Heavy flavour hadronization within a coalescence plus fragmentation approach from pp to AA

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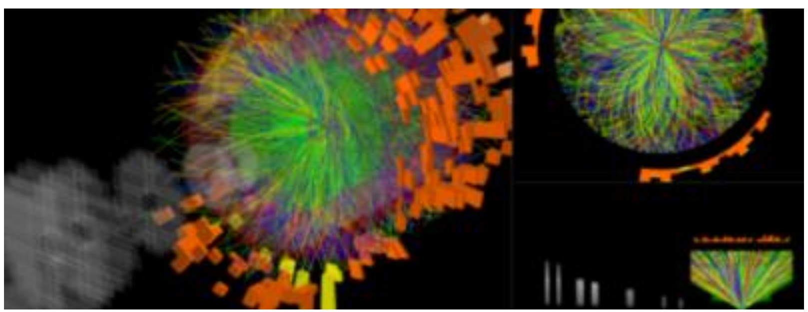


Quark-Gluon Plasma Characterisation with Heavy Flavour Probes (HYBRID)
15-18 November 2021 ECT* - Trento

Outline

- **Hadronization: Fragmentation, Coalescence model**

- **Heavy Quarks in AA collisions:**
  Λ_c and D mesons spectra for RHIC and LHC
  Λ_c/D^0 ratio

- **Heavy Quarks in small systems:**
  Λ_c/D^0 , Ξ_c/D^0, Ω_c/D^0 quite in agreement to ALICE data

- **Predictions for multi-charm production PbPb vs KrKr @5ATeV :**
  Yield, p_T distribution and ratios for Ξ_{cc}, Ω_{scc} , Ω_{ccc}

- **Conclusions**

Relativistic Boltzmann eq. at finite η/s

Bulk evolution

$$p^{\mu}\,\partial_{\mu} f_q(x,p) + m(x)\,\partial^{x}_{\mu} m(x)\,\partial^{\mu}_{p} f_q(x,p) = C[f_q, f_g]$$

$$p^{\mu}\,\partial_{\mu} f_g(x,p) + m(x)\,\partial^{x}_{\mu} m(x)\,\partial^{\mu}_{p} f_g(x,p) = C[f_q, f_g]$$

Equivalent to viscous hydro η/s≈0.1

free-streaming

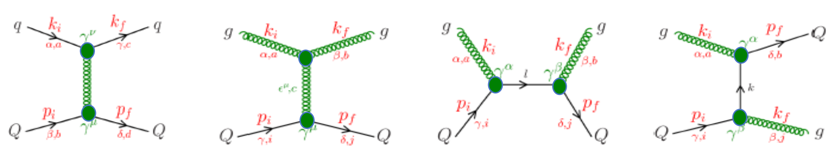
field interaction
ε-3p≠0

collision term
gauged to some η/s≠0

For details:
M.L. Sambataro
(Tue 10:30)

HQ evolution

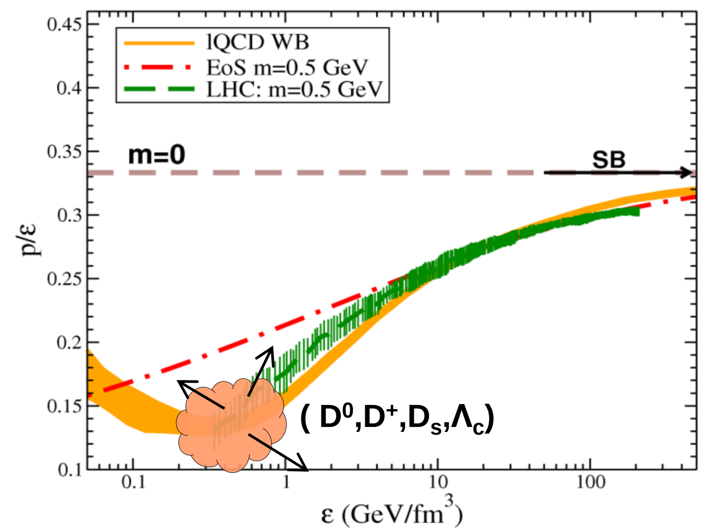
$$p^{\mu}\partial_{\mu} f_Q(x,p) = \mathcal{C}[f_q, f_g, f_Q](x,p)$$



$$\mathcal{C}[f_Q] = \frac{1}{2E_1}\int \frac{d^3p_2}{2E_2(2\pi)^3}\int \frac{d^3p'_1}{2E_{1'}(2\pi)^3} \times [f_Q(p'_1) f_{q,g}(p'_2) - f_Q(p_1) f_{q,g}(p_2)] \times |\mathcal{M}_{(q,g)+Q}(p_1 p_2 \to p'_1 p'_2)|^2 \times (2\pi)^4 \delta^4(p_1 + p_2 - p'_1 - p'_2) \; ,$$

M scattering matrix by QPM model fit to lQCD EoS

S. Plumari et al.,J.Phys.Conf.Ser. 981 012017 (2018).



Λ_c/D^0 ratio in elementary collisions

$$\frac{dN_h}{d^2p_h} = \sum_f \int dz \frac{dN_f}{d^2p_f} D_{f\to h}(z)$$

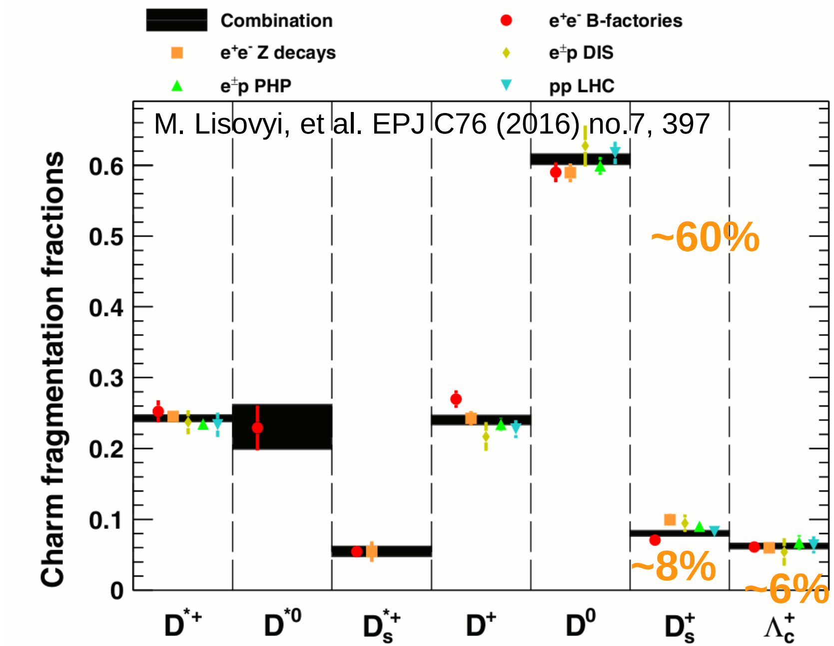
Fragmentation function

The distribution function is evaluated at the Fixed-Order plus Next-to-Leading-Log (FONLL)
M. Cacciari, P. Nason, R. Vogt, PRL 95 (2005) 122001

We use the Peterson fragmentation function
C. Peterson, D. Schalatter, I. Schmitt, P.M. Zerwas PRD **27** (1983) 105

$$D_{f\to h}(z) \propto \frac{1}{z\left[1-\frac{1}{z}-\frac{\epsilon}{1-z}\right]^2}$$

Recent update He-Rapp, PLB795(2019):
Increase ≈ 2 due to added Λ_c resonance not present in PDG, but predicted by RQM [assumed BR with Λ_c dominance]



*** Fragmentation functions**

$$\left(\frac{\Lambda_c^+}{D^0}\right)_{\substack{e^+e^-\\pp}} \simeq 0.1 \qquad \left(\frac{D_s^+}{D^0}\right)_{\substack{e^+e^-\\pp}} \simeq 0.13$$

*** Thermal models about 2 times larger**
A. Andronic et al., Phys. Lett. **B**571, 36 (2003)
I. Kuznetsova, J. Rafelski, EPJ C51, 113 (2007)

$$\left(\frac{\Lambda_c^+}{D^0}\right)_{\substack{e^+e^-\\pp}} \simeq 0.25-0.30$$

Coalescence approach in phase space for HQ

Statistical factor colour-spin-isospin

Parton Distribution function

Hadron Wigner function

$$\frac{dN_{Hadron}}{d^2p_T} = g_H \int \prod_{i=1}^{n} p_i \cdot d\sigma_i \frac{d^3p_i}{(2\pi)^3} f_q(x_i,p_i) f_W(x_1,\ldots,x_n;p_1,\ldots,p_n)\,\delta\left(p_T - \sum_i p_{iT}\right)$$

Thermal+flow for **u,d,s** (p_T< 3 GeV)

$$\frac{dN_{q,\bar{q}}}{d^2r_T\, d^2p_T} = \frac{g_{q,\bar{q}}\tau m_T}{(2\pi)^3} \exp\left(-\frac{\gamma_T(m_T - p_T\cdot\beta_T \mp \mu_q)}{T}\right)$$

$$V = \pi R^2 \tau \cosh(y_z) \quad , R(\tau_f) = R_0(1 + 0.5\,\beta_{max}\tau_f)$$

$$\beta(r) = \frac{r}{R}\beta_{max}$$

PbPb@5ATeV (0-10%) : $\tau_f = 8.6\,\frac{\text{fm}}{\text{c}} \rightarrow V|_{|y|<0.5} = 4580\ fm^3$

+ quenched minijets for u,d,s (p_T >3 GeV)

For **Charm** from the studies of R_{AA} and v_2 of **D-meson** to determine the Space Diffusion coeff.: from parton simulations solving relativistic Boltzmann transport equation

In pp it is FONNL distribution

Coalescence evaluated in a fireball

Space-momentum-time correlation over the freeze-out hypersurfarce of a transport simulation are **not fully** transferred

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Wigner function <-> Wave function

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \varphi_M\left(\mathbf{r} + \frac{\mathbf{r}'}{2}\right) \varphi_M^*\left(\mathbf{r} - \frac{\mathbf{r}'}{2}\right)$$

$\varphi_M(\boldsymbol{r})$ *meson wave function*

Assuming gaussian wave function

$$f_M(x_1, x_2; p_1, p_2) = A_W \exp\left(-\frac{x_{r1}^2}{\sigma_r^2} - p_{r1}^2\sigma_r^2\right)$$

For baryon N_q=3

$$f_H(\ldots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2\sigma_{ri}^2\right)$$

Note: only σ_r coming from $\varphi_M(\boldsymbol{r})$ or $\sigma_r * \sigma_p = 1$ valid for harmonic oscillator with V(r) $\sigma_r * \sigma_p > 1$

Wigner function **width** fixed by root-mean-square charge radius from **quark model**

Meson	$\langle r^2\rangle_{ch}$	σ_{p1}	σ_{p2}
$D^+ = [c\bar{d}]$	0.184	0.282	—
$D_s^+ = [\bar{s}c]$	0.083	0.404	—
Baryon	$\langle r^2\rangle_{ch}$	σ_{p1}	σ_{p2}
$\Lambda_c^+ = [udc]$	0.15	0.251	0.424
$\Xi_c^+ = [usc]$	0.2	0.242	0.406
$\Omega_c^0 = [ssc]$	-0.12	0.337	0.53

C.-W. Hwang, EPJ C23, 585 (2002);
C. Albertus et al., NPA 740, 333 (2004)

$$\langle r^2\rangle_{ch} = \frac{3}{2}\frac{m_2^2Q_1 + m_1^2Q_2}{(m_1+m_2)^2}\sigma_{r1}^2 + \frac{3}{2}\frac{m_3^2(Q_1+Q_2) + (m_1+m_2)^2Q_3}{(m_1+m_2+m_3)^2}\sigma_{r2}^2 \quad (8)$$

$\sigma_{ri} = 1/\sqrt{\mu_i\omega}$ Harmonic oscillator relation

$$\mu_1 = \frac{m_1m_2}{m_1+m_2}, \quad \mu_2 = \frac{(m_1+m_2)m_3}{m_1+m_2+m_3}.$$

Normalization $f_H(\ldots)$ *fixed by requiring* $P_{coal}(p\to 0)=1$ which fixes A_w, additional assumption wrt standard coalescence which does not have confinement

Coalescence approach in phase space for HQ

Statistical factor colour-spin-isospin

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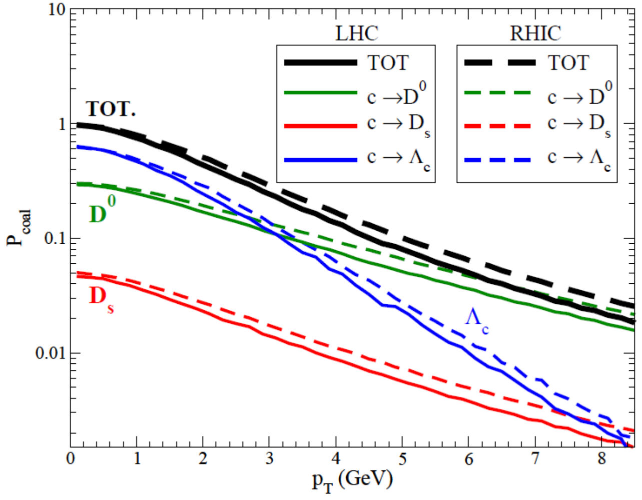
$$\frac{dN_{Hadron}}{d^2p_T} = g_H \int \prod_{i=1}^{n} p_i \cdot d\sigma_i \frac{d^3p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \ldots, x_n; p_1, \ldots, p_n) \delta\left(p_T - \sum_i p_{iT}\right)$$

$$f_H(\ldots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2\sigma_{ri}^2\right)$$



S. Plumari, V. Minissale et al., Eur. Phys. J. **C78** no. 4, (2018) 348

- Normalization in $f_W(\ldots)$ *fixed by requiring* $P_{coal}(p\to 0)=1$:
  ….others modify by hand σ_r to enforce confinement for a charm at rest in the medium

- The charm not “coalescencing” undergo fragmentation:

$$\frac{dN_{had}}{d^2p_T\,dy} = \sum \int dz \frac{dN_{fragm}}{d^2p_T\,dy} \frac{D_{had/c}(z, Q^2)}{z^2}$$

charm number conserved at each p_T ,

we have employed e^+e^- FF now PYTHIA

Heavy flavour (charm): Resonance decay

In our calculations we take into account main hadronic channels, including the ground states and the first excited states for D and Λ_c

Statistical factor

$$\frac{[(2J+1)(2I+1)]_{H*}}{[(2J+1)(2I+1)]_{H}}\left(\frac{m_{H*}}{m_H}\right)^{3/2} e^{-(E_{H*}-E_H)/T}$$

MESONS

D^+ *(I=1/2,J=0)*

D^0 *(I=1/2,J=0)*

D_s^+ *(I=0,J=0)*

Resonances

D^{*+} *(I=1/2,J=1)* ⟶ **$D^0\,\pi^+$** *B.R. 68%*; **D^+ X** *B.R. 32%*

D^{*0} *(I=1/2,J=1)* ⟶ **$D^0\,\pi^0$** *B.R. 62%*; **$D^0\,\gamma$** *B.R. 38%*

D_s^{*+} *(I=0,J=1)* ⟶ **D_s^+ X** *B.R. 100%*

D_{s0}^{*+} *(I=0,J=0)* ⟶ **D_s^+ X** *B.R. 100%*

BARYONS

Λ_c^+ *(I=0, J=1/2)*

Resonances

$\Lambda_c^+(2595)$ *(I=0,J=1/2)* ⟶ **Λ_c^+** *B.R. 100%*

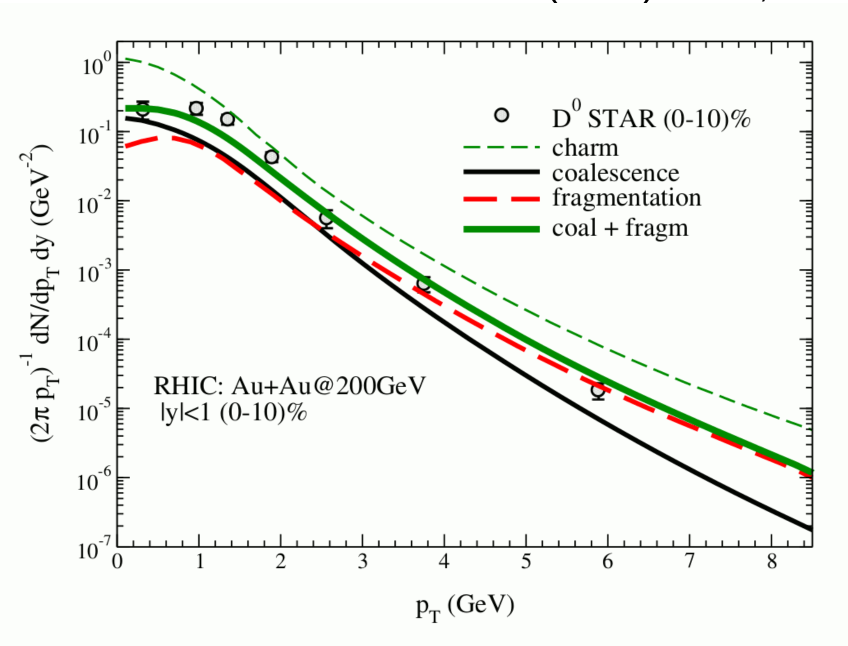
$\Lambda_c^+(2625)$ *(I=0,J=3/2)* ⟶ **Λ_c^+** *B.R. 100%*

$\Sigma_c^+(2455)$ *(I=1,J=1/2)* ⟶ **$\Lambda_c^+\pi$** *B.R. 100%*

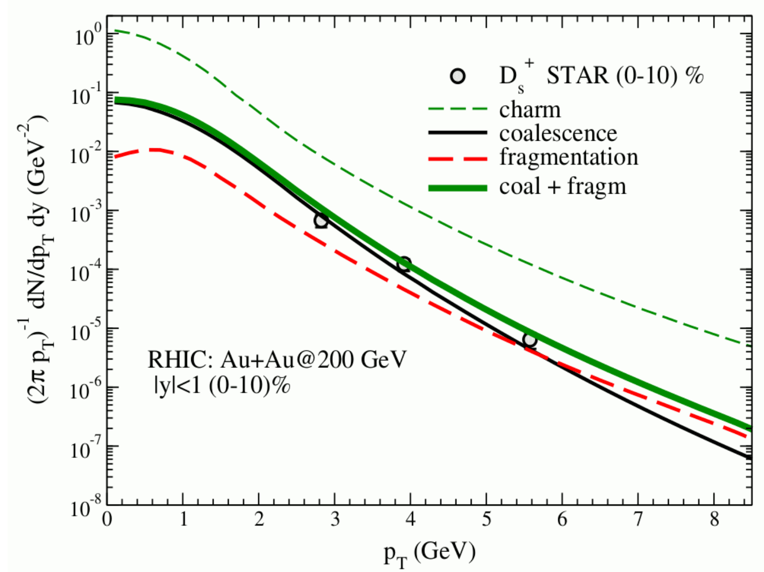
$\Sigma_c^+(2520)$ *(I=1,J=3/2)* ⟶ **$\Lambda_c^+\pi$** *B.R. 100%*

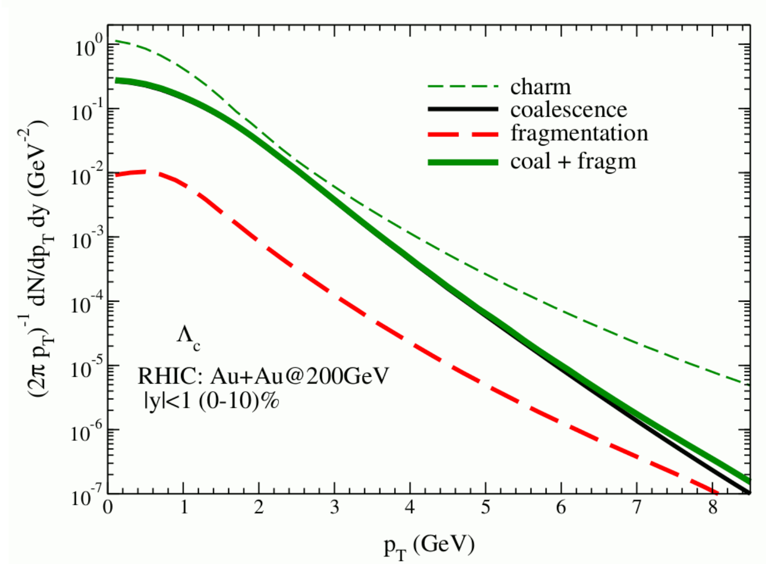
RHIC: results

Data from STAR Coll. PRL **113** (2014) no.14, 142301



Data from STAR Coll., arXiv:1704.04364 [nucl-ex].





S. Plumari, et al., Eur. Phys. J. ***C78*** *no. 4, (2018) 348*

RHIC: Baryon/meson

*S. Plumari, et al., Eur. Phys. J. **C78** no. 4, (2018) 348*

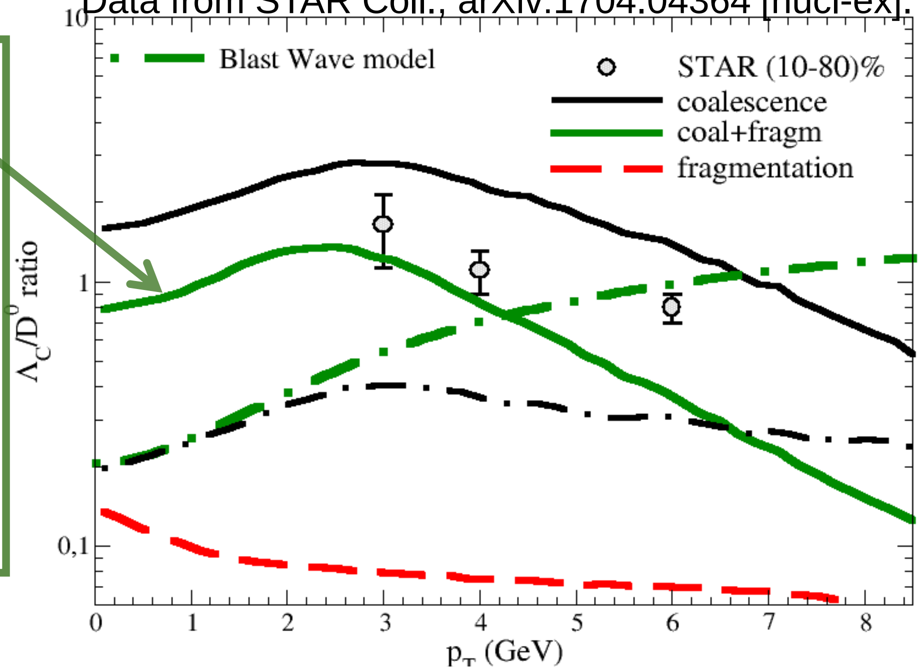
Data from STAR Coll., arXiv:1704.04364 [nucl-ex].

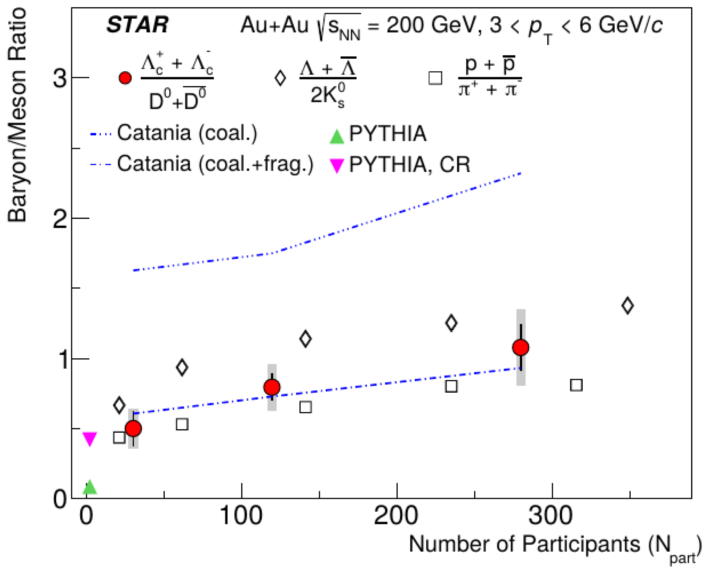
Coalescence

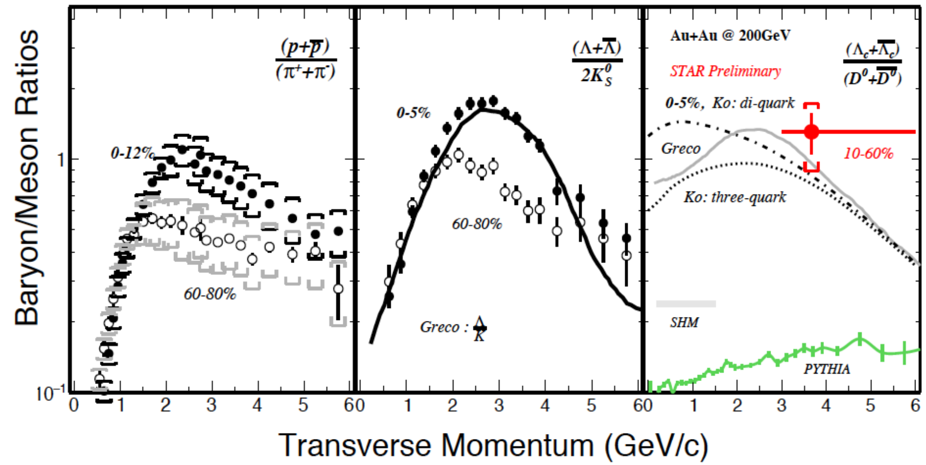
Following: L.W.Chen, C.M. Ko, W. Liu, M. Nielsen, PRC 76, 014906 (2007).
K.-J. Sun, L.-W. Chen, PRC 95, 044905 (2017).
For hypersurface of proper time τ and non relativistic limit:

for $p_T << m$ $$\frac{\Lambda_c^+}{D^0} \propto \frac{g_\Lambda}{g_D}\left(\frac{m_T^\Lambda}{m_T^D}\right) e^{-(m^\Lambda - m^D)/T_C}\, \tau\mu_2$$

$$\mu_2 = \frac{m_3(m_1+m_2)}{m_1+m_2+m_3}$$ Is the reduced mass of the baryon



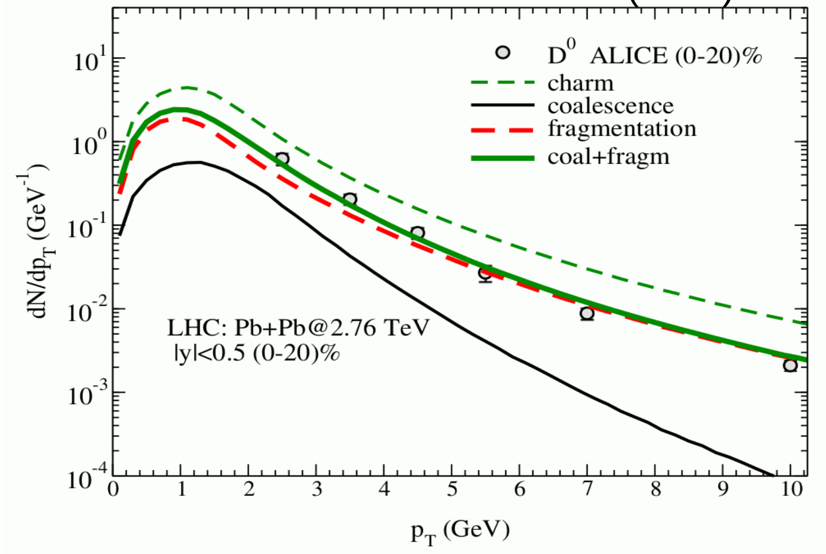


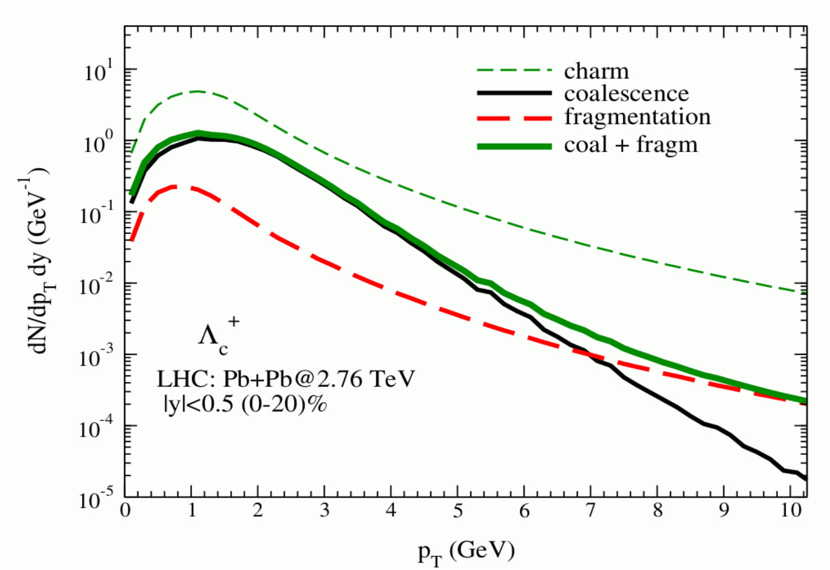


X. Dong and V. Greco.,Prog.Part.Nucl. Phys. (2018)

LHC: results

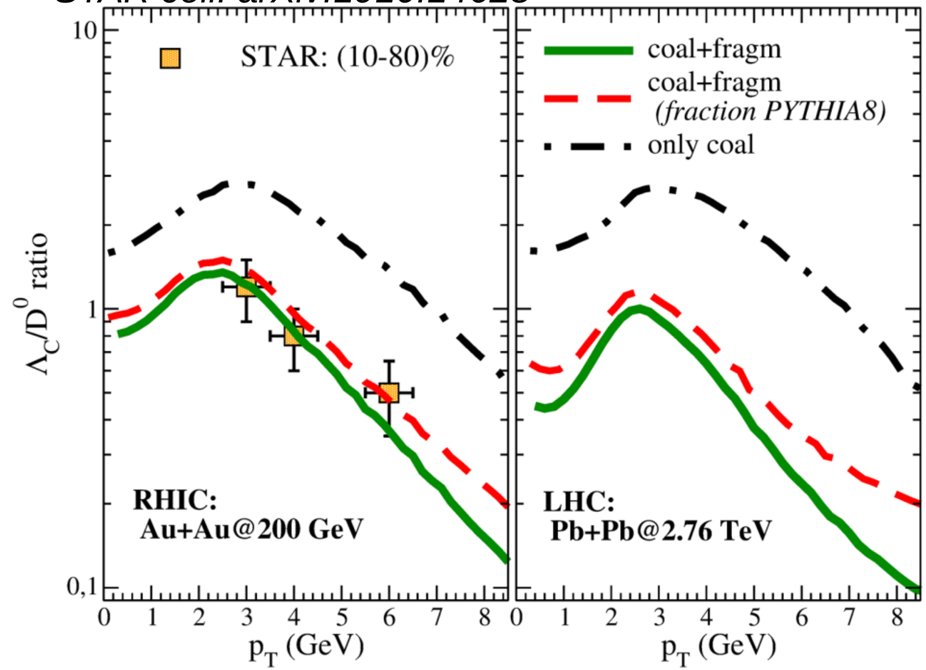
Data from ALICE Coll. JHEP 1209 (2012) 112





wave function widths σ_p of baryon and mesons kept the same at RHIC and LHC!

STAR coll. arXiv:1910.14628



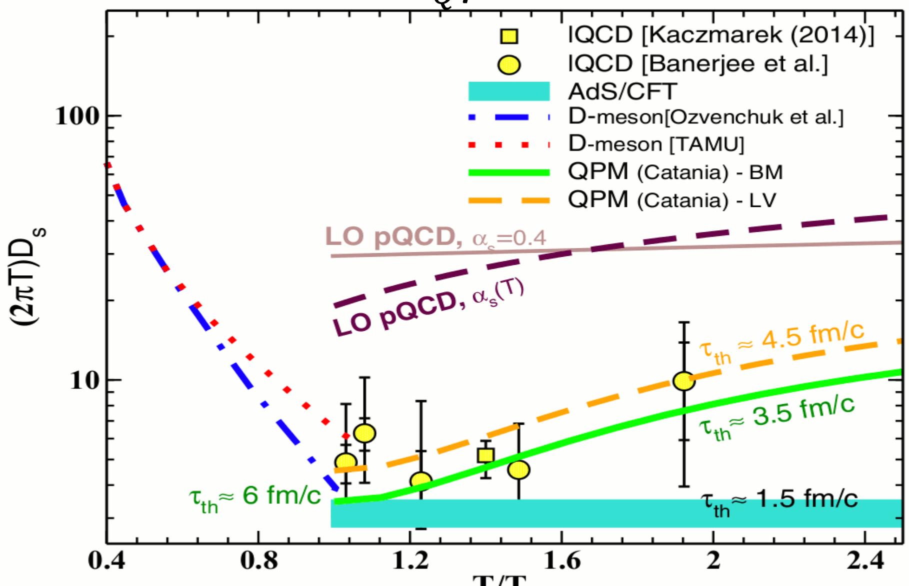
The Λ_c/D^0 ratio is smaller at LHC energies: fragmentation play a role at intermediate p_T

S. Plumari, et al., Eur. Phys. J. ***C78*** *no. 4, (2018) 348*

LHC: results

See talk M.L.Sambataro (Tuesday 10:30)

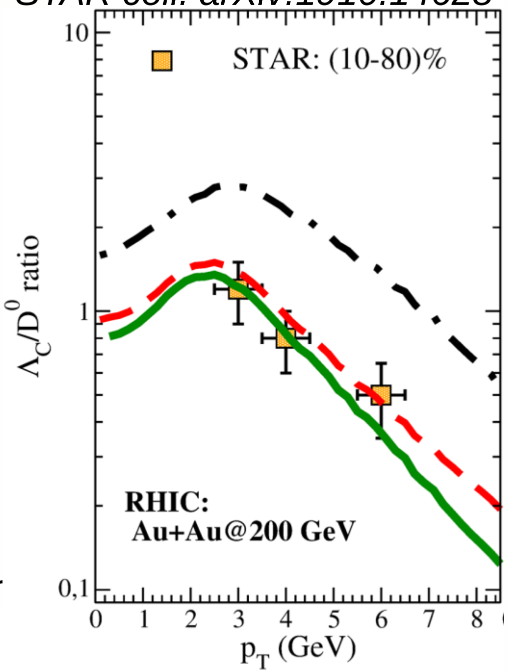
$$D_s(p=0) = \frac{T}{m_Q \gamma} = T m_Q \tau_{th}$$



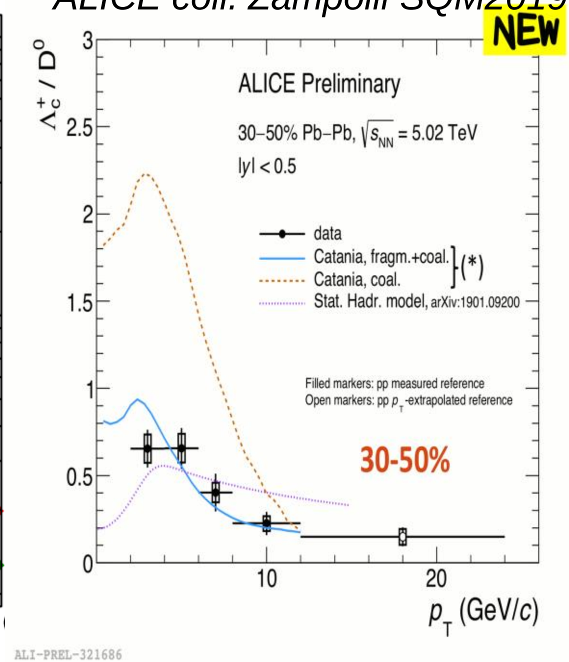
- Not a model fit to lQCD data! but th result from spectra or $R_{AA}(p_T)$ & $v_2(p_T)$

wave function widths σ_p of baryon and mesons kept the same at RHIC and LHC!

STAR coll. arXiv:1910.14628



ALICE coll. Zampolli SQM2019



- With the same coalescence plus fragmentation model we describe the Λ_c/D^0

The Λ_c/D^0 ratio is smaller at LHC energies: fragmentation play a role at intermediate p_T

F. Scardina, S. K. Das, V. Minissale, S. Plumari, V. Greco, PR**C96** (2017) no.4, 044905.

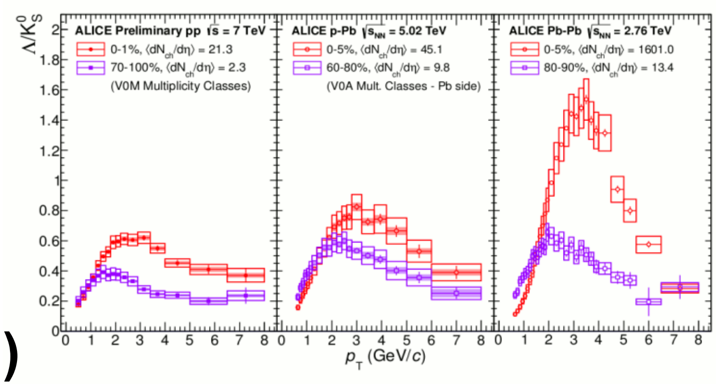
S. Plumari, et al., Eur. Phys. J. ***C78*** *no. 4, (2018) 348*

Small systems

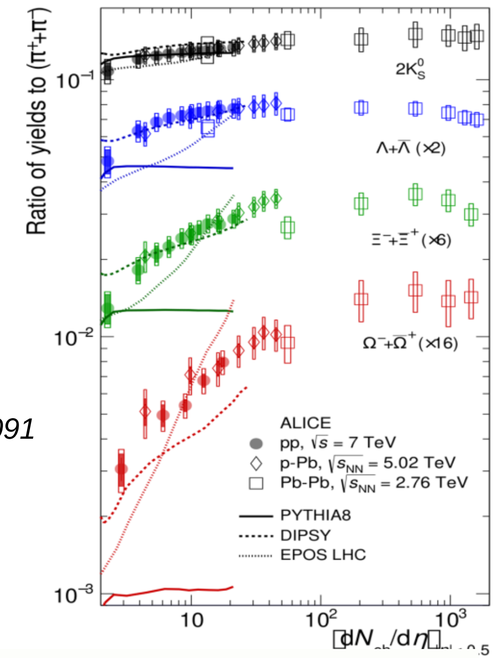
ALICE coll. Nature Phys. 13 (2017) 535

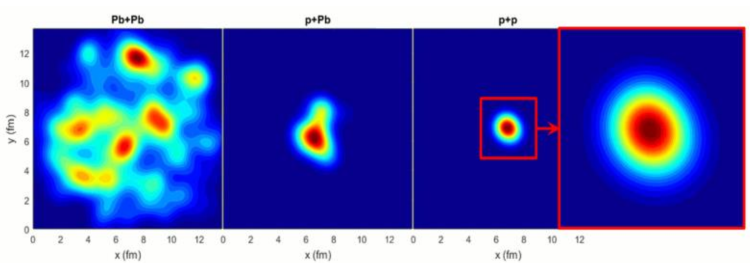
Traditional view:

- **QGP in Pb+Pb**
- **no QGP in p+p (“baseline”)**



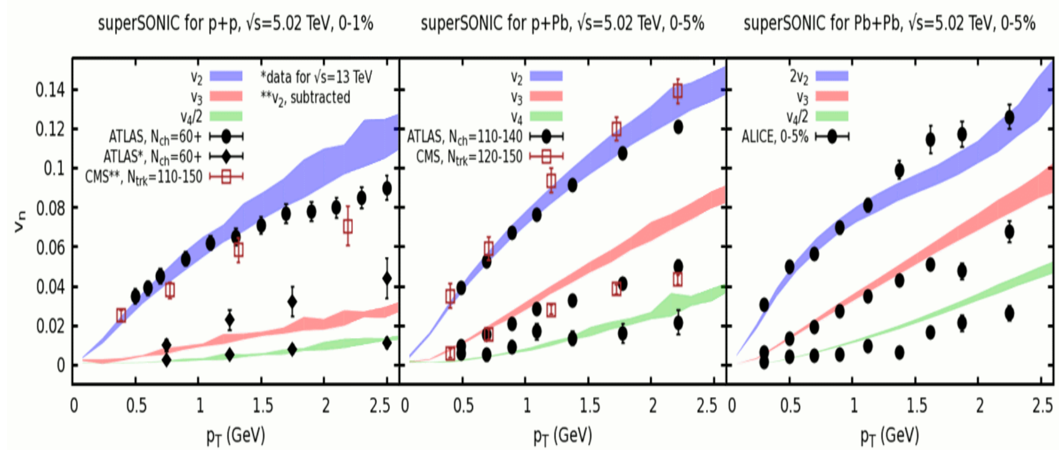
ALICE Coll., PRL 111 (2013) 222301
ALICE Coll., J. Phys.: Conf. Ser. 509 (2014) 012091
ALICE Coll. NPA 956 (2016) 777-780.





<u>Objections to applying hydro in pp</u>

- **Too few particles, cannot be collective**
- **System not in equilibrium**



R. D. Weller, P. Romatschke Phys.Lett. B774 (2017) 351-356

Small systems: Coalescence in pp?

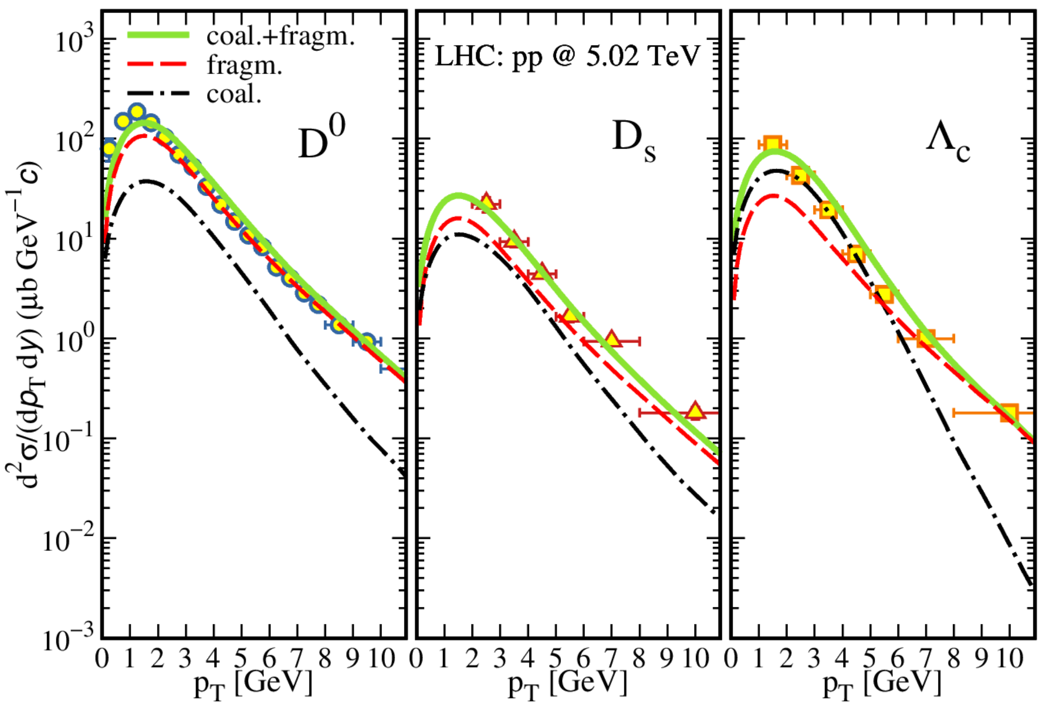
Common consensus of possible presence of QGP in smaller system.

What if:

.Assuming QGP formation also in pp?

.What coal.+frag. predicts in this case?

Data taken from: ALICE coll. EPJ C79 (2019) no.5, 388
ALICE coll. Meninno Hard Probes 2018



V. Minissale et al., *Phys.Lett.B* 821 (2021) 136622

- Thermal Distribution (p_T< 2 GeV)

$$\frac{dN_q}{d^2r_T d^2p_T} = \frac{g_g \tau m_T}{(2\pi)^3} \exp\left(-\frac{\gamma_T(m_T - p_T \cdot \beta_T)}{T}\right)$$

- Collective flow

$$\beta_T = \beta_0 \frac{r}{R}$$

- Fireball radius+radial flow constraints dN_{ch}/dy and dE_T/dy
- Minijet Distribution (p_T> 2 GeV)
- NO QUENCHING

p+p @ 5 TeV

- t_{pp}=1.7 fm/c
- β_0=0.4
- R=2.5 fm
- V~30 fm^3

wave function widths σ_p of baryon and mesons kept the same at RHIC and LHC!

Small systems: Coalescence in pp?

Common consensus of possible presence of QGP in smaller system.

What if:

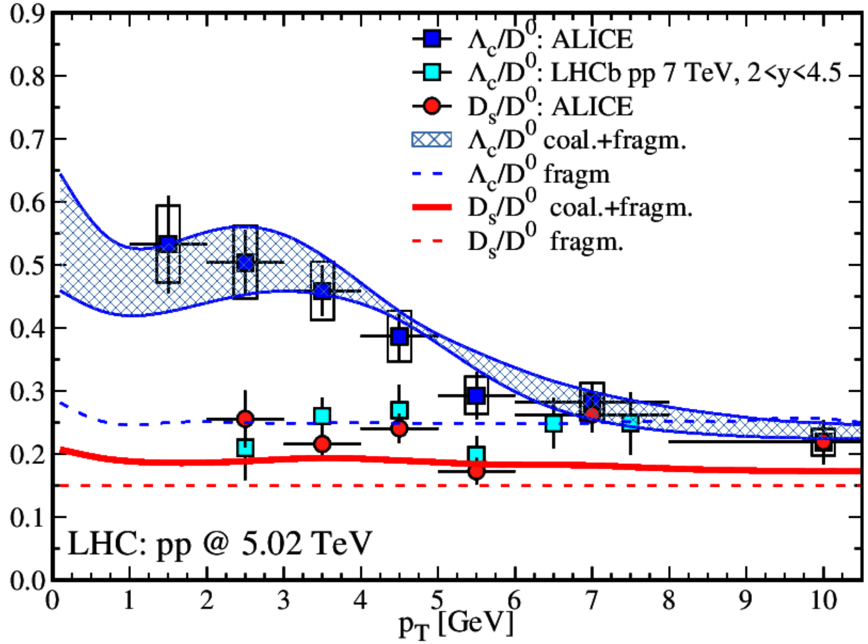
.Assuming QGP formation also in pp?

.What coal.+frag. predicts in this case?

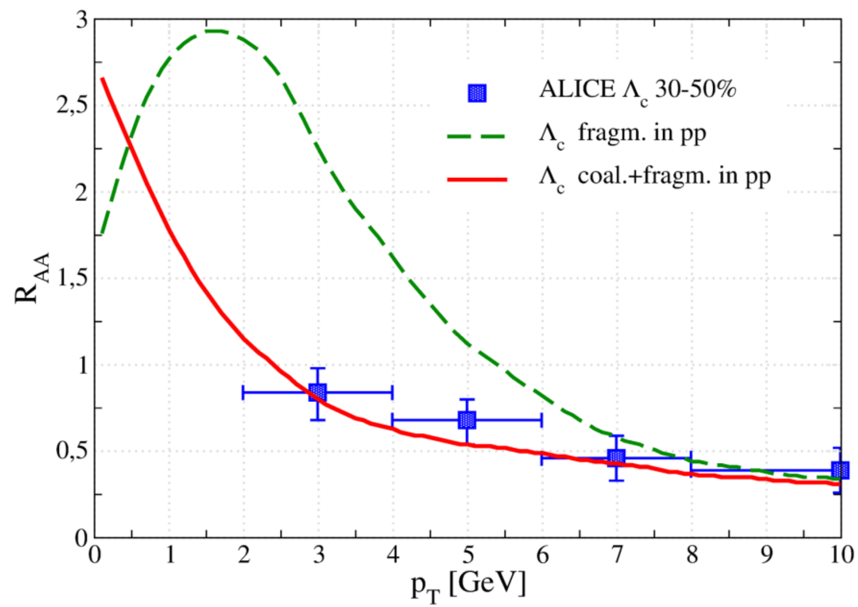
Big effects in AA collisions on R_{AA} of Λ_c -> different behaviour expecially at low momenta.

Data seem to favor model where both coal. and frag. are present in pp

V. Minissale et al., *Phys.Lett.B* 821 (2021) 136622

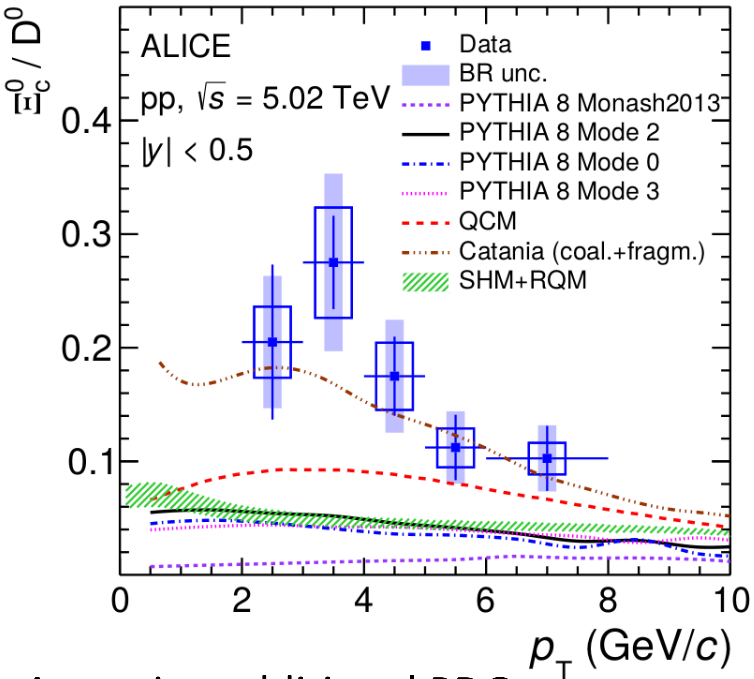


Data taken from: ALICE coll. JHEP 04 (2018) 108
ALICE coll. Rossi SQM2019



Data taken from: ALICE coll. Zampolli SQM2019

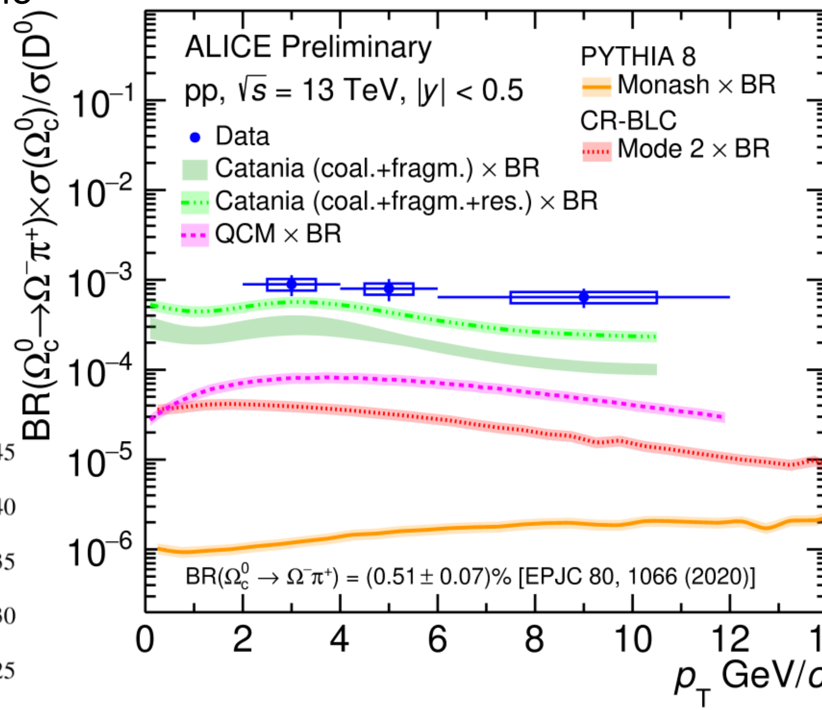
Small systems: Coalescence in pp?



New measurements of heavy hadrons at ALICE:

- Ξ_c/D^0 ratio, same order of Λ_c/D^0: coalescence gives enhancement

- very large Ω_c/D^0 ratio



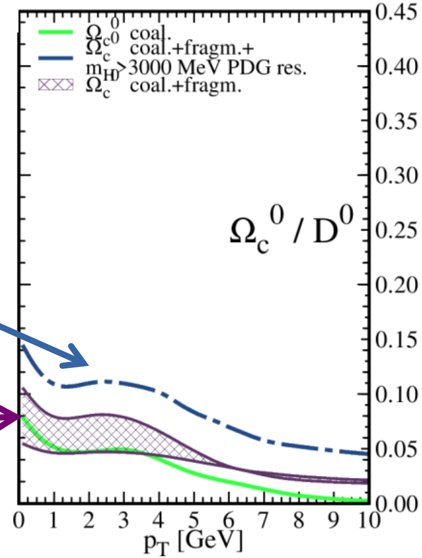
ALICE Collaboration, e-Print: 2109.04326 [hep-ex] V. Minissale et al., *Phys.Lett.B* 821 (2021) 136622

Assuming additional PDG resonances with J=3/2 and decay to Ω_c additional to $\Omega_c^0(2770)$

$\Omega_c^0(3000)$, $\Omega_c^0(3005)$, $\Omega_c^0(3065)$, $\Omega_c^0(3090)$, $\Omega_c^0(3120)$

supply an idea of how these states may affect the ratio

Error band correspond to <r^2> uncertainty in quark model



Multicharm production PbPb and KrKr

$\Xi_{cc}^{+,++}$, Ω_{scc}, Ω_{ccc}

Baryon			
$\Xi_{cc}^{+,++} = dcc, ucc$	3621	$\frac{1}{2}\,(\frac{1}{2})$	
$\Omega_{scc}^{+} = scc$	3679	$0\,(\frac{1}{2})$	
$\Omega_{ccc}^{++} = ccc$	4761	$0\,(\frac{3}{2})$	
Resonances			
Ξ_{cc}^{*}	3648	$\frac{1}{2}\,(\frac{3}{2})$	$1.71 \times g.s$
Ω_{scc}^{*}	3765	$0\,(\frac{3}{2})$	$1.23 \times g.s$

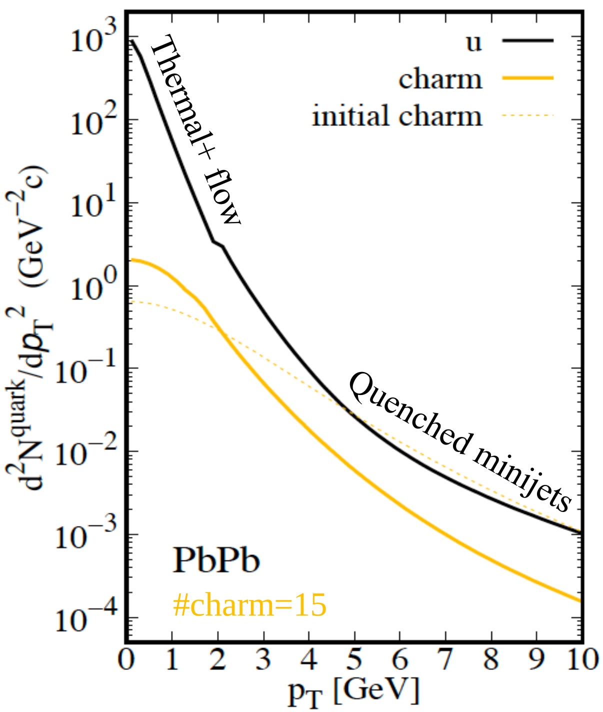
like S.Cho and S.H. Lee, PRC101 (2020)
from R.A. Briceno et al., PRD 86(2012)

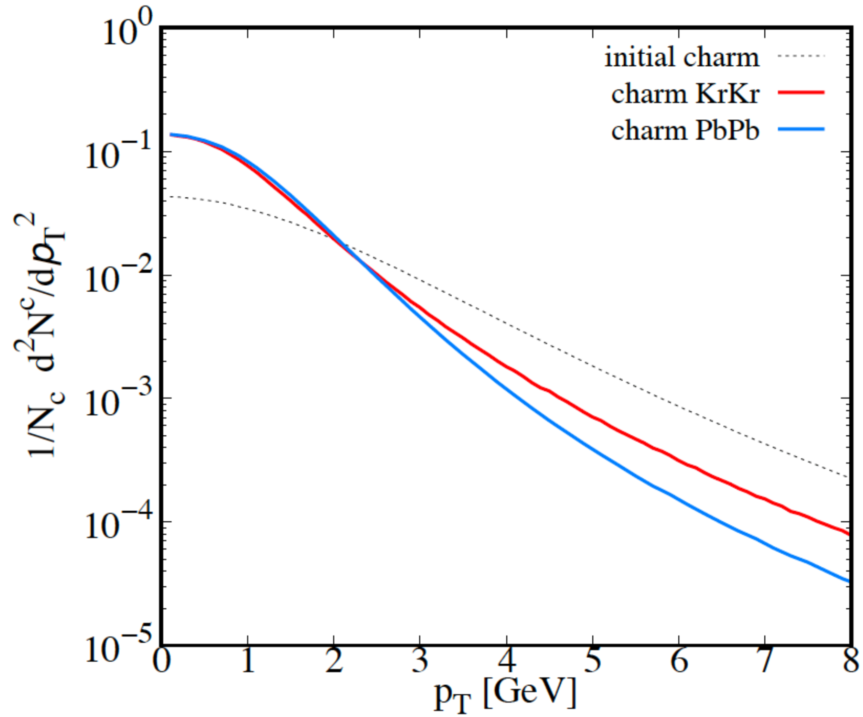
Strengths of the approach:

- Does not rely on distribution in equilibrium for charm
  useful for small AA down to pp collisions and at p_T> 3-4 GeV

- Provide a p_T dependence of spectra and their ratios vs p_T

pT distributions in PbPb vs KrKr from transport approach





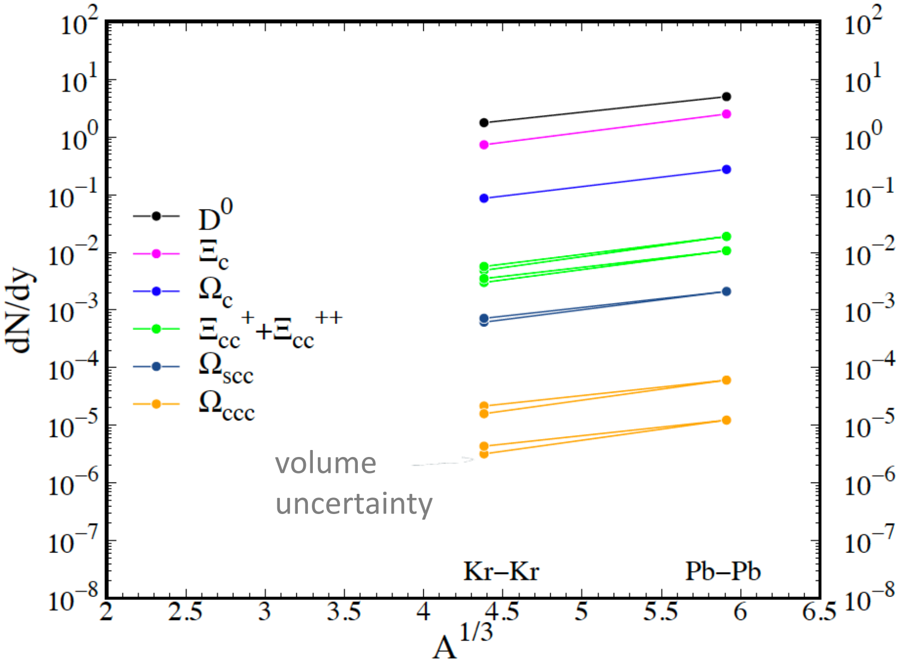
	$KrKr$	$PbPb$
$R_0(fm)$	4.9	6.5
$R_{max}(fm)$	8.6 - 8.9	13.
$\tau(fm)$	5.6 - 6.2	8.3
β_{max}	0.65	0.8
$V_{\lvert y\rvert<0.5}(fm^3)$	1300-1530	4580

KrKr volume scales down by a 3-3.5 factor

Smaller lifetime PbPb: 8.3 fm/c ▯ KrKr:5.6-6.2 fm/c

Shadowing on charm included as a K =0.65 factor [no p_T dependence]
#charm= 15 (PbPb) and 5 (KrKr)

Multi-charm production in PbPb vs KrKr: Yields



In an homogeneous density background in equilibrium at fixed T, discarding flow and wave function effects:

expected scaling with $\approx V \left(\frac{N_c}{V}\right)^c$

c # of charm quarks in the baryon

[work within the same system]

	$KrKr$	$PbPb$
Λ_c	1.361	4.416
Ξ_c	0.737	2.514
Ω_c	0.087	0.275
$\Xi_{cc}^{+,++}$	$4.87 - 5.68 \times 10^{-3}$	1.89×10^{-2}
$\Xi_{cc}^{+,++}$ from ω	$3. - 3.5 \times 10^{-3}$	1.06×10^{-2}
Ω_{scc}	$6.16 - 7.19 \times 10^{-4}$	2.1×10^{-3}
Ω_{ccc}	$3.19 - 4.35 \times 10^{-6}$	1.23×10^{-5}
$\Omega_{ccc}^{\sigma_r\sigma_p=3/2}$	$1.58 - 2.16 \times 10^{-5}$	6.06×10^{-5}

Ξ_{cc}(PbPb)/Ξ_{cc}(KrKr)=3.0-3.5

Ω_{scc}(PbPb)/Ω_{scc}(PbPb)=2.9-3.4

Ω_{ccc}(KrKr)/Ω_{ccc}(PbPb)=2.8-3.8

scaling with $\approx V \left(\frac{N_c}{V}\right)^c$

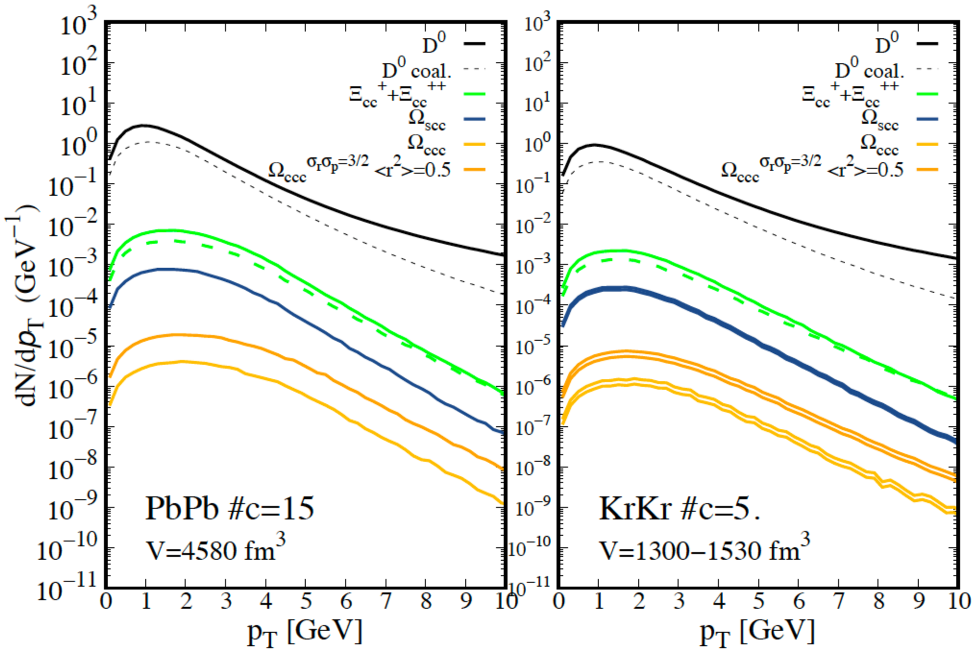
Ξ_{cc} should scale by $N_c \frac{N_c}{V}$ i.e. 2.6-3

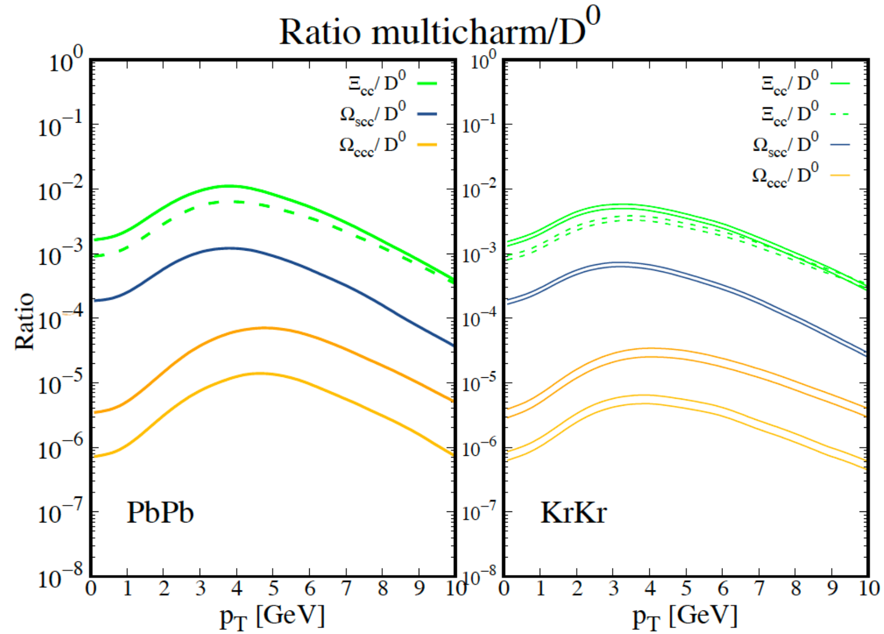
Ω_{ccc} should scale by $N_c \left(\frac{N_c}{V}\right)^2$ i.e. ≈ 2.2-3

*scaling by 2.6 the volume like in SHM one would get 4 for Ω_{ccc} according to the formula

Multi-charm production in PbPb vs KrKr: pT-spectra and Ratios

* No fragmentation contribution included for multi-charm



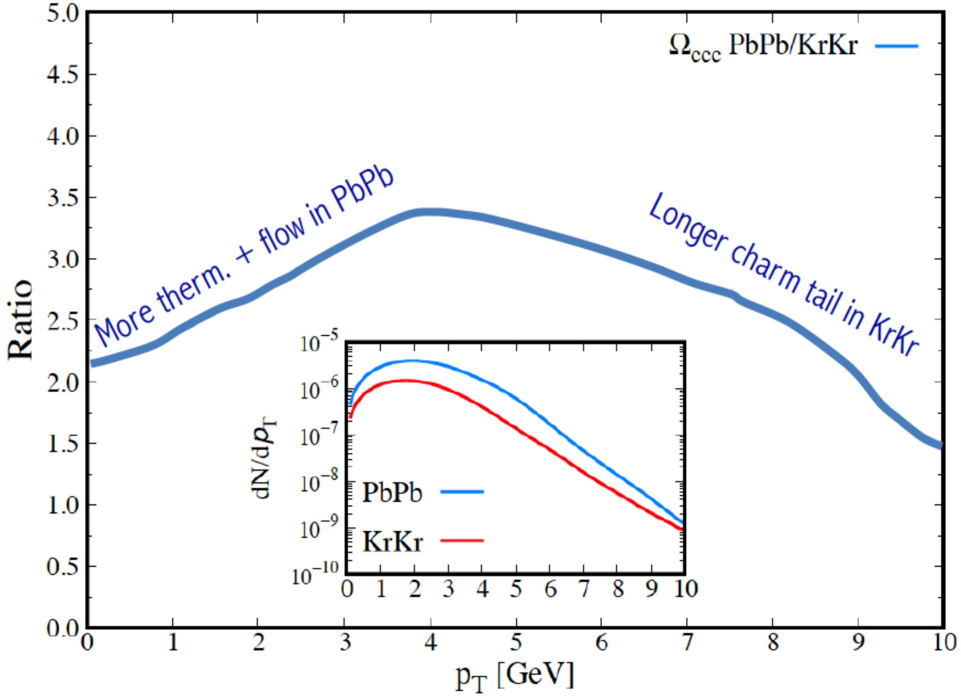


p_T dependence of spectra marginally affected by the discussed uncertainties in the yield

- Ratio Ω_{ccc} /D^0 increases by a factor 12(PbPb) 8(KrKr) peaks at 4.5(3.5) GeV
- Ratio Ξ_{cc} /D^0 increases by a factor 3.5 peaks at 3.5 GeV

A Delicacy from the model

Ratio of Ω_{ccc} spectra in PbPb/KrKr



Conclusions

- **Good agreement with RHIC and LHC data:**

  - **Λ_c production at intermediate p_T dominant role of coalescence mechanism**
  - **Λ_c/D^0 ~1.0 for p_T ~3 GeV with Coal.+fragm. Model**

- ***In p+p assuming a medium like in hydro:***
  - ***Coal.+fragm. good description of heavy baryon/meson ratio.***
  - ***Good description of the recent data: enhancement of Ξ_c/D^0 ratio due coalescence***

- In PbPb (0-10%) from coalescence we have

  $\Xi_{cc}^{++} \approx 5*10^{-3}$, $\Omega_{scc} \approx 2*10^{-3}$, $\Omega_{ccc} \approx 10^{-5}$ (or up to 10^{-4})

  in KrKr (0-10%) the yield are expected scale down by about a factor 3-4

Multi-charm production in PbPb vs KrKr

For multi-charm baryons $\Xi_{cc}^{+}, \Omega_{scc}, \Omega_{ccc}$, we did not have a Quark Model guidance.

We considered two cases:

1) Same σ_r (=σ_p^{-1}) for $\Xi_{cc}^{+,++} = \Omega_{scc} = \Xi_c^0$ and for $\Omega_{ccc} = \Omega_c$ (overestimate?!)
2) Scale the σ_r according to $\sigma_r = 1/\sqrt{\mu_i \omega}$ starting from Ξ_c^0 and Ω_c (harmonic oscillator relation)
3) For Ω_{ccc} we considered that H. He, Y. Liu and P. Zhuang, Phys. Lett. B 746 (2015) studied wave function and Wigner function by solving Schroedinger eq. under Cornell potential

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{i<j} V_{cc}(\mathbf{r}_i, \mathbf{r}_j). \qquad V_{c\bar{c}}(\mathbf{r}_i, \mathbf{r}_j) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma |\mathbf{r}_{ij}|,$$

α, σ vacuum (T=0) values

finding $\boldsymbol{\sigma_r \cdot \sigma_p}$ **≈ 1.5-2** and $\langle r \rangle \approx 0.5\ fm$ --> we adjust our σ_r, σ_p to these values (still assuming Gaussian w.f.)

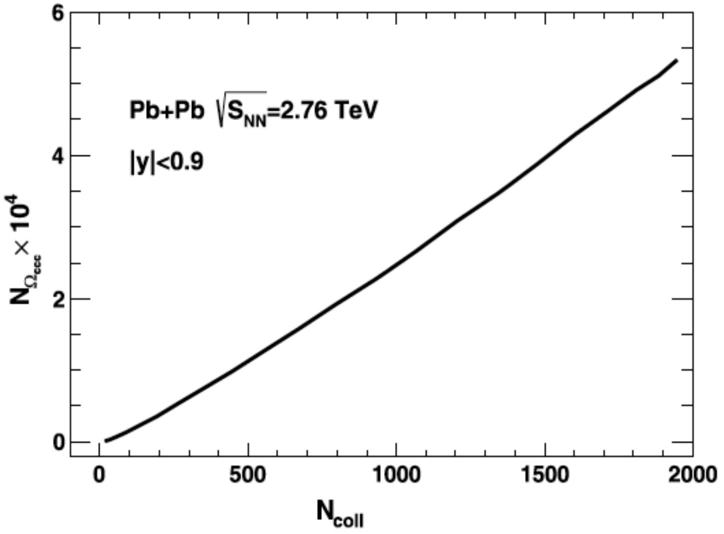
		$\sigma_{p_1}(GeV)$	$\sigma_{p_2}(GeV)$	$\sigma_{r_1}(fm)$	$\sigma_{r_2}(fm)$	$\langle r^2\rangle(fm^2)$
same σ_r	Ξ_{cc}	0.262	0.438	0.751	0.450	0.827
	Ω_{ccc}	0.345	0.557	0.572	0.354	0.371
scaled as H.O. σ_r	Ξ_{cc}^{ω}	0.317	0.573	0.622	0.344	0.545
	$\Omega_{ccc}^{\sigma_r\sigma_p=3/2}$	0.518	$0.595*\sqrt{3}/2$	0.571	$0.571*2/\sqrt{3}$	$\langle r\rangle(fm)=0.5$

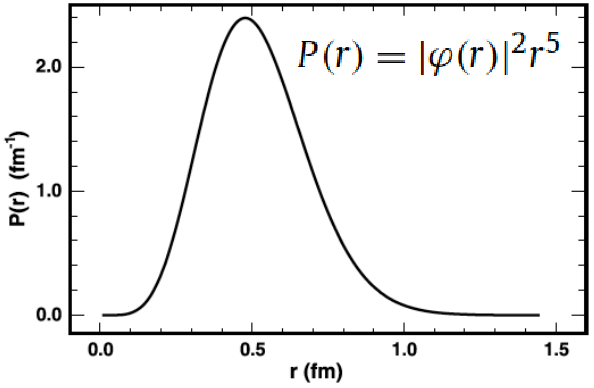
$\boldsymbol{\sigma_r \cdot \sigma_p}$ **≈ 1.5 +** $\langle r \rangle \approx 0.5\ fm$

- Gaussians $\sqrt{\langle r^2\rangle\langle p^2\rangle} = \sqrt{3}\, \boldsymbol{\sigma_r \cdot \sigma_p}$
- PLB746 does not have guassian w.f.

  but has $\sqrt{\langle r^2\rangle\langle p^2\rangle} = 3$ & $\langle r \rangle \approx 0.5\ fm$

Ω_{ccc} Tsignhua approach





$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{i<j} V_{cc}(\mathbf{r}_i, \mathbf{r}_j). \qquad V_{c\bar{c}}(\mathbf{r}_i, \mathbf{r}_j) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma |\mathbf{r}_{ij}|,$$

$$\left[\frac{1}{2m_c}\left(-\frac{d^2}{dr^2} - \frac{5}{r}\frac{d}{dr}\right) + v(r)\right]\varphi(r) = E\varphi(r)$$

$\alpha = \pi/12$ and $\sigma = 0.2$ GeV^2

$$W(\mathbf{r}, \mathbf{p}) = \int d^6\mathbf{y} e^{-i\mathbf{p}\cdot\mathbf{y}} \psi\left(\mathbf{r} + \frac{\mathbf{y}}{2}\right) \psi^*\left(\mathbf{r} - \frac{\mathbf{y}}{2}\right)$$

$$W(r, p, \theta) = \frac{1}{\pi^3} \int d^6\mathbf{y} e^{-ipy_1} \varphi\left(r_y^+\right) \varphi^*\left(r_y^-\right),$$

$$r_y^\pm = \sqrt{r^2 + \frac{1}{4}\sum_{i=1}^{6} y_i^2 \pm (y_1 r\cos\theta + y_2 r\sin\theta)}.$$

$$\frac{dN}{d^2\mathbf{P}_T d\eta} = C \int_\Sigma \frac{P^\mu d\sigma_\mu(R)}{(2\pi)^3} \int \frac{d^4r_x d^4r_y d^4p_x d^4p_y}{(2\pi)^6} \times F(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{p}_1, \tilde{p}_2, \tilde{p}_3) W(r_x, r_y, p_x, p_y),$$

Solve the 3-body problem by a 1-body in higher dimensions hyperspherical coordinates method