Tackling The Real-Time Challenge In Strongly Correlated Systems: Spectral Properties From Euclidean Path Integrals
September 13-17, 2021

Parton Distribution Functions

from Euclidean Correlation functions

September 15, 2021

Kostas Orginos for the HadStruc collaboration

HadStruc

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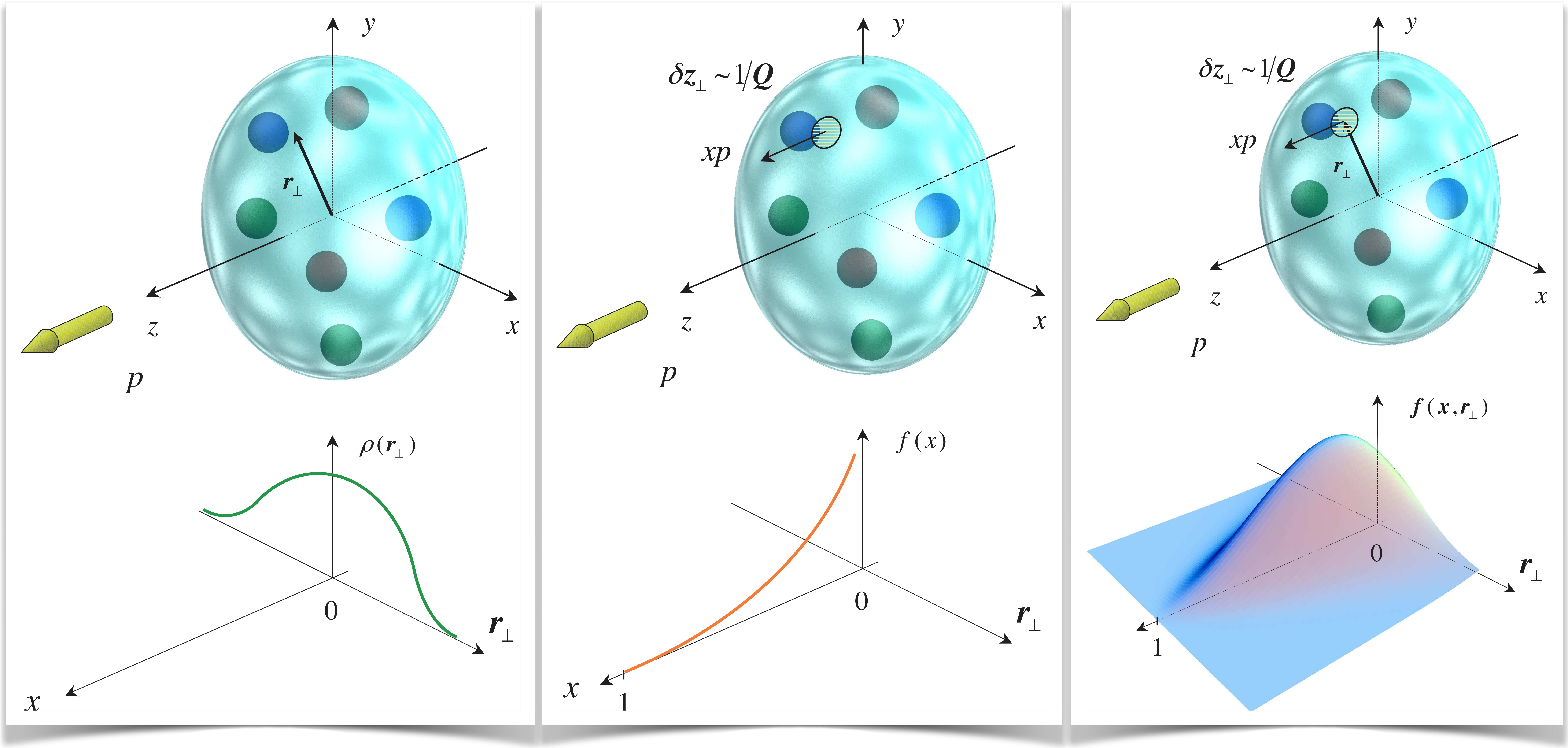
X. Ji, D. Muller, A. Radyushkin (1994-1997)

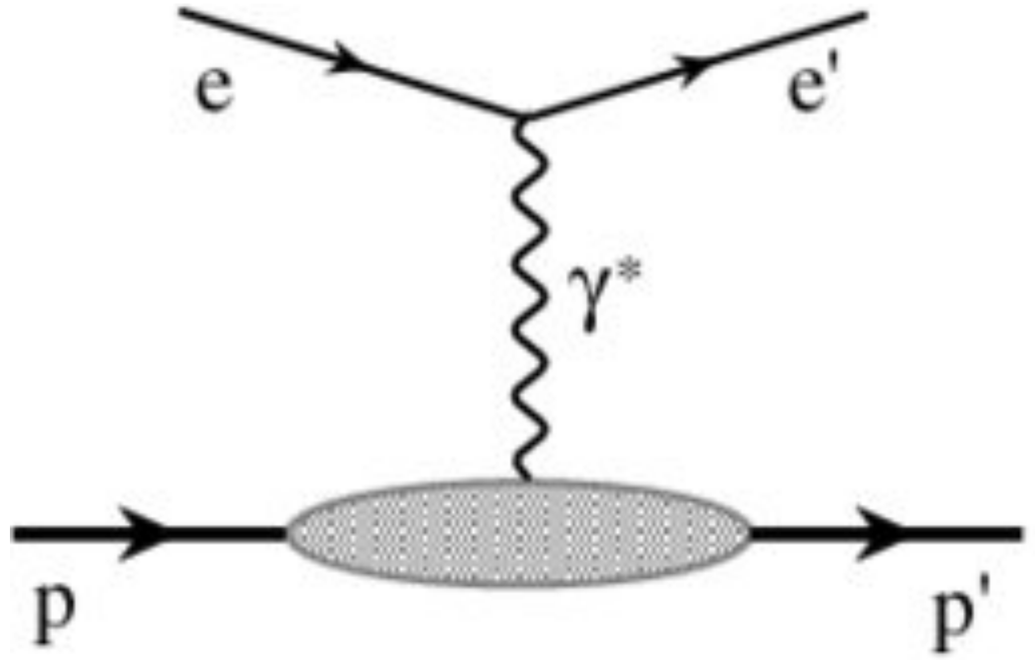


Form Factors

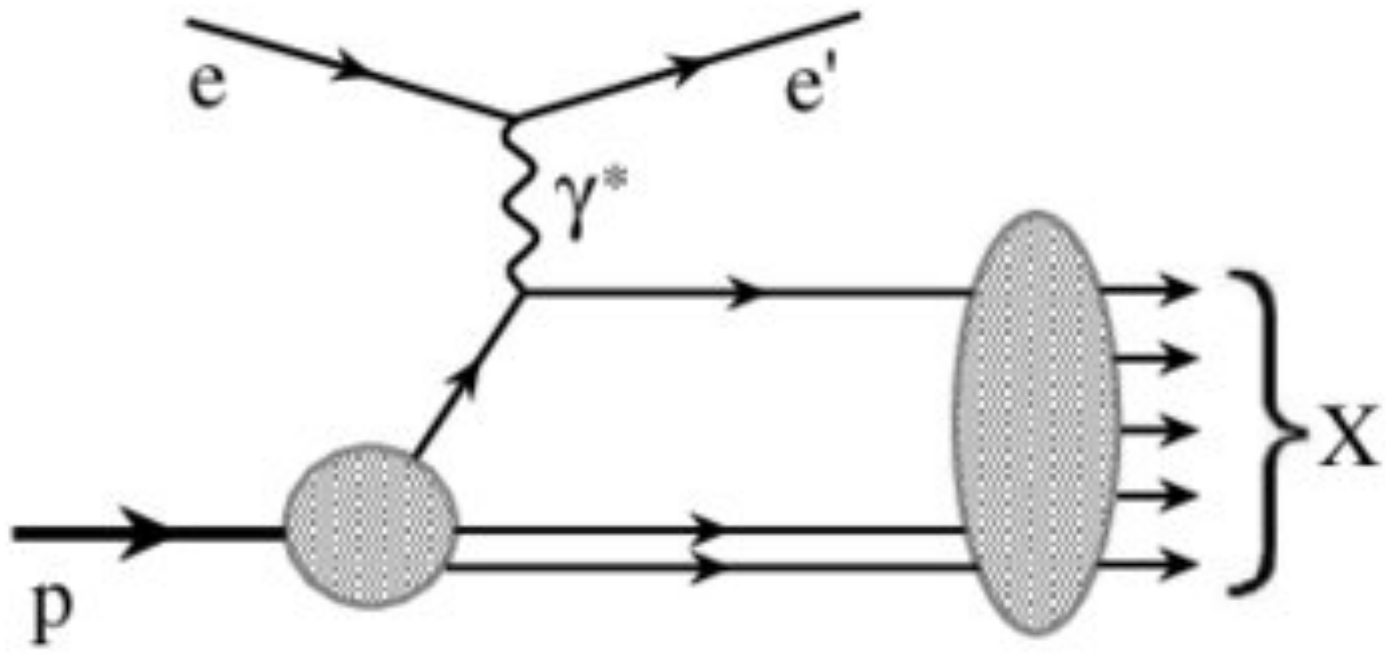
Parton Distribution functions

Generalized Parton Distribution functions

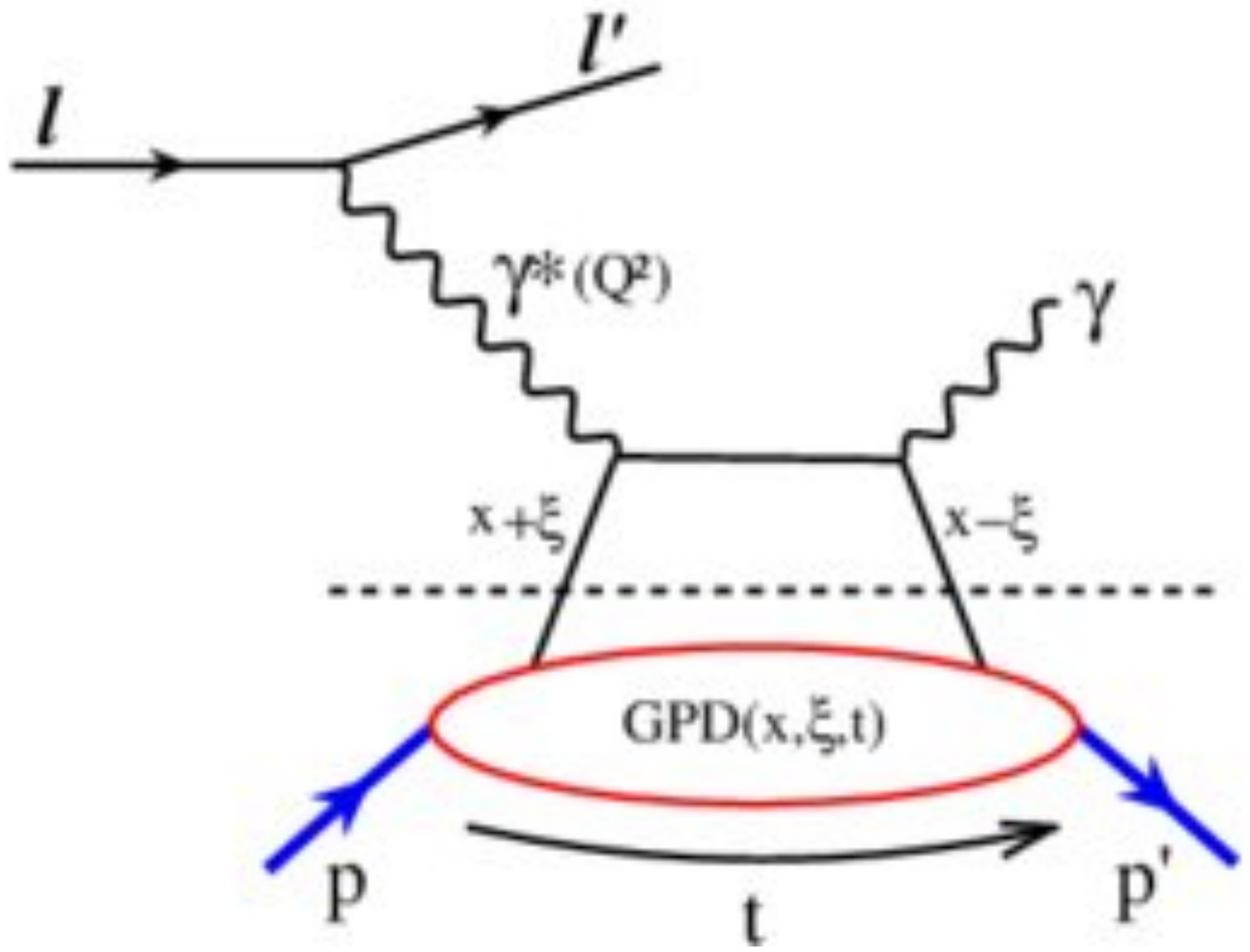
Factorization → non-perturbative structure

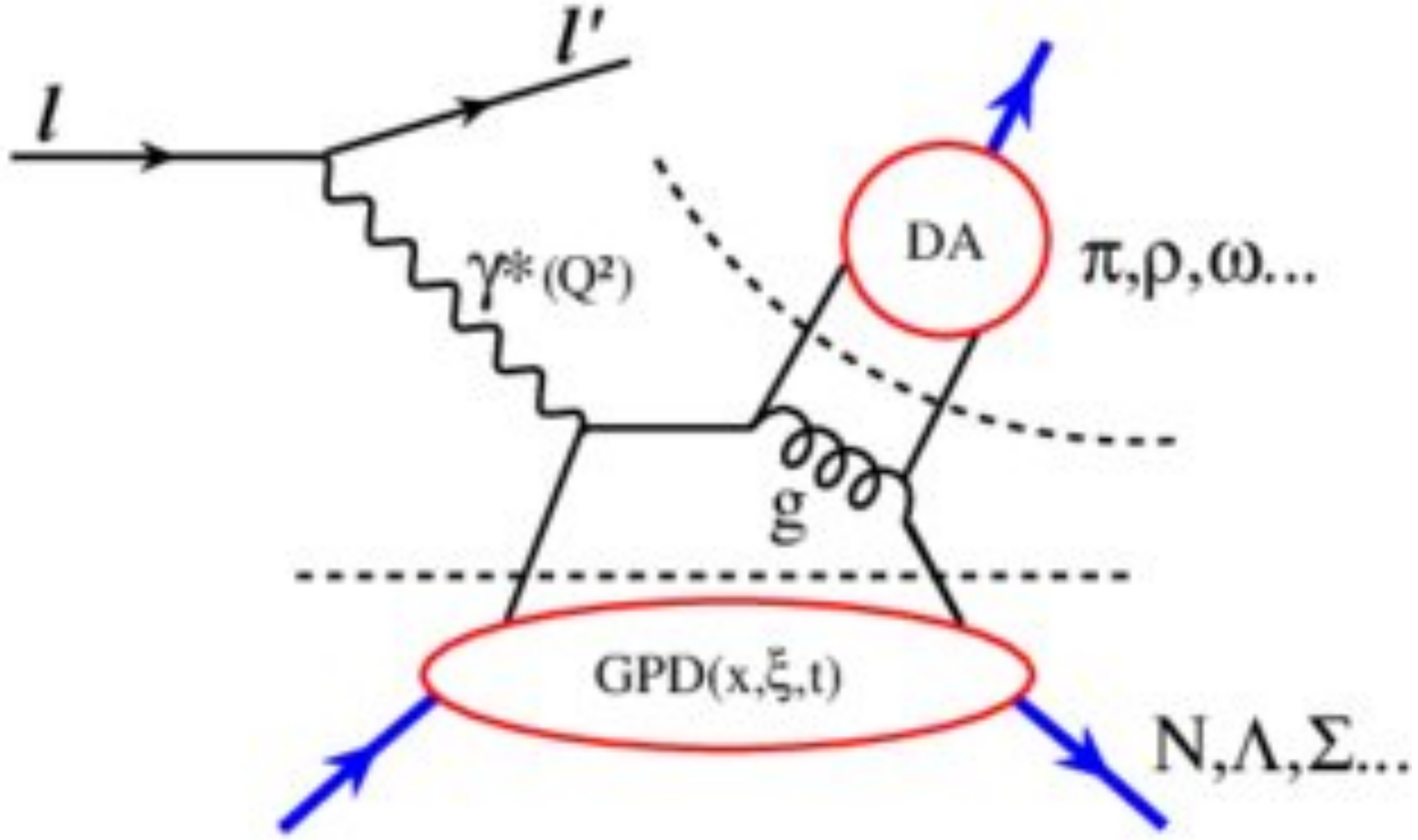


Elastic scattering: Form factor



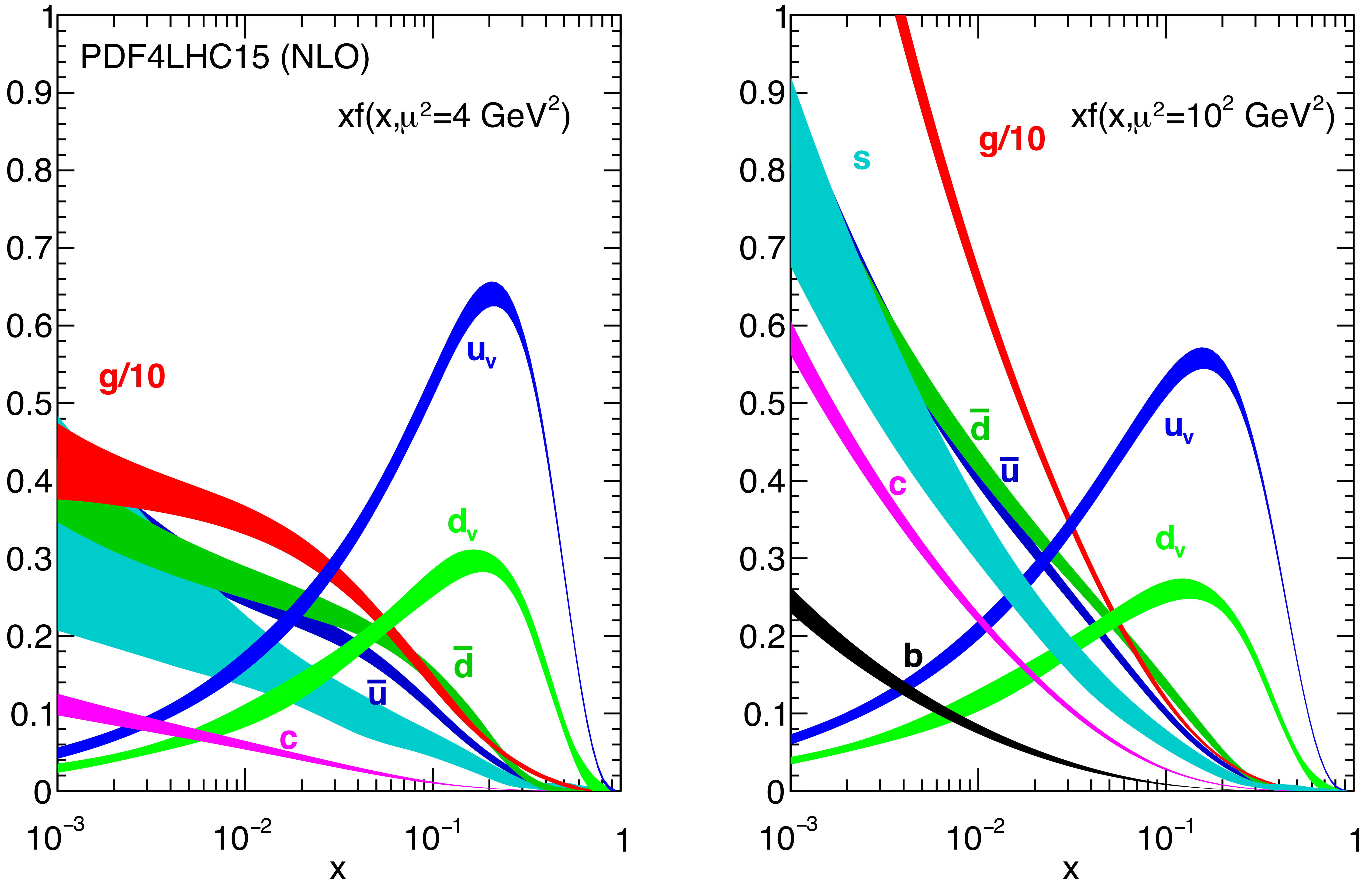
DIS: Parton distributions





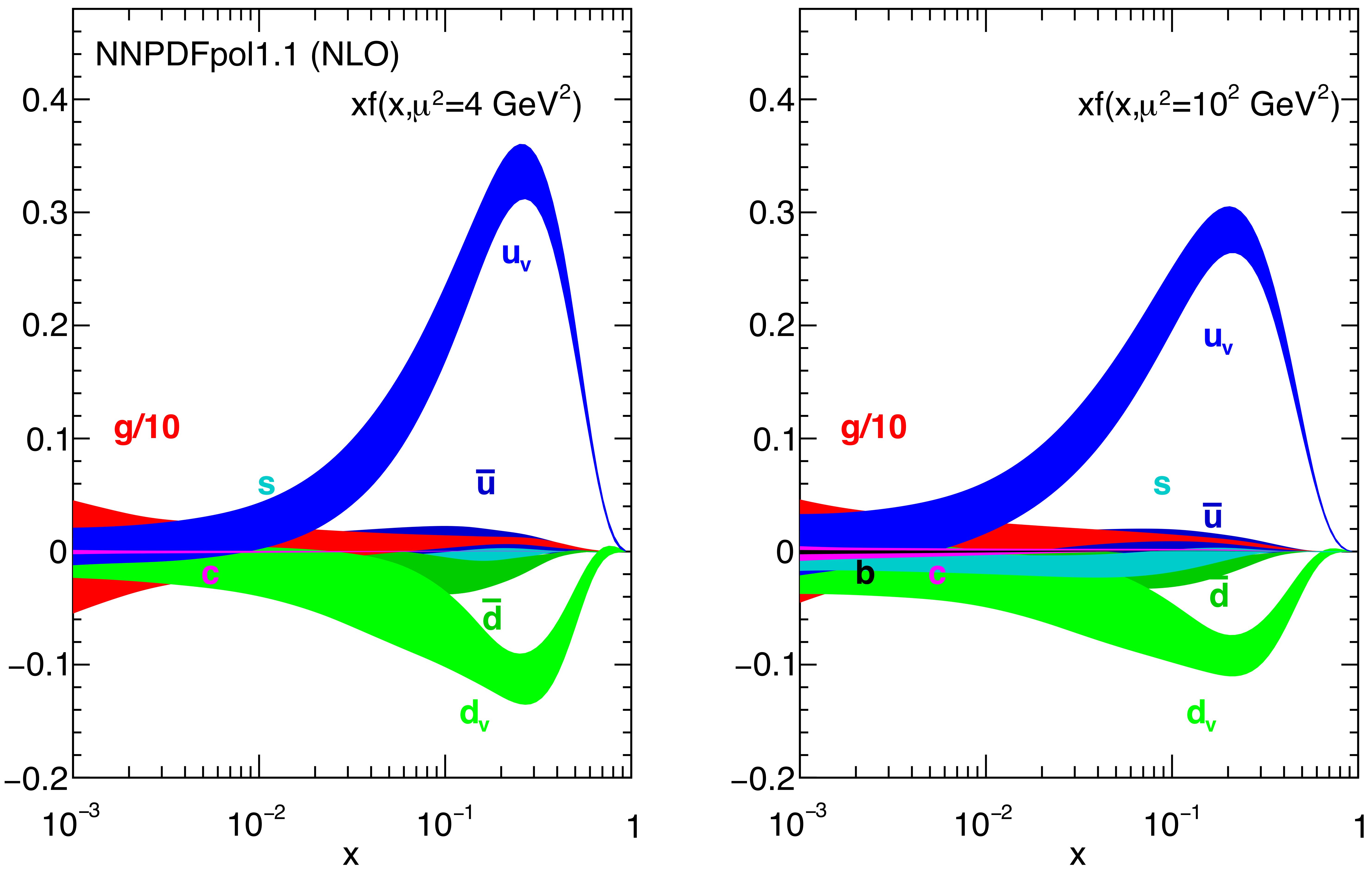
DVCS or DVMP: Generalized Parton distributions

Determination of Parton distribution functions from Experiment



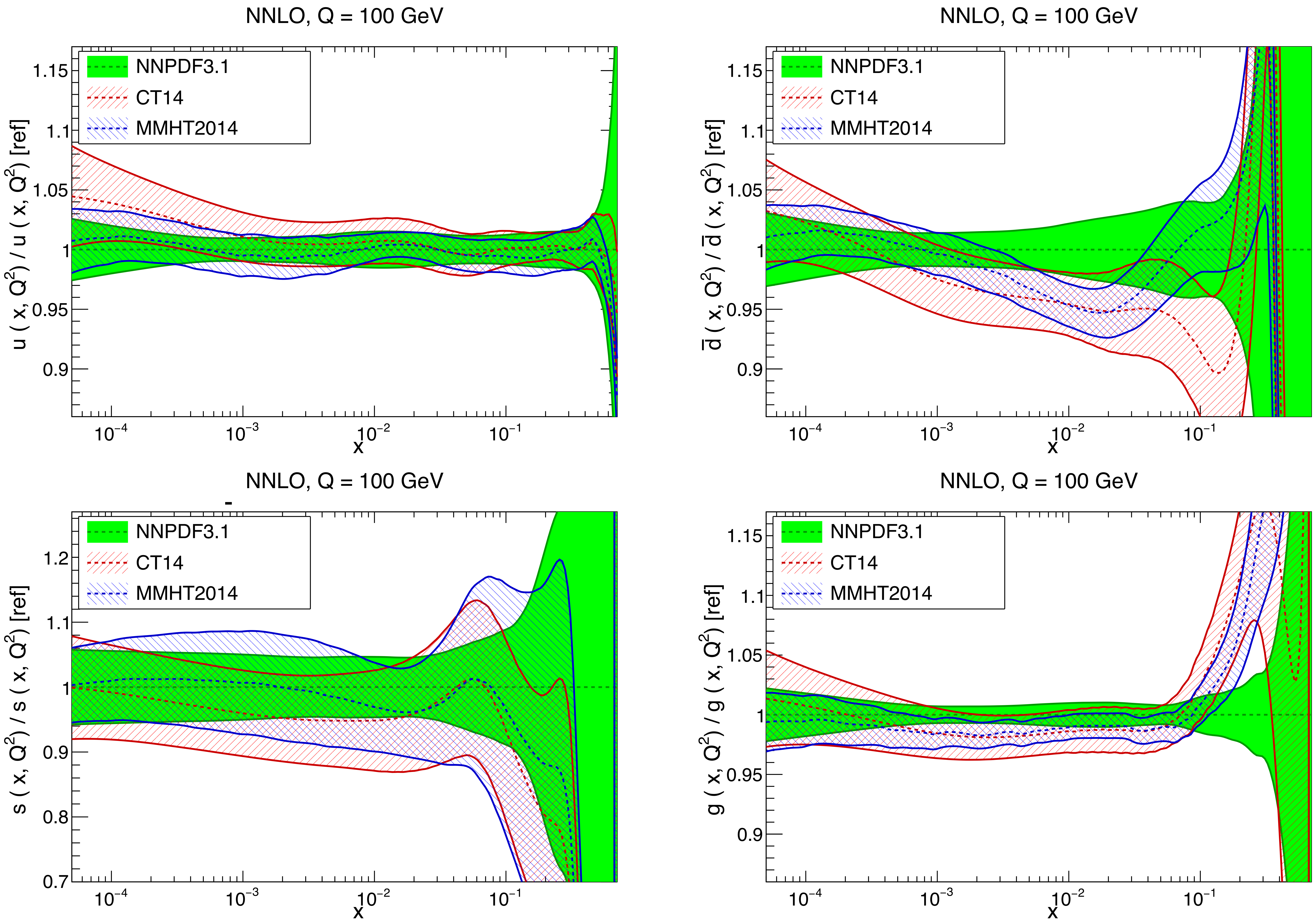
Fits to experimental data

Determination of Parton distribution functions from Experiment



Fits to experimental data

Determination of Parton distribution functions from Experiment

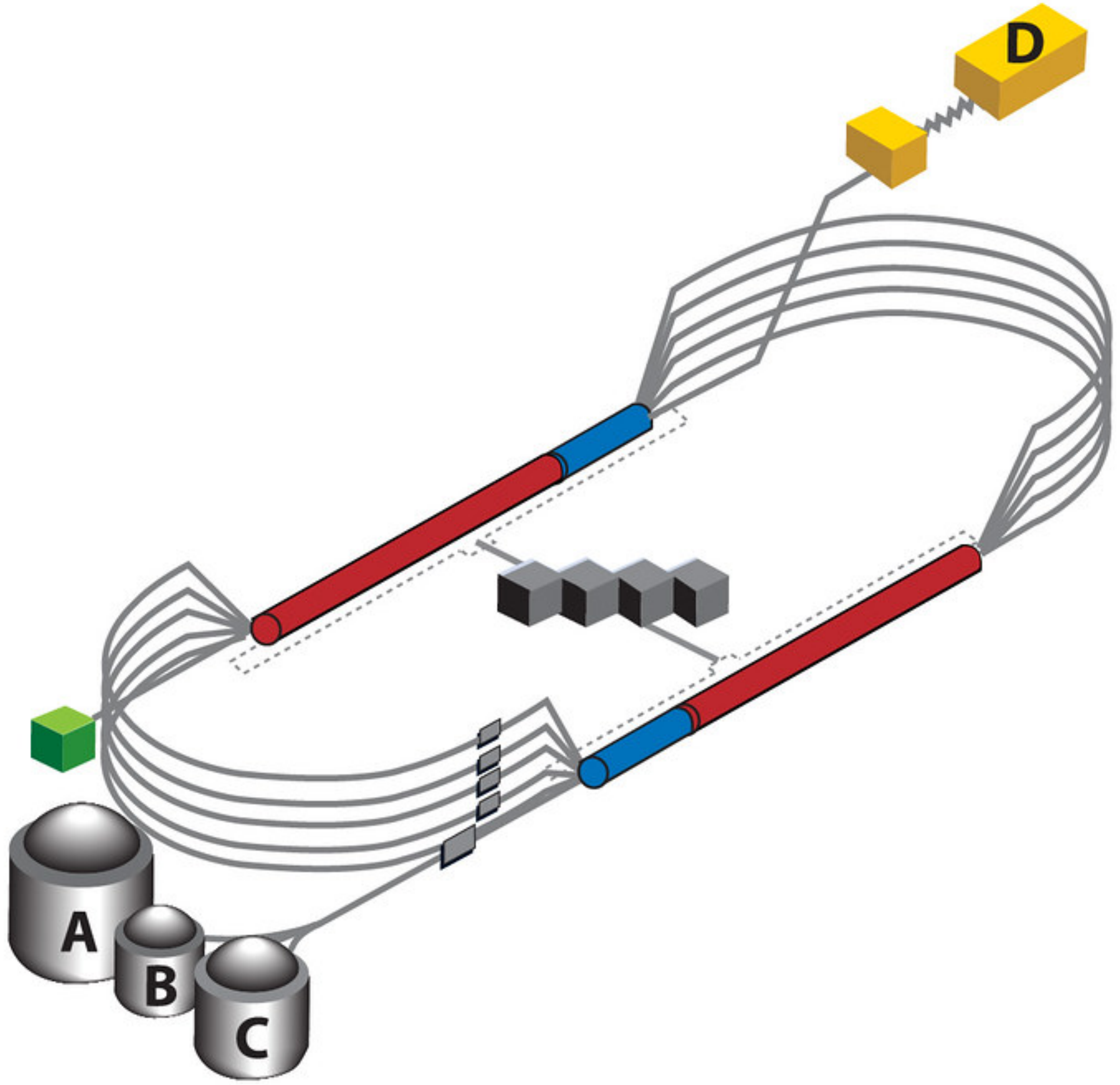


Parton distributions and lattice QCD calculations: a community white paper

arXiv:1711.07916

JLab 12 GeV

Generalized Parton Distributions



The Electron-Ion Collider

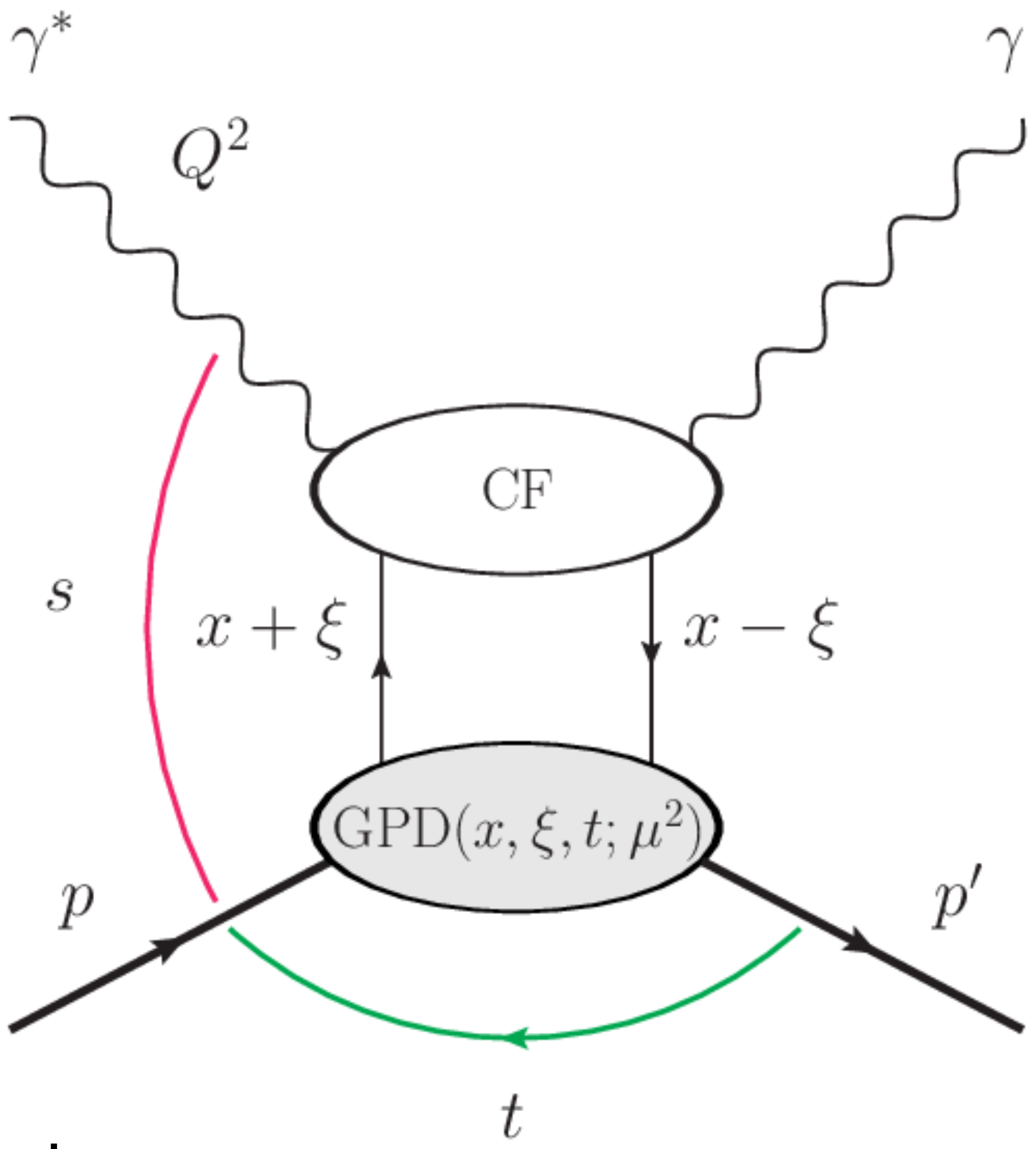
A machine that will unlock the secrets of the strongest force in Nature

The computers and smartphones we use every day depend on what we learned about the atom in the last century. All information technology—and much of our economy today—relies on understanding the electromagnetic force between the atomic nucleus and the electrons that orbit it. The science of that force is well understood but we still know little about the microcosm within the protons and neutrons that make up the atomic nucleus. That's why Brookhaven Lab is building a new machine—an Electron-Ion Collider, or EIC—to look *inside* the nucleus and its protons and neutrons.

The EIC will be a particle accelerator that collides electrons with protons and nuclei to produce snapshots of those particles' internal structure—like a CT scanner for atoms. The electron beam will reveal the arrangement of the quarks and gluons that make up the protons and neutrons of nuclei. The force that holds quarks together, carried by the gluons, is the strongest force in Nature. The EIC will allow us to study this "strong nuclear force" and the role of gluons in the matter within and all around us. What we learn from the EIC could power the technologies of tomorrow.

taken from https://www.bnl.gov/eic/

DVCS factorization



$$\mathcal{F}(\xi,t,Q^2) = \int_{-1}^{1} dx\, C(x,\xi,a_s(\mu),Q/\mu) G(x,\xi,t,\mu)$$

Ill-defined inverse problem —-> Lattice QCD computations are essential

Summary

- Recent developments have made it possible for lattice QCD to compute the x dependence of PDFs, GPDs, and TMDs
  - Nuclear Femtography
  - Jlab 12GeV program : Next 10 years or so
- Computational challenge
- Data analysis challenge
  - Solution of the inverse problem at hand
    - Lattice QCD computations are essential

CENTER for NUCLEAR FEMTOGRAPHY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

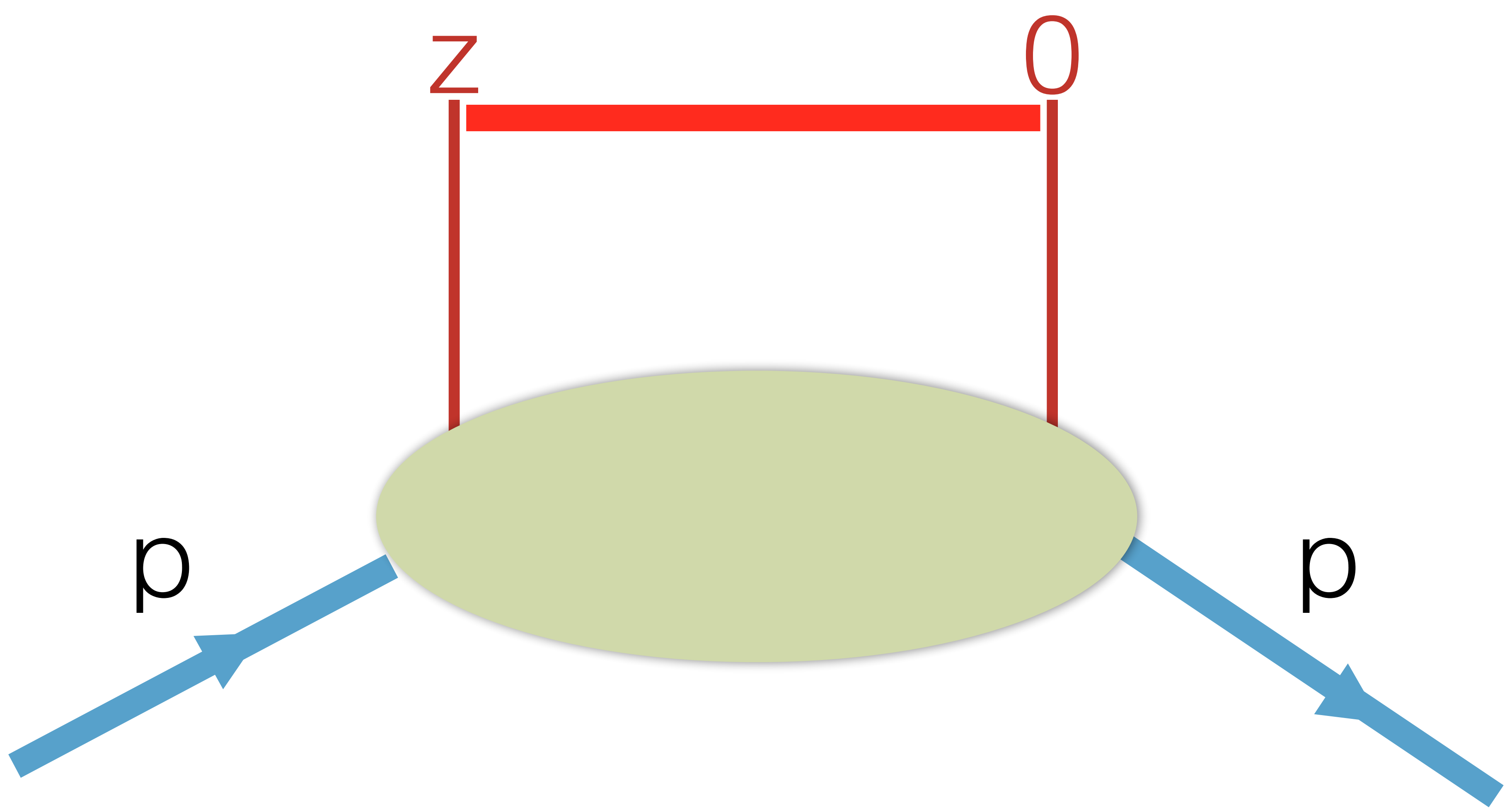
X. Ji, Phys.Rev.Lett. 110, (2013)
Y.-Q. Ma J.-W. Qiu (2014) arXiv:1404.6860
Y.-Q. Ma J.-W. Qiu (2017) arXiv:1709.3018
A. Radyushkin, Phys. Lett. B767 (2017)

Matrix Element Formulation

Unpolarized PDFs proton:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\hat{E}(0,z;A) = \mathcal{P}\exp\left[-ig\int_0^z \mathrm{d}z'_\mu\,A^\mu_\alpha(z')T_\alpha\right]$$



A. Radyushkin Phys.Lett. B767 (2017)

Lorentz decomposition:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha\mathcal{M}_p(-(zp),-z^2) + z^\alpha\mathcal{M}_z(-(zp),-z^2)$$

Collinear PDFs: Choose

$$z = (0, z_-, 0)$$

$$p = (p_+, 0, 0)$$

$$\gamma^+$$

$$\mathcal{M}^+(z,p) = 2p^+\mathcal{M}_p(-p_+z_-,0)$$

Ioffe time $-z\cdot p = \nu$

Definition of PDF:

$$\mathcal{M}_p(-p_+z_-,0) = \int_{-1}^{1} dx\, f(x)\, e^{-ixp_+z_-}$$

$$\mathcal{M}_p(\nu, z^2) = \int_0^1 d\alpha\, \mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\, \mathcal{Q}(\alpha\nu, \mu) \ + \mathcal{O}(z^2\Lambda^2_{qcd})$$

$\mathcal{Q}(\nu, \mu)$ is called the Ioffe time PDF

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

$$\mathcal{Q}(\nu, \mu) = \int_{-1}^1 dx\, e^{-ix\nu} f(x, \mu)$$

Matching to $\overline{MS}$

Factorization of collinear divergence at $-z^2 \to 0$

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Izubuchi et al. Phys.Rev. D98 (2018) no.5, 056004
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

Consider the ratio $$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$$

UV divergences will cancel in this ratio resulting a renormalization group invariant (RGI) function

The collinear divergences at $z_3^2 = 0$ limit only appear in the numerator

The lattice regulator can now be removed

$\mathfrak{M}^{cont}(\nu, z_3^2)$ Universal independent of the lattice

$\mathcal{M}_p(0, 0) = 1$ Isovector matrix element

Continuum limit matching to $\overline{MS}$ computed at 1-loop

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q_v(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k \,.$$

$$\mathcal{K}(x\nu, z^2\mu^2) = \cos(x\nu) - \frac{\alpha_s}{2\pi} C_F \left[\ln(e^{2\gamma_E+1} z^2\mu^2/4)\tilde{B}(x\nu) + \tilde{D}(x\nu)\right]$$

$$\tilde{B}(x) = \frac{1-\cos(x)}{x^2} + 2\sin(x)\frac{x\mathrm{Si}(x)-1}{x} + \frac{3-4\gamma_E}{2}\cos(x) + 2\cos(x)\left[\mathrm{Ci}(x) - \ln(x)\right]$$

$$\tilde{D}(x) = x\mathrm{Im}\left[e^{ix}{}_3F_3(111;222;-ix)\right] - \frac{2-(2+x^2)\cos(x)}{x^2}$$

Polynomial corrections to the Ioffe time PDF may be suppressed

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)
M. Anselmino et al. 10.1007/JHEP04(2014)005
A. Radyushkin Phys.Lett. B767 (2017)

However on the Lattice after expanding in lattice spacing we have

$$\mathfrak{M}(p,z,a) = \mathfrak{M}_{\text{cont}}(\nu, z^2) + \sum_{n=1} \left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\text{QCD}})^n R_n(\nu)\,.$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q_v(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k\,.$$

Ioffe time $\quad -z\cdot p = \nu$

- All coefficient functions respect continuum symmetries
- Lattice spacing corrections to higher twist effects are ignored
- On dimensional ground a/z terms must exist
- Additional O(a) effects (last term)

The inverse problem to solve: Obtain q(x,μ) from the lattice matrix elements

see dicussion in J. Karpie *et al JHEP* 04 (2019) 057 and L. DelDebio *et al JHEP* 02 (2021) 138

Exploration of various methods for LO matching

Exploration of the NNPDF approach applied to lattice data

Our inverse problem

$$\mathfrak{M}(p, z, a) = \mathfrak{M}_{\text{cont}}(\nu, z^2) + \sum_{n=1} \left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\text{QCD}})^n R_n(\nu)\,.$$

$$\text{Re}\,\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}_R(x\nu, \mu^2 z^2) q_-(x, \mu^2) + \mathcal{O}(z^2)$$

$$\text{Im}\,\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}_I(x\nu, \mu^2 z^2) q_+(x, \mu^2) + \mathcal{O}(z^2)\,,$$

- Obtain the PDF from a limited set of matrix elements obtained from lattice QCD
- z^2 is a physical length scale sampled on discrete values
- z^2 needs to be sufficiently small so that higher twist effects are under control
- ν is dimensionless also sampled in discrete values
- the range of ν is dictated by the range of z and the range of momenta available and is typically limited
- Parametrization of unknown functions

Sample data

arXiv:2105.13313 **[hep-lat]** **J. Karpie** *et. al.*

ID	a(fm)	M_π(MeV)	β	c_{SW}	κ	$L^3 \times T$	N_{cfg}
$\widetilde{A}5$	0.0749(8)	446(1)	5.2	2.01715	0.13585	$32^3 \times 64$	1904
E5	0.0652(6)	440(5)	5.3	1.90952	0.13625	$32^3 \times 64$	999
N5	0.0483(4)	443(4)	5.5	1.75150	0.13660	$48^3 \times 96$	477

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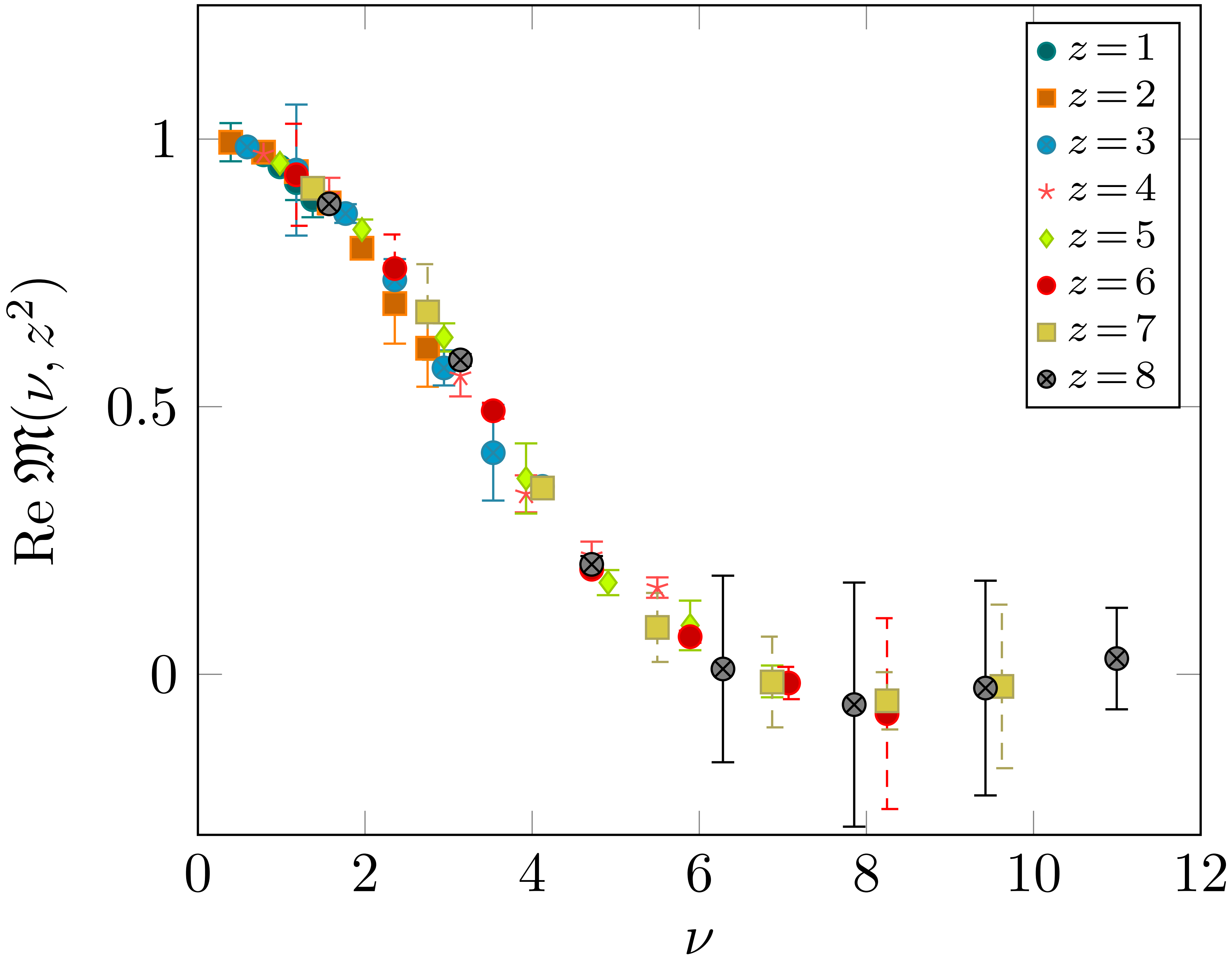
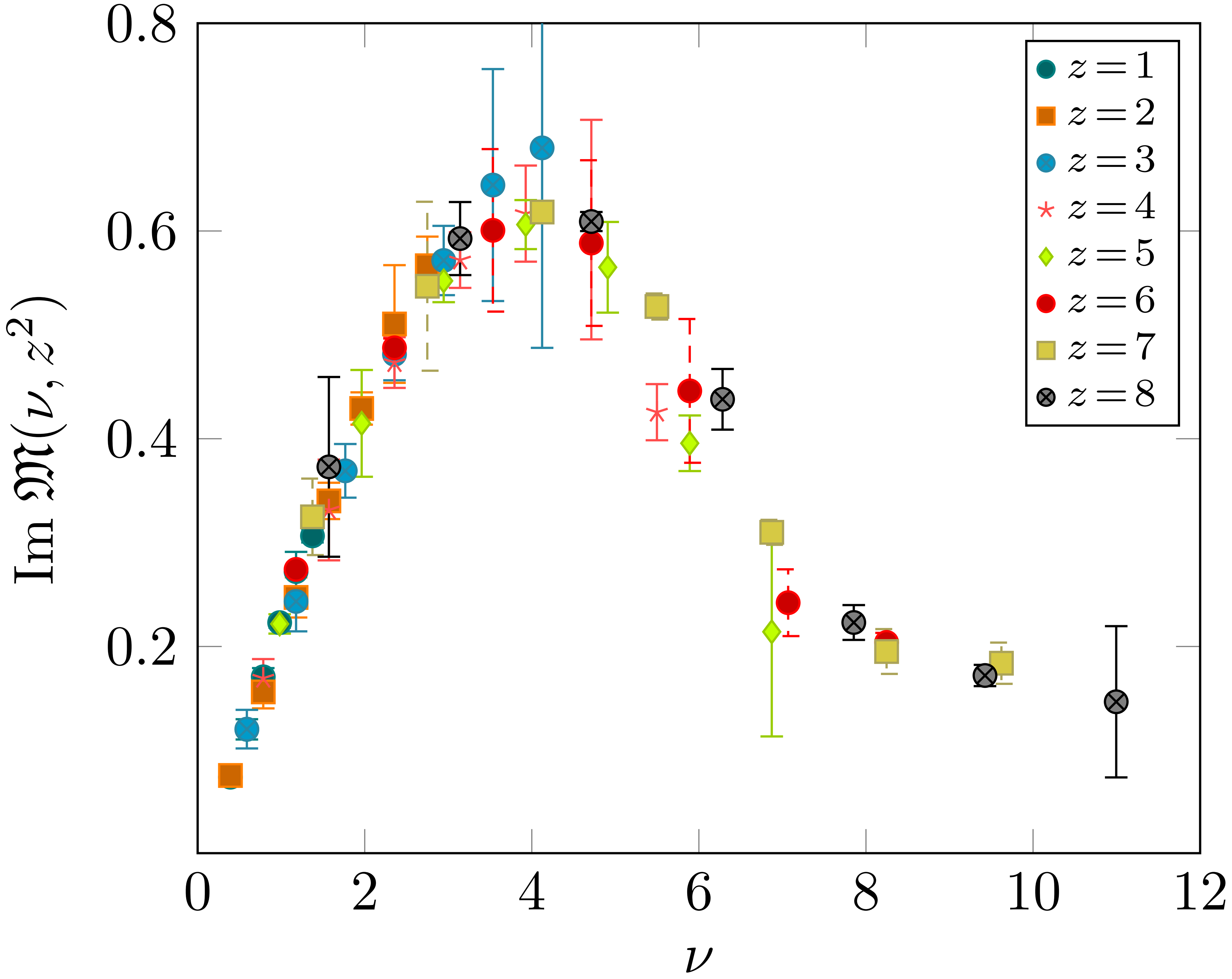
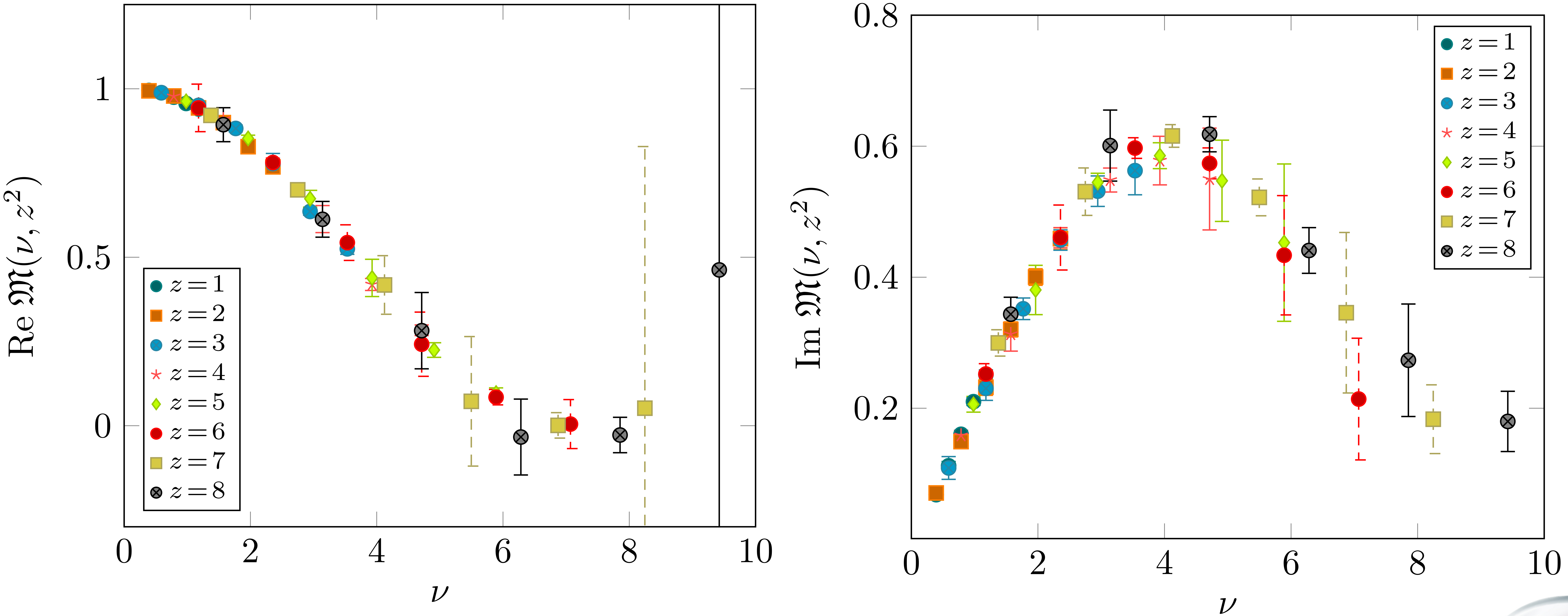




a= 0.075fm

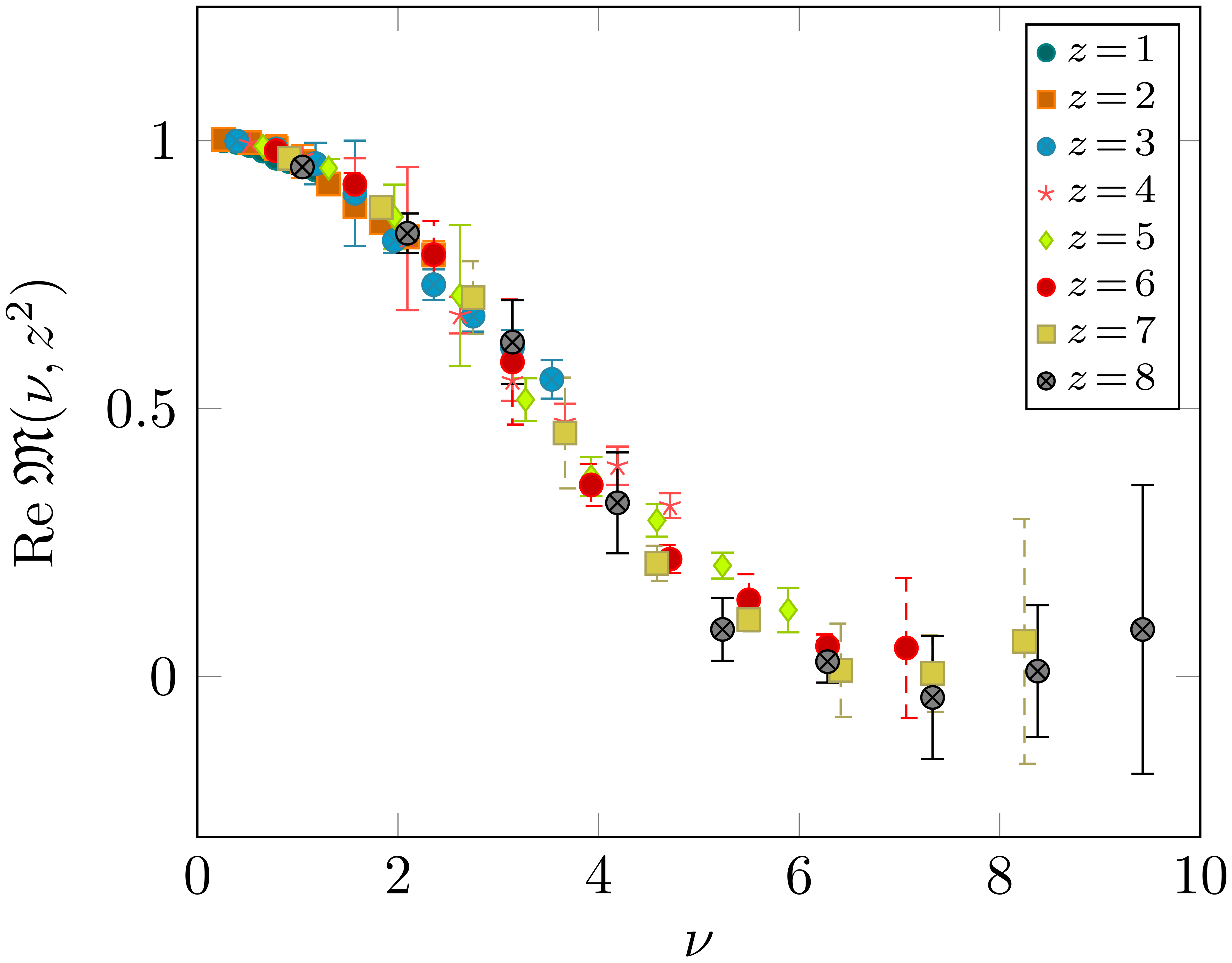
arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

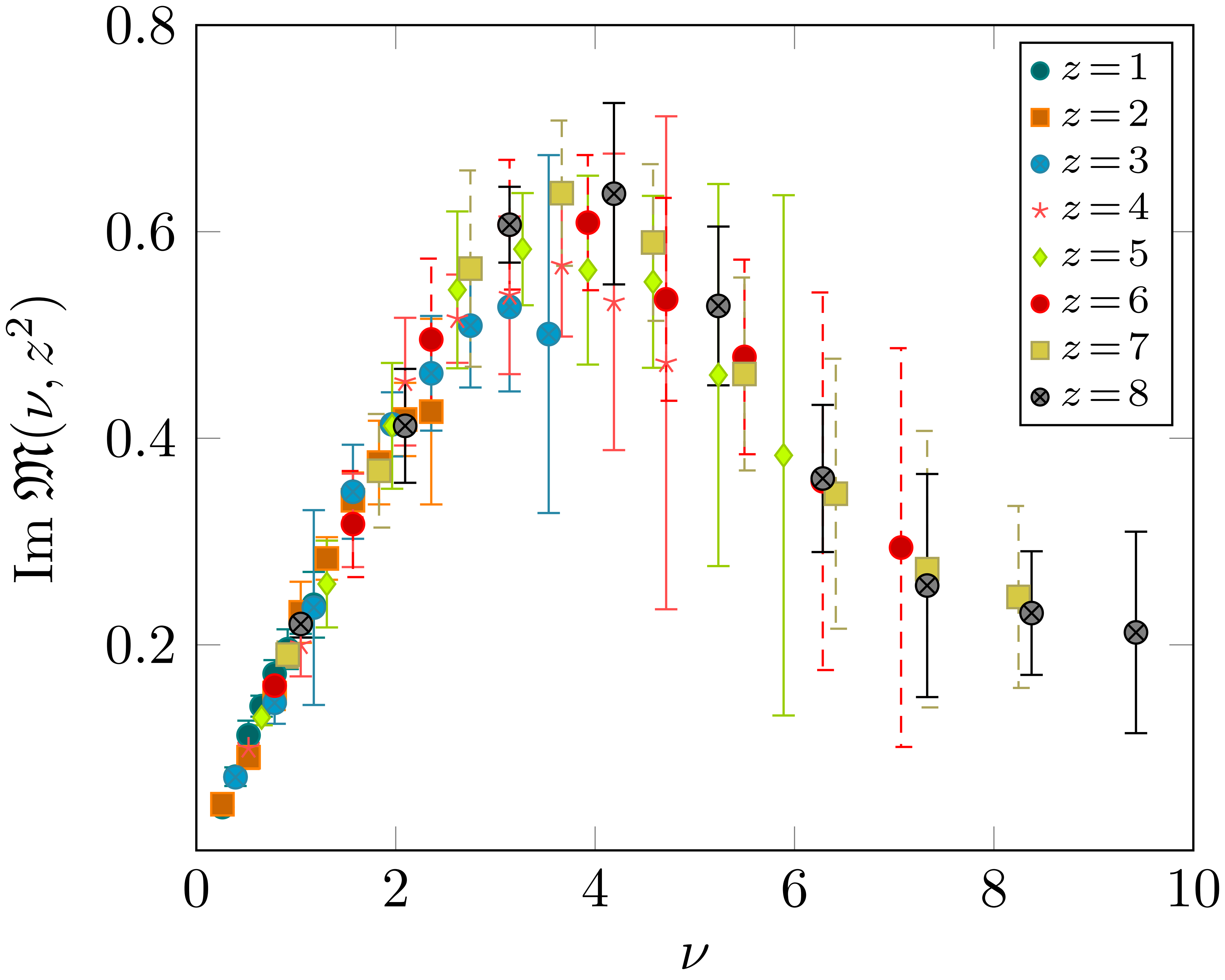
HadStruc



a= 0.065fm

arXiv:2105.13313 [hep-lat] **J. Karpie** *et. al.*



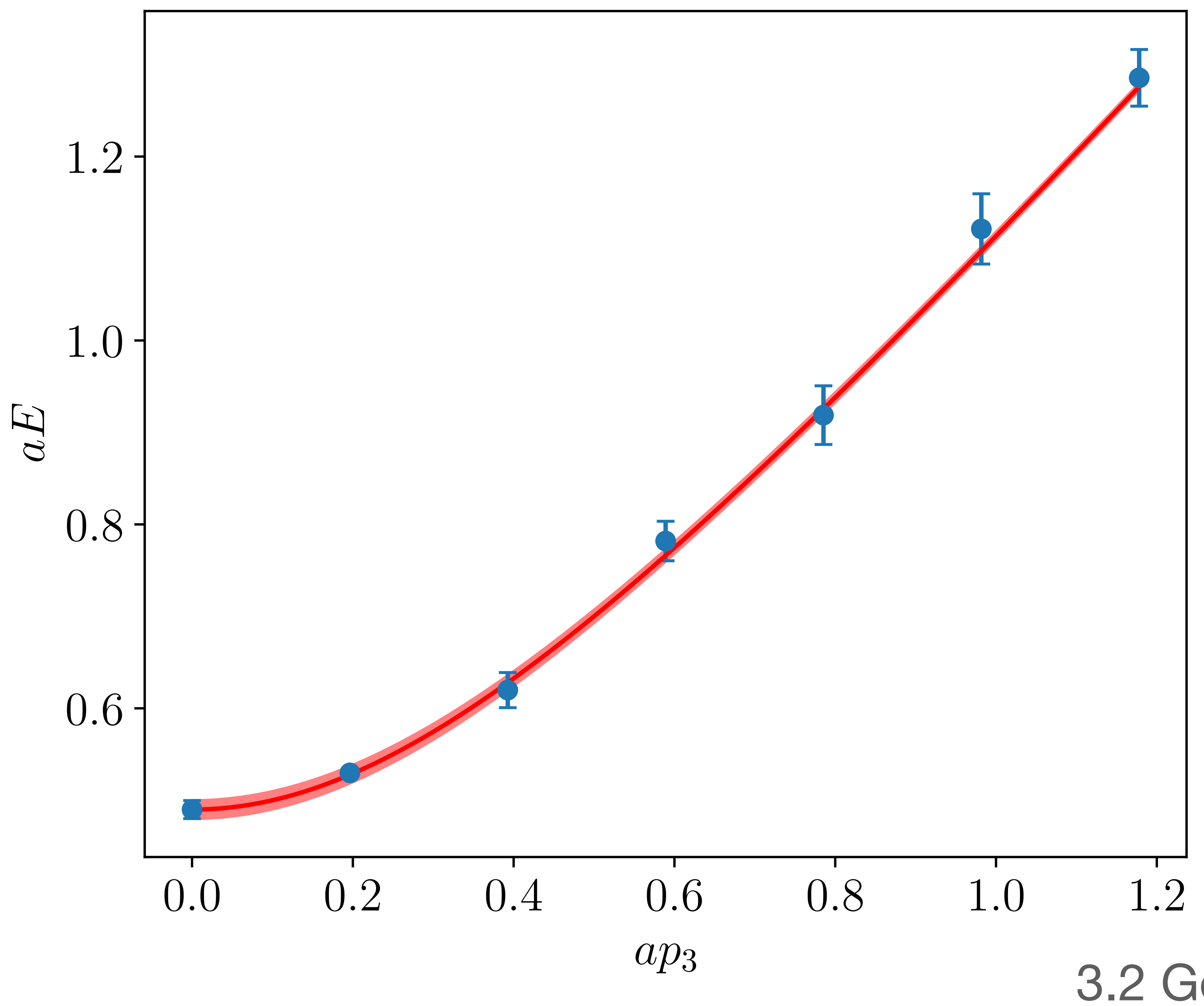


a= 0.048fm

arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

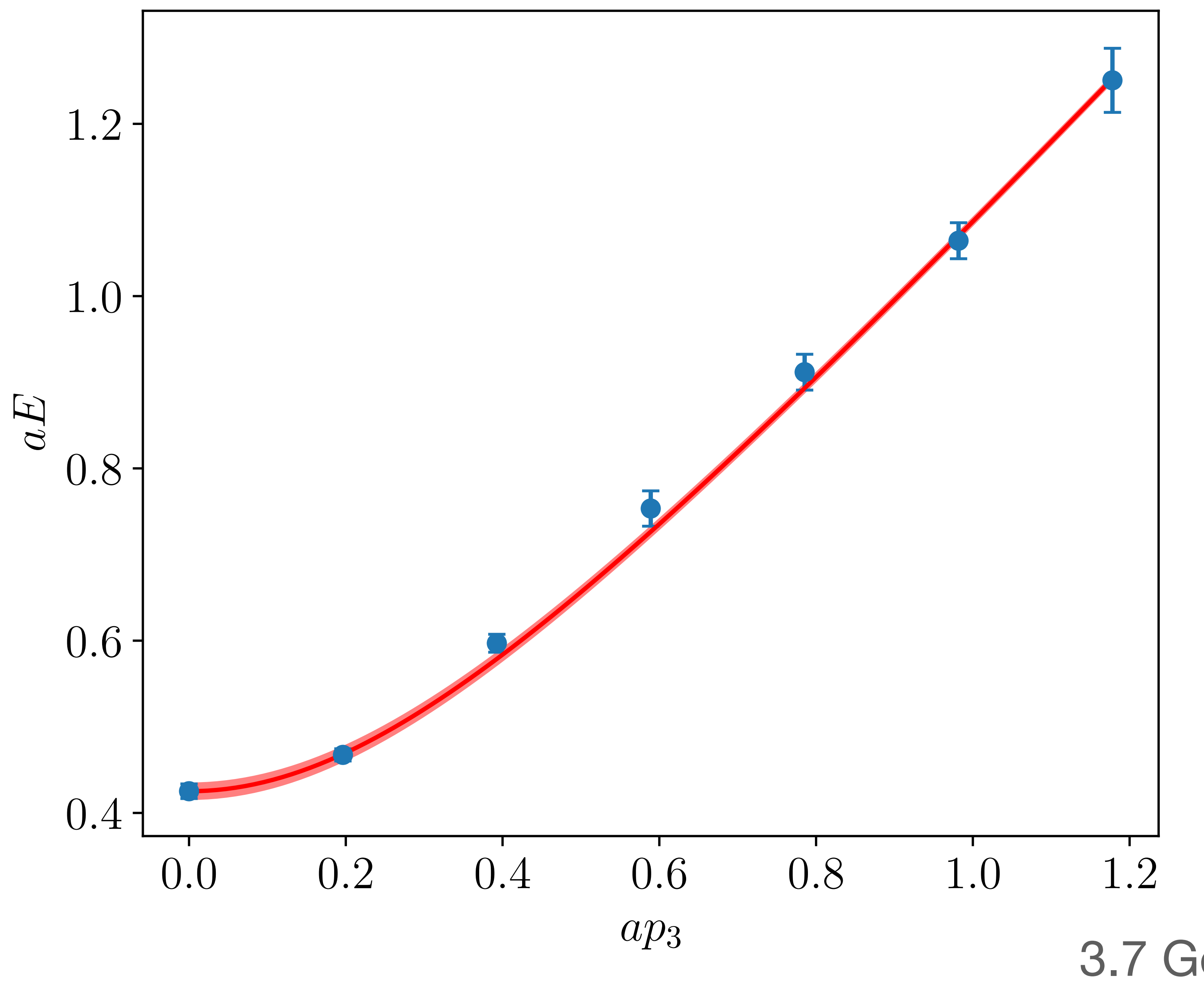
Nucleon Momentum scan

Energy *vs* momentum



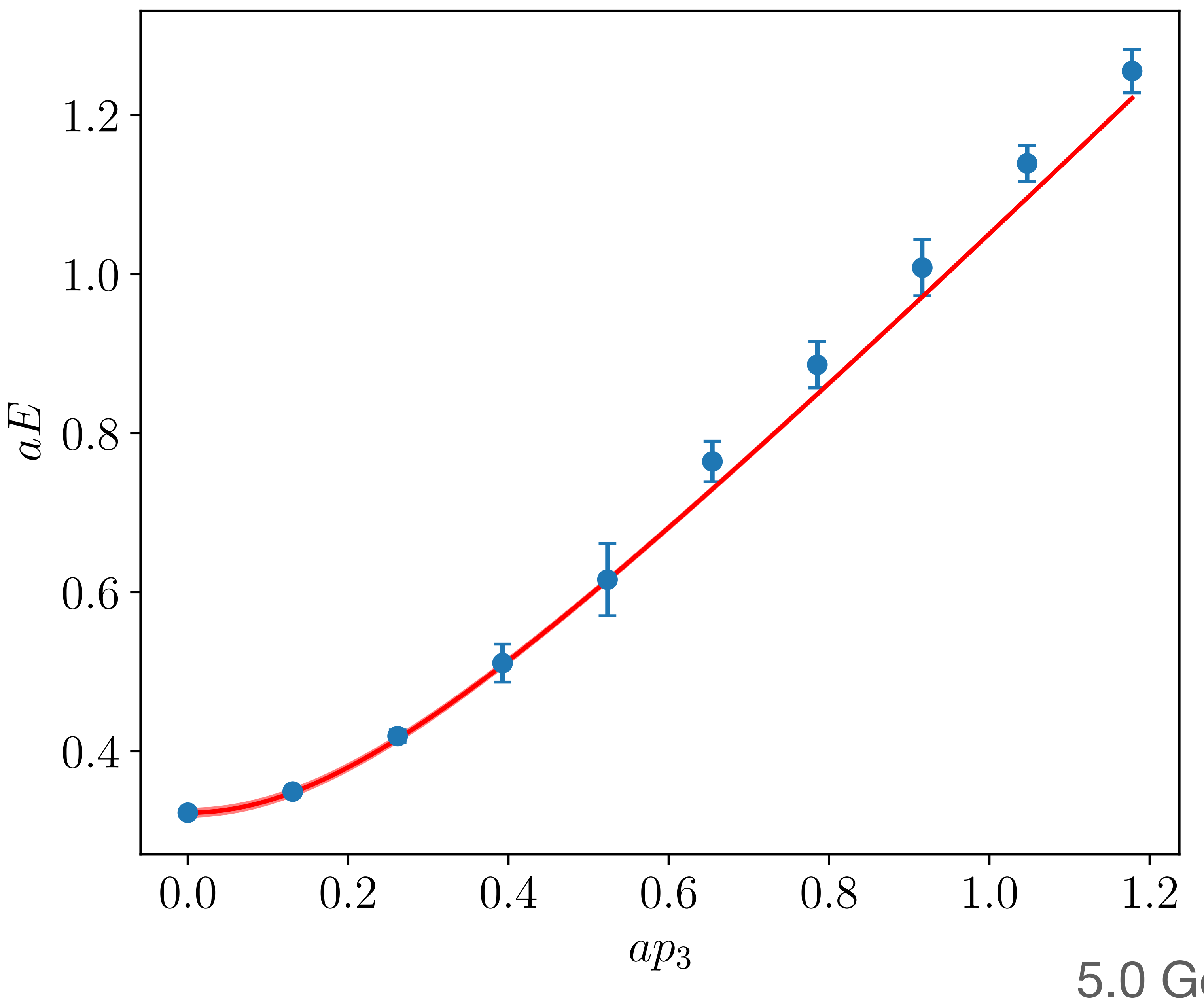
3.2 GeV

a=0.075 fm



3.7 GeV

a=0.065 fm



5.0 GeV

a=0.048 fm

Maximum attainable momentum in lattice units can be up to $\mathcal{O}(1)$

Smaller lattice spacing allows for physically larger momentum

arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

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Jacobi Polynomials

Inverse problem

$$q_+(x) = q(x) + \bar{q}(x)$$

$$q_-(x) = q(x) - \bar{q}(x)$$

PDF parametrization

$$q_\pm(x) = x^\alpha (1-x)^\beta \sum_{n=0}^{\infty} {}_\pm d_n^{(\alpha,\beta)} J_n^{(\alpha,\beta)}(x)$$

$J_n^{(\alpha,\beta)}(x)$ Jacobi Polynomials: Orthogonal and complete in the interval [0,1]

$$\int_0^1 dx\, x^\alpha (1-x)^\beta J_n^{(\alpha,\beta)}(x) J_m^{(\alpha,\beta)}(x) = N_n^{(\alpha,\beta)} \delta_{n,m}$$

Complete basis of functions in the interval [0,1] for any α and β

$$J_n^{(\alpha,\beta)}(x) = \sum_{j=0}^{n} \omega_{n,j}^{(\alpha,\beta)} x^j \ ,$$

$$\omega_{n,j}^{(\alpha,\beta)} = \binom{n}{j} \frac{(-1)^j}{n!} \frac{\Gamma(\alpha+n+1)\Gamma(\alpha+\beta+n+j+1)}{\Gamma(\alpha+\beta+n+1)\Gamma(\alpha+j+1)} \, .$$

$$\mathrm{Re}\,\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}_R(x\nu, \mu^2 z^2) q_-(x, \mu^2) \qquad \mathrm{Im}\,\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}_I(x\nu, \mu^2 z^2) q_+(x, \mu^2)$$

$$\mathcal{K}_R(x\nu, \mu^2 z^2) = \sum_{n=0}^{\infty} \frac{\sigma_n^{(\alpha,\beta)}(\nu, \mu^2 z^2)}{N_n^{(\alpha,\beta)}} J_n^{(\alpha,\beta)}(x)$$

$$\mathcal{K}_I(x\nu, \mu^2 z^2) = \sum_{n=0}^{\infty} \frac{\eta_n^{(\alpha,\beta)}(\nu, \mu^2 z^2)}{N_n^{(\alpha,\beta)}} J_n^{(\alpha,\beta)}(x)\,,$$

$$\sigma_n^{(\alpha,\beta)}(\nu, z^2\mu^2) = \sum_{j=0}^{n} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} c_{2k}(z^2\mu^2) \omega_{n,j}^{(\alpha,\beta)}\, B(\alpha + 2k + j + 1, \beta + 1)\, \nu^{2k}$$

$$\eta_n^{(\alpha,\beta)}(\nu, z^2\mu^2) = \sum_{j=0}^{n} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} c_{2k+1}(z^2\mu^2) \omega_{n,j}^{(\alpha,\beta)} B(\alpha + 2k + j + 2, \beta + 1) \nu^{2k+1}($$

$$\mathrm{Re}\,\mathfrak{M}_{\mathrm{lt}}(\nu, z^2) = 1 + \sum_{n=1}^{N_-} \sigma_n^{(\alpha,\beta)}(\nu, z^2\mu^2)_- d_n^{(\alpha,\beta)}$$

$$\mathrm{Im}\,\mathfrak{M}_{\mathrm{lt}}(\nu, z^2) = \sum_{n=0}^{N_+ - 1} \eta_n^{(\alpha,\beta)}(\nu, z^2\mu^2)_+ d_n^{(\alpha,\beta)}\,.$$

$$\mathfrak{M}(p,z,a) = \mathfrak{M}_{\mathrm{cont}}(\nu, z^2) + \sum_{n=1} \left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\mathrm{QCD}})^n R_n(\nu)\,.$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q_v(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) \; + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k\,.$$

Parametrization of correction terms - Only use one of each kind

Higher Twist

$$\mathrm{Re}\, B_1(\nu) = \sum_{n=1}^{N_{R,b}} \sigma_{0,n}^{(\alpha,\beta)}(\nu) b_{R,n}^{(\alpha,\beta)} \quad , \qquad \mathrm{Im}\, B_1(\nu) = \sum_{n=1}^{N_{I,b}} \eta_{0,n}^{(\alpha,\beta)}(\nu) b_{I,n}^{(\alpha,\beta)}$$

z-dependent lattice spacing

$$\mathrm{Re}\, P_1(\nu) = \sum_{n=1}^{N_{R,p}} \sigma_{0,n}^{(\alpha,\beta)}(\nu) p_{R,n}^{(\alpha,\beta)} \quad , \qquad \mathrm{Im}\, P_1(\nu) = \sum_{n=1}^{N_{I,p}} \eta_{0,n}^{(\alpha,\beta)}(\nu) p_{I,n}^{(\alpha,\beta)}$$

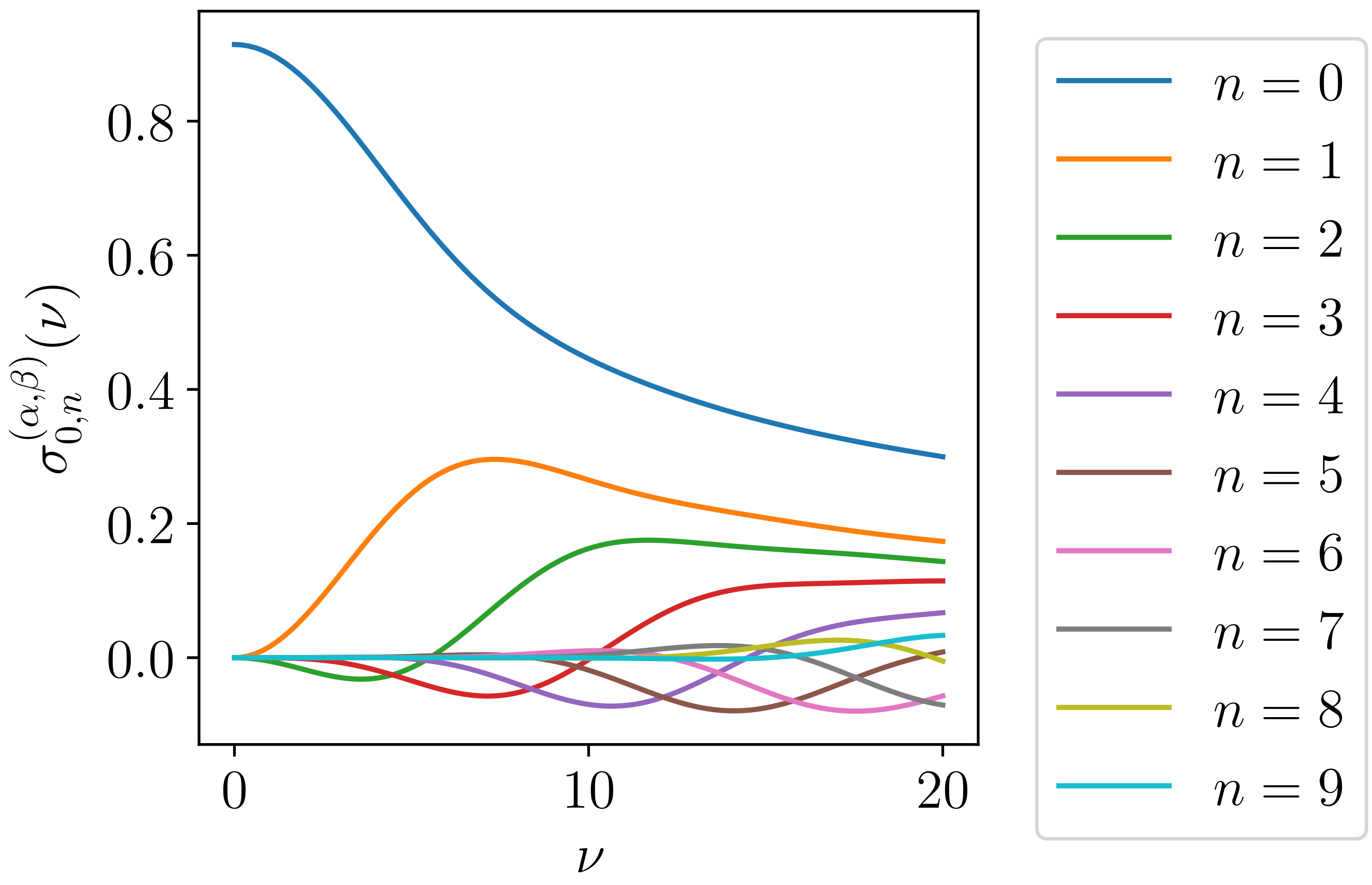
z-independent lattice spacing

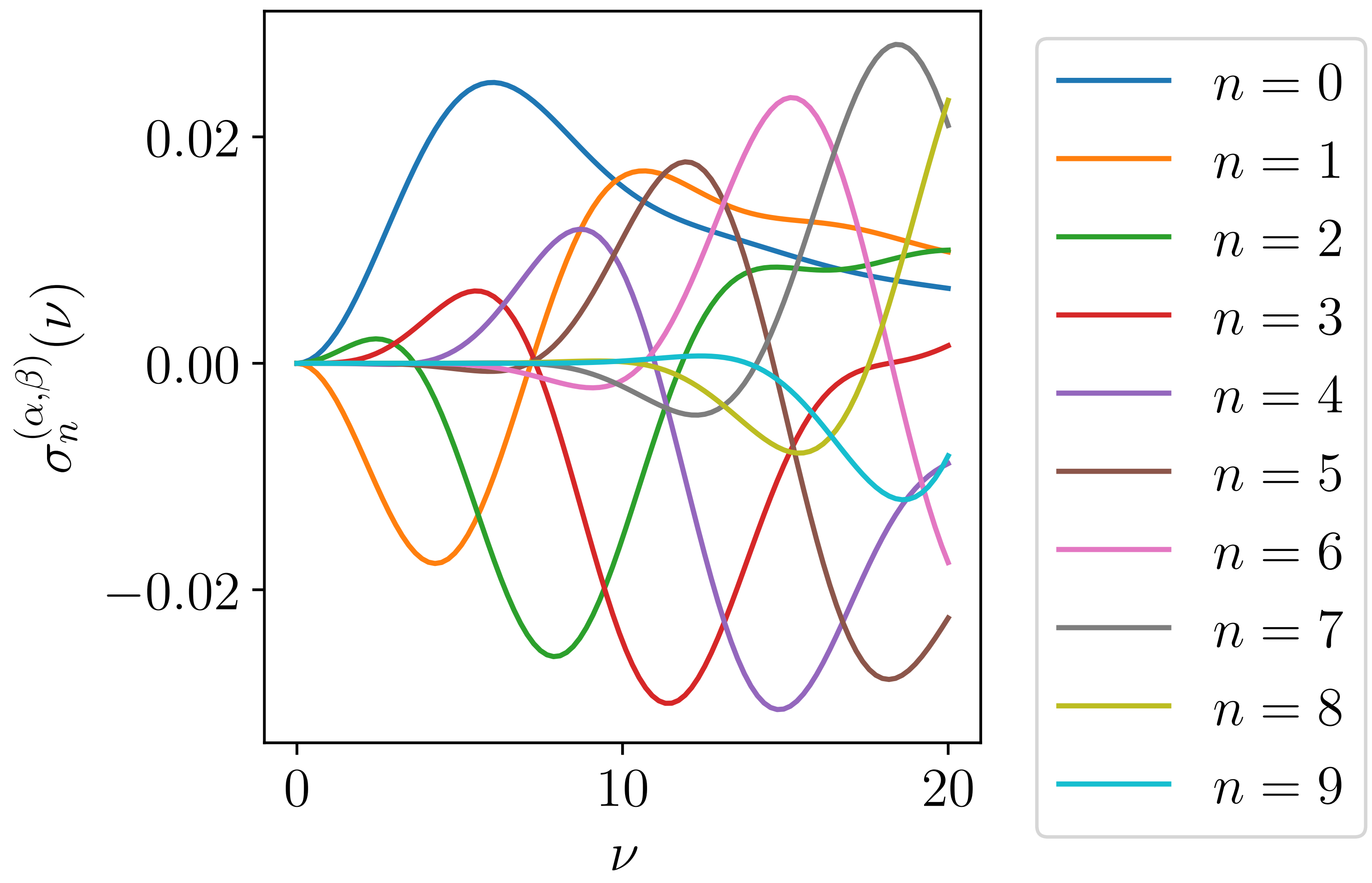
$$\mathrm{Re}\, R_1(\nu) = \sum_{n=1}^{N_{R,r}} \sigma_{0,n}^{(\alpha,\beta)}(\nu) r_{R,n}^{(\alpha,\beta)} \quad , \qquad \mathrm{Im}\, R_1(\nu) = \sum_{n=1}^{N_{I,r}} \eta_{0,n}^{(\alpha,\beta)}(\nu) r_{I,n}^{(\alpha,\beta)}\,,$$

$$\sigma_{0,n}^{(\alpha,\beta)}(\nu) = \int_0^1 dx\, \cos(\nu x) x^\alpha (1-x)^\beta J_n^{(\alpha,\beta)}(x)$$

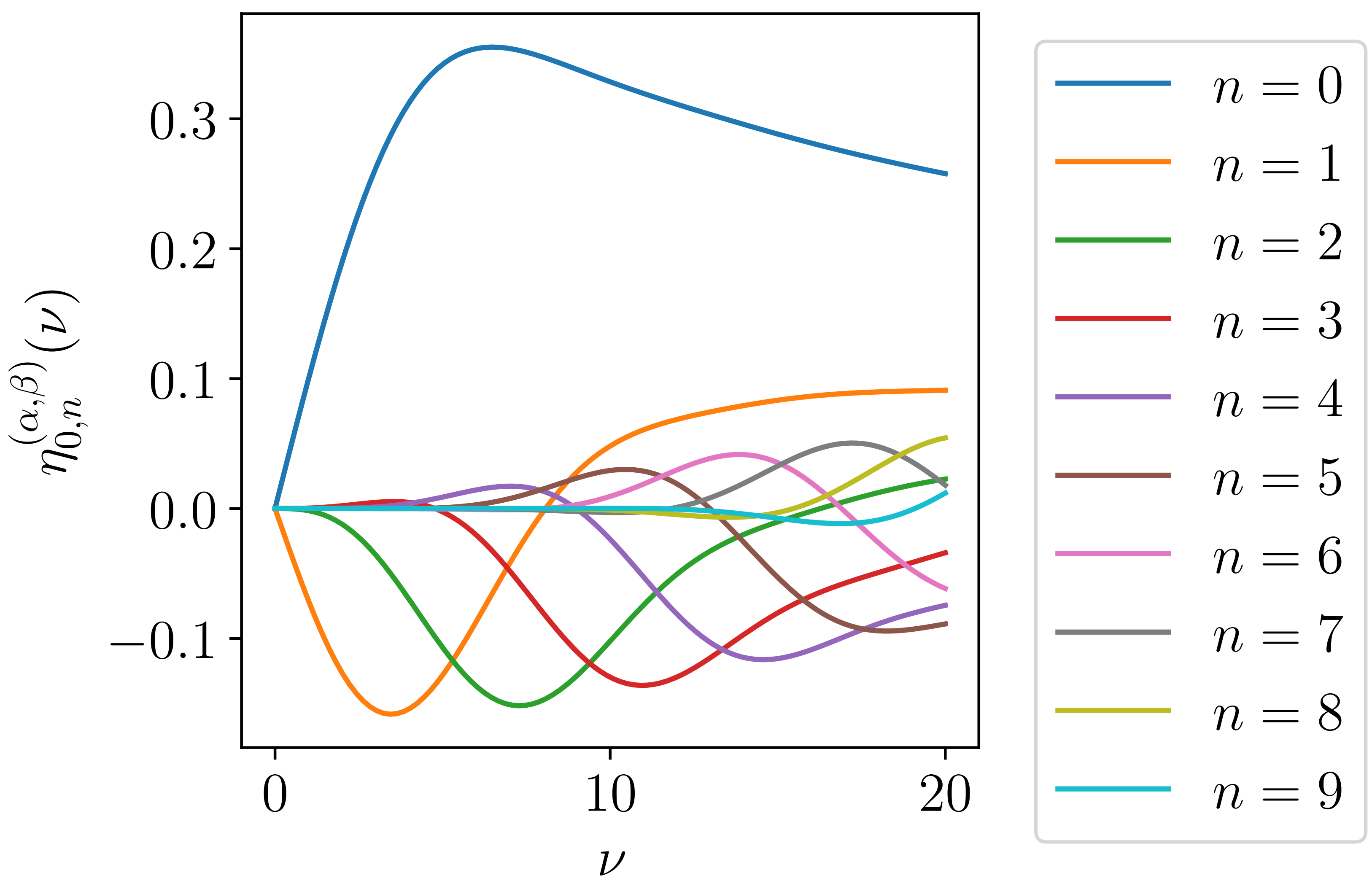
$$\eta_{0,n}^{(\alpha,\beta)}(\nu) = \int_0^1 dx\, \sin(\nu x) x^\alpha (1-x)^\beta J_n^{(\alpha,\beta)}(x)\,,$$

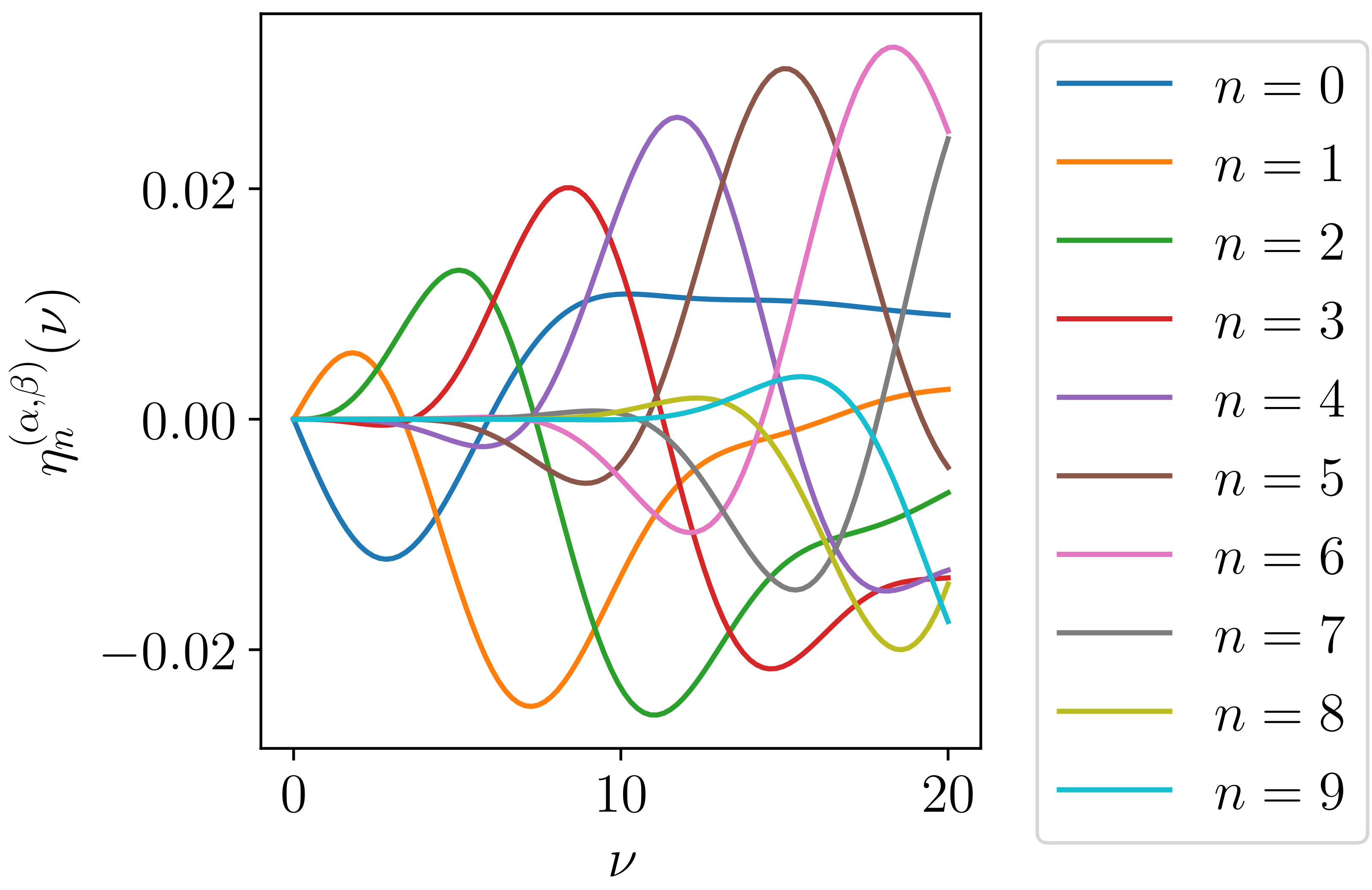
$$\sigma_n^{(\mathrm{NLO})}(\nu, z^2\mu^2) = \sigma_n^{(\alpha,\beta)}(\nu, z^2\mu^2) - \sigma_{0,n}^{(\alpha,\beta)}(\nu)$$





$$\eta_n^{(\mathrm{NLO})}(\nu, z^2\mu^2) = \eta_n^{(\alpha,\beta)}(\nu, z^2\mu^2) - \eta_{0,n}^{(\alpha,\beta)}(\nu)$$





Bayesian Inference

Optimize model parameters

- Fix the expansion order in the Jacobi polynomial expansion
- Optimize α,β and the expantion of coefficients by maximizing the posterior probability
- Average over models using AICc
- Note that one could fix α,β at a reasonable value and the vary the order of trancation in the Jacobi polynomial expansion

$$P\left[\theta|\mathfrak{M}^L, I\right] = \frac{P\left[\mathfrak{M}^L|\theta\right] P\left[\theta|I\right]}{P\left[\mathfrak{M}^L|I\right]} .$$

$$P\left[\theta|\mathfrak{M}^L, I\right] = \frac{P\left[\mathfrak{M}^L|\theta\right] P\left[\theta|I\right]}{P\left[\mathfrak{M}^L|I\right]}\,.$$

Probability distribution of the data given the parameters

$$P[\mathfrak{M}^L|\theta] \propto \exp[-\frac{\chi^2}{2}]$$

$$\chi^2 = \sum_{k,l}(\mathfrak{M}_k^L - \mathfrak{M}_k)C_{kl}^{-1}(\mathfrak{M}_l^L - \mathfrak{M}_l),$$

Prior distributions

Shifted lognormal for α,β so that α>-1 and b>-1

Normal distribution for all linear parameters (expansion coefficients)

Optimize parameters using non-linear optimizer for α,β only

VarPro (Variable projection method) allows for exact optimization of all expansion coefficients given α,β

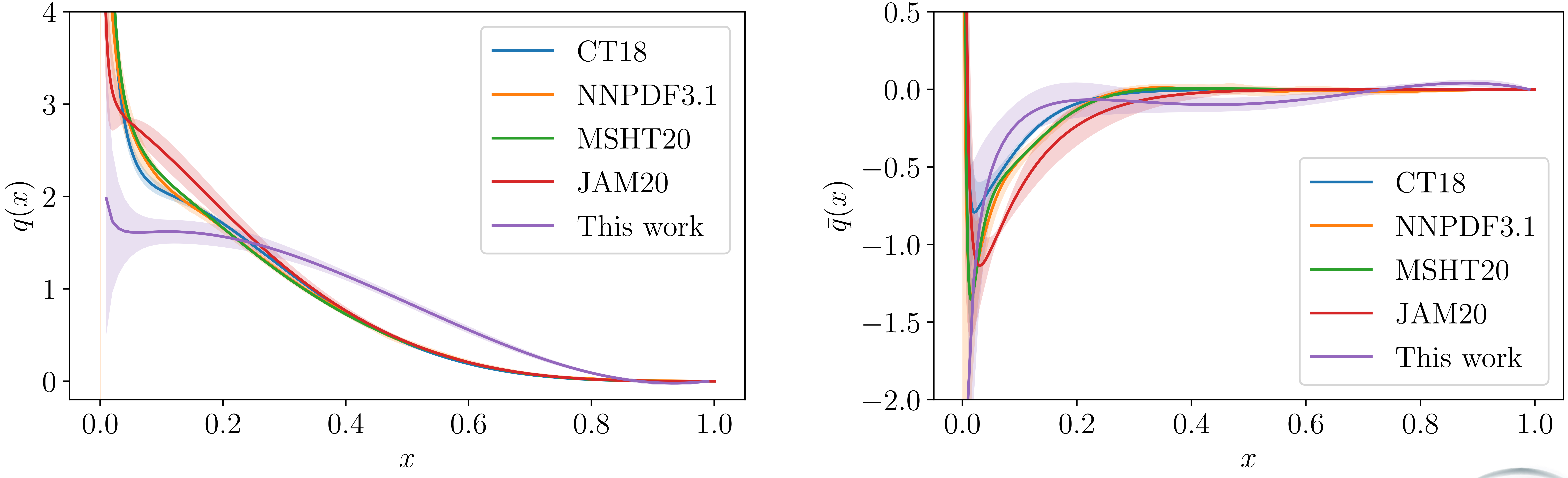
model	Real L^2/d.o.f.	Real χ^2/d.o.f.	Imag L^2/d.o.f.	Imag χ^2/d.o.f.
Q only	3.173	3.094	3.146	3.095
Q and B_1	2.721	2.479	3.054	2.969
Q and R_1	3.028	2.748	3.068	2.871
Q and P_1	0.876	0.809	1.186	1.088
Q, B_1, and R_1	2.610	2.057	2.917	2.619
Q, B_1, and P_1	0.852	0.723	1.020	0.888
Q, R_1, and P_1	0.881	0.763	1.289	1.063
All terms	0.857	0.727	1.026	0.893

the z-dependent lattice spacing effect seems the most important systematic error

arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

Isovector quark and anti-quark distributions

Comparison with phenomenology



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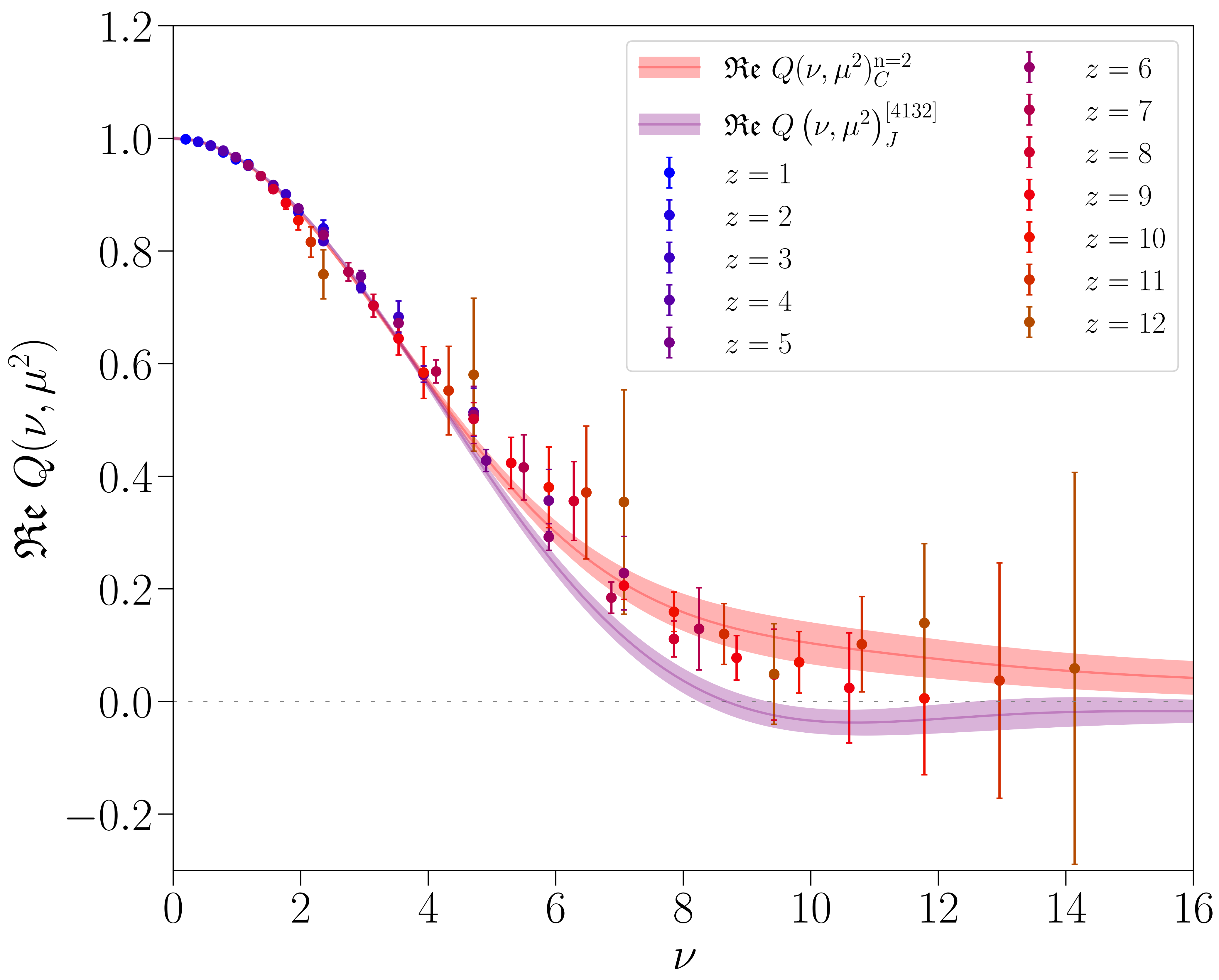
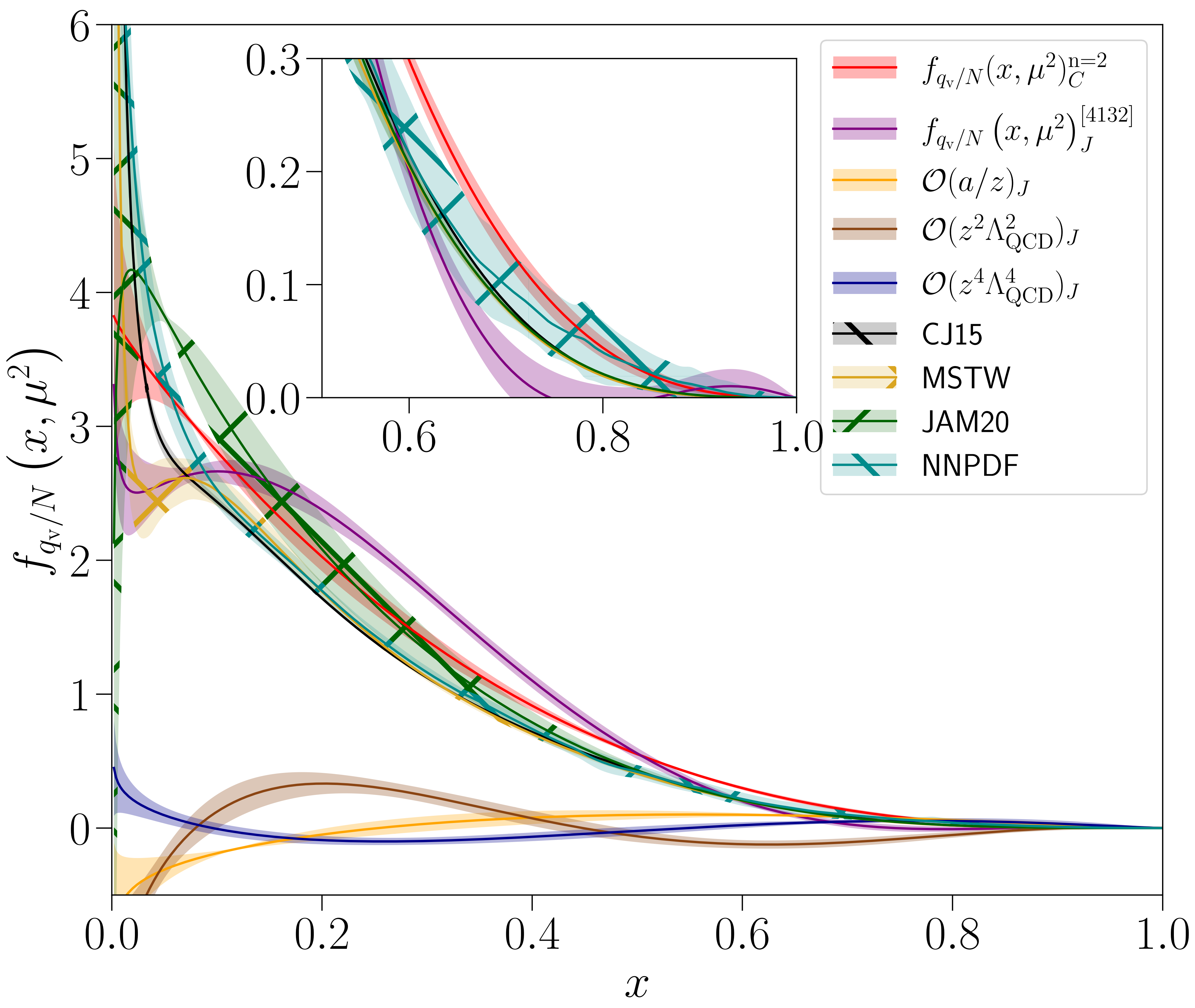
First distillation results

2+1 flavors single lattice spacing





arXiv:2107.05199 [hep-lat] C. Egerer *et. al.*

HadStruc

Conclusions

Outlook

- Understanding hadronic structure is a major goal in nuclear physics
  - Large experimental effort: JLab 12 GeV and future EIC
- Lattice QCD calculations can in principle compute (Generalized) parton distribution functions from first principles
- Controling all systematics of the calculation is important and that complicates the solution of the inverse problem at hand
- A particular approach that relies on Jacobi polynomial parametrization of unknown functions was presented
  - Both lattice spacing and higher twist effects need to be controled
- Synergy between lattice and experiment may be proven essential in providing precision estimates of (Generalized) parton distribution functions

END

Global Lattice QCD effort

ETMC

Flavor decomposition of the nucleon unpolarized, helicity and transversity parton distribution functions from lattice QCD simulations C. Alexandrou et al arXiv:2106.16065

Lattice continuum-limit study of nucleon quasi-PDFs C. Alexandrou et al arXiv:2011.00964

Flavor decomposition for the proton helicity parton distribution functions C. Alexandrou et al arXiv:2009.13061

Quasi-PDFs with twisted mass fermions C. Alexandrou et al arXiv:1909.10744

LP3

Gluon Parton Distribution of the Pion from Lattice QCD, Z Fan, H-W Lin, arXiv:2104.06372

Lattice Nucleon Isovector Unpolarized Parton Distribution in the Physical-Continuum Limit H-W Lin et al arXiv:2011.14971

Pion generalized parton distribution from lattice QCD J. Chen et al arXiv:1904.12376

Nucleon Transversity Distribution at the Physical Pion Mass from Lattice QCD Y Liu et al arXiv:1810.05043

BNL

Towards studying the structural differences between the pion and its radial excitation X. Gao et al arXiv:2101.11632

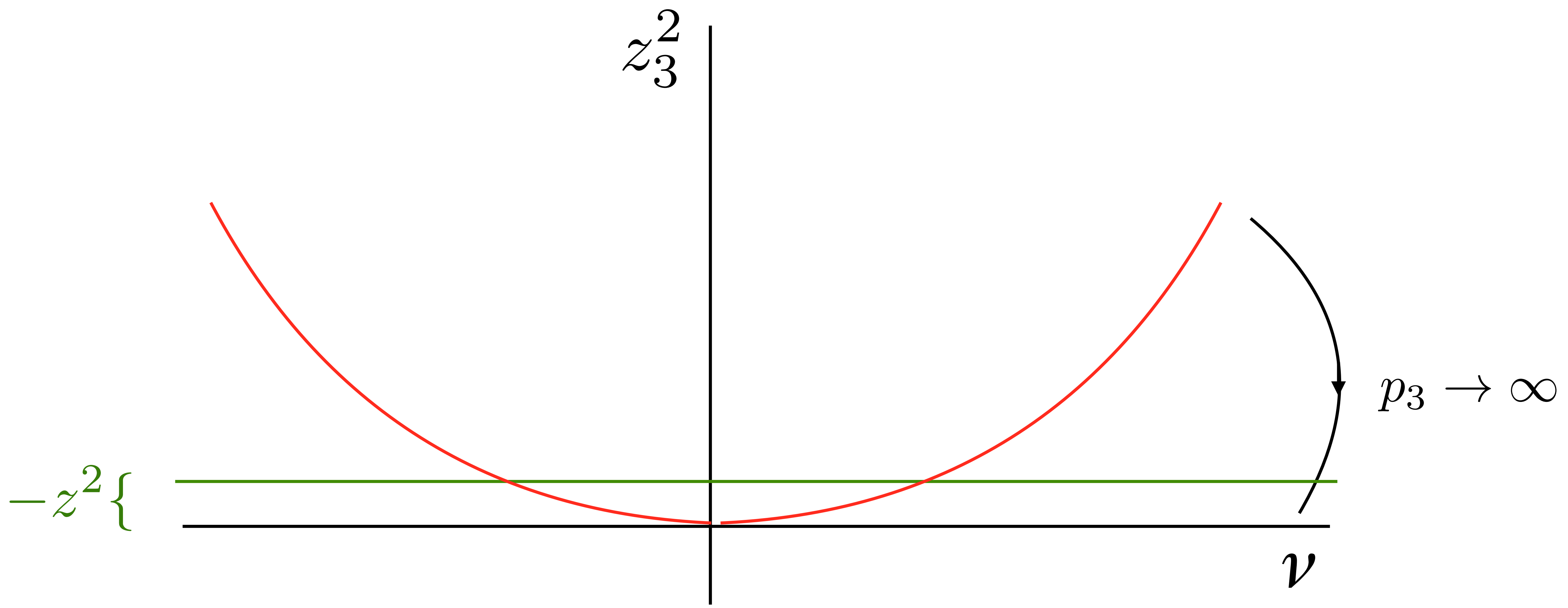
Isovector parton distribution functions of the proton on a superfine lattice Z. Fan, arXiv:2005.12015

Pion valence quark PDF from lattice QCD, C. Shugert et al, arXiv:2001.11650

$$Q(y,p_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu \mathcal{M}_p(\nu, \nu^2/p_3^2) e^{-iy\nu}$$ Ji's quasi-PDF

Large values of $z_3 = \nu/p_3$ are problematic

Alternative approach to the light-cone:

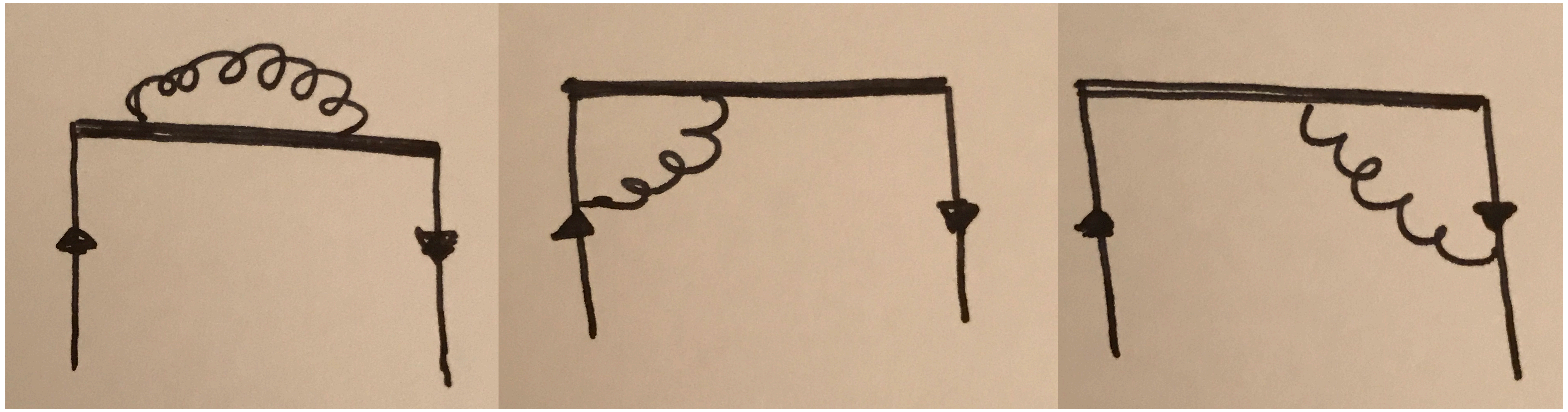


$$\mathcal{P}(x,-z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, \mathcal{M}_p(\nu, -z^2) e^{-ix\nu}$$

PDFs can be recovered $-z^2 \to 0$

Note that $x \in [-1,1]$

Radyusking Phys.Lett. B767 (2017) 314-320



One loop calculation of the UV divergences results in

$$\mathcal{M}^0(z,P,a) \sim e^{-m|z|/a}\left(\frac{a^2}{z^2}\right)^{2\gamma_{end}}$$

after re-summation of one loop result resulting exponentiation

- J.G.M.Gatheral,Phys.Lett.133B,90(1983)
- J.Frenkel, J.C.Taylor,Nucl.Phys.B246,231(1984),
- G.P.Korchemsky, A.V.Radyushkin,Nucl.Phys.B283,342(1987).

UV divergences appear multiplicatively