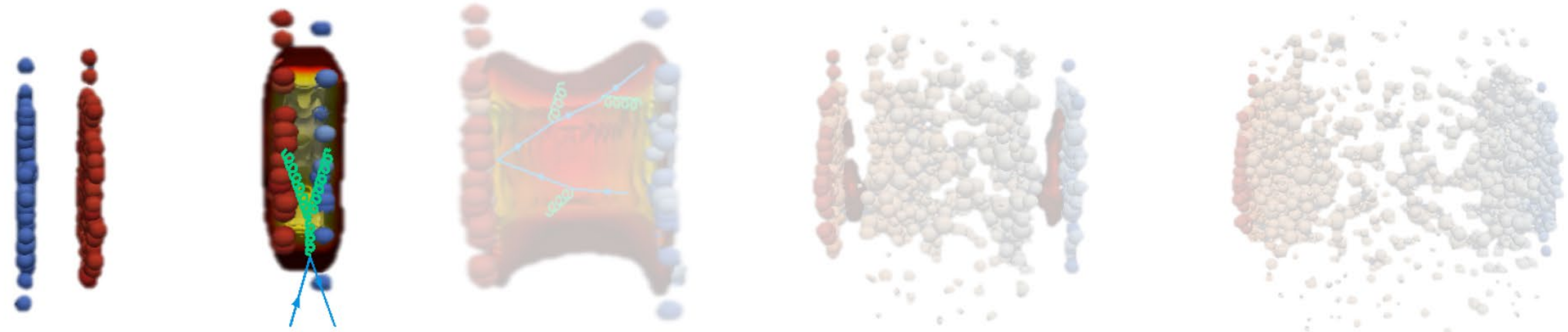
Data-driven analysis for the heavy-quark diffusion coefficient in HIC

Wenkai Fan, Yingru Xu, Steffen A. Bass

Duke University

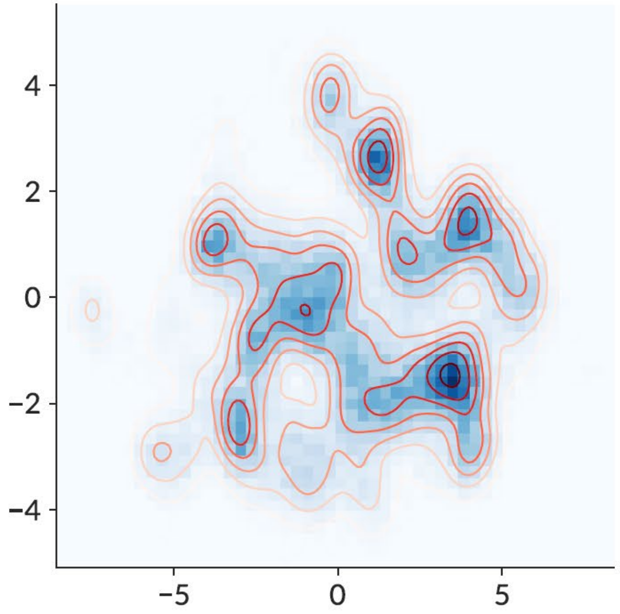
This work has been supported by the U.S Department of Energy under grand DE-FG02-05ER41367. Computational resources were provided by the Open Science Grid (OSG) and NERSC.

PART 1 **Simulation Setup**



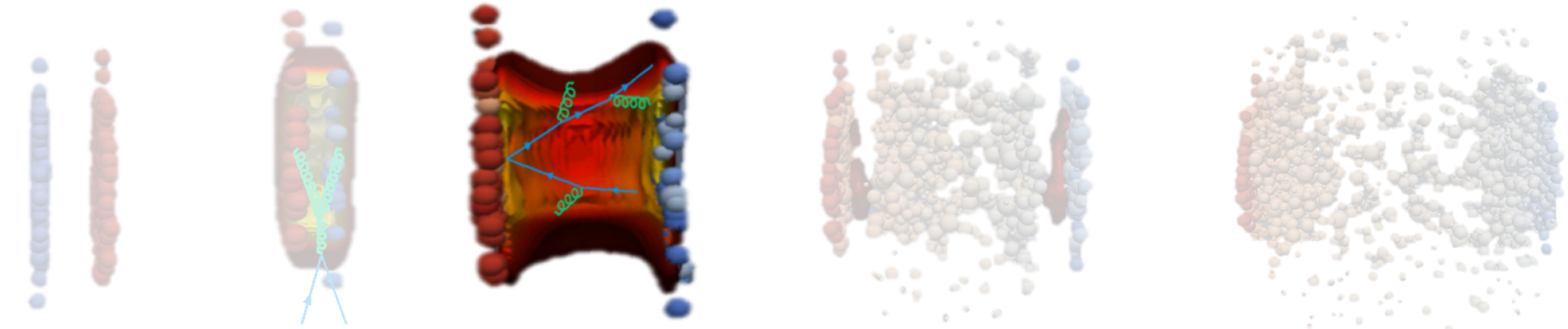
Initial conditions

- Soft matter: T_RENTo
  - Entropy deposition proportional to empirical parametrization: $\frac{ds}{dy}|_{\tau=\tau_0} \propto \sqrt{T_A T_B}$, T_A (nucleon thickness function)
- Heavy quarks
  - Position space: binary collision density
  - Momentum space: (initial hard scattering) Fixed-Order + Next-to-Leading Log (FONLL)



J.S.Moreland,J.Bernhard, and S.A.Bass, Phys.Rev.C 92, 011901(2015)

PART 1 Simulation Setup



Soft medium evolution

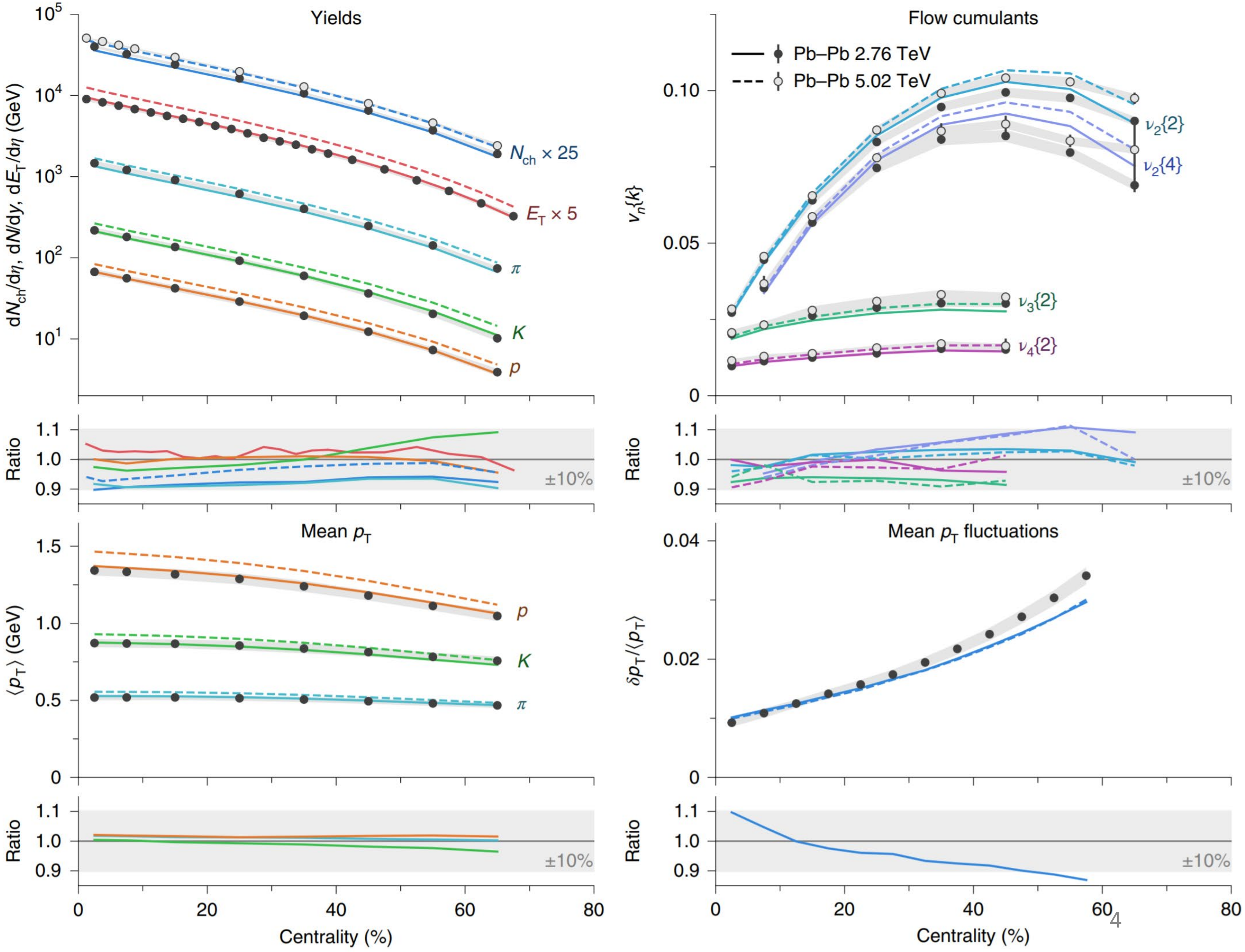
- Event-by-event (2+1)D viscous hydrodynamic model: `iEbE-VISHNU`
  - Shear and bulk viscosities: $\eta/s(T), \zeta/s(T)$
  - All the soft medium related parameters are calibrated on soft hadronic observables by Bayesian analysis

H.Song and U.W.Heinz, Phys.Rev.C 77, 064901(2008)

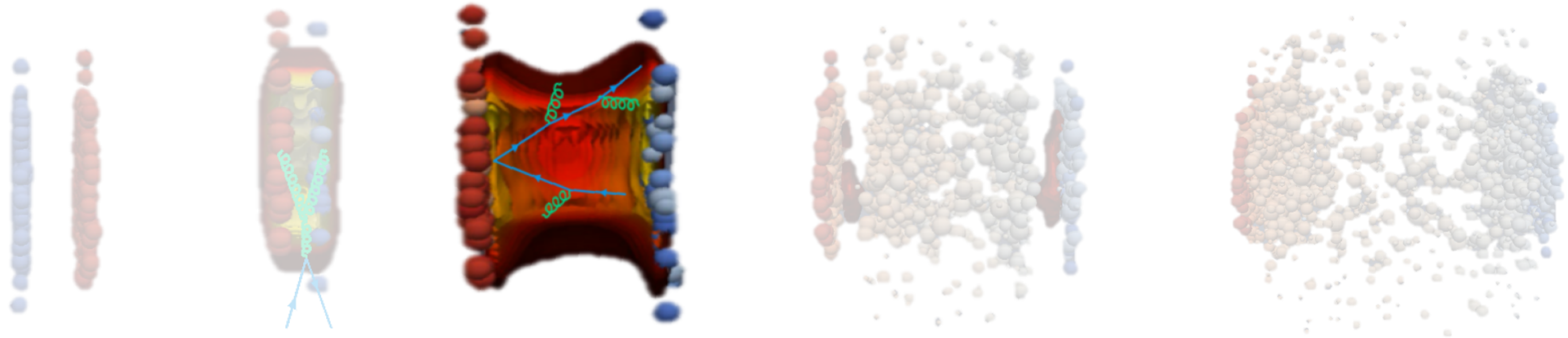
J.Bernhard,J.S.Moreland,S.A.Bass,J.Liu, and U.Heinz Phys.Rev.C 94, 024907(2015)

PART 1 Simulation Setup

Bulk profile calibrated on several soft observables at two collision energys and different centralities. *[Nature Physics* 15.11 (2019): 1113-1117]



PART 1 **Simulation Setup**



Heavy quark in-medium transport

- Model A: improved Langevin model
- Model B: `Lido` - linearized Boltzmann + diffusion model

PART 1 **Simulation Setup - Improved Langevin Equation**

$$\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi} + \vec{f}_g$$

- **Drag force:**

$$\eta_D(p) = \kappa/(2TE)$$

- **Thermal random force:**

$$\left\langle \xi^i(t)\xi^j(t')\right\rangle = \kappa\delta^{ij}\delta(t-t')$$

- **Recoil force from gluon emission:**

$$\vec{f}_g = -\mathrm{d}\vec{p}_g/dt$$

$$\frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s P(x) C_A/C_F}{\pi k_\perp^4}\hat{q}\sin^2\left(\frac{t-t_i}{2\tau_f}\right)\left(\frac{k_\perp^2}{k_\perp^2 + x^2M^2}\right)^4$$

where $\eta_D(p) \approx \frac{\kappa}{2TE}, \kappa = \kappa_\perp = \kappa_\parallel, \hat{q} = 2\kappa$

[S.Cao, G.Qin, and S.A.Bass, Phys.Rev.C 92, 024907(2015)]

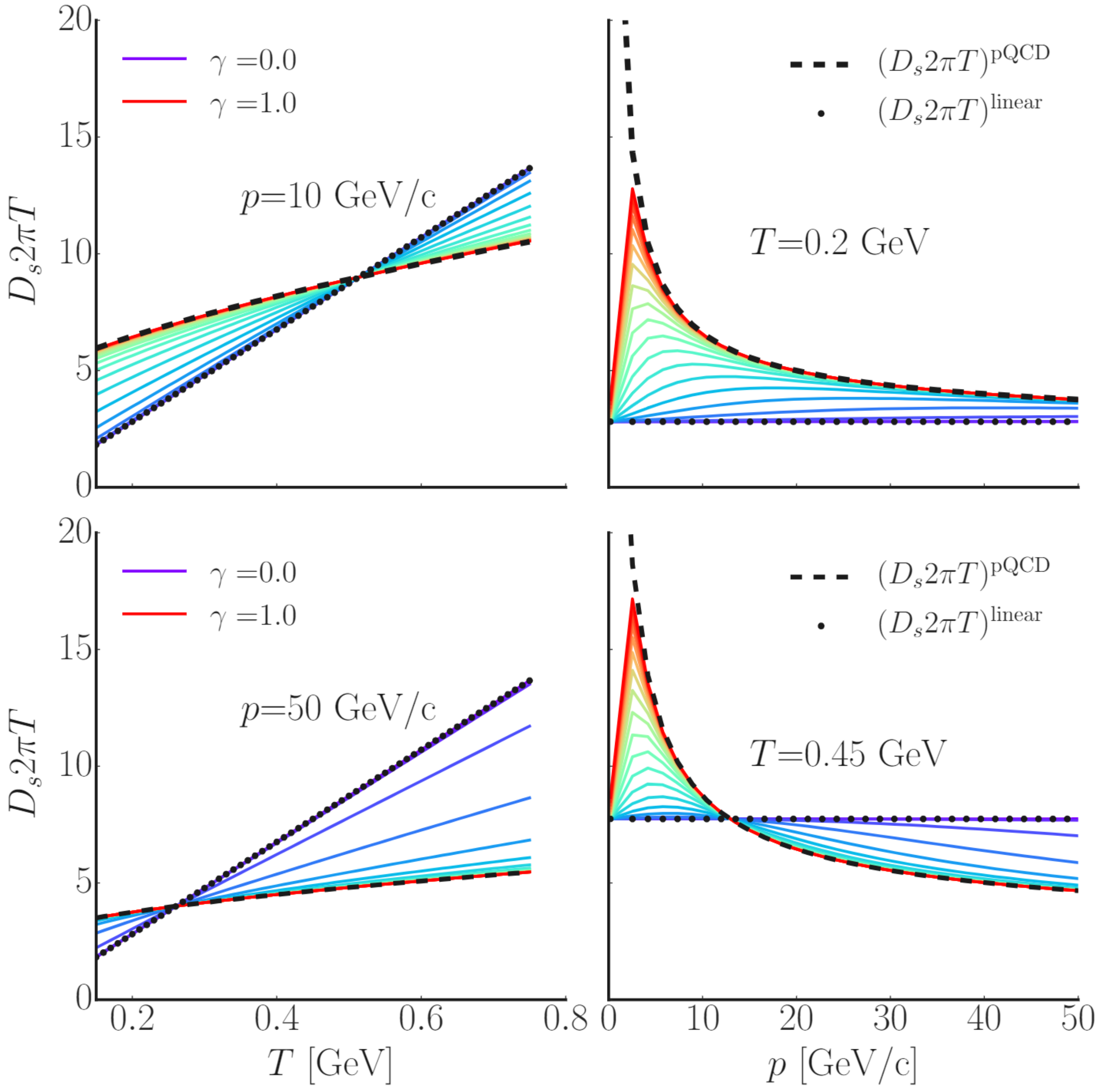
PART 1 **Improved Langevin Equation**

$$D_s 2\pi T = 8\pi/(\hat{q}/T^3)$$

$$D_s 2\pi T = \frac{1}{1+(\gamma^2 p)^2}(D_s 2\pi T)^{soft} + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2}(D_s 2\pi T)^{pQCD}$$

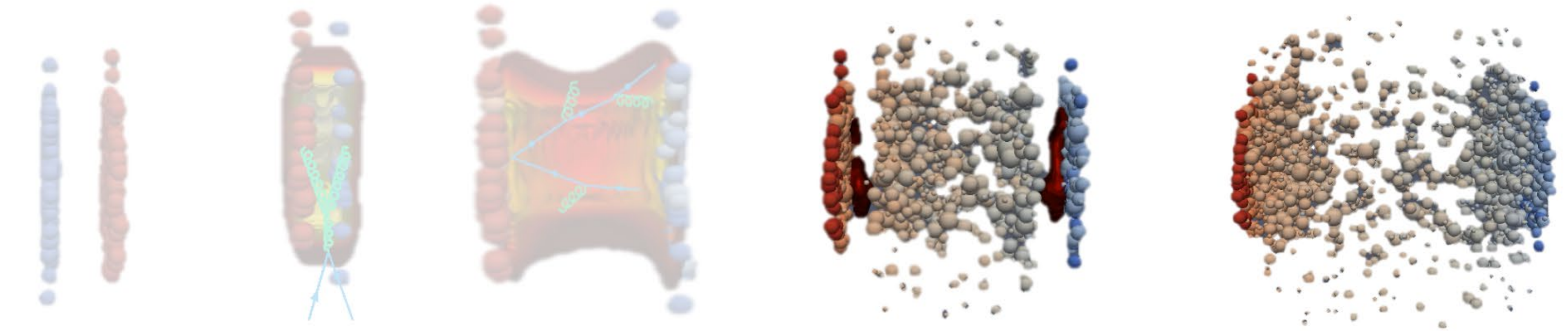
$$(D_s 2\pi T)^{soft} = \alpha(1+\beta(\frac{T}{T_c}-1))$$

γ ranges from 0 to 1 and controls the relative magnitude between $(D_s 2\pi T)^{soft}$ and $(D_s 2\pi T)^{pQCD}$.



PART 1

Simulation Setup



Hadronization/particalization

- Soft medium: particlization (hydrodynamic model $\to$ hadron gas) at T_{switch}
- $c \to D$-meson, charmed baryons at $T_c = 154$ MeV:
  combined model of recombination and fragmentation

Hadronic re-scattering

S. A. Bass et al., Prog. Part. Nucl. Phys. 41 (1998)

M. Bleicher et al. J. Phys. G: Nucl. Part. Phys. 25 (1999)

- UrQMD: solving the Boltzmann equation of hadron scattering
- D-mesons scatter with π, ρ:
  $\pi D \to \pi D, \pi D^* \to \pi D^*, \pi D \leftrightarrow \rho D^*$
  $\rho D \to \rho D, \rho D^* \to \rho D^*, \rho D \leftrightarrow \pi D^*$

Z.-W. Lin, T. Di, and C. Ko, Nucl. Phys. A689, 965 (2001)

PART 1

Simulation Setup - Heavy quark hadronization and hadronic re-scattering

Heavy quark hadronization: **fragmentation** (high p_T) + **recombination** (low p_T)

Momentum spectra of recombined mesons and baryons:

$$\frac{dN_M}{d^3p_M} = \int d^3p_1 d^3p_2 \frac{dN_1}{d^3p_1}\frac{dN_2}{d^3p_2} f_M^W(\vec{p}_1,\vec{p}_2)\delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2)$$

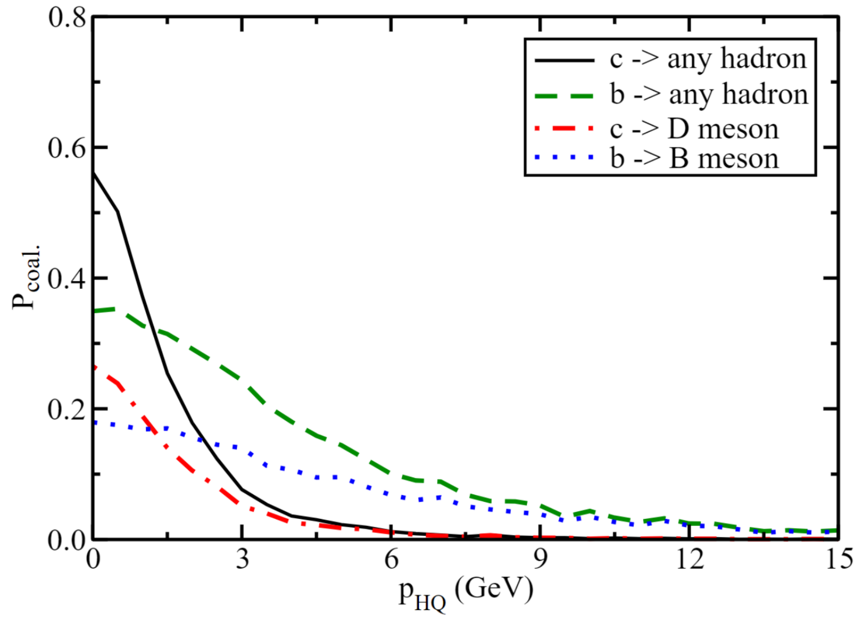
$$\frac{dN_B}{d^3p_B} = \int d^3p_1 d^3p_2 d^3p_3 \frac{dN_1}{d^3p_1}\frac{dN_2}{d^3p_2}\frac{dN_3}{d^33} f_B^W(\vec{p}_1,\vec{p}_2,\vec{p}_3)\delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

where the Wigner functions reads:

$$f_M^W(q^2) = \frac{g_M\left(2\sqrt{\pi}\sigma\right)^3}{V} e^{-q^2\sigma^2}$$

$$f_B^W(q_1^2,q_2^2) = \frac{g_B\left(2\sqrt{\pi}\sigma_1\right)^3\left(2\sqrt{\pi}\sigma_2\right)^3}{V^2} e^{-q_1^2\sigma_1^2-q_2^2\sigma_2^2}$$

$\sigma = 1/\sqrt{\mu\omega}$, μ is the reduced mass and ω is the oscillator frequency.



Probability of a heavy quark recombine into a hadron

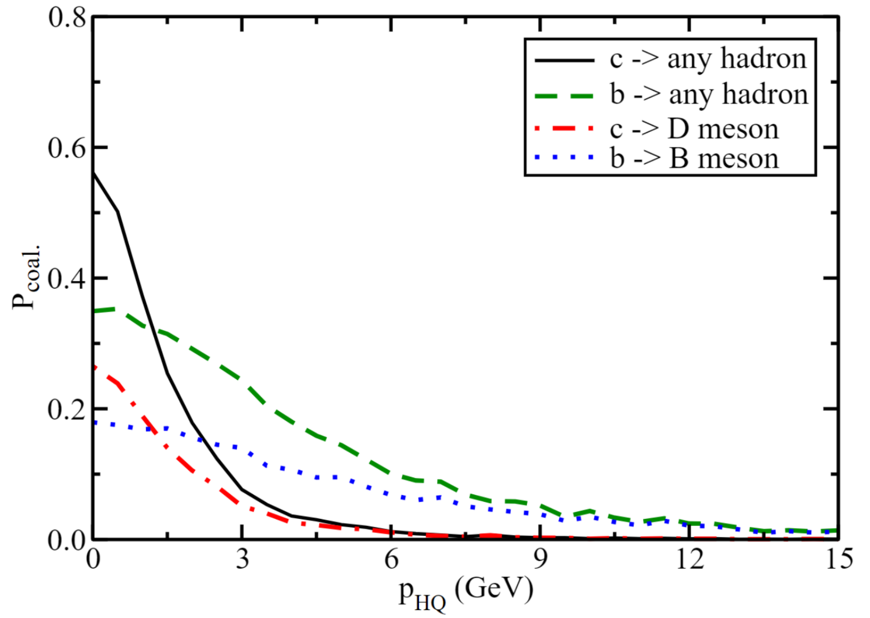
Simulation Setup - Heavy quark hadronization and hadronic re-scattering

The oscillator frequency ω is fitted to charge radii of the corresponding charged hadrons:

$$\langle r_M^2 \rangle_{ch} = \frac{3}{2\omega} \frac{1}{(m_1+m_2)(Q_1+Q_2)} (\frac{m_2}{m_1} Q_1 + \frac{m_1}{m_2} Q_2)$$

$$\langle r_B^2 \rangle_{ch} = \frac{3}{2\omega} \frac{1}{(m_1+m_2+m_3)(Q_1+Q_2+Q_3)}$$
$$(\frac{m_2+m_3}{m_1} Q_1 + \frac{m_3+m_1}{m_2} Q_2 + \frac{m_1+m_2}{m_3} Q_3)$$

We get $\omega = 0.33$ GeV for charm and beauty mesons, $\omega = 0.43$ GeV for charm baryons and $\omega = 0.41$ GeV for beauty baryons.

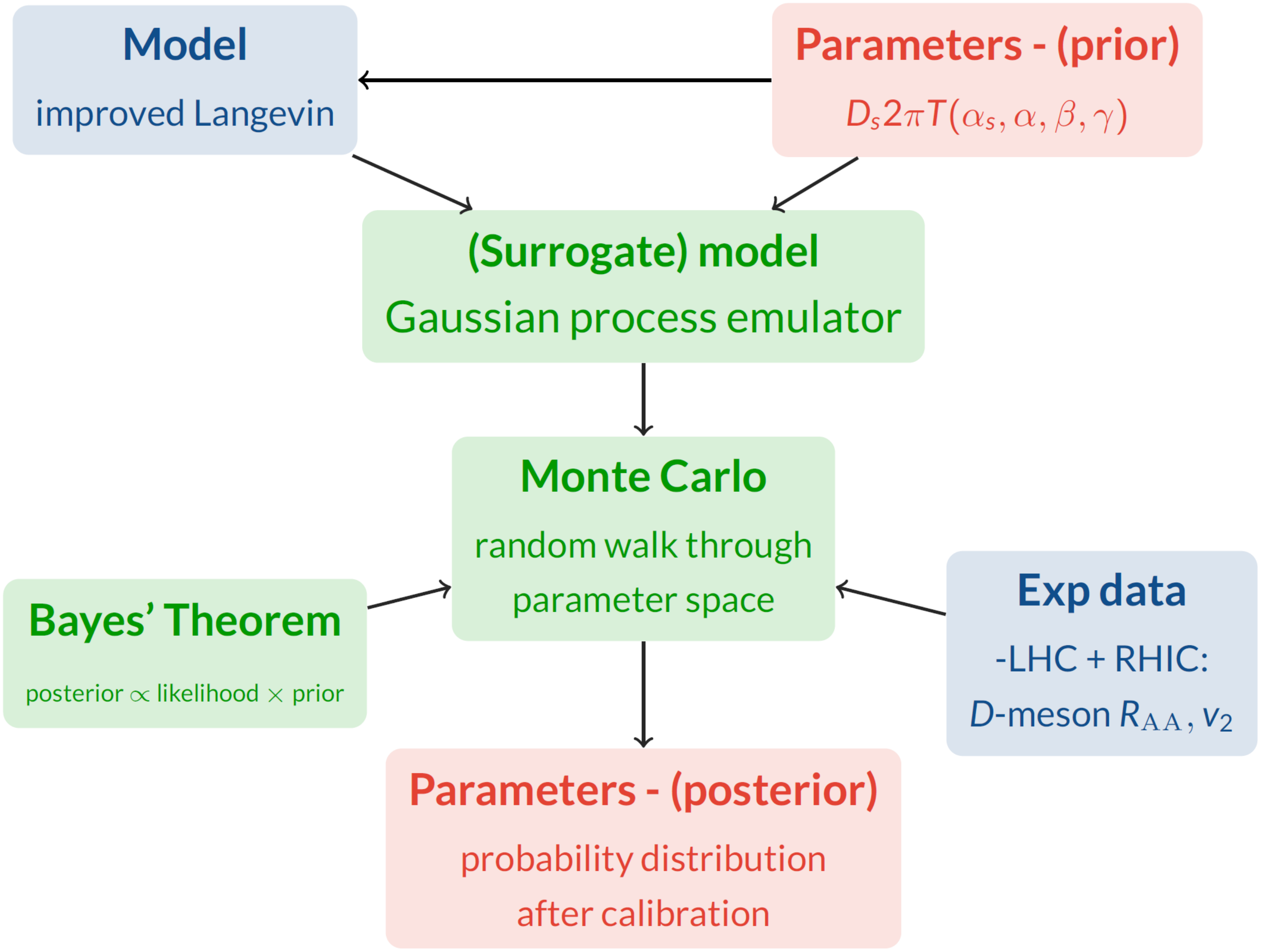


Probability of a heavy quark recombine into a hadron

Simulation Setup – Bayesian Analysis

Answer the following question:

Given the model and the experimental data, find the posterior distribution of the model parameters.



Simulation Setup – Bayesian Analysis

$$p(\theta|y = y_{\mathrm{exp}}) \propto \mathcal{L}(y = y_{\mathrm{exp}}|\theta) \times p(\theta)$$

- Posterior distribution: probability of θ given observation y_{exp}
- Likelihood: $\mathcal{L}(y = y_{\mathrm{exp}}|\theta) \propto \exp[-(y(\theta) - y_{\mathrm{exp}})\Sigma^{-1}(y(\theta) - y_{\mathrm{exp}})^T]$
- Covariance matrix: $\Sigma = \Sigma_{\mathrm{exp}} + \Sigma_{\mathrm{model}} + \Sigma_{\mathrm{GP}}$
- Prior distribution $P(\theta)$: prior knowledge of parameters

Gaussian process emulator

- Non-parametric regression
- Quickly predict model output given input $\rightarrow y(\theta)$
- Returns not only mean of prediction $\hat{y}(\theta)$, but also uncertainty σ_{GP}

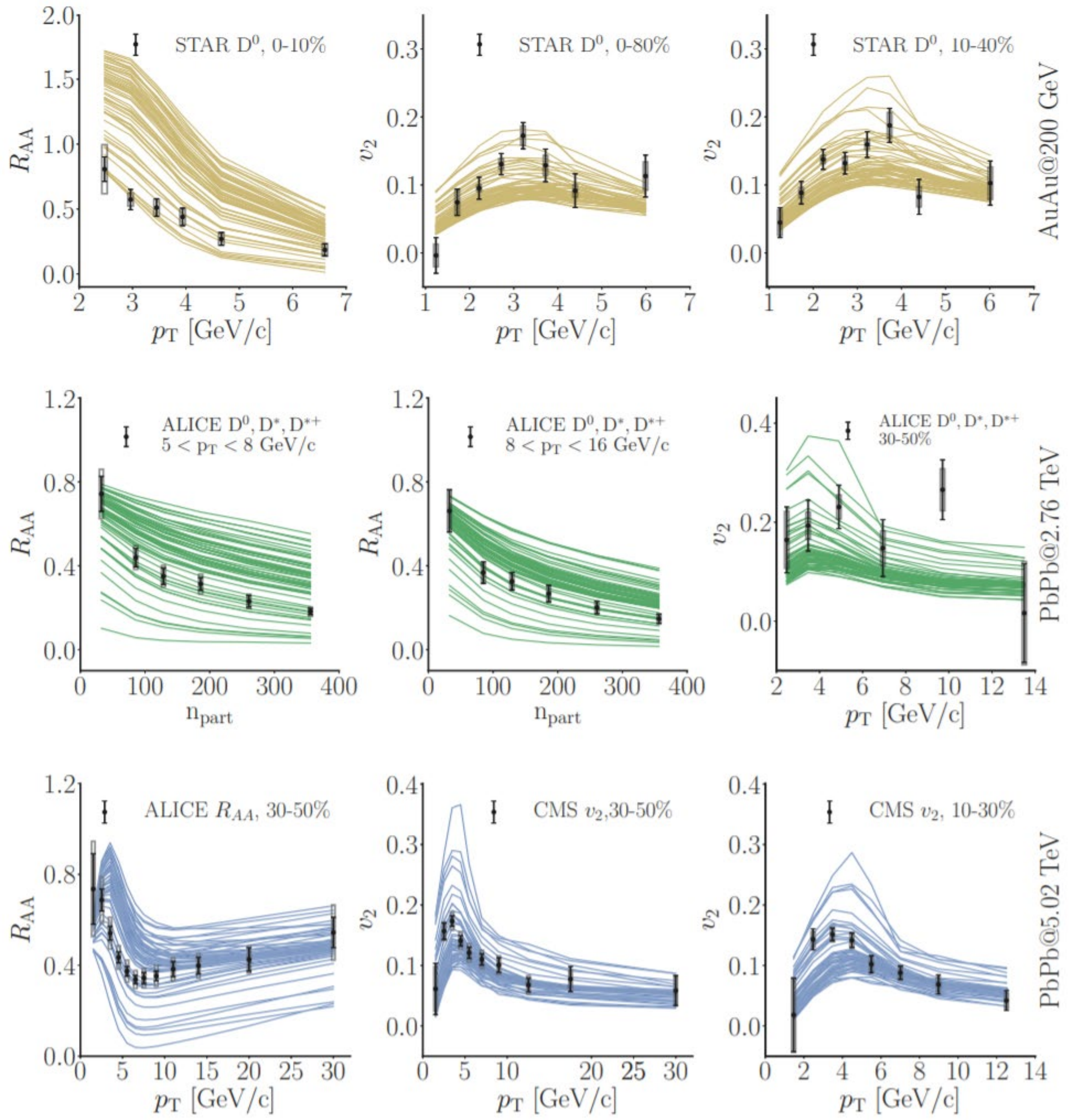
Markov chain Monte Carlo

- Random walk through the parameter space
- Accepted/rejected based on likelihood
- Posterior ensembles achieved after equilibrium

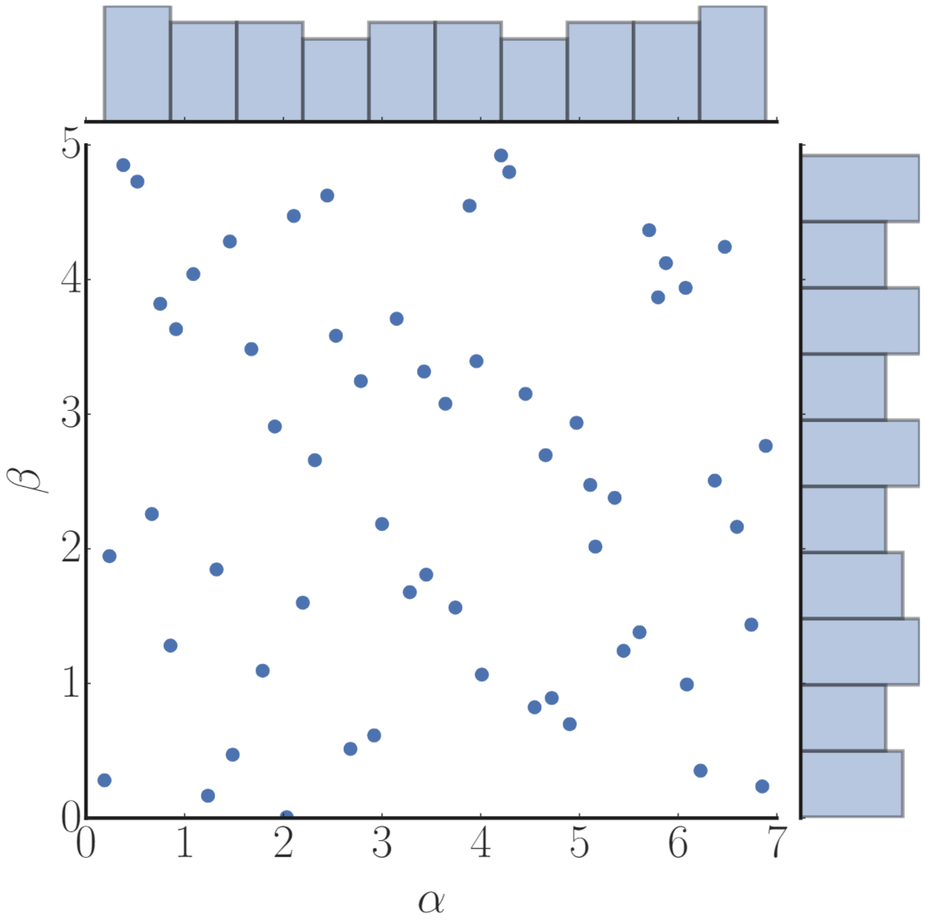
PART 2

Results - Bayesian Analysis

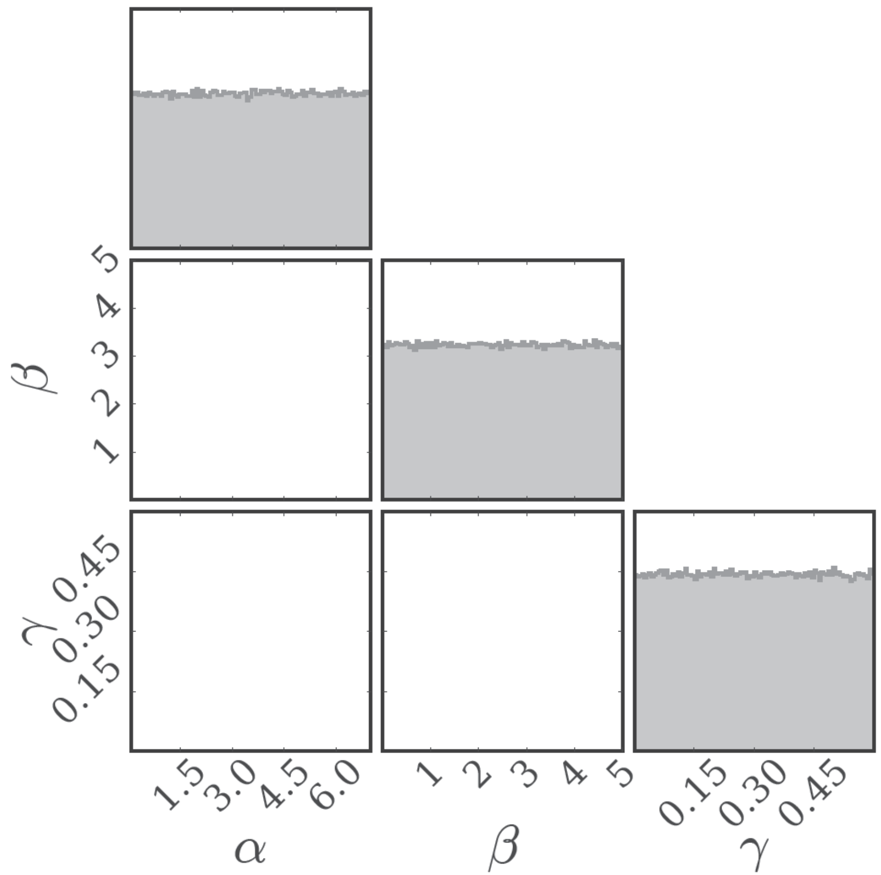
Parameter	Description	Range
α	$D_s 2\pi T$ at T_c	0.1–7.0
β	Slope of $(D_s 2\pi T)^{\text{linear}}$ above T_c	0–5.0
γ	Ratio between D_s^{linear} and D_s^{pQCD}	0.0–0.6



PART 2 Results - Bayesian Analysis



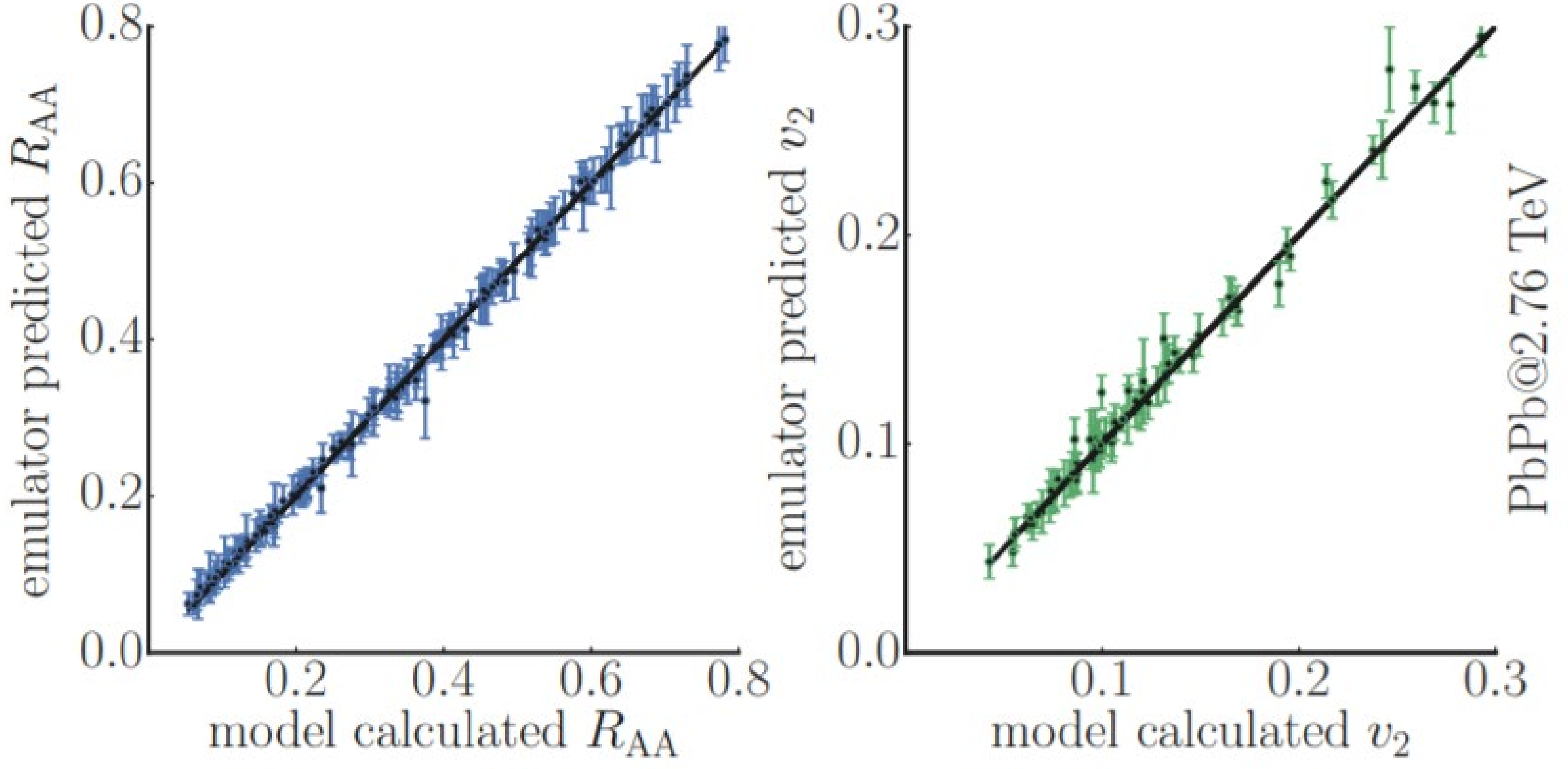
Latin hyper cube design in the parameter space



Uniform prior distributions

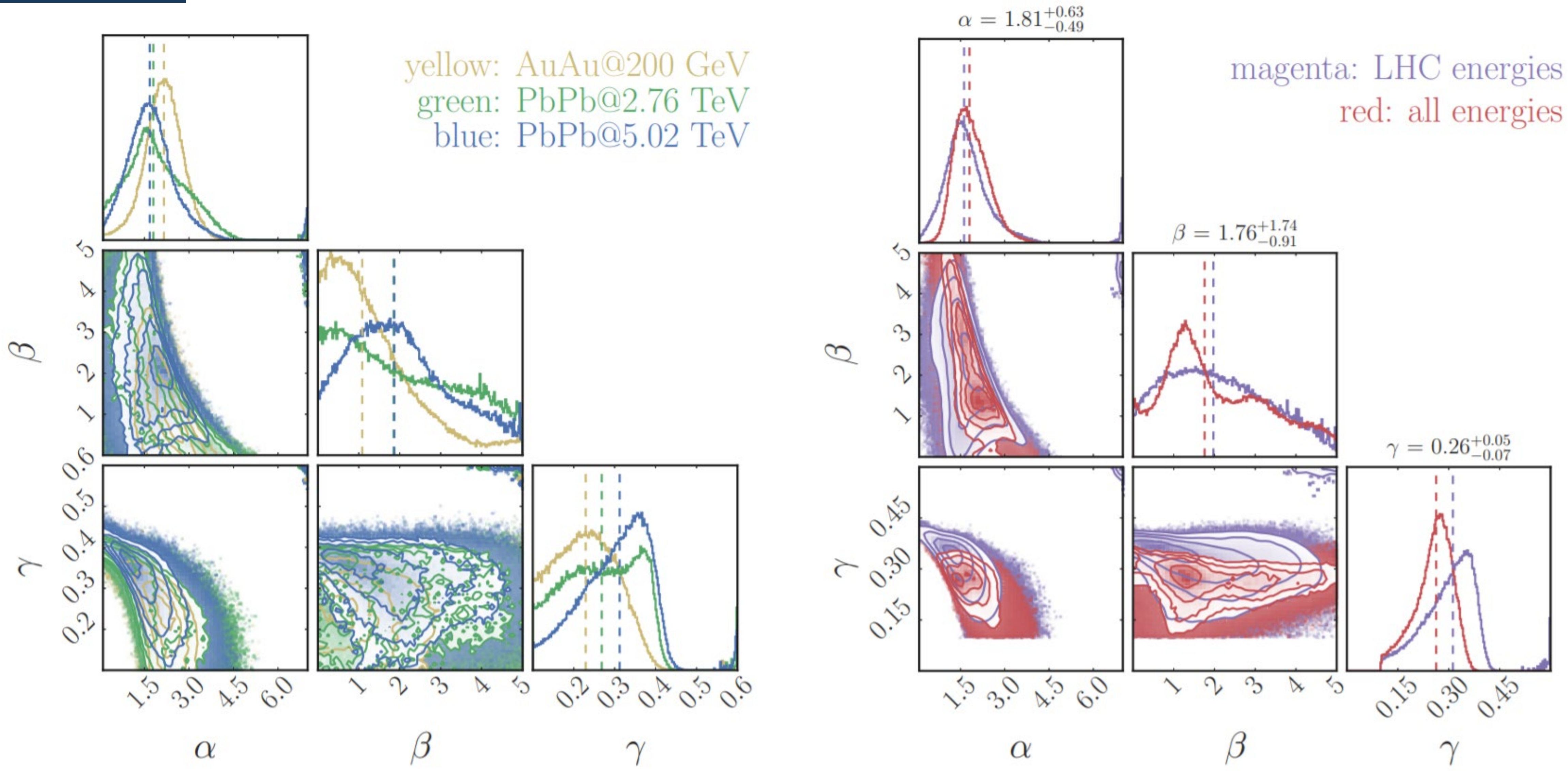
PART 2

Results - Bayesian Analysis



Emulator validation with model calculation

Results - Bayesian Analysis

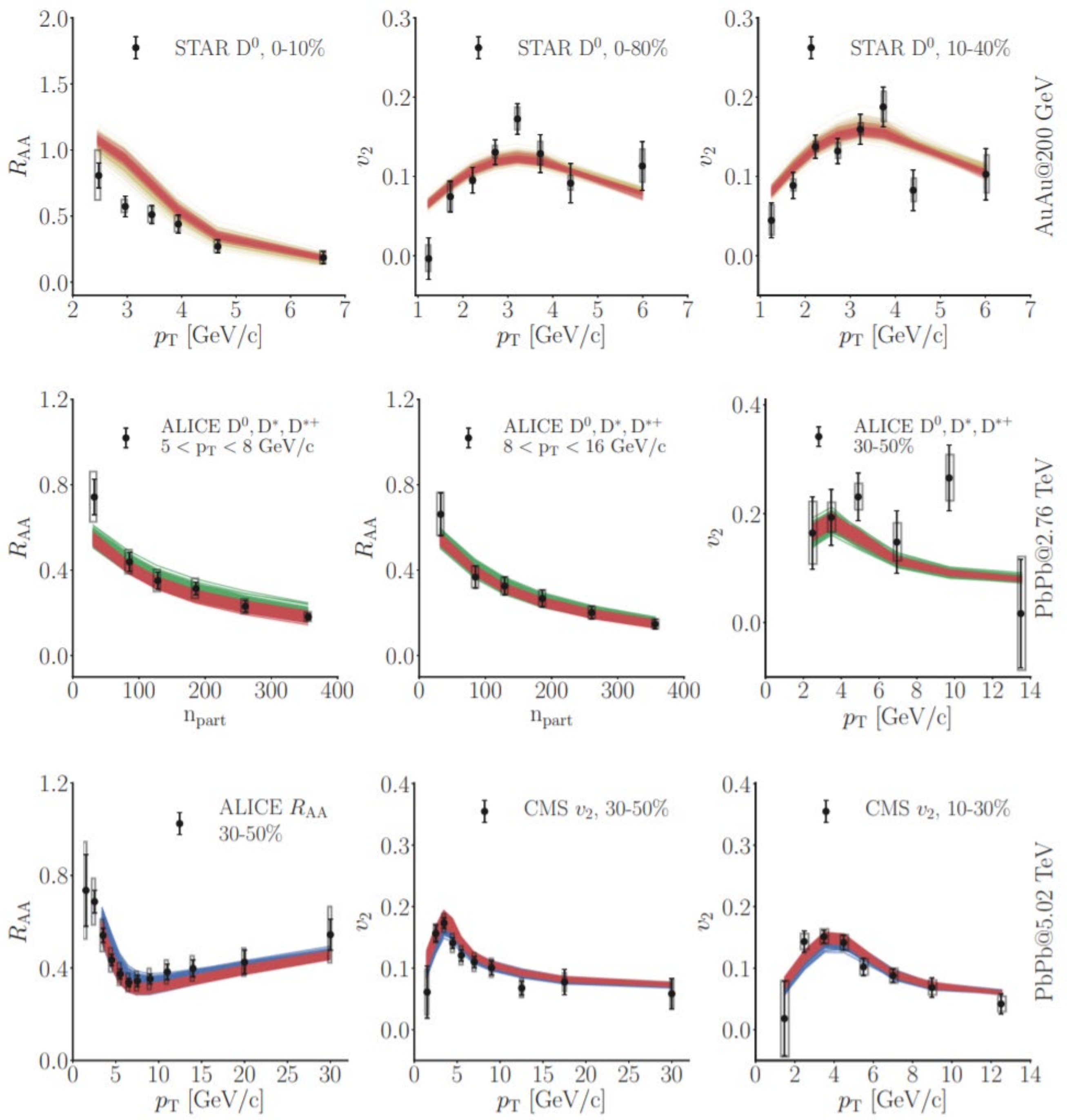


Posterior distribution with different combination of training data sets

PART 2

Results - Bayesian Analysis

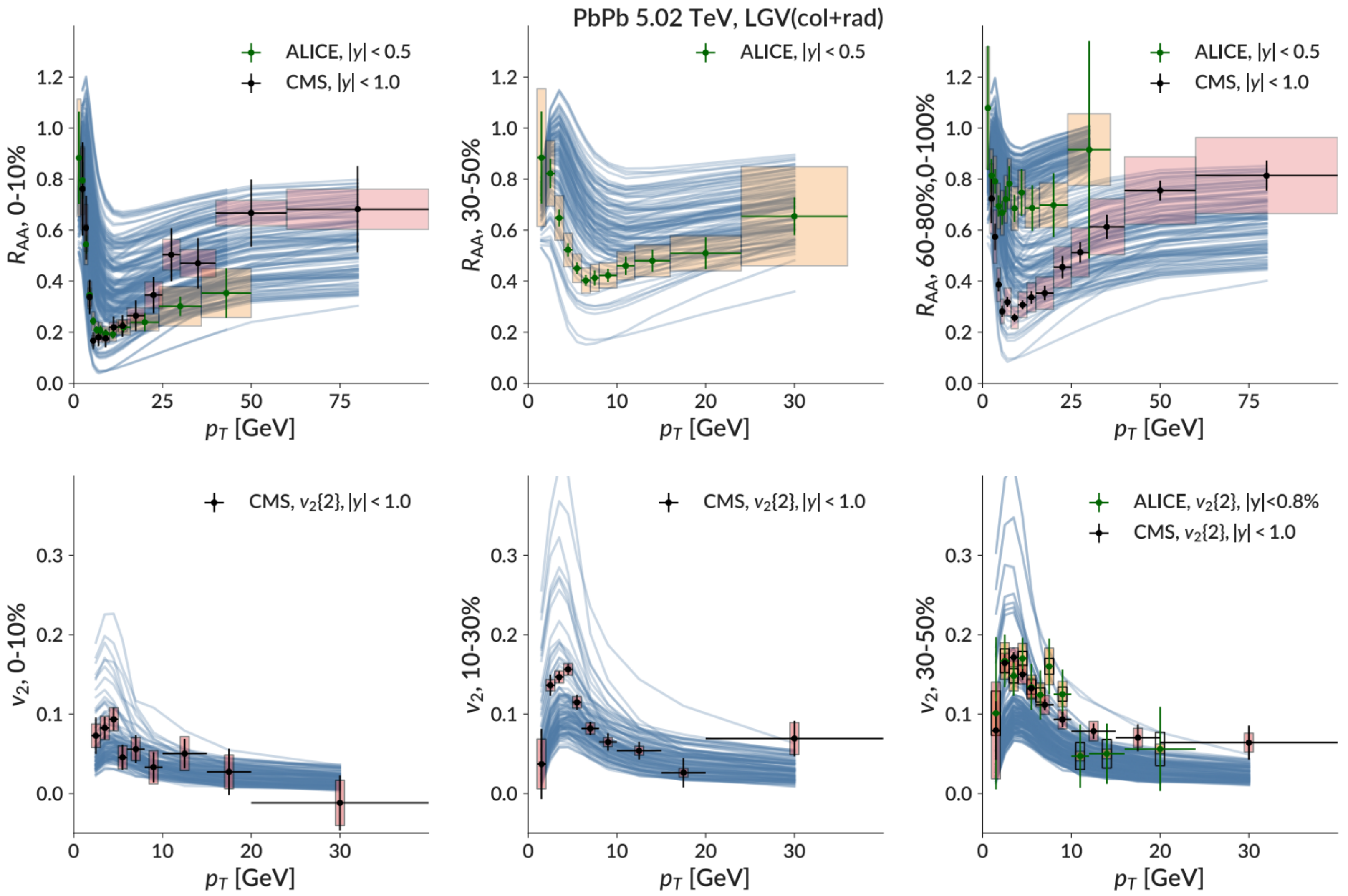
Emulator predictions for 200 random input parameters sampled from the posterior distributions.



PART 2

Results - Bayesian Analysis

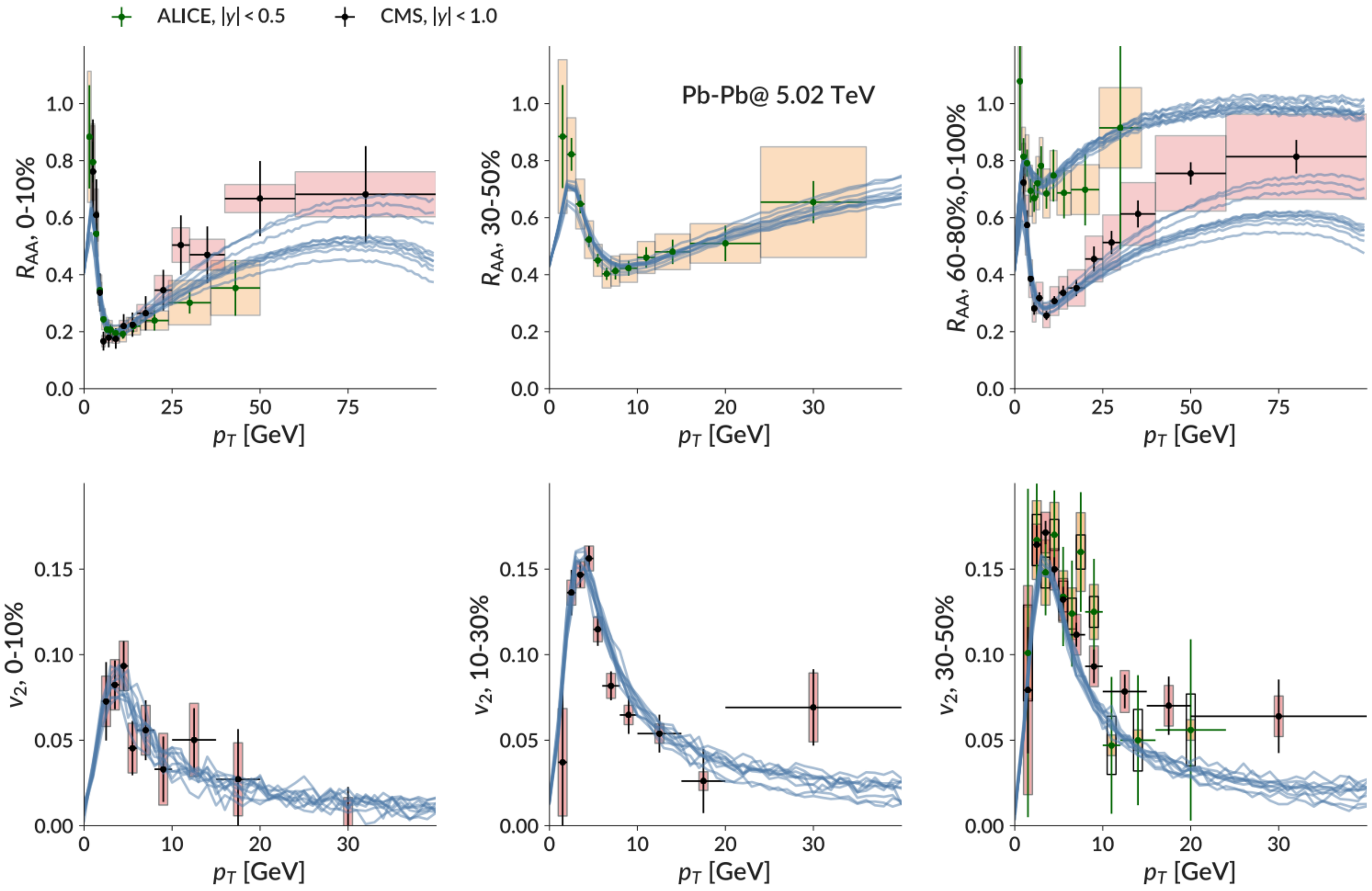
Parameter	Description	Range
α_s	strong coupling constant	0.1 - 0.6
α	$D_s 2\pi T$ at T_c	0.1 - 7
β	slope of $D_s 2\pi T^{\text{soft}}$ above T_c	0 - 5.0
γ	ratio between $D_s 2\pi T^{\text{soft}}$ and $D_s 2\pi T^{\text{pQCD}}$	0 - 0.6
τ_f	free-streaming time	0.1 - 1



PART 2

Results - Bayesian Analysis

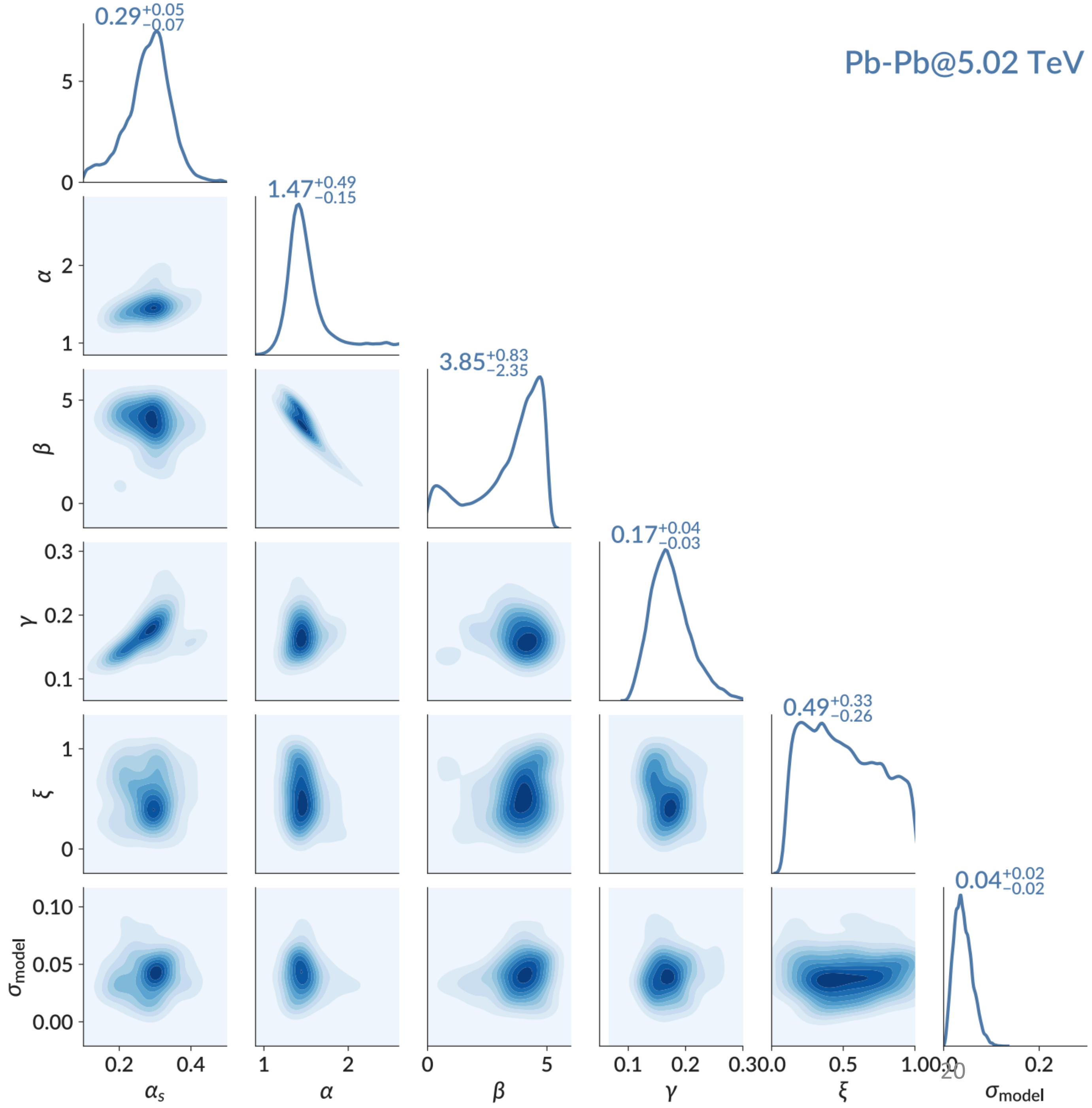
Comparison with experimental data using several sets of parameters sampled from the posterior distribution.



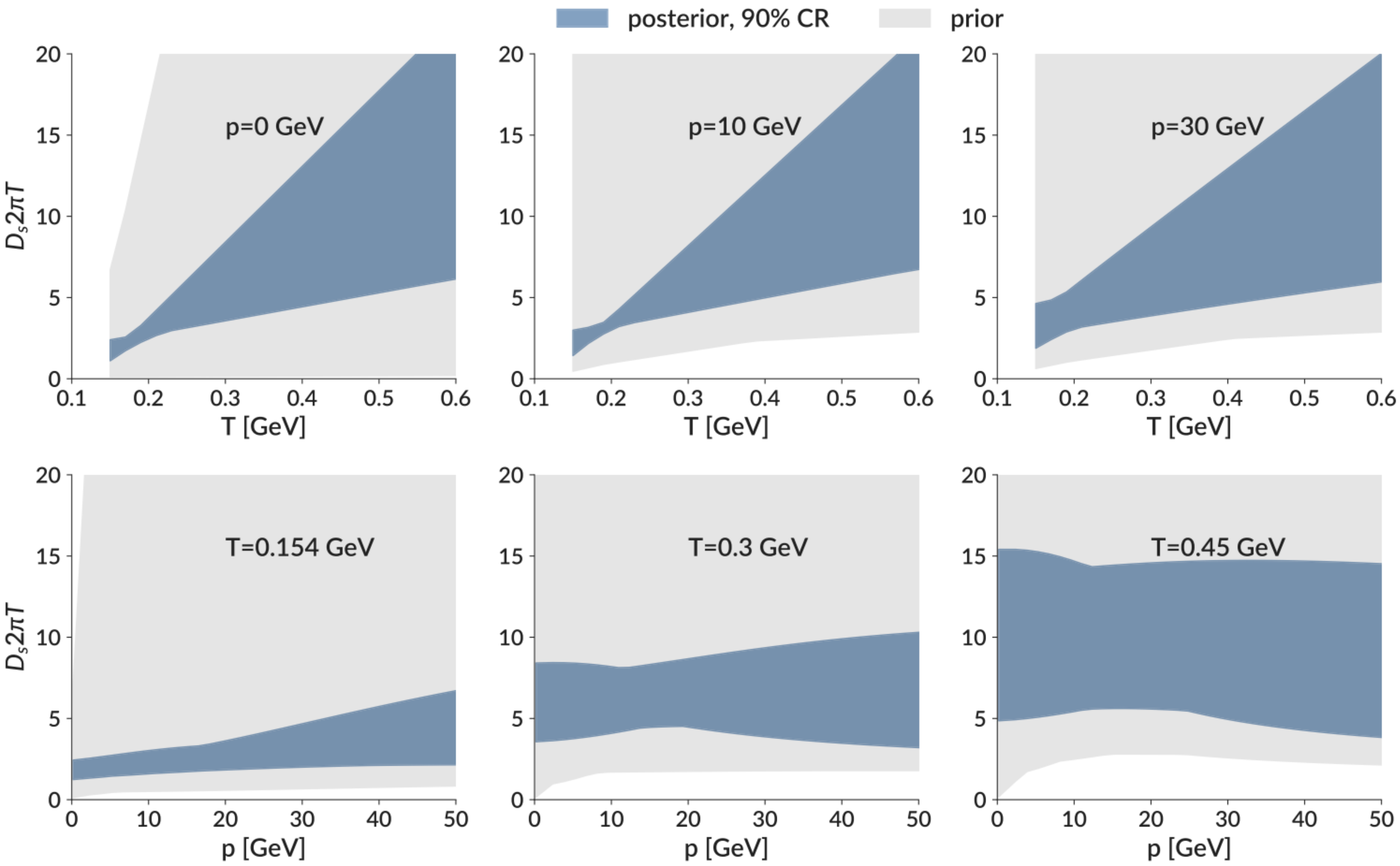
PART 2

Results - Bayesian Analysis

Posterior distribution of model parameters.

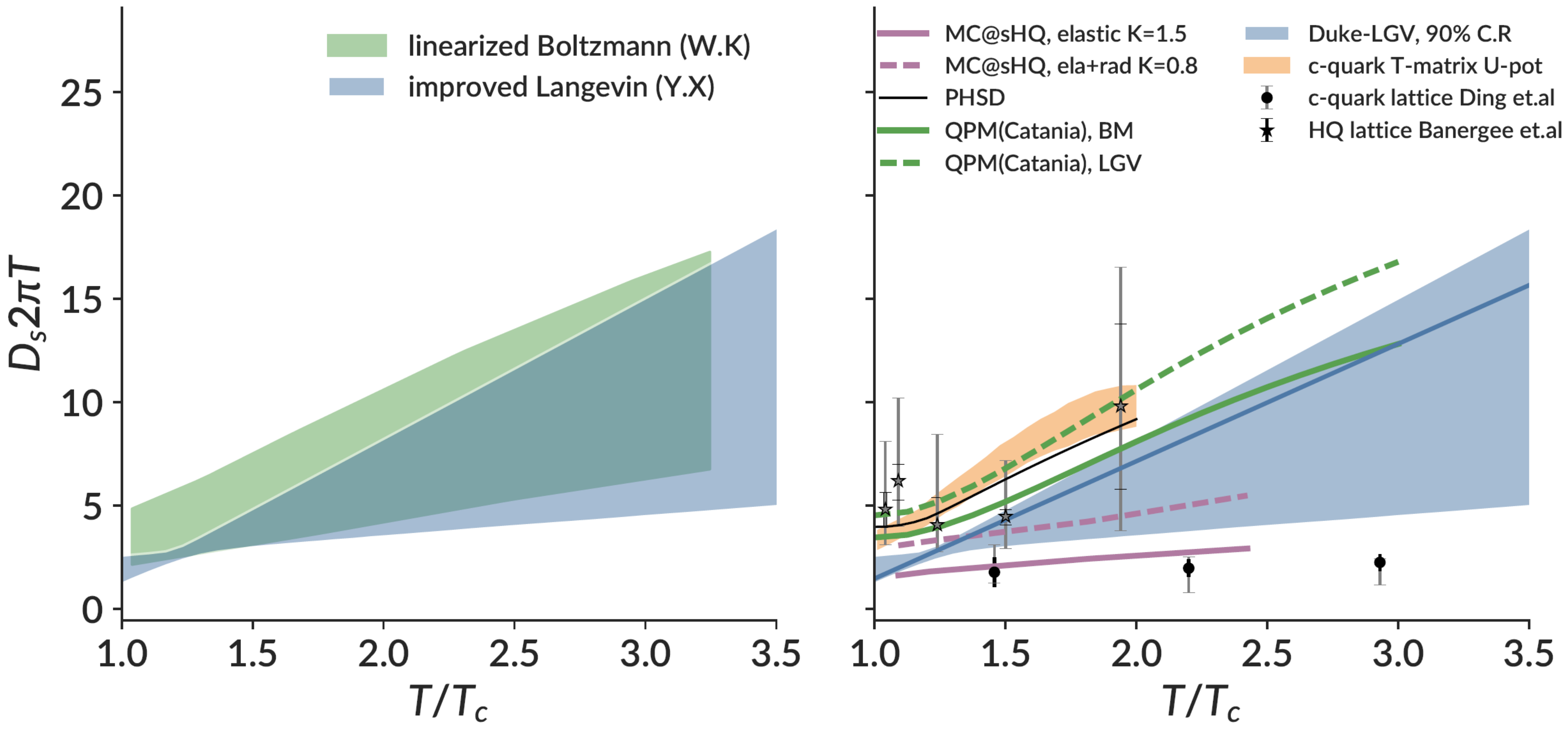


Results - Bayesian Analysis



Posterior prediction of $D_s 2\pi T$

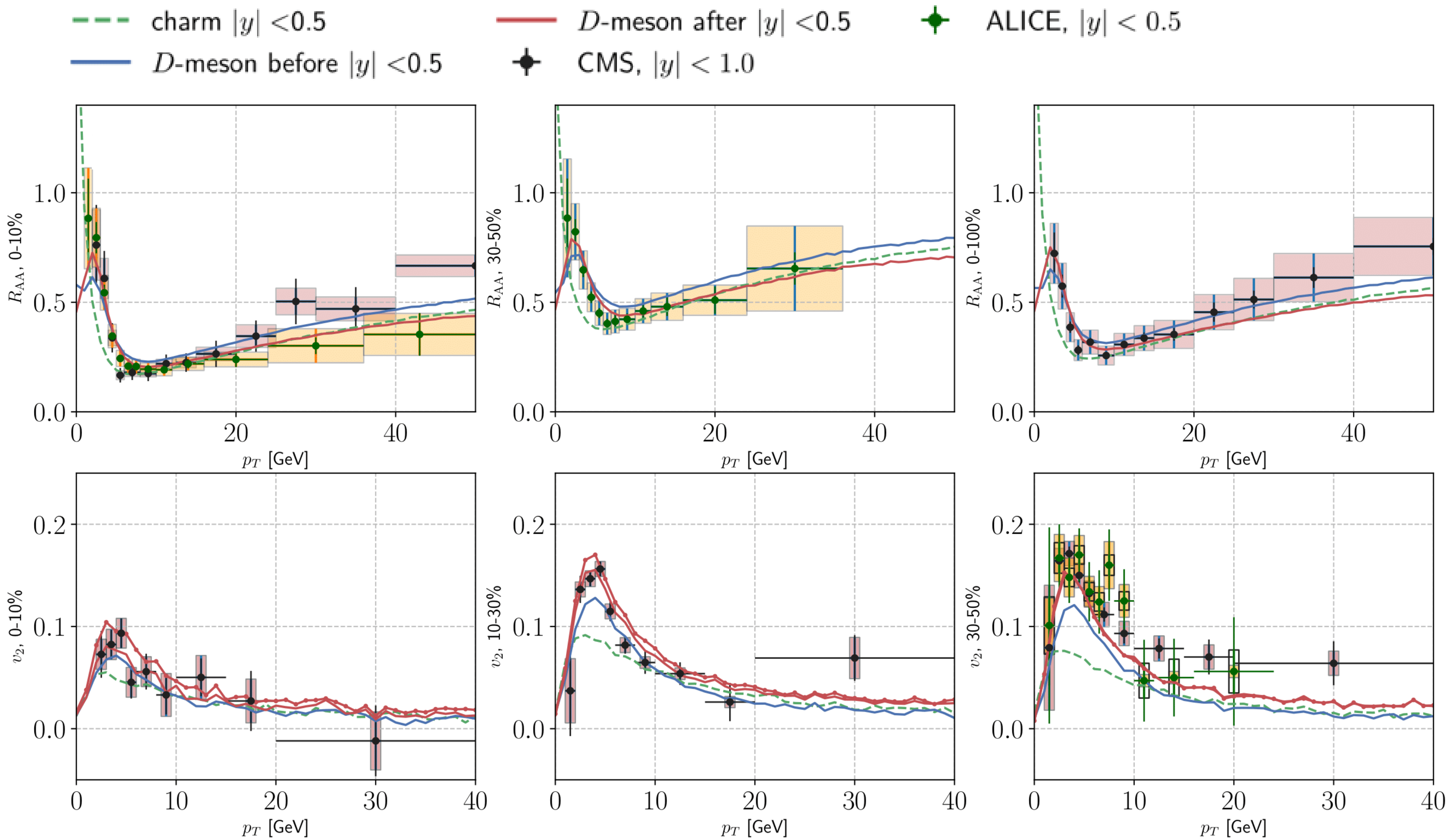
Results - Bayesian Analysis



Posterior prediction of $D_s 2\pi T$ compared with other models

PART 2

Results – Best estimation

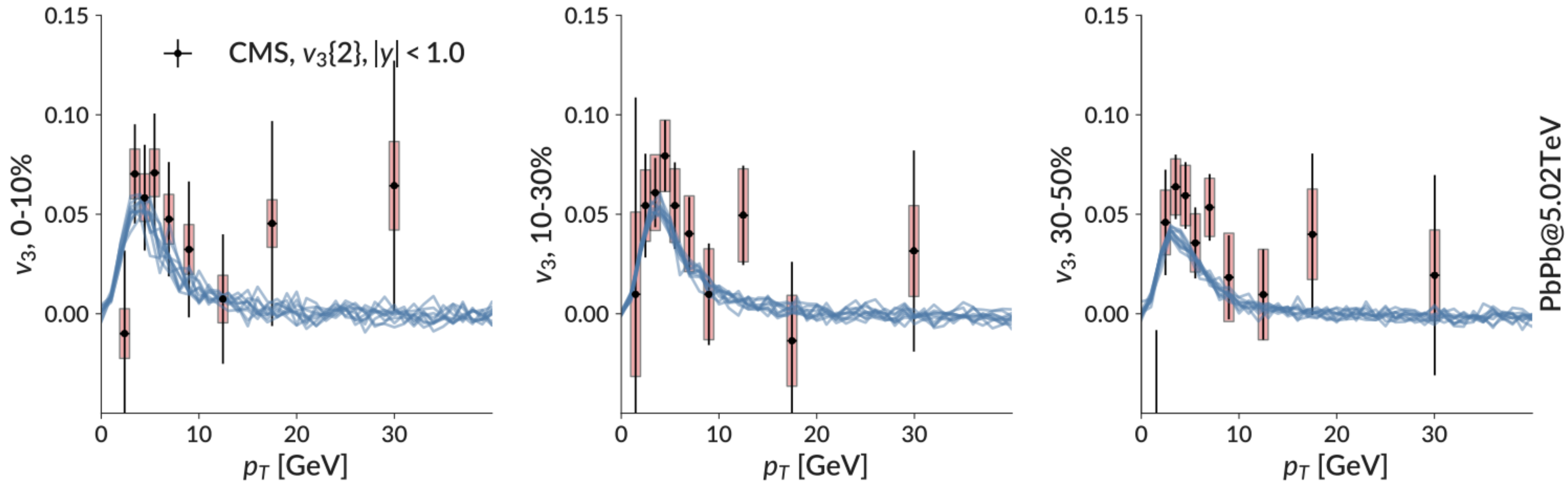


$$\alpha_s = 0.32, \alpha = 1.74, \beta = 2.43, \gamma = 0.18$$

PART 2

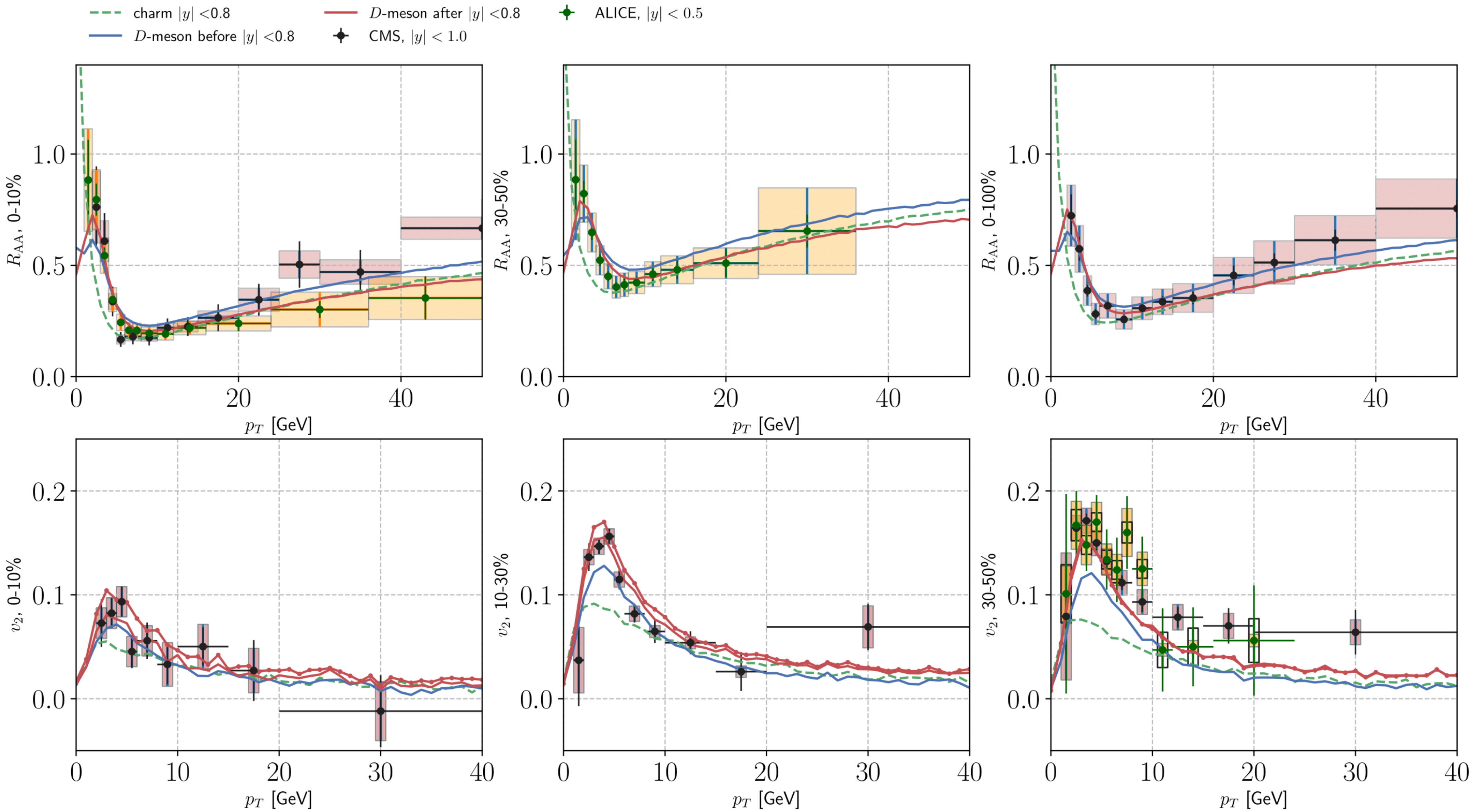
Results – Best estimation

We can also use the extracted parameters to predict other observables:



Best estimate of v_3

Homework Results

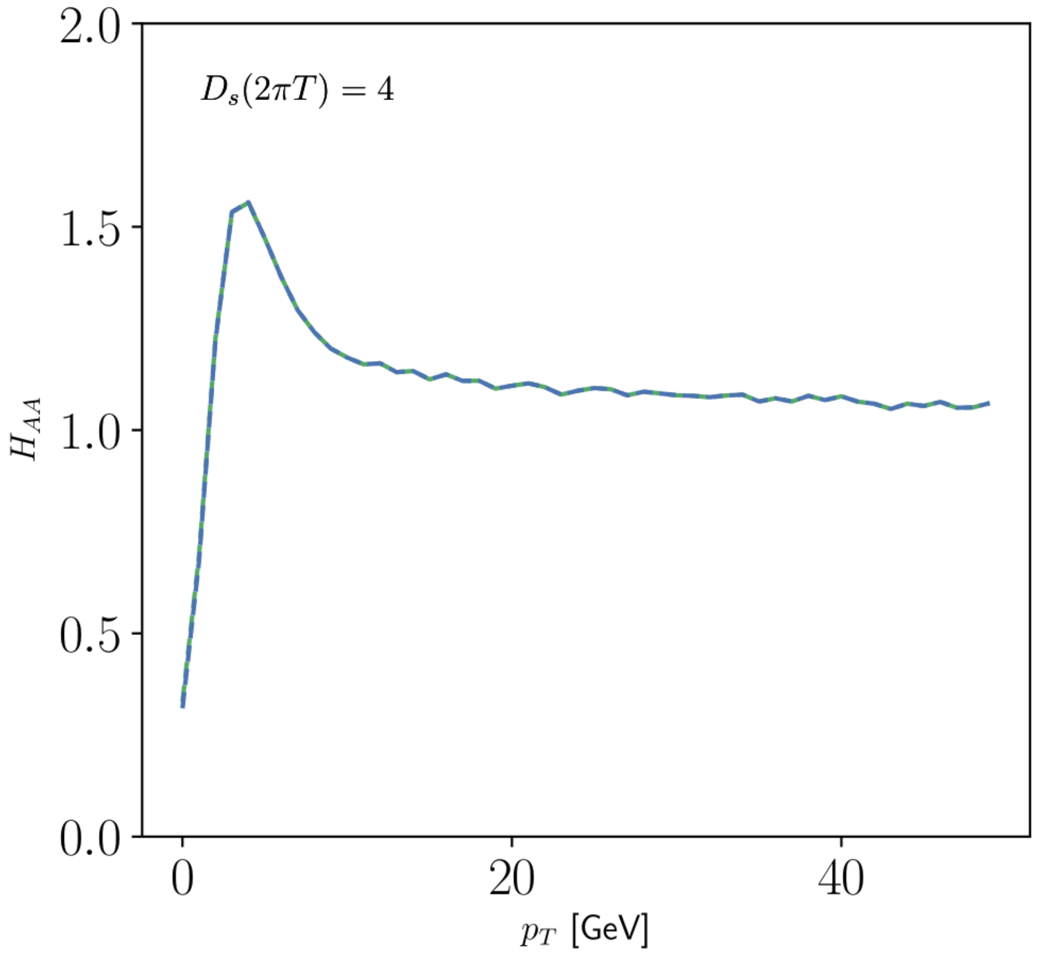


Comparison with data using a set of optimal parameters

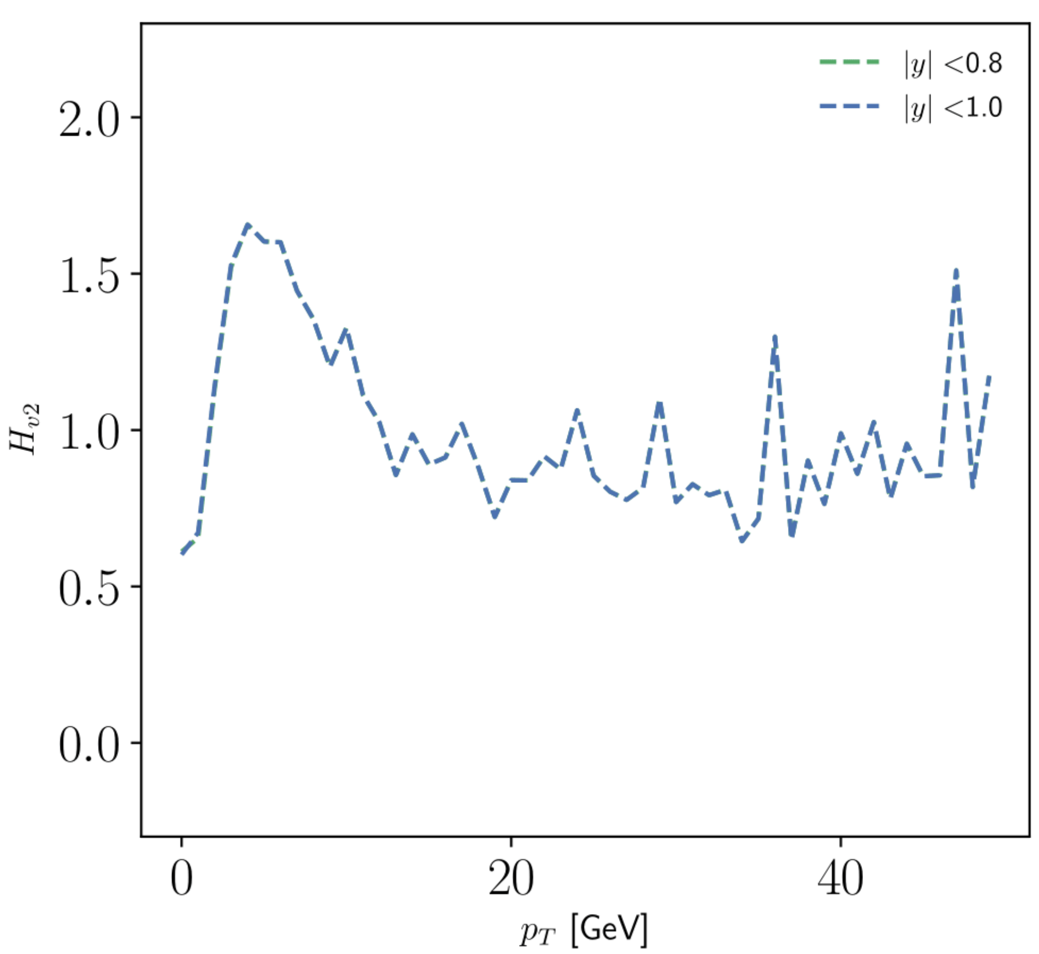
PART 3

Homework Results

$H_{AA} = R_{AA}^{D}/R_{AA}^{c}$



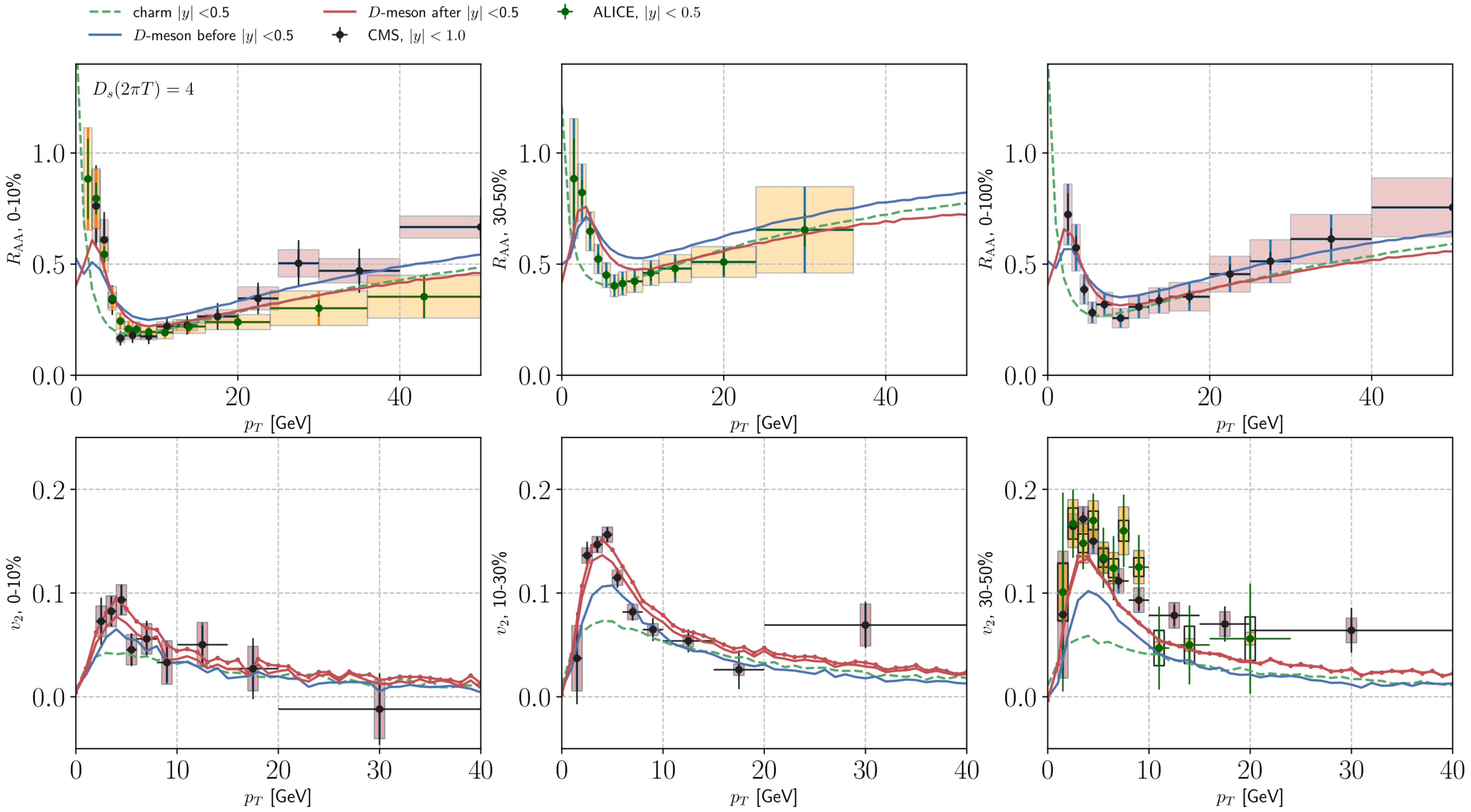
$H_{v_2} = v_2^D/v_2^c$



H_{AA} and H_{v_2} using a set of optimal parameters

PART 3

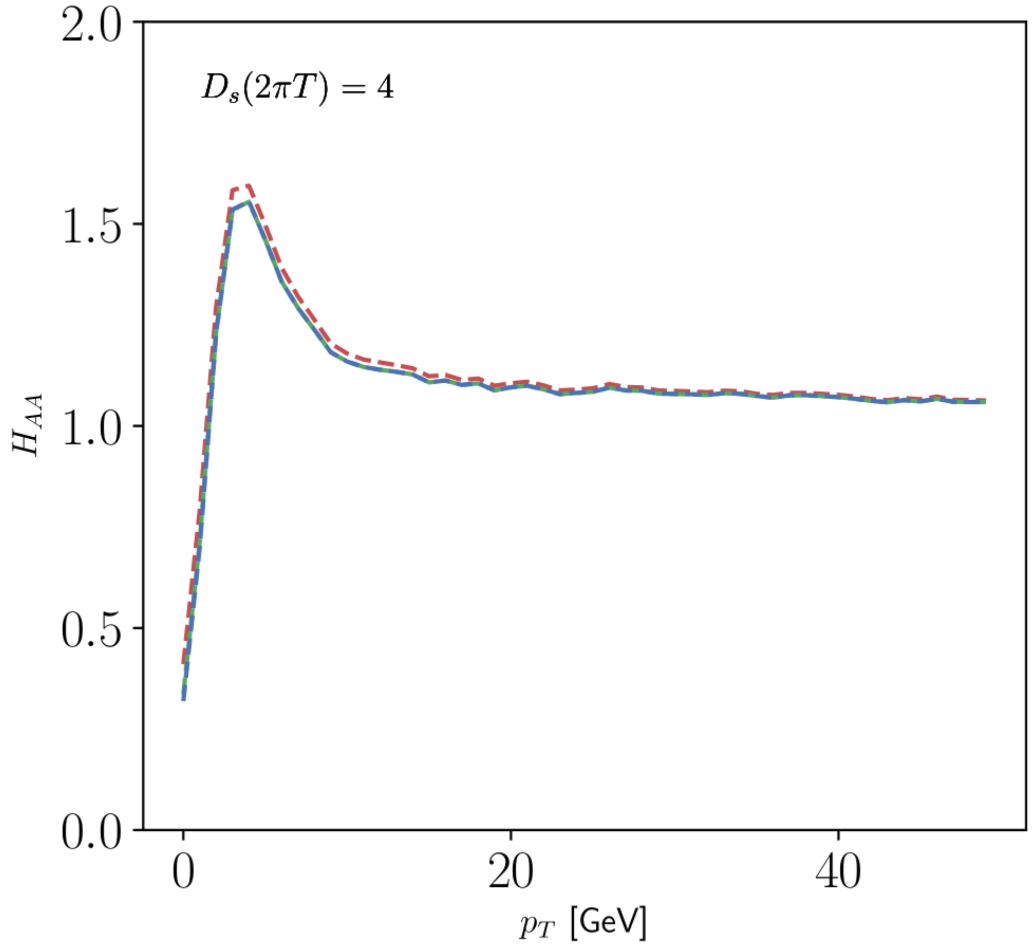
Homework Results



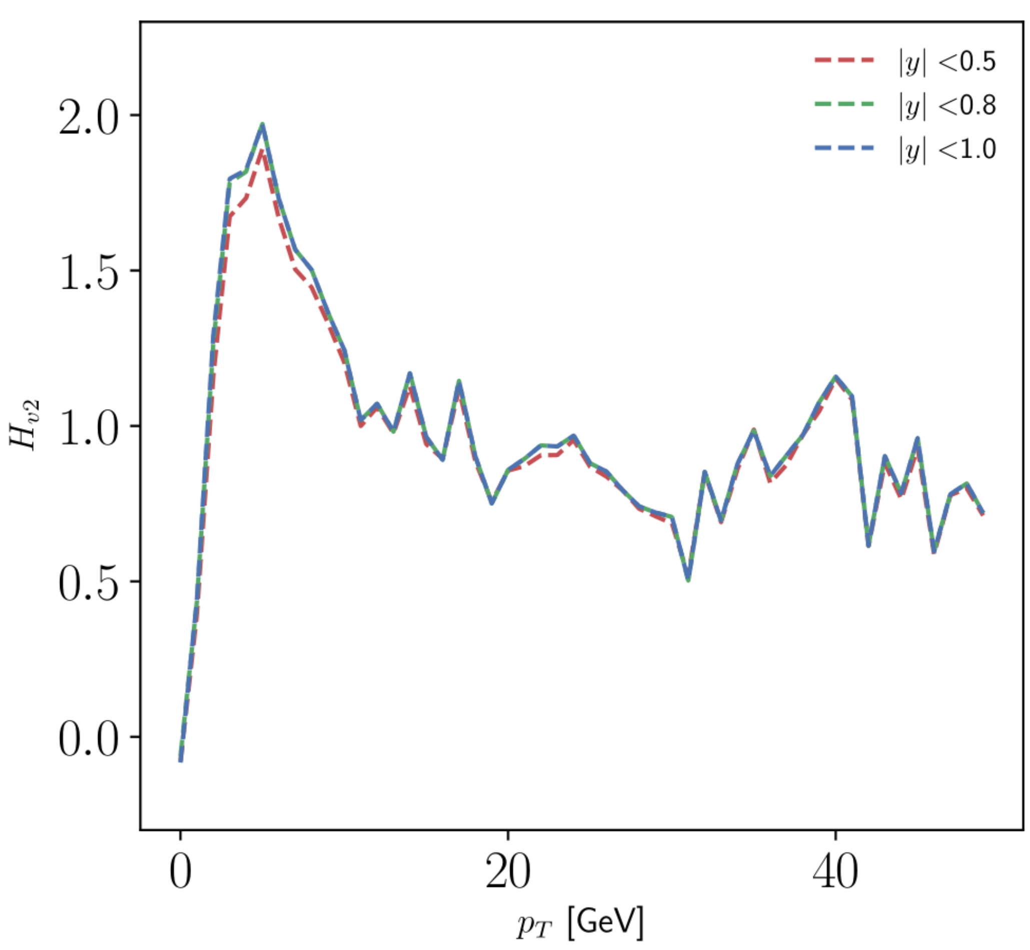
Comparison with data using a fixed $D_s 2\pi T = 4$

Homework Results

$H_{AA} = R_{AA}^D / R_{AA}^c$



$H_{v_2} = v_2^D / v_2^c$



H_{AA} and H_{v_2} using a fixed $D_s 2\pi T = 4$

Conclusion

- We performed a data driven analysis on the diffusion coefficient of charm quark.

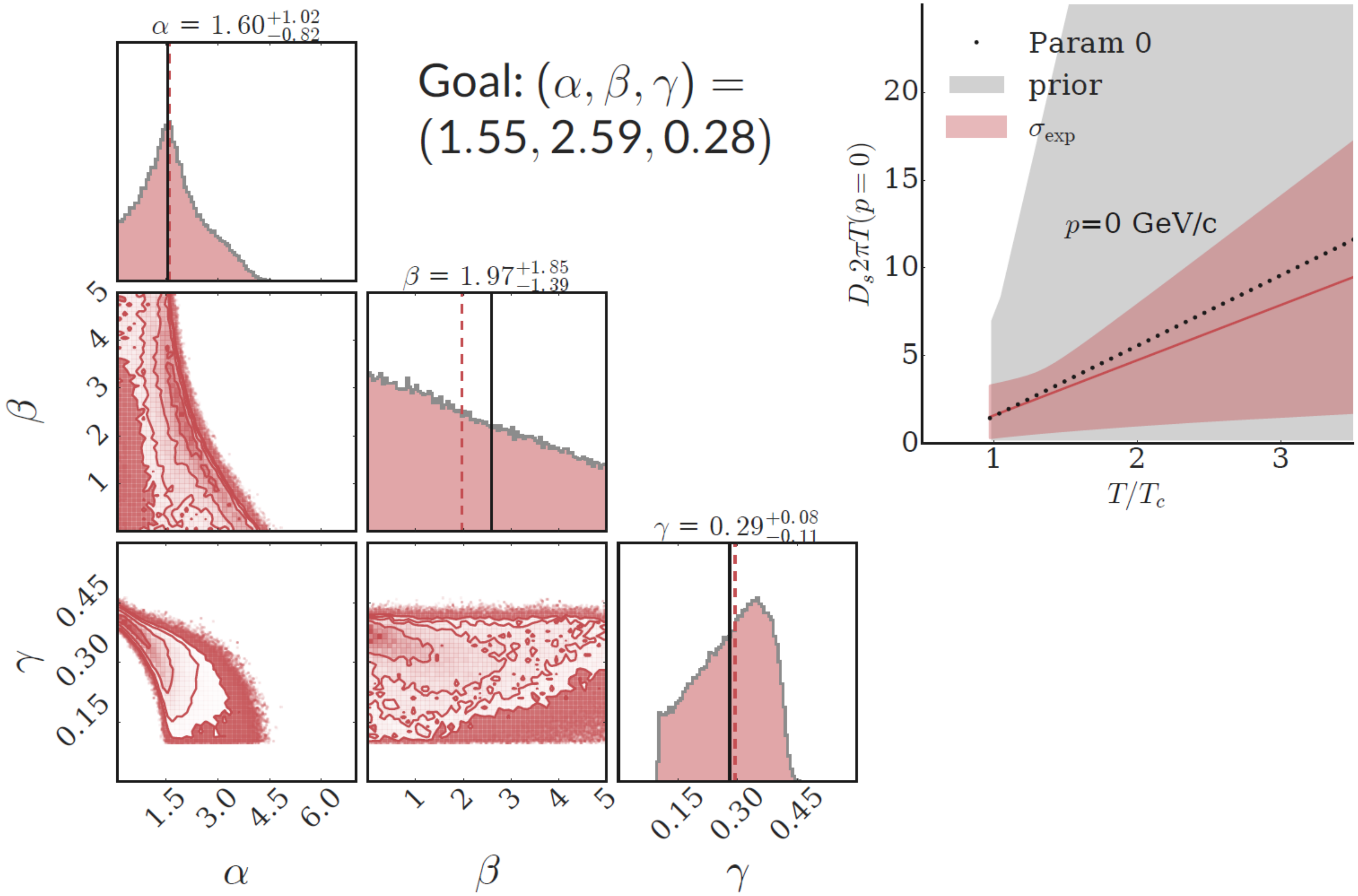
- This study mainly focuses on heavy quark diffusion. Initial condition, hydrodynamics, hadronization and hadronic transport all uses calibrated results.

- Our current estimation is compared to lattice and other model calculations, and we see some tension near T_c. To further reduce the uncertainties, both experiment and systematic/emulator uncertainties should be improved.

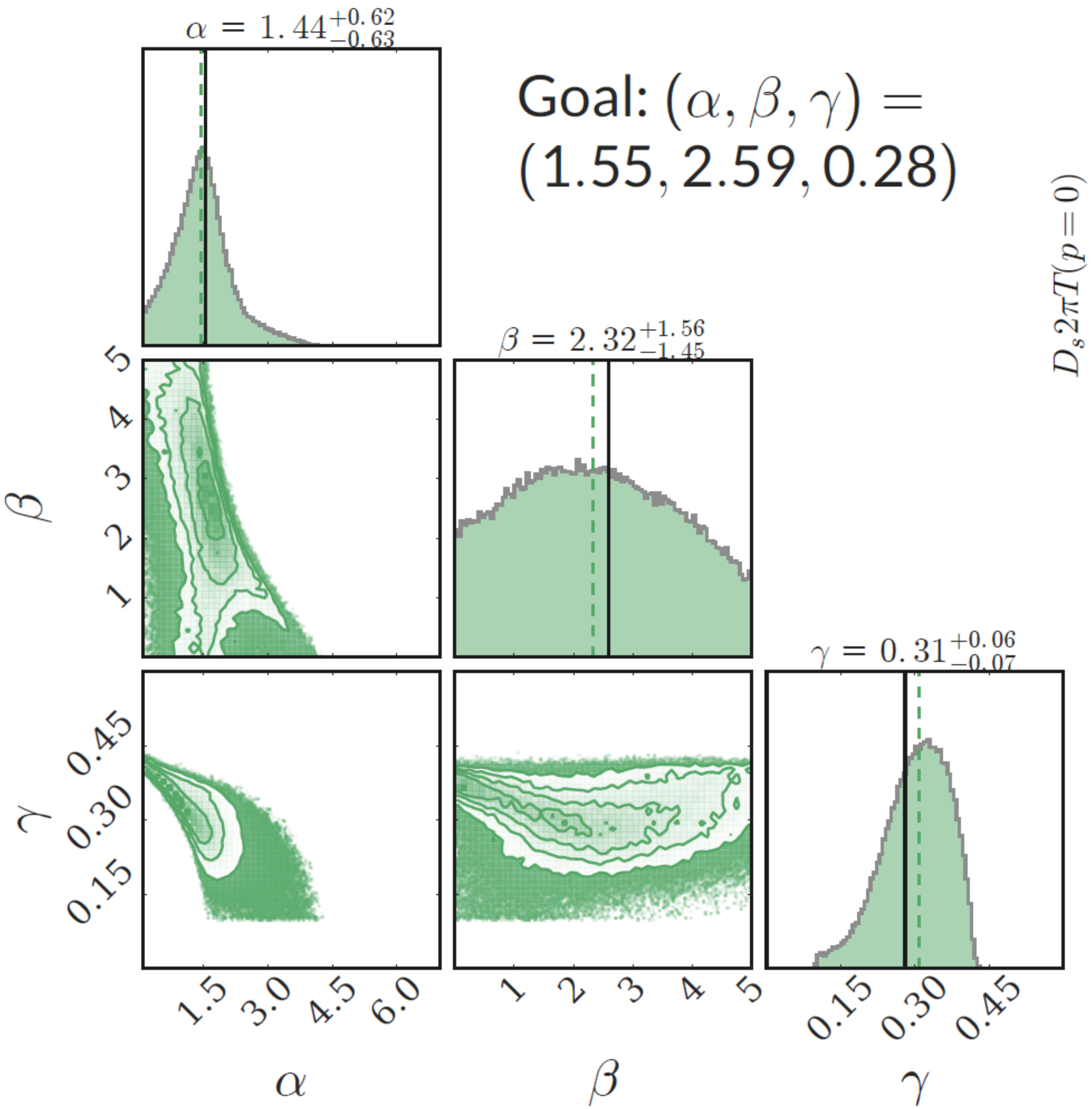
- A further systematic comparison between different transport models, initial condition and hydro can be found in *Physical Review C* 99.1 (2019): 014902.

Thank you!

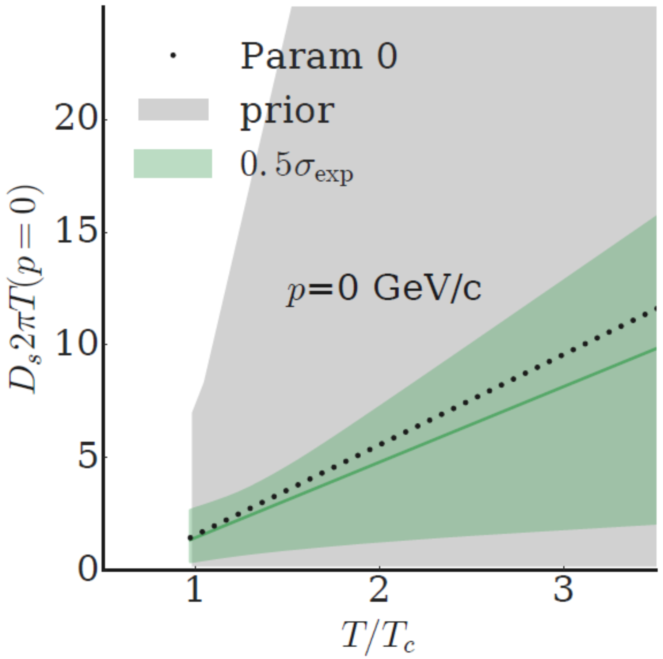
Higher precision: current σ_{exp}



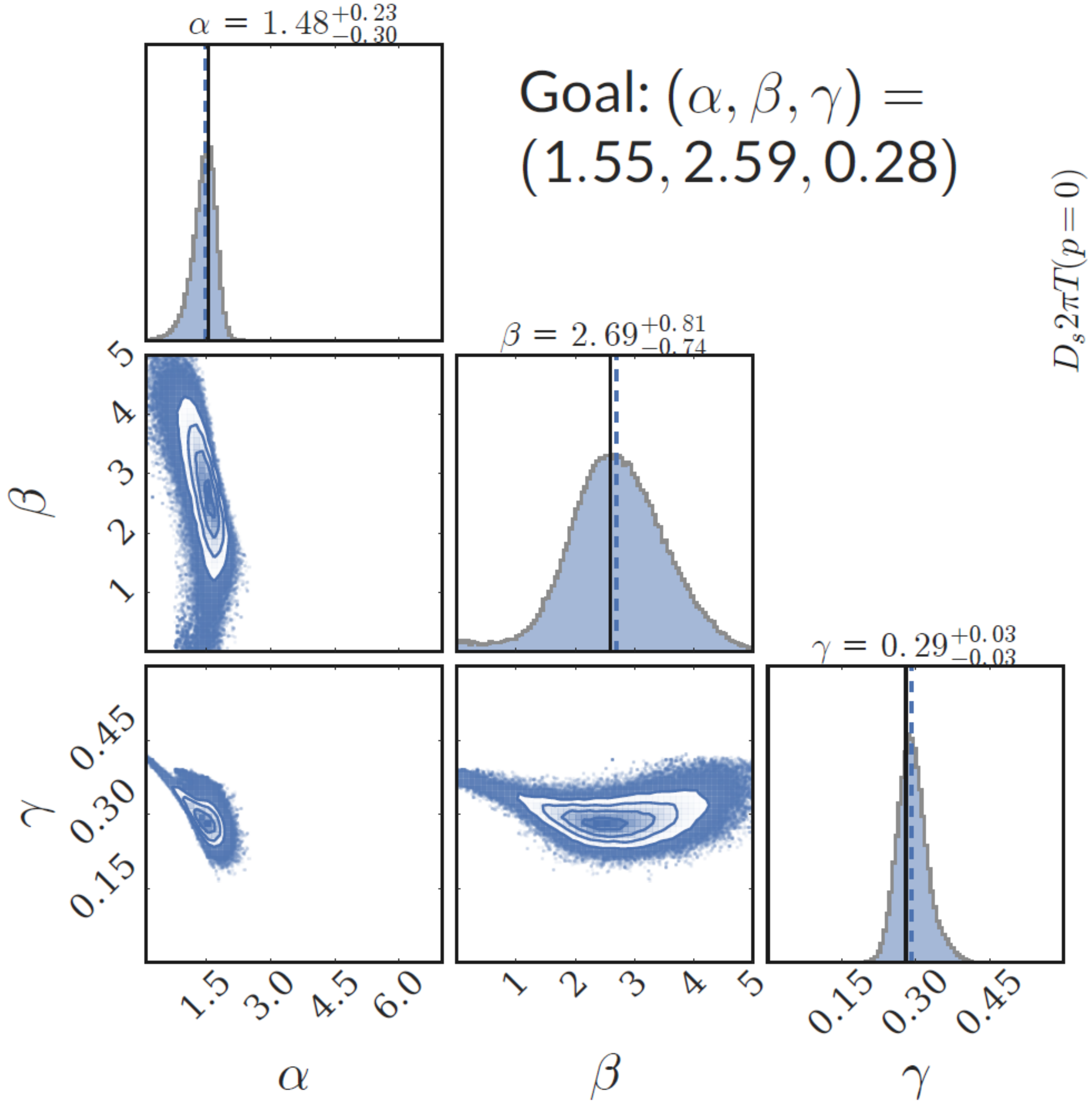
Higher precision: $0.5\sigma_{\mathrm{exp}}$



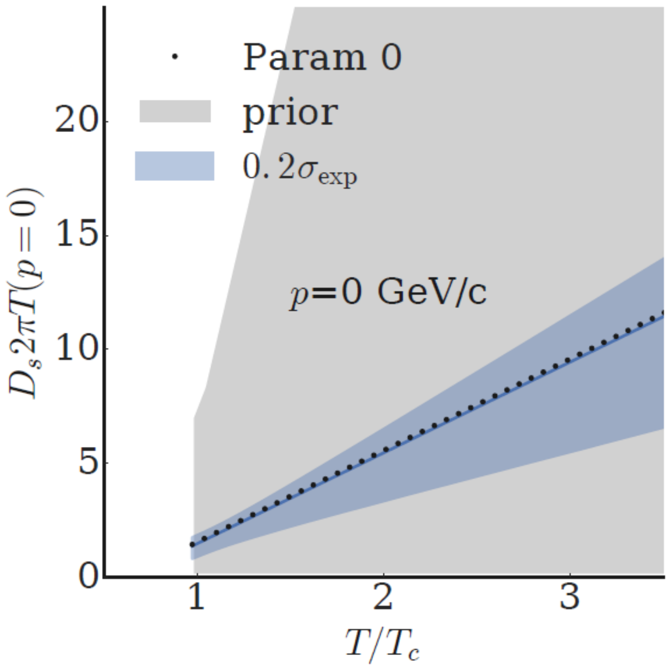
Goal: $(\alpha, \beta, \gamma) =$
$(1.55, 2.59, 0.28)$



Higher precision: $0.2\sigma_{\rm exp}$



Goal: $(\alpha, \beta, \gamma) = (1.55, 2.59, 0.28)$



0.2 of current $\sigma_{\rm exp}$!
(considering PbPb @ 5.02 TeV only)