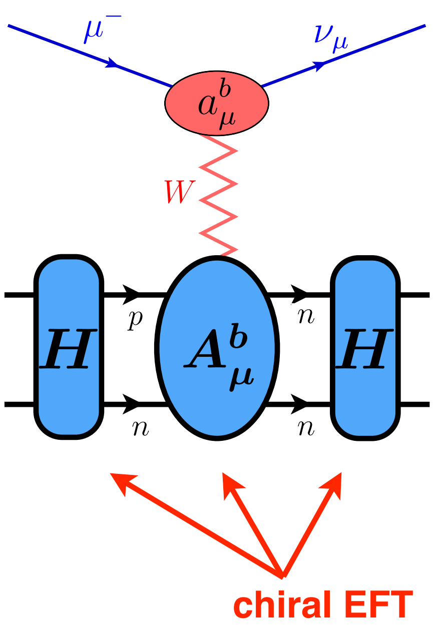
Evgeny Epelbaum, RUB

Exploring the role of electro-weak currents in atomic nuclei
ECT*, Trento, Italy, April 23-27, 2018

Electroweak currents from chiral EFT

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317
Hermann Krebs, review article, to appear



- Derivation of nuclear forces & currents
- Selected applications
- Summary and outlook

From QCD to nuclei

QCD

symmetries (especially the chiral symmetry);
lost of information (LECs)

Method of Unitary Transformation

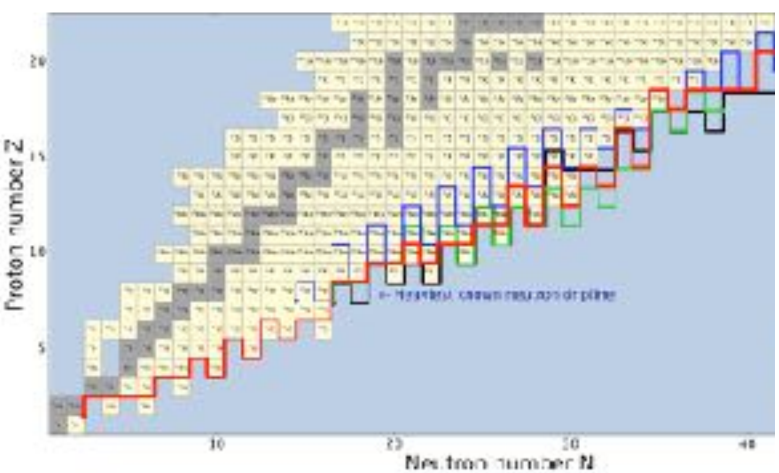
effective chiral Lagrangian $\mathcal{L}_{\text{eff}}(\pi, N)$

integrate out $|\vec{p}| \sim \sqrt{M_\pi m_N}$ (but retain $|\vec{p}| \sim M_\pi$):
Chiral Perturbation Theory

nuclear forces and currents

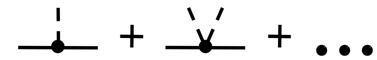
ab initio few-body methods:
lattice, FY, NCSM,…

nuclear structure and dynamics



Method of Unitary Transformation

Taketani, Mashida, Ohnuma'52; Okubo '54; EE, Glöckle, Meißner, Krebs, ...

- **Canonical transformation & quantization:** $\mathcal{L}_{\pi N} \longrightarrow \mathcal{H}_{\pi N} =$ 

EOM:

$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$

nucleonic states $|N\rangle, |NN\rangle, \ldots$

projectors

states with mesons $|N\pi\rangle, |N\pi\pi\rangle, \ldots$

← can not solve (infinite-dimensional eq.)

- **Decouple pions via a suitable UT:** $\tilde{H} \equiv U^\dagger \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$

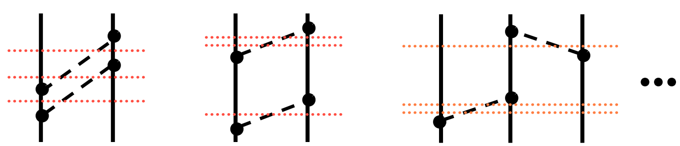
(Minimal) ansatz: $U = \begin{pmatrix} \eta(1+A^\dagger A)^{-1/2} & -A^\dagger(1+AA^\dagger)^{-1/2} \\ A(1+A^\dagger A)^{-1/2} & \lambda(1+AA^\dagger)^{-1/2} \end{pmatrix}, \quad A = \lambda A \eta$

Okubo '54

Require: $\eta \tilde{H} \lambda = \lambda \tilde{H} \eta = 0 \longrightarrow$ $\boxed{\lambda \left(H - [A,\, H] - A H A \right) \eta = 0}$

The decoupling equation is solved perturbatively (chiral expansion)

E.g., for the 2-pion exchange $\propto g_A^4$ one finds:



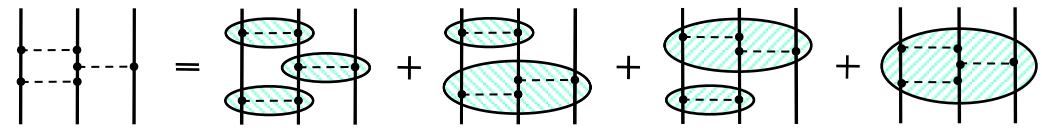
$$V^{(2)} = \eta \Big[- H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} + \frac{1}{2} H_I^{(1)} \frac{\lambda}{E_\pi^2} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} + \frac{1}{2} H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi^2} H_I^{(1)} \Big] \eta$$

Method of Unitary Transformation

Taketani, Mashida, Ohnuma'52; Okubo '54; EE, Glöckle, Meißner, Krebs, ...

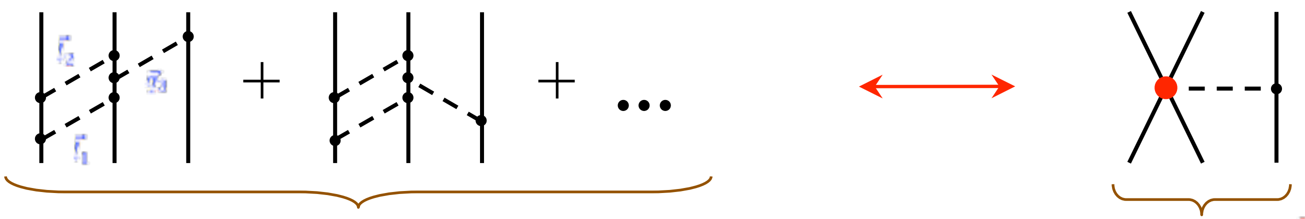
Contrary to S-matrix, renormalizability of nuclear potentials is not guaranteed:

UV finite →



← not necessarily UV finite

Indeed, explicit calculations of e.g. the 3NF $\propto g_A^6$ yield:



time-ordered-like graphs from the method of UT

$\propto 1/\omega_3^2$

$$V = \ldots = \int d^3 l_1 \, d^3 l_2 \, \delta(\vec{l}_1 - \vec{l}_2 - \vec{q}_1) \left[\ldots \right]$$

$$\times \left[2 \frac{\omega_1^2 + \omega_2^2}{\omega_1^4 \, \omega_2^4 \, \omega_3^2} + \frac{8}{\omega_1^2 \, \omega_2^2 \, \omega_3^4} - \frac{\omega_1 + \omega_2}{\omega_1^3 \, \omega_2^3 \, \omega_3^3} - \frac{2}{\omega_1^4 \, \omega_2^2 \, \omega_3 \, (\omega_1 + \omega_3)} - \frac{2}{\omega_1^2 \, \omega_2^4 \, \omega_3 \, (\omega_2 + \omega_3)} \right]$$

→ cannot renormalize the potential !

Method of Unitary Transformation

Taketani, Mashida, Ohnuma'52; Okubo '54; EE, Glöckle, Meißner, Krebs, ...

Solution [EE '06]

Nuclear potentials are not uniquely defined. Starting from N^{3}LO, can construct additional UTs in Fock space beyond the (minimal) Okubo UT.

The UTs relevant for the N^{3}LO contributions $\propto g_A^6$ are $U = e^{\alpha_1 S_1 + \alpha_2 S_2}$, with the generators given by:

$$S_1 = \eta \Big[H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi^3} H_I^{(1)} - \text{h. c.} \Big] \eta$$

$$S_2 = \eta \Big[H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \frac{\lambda}{E_\pi^2} H_I^{(1)} - \text{h. c.} \Big] \eta$$

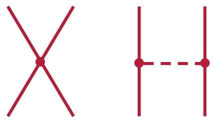
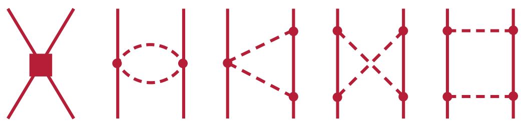
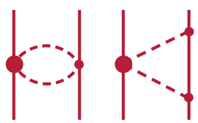
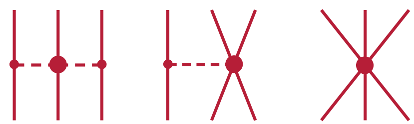
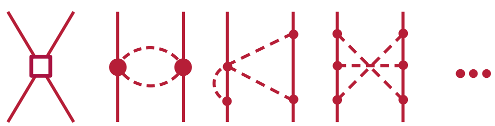
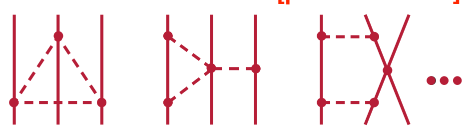
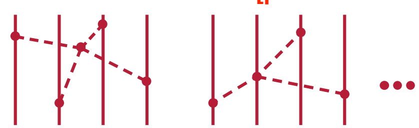
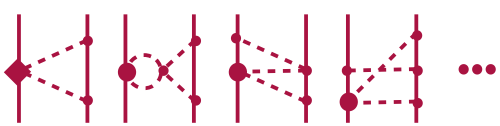
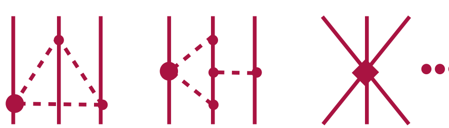
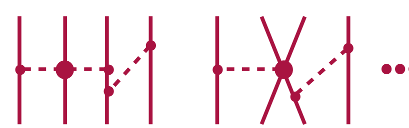
They induce additional contributions in the Hamiltonian starting from N^{3}LO

$$\delta V^{(4)} = [(H_{\rm kin} + V^{(0)}), \ S] = -\alpha_1 \, H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi} H_I^{(1)} \, \eta \, H_I^{(1)} \frac{\lambda}{E_\pi^3} H_I^{(1)} + \ \ldots .$$

Demanding renormalizability constrains α_1, α_2 and leads to unique static results.

So far, it was always possible to get finite nuclear potentials & currents.

Chiral expansion of the nuclear forces [W-counting]

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	 Weinberg '90	—	—
NLO (Q^2)	 Ordonez, van Kolck '92	—	—
N^2LO (Q^3)	 Ordonez, van Kolck '92	 van Kolck '94; EE et al. '02	—
N^3LO (Q^4)	 Kaiser '00 - '02	[parameter-free]  Bernard, EE, Krebs, Meißner,'08, '11	[parameter-free]  EE '06
N^4LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15	 Girlanda, Kievsky, Viviani '11 Krebs, Gasparyan, EE '12,'13 (short-range loop contrib. still missing)	 still have to be worked out

Electroweak currents using the method of UT

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

- Switch on external sources $s,\ p,\ r_\mu,\ l_\mu$ and consider ***local*** chiral rotations:

$$r_\mu \rightarrow r'_\mu = R\, r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\,, \qquad l_\mu \rightarrow l'_\mu = L\, l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\,,$$
$$s + i\,p \rightarrow s' + i\,p' = R(s + i\,p)L^\dagger\,, \qquad s - i\,p \rightarrow s' - i\,p' = L(s - i\,p)R^\dagger$$

The sources can be conveniently rewritten via $v_\mu = \frac{1}{2}(r_\mu + l_\mu)\ , \quad a_\mu = \frac{1}{2}(r_\mu - l_\mu)$ with:

$$v_\mu = v_\mu^{(s)} + \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{v}_\mu, \qquad a_\mu = \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{a}_\mu, \qquad s = s_0 + \boldsymbol{\tau}\cdot\boldsymbol{s}, \qquad p = p_0 + \boldsymbol{\tau}\cdot\boldsymbol{p}$$

already known from the strong sector…

- (Naive) attempt: calculate $\tilde H \rightarrow \tilde H[a,v,s,p] = U^\dagger H[a,v,s,p] U$ and extract the nuclear currents via $V_\mu^a(\vec x\,) = \frac{\delta \tilde H}{\delta v_a^\mu(\vec x, t)}, \quad A_\mu^a(\vec x\,) = \frac{\delta \tilde H}{\delta a_a^\mu(\vec x, t)}$ at $v = a = p = \boldsymbol{s} = 0, \quad s_0 = m_q$.

However, the resulting currents turn out to be non-renormalizable…

$\longrightarrow$ Need to consider a more general class of UTs

Specifically, employ additional η-space UTs $U[a,v,s,p]$ subject to the constraint $U[0,0,m_q,0] = 1$

Notice: the resulting UTs are time-dependent, thus $H' \neq U^\dagger H U$. Indeed:

$$i\frac{\partial}{\partial t}\Psi = H\Psi \quad \longrightarrow \quad i\frac{\partial}{\partial t}\Big(U^\dagger(t)\Psi\Big) = \left[U^\dagger(t) H U(t) - U^\dagger(t)\left(i\frac{\partial}{\partial t}U(t)\right)\right]\Big(U^\dagger(t)\Psi\Big)$$

Electroweak currents using the method of UT

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- Thus, we have:

$$H_{\rm eff}[a,\dot a,v,\dot v,s,\dot s,p,\dot p] \;=\; U^\dagger[a,v,s,p]U^\dagger_{\rm str}H[a,v,s,p]U_{\rm str}U[a,v,s,p] \;+\; \left(i\frac{\partial}{\partial t}U^\dagger[a,v,s,p]\right)U[a,v,s,p]$$

(to the order we are working [leading 1-loop for 2-body operators], can write 33 such UTs…)

Nuclear potentials are given by

$$V \;:=\; H_{\rm eff}[v=\dot v=a=\dot a=p=\dot p=\dot s=0,\, s=m_q] \;-\; H_0$$

while the current operators in momentum space are defined as (in the Schrödinger picture):

$$V_\mu^j(\vec k,k_0) := \frac{\delta H_{\rm eff}}{\delta v_j^\mu(\vec k,k_0)}, \qquad A_\mu^j(\vec k,k_0) := \frac{\delta H_{\rm eff}}{\delta a_j^\mu(\vec k,k_0)}, \qquad P^j(\vec k,k_0) := \frac{\delta H_{\rm eff}}{\delta p^j(\vec k,k_0)},$$

where the FT of the sources are given by $f(x) =: \int d^4q\, e^{-iq\cdot x}\, f(q)$ with $f=\{v_\mu^j,\, a_\mu^j,\, p^j\}$, $H_{\rm eff}$ is taken at $t=0$ & the functional derivatives are taken at $v=\dot v=a=\dot a=p=\dot p=\dot s=\boldsymbol{s}=0$ & $s_0=m_q$.

- It is straightforward to verify the proper relation to the S-matrix, e.g.:

$$\frac{\delta}{\delta a^{j\mu}(k_0,\vec k)}\langle\alpha|S|\beta\rangle \;=\; -i\,2\pi\delta(E_\alpha-E_\beta-k_0)\langle\alpha|A_\mu^j(k_0,\vec k)|\beta\rangle$$

Electroweak currents using the method of UT

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

- **Manifestations of the symmetry (= continuity equation)**

The transformation properties of the external sources

$$v_\mu = v_\mu^{(s)} + \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{v}, \quad a_\mu = \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{a}, \quad s = s_0 + \boldsymbol{\tau}\cdot\boldsymbol{s}, \quad p = p_0 + \boldsymbol{\tau}\cdot\boldsymbol{p}$$

are given by

$$\begin{aligned}
\boldsymbol{v}_\mu \quad &\rightarrow \quad \boldsymbol{v}'_\mu = \boldsymbol{v}_\mu + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_V, \\
\boldsymbol{a}_\mu \quad &\rightarrow \quad \boldsymbol{a}'_\mu = \boldsymbol{a}_\mu + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_A, \\
s_0 \quad &\rightarrow \quad s'_0 = s_0 - \boldsymbol{p}\cdot\boldsymbol{\epsilon}_A, \\
\boldsymbol{s} \quad &\rightarrow \quad \boldsymbol{s}' = \boldsymbol{s} + \boldsymbol{s}\times\boldsymbol{\epsilon}_V - p_0\boldsymbol{\epsilon}_A, \\
i\,p_0 \quad &\rightarrow \quad i\,p'_0 = i(p_0 + \boldsymbol{s}\cdot\boldsymbol{\epsilon}_A), \\
i\,\boldsymbol{p} \quad &\rightarrow \quad i\,\boldsymbol{p}' = i(\boldsymbol{p} + \boldsymbol{p}\times\boldsymbol{\epsilon}_V + s_0\,\boldsymbol{\epsilon}_A)
\end{aligned}$$

where the chiral rotation angles are given by $\boldsymbol{\epsilon}_V = \frac{1}{2}(\boldsymbol{\epsilon}_R + \boldsymbol{\epsilon}_L)$ and $\boldsymbol{\epsilon}_A = \frac{1}{2}(\boldsymbol{\epsilon}_R - \boldsymbol{\epsilon}_L)$

Electroweak currents using the method of UT

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Start with the Schrödinger equation

$$i\frac{\partial}{\partial t}\Psi = H_{\rm eff}[a, \dot a, v, \dot v, s, \dot s, p, \dot p]\Psi$$

and perform a chiral rotation $a, \dot a, v, \dot v, s, \dot s, p, \dot p \;\longrightarrow\; a', \dot a', v', \dot v', s', \dot s', p', \dot p'$. After the rotation, the dynamics of the systems is given by:

$$i\frac{\partial}{\partial t}\Psi' = H_{\rm eff}[a', \dot a', v', \dot v', s', \dot s', p', \dot p']\,\Psi'$$

For observables to remain unaffected, the two Hamiltonians must be unitary equivalent, i.e. there must exist a UT on the Fock space such that:

(*) $$H_{\rm eff}[a', \dot a', v', \dot v', s', \dot s', p', \dot p'\,] \;=\; U^\dagger H_{\rm eff}[a, \dot a, v, \dot v, s, \dot s, p, \dot p]U \;+\; \left(i\frac{\partial}{\partial t}U^\dagger\right)U$$

Make an ansatz for the unitary operator in the form:

$$U = \exp\left(i\int d^3x[\boldsymbol{R}_0^v(\vec x)\cdot\boldsymbol{\epsilon}_V(\vec x, t) + \boldsymbol{R}_1^v(\vec x)\cdot\dot{\boldsymbol{\epsilon}}_V(\vec x, t) + \boldsymbol{R}_0^a(\vec x)\cdot\boldsymbol{\epsilon}_A(\vec x, t) + \boldsymbol{R}_1^a(\vec x)\cdot\dot{\boldsymbol{\epsilon}}_A(\vec x, t)]\right)$$

The unknown operators **R** can be determined by expanding both sides of (*) in powers of the rotation angles and their derivatives [keep only linear terms] and matching the r.h.s. with the l.h.s. for the case $v{=}\dot v{=}a{=}\dot a{=}p{=}\dot p{=}\dot s{=}0, s{=}m_q$.

Electroweak currents using the method of UT

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Right-hand side:

$$W + \int d^3x \Big(i[W, \boldsymbol{R}_0^v(\vec{x})] \cdot \boldsymbol{\epsilon}_V(\vec{x},t) + i[W, \boldsymbol{R}_1^v(\vec{x})] \cdot \dot{\boldsymbol{\epsilon}}_V(\vec{x},t) + i[W, \boldsymbol{R}_0^a(\vec{x})] \cdot \boldsymbol{\epsilon}_A(\vec{x},t)$$

$$+ i[W, \boldsymbol{R}_1^a(\vec{x})] \cdot \dot{\boldsymbol{\epsilon}}_A(\vec{x},t) + \boldsymbol{R}_0^v(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_V(\vec{x},t) + \boldsymbol{R}_1^v(\vec{x}) \cdot \ddot{\boldsymbol{\epsilon}}_V(\vec{x},t) + \boldsymbol{R}_0^a(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_A(\vec{x},t) + \boldsymbol{R}_1^a(\vec{x}) \cdot \ddot{\boldsymbol{\epsilon}}_A(\vec{x},t) \Big)$$

where we have introduced $W \equiv H_0 + V$. For the left-hand side, we get:

$$W + \int d^3x \Big(\boldsymbol{V}_\mu^{(0)}(\vec{x}) \cdot \partial^\mu \boldsymbol{\epsilon}_V(\vec{x},t) + \boldsymbol{V}_\mu^{(1)}(\vec{x}) \cdot \partial^\mu \dot{\boldsymbol{\epsilon}}_V(\vec{x},t) + \boldsymbol{A}_\mu^{(0)}(\vec{x}) \cdot \partial^\mu \boldsymbol{\epsilon}_A(\vec{x},t) + \boldsymbol{A}_\mu^{(1)}(\vec{x}) \cdot \partial^\mu \dot{\boldsymbol{\epsilon}}_A(\vec{x},t)$$

$$+ m_q \boldsymbol{P}^{(0)}(\vec{x}) \cdot \boldsymbol{\epsilon}_A(\vec{x},t) + m_q \boldsymbol{P}^{(1)}(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_A(\vec{x},t) \Big)$$

where the various currents are defined according to:

$$V_\mu^{(0)j}(\vec{x}) := \frac{\delta H_{\rm eff}}{\delta v_j^\mu(\vec{x},t)}, \quad V_\mu^{(1)j}(\vec{x}) := \frac{\delta H_{\rm eff}}{\delta \dot{v}_j^\mu(\vec{x},t)}, \quad A_\mu^{(0)j}(\vec{x}) := \frac{\delta H_{\rm eff}}{\delta a_j^\mu(\vec{x},t)},$$

$$A_\mu^{(1)j}(\vec{x}) := \frac{\delta H_{\rm eff}}{\delta \dot{a}_j^\mu(\vec{x},t)}, \quad P_j^{(0)}(\vec{x}) := \frac{\delta H_{\rm eff}}{\delta p_j(\vec{x},t)}, \quad P_j^{(1)}(\vec{x}) := \frac{\delta H_{\rm eff}}{\delta \dot{p}_j(\vec{x},t)}$$

(all functional derivatives are taken at $v = \dot{v} = a = \dot{a} = p = \dot{p} = \dot{s} = 0, s = m_q$)

Electroweak currents using the method of UT

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Matching the terms proportional to $\ddot{\boldsymbol{\epsilon}}_V(\vec{x},t)$ and $\ddot{\boldsymbol{\epsilon}}_A(\vec{x},t)$ one obtains:

$$\boldsymbol{R}_1^v(\vec{x}) = \boldsymbol{V}_0^{(1)}(\vec{x}), \quad \boldsymbol{R}_1^a(\vec{x}) = \boldsymbol{A}_0^{(1)}(\vec{x})$$

Matching the terms proportional to $\dot{\boldsymbol{\epsilon}}_V(\vec{x},t)$ and $\dot{\boldsymbol{\epsilon}}_A(\vec{x},t)$ one obtains:

$$\begin{aligned} \boldsymbol{R}_0^v(\vec{x}) + i\,[W, \boldsymbol{R}_1^v(\vec{x})] &= \boldsymbol{V}_0^{(0)}(\vec{x}) - \vec{\nabla}\cdot\vec{\boldsymbol{V}}^{(1)}(\vec{x}), \\ \boldsymbol{R}_0^a(\vec{x}) + i\,[W, \boldsymbol{R}_1^a(\vec{x})] &= \boldsymbol{A}_0^{(0)}(\vec{x}) - \vec{\nabla}\cdot\vec{\boldsymbol{A}}^{(1)}(\vec{x}) + m_q \boldsymbol{P}^{(1)}(\vec{x}) \end{aligned}$$

Matching the remaining terms one obtains:

$$\begin{aligned} i[W, \boldsymbol{R}_0^v(\vec{x})] &= -\vec{\nabla}\cdot\vec{\boldsymbol{V}}^{(0)}(\vec{x}), \\ i[W, \boldsymbol{R}_0^a(\vec{x})] &= -\vec{\nabla}\cdot\vec{\boldsymbol{A}}^{(0)}(\vec{x}) + m_q \boldsymbol{P}^{(0)}(\vec{x}) \end{aligned}$$

Combining these equations together, the final result is:

$$\begin{aligned} i\,[W, \boldsymbol{V}_0^{(0)}(\vec{x}) - \vec{\nabla}\cdot\vec{\boldsymbol{V}}^{(1)}(\vec{x}) - i\,[W, \boldsymbol{V}_0^{(1)}(\vec{x})]] &= -\vec{\nabla}\cdot\vec{\boldsymbol{V}}^{(0)}(\vec{x}), \\ i\,[W, \boldsymbol{A}_0^{(0)}(\vec{x}) - \vec{\nabla}\cdot\vec{\boldsymbol{A}}^{(1)}(\vec{x}) - i\,[W, \boldsymbol{A}_0^{(1)}(\vec{x})] + m_q \boldsymbol{P}^{(1)}(\vec{x})] &= -\vec{\nabla}\cdot\vec{\boldsymbol{A}}^{(0)}(\vec{x}) + m_q \boldsymbol{P}^{(0)}(\vec{x}) \end{aligned}$$

Electroweak currents using the method of UT

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

In momentum space, the continuity equations take the form:

$$\vec{k}\cdot\vec{A}^i(\vec{k},0) = \Big[H_{\rm str},\, A_0^i(\vec{k},0) - \frac{\partial}{\partial k_0}\Big(\vec{k}\cdot\vec{A}^i(k) + [H_{\rm str},\, A_0^i(k)] + i m_q P^i(k)\Big)\Big] + i m_q P^i(\vec{k},0)$$

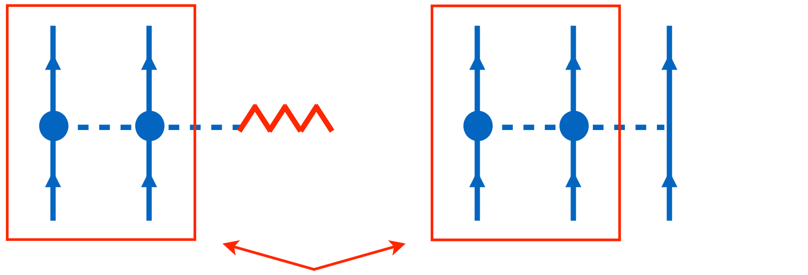
$$\vec{k}\cdot\vec{V}^i(\vec{k},0) = \Big[H_{\rm str},\, V_0^i(\vec{k},0) - \frac{\partial}{\partial k_0}\Big(\vec{k}\cdot\vec{V}^i(k) + [H_{\rm str},\, V_0^i(k)]\Big)\Big]$$

Notice: the (linear) k_0-dependence of the currents, which is an off-shell effect, is induced by the additional unitary transformations depending on external sources (renormalizability…)

- Unitary ambiguity [33 UT's@N^{3}LO] is strongly reduced but not completely eliminated by the renormalizability requirement for the currents.

We further require that ∀ pion-pole contributions to the axial currents match the corresponding 1π-exchange contributions to the nuclear forces at the pion pole:

$$\lim_{q_i^2\to -M_\pi^2}(q_i^2+M_\pi^2)\Big[H_{\rm str}\;-\;\vec{A}(-\vec{q}_i)\cdot\Big(-\frac{g_A}{2F_\pi^2}\vec{\sigma}_i\boldsymbol{\tau}_i\Big)\Big] = 0$$

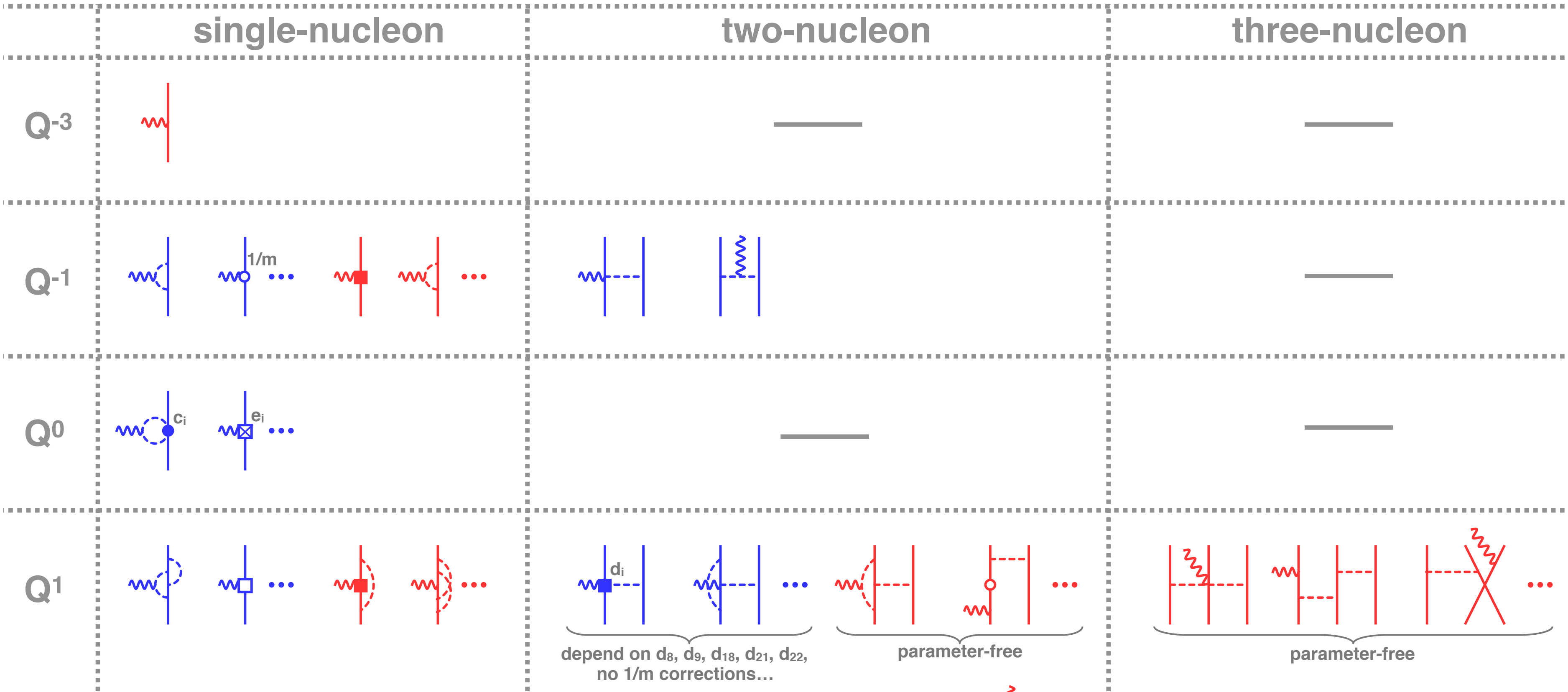


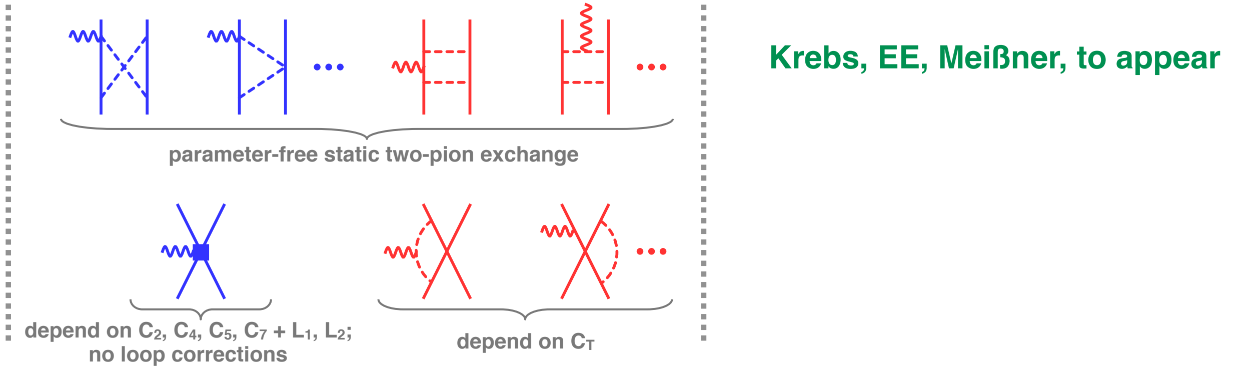
With these constraints, the expressions for the currents are determined unambiguously.

Electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502;
PRC 86 (12) 047001

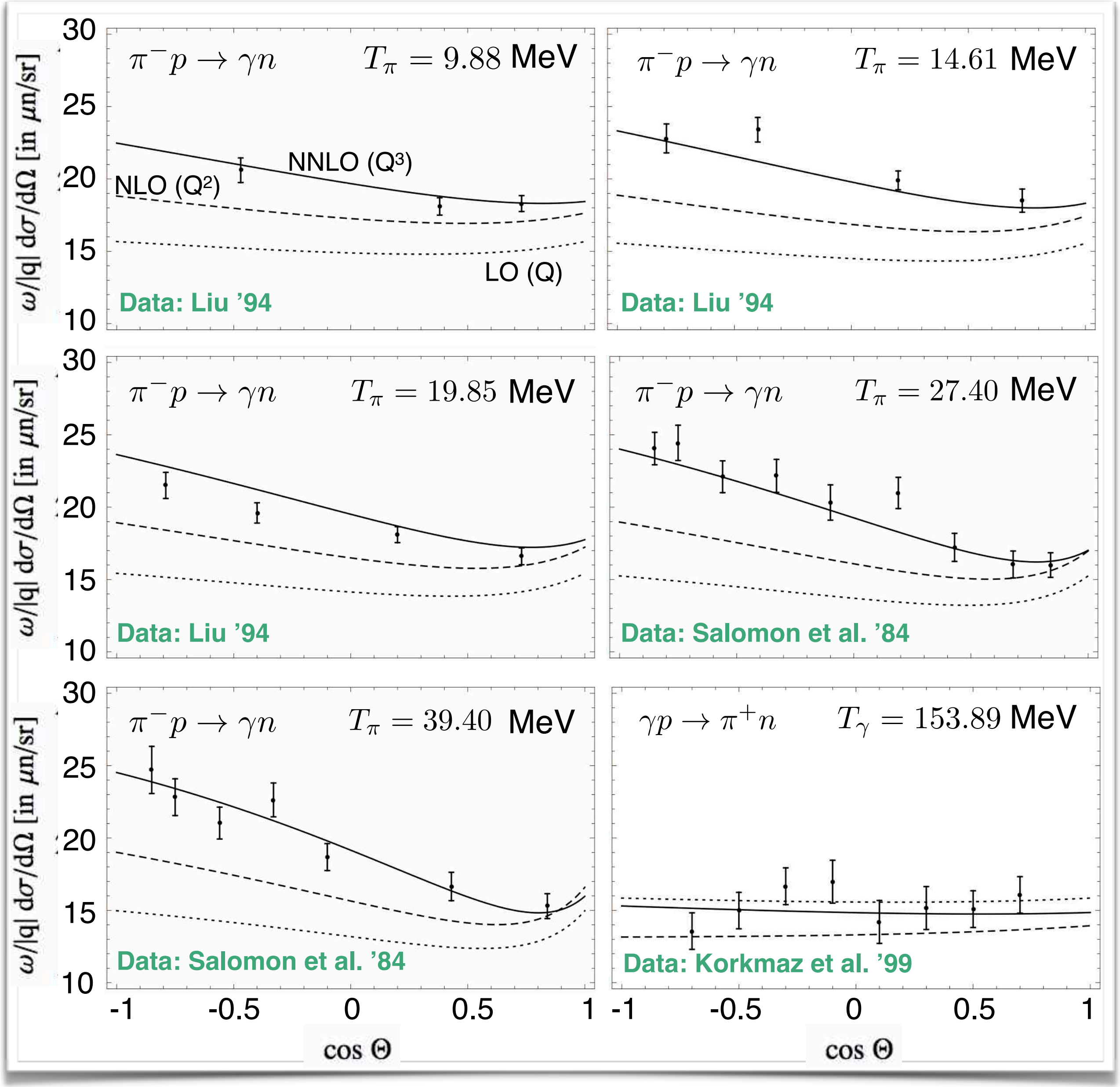
Chiral expansion of the electromagnetic **current** and **charge** operators





- **Exchange currents do not depend on k_0.**
- **Our results differ from the ones of the JLab-Pisa group (Pastore et al., 08-11)**

Low-energy constants



LECs entering the 1**π** current:

$$\bar{l}_6,\ \bar{d}_8,\ \bar{d}_9,\ \bar{d}_{18},\ \bar{d}_{21},\ \bar{d}_{22}$$

$\bar{l}_6$ - known from the **π** sector

$\bar{d}_{18}$ - known from GTD

$\bar{d}_{22}$ - from the axial radius:

$$\bar{d}_{22} = 2.2 \pm 0.2\ \mathrm{GeV}^{-2}$$

$\bar{d}_9,\ \bar{d}_{21},\ \bar{d}_{22}$ - contribute to charged pion photoproduction (radiative capture)

Fearing et al.'00
Till Wolf, master thesis, Bochum, 2013

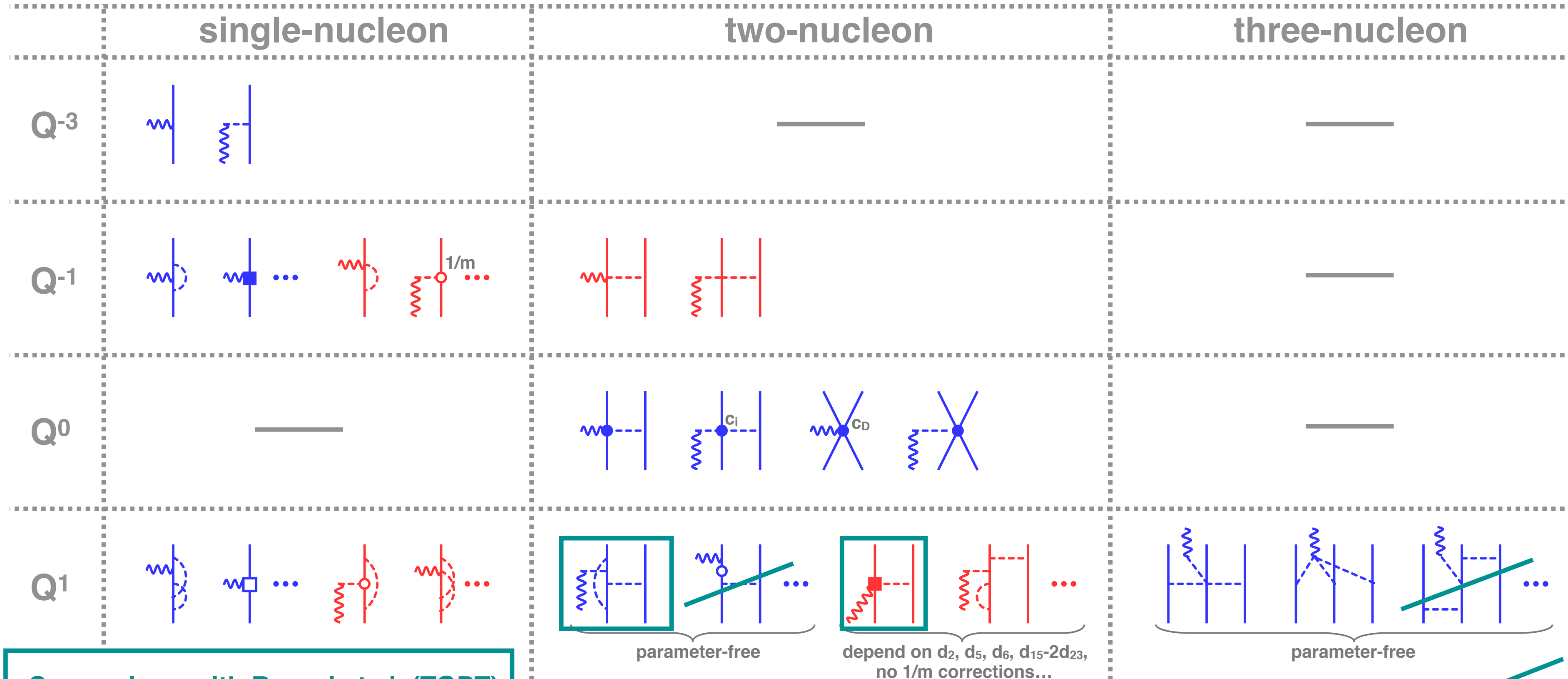
LEC [GeV^{-2}]	Fearing *et al.*	Wolf
$\bar{d}_9$	2.5 ± 0.8	2.2 ± 0.9
$\bar{d}_{20}$	-1.5 ± 0.5	-3.2 ± 0.5
$2\bar{d}_{21} - \bar{d}_{22}$	5.7 ± 0.8	6.8 ± 1.0

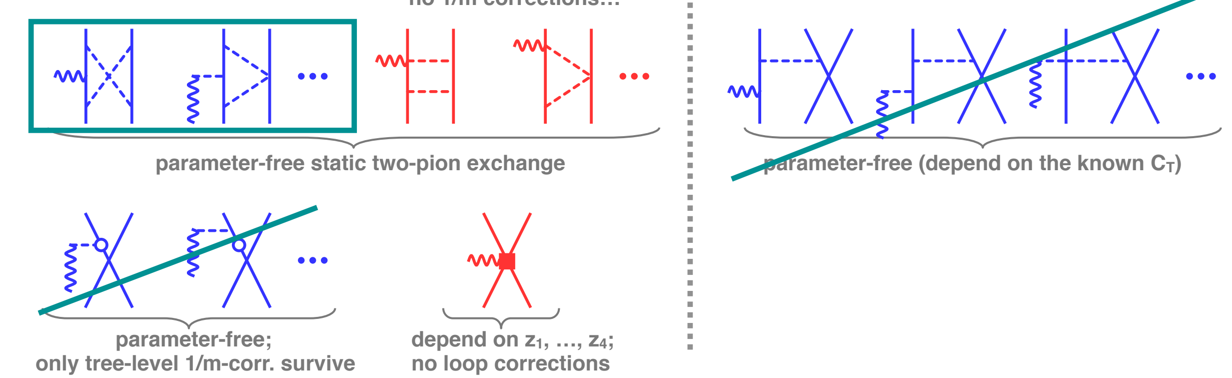
Some d_i's have been determined by Gasparyan, Lutz '10 (ChPT + disp. relations)

Exchange axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial **current** and **charge** operators





Comparison with Baroni et al. (TOPT)

- didn't consider 1/m-corrections at order Q^1
- looked only at irred. 3N graphs
- different results for π-exchange current contributions
- differences in tree-level 1π-terms

Selected applications

(Here main focus on ^{3}H beta decay…)

✅ **Accurate & precise NN potentials**

— new generation of semilocal r- and p-space NN potentials up to N^{4}LO$^+$
— currently the best description of the 2013 Granada data

☑️ **Consistent 3NFs**

— worked out to N^{3}LO (and even beyond), numerical PWD has been developed
— regularization nontrivial starting from N^{3}LO, 3NF@N^{2}LO ready to use

☑️ **Consistent currents**

— worked out to N^{3}LO, numerical PWD has been developed (at the 2N level…)
— consistent regularization to be done, axial currents@N^{2}LO ready to use

✅ **Error analysis**

— statistical & some systematics (truncation errors); Bayesian analysis…

State-of-the-art NN potentials

P. Reinert, H. Krebs, EE, arXiv:1711.08821[nucl-th], to appear in EPJA

— local regularization of the long-range interaction minimizes Λ-artifacts

$$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{p'^4+p^4}{\Lambda^4}}}{\vec{q}^{\,2}+M_\pi^2} \longrightarrow \frac{1}{\vec{q}^{\,2}+M_\pi^2}\left(1-\frac{p'^4+p^4}{\Lambda^4}+\mathcal{O}(\Lambda^{-8})\right)$$

$$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{\vec{q}^{\,2}+M_\pi^2}{\Lambda^2}}}{\vec{q}^{\,2}+M_\pi^2} \longrightarrow \frac{1}{\vec{q}^{\,2}+M_\pi^2}\left(1+\text{short-range terms}\right)$$

— p-space implementation of the regulator straightforwardly applicable to 3NFs & currents

— 3 contacts @ N^3LO out of 15 are found to be redundant ⟶ drastic simplification of the fits…

— πN LECs from the RS determination (no fine tuning)

— contact interactions are fitted to the 2013 Granada data base

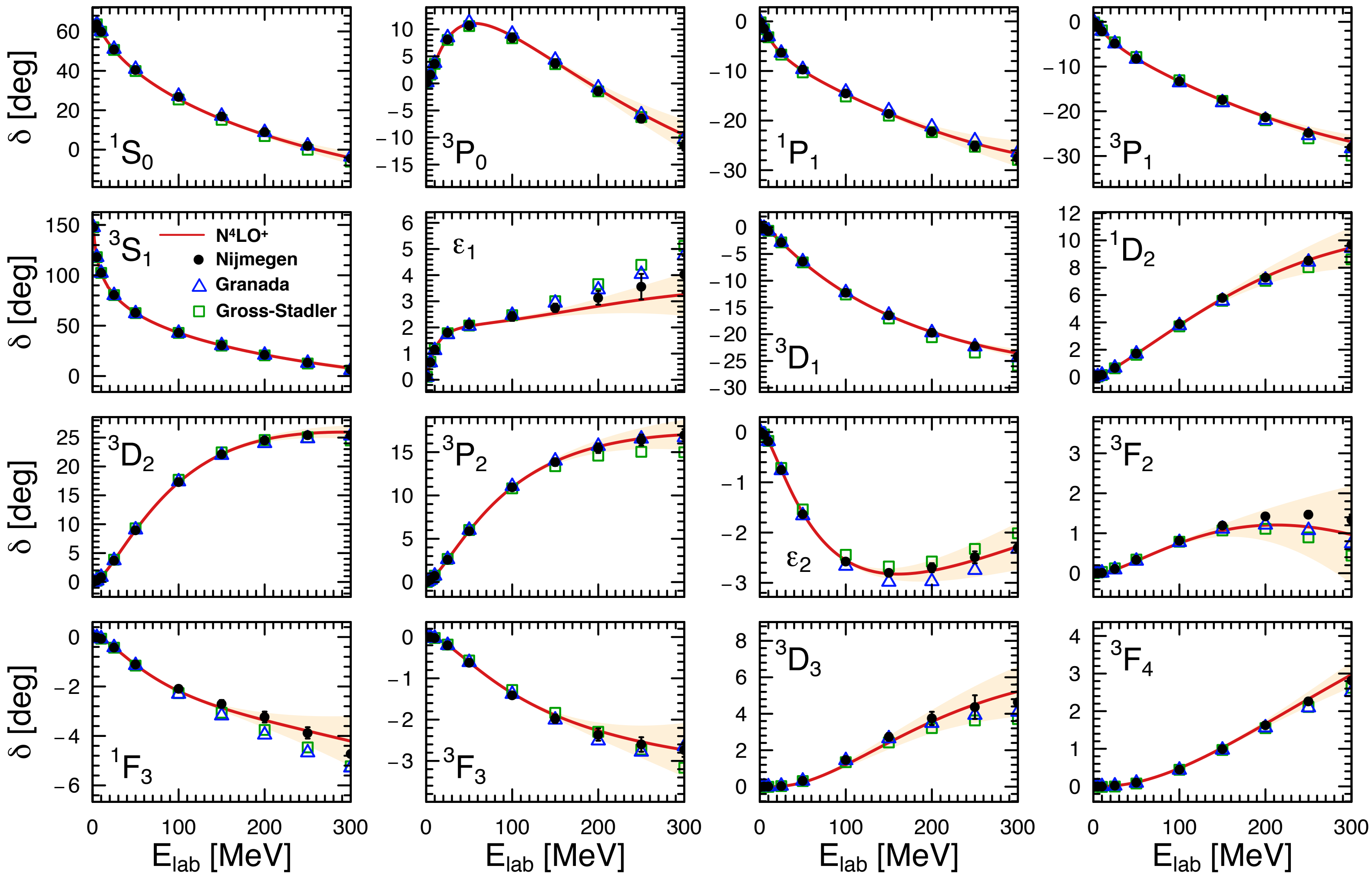
— developed for 5 cutoffs Λ = 350, 400, 450, 500 and 550 MeV and for LO…N^4LO^+ (N^4LO^+ includes 4 F-wave N^5LO contact interactions)

— the adopted choice of the redundant contacts at N^3LO leads to soft potentials

— comprehensive error analysis (statistical & systematic…)

State-of-the-art NN potentials

P. Reinert, H. Krebs, EE, arXiv:1711.08821[nucl-th], to appear in EPJA



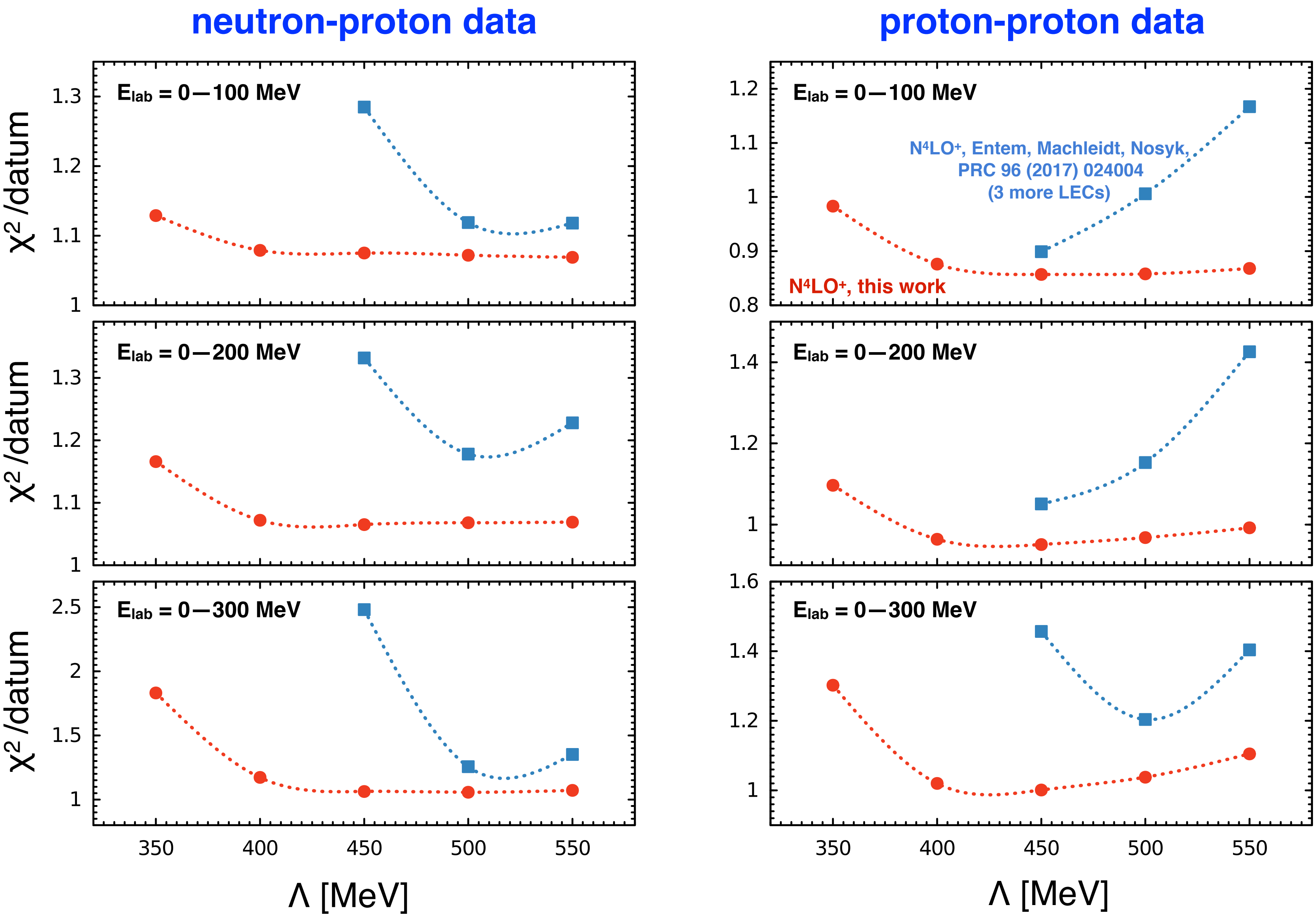
— N^4LO^+ yields currently the best description of np+pp data below E_{lab} = 300 MeV

— 40% less parameters (27+1) compared to high-precision potentials

— Clear evidence of the parameter-free chiral 2π exchange

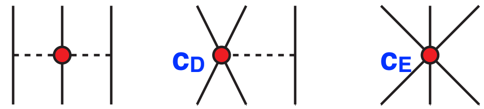
State-of-the-art NN potentials

P. Reinert, H. Krebs, EE, arXiv:1711.08821[nucl-th], to appear in EPJA



Three-nucleon forces

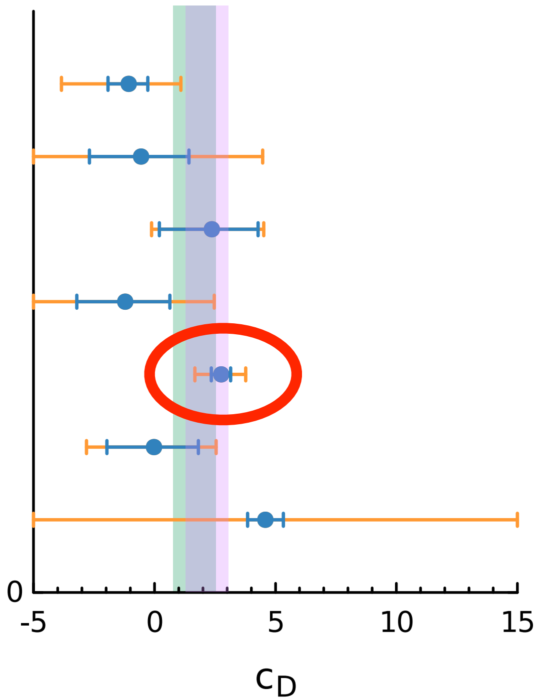
N²LO: tree-level graphs, 2 new LECs
van Kolck '94; EE et al '02



Determination of the LECs c_D, c_E

— Triton BE (c_D-c_E correlation)

— Explore various possibilities and let theory and/or data decide…



pd minimum of dσ/dθ at 135 MeV [Sekiguchi et al.'02]

nd σ_{tot} at 135 MeV [Abfalterer et al.'01]

pd minimum of dσ/dθ at 108 MeV [Ermisch et al.'03]

nd σ_{tot} at 108 MeV [Abfalterer et al.'01]

pd minimum of dσ/dθ at 70 MeV [Sekiguchi et al.'02]

nd σ_{tot} at 70 MeV [Abfalterer et al.'01]

nd scattering length ^2a [Schoen et al.'03]

LENPIC, to appear
(based on r-space-regularized potentials, R = 0.9 fm)

yields the strongest constraint…

LENPIC: Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration

Determination of c_D, c_E (preliminary)

Nd total cross section at 70 MeV (preliminary)

^{3}H beta decay (preliminary)

Summary and outlook

Nuclear Hamiltonian:

— derivation of contributions up to N³LO completed already in 2011; derivation of N⁴LO corrections done for V_{2N} and almost done for V_{3N} (new LECs…) and V_{4N}

— accurate & precise NN potentials at N^4LO^+ are available, implementation of many-body forces beyond N^2LO in progress **[LENPIC]**

Electroweak current operators:

— have been worked out completely to N^3LO

— 1N contributions expressible in terms of form factors

— some πN LECs in 1π axial charge at N^3LO are unknown…
[lattice QCD? ν-induced π-production? resonance saturation? large-N_c?…]

— 2N short-range e.m. current/axial charge involve a few new LECs

Next steps (in progress):

— Precision tests of the theory for ^{3}H β decay & μ capture (validation)

— Extension to other processes, heavier nuclei, N^4LO, explicit Δ's, …