

Inverse Problems – Methods Discussion

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Norwegian Particle, Astroparticle
& Cosmology Theory network

Three Strategies

Central goal: reconstruct spectral function encoded in simulation data

Strategy I

do not accept the premise. (smeared SPF or purely Euclidean quantity)

Challenges

Identify the physical problems that can benefit from knowledge of smeared SPF or avoid inverse problem all together.

Strategy II

accept premise and focus on information within the data.

Challenges

Need to find optimal bases for sparse modelling / correlation structure for GP.

What about non pos.?

Strategy III

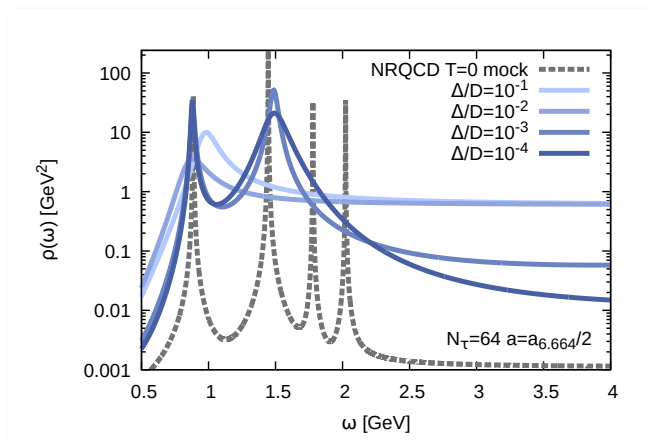
accept premise and attempt to supply as much extra info as possible.

Challenges

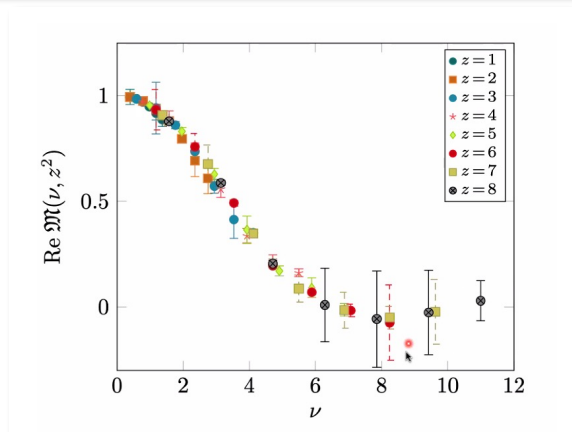
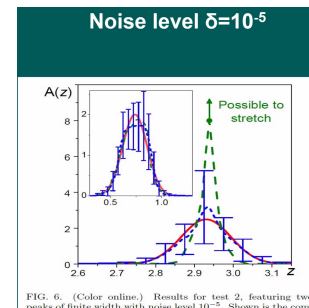
How can Bayesian methods encode more specific prior info? Beyond positivity and smoothness.

Challenge I: Information scarcity

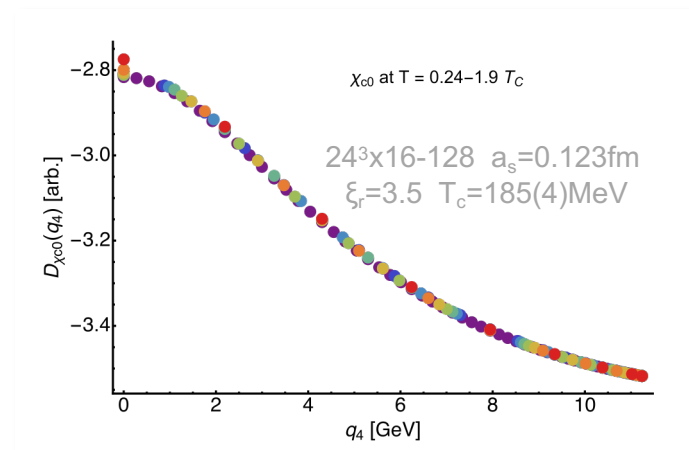
- Chances for improvement with current methods:



Resolving excited states from generic operators challenging.



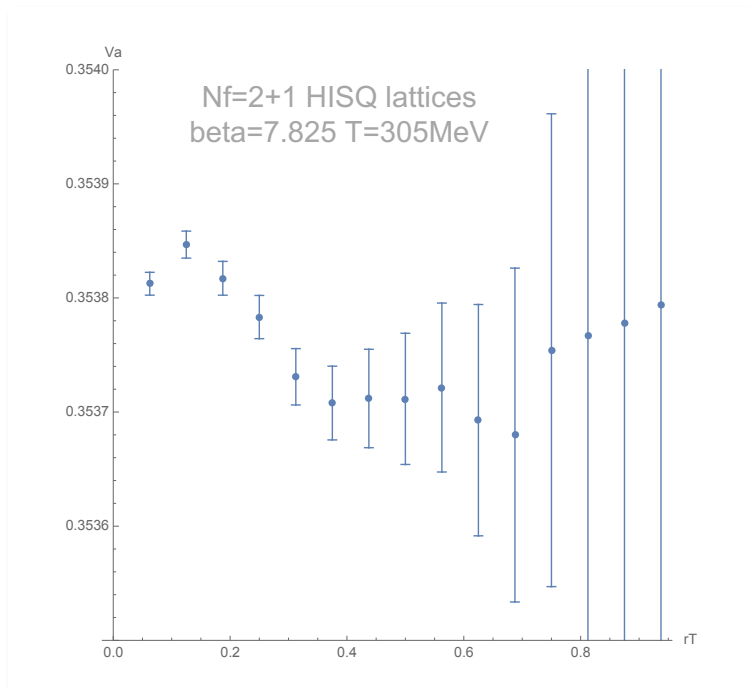
For loffe-time PDFs challenge to cover the whole Brillouin zone



Standard correlators encode thermal physics in essentially two datapoints.

Challenge II: Competing interests

- Improved lattice actions not designed with spectral reconstruction in mind



Precision data reveals presence of positivity violation

- Loss of positivity blunts the sharpest sword in the the arsenal
- Reflection positive actions needed
- Similar problem with Gradient flow, extended operator basis etc.

effective mass of Wilson line correlators

Challenge III: Specificity

- Most reconstruction methods deploy generic concepts of smoothness or resolution but no further analytic input from underlying field theory.
- How can we utilize machine learning to incorporate analytic insight, e.g. admissible pole structure of correlation functions etc. (c.f. manual choice of basis function)
- Can we identify complementary observables to constrain target quantities. At $T=0$ the extended operator bases play such a role. What about $T>0$?

Challenge IV: Comparability

- Some reconstruction methods can be recast e.g. in a Bayesian statistical language: MEM, BR, SAI, SOR, Tikhonov, (can test dependence on input data and regulator)
- Backus-Gilbert offers direct control over resolution scale. Similar concept can be implemented in Bayesian methods by a choice of basis (e.g. Fourier basis).
- Prior information in machine learning often resides in the training data. How can we quantify systematic uncertainty in this framework?