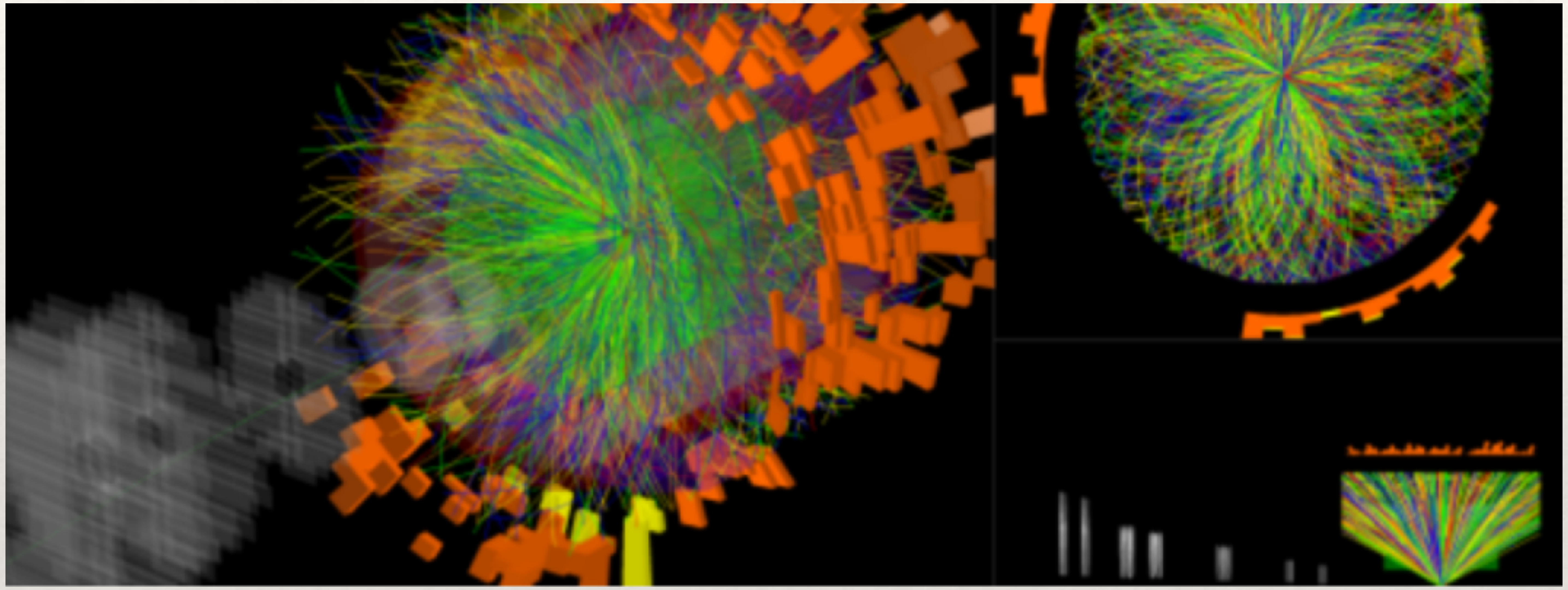
Boltzmann Transport of Heavy Quarks



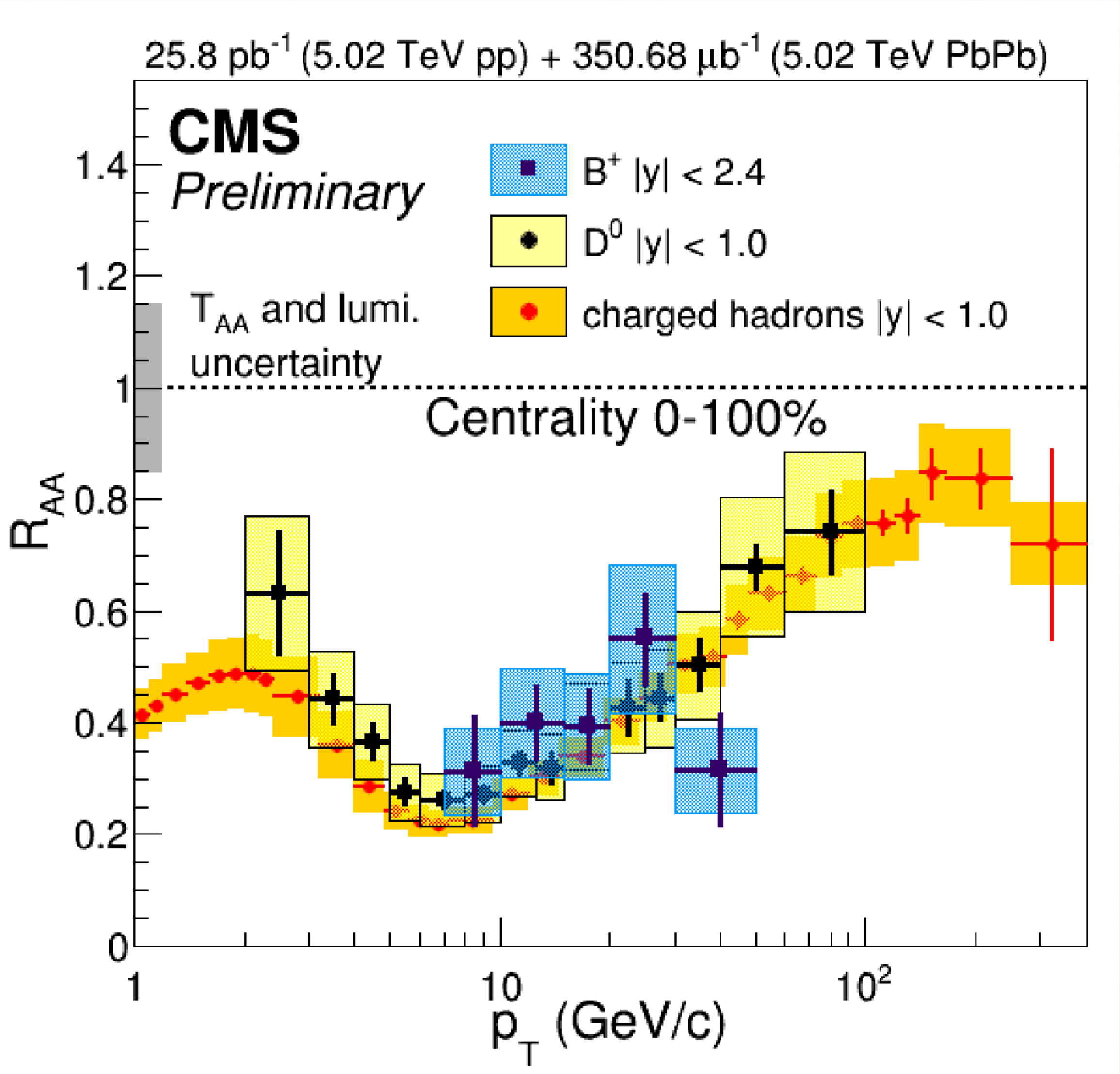
Shanshan Cao
Shandong University

Outline

- Parton energy loss at high p_T
- Hadronization at medium to low p_T
- Color potential interaction at low p_T
- Probing QGP properties in different collision systems

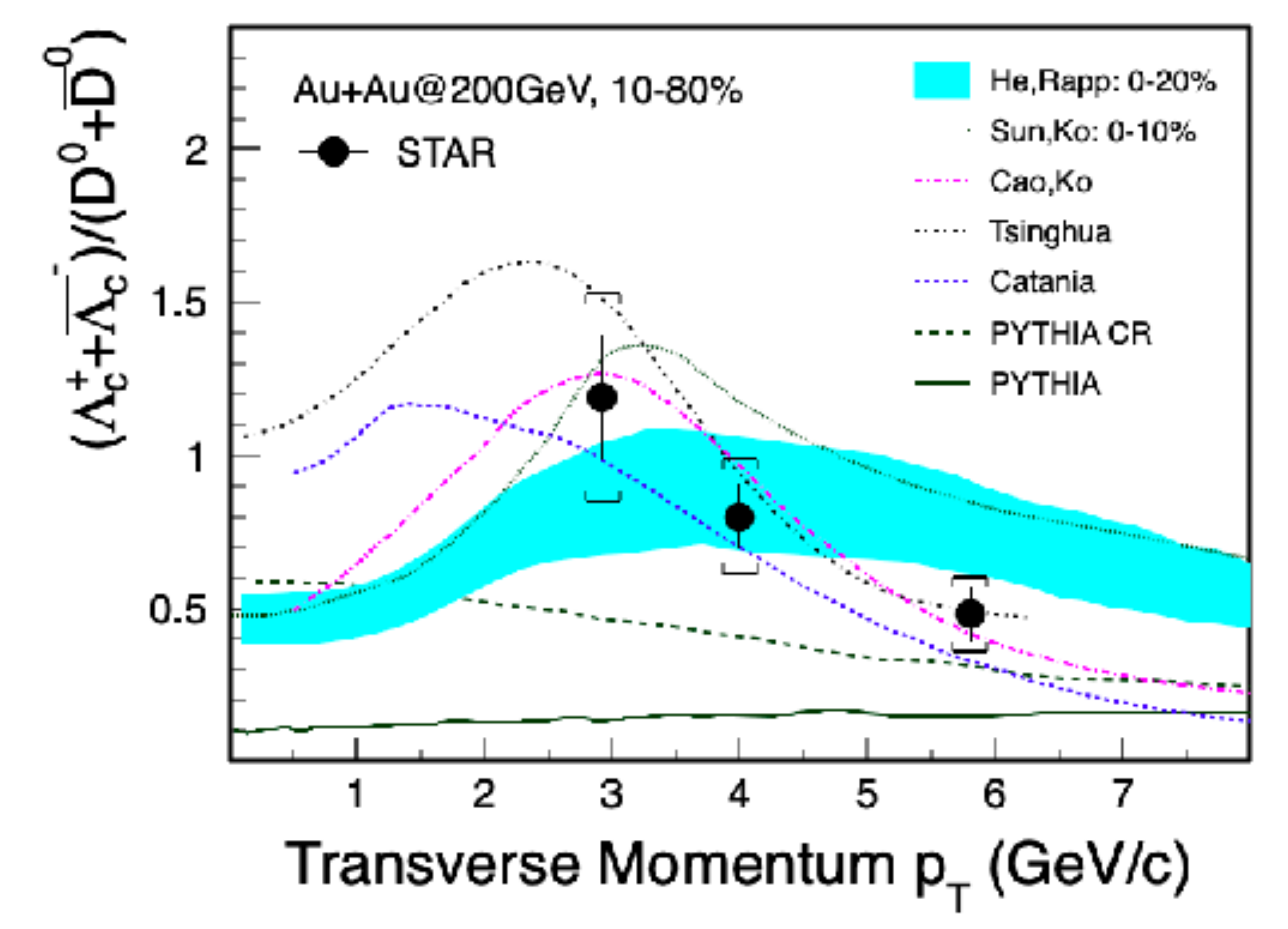
Heavy and light flavor hadrons at different scales

high p_T



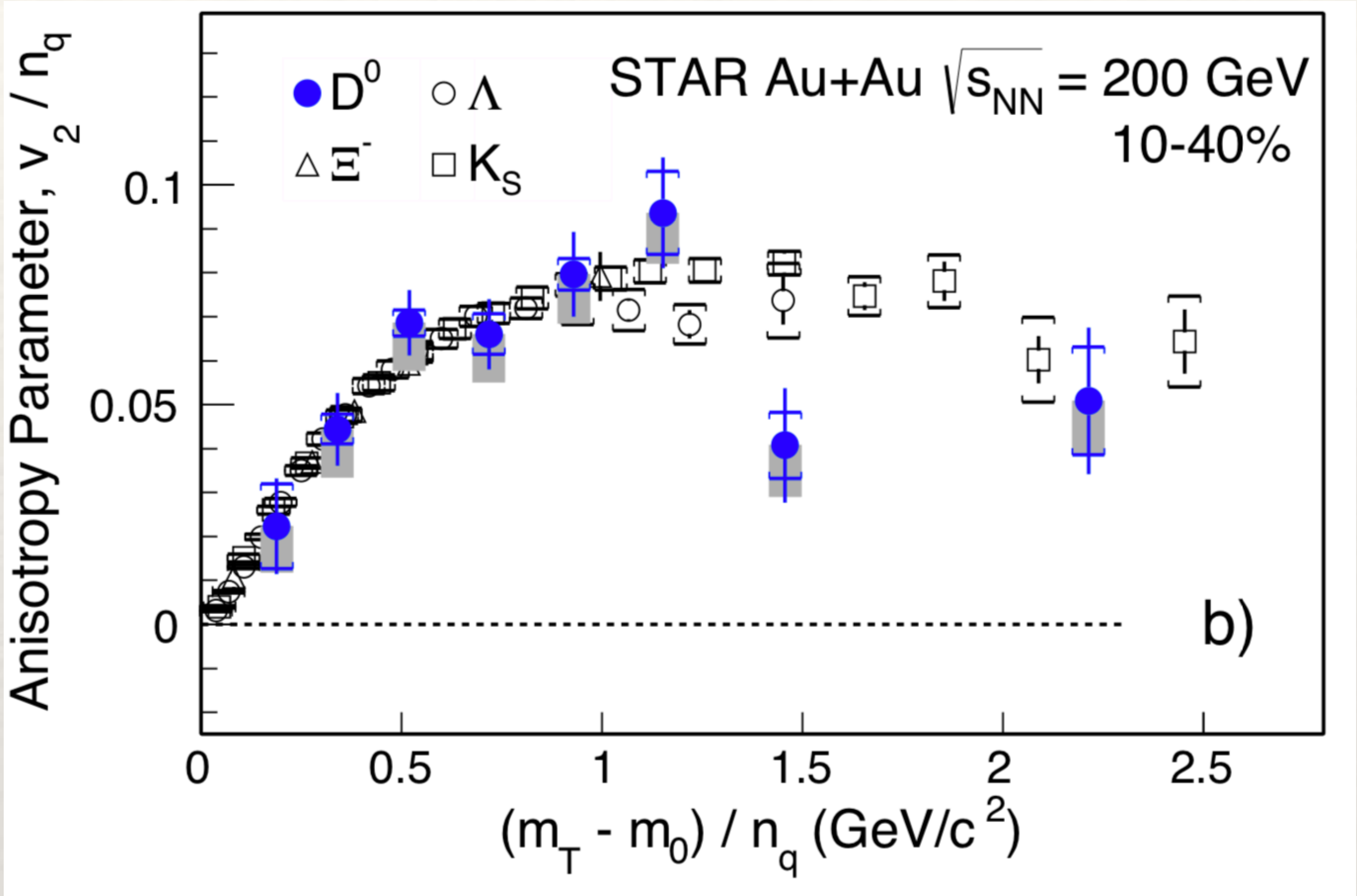
- Mass and flavor hierarchy of parton energy loss

medium p_T



- Hadronization process and in-medium hadron wave-function

low p_T



- Thermalization process and color potential of heavy-quark-medium interaction

High p_T parton-medium interaction

Linear Boltzmann Transport

$$p_a \cdot \partial f_a(x_a, p_a) = E_a(\mathscr{C}_a^{\mathrm{el}} + \mathscr{C}_a^{\mathrm{inel}})$$

Elastic energy loss ($ab \rightarrow cd$)

$$\mathscr{C}_a^{\mathrm{el}} = \sum_{b,c,d} \int \prod_{i=b,c,d} \frac{d[p_i]}{2E_a} (\gamma_d f_c f_d - \underline{\gamma_b f_a f_b}) \cdot (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) \left| \mathscr{M}_{ab\rightarrow cd} \right|^2$$

$2 \rightarrow 2$ scattering matrices

loss term: **scattering rate**
(for Monte-Carlo simulation)

$$\Gamma_a^{\mathrm{el}}(\mathbf{p}_a, T) = \sum_{b,c,d} \frac{\gamma_b}{2E_a} \int \prod_{i=b,c,d} d[p_i] f_b \cdot (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d) \left| \mathscr{M}_{ab\rightarrow cd} \right|^2$$

Recent improvement LBT → QLBT

[Liu, Xing, Wu, Qin, Cao, Wang, arXiv:2107.11713]

Introduce thermal mass of light flavor partons (quasi-particle model)

$$m_g^2 = \frac{1}{6}g^2\left[(N_c+\frac{1}{2}n_f)T^2+\frac{N_c}{2\pi^2}\Sigma_q\mu_q^2\right]$$

$$m_{u,d}^2 = \frac{N_c^2-1}{8N_c}g^2\left[T^2+\frac{\mu_{u,d}^2}{\pi^2}\right]$$

$$m_s^2 - m_{0s}^2 = \frac{N_c^2-1}{8N_c}g^2\left[T^2+\frac{\mu_s^2}{\pi^2}\right]$$

affect EoS →

$$P_{qp}(m_u, m_d, ..., T) = \Sigma_{i=u,d,s,g}d_i\int\frac{d^3p}{(2\pi)^3}\frac{|\vec{p}^2|}{3E_i(p)}f_i(p) - B(T)$$
$$= \Sigma_i P^i_{kin}(m_i, T) - B(T)$$

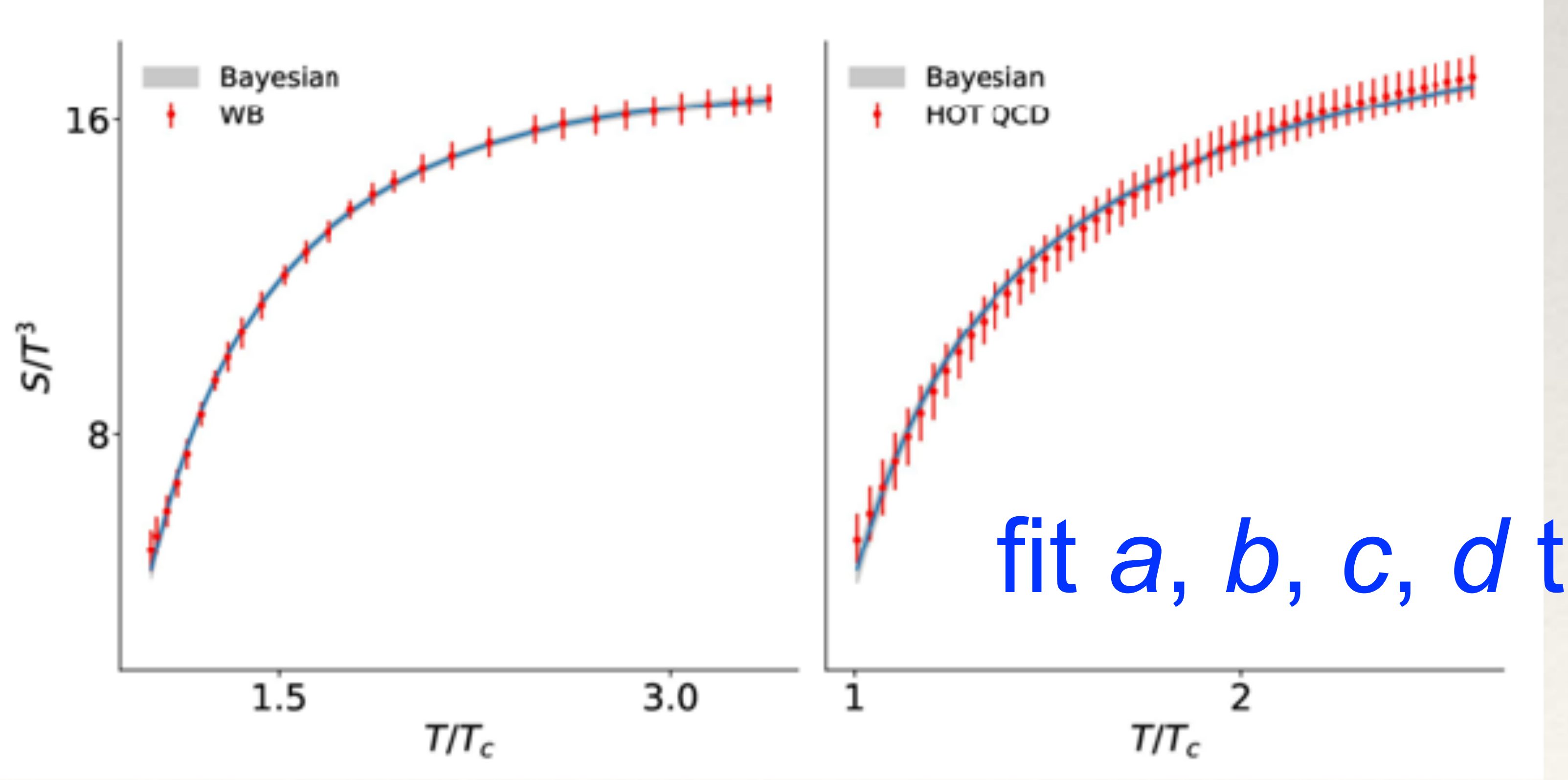
$$\epsilon = TdP(T)/dT - P(T), \qquad s = (\epsilon + P)/T$$

Parametrization of $g(T)$:

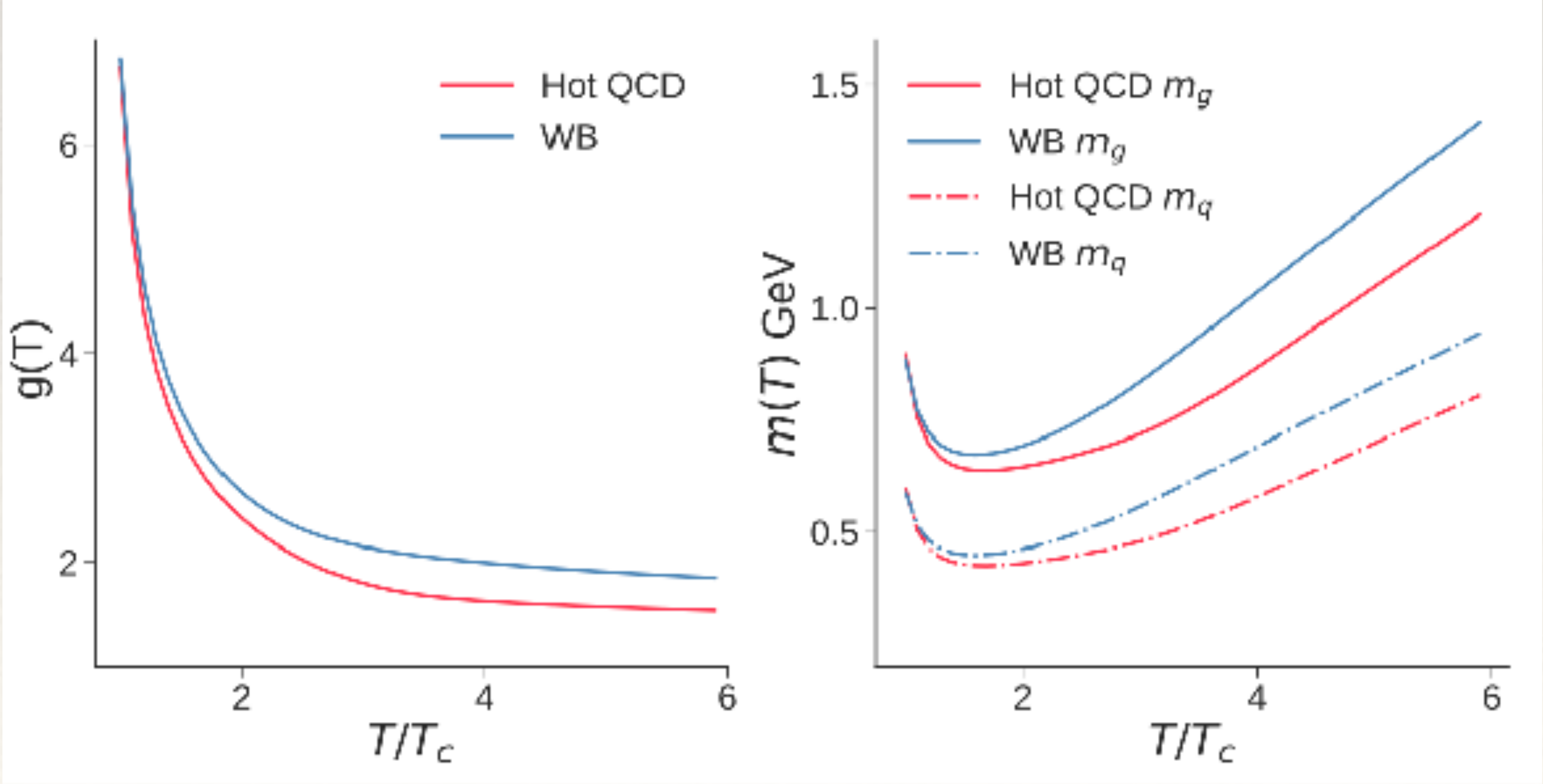
$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\log\left(\frac{1}{ce^{-d(T/T_c)^2}+1}(a\frac{T}{T_c}+b)^2\right)}$$

obtain $g(T)$ and $m(T)$

enter scattering rate

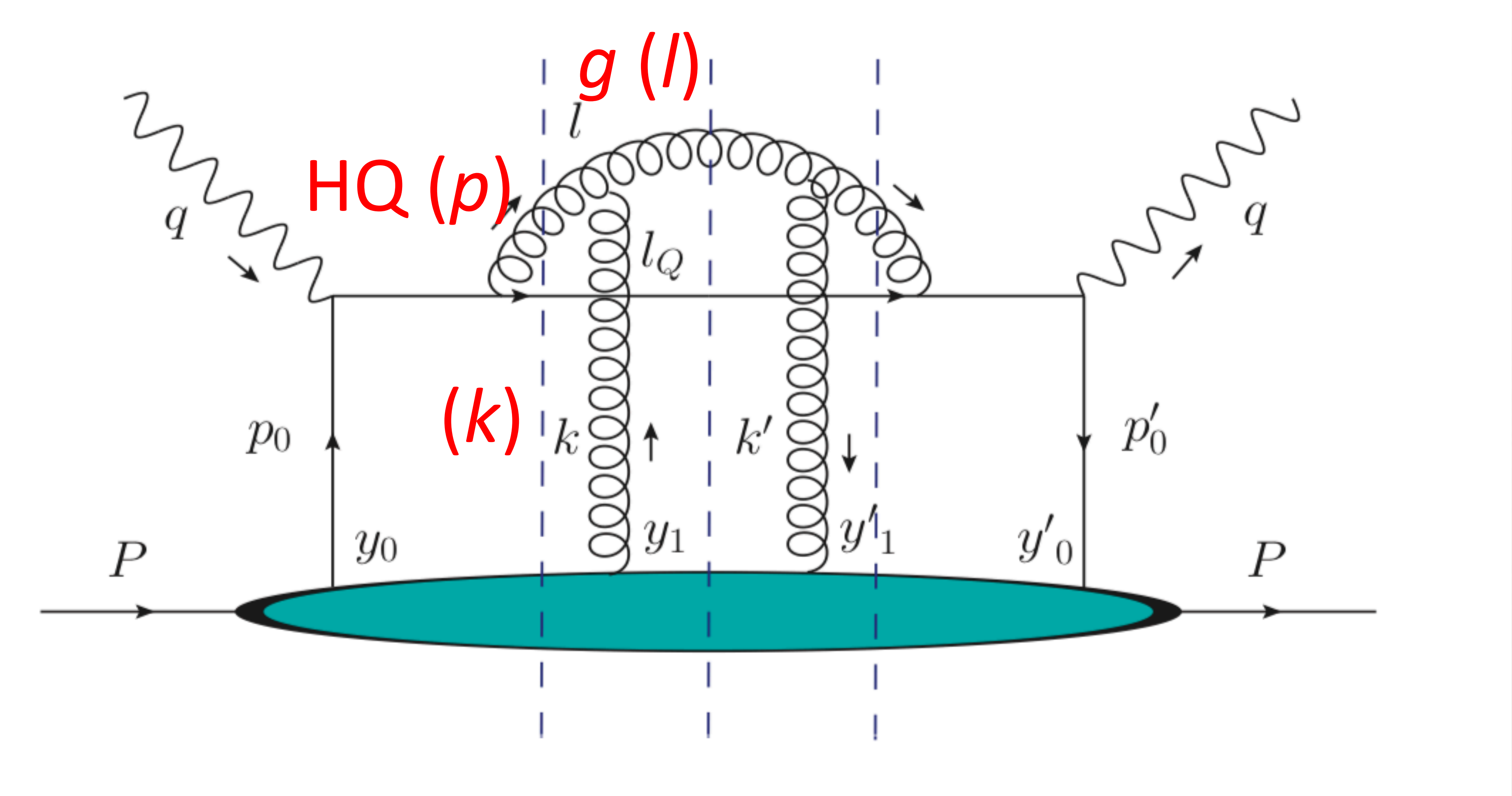


fit a, b, c, d to lattice EOS →



Inelastic energy loss

- **Inelastic scattering with a general medium**



[Majumder PRD 85 (2012); Zhang, Wang and Wang, PRL 93 (2004)]

- **Higher-twist**: collinear expansion ($\langle k_\perp^2 \rangle \ll l_\perp^2 \ll Q^2$)

$$\frac{d\Gamma_a^{\text{inel}}}{dz dl_\perp^2} = \frac{dN_g}{dz dl_\perp^2 dt} = \frac{6\alpha_s P(z) l_\perp^4 \hat{q}}{\pi (l_\perp^2 + z^2 M^2)^4} \sin^2\left(\frac{t - t_i}{2\tau_f}\right)$$

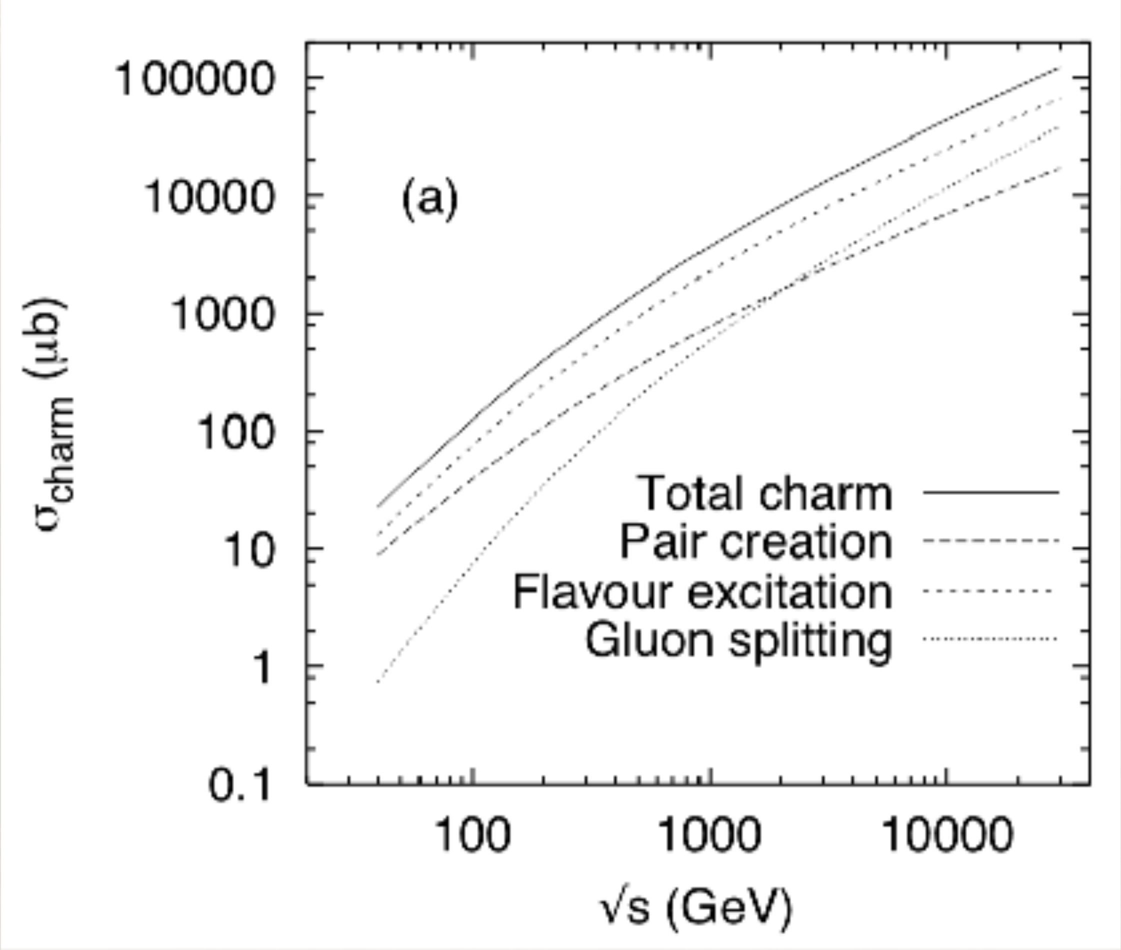
- Medium information absorbed in $\hat{q} \equiv d\langle p_\perp^2 \rangle / dt$

Flavor hierarchy of jet quenching

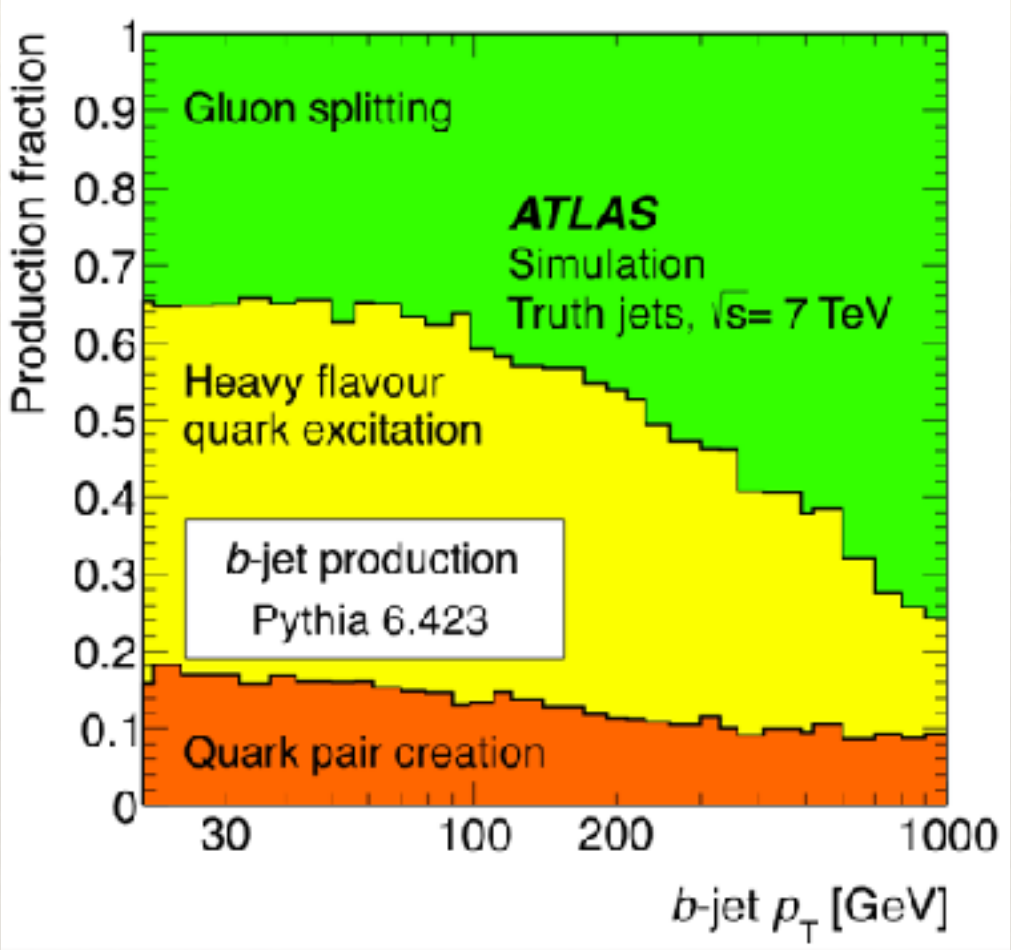
Clean perturbative framework is sufficient for describing the flavor hierarchy at high p_T ($> 8\ \mathrm{GeV}$)

[Xing, Cao, Qin and Xing, Phys. Lett. B 805 (2020) 135424]

NLO contribution to HQ production in Pythia simulation (gluon splitting)



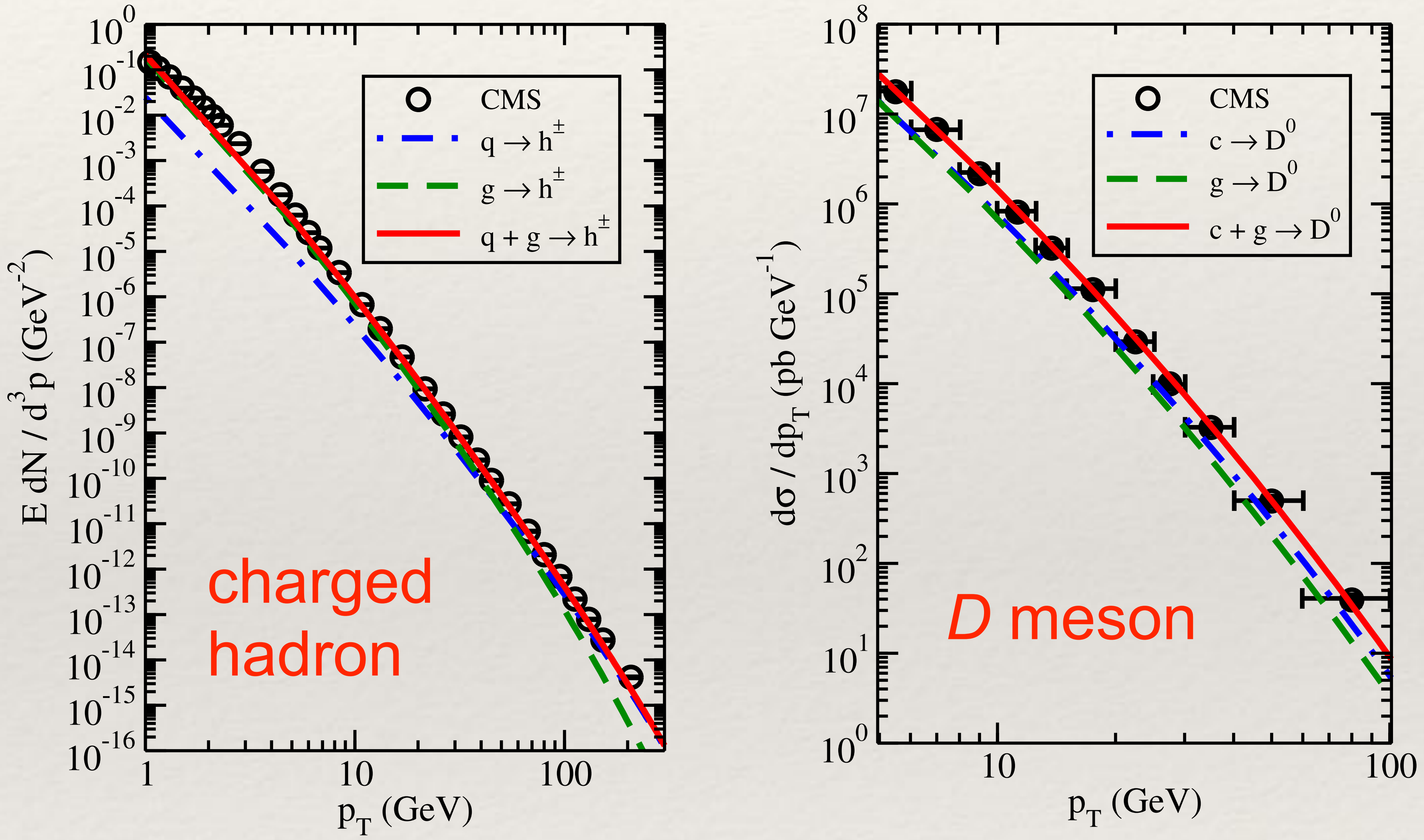
[Norrbin and Sjostrand, EPJC 17 (2000)]



[ATLAS, EPJC 73 (2013)]

- NLO contribution increases with $\sqrt{s}$
- NLO contribution increases with *b*-jet p_T

Light vs. heavy hadron spectra within NLO production + fragmentation framework



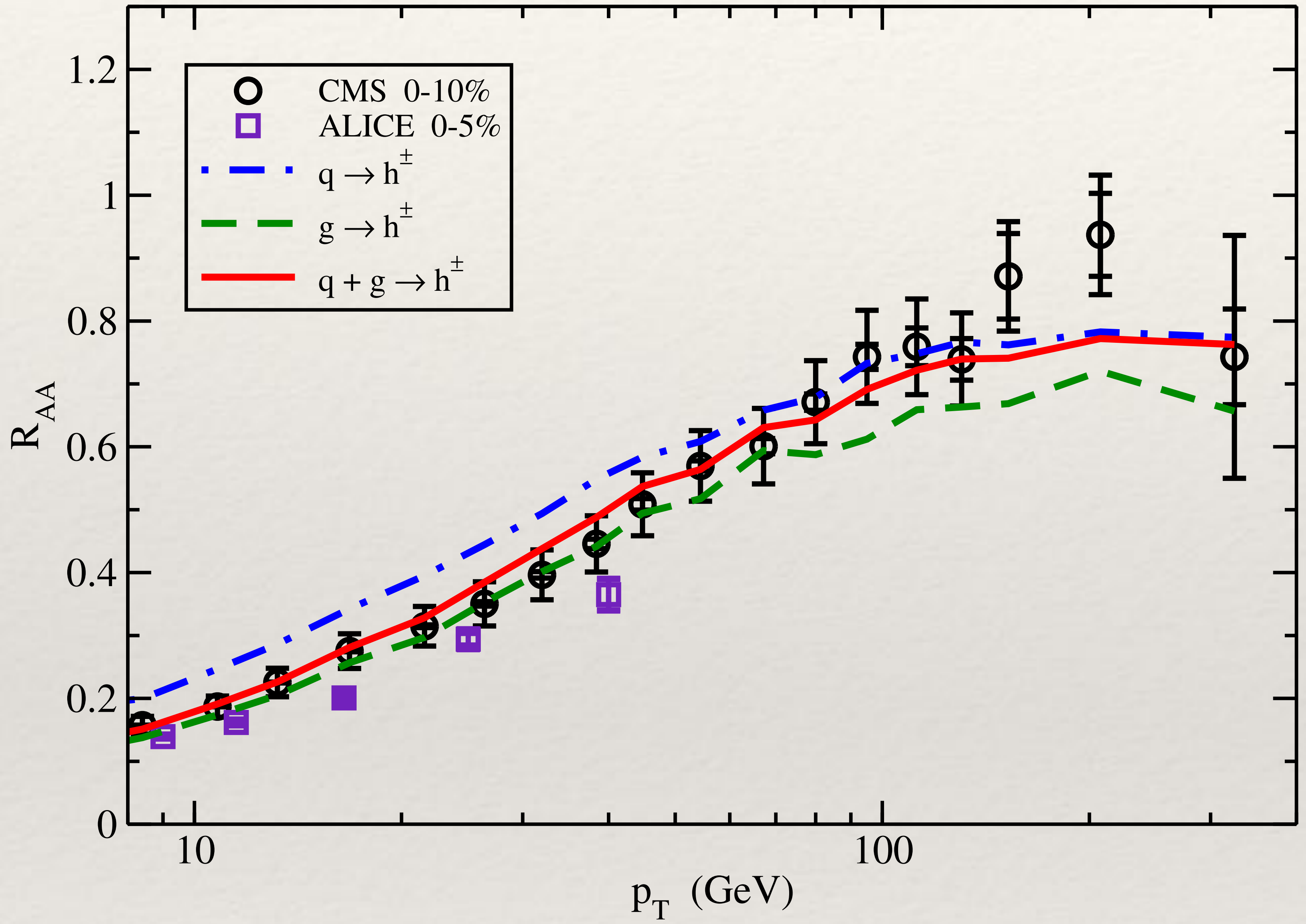
Gluon fragmentation

- dominates $h^{\pm}$ production up to 50 GeV
- contributes to over 40% *D* up to 100 GeV

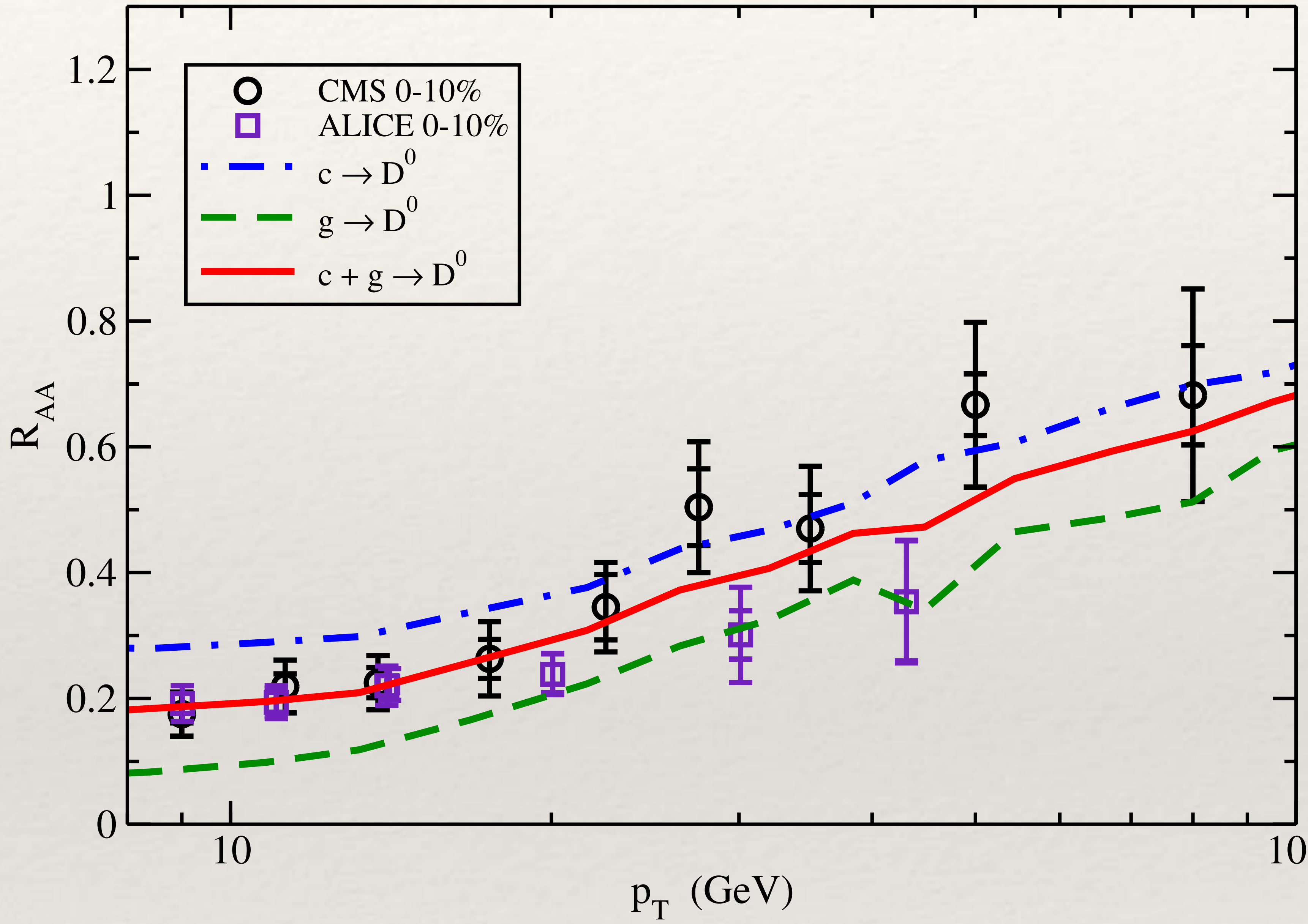
Flavor hierarchy of jet quenching

NLO initial production and fragmentation + Boltzmann transport (elastic and inelastic energy loss) + hydrodynamic medium for QGP

charged hadron

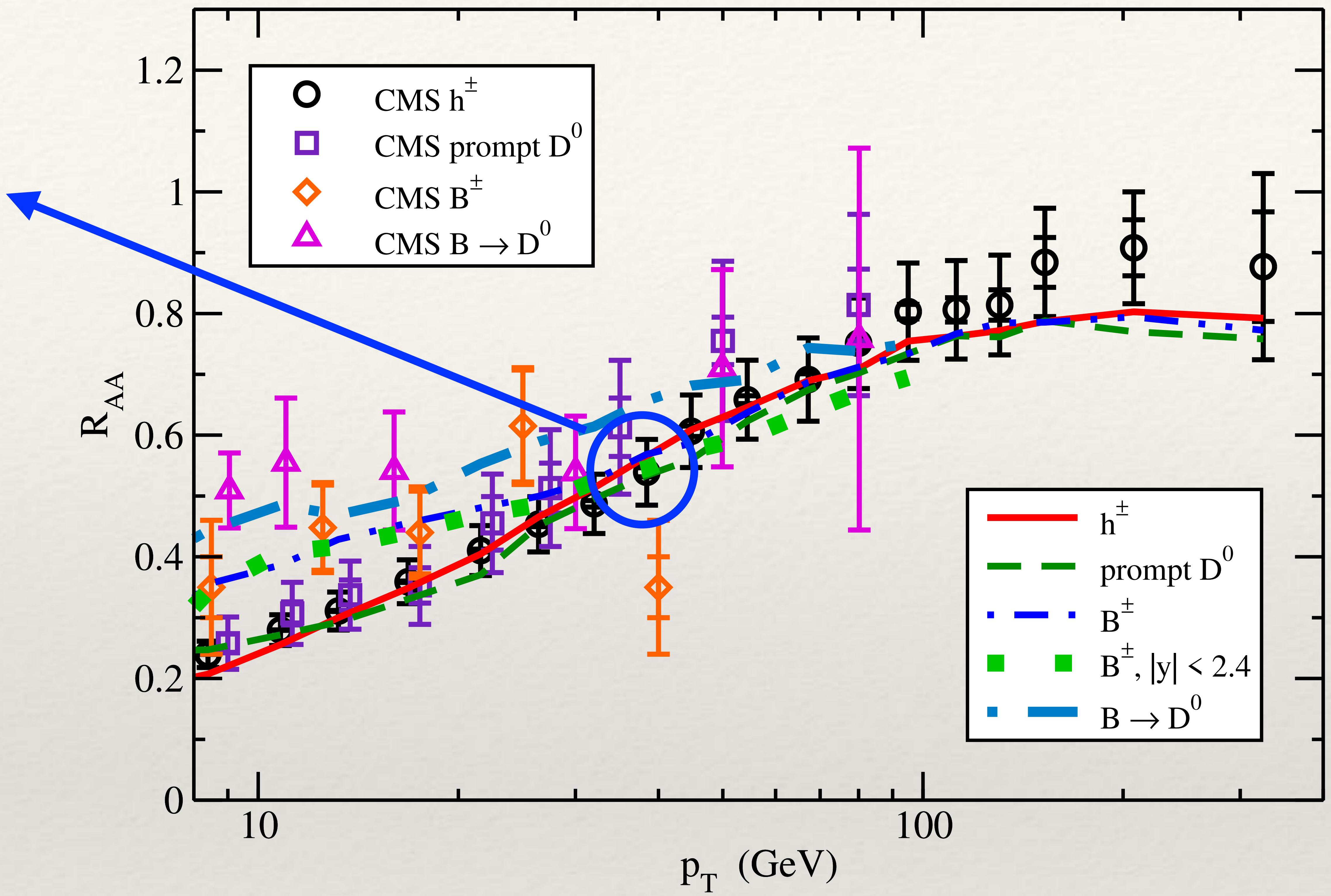


D meson



- *g*-initiated *h* & *D* R_{AA} < *q*-initiated *h* & *D* R_{AA} => **$\Delta E_g > \Delta E_q > \Delta E_c$ holds**
- Although R_{AA} (*c*->*D*) > R_{AA} (*q*->*h*), R_{AA} (*g*->*D*) < R_{AA} (*g*->*h*) due to different fragmentation functions => **R_{AA} (*h*) ≈ R_{AA} (*D*)**

Flavor hierarchy of jet quenching

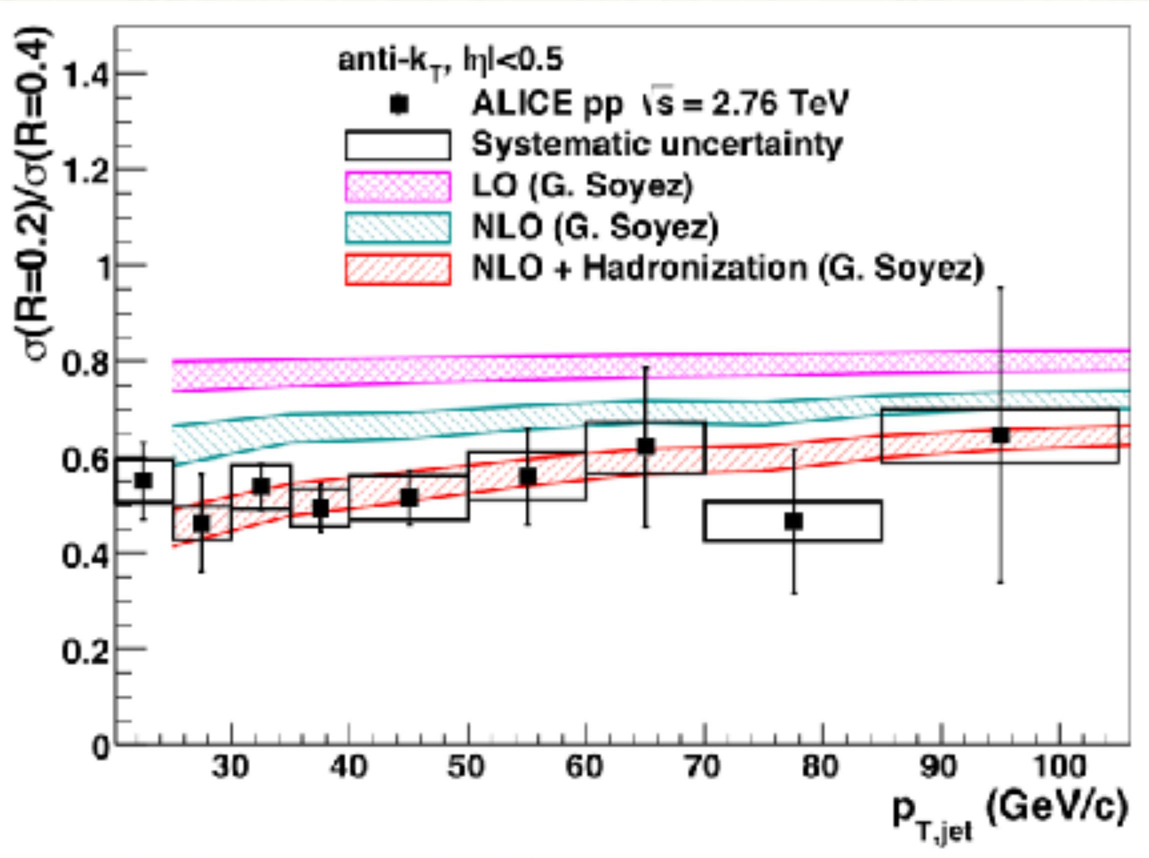


[Xing, Cao, Qin and Xing, Phys. Lett. B 805 (2020) 135424]

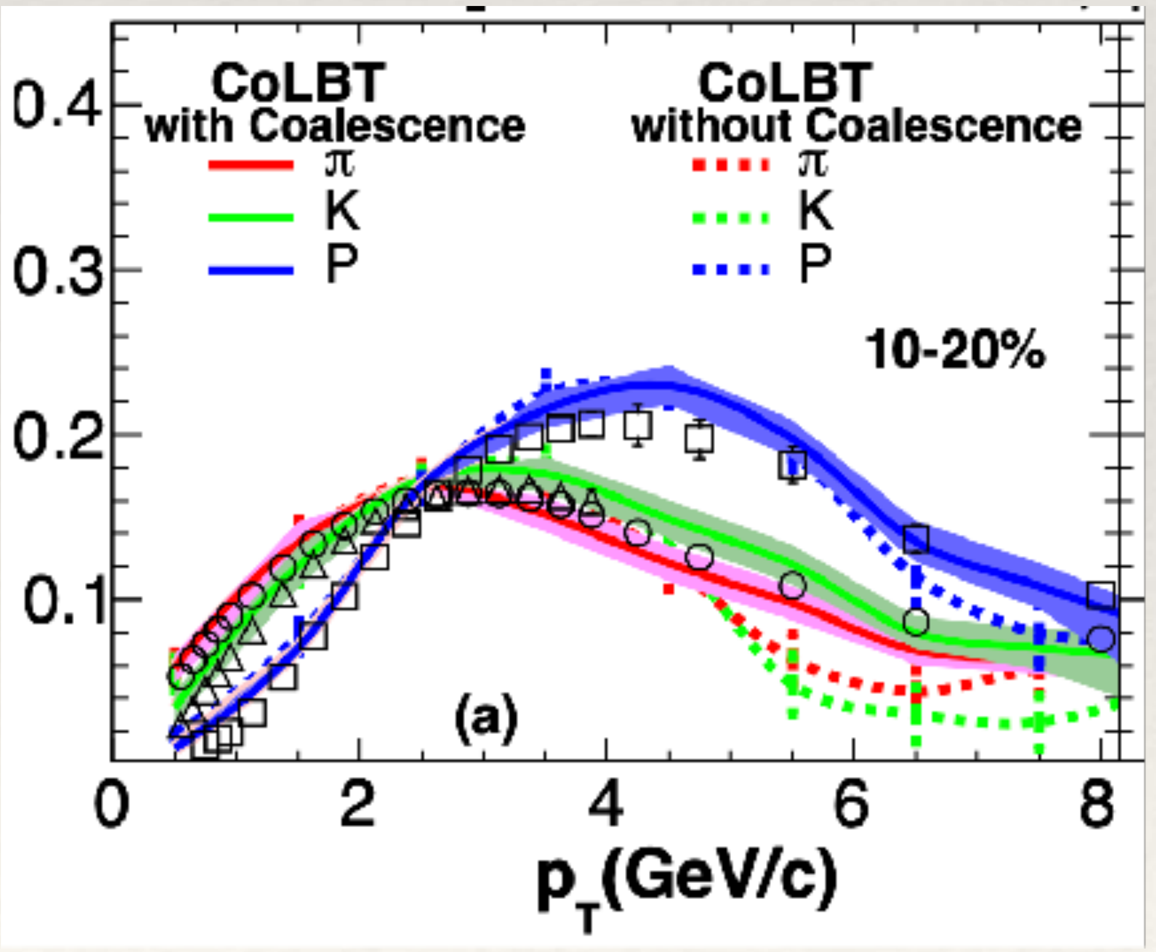
- A simultaneous description of charged hadron, *D* meson, *B* meson, *B*-decay *D* meson R_{AA}'s starting from p_T ~ 8 GeV
- Predict R_{AA} separation between *B* and *h* / *D* below 40 GeV, but similar values above – **wait for confirmation from future precision measurement**

Medium to low p_T hadrons — hadronization

Importance of hadronization



[ALICE, PLB 722 (2013)]



[Zhao, Ke, Chen, Luo, Wang, arXiv:2103.14657]

Soft hadrons:

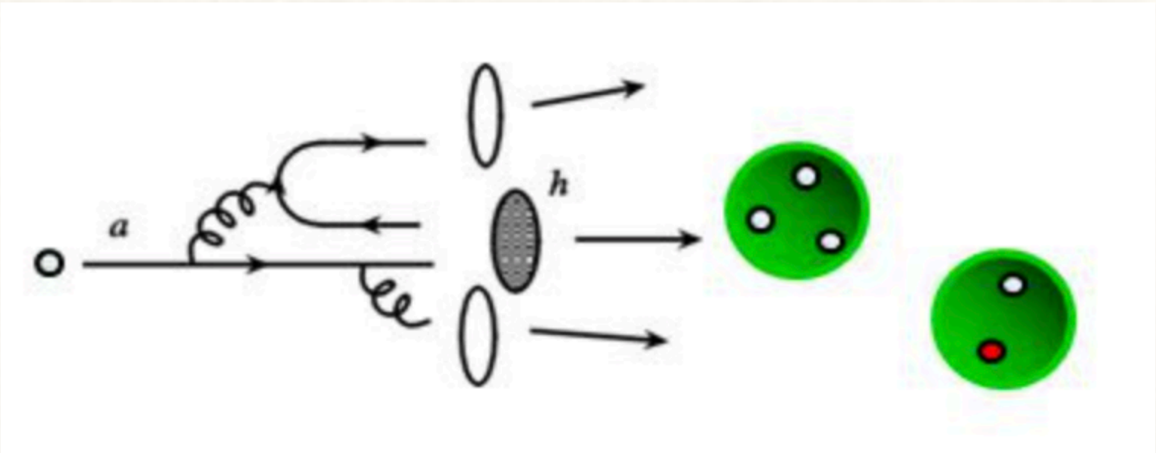
NCQ scaling of v_2

Jets:

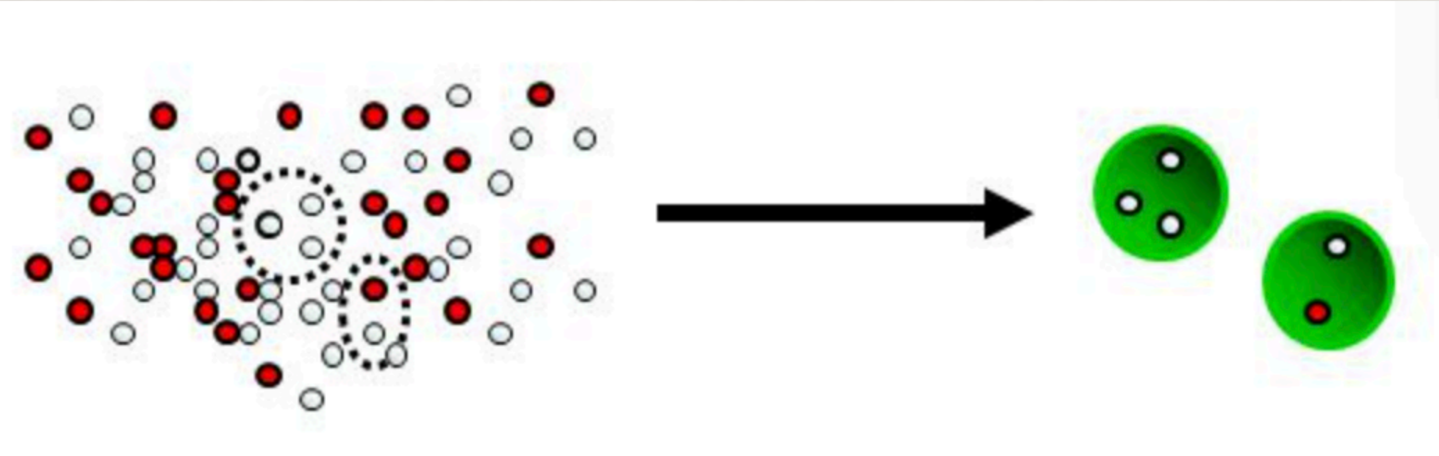
equal importance as the NLO effects

Medium p_T hadrons: crucial for a simultaneous description of R_{AA} and v_2

Hadronization mechanisms



Fragmentation:
High *p* partons fragment into hadrons
[Peterson, FONNL, Pythia, etc.]

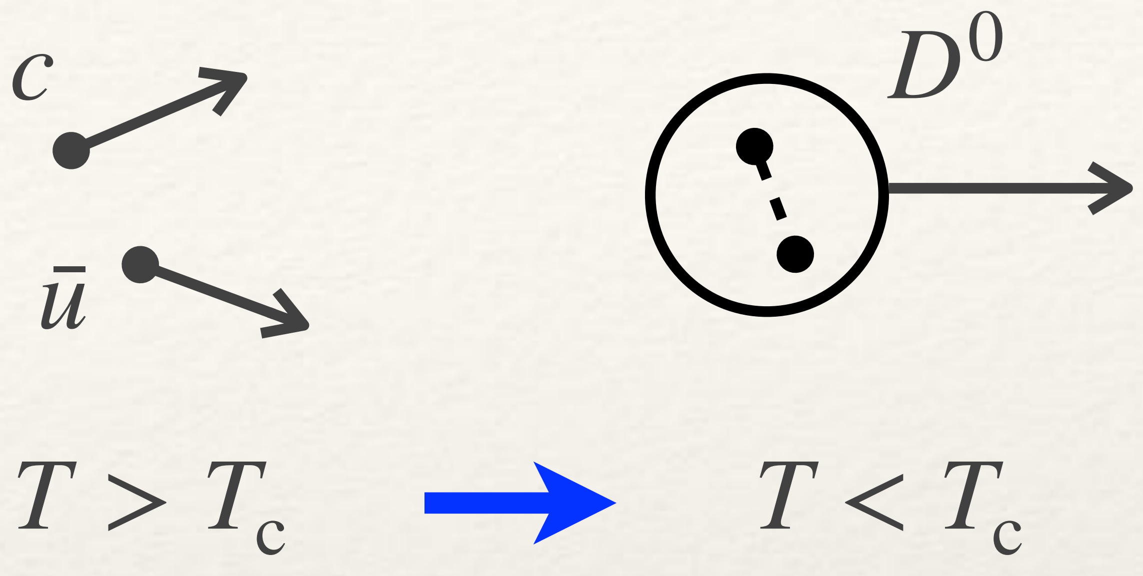


Coalescence (***recombination***):
Low *p* partons combine with thermal partons into hadrons

Instantaneous coalescence model

Coalescence probability ~ wavefunction overlap

- Sudden approximation: $|q, g\rangle \to |h\rangle$ as T drops across T_c
- Probability: wave function projection $W_M \equiv |\langle M | q_1, q_2\rangle|^2$
- Encodes information of microscopic hadron structures



Hadron spectrum from coalescence (e.g. meson formation)

$$f_M(\vec{p}\,'_M) = \int d^3p_1 d^3p_2 f_1(\vec{p}_1) f_2(\vec{p}_2) W(\vec{p}_1, \vec{p}_2) \delta(\vec{p}\,'_M - \vec{p}_1 - \vec{p}_2)$$

$f_i(\vec{p}_i)$: distribution of constituent quarks

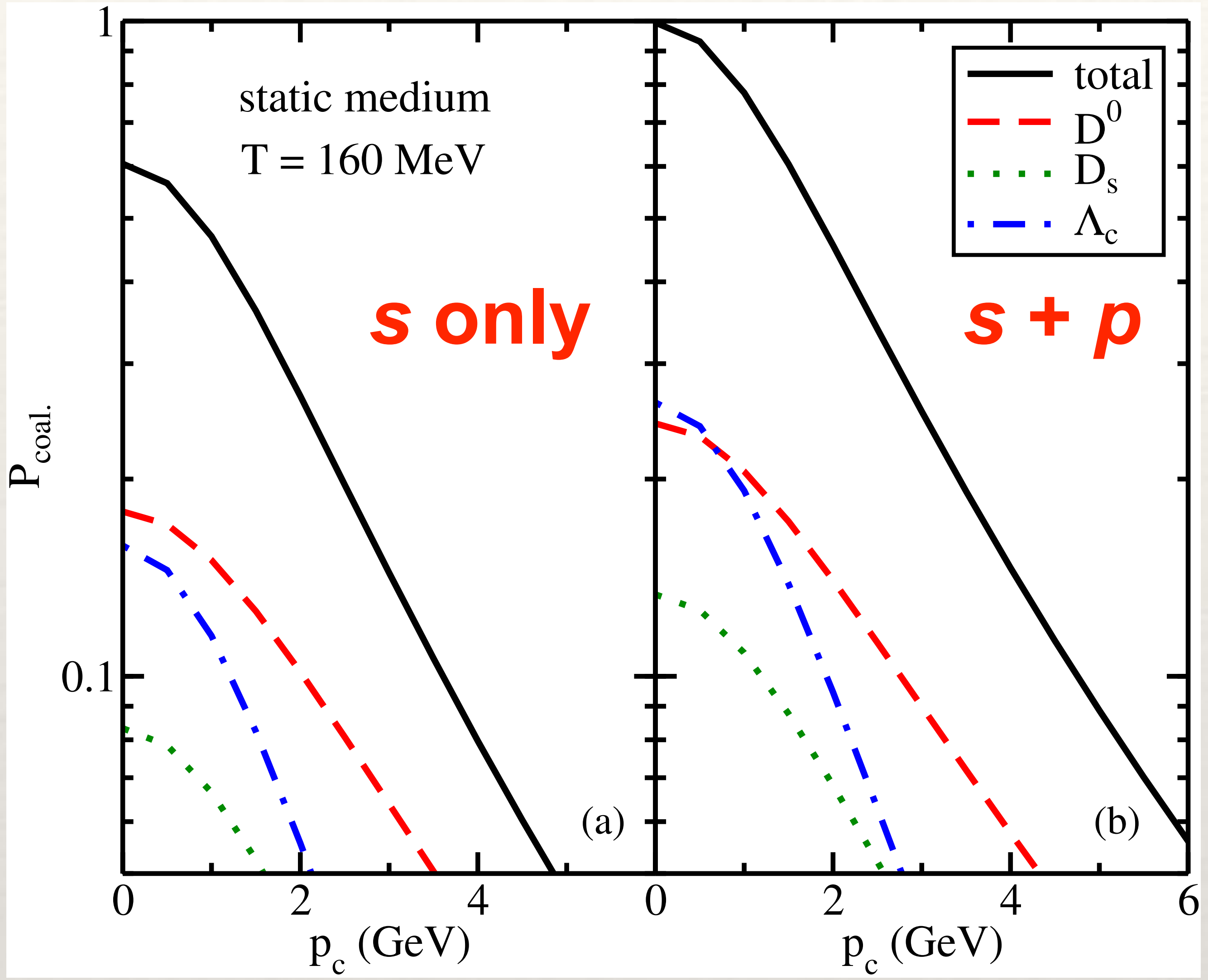
Light quarks: thermal distribution in the local rest frame of the QGP

Heavy quarks: from a transport model simulation

Recent improvements of the coalescence model

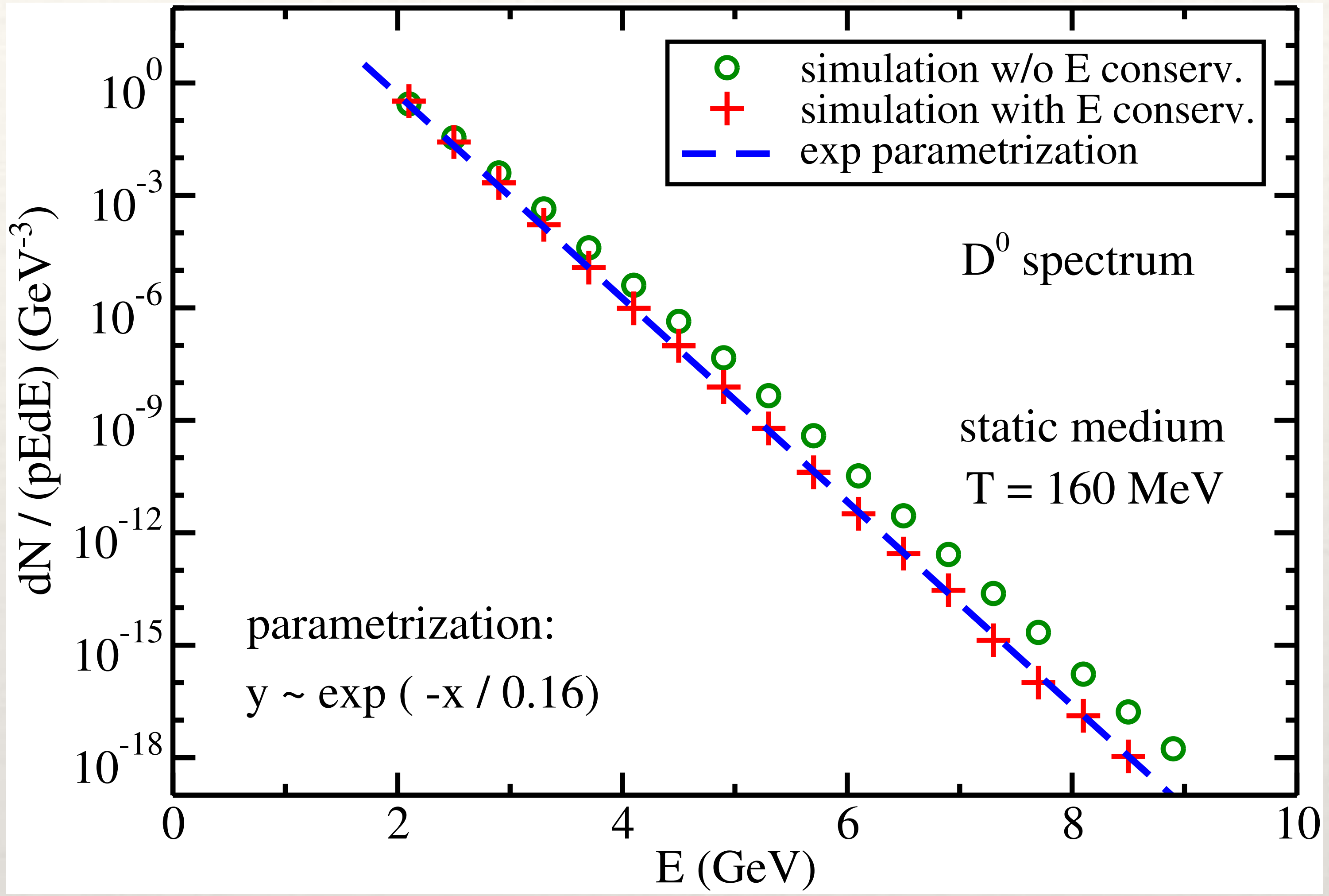
[Cao, Sun, Li, Liu, Xing, Qin and Ko, Phys. Lett. B 807 (2020) 135561]

Including both *s* and *p*-wave states



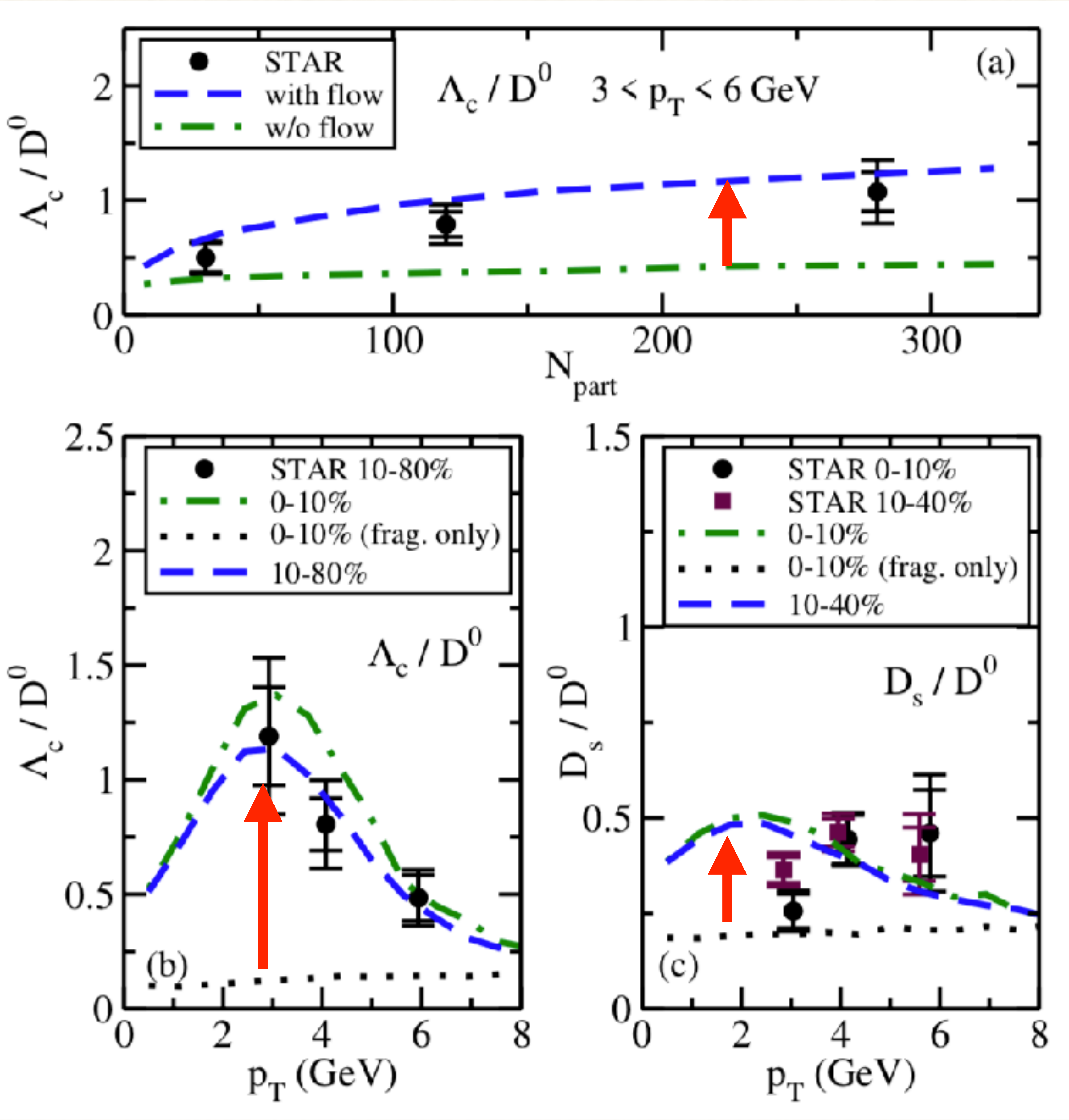
- Enhances the total $P_{\rm coal}$
- Includes nearly all *c*-hadrons in PDG
- Allows normalizing $P_{\rm coal}(p_c = 0)$ with proper in-medium hadron sizes ($r_{D^0} = 0.97$ fm)

Energy conservation and thermal limit



- 3-*p* → 4-*p* conservation by coalescing to off-shell excited states and then decay to on-shell ground states
- Guarantees boost invariance
- Guarantees the thermal limit of *c*-hadrons: thermal *c* + thermal *q* → thermal D^0

Charmed hadron chemistry at RHIC



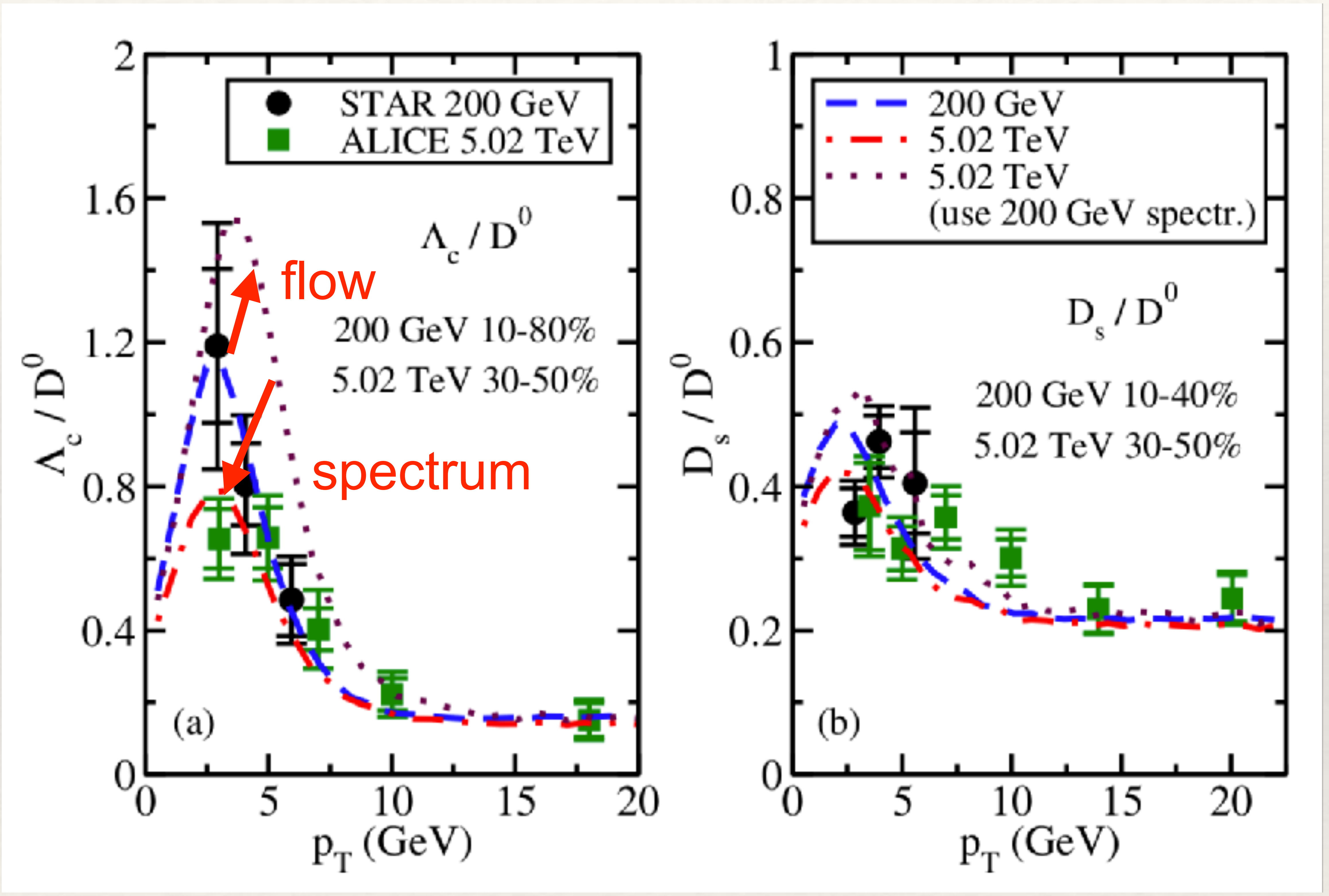
effects of the QGP flow

effects of coalescence

- (a) Stronger QGP flow boost on heavier hadrons => increasing Λ_c/D^0 with N_{part}
- (b) Coalescence significantly increases Λ_c/D^0, larger value in more central collisions (stronger QGP flow)
- (c) Enhanced D_s/D^0 due to strangeness enhancement in QGP and larger D_s mass than D^0

RHIC vs. LHC

Λ_c/D^0 D_s/D^0

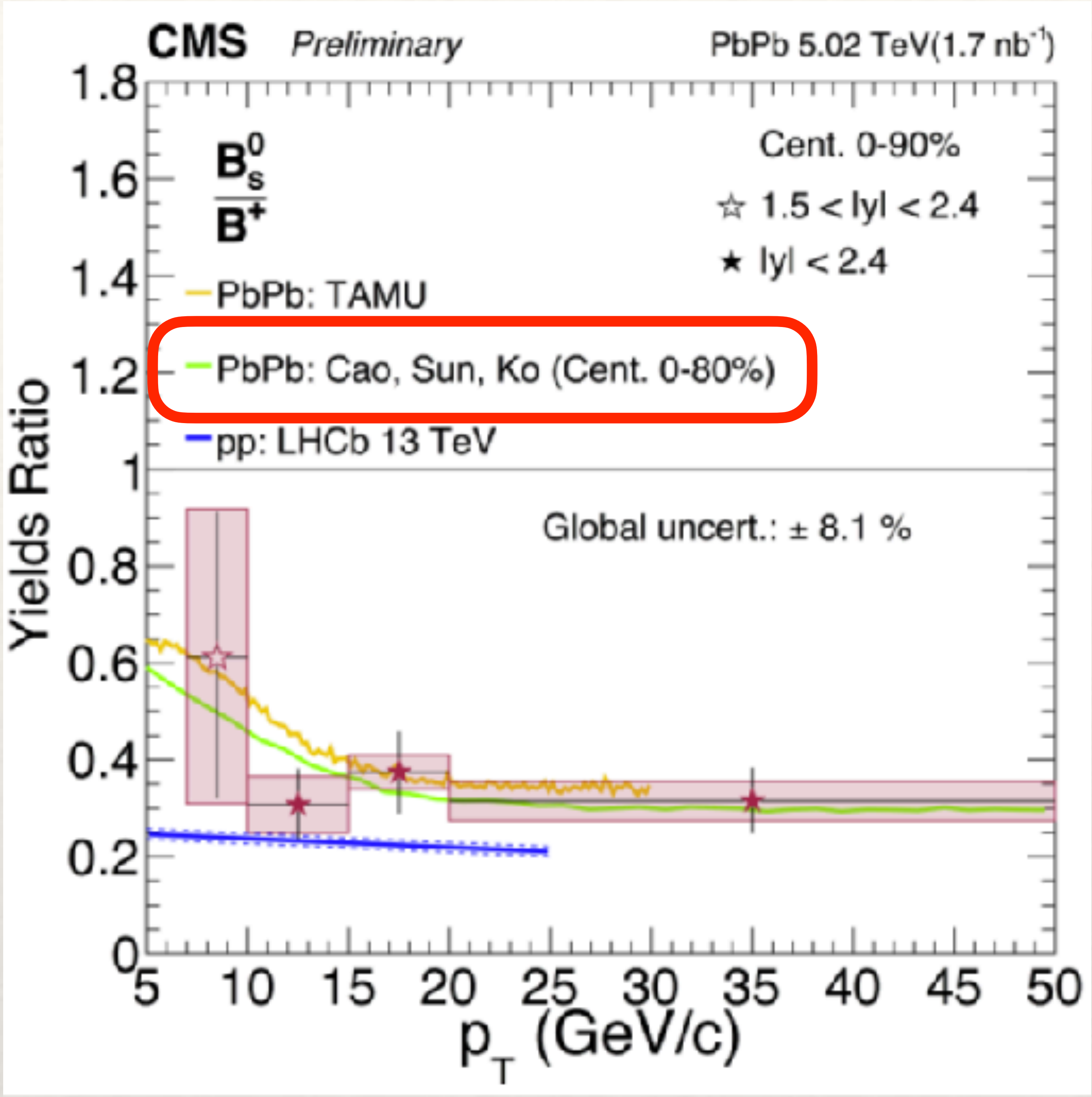


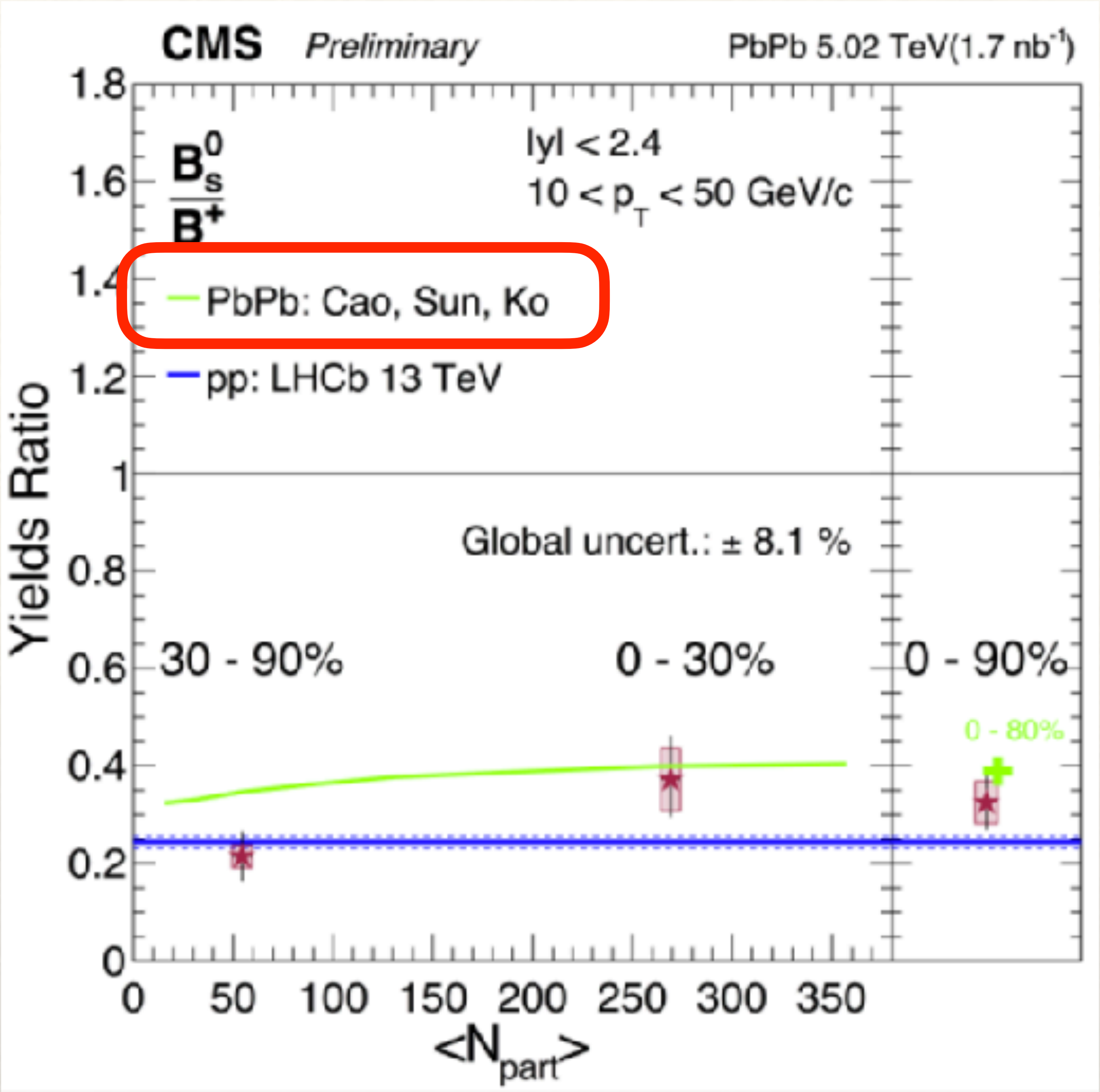
- IF charm quarks have the same initial spectrum at RHIC and LHC, Λ_c/D^0 would be larger at LHC than RHIC due to the flow effect

- The harder initial charm quark spectra at LHC reduces Λ_c/D^0

- Similar theoretical prediction on D_s/D^0

Prediction on bottom hadron chemistry



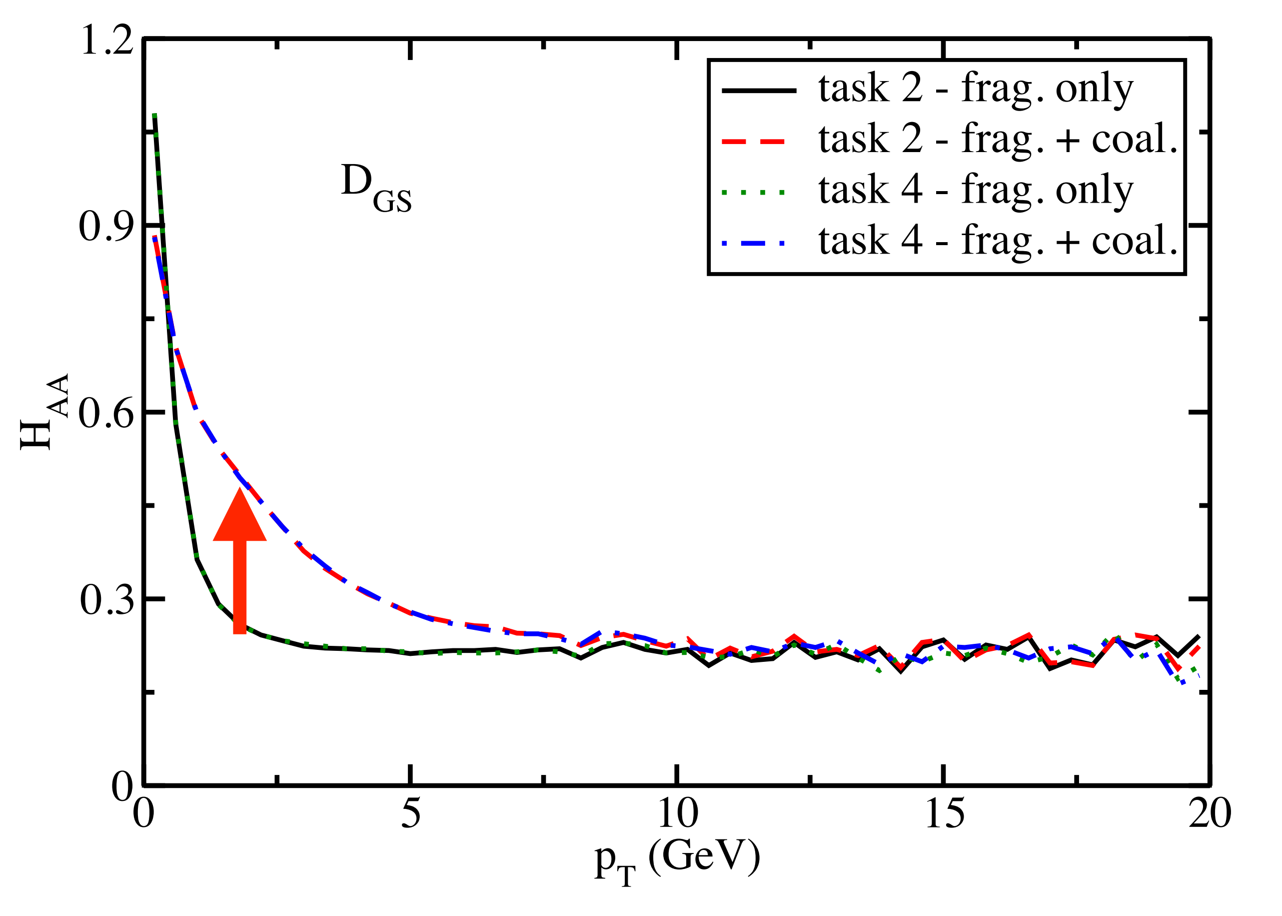


taken from CMS presentation at HP2020 by Z. Shi

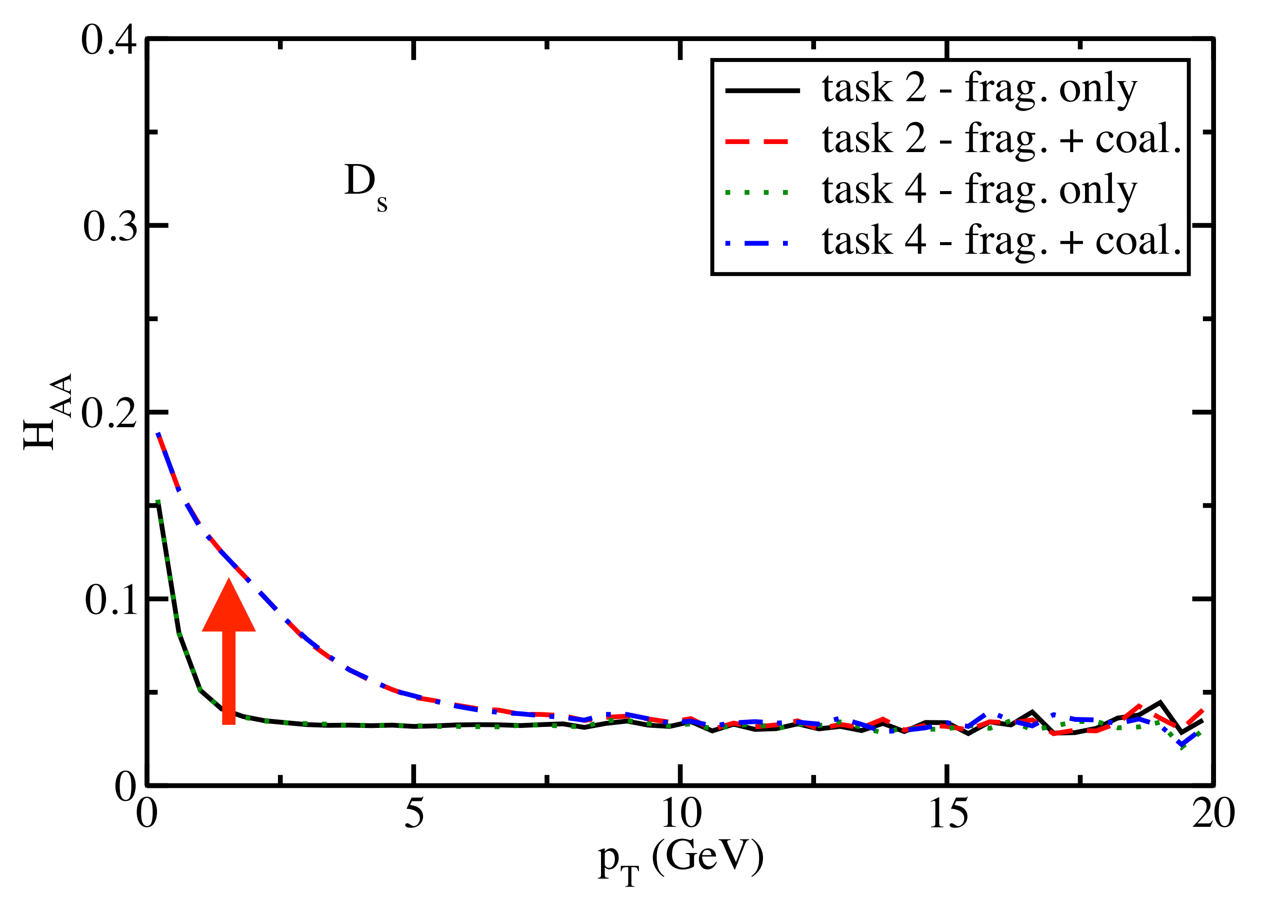
- More constraints on the mass (velocity/momentum) dependence of hadronization models
- Assume same diffusion coefficient D_s between *c* and *b* quarks

Homework from this workshop ($H_{\rm AA}$)

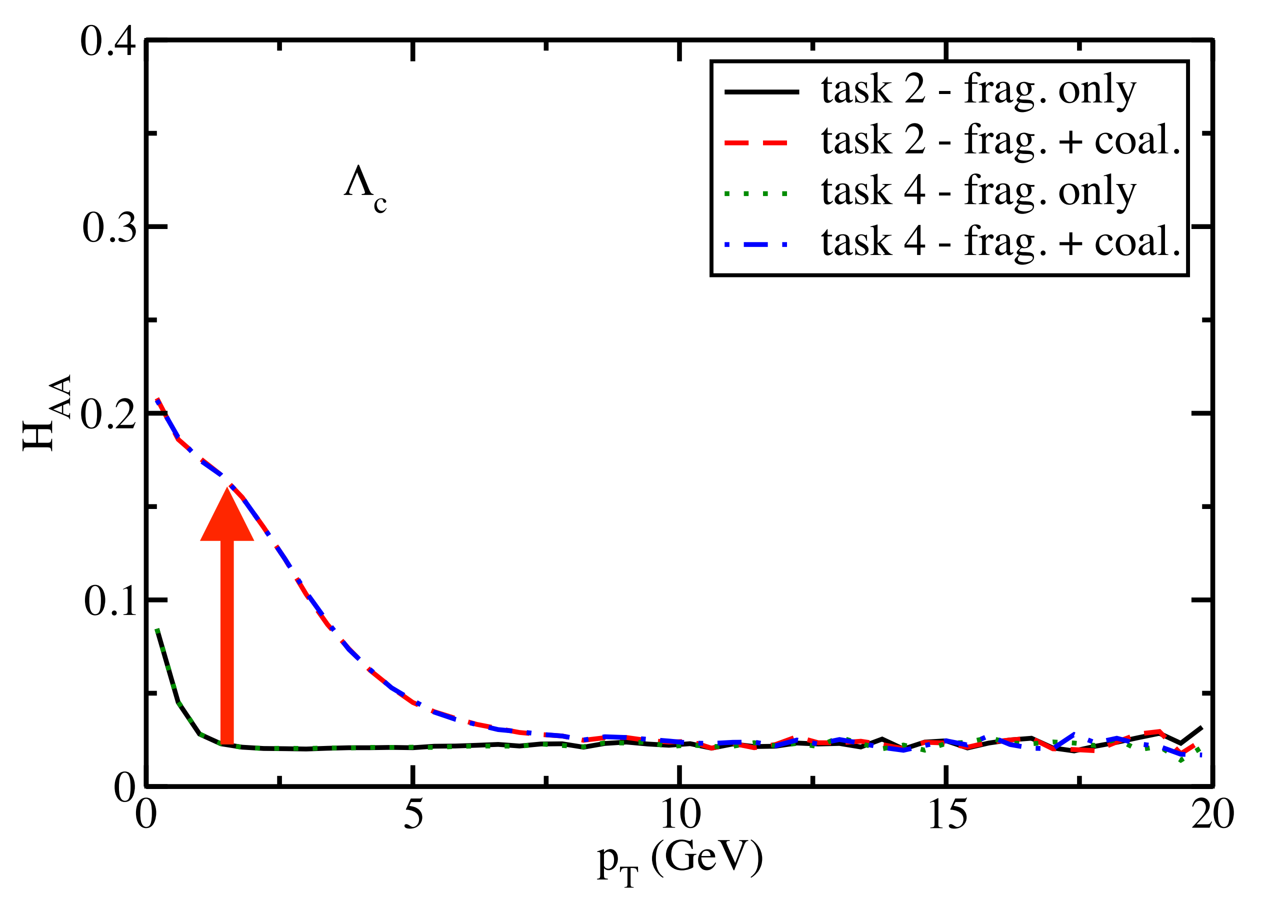
$D^0 + D^+$



D_s



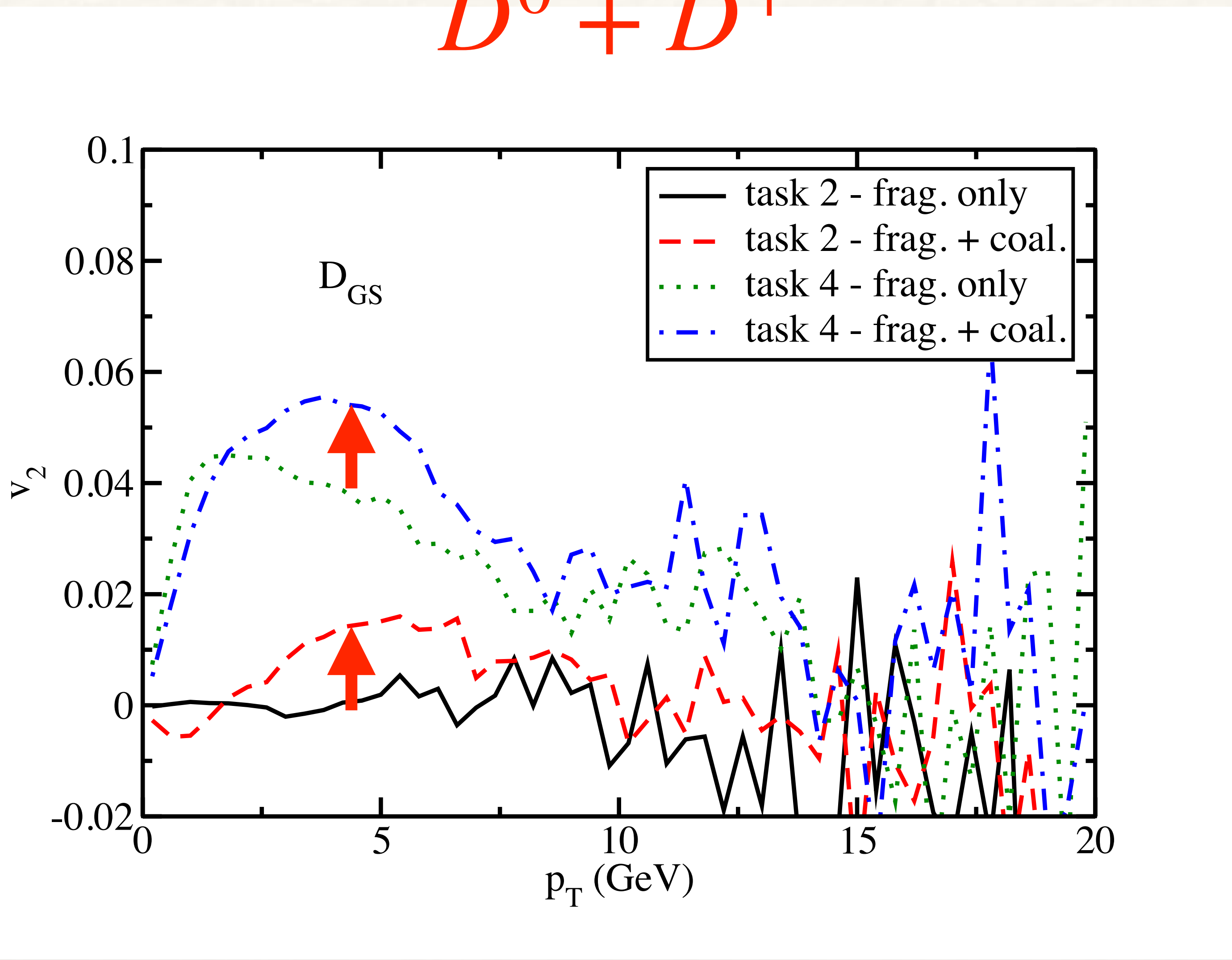
Λ_c



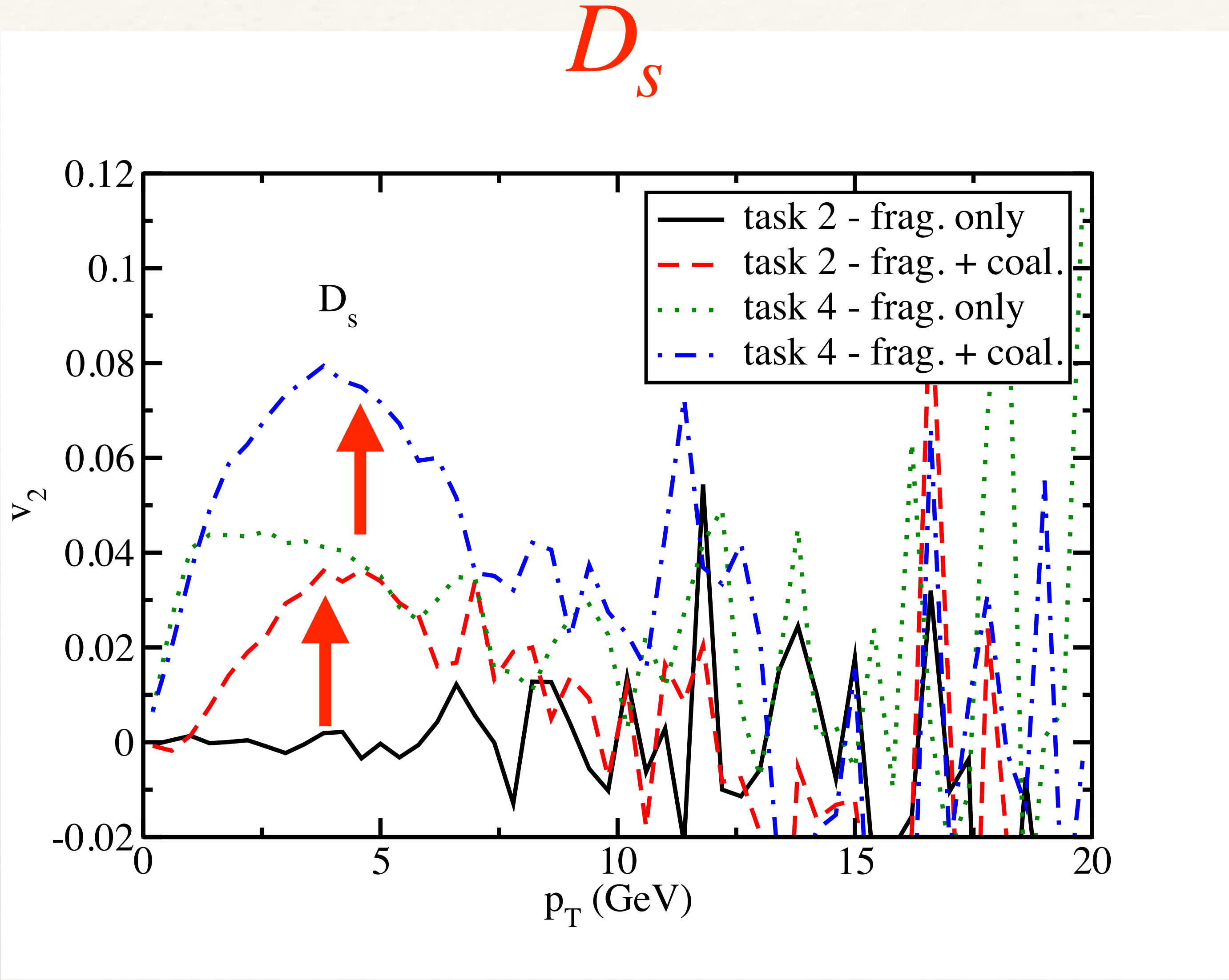
- $H_{\rm AA} = (dN_h/dp_{\rm T})/(dN_c/dp_{\rm T})$
- Coalescence increase $H_{\rm AA}$ at low $p_{\rm T}$, stronger increase for Λ_c than D
- Same results between task 2 and task 4 — same $p_{\rm T}$ different ϕ distribution of charm quarks

Homework from this workshop (v_2)

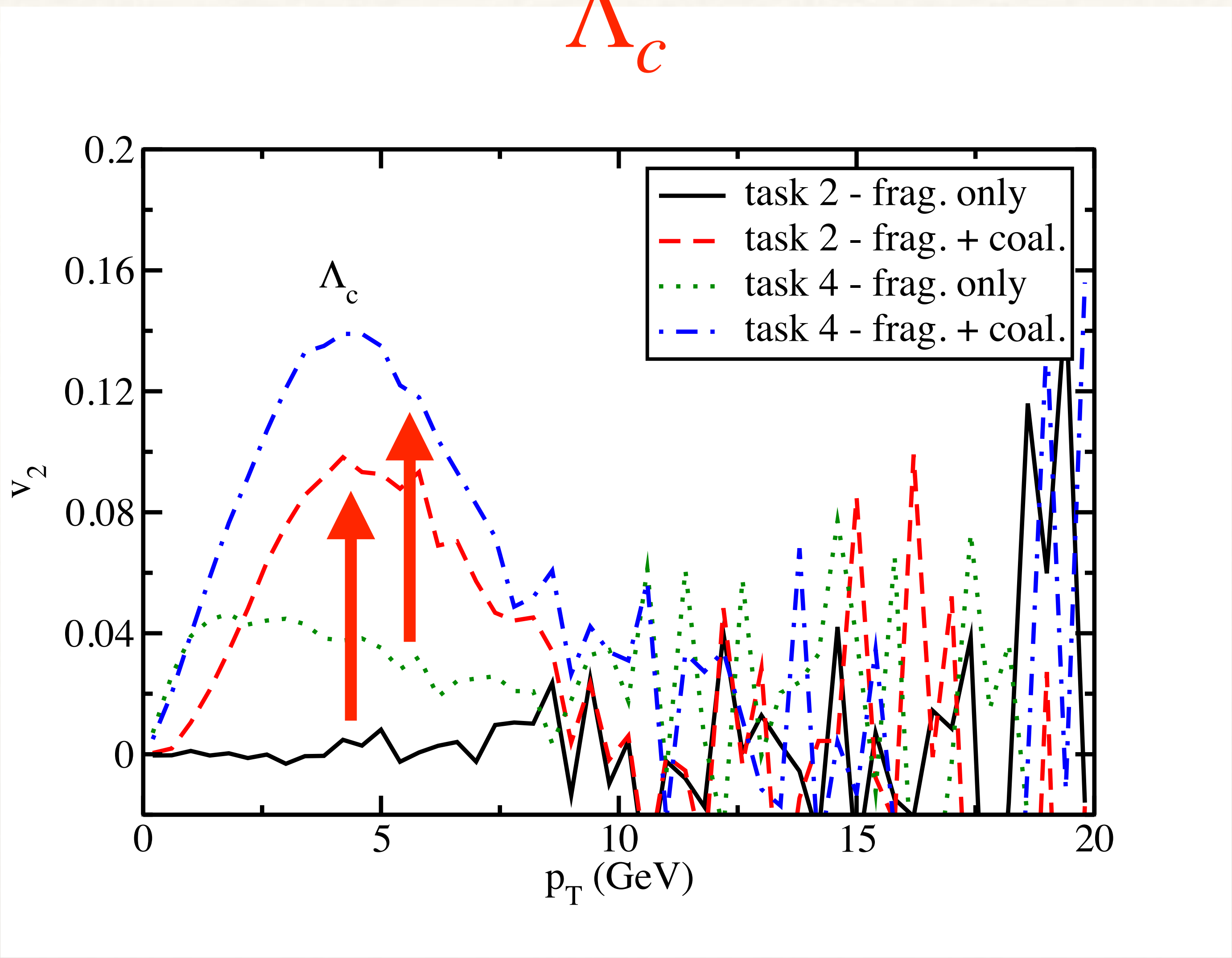
$D^0 + D^+$



D_s



Λ_c

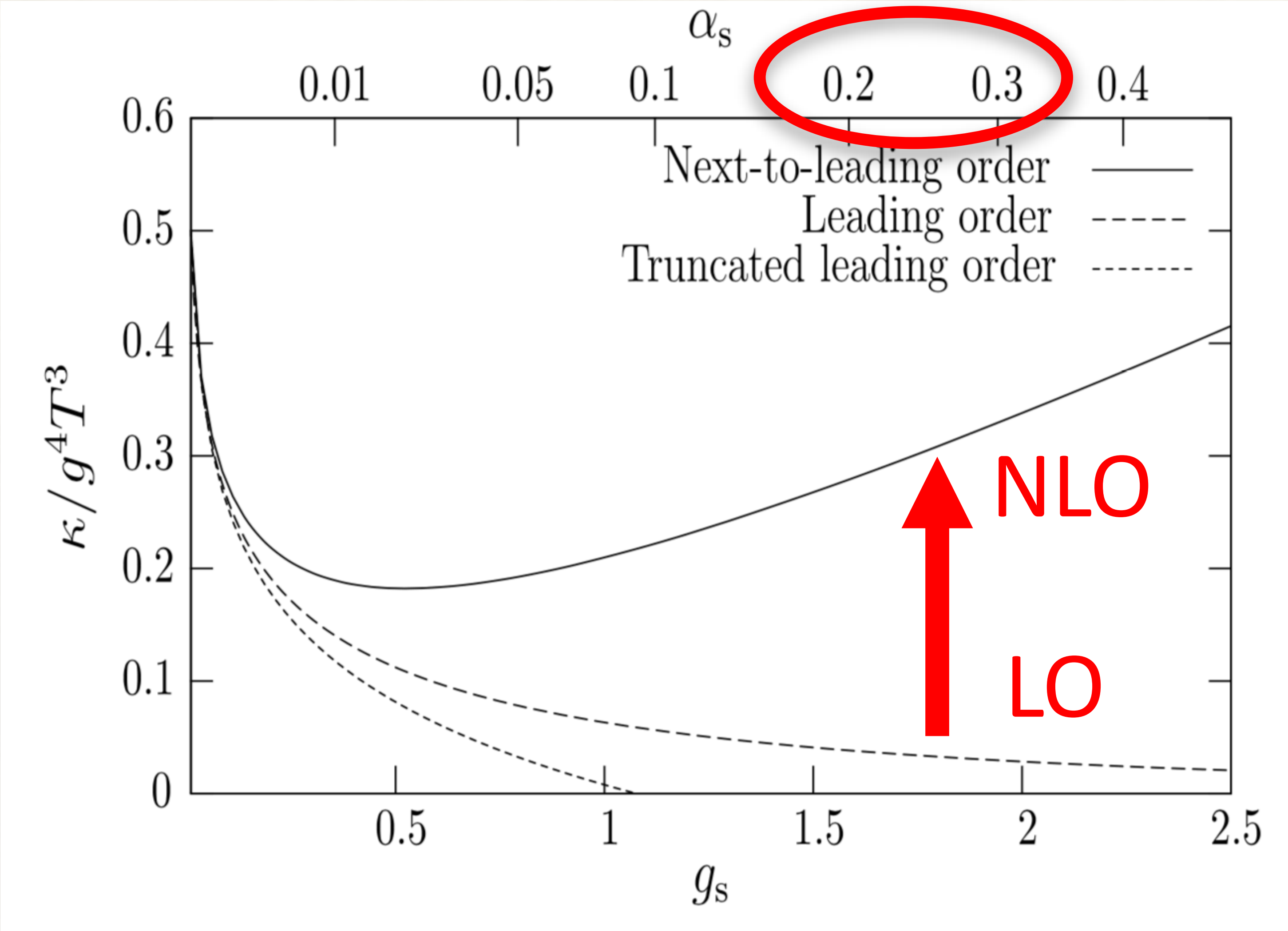


- Task 2 (zero v_2 for *c*-quarks): zero v_2 from fragmentation, finite v_2 after including coalescence
- Task 4 (finite v_2 for *c*-quarks): finite v_2 from fragmentation, larger v_2 after including coalescence
- Coalescence enhancement of v_2: $\Lambda_c > D_s > D^0\,(D^+)$

Low p_T HQ's — color potential interaction

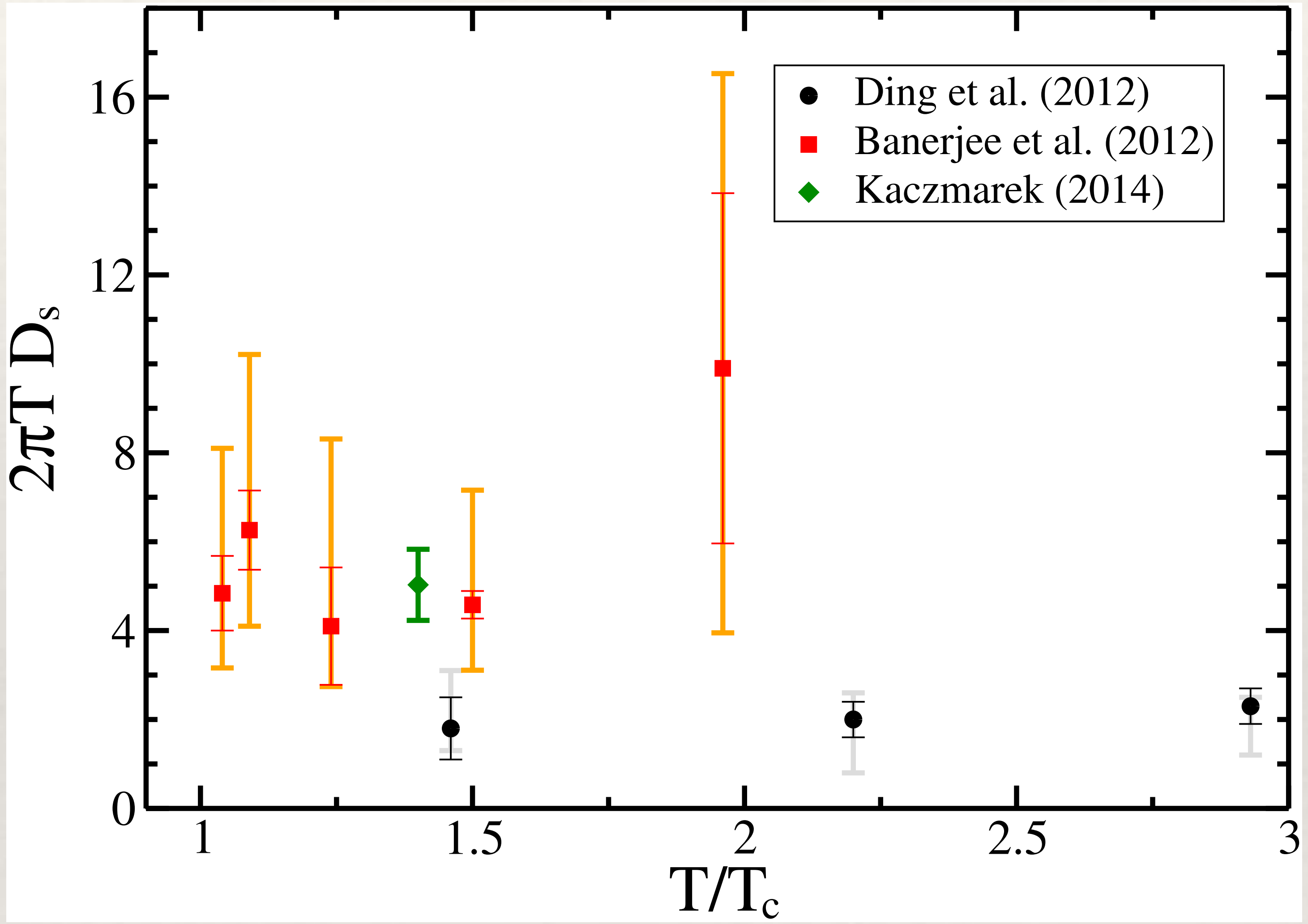
- Suppression of radiative energy loss due to the "dead cone effect"
- Heavy quark diffusion, **diffusion coefficient** κ or D_s as important input into transport models

Perturbation calculation fails at low p_T



- LO: Svetitsky, PRD 37 (1988)
  Moore and Teaney, PRC 71 (2005)
- NLO: Caron-Huot and Moore, JHEP 02 (2008)
- A factor of over 5 increase at NLO

Inputs from lattice calculations



- Uncertainty is still large
- No results for finite momentum HQ yet

Perturbative calculation with effective propagator approach

- Parametrization of the heavy-quark-QGP interaction potential:

$$V(r,T) = -\boxed{\frac{4}{3}\alpha_s\frac{e^{-m_d r}}{r}} - \boxed{\frac{\sigma}{m_s}e^{-m_s r}}$$

Yukawa (color coulomb) String

Liu and Rapp, Phys. Rev. C 97 (2018) 034918

Parameters fit to the lattice potential from

Burnier, Kaczmarek and Rothkopf , Phys. Rev. Lett. 114 (2015) 082001

in which $m_d = a + b * T$ and $m_s = \sqrt{a_s + b_s * T}$ are the respective screening masses, α_s and σ are the respective Yukawa and confining interaction strength.

- By Fourier transformation,

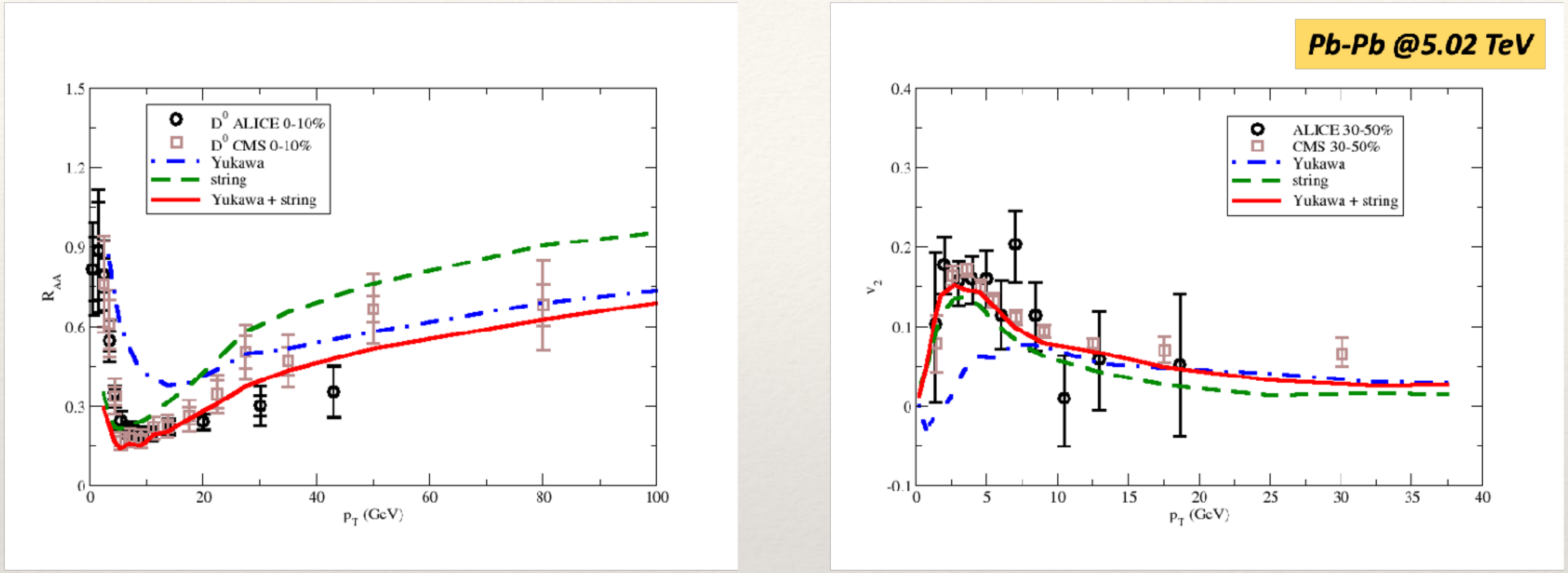
$$V(\vec{q},T) = -\boxed{\frac{4\pi\alpha_s C_F}{m_d^2 + |\vec{q}|^2}} - \boxed{\frac{8\pi\sigma}{\left(m_s^2 + |\vec{q}|^2\right)^2}}$$

- For $Qq \longrightarrow Qq$ process, we express the scattering amplitude with effective potential propagator,

Riek and Rapp, Phys. Rev. C 82 (2010) 035201

$$iM = iM_C + iM_S = \bar{u}\gamma^\mu u V_c \bar{u}\gamma^\nu u + \bar{u}u V_S \bar{u}u$$

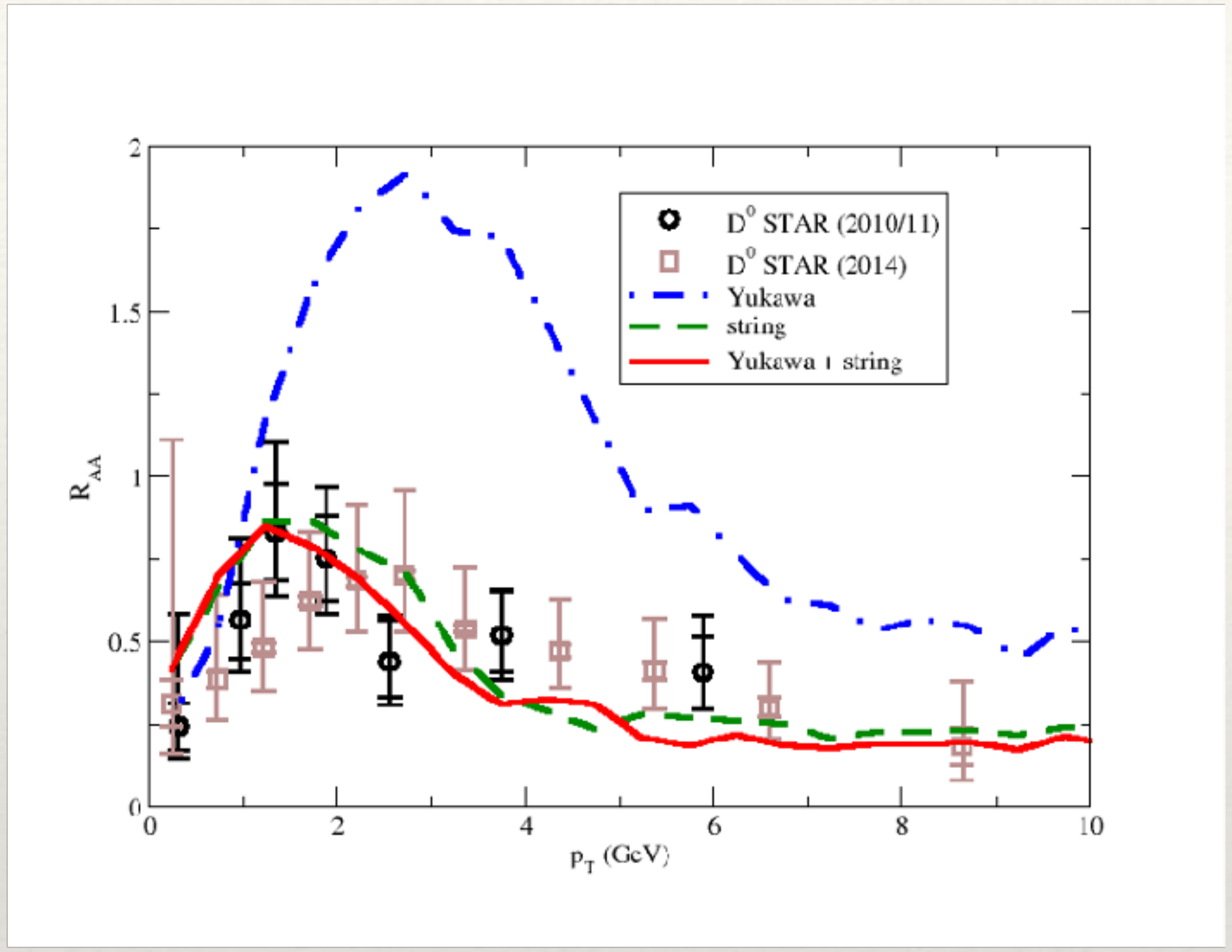
R_{AA} and v_2 of *D* mesons at LHC

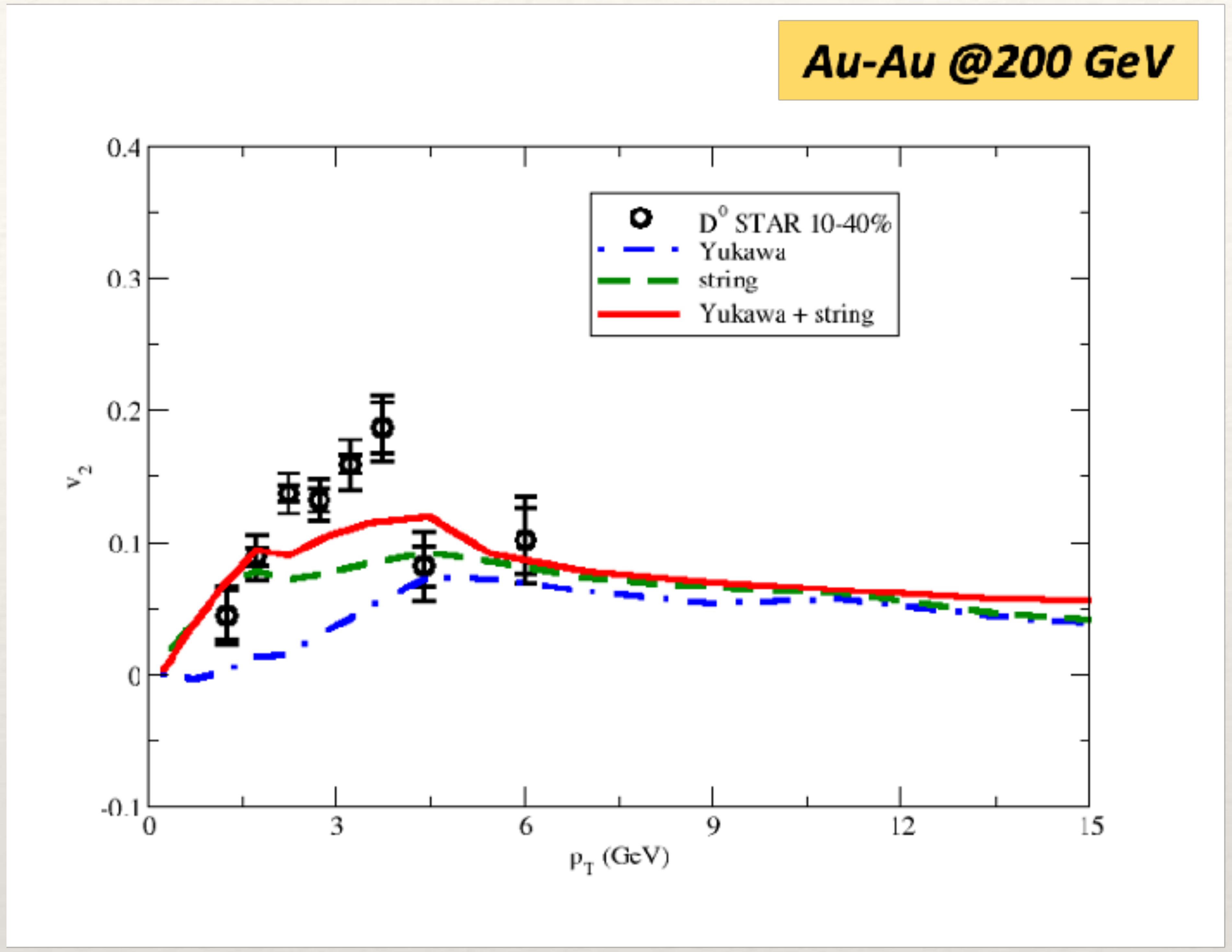


Xing, Qin, Cao, in preparation

- At high p_T, the Yukawa interaction dominates heavy-quark-medium interaction
- At low to intermediate p_T, the string interaction dominates, stronger contribution at later evolution stage (near T_c)

R_{AA} and v_2 of *D* mesons at RHIC



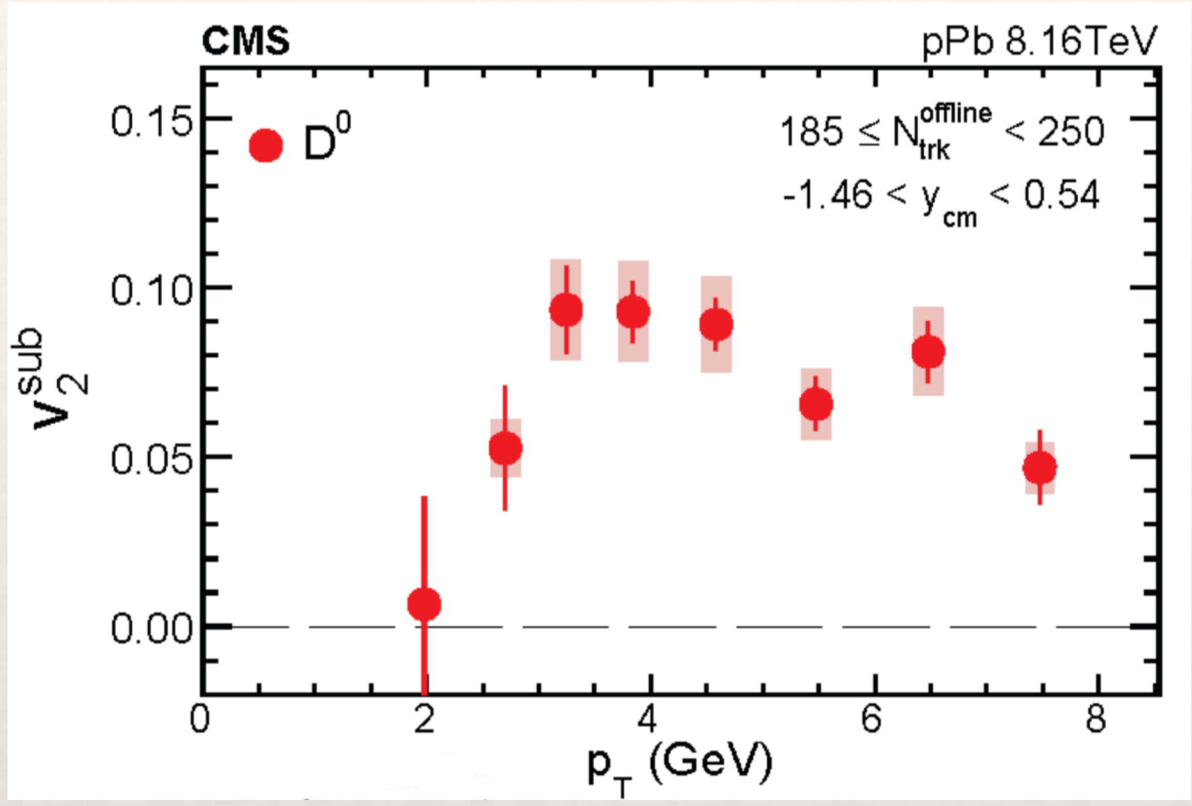


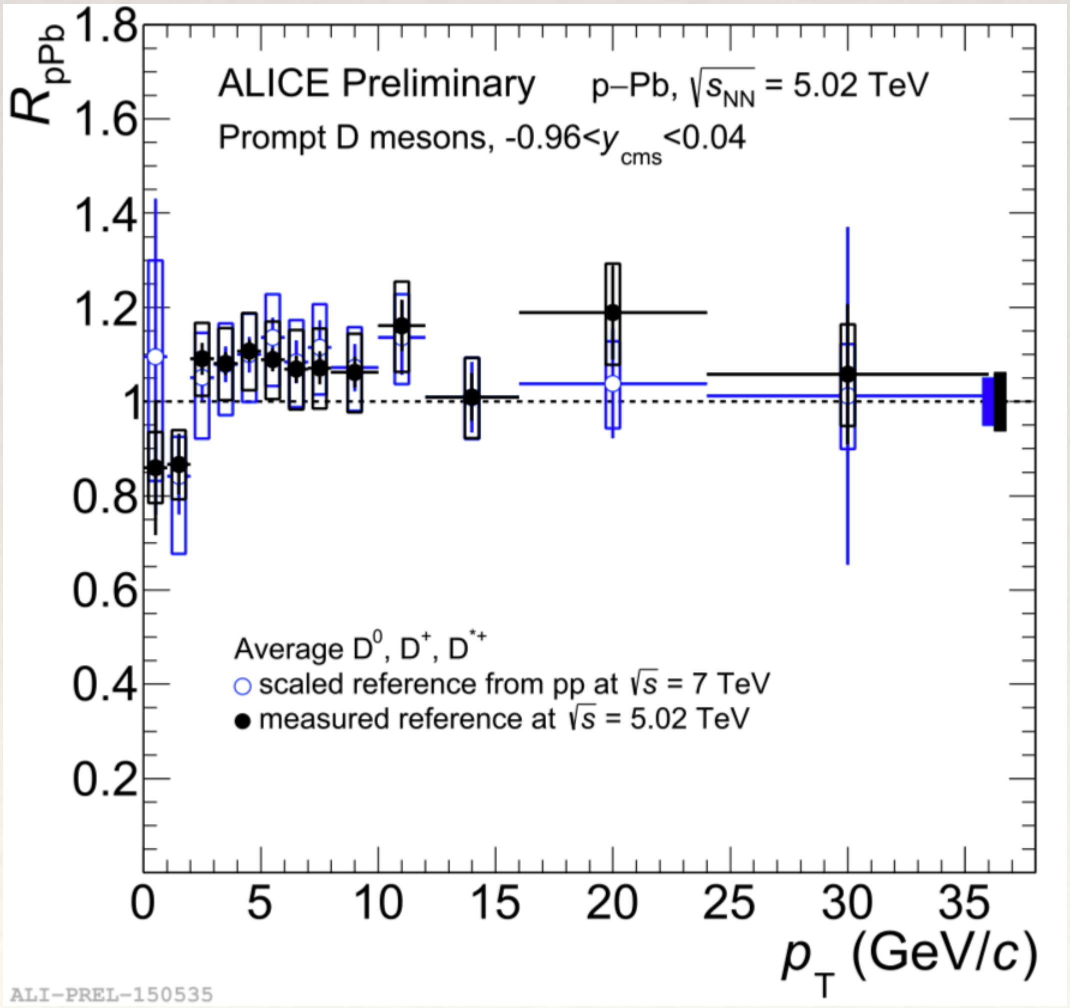
Xing, Qin, Cao, in preparation

- Effects of string interaction are crucial for the p_T regime studied at RHIC
- Combination of short-range Yukawa and long-range string interactions provide a reasonable description of the *D* meson R_{AA} and v_2

Probing system size dependence of jet quenching

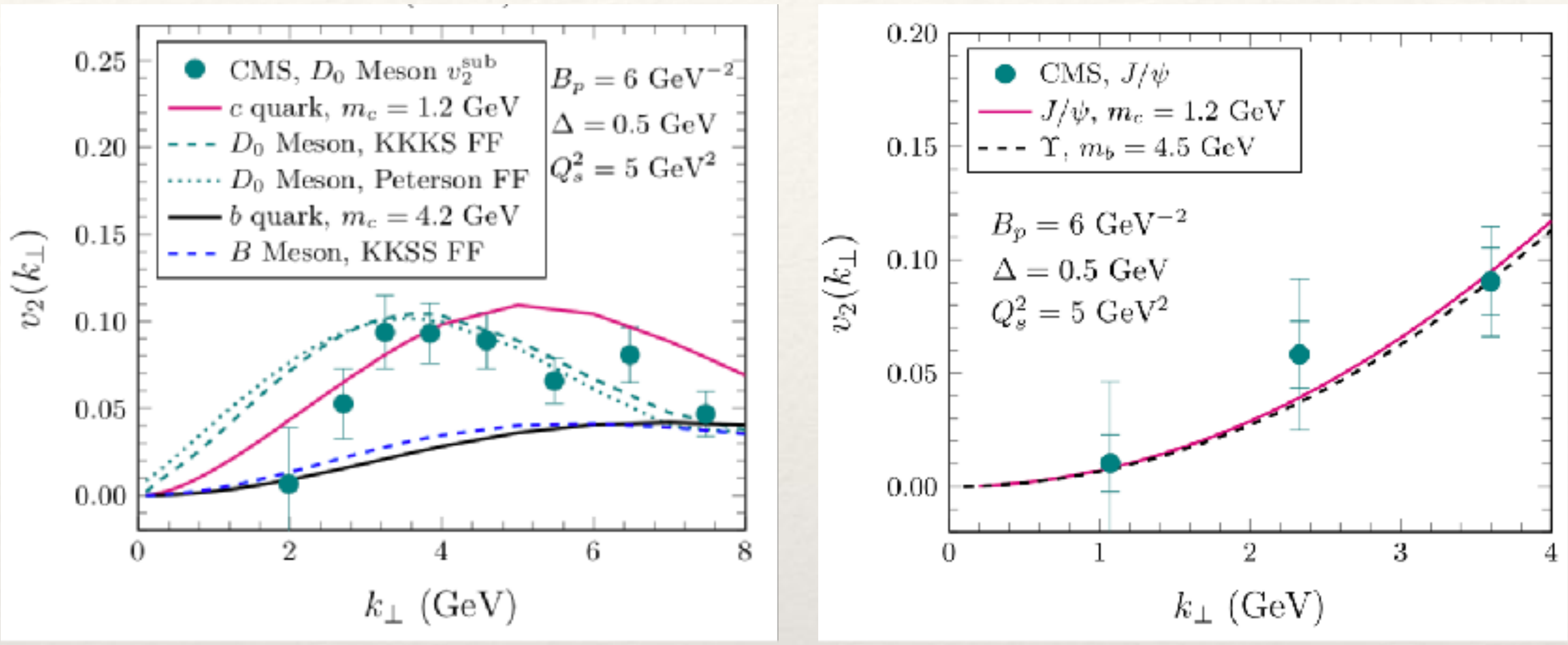
Small system (p-Pb) puzzle





- Large *D* meson v_2 up to 8 GeV
- Almost no suppression
- Should not be QGP effects
- Could it be initial state effects?

Initial state effects

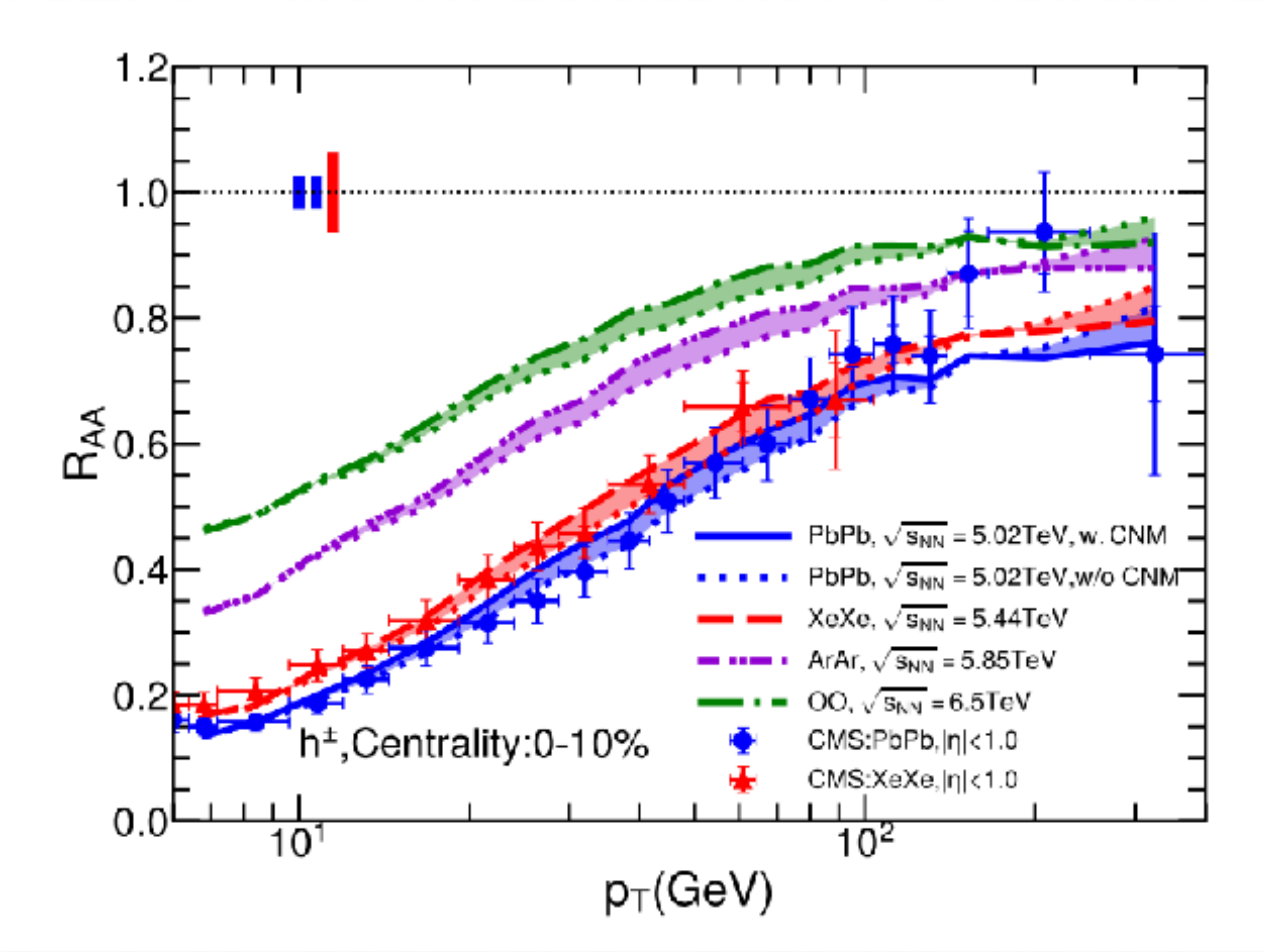


[Zhang, Marquet, Qin, Wei and Xiao, PRL 122 (2019)]

- Initial state interactions (CGC) successfully explain the large v_2 of both open charmed meson and charmonium in p-Pb collisions.
- How to separate initial state and QGP effect — a system size scan of jet quenching to bridge large and small systems

Charged hadron and *D* meson *R_{AA}* in different systems

charged hadron



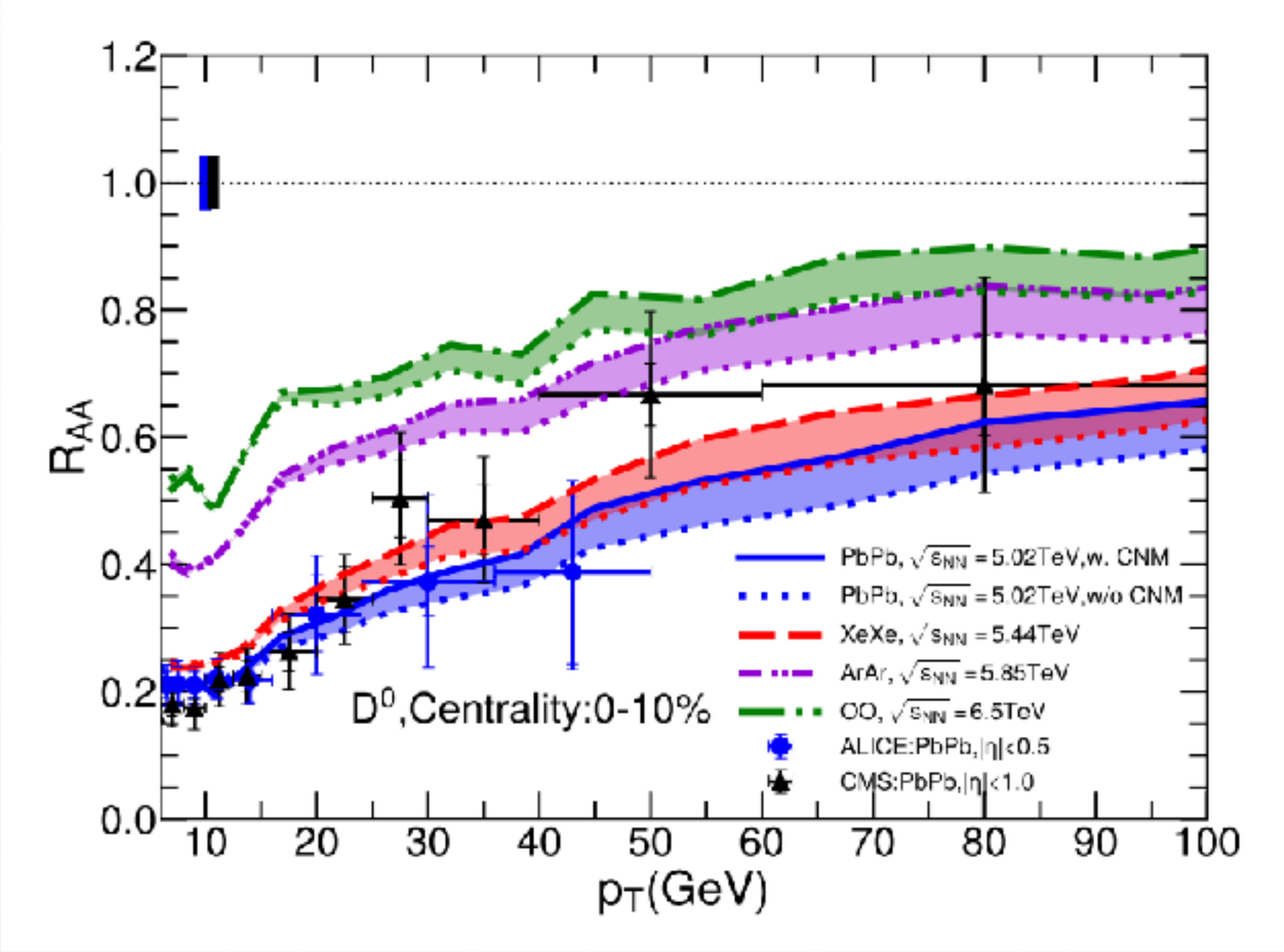
O-O

Ar-Ar

Xe-Xe

Pb-Pb

D meson

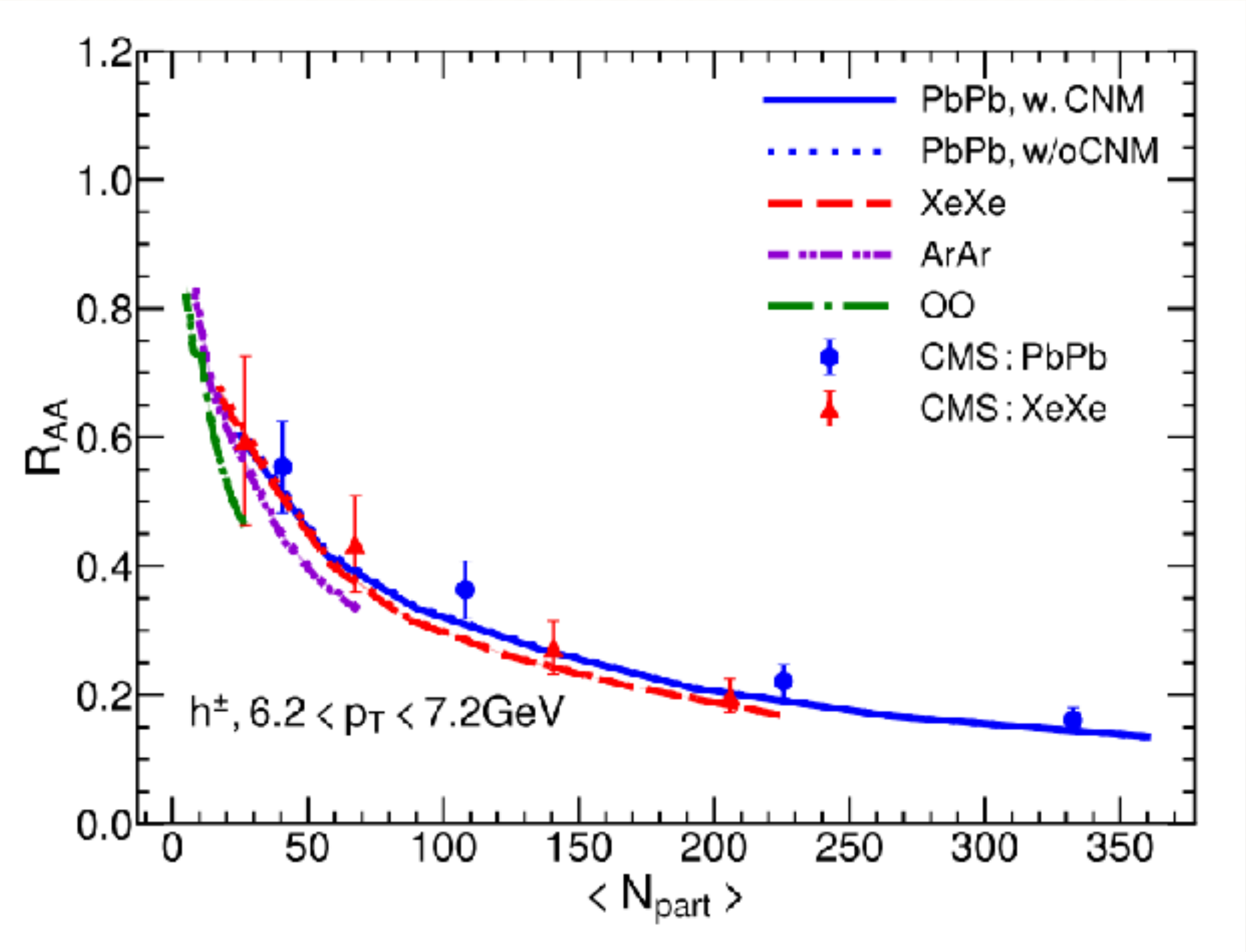


Liu, Xing, Wu, Qin, Cao, Xing, arXiv:2107.01522

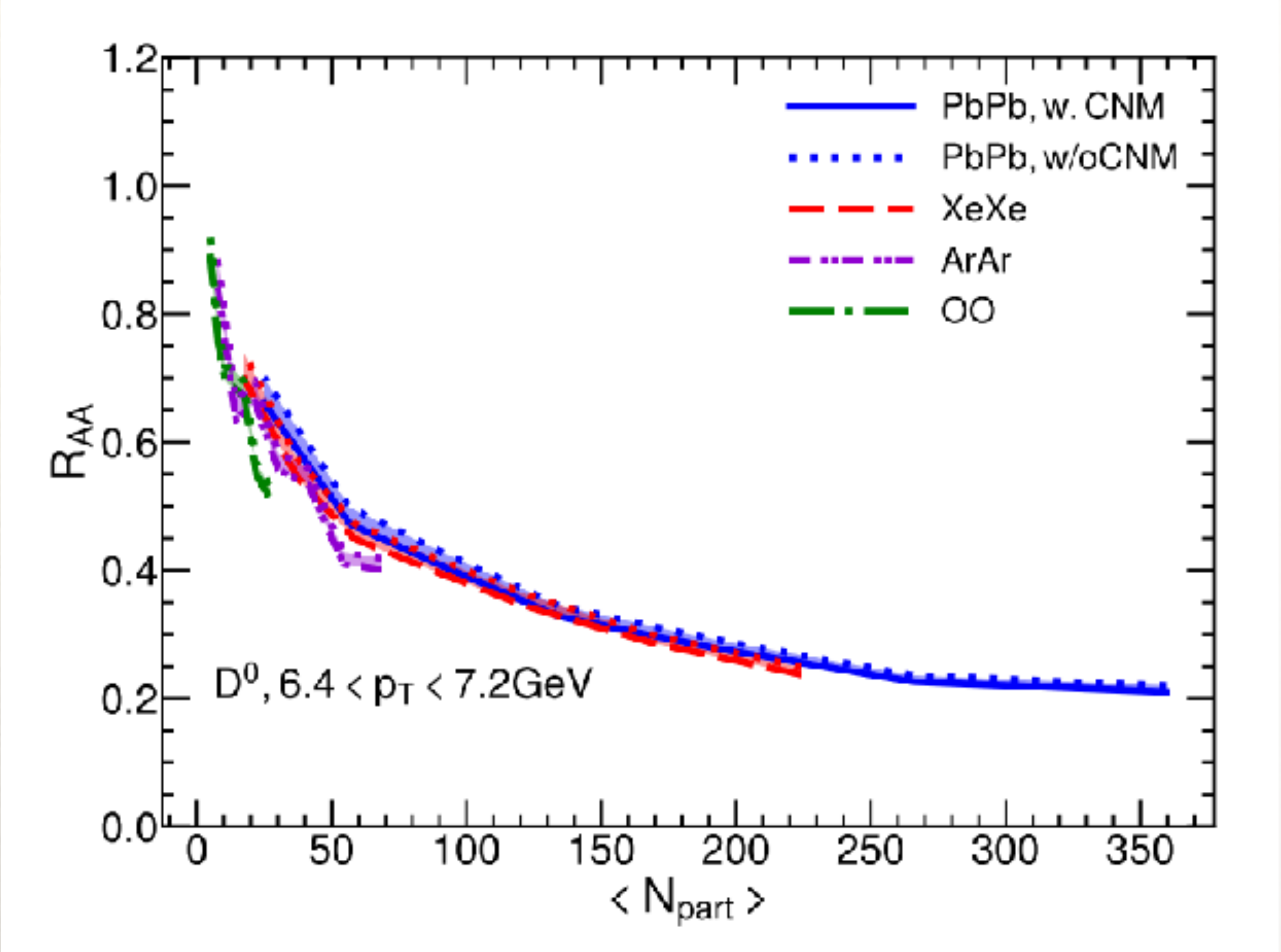
- Clear hierarchy of hadron *R_{AA}* with respect to the system size
- Significant hadron *R_{AA}* in the small O-O system, existence of QGP

Scaling of R_{AA} with respect to N_{part}

charged hadron



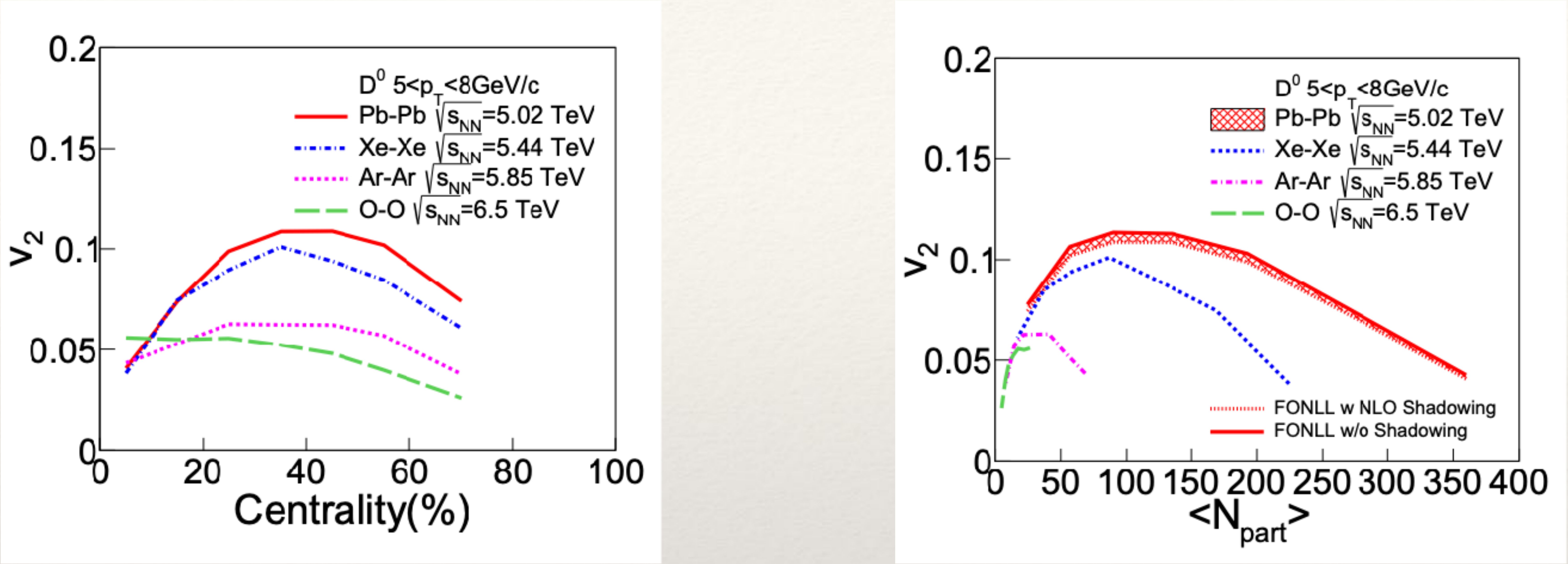
D meson



Liu, Xing, Wu, Qin, Cao, Xing, arXiv:2107.01522

- Scaling of the hadron R_{AA} with the system size (quantified by N_{part}) across different collision systems
- R_{pA} ~ 1 in proton-nucleus collisions is mainly due to the small size of the medium

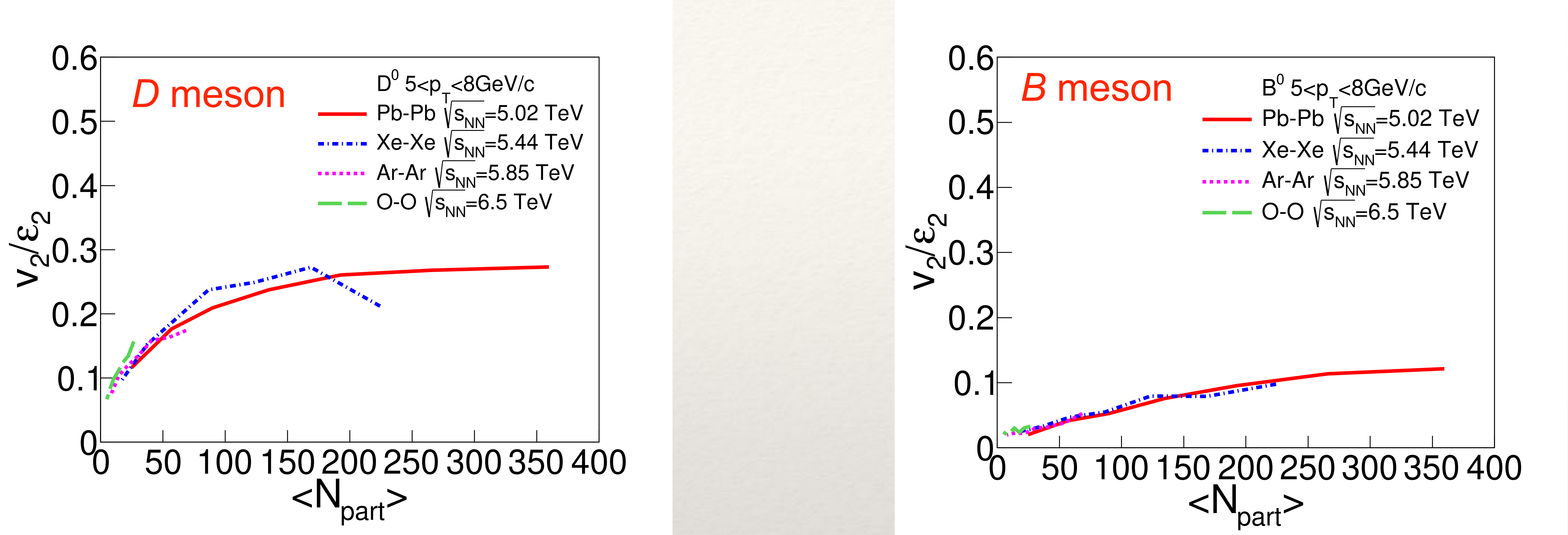
D meson v_2 in different systems



Li, Xing, Wu, Cao, Qin, arXiv:2108.06648

- Energy loss effect: for a given centrality, v_2 increases with the system size
- Geometry effect: for a given N_{part}, v_2 increases from O-O, Ar-Ar, Xe-Xe to Pb-Pb

Scaling of v_2/ε_2 with respect to N_{part}



Li, Xing, Wu, Cao, Qin, arXiv:2108.06648

- Separate energy loss and geometry effects by rescaling heavy quark v_2 with bulk ε_2
- v_2/ε_2 scales with the system size across different collision systems
- Search for the breaking of the scaling with future experiments — initial state effect overwhelms QGP effect

Summary

Jet-medium interaction at different p_T and in different collision systems

- pQCD is sufficient to describe flavor hierarchy of jet quenching above 8 GeV
- Coalescence + fragmentation hadronization is crucial for understanding hadron chemistry at medium p_T
- Color potential interaction significantly improves model calculation at low p_T
- Scaling behaviors of jet R_{AA} and v_2 across different collision systems may help distinguish initial state and QGP effects at different system size