

# The statistical hadronization model for heavy quarks

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A. Andronic - University of Münster



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- The statistical model and the thermal fits
  - The charm quarks
  - The beauty quarks

Andronic, Braun-Munzinger, Redlich, Stachel, [Nature 561 \(2018\) 321](#)

...+ Köhler, Mazeliauskas, Vislavicius, [JHEP 07 \(2021\) 035](#)

# The statistical (thermal) model

grand canonical partition function for specie (hadron)  $i$  :

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$  spin degeneracy factor;  $T$  temperature;

$E_i = \sqrt{p^2 + m_i^2}$  total energy; (+) for fermions (-) for bosons

$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$  chemical potentials

$\mu$  ensure conservation (on average) of quantum numbers, fixed by “initial conditions”

i) isospin:  $\sum_i n_i I_{3i} / \sum_i n_i B_i = I_3^{tot} / N_B^{tot}$ ,  $N_B^{tot} \sim \mu_B$

$I_3^{tot}$ ,  $N_B^{tot}$  isospin and baryon number of the system (=0 at high energies)

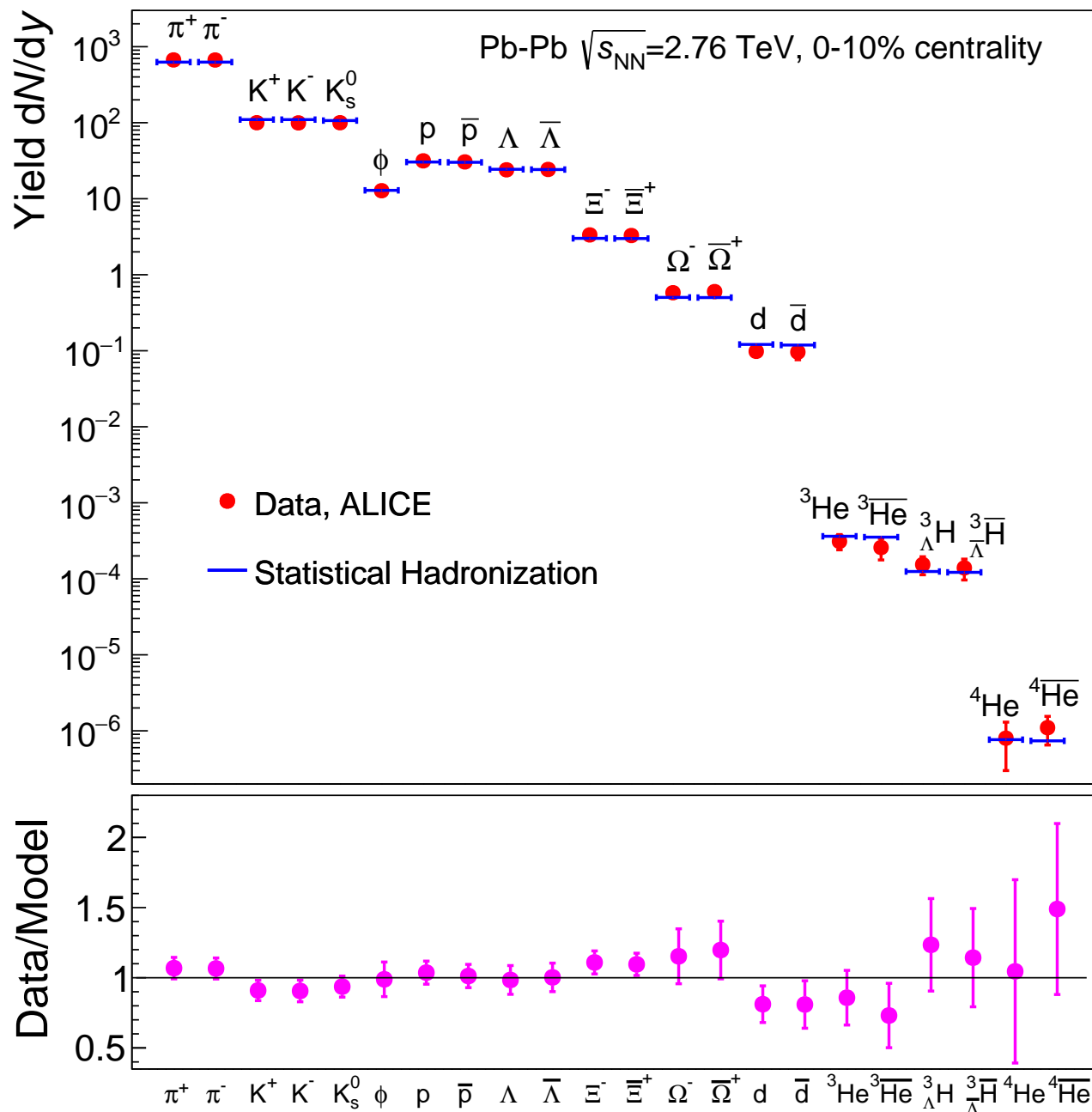
ii) strangeness:  $\sum_i n_i S_i = 0$

iii) charm:  $\sum_i n_i C_i = 0$ .

# Thermal fit – LHC, Pb–Pb, 0-10%

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matter and antimatter produced in equal amounts

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu_B = 0.7 \pm 3.8 \text{ MeV}$$

$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

$$\chi^2/N_{df} = 16.7/19$$

*S-matrix treatment*

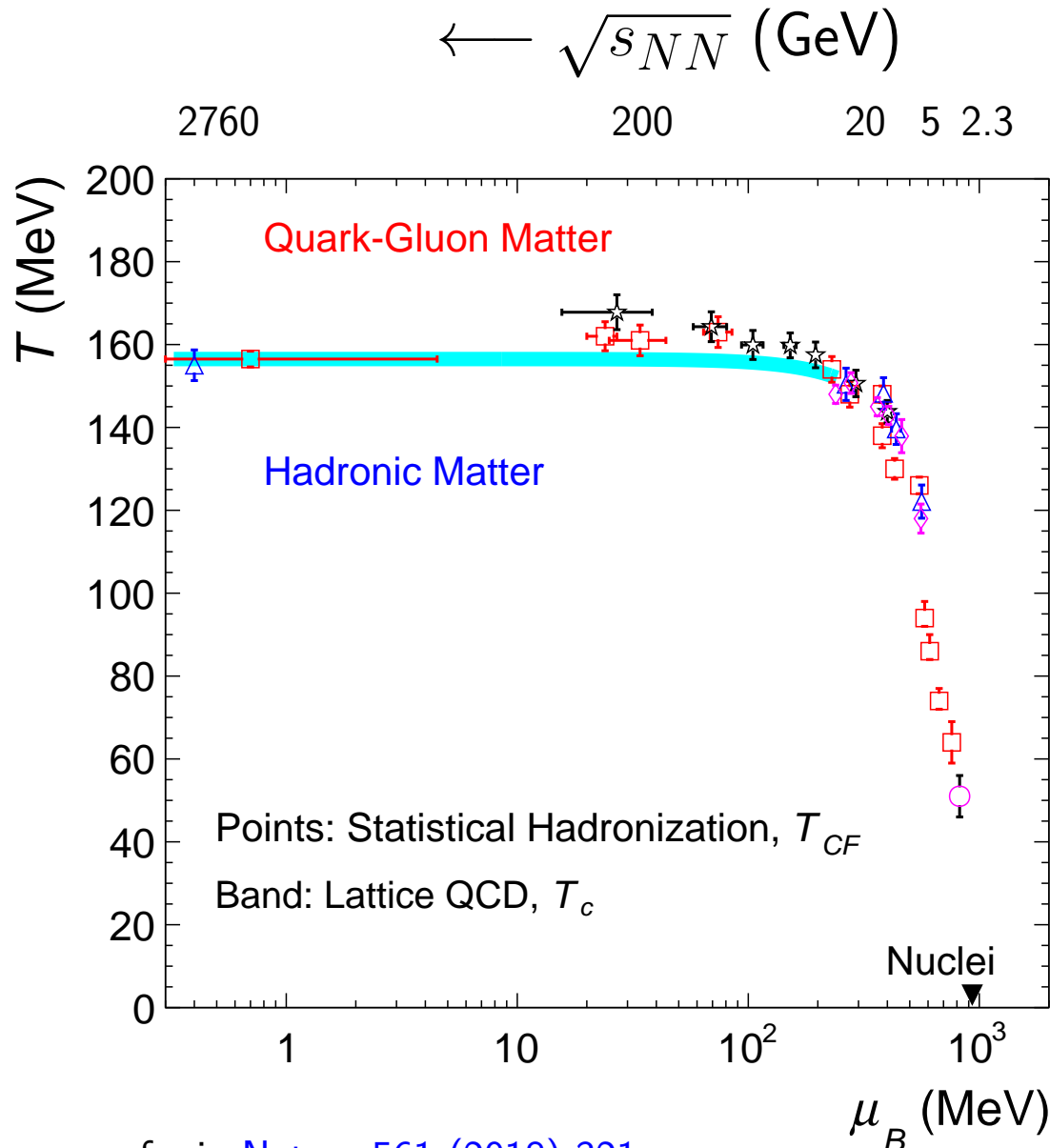
remarkably, loosely-bound objects are also well described ( ${}^3_{\Lambda}\text{H}$  with 25% B.R.)

hadronization as bags of quarks and gluons?

# The phase diagram of QCD

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at LHC, remarkable “coincidence” with Lattice QCD results

at LHC ( $\mu_B \simeq 0$ ): purely-produced (anti)matter ( $m = E/c^2$ ), as in the Early Universe

$\mu_B > 0$ : more matter, from “remnants” of the colliding nuclei

$\mu_B \gtrsim 400 \text{ MeV}$ : *the critical point awaiting discovery*  
(RHIC BES / FAIR)

see refs. in [Nature 561 \(2018\) 321](#)

points: independent analyses of same data  $\rightarrow$  “model/code uncert.” are small

# SHM for charm (SHMc)

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pQCD production, "throw in":  $N_{c\bar{c}} = 9.6 \rightarrow g_c = 30.1$  ( $I_1/I_0 = 0.974$ )

LHC, central collisions

assume:

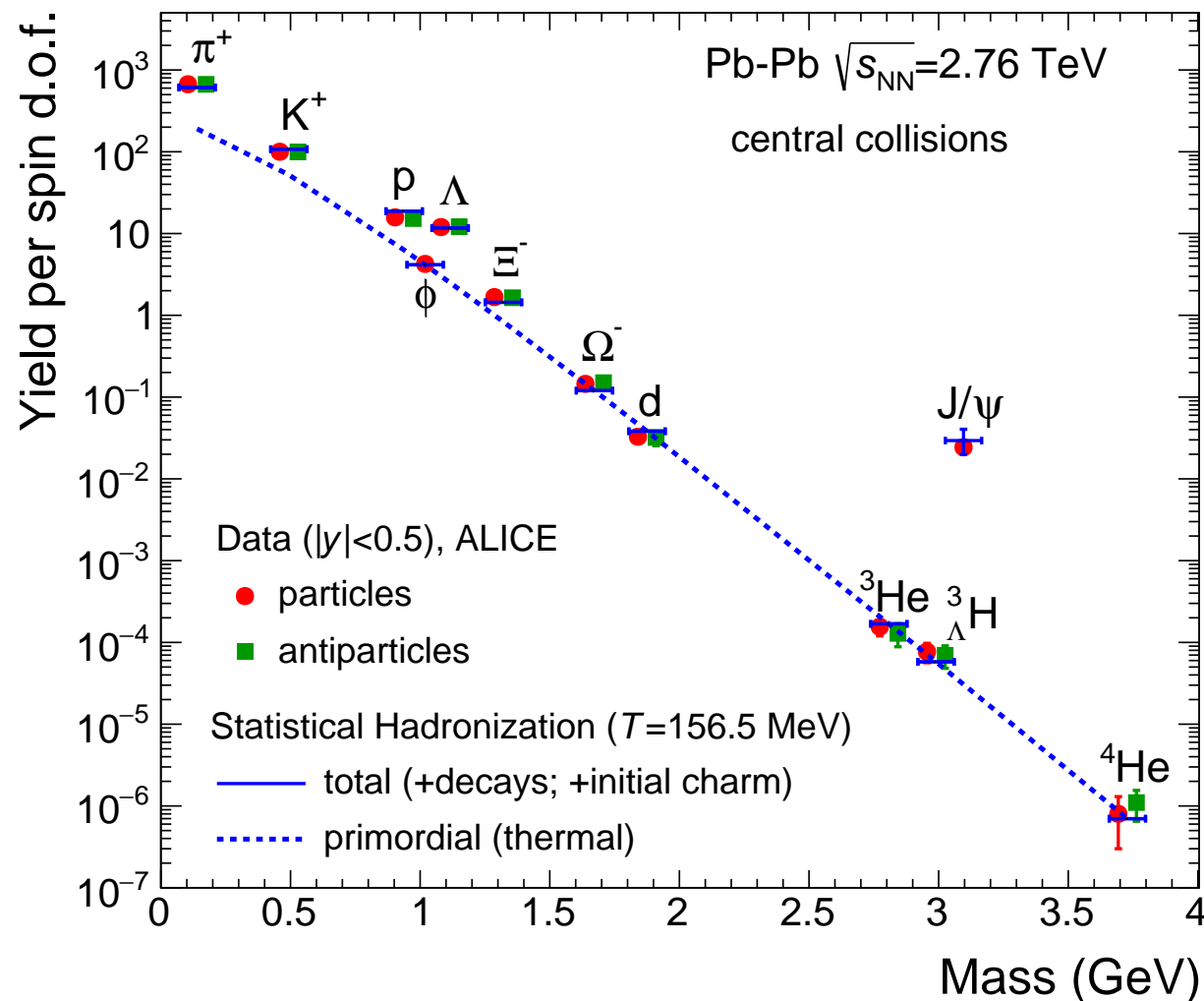
- full thermalization of  $c, \bar{c}$   
("mobility" in  $V \simeq 4000 \text{ fm}^3$ )

- full color screening  
(Matsui-Satz)

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

Model predicts all charm  
chemistry ( $\psi(2S), X(3872)$ )

$\pi, K^\pm, K^0$  from charm included in the thermal fit  
(0.7%, 2.9%, 3.1% for  $T=156.5 \text{ MeV}$ )



[PLB 797 \(2019\) 134836](#)

# SHMc: method and inputs

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#), [NPA 690 \(2001\) 119](#)

- Thermal model calculation (grand canonical)  $T, \mu_B$ :  $\rightarrow n_X^{th}$
- $N_{c\bar{c}}^{dir} = \frac{1}{2}g_c V (\sum_i n_{D_i}^{th} + n_{\Lambda_i}^{th}) + g_c^2 V (\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th})$
- $N_{c\bar{c}} \ll 1 \rightarrow$  Canonical (Cleymans, Redlich, Suhonen, Z. Phys. C51 (1991) 137):

Gorenstein, Kostyuk, Stöcker, Greiner, [PLB 509 \(2001\) 277](#)

$$N_{c\bar{c}}^{dir} = \frac{1}{2}g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \rightarrow g_c \text{ (charm fugacity)}$$

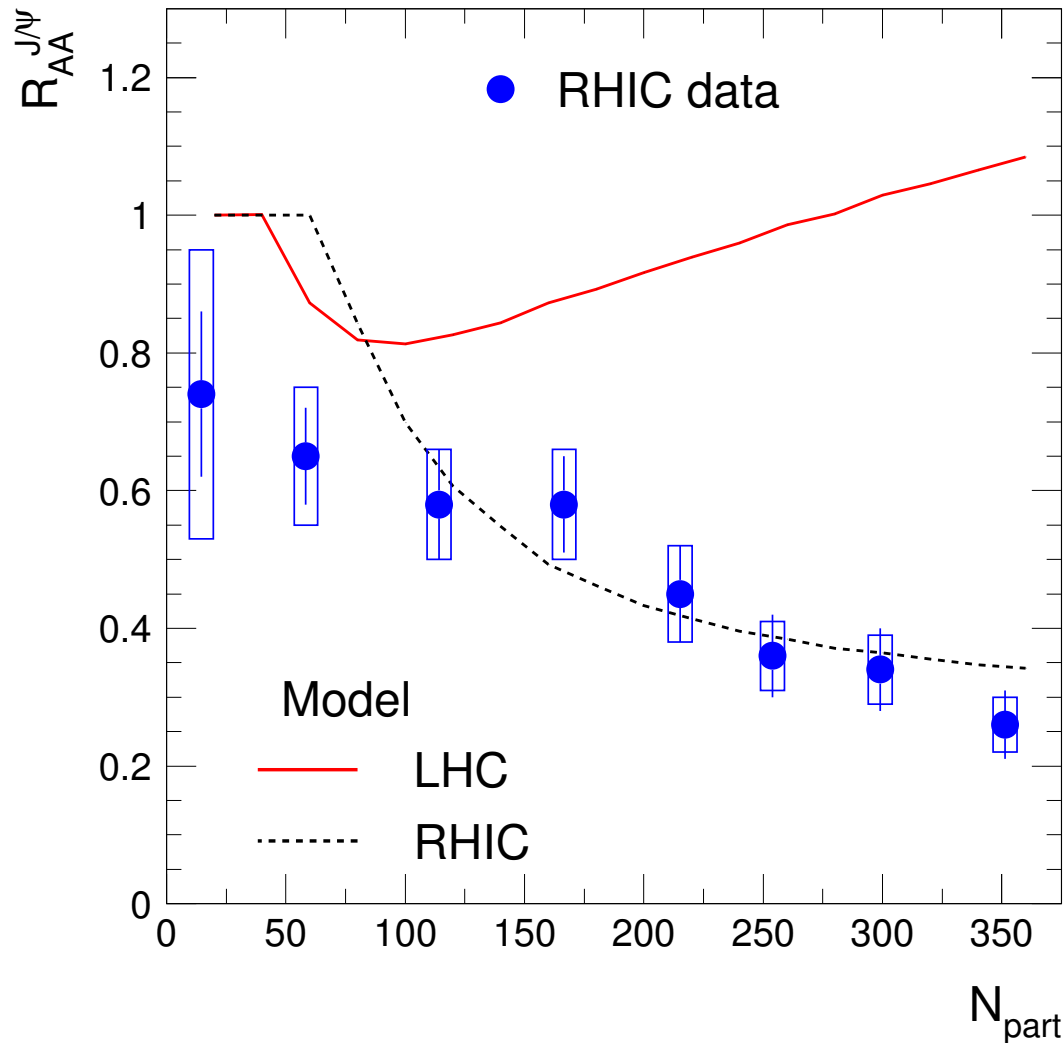
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$$\text{Outcome: } N_D = g_c V n_D^{th} I_1/I_0 \quad N_{J/\psi} = g_c^2 V n_{J/\psi}^{th}$$

$$\text{Inputs: } T, \mu_B, \quad V_{\Delta y=1} (= (dN_{ch}^{exp}/dy)/n_{ch}^{th}), \quad N_{c\bar{c}}^{dir} \text{ (exp. or pQCD)}$$

Assumed minimal volume for QGP:  $V_{QGP}^{min} = 200 \text{ fm}^3$

# High hopes for charmonium at the LHC



$$R_{AA}^{J/\psi} = \frac{dN_{J/\psi}^{AuAu}/dy}{N_{coll} \cdot dN_{J/\psi}^{pp}/dy}$$

- "suppression" at RHIC
- "enhancement" at LHC

$$N_{J/\psi} \sim (N_{c\bar{c}}^{dir})^2$$

What is so different at LHC?  
(compared to RHIC)

$\sigma_{c\bar{c}}$ :  $\sim 10x$ , Volume: 2-3x

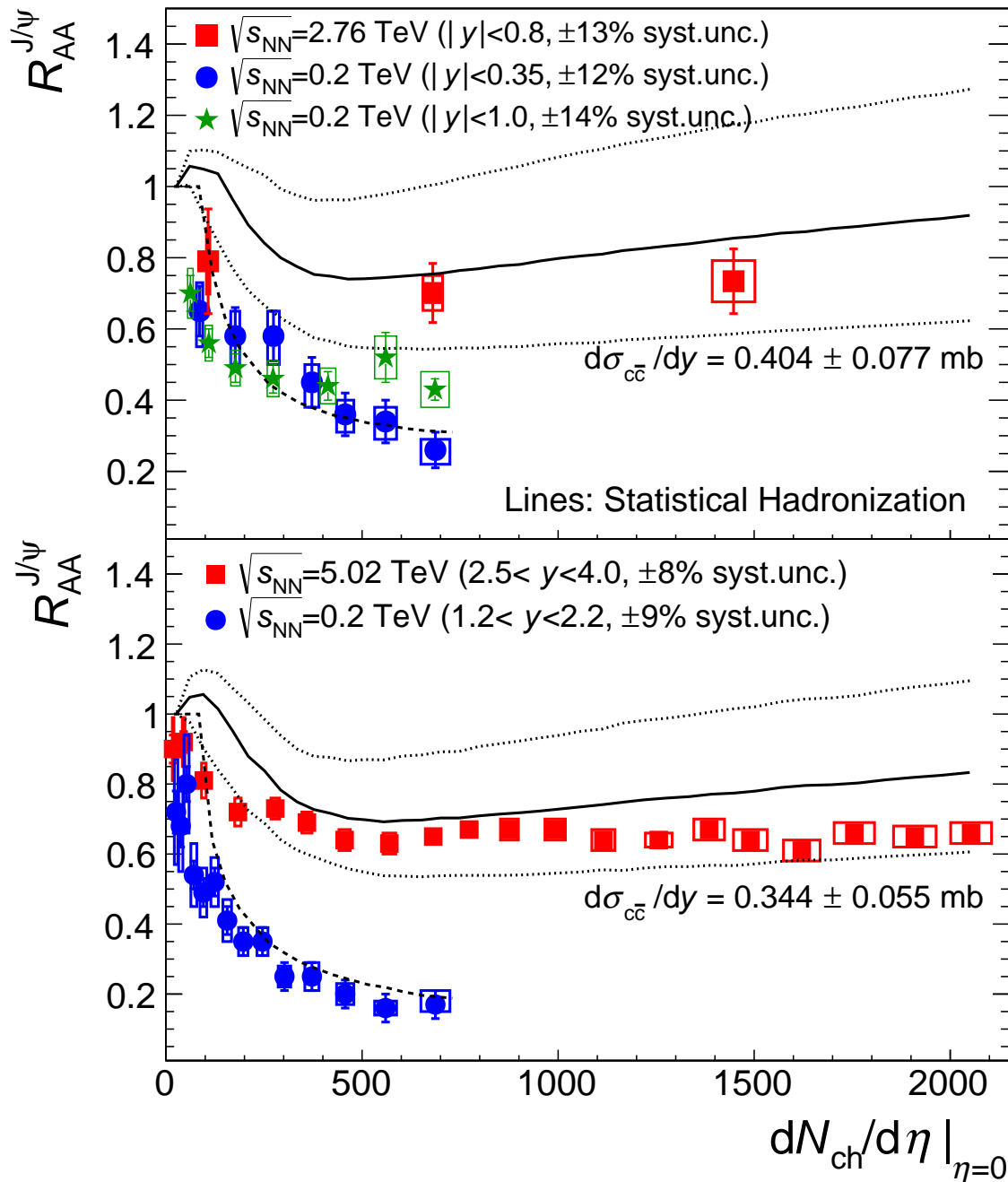
PLB 652 (2007) 259

this is a generic prediction of SHM ...was confirmed by data

# Charmonium data at RHIC and the LHC

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- "suppression" at RHIC (PHENIX)
- dramatically different at the LHC

$J/\psi$  is another observable (charm) for the phase boundary  
calculations are for  $T=156$  MeV

$\sigma_{c\bar{c}}$ : current knowledge at LHC  
 (data in pp, p-Pb; ALICE, LHCb)

$$dN_{ch}/d\eta \sim \varepsilon$$

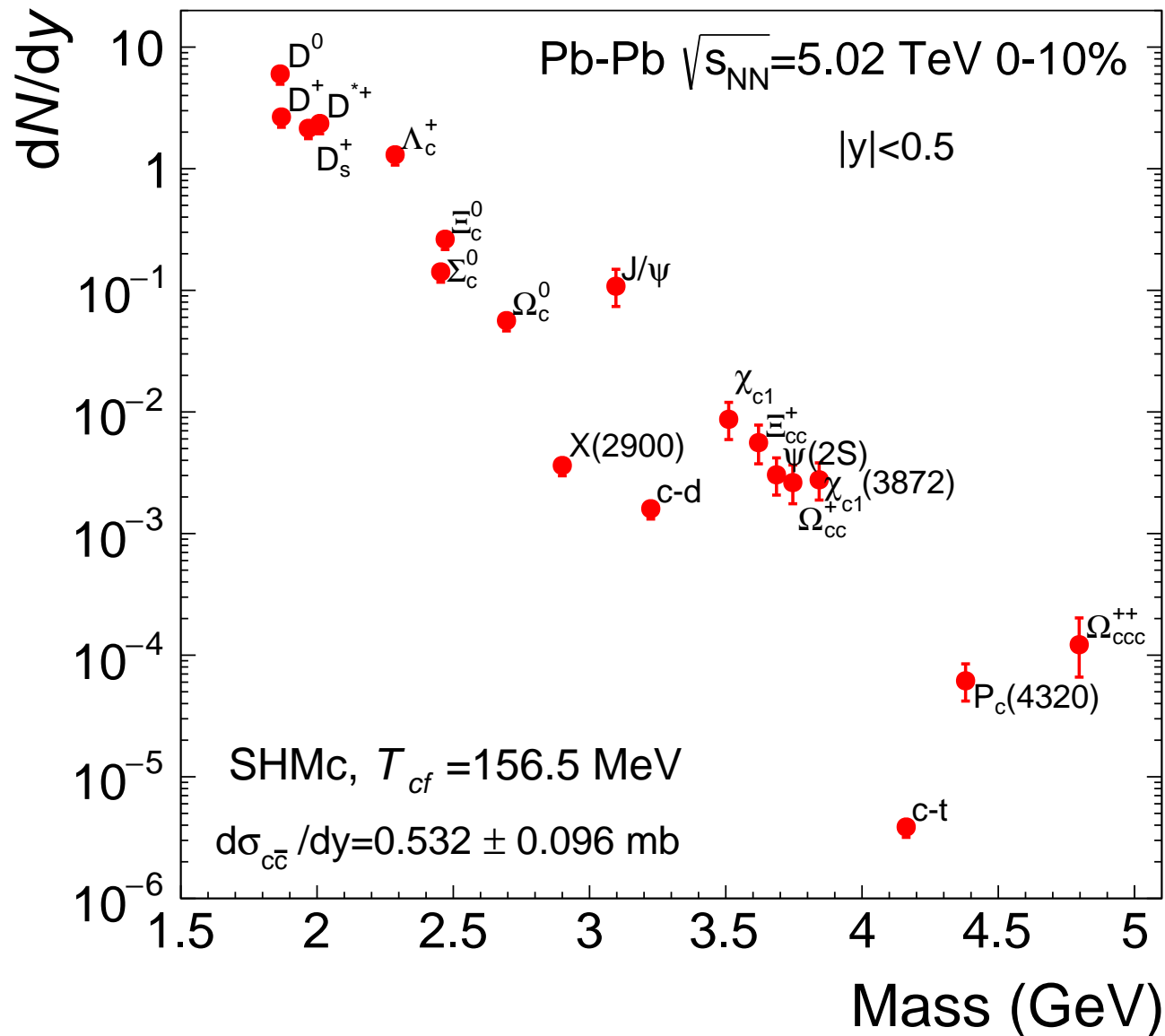
( $>20$  GeV/fm<sup>3</sup>, for  $dN_{ch}/d\eta \simeq 2000$ )



# Full charm predictions for the LHC

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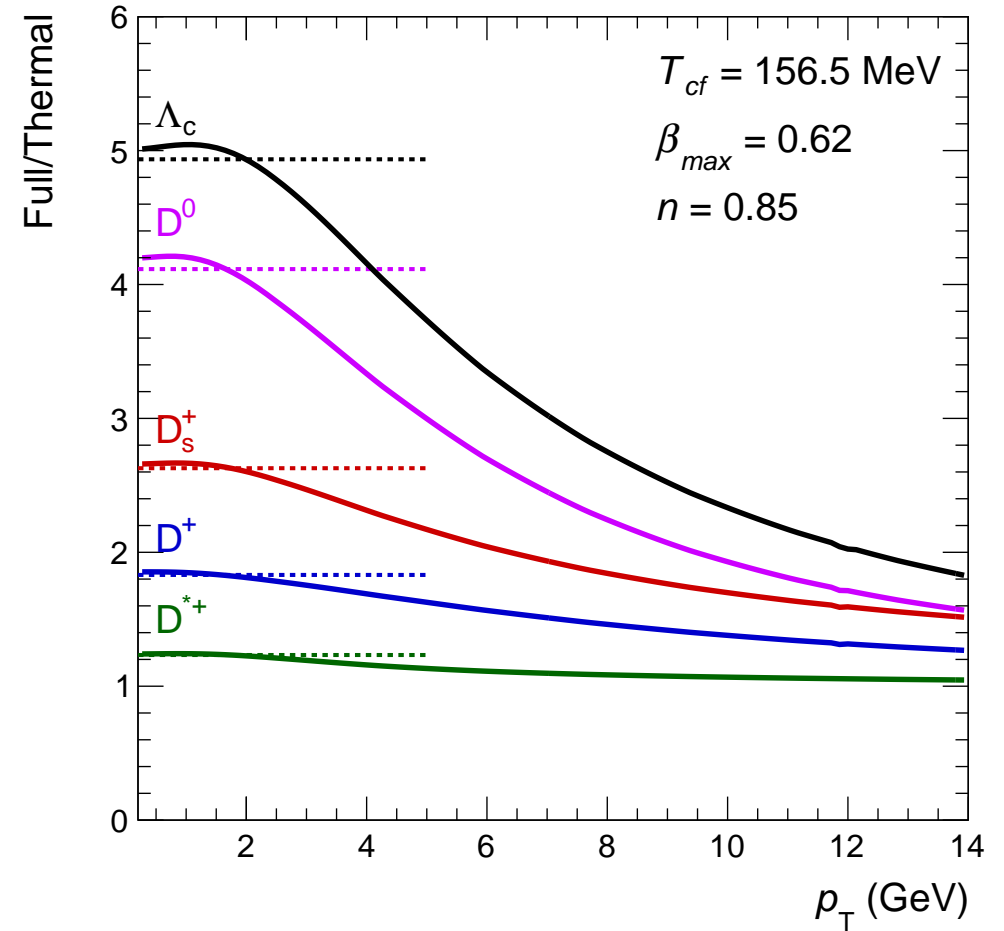
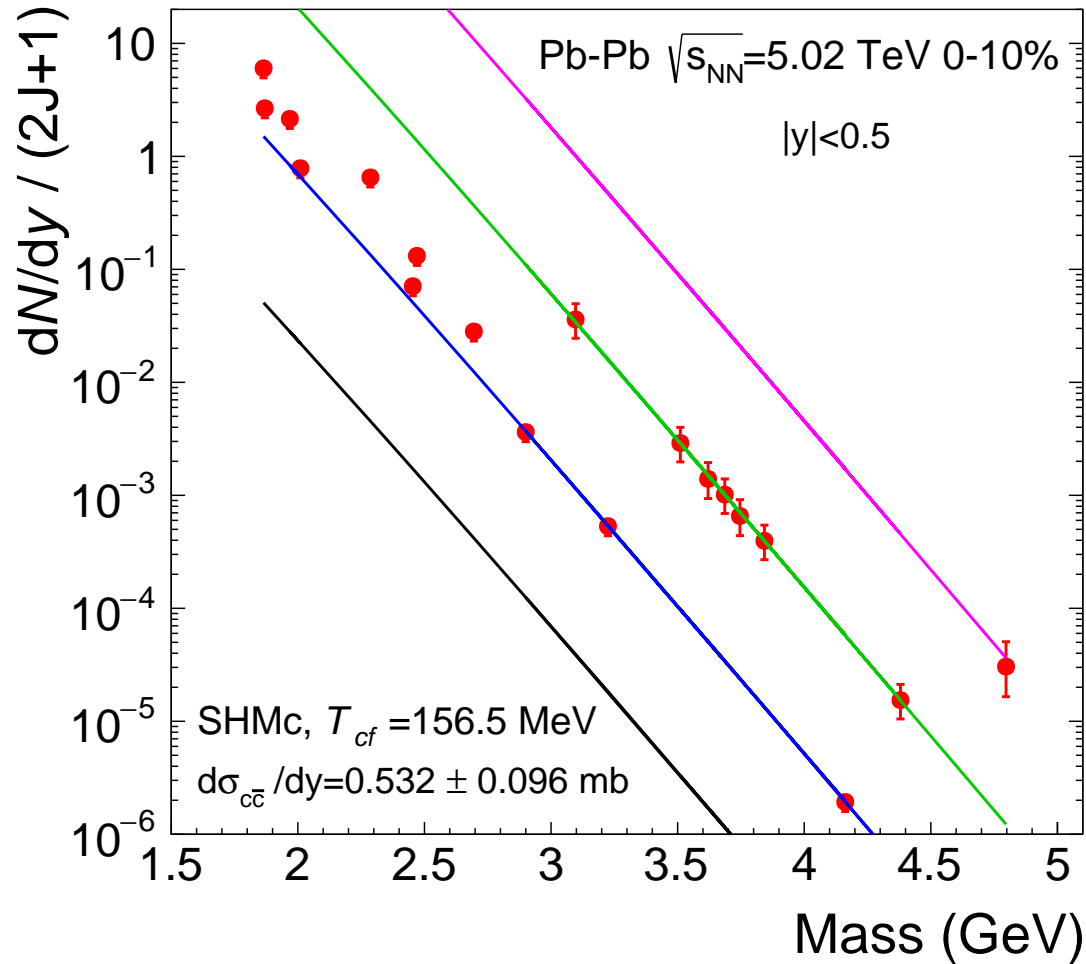


We do have predictions for different collision systems and  $y$  ranges (+30-50%)

# Full charm predictions for the LHC

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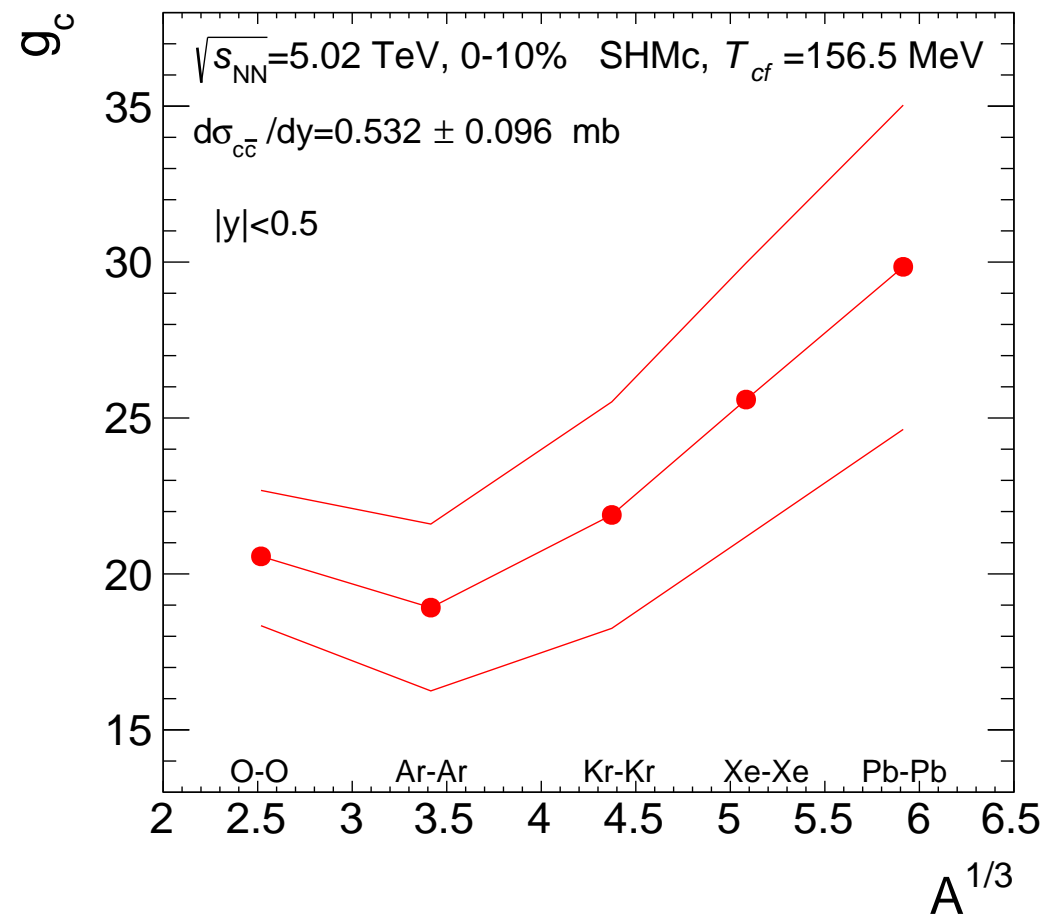
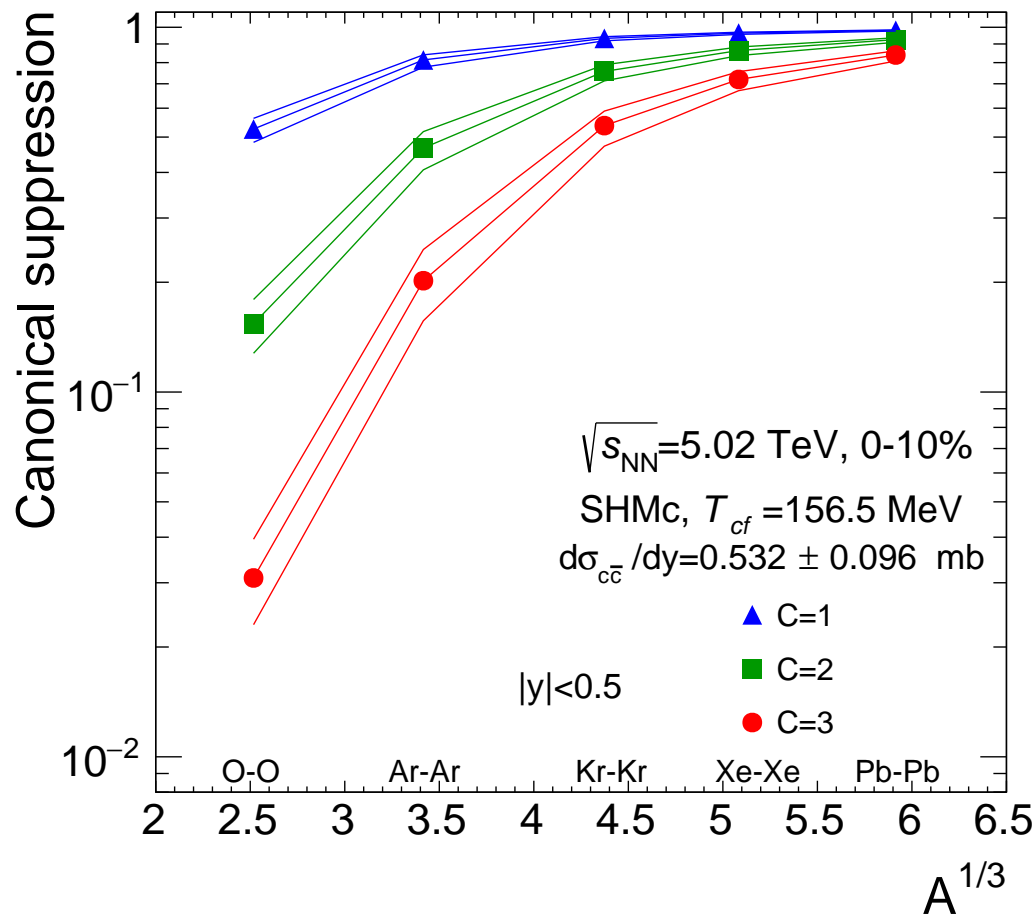


Charm-hadron spectrum as in PDG: 55 c-mesons, 74 c-baryons (part.+antipart.)

# SHMc: system dependence (central, 0-10%)

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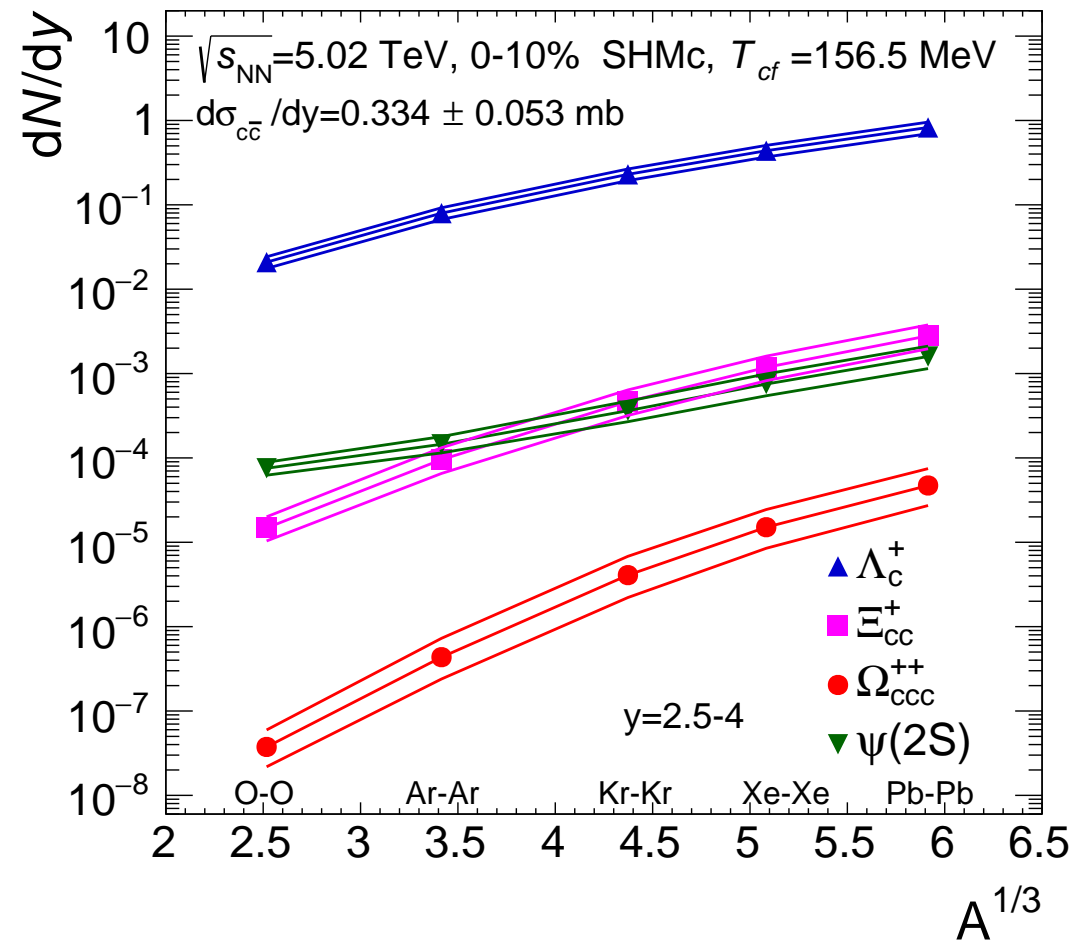
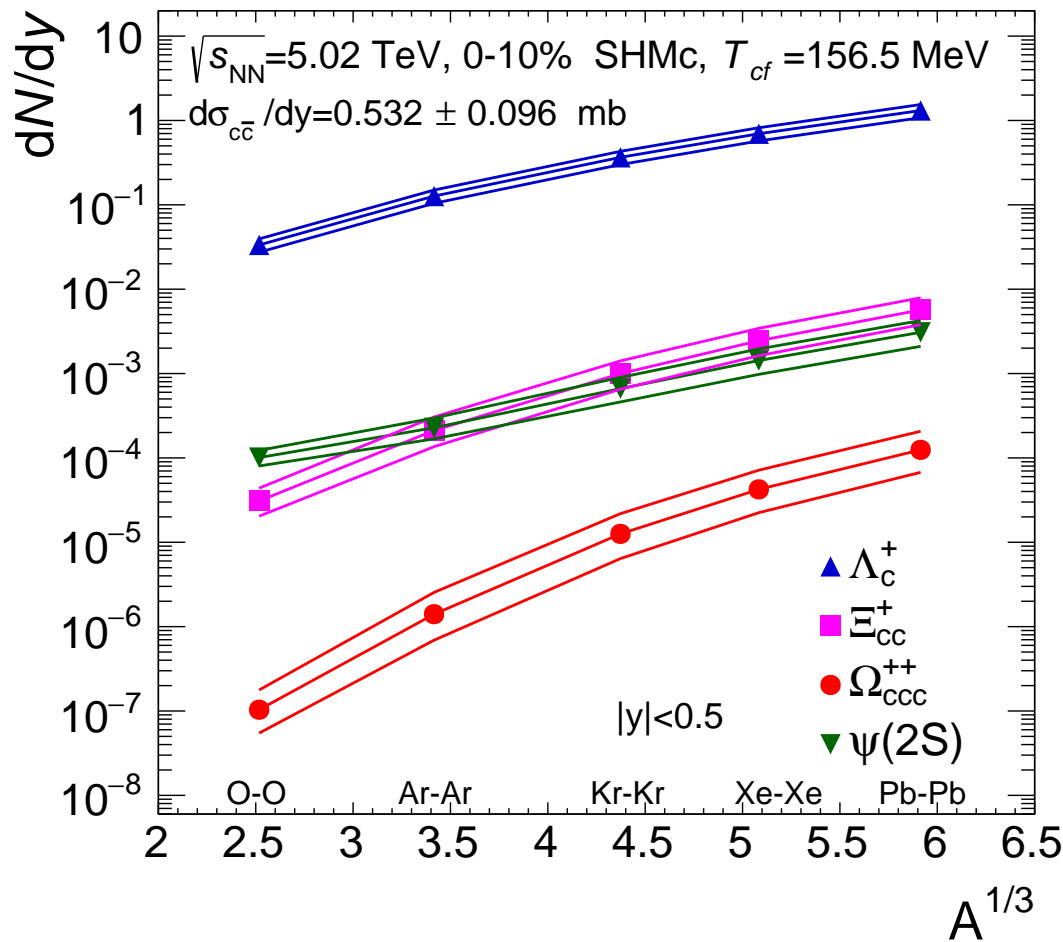


Strong canonical suppression for light systems (for multi-charm hadrons)

# SHMc: system dependence (central, 0-10%)

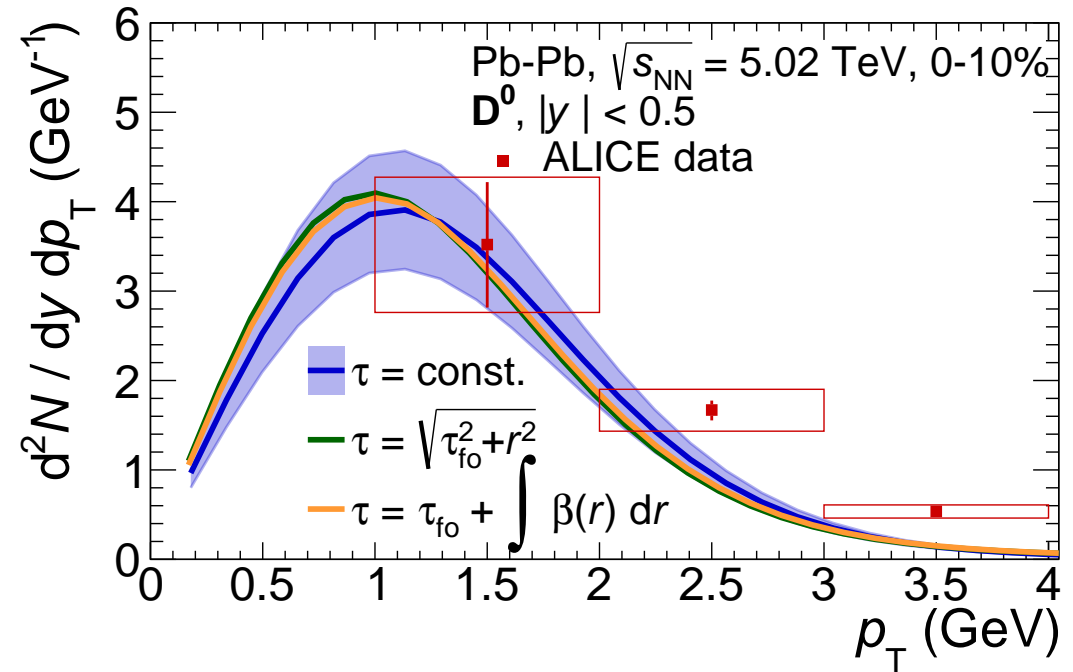
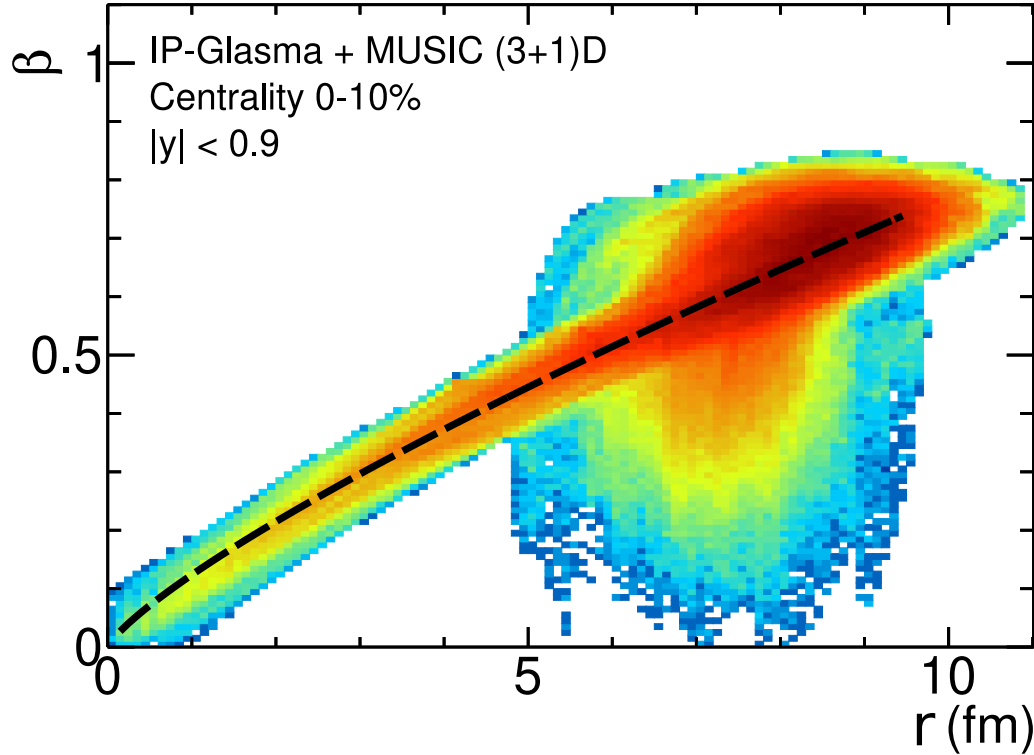
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# SHMc: $p_T$ dependence

full hydrodynamic flow (MUSIC(3+1)D, IP-Glasma; parametrized via blast-wave



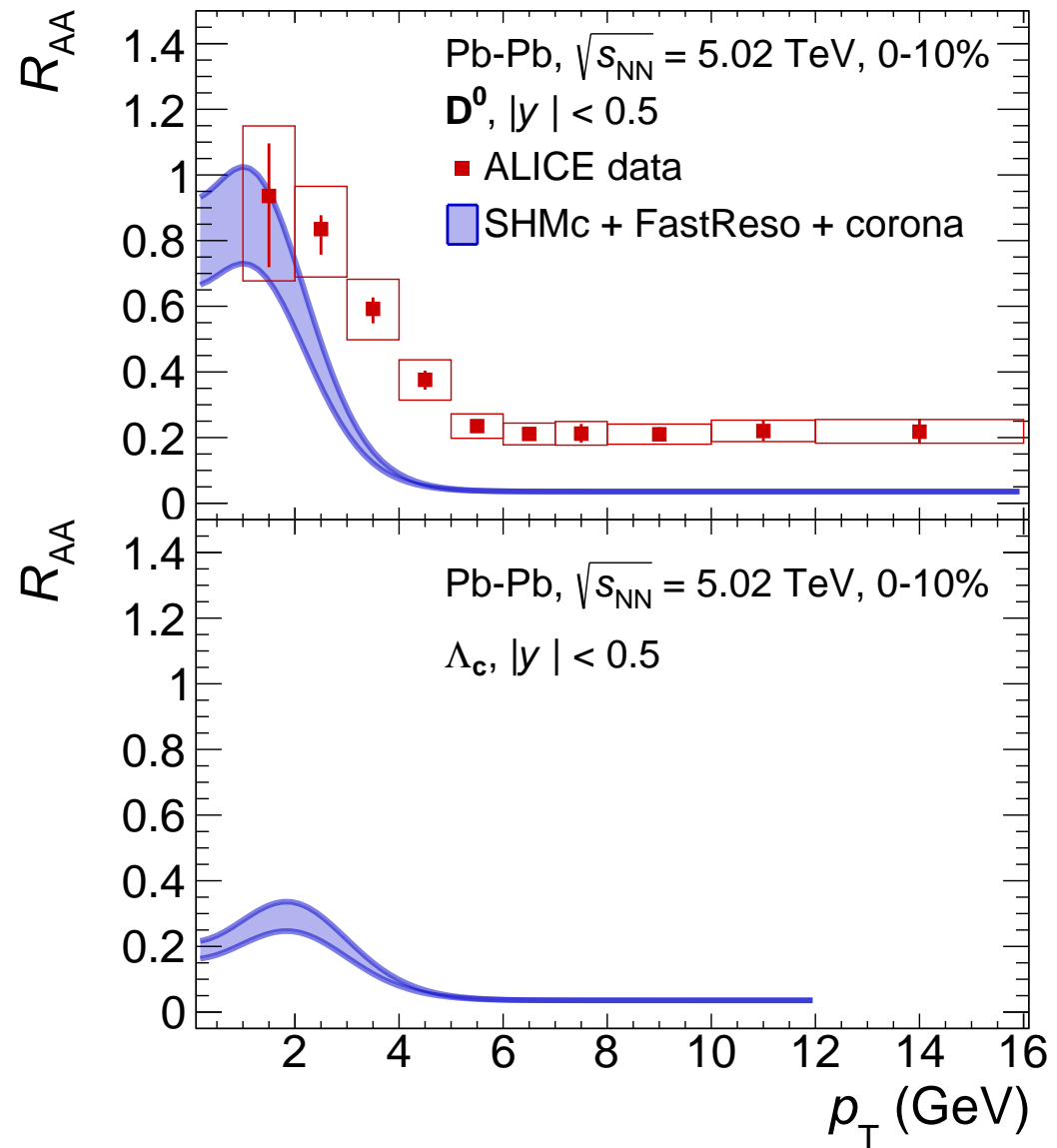
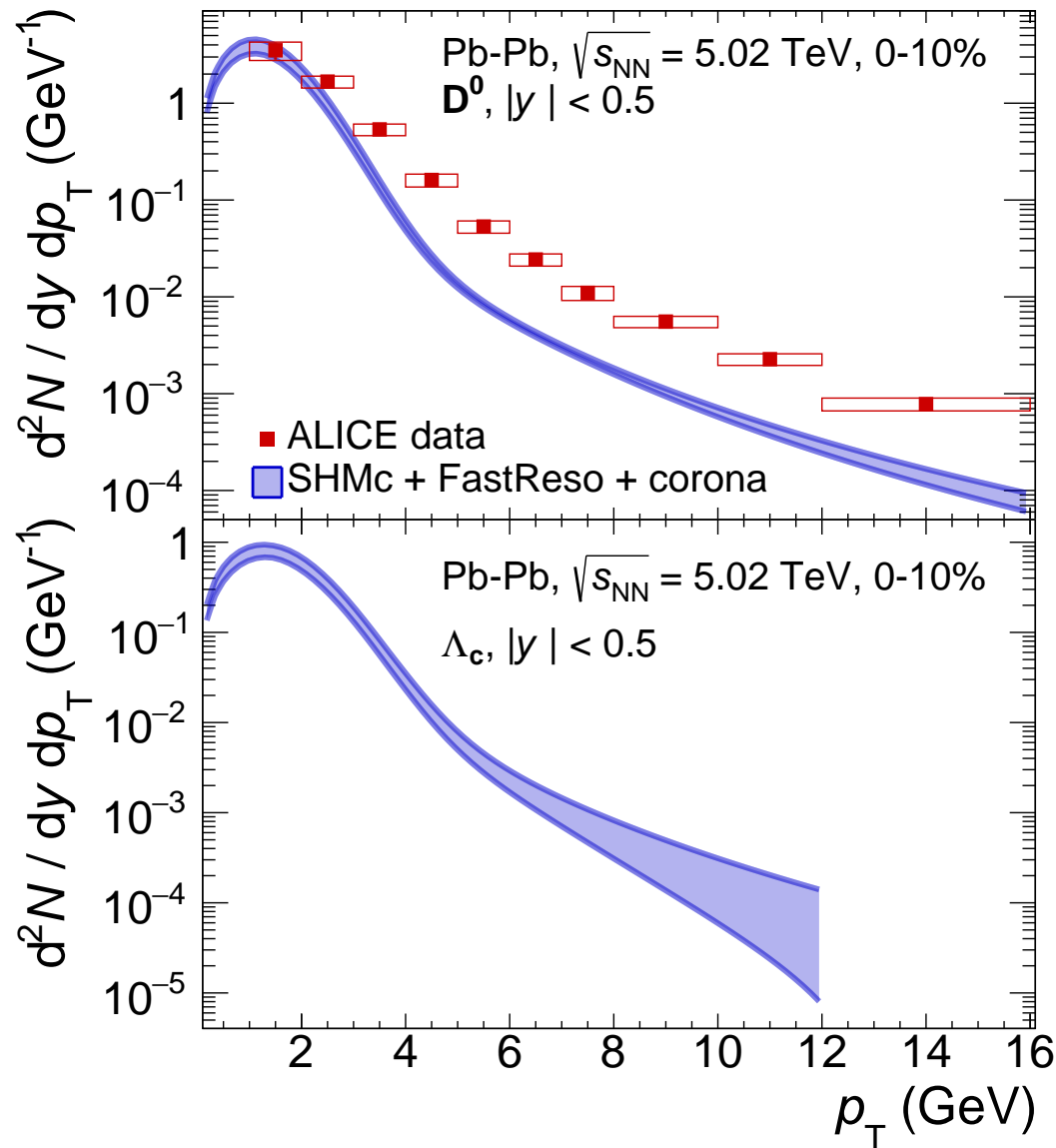
$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{2J+1}{(2\pi)^3} \int d\sigma_\mu p^\mu f(p) = \frac{2J+1}{(2\pi)^3} \int_0^{r_m} dr \tau(r) r \left[ K_1^{\text{eq}} - \frac{\partial \tau}{\partial r} K_2^{\text{eq}} \right]$$

$$K_1^{\text{eq}}(p_T, u^r) = 4\pi m_T I_0 \left( \frac{p_T u^r}{T} \right) K_1 \left( \frac{m_T u^r}{T} \right), \quad K_2^{\text{eq}}(p_T, u^r) = 4\pi p_T I_1 \left( \frac{p_T u^r}{T} \right) K_0 \left( \frac{m_T u^r}{T} \right)$$

# SHMc: $p_T$ dependence

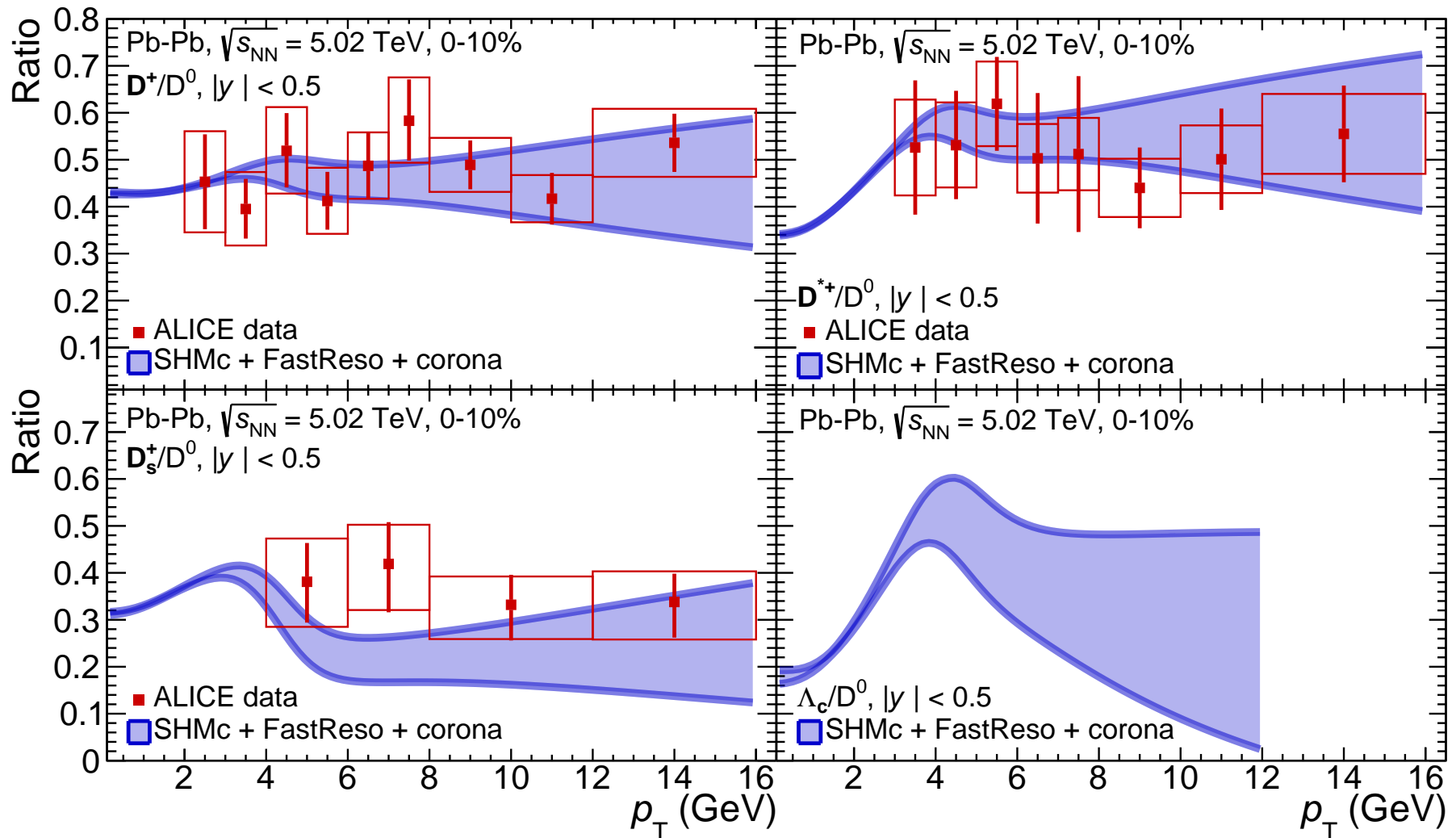
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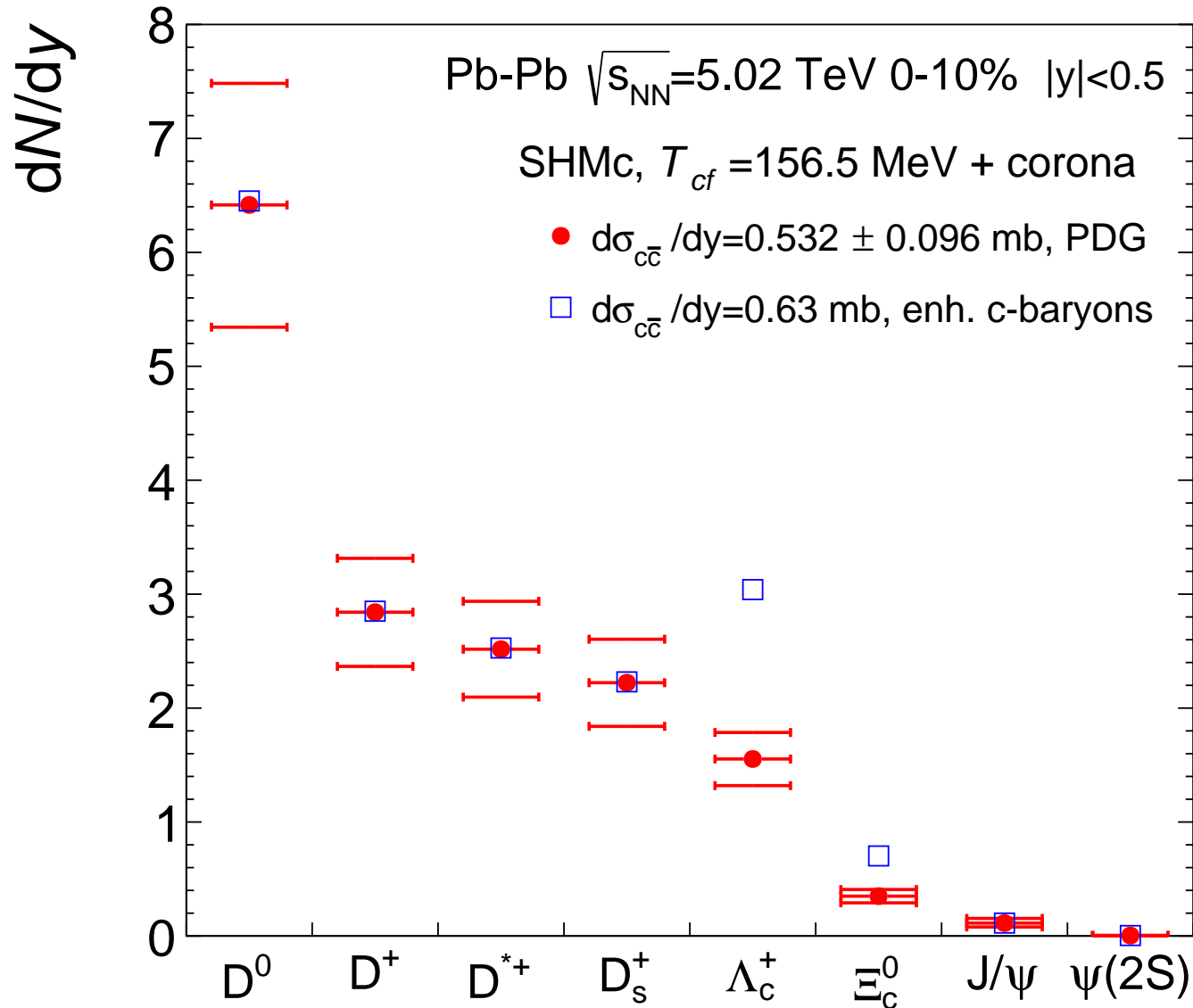
Nuclear-corona contribution added whenever pp measurements available

# Ratios to $D^0$



# Modified charm-hadron spectrum

...ad-hoc: tripled the excited charm-baryon states, enhanced  $d\sigma_{c\bar{c}}/dy$  by 19%

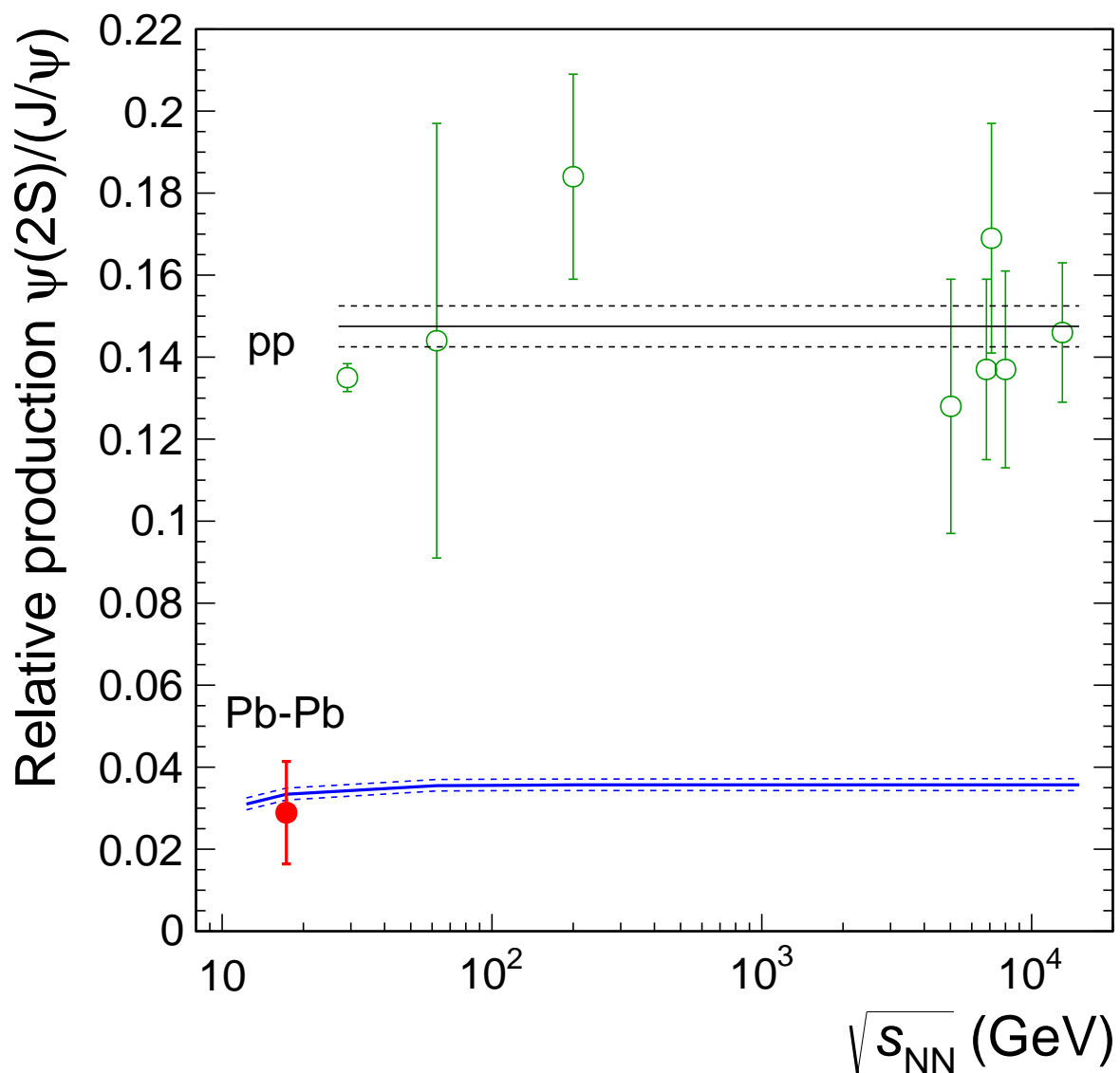




# IF charm thermalizes (fully) at lower energies

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...SHMc is easy to be extended to lower energies

AA, Braun-Munzinger, Redlich, Stachel, [PLB 659 \(2008\) 149](#)

..litmus test:  $\psi(2S)$  ( $+v_2$ ,  $R_{AA}$ )

SHMc works (was proposed) at SPS

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

for D, stat. hadronization is a simpler act, may be at work in pp and in  $e^+e^-$

[PLB 678 \(2009\) 350](#)

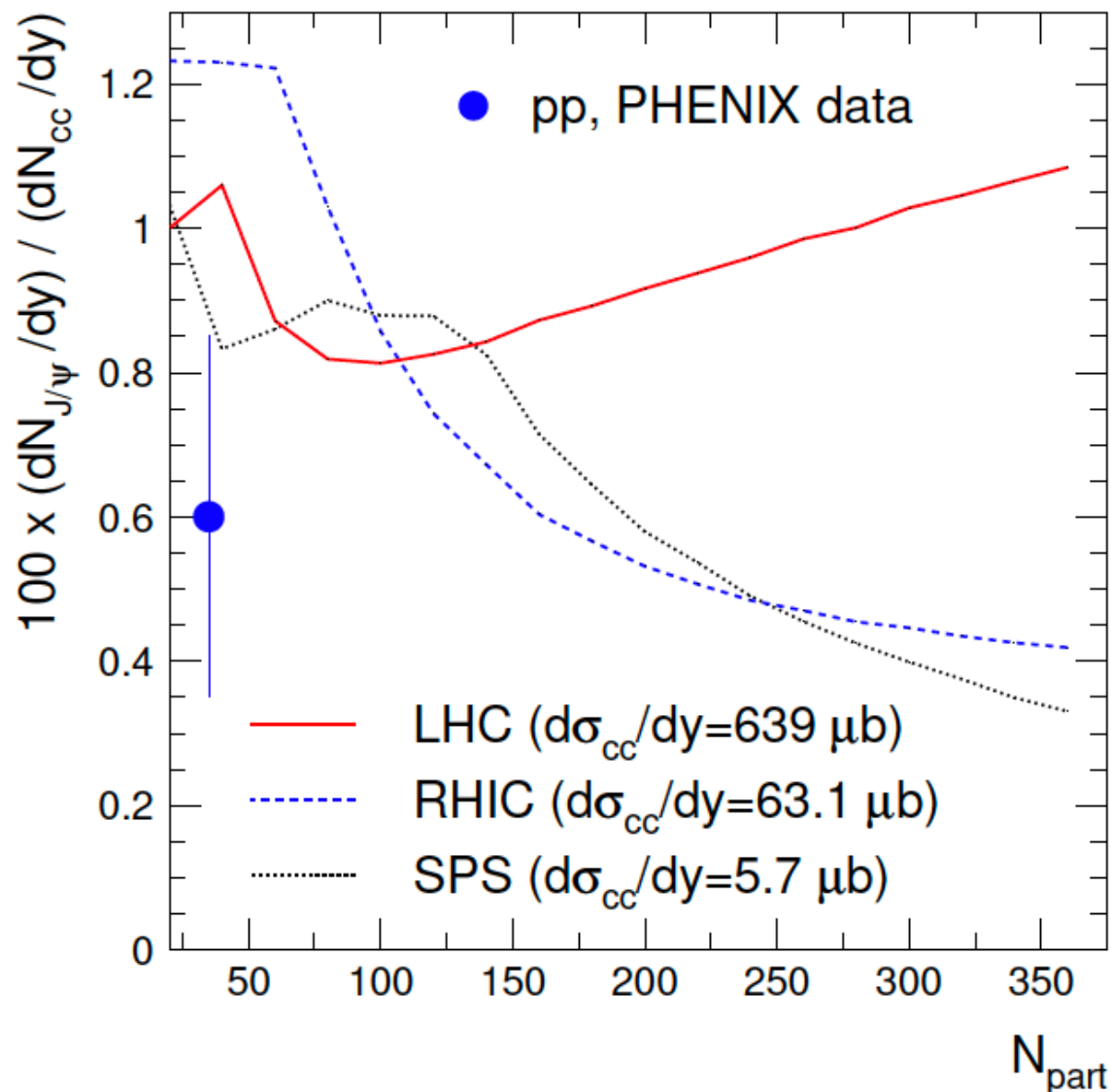
The measurement in Pb-Pb at LHC is a central goal for Run 3,4 ([YR, WG5 HL-LHC](#))

# $J/\psi$ production relative to charm

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...an observable with similar features as  $R_{AA}$



NPA 789 (2007) 334

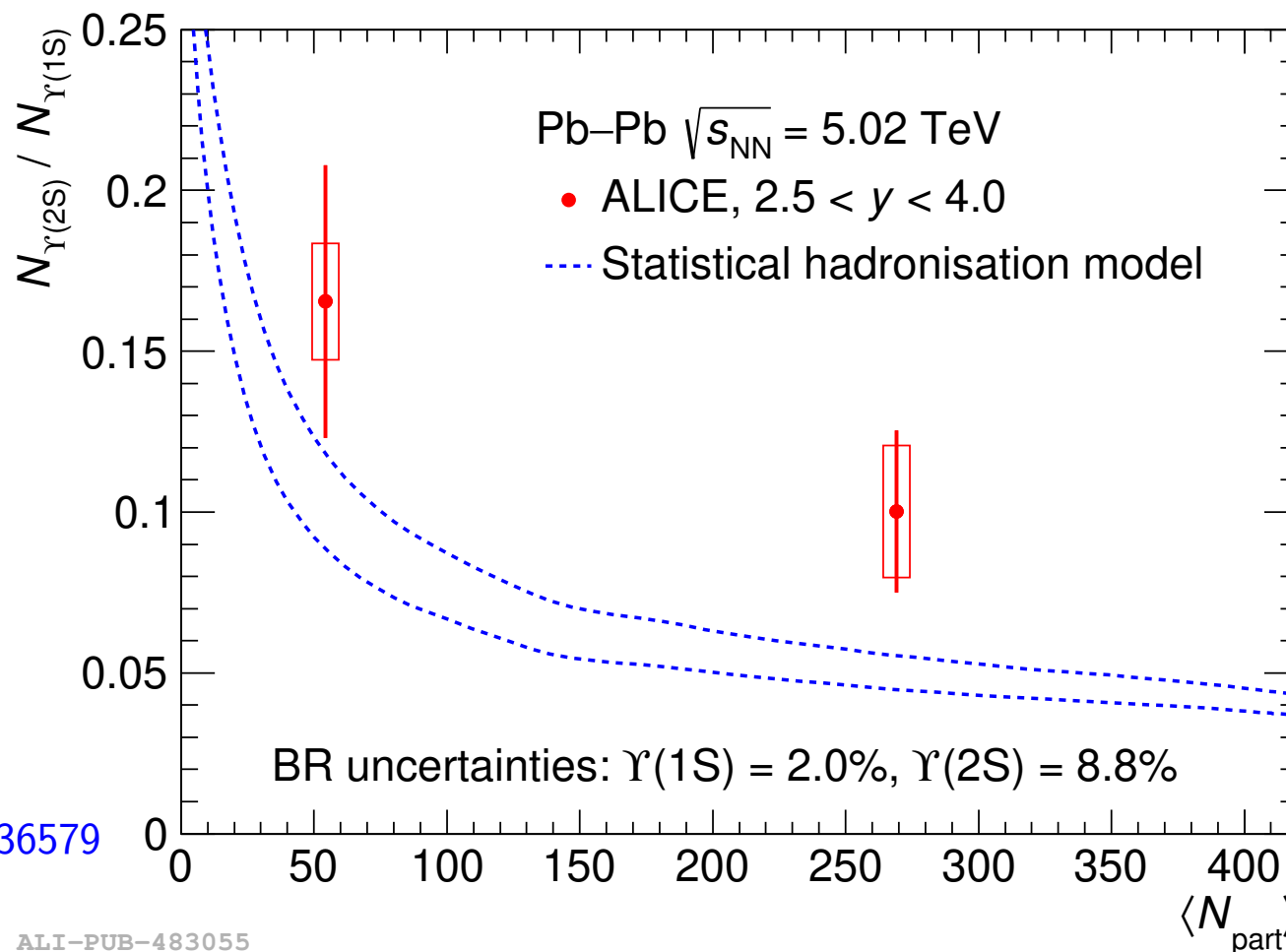
(see also: Satz, *Adv.HEP* 2013 (2013) 242918)

- similar values at RHIC and SPS  
...with differences in fine details  
...determined by canonical suppression of open charm
- enhancement-like at LHC  
can. suppr. lifted, quadratic term dominant

# A word on beauty (at the LHC)

Beauty is expected (data: scarce) to be less thermalized compared to charm

The beauty-hadron spectrum is less well known (PDG: 48 b-mes, 46 b-bar total)



ALICE,

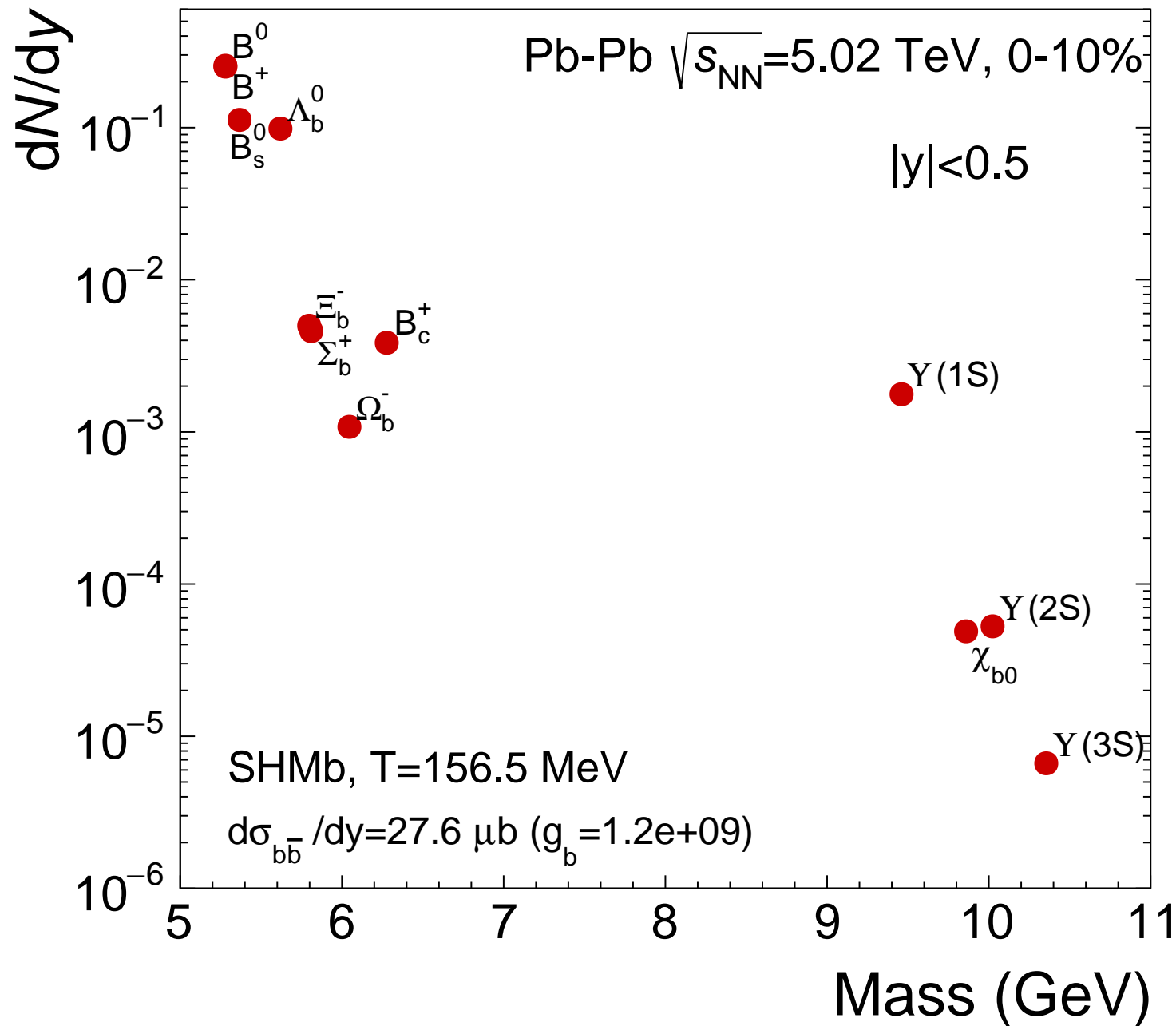
[PLB 822 \(2021\) 136579](#)

ALI-PUB-483055

Color screening may not destroy all  $\Upsilon$  mesons in QGP

Uncertainty band determined by nuclear-corona

# The limiting case: full beauty thermalization



# Summary / Conclusions

In the (our) statistical hadronization model:

- The hadronization is a rapid process in which all quark flavors take part concurrently
- All charmonium and open charm states are generated exclusively at hadronization (chemical freeze-out) ...full color screening

The model is very successful in reproducing the  $J/\psi$  and open charm data  
*A handle for hadronization  $T$  with a mass scale well above  $T$*

"The competition":

the kinetic model, continuous  $J/\psi$  destruction and (re)generation in QGP

(only up to 2/3 of the  $J/\psi$  yield (LHC, central collisions) originates from deconfined  $c$  and  $\bar{c}$  quarks)

*Discriminating the two pictures implies providing an answer to fundamental questions related to the fate of hadrons in a hot deconfined medium.*

A precision ( $\pm 10\%$ ) measurement of  $d\sigma_{c\bar{c}}/dy$  in Pb-Pb (Au-Au) collisions needed for a stringent test  
(within reach with the upgraded detectors at the LHC and RHIC)

# Summary / Conclusions

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In the (our)statistical hadronization model:

- The hadronization is a rapid process in which all quark flavors take part concurrently
  - Beauty may be only partially ( $\sim 50\%$ ?) thermalized
  - $\Upsilon$  states may not be fully destroyed by color screening in QGP
- consequently, our predictions in the beauty sector are upper limits

# Extra slides

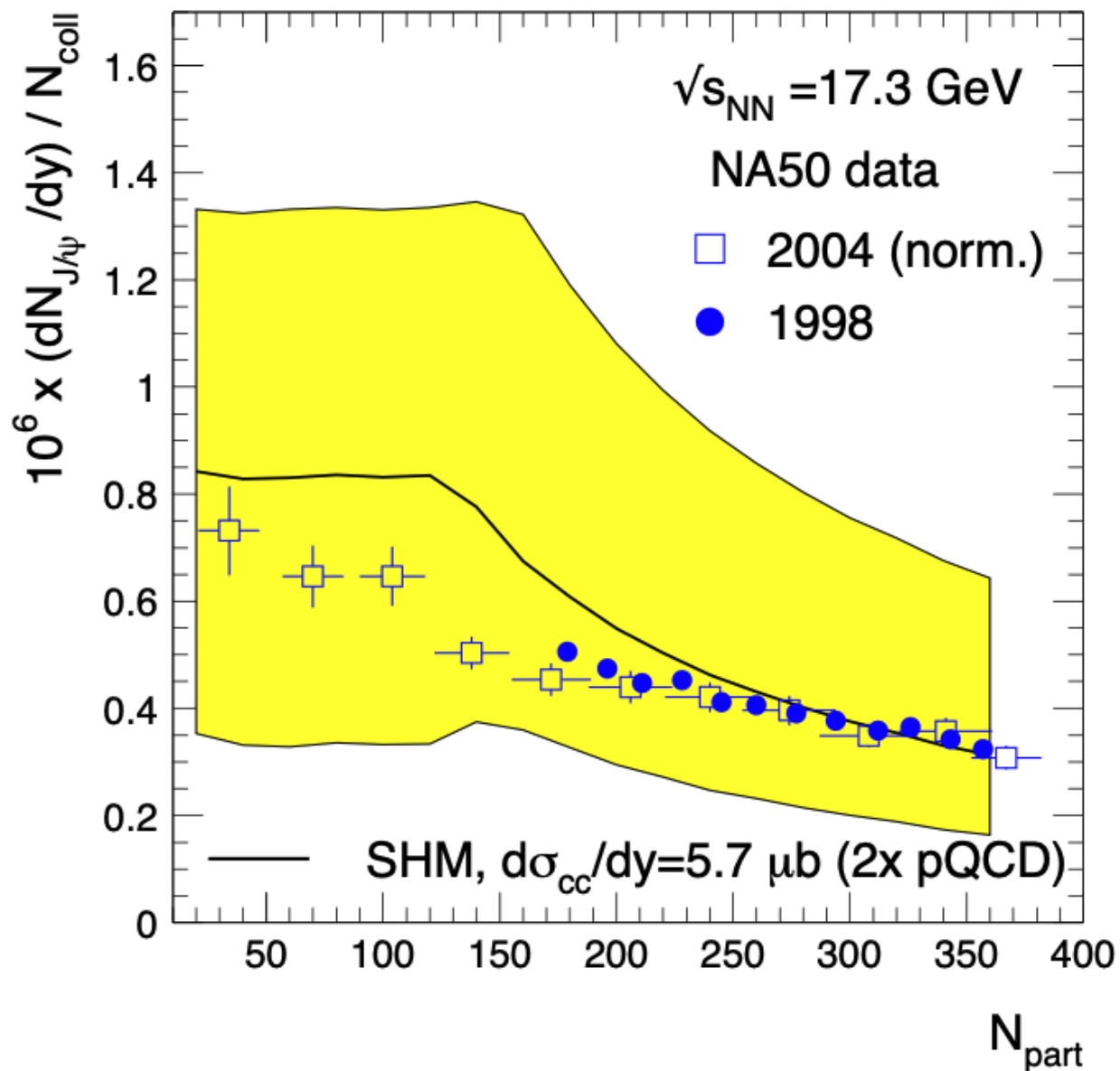
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# $J/\psi$ production at SPS and SHMc

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NPA 789 (2007) 334

Trend determined by canonical suppression of open charm

Model uncertainties large ( $d\sigma_{c\bar{c}}/dy$ )

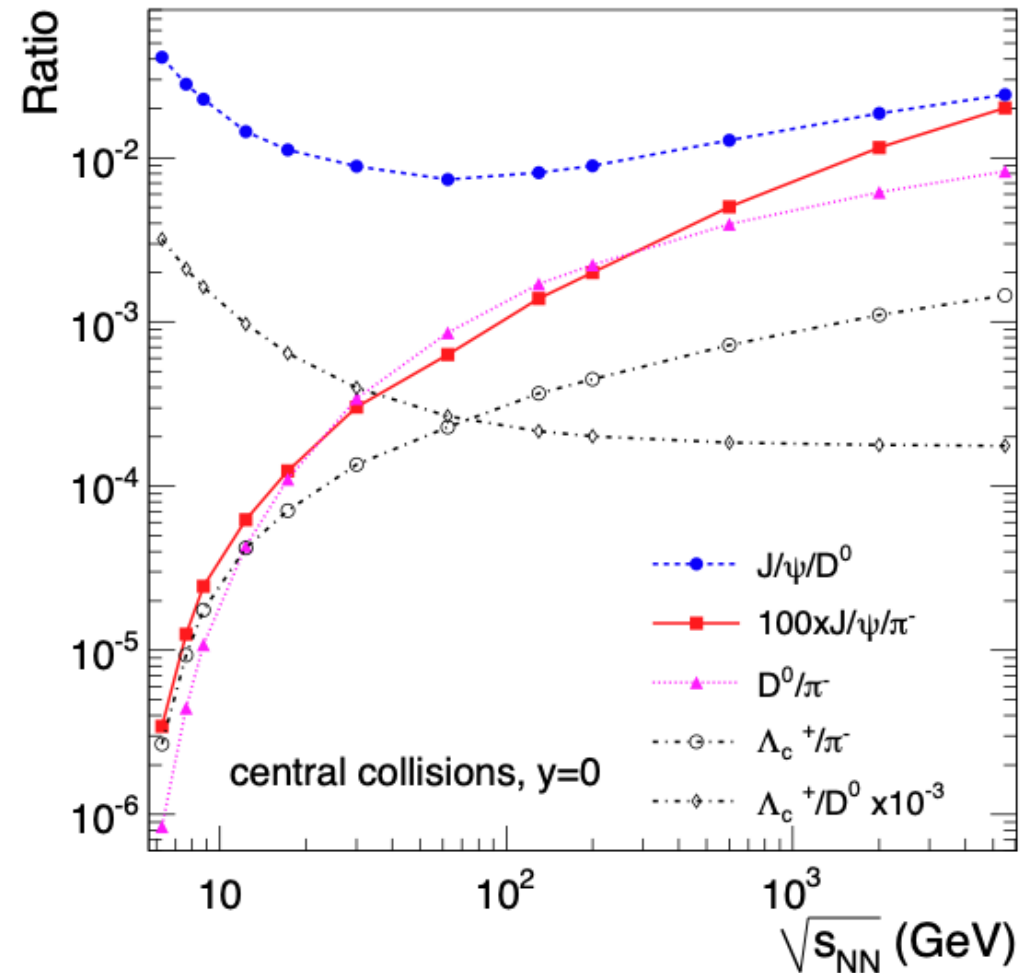
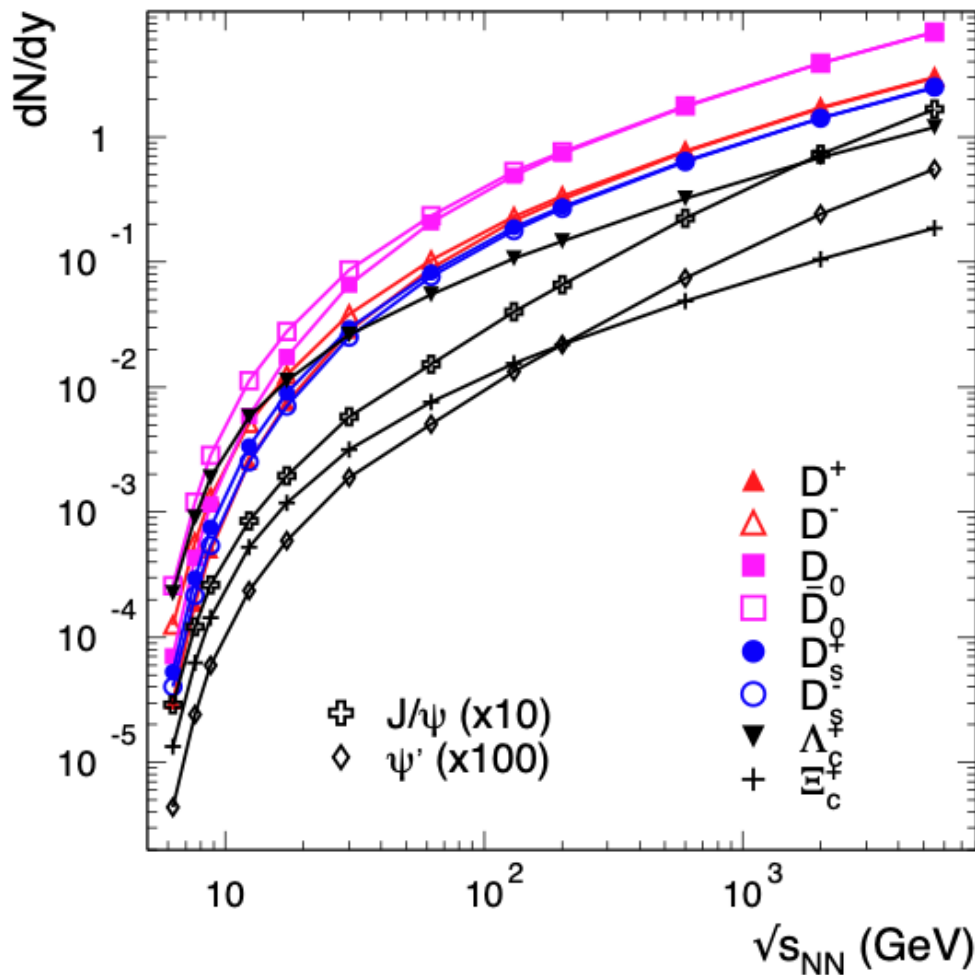
...constraining the model needs a good measurement of  $d\sigma_{c\bar{c}}/dy$



# Charm chemistry at lower energies

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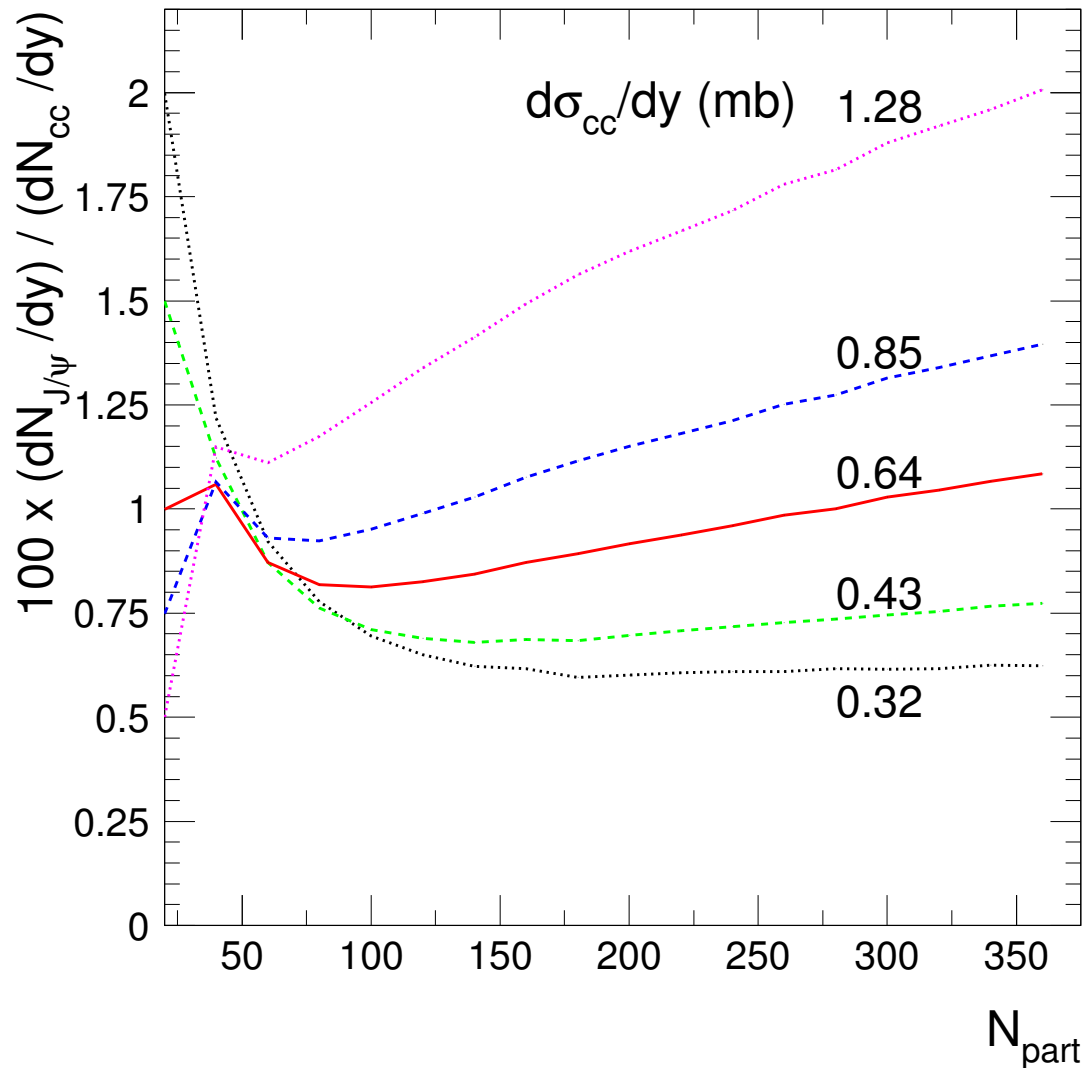


AA, Braun-Munzinger, Redlich, Stachel, [JPG 37 \(2010\) 094014](#)

NB: uncertainties from  $\sigma_{c\bar{c}}$  are large

There are non-monotonic ratios ( $J/\psi/D$ ), determined by canonical suppression

# SHM and charmonium production at the LHC



$$\frac{dN_{J/\psi}^{AA}/dy}{dN_{c\bar{c}}^{AA}/dy}$$

(“proxy” for  $R_{AA}$ )

- “enhancement” at the LHC

$$N_{J/\psi} \sim (N_{c\bar{c}}^{dir})^2$$

canonical suppression (mostly)  
lifted, quadratic term dominant

it can be more dramatic at FCC

AA et al., in N. Armesto et al., “Last Call...”, [JPG 35 \(2008\) 054001](#)

this was for  $\sqrt{s_{NN}} = 5.5$  TeV ... but is a generic prediction of the model

# The charm production cross section at the LHC

- LHCb  $p_T < 8 \text{ GeV}/c$ ,  $2.0 < y < 4.5$

pp 5 TeV, [JHEP 06 \(2017\) 147](#)

$$\sigma(c\bar{c}) = 1193 \pm 3(\text{stat}) \pm 67(\text{syst}) \pm 58(\text{frag}) \mu\text{b}, \quad \rightarrow \frac{d\sigma_{c\bar{c}}}{dy} = 0.477 \pm 0.036 \text{ mb}$$

pp 7 TeV, [NPB 871 \(2013\) 1](#)

$$\sigma(c\bar{c}) = 1419 \pm 12(\text{stat}) \pm 116(\text{syst}) \pm 65(\text{frag}) \mu\text{b}$$

$$\Lambda_c/D^0 = 0.140 \pm 0.045$$

- ALICE,  $|y| < 0.5$ ,  $p_T > 0 \text{ GeV}/c$

pp 5 TeV, [arXiv:2105.06335](#), [arXiv:2102.13601](#) :

$$\frac{d\sigma_{c\bar{c}}}{dy} = 1.165 \pm 0.044(\text{stat}) \pm 0.065(\text{syst})_{-0.038}^{+0.098}(\text{extr}) \pm 0.043(\text{BR}) \pm 0.042(\text{RS}) \pm 0.024(\text{lumi}) \text{ mb}$$

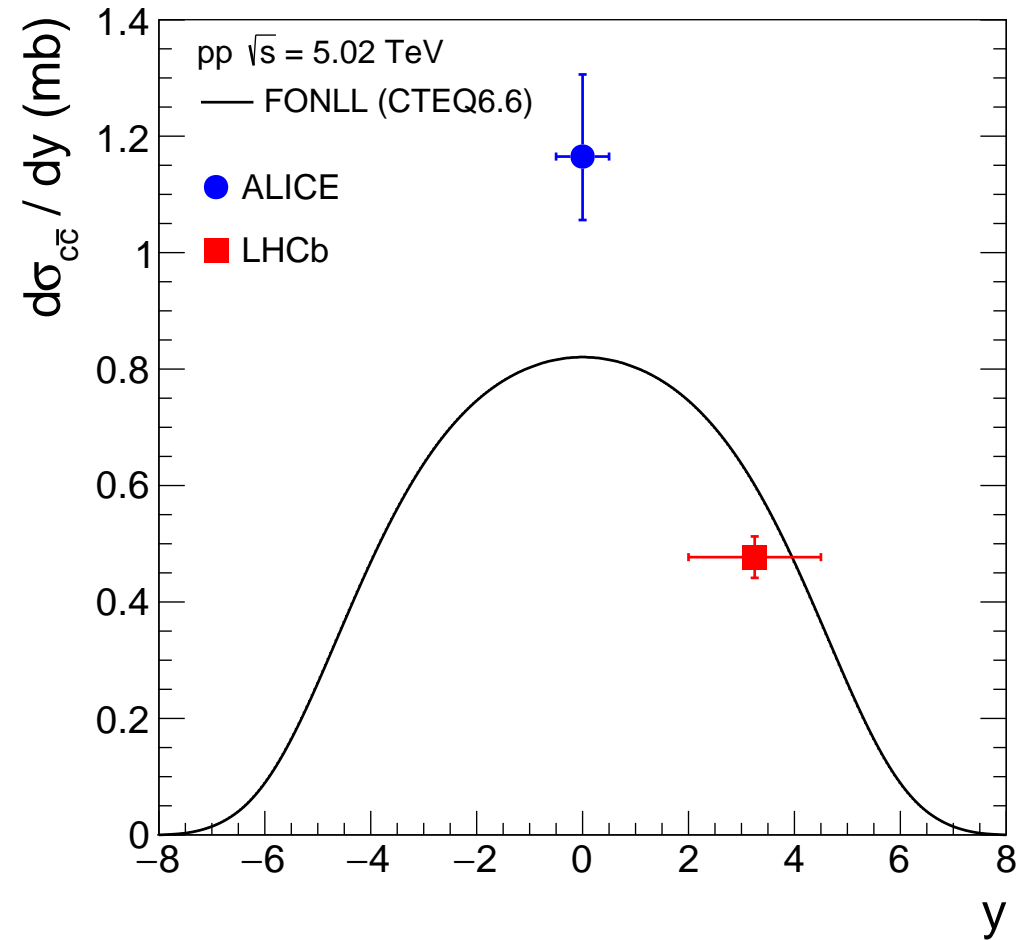
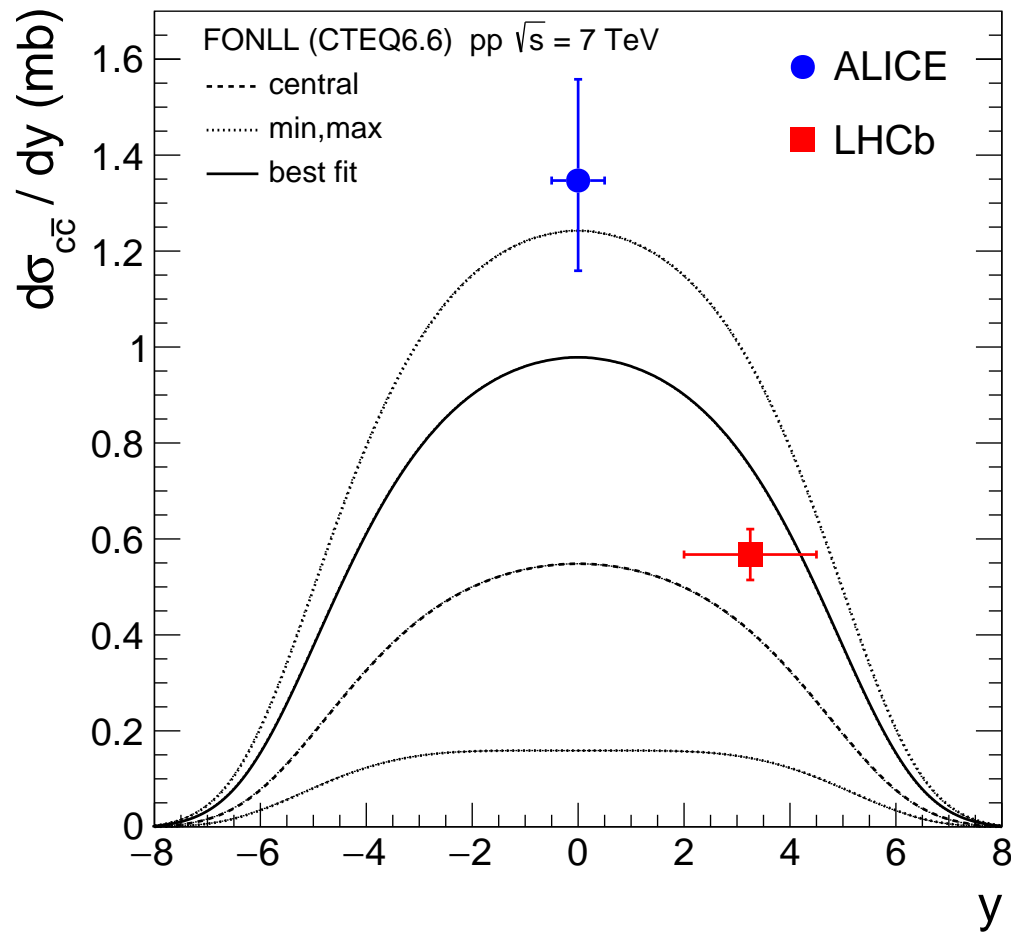
$$\Lambda_c/D^0 = 0.51 \pm 0.06$$

pp 7 TeV, [JHEP 04 \(2018\) 108](#)

$$\frac{d\sigma_{c\bar{c}}}{dy} = 1.347 \pm 0.097(\text{stat}) \pm 0.104(\text{syst})_{-0.105}^{+0.142}(\text{FF}) \pm 0.011(\text{BR}) \pm 0.044(\text{RS}) \pm 0.047(\text{lumi}) \text{ mb}$$

# Data in comparison to FONLL

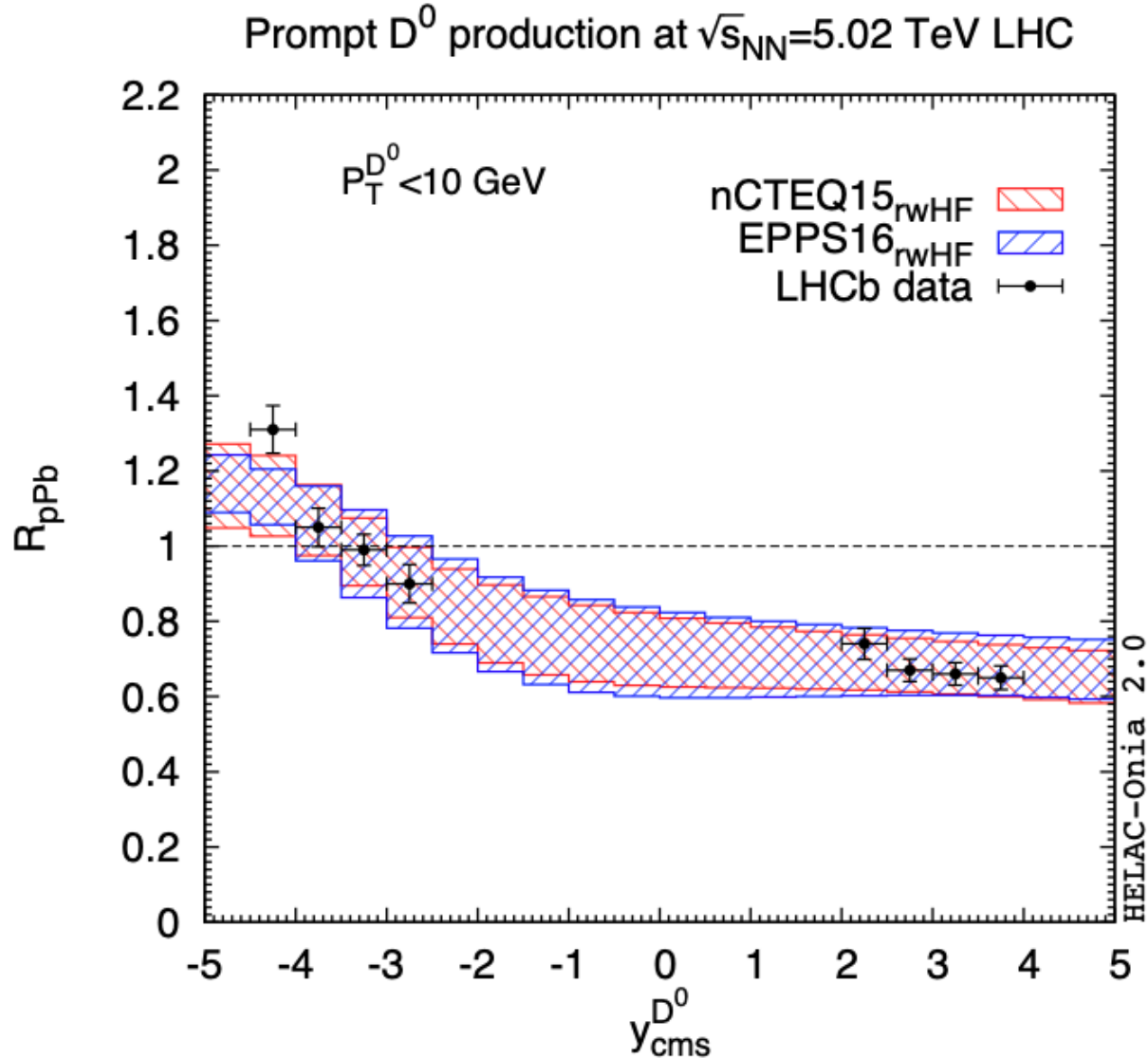
... $p_T$ -integrated ( $p_T > 0$ ); FONLL web interface



# $R_{pPb}$ and shadowing

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weighted nPDFs (90% CL):

Kusina et al, [PRD 104 \(2021\) 014010](#)

Data: LHCb, [JHEP 10 \(2017\) 090](#)

ALICE:  $0.960 \pm 0.086$ , [JHEP 12 \(2019\) 092](#)

$y=0$ :

$$R_{pPb} = 0.73 \pm 0.067 (1\sigma)$$

$$\Rightarrow S_{PbPb} = 0.53 \pm 0.097$$

in SHMc (earlier value):

$$S_{PbPb} = 0.65 \pm 0.12$$

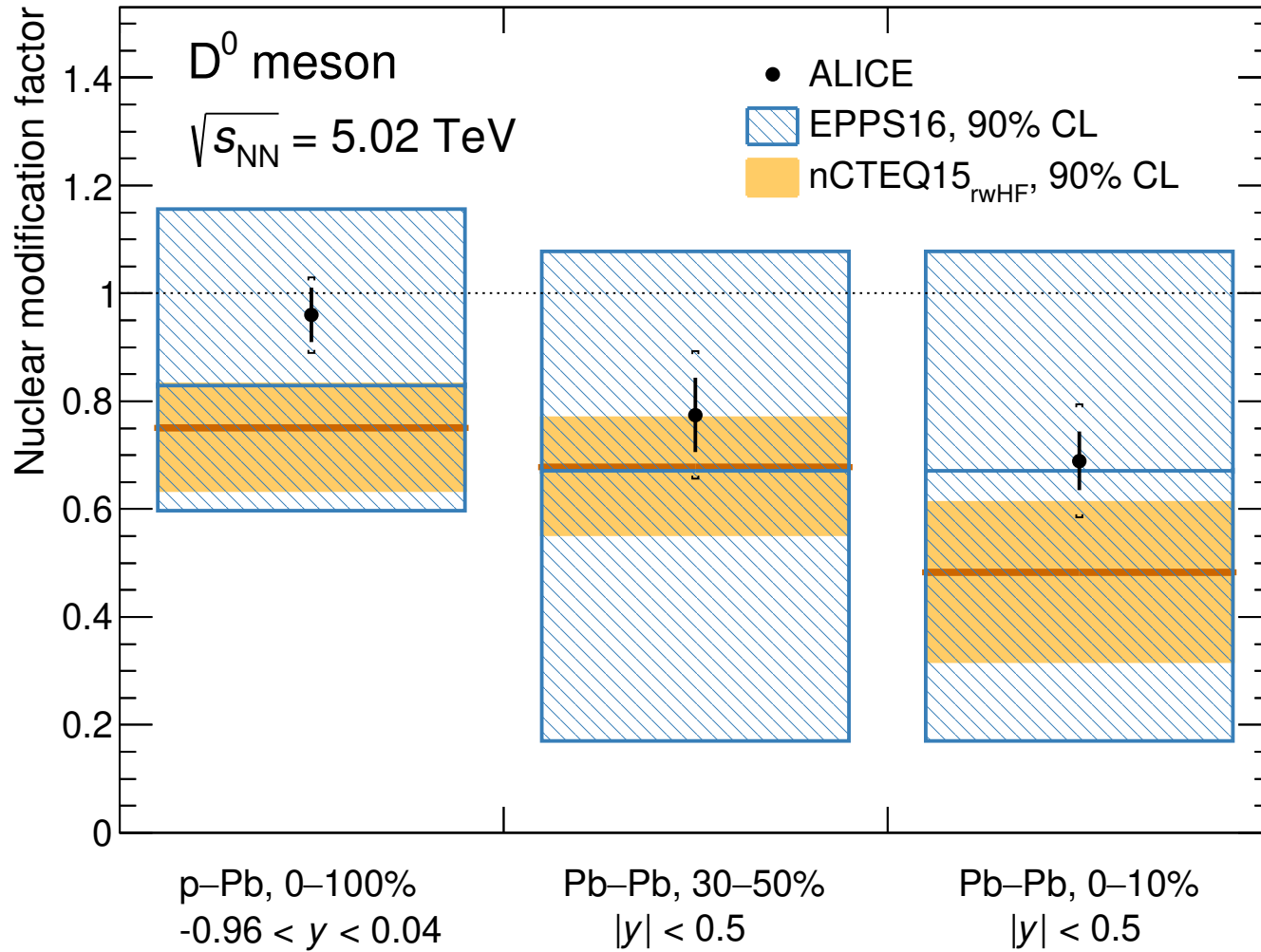
assumed shadowing in Pb–Pb

$$S_{PbPb} = R_{pPb}(-y) \cdot R_{pPb}(y)$$

$$\Rightarrow \frac{d\sigma_{c\bar{c}}}{dy} = 0.532 \pm 0.096 \text{ mb for } y=0 \text{ (Pb–Pb at } \sqrt{s_{NN}} = 5.02 \text{ TeV, SHMc)}$$

$$0.334 \pm 0.053 \text{ for } y=2.5-4$$

# A fresh ALICE shadowing in Pb-Pb



0-10%

Data (stat+sys):

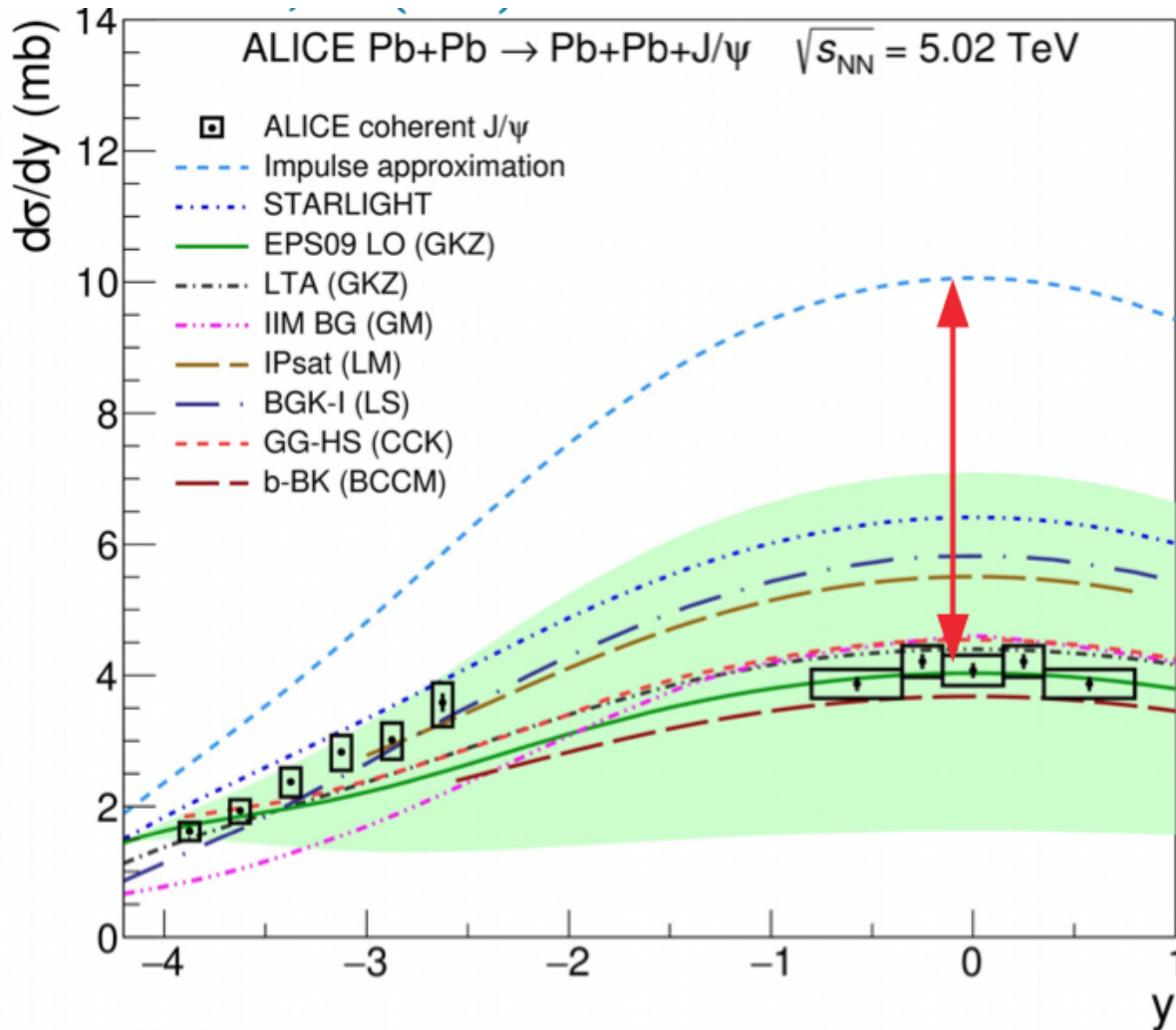
$$S_{PbPb} = 0.69 \pm 0.11$$

nCTEQ15<sub>rWHF</sub> ( $1 \sigma$ ):

$$S_{PbPb} = 0.47 \pm 0.09$$

(centrality-dependent)

# Another argument for large shadowing in Pb-Pb



J/ψ photoproduction

$$S_{PbPb} = \frac{d\sigma/dy(data)}{d\sigma/dy(imp.approx.)}$$

$$S_{PbPb} = 0.42 \pm 0.04$$

$$(x_{Bj} \simeq (0.3 - 1.4) \cdot 10^{-4})$$

ALI-PUB-482756

ALICE, EPJC 81 (2021) 712

Impulse approximation: STARLIGHT, no nuclear effects except coherence