Vector spectral functions of the quark-gluon plasma

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Thermal QCD vector spectral functions

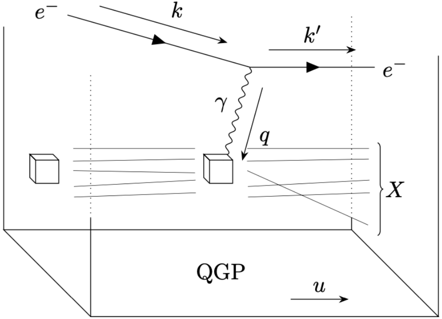
$$\rho^{\mu\nu}(q) = \int d^4x\, e^{iq\cdot x}\, \frac{1}{Z}\sum_n e^{-E_n/T}\langle n|[j^\mu(x), j^\nu(0)]|n\rangle$$

- Two kinematic variables: 'energy' $\omega \equiv q^0$ and virtuality q^2.
- Vanishing virtuality: $\rho^{\mu\nu}$ determines the photon emissivity of the thermal medium.
- Timelike virtuality $q^2 > 0$: dilepton emission rate. [McLerran & Toimela 1985]
- Spacelike virtuality $q^2 < 0$: vector spectral functions measure the ability of the medium to convert the energy stored in external electromagnetic fields into heat:
  - Couple the plasma to a harmonic external vector potential $\boldsymbol{A}(t,\boldsymbol{x}) = \mathrm{Re}(\boldsymbol{A_q} e^{i(\boldsymbol{q}\cdot\boldsymbol{x}-\omega t)})$, via Hamiltonian $\Delta H = -e\int d^3x\, \boldsymbol{j}\cdot\boldsymbol{A}$.
  - If energy of external electromagnetic fields: $E_{\rm e.m.} = \frac{1}{2}\int d^3x\,(\boldsymbol{E}^2 + \boldsymbol{B}^2)$,

$$\boldsymbol{A_q} \perp \boldsymbol{q} \;:\qquad \frac{-1}{E_{\rm e.m.}}\frac{dE_{\rm e.m.}}{dt} = \alpha\frac{2\pi\omega\,(\delta^{ij} - \hat{q}^i\hat{q}^j)\rho^{ij}(\omega,\boldsymbol{q})}{\omega^2 + \boldsymbol{q}^2},$$

$$\boldsymbol{A_q} \parallel \boldsymbol{q} \;:\qquad \frac{-1}{E_{\rm e.m.}}\frac{dE_{\rm e.m.}}{dt} = \alpha\frac{4\pi\hat{q}^i\hat{q}^j\,\rho^{ij}(\omega,\boldsymbol{q})}{\omega}$$

Spacelike virtuality: possible source of external e.m. fields



Lepton scattering on quark-gluon plasma

$$\frac{d^2\sigma}{dE' d\Omega} = \frac{e^4\, L^3}{8\pi^2 Q^4}\left(\frac{E'}{E}\right) \ell_{\mu\nu} W_>^{\mu\nu}(u,q),$$

$\ell_{\mu\nu}$ = leptonic tensor, $W_>^{\mu\nu}$ = ‘hadronic’ tensor.
2012.007522 (JHEP) Cè, Harris, HM, Toniato

Structure functions of the medium

Hadronic tensor:

$$W_>^{\mu\nu}(u,q) = \frac{1}{4\pi\, Z} \sum_n e^{-\beta E_n} \int d^4x\; e^{iq\cdot x}\, \langle n| j^\mu(x)\, j^\nu(0) |n\rangle,$$

Kubo–Martin-Schwinger: relation to spectral function in the rest frame of the medium,

$$W_>^{\mu\nu}(u,q) = \frac{1}{4\pi(1-e^{-\beta q^0})}\, \rho^{\mu\nu}(q^0,\boldsymbol{q}).$$

Tensor decomposition suitable for studying DIS limit: $u =$ medium 4-velocity

$$\begin{aligned} W_>^{\mu\nu}(u,q) &= F_1(u\cdot q, Q^2)\Big(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big) \\ &\quad + \frac{T}{u\cdot q} F_2(u\cdot q, Q^2)\Big(u^\mu - (u\cdot q)\frac{q^\mu}{q^2}\Big)\Big(u^\nu - (u\cdot q)\frac{q^\nu}{q^2}\Big). \end{aligned}$$

F_1, $F_2 =$ structure functions of the medium.

Deep-inelastic scattering limit

$Q^2 \to \infty$ at fixed $x = \frac{Q^2}{2Tq^0}$

"There is a large cancellation in calculating Q^2 as the difference $\boldsymbol{q}^2 - (q^0)^2$, but Q^2 is still large compared to T^2 ".

NB. $0 < x < \infty$, unlike in DIS on the proton, where $0 < x < 1$.

Partonic interpretation of the structure functions

Let $f_f(\xi)\, d\xi$ represent the number of partons of type f in the fluid cell carrying momentum $\xi\, T\, u$. This number is of order volume, L^3.

DIS limit of the structure functions:

$$F_1(u \cdot q, Q^2) = \frac{1}{4L^3T} \sum_f \mathcal{Q}_f^2\ f_f(x).$$

"$4\, F_1 \cdot dx$ is the square-electric-charge weighted number of partons carrying a momentum xT times the fluid four-velocity u^μ per unit transverse area in a slab of fluid which in its rest frame has thickness $1/T$ in the longitudinal direction."

Illustration: non-interacting plasma of quarks

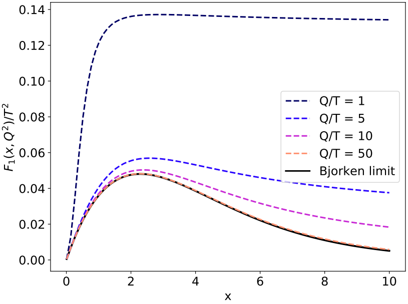
Explicit calculation [based on Laine 1310.0164]:

$$\lim_{Q^2\to\infty} F_1(x,Q^2) = \frac{(\sum_f \mathcal{Q}_f^2) N_c T^2}{4\pi^2} x \log(1+e^{-x/2}).$$

Also: the Callan-Gross relation holds,

$$\lim_{Q^2\to\infty} (F_2 - 2xF_1) = 0.$$

i.e. the scattering occurs on spin 1/2 constituents of the plasma.



Moment sum rules

Using the twist expansion:

$$\int_0^\infty dx\, x^{n-1} [F_1(x, Q^2)]_{\text{leading-twist}} = \sum_{f,j} \frac{\mathcal{Q}_f^2}{2} M_{fj}(Q, \tilde{\mu}) \langle O_{nj} \rangle, \quad n = 2, 4, \ldots,$$

where

$$\langle O_{nj}^{\mu_1 \ldots \mu_n} \rangle = T^n [u^{\mu_1} \ldots u^{\mu_n} - \text{traces}] \langle O_{nj} \rangle \, .$$

and the $O_{nj}^{\mu_1 \ldots \mu_n}$ are the usual twist-two operators of ordinary DIS (j runs over the quark flavours and the gluon operator).

NB. For n not too large, the right-hand side is computable on the lattice.

Probing the structure functions with Euclidean correlation functions

Focus here on the transverse channel:

$$H_E^T(\omega_n;Q^2) \quad = \quad \int d^4x\, e^{\sqrt{\omega_n^2-Q^2}\hat{\boldsymbol{q}}\cdot\boldsymbol{x}+i\omega_n x_0} \left(\tfrac{1}{2}\left(\delta_{ik}-\hat{q}_i\hat{q}_k\right)\langle j_i(x)\,j_k(0)\rangle\right).$$

Note the 'imaginary spatial momentum' required to reach the DIS regime.

Dispersive representation:

$$H_E^T(\omega_n;Q^2) - H_E^T(\omega_r;Q^2) = \int_0^\infty \frac{dx}{\pi}\, x\,\hat{\sigma}^T(x,Q^2)\frac{a_r^2-a_n^2}{(1+a_n^2x^2)(1+a_r^2x^2)}\,,$$

where $a_n = 2T\omega_n/Q^2$ are to be kept fixed and
$\hat{\sigma}^T(x,Q^2) = -4\pi(1-e^{-\beta q^0})F_1(q^0,Q^2)$.

- Then the only Q^2 dependence comes from $\hat{\sigma}^T(x,Q^2)$.
- The approach to the Bjorken limit and scaling violations can in principle be studied (tough project).
- Similar to nucleon forward Compton amplitude [Ji,Jung hep-lat/0101014], but the imaginary momentum is injected in spatial, not temporal component.

Dispersion relation for Euclidean correlator at zero virtuality

- Let $\sigma(\omega) \equiv -\hat{\sigma}^T(\omega, Q^2 = 0) \geq 0$ be the relevant spectral function proportional to the photon emission rate ($\omega \equiv q^0$);

- let $H_E(\omega_n) \equiv G_E(\omega_n, k = i\omega_n)$ the momentum-space Euclidean correlator with Matsubara frequency ω_n and imaginary spatial momentum $k = i\omega_n$;

- once-subtracted dispersion relation: ($\sigma(\omega) \sim \omega^{1/2}$ at weak coupling*)

$$H_E(\omega_n) - H_E(\omega_r) = \int_0^\infty \frac{d\omega}{\pi}\, \omega\, \sigma(\omega) \Big[\frac{1}{\omega^2 + \omega_n^2} - \frac{1}{\omega^2 + \omega_r^2}\Big], \quad n, r \neq 0.$$

Photon emission rate per unit volume plasma:

$$\frac{d\Gamma_\gamma}{d\omega} = \frac{\alpha}{\pi}\, \frac{1}{e^{\beta\omega} - 1} \cdot (\omega\, \sigma(\omega)).$$

Try compute power carried away by photons: $\int_0^\infty d\omega\, \omega \frac{d\Gamma_\gamma}{d\omega}$ (cf. Hashimoto (Tue)).

[HM, 1807.00781. (*) Caron-Huot et al, hep-th/0607237]

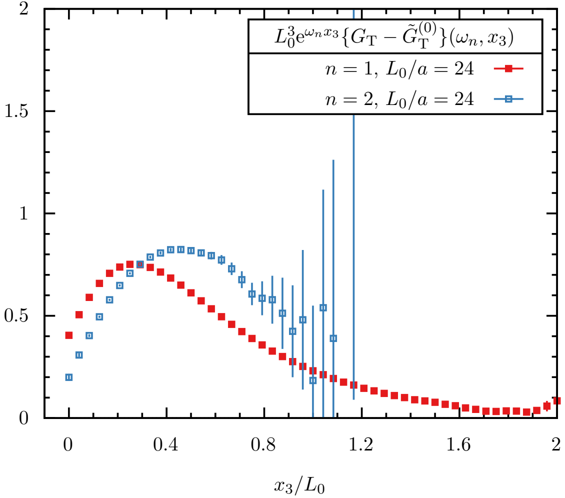
Computing $H_E(\omega_n)$ in lattice QCD

New since 1807.00781: Use the estimator

$$H_E(\omega_n) = \int_0^\beta dx_0 \int d^3x \, (e^{i\omega_n x_0} - e^{i\omega_n x_2}) \, e^{\omega_n x_3} \, \langle j_1(x) j_1(0) \rangle \, .$$

- The subtracted term ($e^{i\omega_n x_2}$) vanishes in the continuum theory, but on the lattice removes an ultraviolet divergence associated with the lack of Lorentz symmetry at finite lattice spacing.
- No cumbersome subtraction of vacuum correlators is needed!
- The above estimator of $H_E(\omega_n)$ automatically vanishes in the vacuum.

Integrand to obtain $H_E(\omega_n)$



Difficult observable, but worthwhile ...

Conclusion

- dispersion relations at fixed virtuality, rather than fixed spatial momentum, open up new perspectives on the thermal spectral functions
- certain moments of the spectrum of emitted photons can be computed in lattice QCD without solving an inverse problem
- in the same way, moments of the structure functions at fixed spacelike virtuality can be computed in lattice QCD.

Integrand for three discretizations of the correlator