ + q^-}$$

$\hat{Q}(q^+ \approx -q^-) \Rightarrow$ turn $1/(q+k)^2$ into **geometric series** in deep space-like region

$$\hat{Q}(q^+ = -q^-) = \frac{1}{q^-} c_0 \sum_{m=0}^{\infty} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n | \operatorname{tr}\left[F^{+j}(0) \left[\frac{i\sqrt{2}D_3}{q^-}\right]^m F_j^+(0)\right] | n \rangle$$

Gauge-invariant, local OPE for $\hat{q}$ – in imaginary time

$$\frac{\hat{q}}{T^3} = \sum_{m=0}^{\infty} \left[\frac{T}{q^-}\right]^{2m} c_0 \frac{T}{T_1+T_2} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{+j}(0)\Delta^{2m}F_j^{+}(0)\right]}{T^4} |n\rangle_{T-V} \quad \text{with} \quad \Delta \equiv \mathrm{i}\sqrt{2}D_3/T$$

- Thermal disc.: width $= T_1+T_2 \simeq 2\sqrt{2}T$; $[T/q^-]^{2m}$ suppresses **higher twist**
- **Parity xor time reflection odd terms vanish for QGP at rest**

Wick rotation: $x^0 \to ix^4, \quad A^0 \to iA^4 \Rightarrow F^{0j} \to iF^{4j}, \quad F^{+j} \to iF^{4j} - F^{3j}$

$$\Rightarrow F^{+j}\Delta^{2m}F^{+j} \to \left[F^{3j}\Delta^{2m}F^{3j} - F^{4j}\Delta^{2m}F^{4j}\right] + i\left[F^{3j}\Delta^{2m}F^{4j} + F^{4j}\Delta^{2m}F^{3j}\right]$$

$$O_m \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}(0)\Delta^{2m}F^{3j}(0) - F^{4j}(0)\Delta^{2m}F^{4j}(0)\right]}{T^4} |n\rangle_{T-V}$$

- Applies for **both pure gluon plasma or full QGP** at this stage
- Leading-twist operator O_0 is just **gluon contribution** to non-singlet component of **energy-momentum-tensor** (EMT) $T_G^{(9)}$ (up to $T_F = 1/2$)
- In **QCD**, $T_G^{(9)}$ mixes with contribution from **sea quarks** $T_Q^{(9)}$
- Continuum EMT conserved, i.e. $T_G^{(9)}$ in pure gauge, or $T_G^{(9)} + T_Q^{(9)}$ in QCD

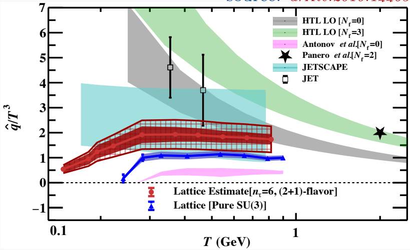
$\Rightarrow$ lattice EMT needs renormalization; in QCD scheme dependent $O_0 \Rightarrow \hat{q}$

Quantitative comparison: HTL, lattice, models, phenomenology

$$\frac{\hat{q}_\infty}{T^3} \equiv \frac{\hat{q}}{T^3}\left(q^- \to \infty\right) = \underbrace{c_0 \frac{T}{T_1+T_2}}_{=\pi\ C_R\ g_R^2} \underbrace{\sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}F^{3j} - F^{4j}F^{4j}\right]}{T^4} |n\rangle_{T-V}}_{=O_0}$$

- 1-loop, $N_f = \{0,3\}$, $\overline{\mathrm{MS}}$ running coupling $g_R^2(\mu)$, scale $\mu = (2\ldots4)\pi T$
- Volume $V \cdot T^3 = {}^{N_\sigma}/_{N_\tau} = 4$ for $T > 0$
- $T = 0$ ensembles with $N_\tau = N_\sigma$
- **Pure gauge**: $0.2\,\mathrm{GeV} \lesssim T \lesssim 1\,\mathrm{GeV}$
- **QCD** $N_\tau = 6$: $0.15\,\mathrm{GeV} \lesssim T \lesssim 0.8\,\mathrm{GeV}$
- $\hat{q}_\infty/T^3$ is nearly **flat** at $T > 0.3\,\mathrm{GeV}$

source: *arXiv:2010.14463*

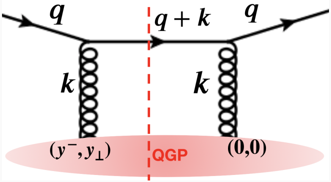


- **Phenomenological** results by **JET** & **JETSCAPE** collaborations
  Burke, et al. (JET), PRC 90 (2014); Soltz (JETSCAPE), PoS HardProbes 2018
- **Stochastic vacuum model** $N_f = 0$ (lattice input): Landau damping
  Antonov, Pirner, EPJC55 (2008)
- **Soft contribution in** $N_f = 2$ **EQCD**: $\hat{q}_{\mathrm{soft}} \approx \hat{q}_{\mathrm{NLO}}\big|_{m_D^{\mathrm{LO}} \to m_D^{\mathrm{NP}}}$
  Panero, et al., PRL 112 (2014)
- **LO HTL** with $q^- = 100\,\mathrm{GeV}$ for $N_f = 0$ or $N_f = 3$

Explict derivation of $\hat{q}$

- **Ergodicity** implies that we can replace an average over N_e events by an average over **Boltzmann-weighted initial medium states**

$$\hat{q} = \sum_{n,X} \frac{e^{-\beta E_n}}{Z T_I} \int d^2 k_\perp \, k_\perp^2 \, \frac{d^2 W^{n,X}(k_\perp)}{d^2 k_\perp}$$



source: *arXiv:2010.14463*

- SCET power counting of A_μ fields:

$$A_\mu^+ \sim \lambda^2 Q, \quad A_\mu^\perp \sim \lambda^3 Q$$

- Roll over $k_\perp^2$ to derivatives $\partial_\perp$ on A_μ
- Prop up to field strength tensors F_j^+

  from here: transverse comp. $j \in \{1, 2\}$

- Use finite volume wave-functions for the spinors
- Shift one vertex to origin, eliminate $\int d^4 x$ against prefactor $1/VT_I$
- Eliminate final medium state $|X\rangle$ via completeness $1 = \sum_X |X\rangle \langle X|$
- k^+-integral is eliminated by on-shell delta function $\delta[k^+ - k_\perp^2/(2q^-)]$
- Integrate over $k^- \ll q^- \Rightarrow$ delta function $2\pi\delta[y^+]$, integrate $y^+ \to 0$

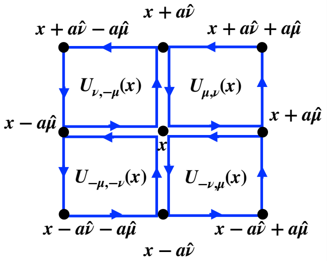
Sufficiently-improved field-strength operator on the lattice

- Plaquette operator is most simple, but has real and imaginary parts

$$U_{\mu,\nu}(x) = \exp\left[a^2 i g_0 F_{\mu\nu}(x)\right] + \mathcal{O}(a^3)$$

- Clover operator is more symmetric, suppresses lattice artifacts

$$Q_{\mu\nu} = \frac{U_{\mu,\nu} + U_{\nu,-\mu} + U_{-\mu,-\nu} + U_{-\nu,\mu}}{4} = \exp\left[a^2 i g_0 F_{\mu\nu}\right] + \mathcal{O}(a^4)$$



source: dissertation A. Kumar

- Traceless-antihermitean **projection**

$$[Q]_{\mathrm{TA}} = \frac{Q - Q^\dagger - \frac{\mathrm{tr}\,[Q - Q^\dagger]}{N_c}}{2}$$

- Weak-coupling picture dysfunctional in practice – $[Q_{\mu,\nu}]_{\mathrm{TA}} \not\simeq a^2 i g_0 F_{\mu\nu}$
- **Tadpole improvement** with factor $u_0 = \sqrt[4]{\langle \mathrm{Tr}[U_{\mu,\nu}] \rangle / N_c} \qquad (\lim_{a,g_0 \to 0} u_0 = 1)$

- Traceless-antihermitian projected, tadpole-improved clover operator

$$i g_0 \mathcal{F}_{\mu\nu}(x) = \frac{[Q_{\mu\nu}(x)]_{\mathrm{TA}}}{a^2 u_0^4} = i g_0 F_{\mu\nu} + \mathcal{O}(a^2)$$

Renormalization and mixing on quenched lattices

$$O_m \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}(0)\Delta^{2m}F^{3j}(0) - F^{4j}(0)\Delta^{2m}F^{4j}(0)\right]}{T^4} |n\rangle_{T-V}$$

- **Reduced symmetry group** in the lattice formulation: $SO(4) \overset{\text{broken}}{\to} SW_4$

$\Rightarrow$ Analyze operators in terms of irreps of SW_4 of given mass dimension

Gluonic contributions to EMT	dim.	irrep.	–
$\mathbf{1}\ \delta_{\mu\nu}$	**0**	**singlet**	cancels against $T=0$
$T_F{}^1/4\ \mathrm{tr}\ [F^{\mu\rho}F^{\mu\rho}]\ \delta_{\mu\nu}$	**4**	**singlet**	mixes with dim $=0$
$T_F \mathrm{tr}\ [F^{\mu\rho}F^{\mu\rho} - F^{\nu\rho}F^{\nu\rho}]\ [1-\delta_{\mu\nu}]$	**4**	**triplet**	$O_0 \equiv T_F\, T_G^{(3)}$
$T_F \mathrm{tr}\ [F^{\mu\rho}F^{\nu\rho} + F^{\nu\rho}F^{\mu\rho}]\ [1-\delta_{\mu\nu}]$	**4**	**sextet**	vanishes at rest

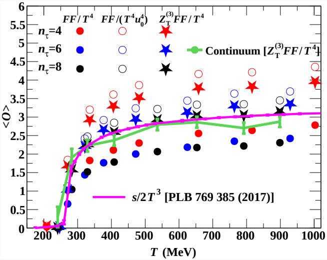
- EMT components on lattice need **renormalization** $T_G^{(3)R} = Z_T^{(3)} T_G^{(3)B}$
- $Z_T^{(3)} \equiv z_T Z_T^{(6)}$: non-perturbative finite momentum **Ward Identities** (WI)
  $z_T, Z_T^{(6)}$ *in* $\overline{\mathrm{MS}}$ *for plaquette action from Giusti, Pepe, PRD 91 (2015); PLB 769 (2017)*
- **Higher-twist**: $O_m,\ m \geq 1$ mix with lower-twist, T-dependent operators
  $\Rightarrow$ **no cancellation** vs $T=0$! mixing not studied systematically yet

$\Rightarrow$ Consider $q^- \to \infty$ first: higher-twist operators do **NOT** contribute!

Leading-twist operator for a pure gauge plasma

$$O_0 \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}F^{3j} - F^{4j}F^{4j}\right]}{T^4} |n\rangle_{T-V}$$

- We use Wilson plaquette action in pure gauge theory
- The renormalized lattice result
  $$O_0^R[a(g_0^2), T = 1/N_\tau a(g_0^2)] = Z^{(3)}(g_0^2) O_0^B[a(g_0^2), T = 1/N_\tau a(g_0^2)]$$
  is extrapolated at fixed T to the continuum via $1/N_\tau^2 = (aT)^2 \to 0$
- For $N_\tau > 4$: cutoff effects $\sim 10\%$



source: *arXiv:2010.14463*

Renormalized triplet comp. in rest frame ⇒ entropy density $T^{(3)R} = Ts = 2O_0^R$

Common approximation in lattice gauge theory: use tadpole factors

$$u_0(g_0^2) = \sqrt[4]{\langle \mathrm{tr}\ [U_{\mu,\nu}]\rangle / N_c}$$

in place of renormalization factor: $1/u_0^4(g_0^2) \approx Z_T^{(3)}(g_0^2)$: overestimation $\sim 10\%$

QCD: explicit sea quarks as uninvited guests

$$O_0 \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}F^{3j} - F^{4j}F^{4j}\right]}{T^4} |n\rangle_{T-V}$$

- Symmetry breaking $SO(4) \overset{\text{broken}}{\rightarrow} SW_4$ in QCD as in pure gauge theory

$\Rightarrow$ Quark contributions in same irreps of SW_4: $T_Q^{(1)} \rightarrow T_Q^{(1)}$, $T_Q^{(9)} \rightarrow \{T_Q^{(3)}, T_Q^{(6)}\}$

Fermionic contribution to EMT	dim.	irrep.	–
$\mathbf{1}\ \delta_{\mu\nu}$	0	singlet	cancels against $T = 0$
$m\ \bar{\psi}\psi\ \delta_{\mu\nu}$	4	singlet	mixes with dim $= 0$
$\bar{\psi}\left[\gamma_\mu D_\mu - \gamma_\nu D_\nu\right]\psi\ [1 - \delta_{\mu\nu}]$	4	triplet	mixes with $T_G^{(3)}$
$\bar{\psi}\left[\gamma_\mu D_\nu + \gamma_\nu D_\mu\right]\psi\ [1 - \delta_{\mu\nu}]$	4	sextet	vanishes at rest

- Renormalization of EMT in QCD requires complete mixing matrix

$$\begin{pmatrix} T_G^{(3)R} \\ T_Q^{(3)R} \end{pmatrix} = \mathcal{Z} \begin{pmatrix} T_G^{(3)B} \\ T_Q^{(3)B} \end{pmatrix}, \quad \mathcal{Z} \equiv \begin{pmatrix} \mathcal{Z}_{GG}^{(3)} & \mathcal{Z}_{GQ}^{(3)} \\ \mathcal{Z}_{QG}^{(3)} & \mathcal{Z}_{QQ}^{(3)} \end{pmatrix}$$

1. Still missing the bare quark contribution $\Rightarrow$ straightforward to compute
2. All four renormalization factors unknown for choice of action $\Rightarrow$ could obtain 2 out of 4 via finite momentum WI via QCD in a moving frame

Dalla Brida, et al., JHEP 04 (2020)

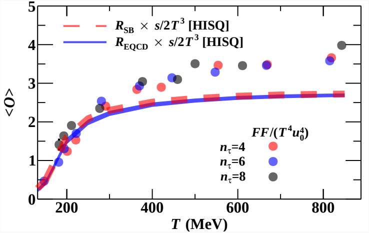
QCD: estimating the influence of the sea quarks

$$O_0 \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}F^{3j} - F^{4j}F^{4j}\right]}{T^4} |n\rangle_{T-V}$$

- We use Symanzik + HISQ action in (2+1)-flavor QCD
- Mixing between $T_G^{(i)}$ and $T_Q^{(i)}$: N_f-dependent coefficients smaller than N_f independent ones $Z_T^{(i)}$ at 1-loop level (plaquette+Wilson)

  Dalla Brida, et al., JHEP 04 (2020)

$\Rightarrow$ mixing $\sim 10\%$ correction to $T_G^{(i)R}$



source: *arXiv:2010.14463*

Compare o_0^B/u_0^4 to entropy density $T^{(3)R} = Ts$ rescaled by weak-coupling ratio

$$R\left(\frac{T}{T_c}\right) = \frac{[s^{N_f=0}/T^3]\,(T/T_c)}{[s^{N_f=3}/T^3]\,(T/T_c)} \quad \text{using} \quad T_c \approx \begin{cases} 270\text{ MeV} & N_f = 0 \\ 155\text{ MeV} & N_f = 3 \end{cases}$$

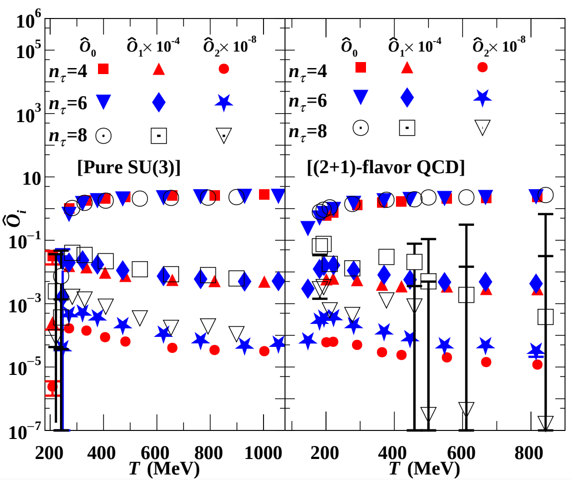
N_f-dependent "critical" temperatures and $T_F = 1/2$; deviation $\lesssim 30\%$

We employ $N_\tau = 6$ assuming $\mathcal{Z}_{GG}^{(3)} \approx 1/u_0^4$, $\mathcal{Z}_{GQ}^{(3)} \approx 0$, 30% systematic error

Higher twist on the lattice (I)

$$O_m = \frac{1}{q^-} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n|\, \mathrm{tr} \left[F^{3j}(0) \Delta^{2m} F^{3j}(0) - F^{4j}(0) \Delta^{2m} F^{4j}(0) \right] |n\rangle \quad \text{suppressed by} \quad \left[\frac{T}{q^-} \right]^{2m}$$

assuming hard parton of $q^- = 100$ GeV



source: *arXiv:2010.14463*

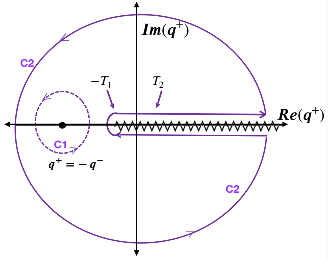
Higher twist on the lattice (II)

$$O_m = \frac{1}{q^-} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \,\mathrm{tr}\, \left[F^{3j}(0) \Delta^{2m} F^{3j}(0) - F^{4j}(0) \Delta^{2m} F^{4j}(0) \right] |n\rangle$$

- Dimension six operators have many more SW_4 irreps – not fully sorted
- Covariant derivatives via finite differences on the lattice

⇒ Gauge-field dependence implies different renormalization of hopping and on-site terms, giving rise to obvious power-law divergences of form

$$O_1^{\mathrm{B}} \equiv X_6^{\mathrm{B}} = \#_{6,i} Z_{6,i}^{-1} X_{6,i}^{\mathrm{R}}(T) + \#_{4,i} Z_{4,i}^{-1} X_{4,i}^{\mathrm{R}}(T)\, a^{-2} + \#_{1,i} \mathbf{1}\, a^{-6}$$



source: dissertation A. Kumar

- Instead could use $q^+ = -xq^-$, $x \sim 1$

$$\frac{1}{(q+k)^2} \simeq \frac{-1}{2x(q^-)^2} \sum_{m=0}^{\infty} \left[\frac{\frac{1-x}{x} k_0 + \frac{1+x}{x} k_3}{\sqrt{2} q^-} \right]^m$$

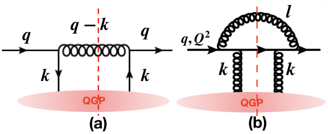
- Linear combination of different integrals could alleviate mixing

Missing contributions

- Hard parton at NLO interacting with non-perturbative medium

$\Rightarrow$ Quite possible that NLO contribution is larger than the tree-level one

- **Flavor-changing scattering** diagrams generate corrections, suppressed by odd powers $[T/q^-]^{2m+1}$ $\Rightarrow$ more relevant than O_m, $m > 0$.
- **not even considered yet** in published results using strict HTL perturbation theory or phenomenology
- Contribute both in QCD and in pure gauge theory: a presence of parton's flavor in the quark sea not necessary (but certainly important)



source: *arXiv:2010.14463*

- Emissions within the medium yield more complicated lattice operators

$\Rightarrow$ similar issues as higher-twist ops.?

All approaches so far consider only **independent scattering** events.