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Two-body currents and weak decays in light nuclei

Exploring the role of electro-weak currents in Atomic Nuclei ECT* Trento, April 23-27, 2018

Petr Navratil

TRIUMF

Collaborators: P. Gysbers (UBC/TRIUMF), S. Quaglioni, K. Wendt (LLNL)



Discovery, accelerate

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11/05/2018

- Chiral forces and currents
- SRG evolution of forces and transition operators
- Calculations with the new N⁴LO NN by Entem, Machleidt and Nosyk
 - Beta decays of light nuclei in NCSM
- Modification of the c_D in the N³LO NN + 3N
 - Impact on medium mass nuclei
 - Beta decays of light nuclei
- Beta decay results for light nuclei with the N²LO_{sat}, Hebeler's EM 1.8/2.0, and an inconsistent N⁴LO NN + 3N

From QCD to nuclei





Nuclear structure and reactions

Chiral Effective Field Theory

- Inter-nucleon forces from chiral effective field theory
 - Based on the symmetries of QCD
 - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
 - Degrees of freedom: nucleons + pions
 - Systematic low-momentum expansion to a given order (Q/Λ_x)
 - Hierarchy
 - Consistency
 - Low energy constants (LEC)
 - Fitted to data
 - Can be calculated by lattice QCD

Λ_x~1 GeV : Chiral symmetry breaking scale



N⁴LO500 NNN³LO NN+N²LO 3N (NN+3N400, NN+3N500) + N²LO 3N

Currents in chiral EFT

Meson-exchange current



- weak axial current
 - one-body: LO Gamow-Teller

$$\boldsymbol{A}_{l} = -g_{A}\tau_{l} \boldsymbol{\sigma}_{l} \boldsymbol{\sigma}_{l} + \frac{2(\boldsymbol{p}_{l}\boldsymbol{\sigma}_{l} \cdot \boldsymbol{p}_{l} - \boldsymbol{\sigma}_{l} \boldsymbol{p}_{l}^{2}) + i\boldsymbol{q} \times \boldsymbol{p}_{l}}{4m_{N}^{2}} \right]$$

two-body: MEC

$$A_{12} = \frac{g_A}{2m_N f_\pi^2} \frac{1}{m_\pi^2 + k^2} \bigg[-\frac{i}{2} \tau_{\times} \boldsymbol{p} (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \boldsymbol{k} \\ + 4\hat{c}_3 \boldsymbol{k} \boldsymbol{k} \cdot (\tau_1^- \boldsymbol{\sigma}_1 + \tau_2^- \boldsymbol{\sigma}_2) + \left(\hat{c}_4 + \frac{1}{4}\right) \tau_{\times} \boldsymbol{k} \times [\boldsymbol{\sigma}_{\times} \times \boldsymbol{k}] \bigg] \\ + \frac{g_A}{m_N f_\pi^2} [2\hat{d}_1 (\tau_1^- \boldsymbol{\sigma}_1 + \tau_2^- \boldsymbol{\sigma}_2) + \hat{d}_2 \tau_{\times}^a \boldsymbol{\sigma}_{\times}],$$



From QCD to nuclei





No-core shell model

- No-core shell model (NCSM)
 - A-nucleon wave function expansion in the harmonic-oscillator (HO) basis
 - Short- and medium range correlations
 - Bound-states, narrow resonances
 - Equivalent description in relative-coordinate and Slater determinant basis

$$(A) \quad \Psi^{A} = \sum_{N=0}^{N_{\text{max}}} \sum_{i} c_{Ni} \Phi^{HO}_{Ni}(\vec{\eta}_{1}, \vec{\eta}_{2}, ..., \vec{\eta}_{A-1})$$

$$(A) \quad \Psi_{SD}^{A} = \sum_{N=0}^{N_{max}} \sum_{j} c_{Nj}^{SD} \Phi_{SDNj}^{HO}(\vec{r}_{1}, \vec{r}_{2}, \dots, \vec{r}_{A}) = \Psi^{A} \varphi_{000}(\vec{R}_{CM})$$



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Leading terms of the chiral NNN force and axial currents



Leading terms of the chiral NNN force and axial currents



Chiral EFT provides a link between the medium-range ($c_{\rm D}$ term) NNN force and the meson-exchange current appearing in nuclear beta decay



From QCD to nuclei



Similarity Renormalization Group (SRG) evolution

- Continuous transformation driving Hamiltonian to band-diagonal form with respect to a chosen basis
- Unitary transformation $H_{a} = U_{a} H U_{a}^{+} \qquad U_{a} U_{a}^{+} = U_{a}^{+} U_{a} = 1$ $\frac{dH_{a}}{da} = \frac{dU_{a}}{da} H U_{a}^{+} + U_{a} H \frac{dU_{a}^{+}}{da} = \frac{dU_{a}}{da} U_{a}^{+} U_{a} H U_{a}^{+} + U_{a} H U_{a}^{+} U_{a} \frac{dU_{a}^{+}}{da}$ $= \frac{dU_{a}}{da} U_{a}^{+} H_{a} + H_{a} U_{a} \frac{dU_{a}^{+}}{da} = [h_{a}, H_{a}]$ $= \text{Setting } h_{a} = [G_{a}, H_{a}] \text{ with Hermitian } G_{a}$ $\frac{dH_{a}}{da} = \oint [G_{a}, H_{a}], H_{a} \oiint$
- Customary choice in nuclear physics $G_{\alpha} = T$...kinetic energy operator
 - band-diagonal in momentum space plane-wave basis
- Initial condition $H_{\partial=0} = H_{/=1} = H$ $/^2 = 1/\sqrt{\partial}$
- Induces many-body forces
 - In applications to chiral interactions three-body induced terms large, four-body small



SRG evolution for A-nucleon system

Evolution induces many-nucleon terms (up to A)

$$H_{a} = H_{a}^{[1]} + H_{a}^{[2]} + H_{a}^{[3]} + H_{a}^{[4]} + \dots + H_{a}^{[A]}$$

SRG "magic" – $\tilde{H}_{\alpha}^{[2]}$ determined completely in A=2 system, $H_{\alpha}^{[3]}$ determined in A=3 system, etc.

• In actual calculations so far only terms up to $H_{a}^{[3]}$ kept

- Three types of SRG-evolved Hamiltonians used
 - NN only: Start with initial T+V_{NN} and keep
 - **NN+3N-induced**: Start with initial T+V_{NN} and keep
 - **NN+3N-full**: Start with initial T+V_{NN}+V_{NNN} and keep

 $\begin{array}{l} H_{a}^{[1]} + H_{a}^{[2]} \\ H_{a}^{[1]} + H_{a}^{[2]} + H_{a}^{[3]} \\ H_{a}^{[1]} + H_{a}^{[2]} + H_{a}^{[3]} \end{array}$

 α variation (\wedge variation) provides a diagnostic tool to asses the contribution of omitted many-body terms, tests the unitarity of the SRG transformation

SRG evolution of general operators

The SRG transformation maintains the same eigenvalues for the Hamiltonian

$$\hat{H} |\psi_k\rangle = E_k |\psi_k\rangle \to \hat{H}_\alpha |\psi_{k,\alpha}\rangle = E_k |\psi_{k,\alpha}\rangle$$

But to extract additional observables from the wavefunction while taking advantage of the SRG tranformation, the corresponding operators must be transformed

$$\left\langle \psi_{i}\right|\hat{O}\left|\psi_{f}\right\rangle =\left\langle \psi_{i,\alpha}\right|\hat{O}_{\alpha}\left|\psi_{f,\alpha}\right\rangle \text{where } \hat{O}_{\alpha}=U_{\alpha}\hat{O}U_{\alpha}^{\dagger}$$

The transformation matrix can be extracted from the eigenfunctions of the Hamiltonian

$$U_{\alpha} = \sum_{k} |\psi_{k,\alpha}\rangle \langle \psi_{k}|$$

 H_{α} , O_{α} : 2-body part determined in *A*=2 system, 3-body part determined in *A*=3 system,

. . .

SRG evolution of general operators

Peter Gysbers (UBC/TRIUMF)

Implementation up to two-body terms:

The matrix U is calculated blockwise, for relative coordinate two-nucleon eigenstates:

$$\left| (A=2)kJ^{\pi}TT_{z} \right\rangle = \sum_{n,\ell} c_{n\ell s}^{k} \left| n\ell s J^{\pi}TT_{z} \right\rangle$$

The corresponding submatrix of \hat{H} is evolved then diagonalized to produce a matrix $U_{\alpha}^{J^{\pi}TT_{z}}$

Compute the matrix elements of the bare operator: $\langle k'J'^{\pi'}T'T_z'||\hat{O}^{(K)}||kJ^{\pi}TT_z\rangle$

Matrix elements of the evolved operator are:

$$\langle k'J'^{\pi'}T'T_z', \alpha || U_{\alpha}^{J'^{\pi'}T'T_z'} \hat{O}^{(K)} U_{\alpha}^{\dagger J^{\pi}TT_z} || kJ^{\pi}TT_z, \alpha \rangle$$

SRG evolution of general operators

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Implementation up to two-body terms:

The matrix U is calculated blockwise, for relative coordinate two-nucleon eigenstates:

$$\left| (A=2)kJ^{\pi}TT_{z} \right\rangle = \sum_{n,\ell} c_{n\ell s}^{k} \left| n\ell s J^{\pi}TT_{z} \right\rangle$$

The corresponding submatrix of \hat{H} is evolved then diagonalized to produce a matrix $U_{\alpha}^{J^{\pi}TT_{z}}$

Compute the matrix elements of the bare operator: $\langle k'J'^{\pi'}T'T'_z||\hat{O}^{(K)}||kJ^{\pi}TT_z\rangle$

Matrix elements of the evolved operator are:

$$\langle k'J'^{\pi'}T'T'_z, \alpha || U_{\alpha}^{J'^{\pi'}T'T'_z} \hat{O}^{(K)} U_{\alpha}^{\dagger J^{\pi}TT_z} || kJ^{\pi}TT_z, \alpha \rangle$$

Converting from the two-nucleon Jacobi basis to the single particle basis:

$$\langle a'b'J'^{\pi'}T'T'_{z}||\,\hat{O}_{\alpha}^{(K)}\,||abJ^{\pi}TT_{z}\rangle \qquad a \equiv \{n_{a},\ell_{a},j_{a}\}$$
$$= \sum C_{n'\ell's'}^{*a'b'}C_{n\ell s}^{ab}\,\langle n'\ell's'J'^{\pi'}T'T'_{z}||\,\hat{O}_{\alpha}^{(K)}\,||n\ell sJ^{\pi}TT_{z}\rangle$$

Code NCSMV2B

The NN interaction from chiral EFT

PHYSICAL REVIEW C 96, 024004 (2017)

High-quality two-nucleon potentials up to fifth order of the chiral expansion

D. R. Entem,^{1,*} R. Machleidt,^{2,†} and Y. Nosyk²



- Chiral NN potential up to N⁴LO
- Set of five potentials constructed
 - Sequence of LO, NLO,...,N⁴LO
 - Uncertainty quantification
- At N³LO and N⁴LO:
 - 24 LECs fitted to the *np* scattering data and the deuteron properties
 - Including c_i LECs (i=1-4) from pionnucleon scattering
- N⁴LO NN fitted to data up to pion production threshold with χ²/datum~1.15



³H and ⁴He with chiral EFT interactions up to N⁴LO

Properties of ³H, ³He and ⁴He with the new N⁴LO NN

- N⁴LO500 NN
 - ³H
 - E = -8.08 MeV
 - point-proton radius = 1.62 fm
 - ³He
 - E = -7.33 MeV
 - point-proton radius = 1.82 fm
 - ⁴He
 - E= -26.61(2) MeV
 - point-proton radius = 1.475(5) fm

Peter Gysbers (UBC/TRIUMF)

$$\hat{O} = GT^{(1)} ! \hat{O}_{e'} = GT^{(1)} + GT^{(2)}_{e'} + \dots$$

Operator:

Gamow-Teller (1-body) $hGT_{e'}^{(2)}i_{A=2} = h(GT^{(1)})_{e'}i_{A=2} - hGT^{(1)}i_{A=2}$

Potential: "N⁴LO NN"

■ chiral NN @N⁴LO, Machleidt PRC96 (2017), 500MeV cuto



Hamiltonian: chiral NN with SRG 2- and 3-body induced (except orange line: bare chiral NN)

Peter Gysbers (UBC/TRIUMF)



chiral NN with SRG 2- and 3-body induced

³H \rightarrow ³He β decay

Peter Gysbers (UBC/TRIUMF)

 \rightarrow $GT^{(1)} + GT^{(2)}_{\lambda}$

 $----- GT^{(1)} + MEC^{(2)}$

- $GT^{(1)} + GT^{(2)}_{\lambda} + MEC^{(2)}_{\lambda}$

 $\lambda = 1$, $\overline{h} \boxtimes = 28$

= 1.6

 $\lambda = 1.8$

 $\lambda = 2.0$

$$\hat{O} = GT^{(1)} + MEC^{(2)}$$
 ! $\hat{O}_{e'} = GT^{(1)} + GT^{(2)}_{e'} + MEC^{(2)}_{e'} + \dots$

1.74

1.72

Operator:

Gamow-Teller (1-body) + chiral meson exchange current (2-body) Park (2003)



Properties of ³H, ³He and ⁴He with the new N⁴LO NN + N²LO 3N_{InI}

Preliminary

N⁴LO500 NN

```
■ <sup>3</sup>H
               E = -8.08 \text{ MeV}
               point-proton radius = 1.62 \text{ fm}
       <sup>3</sup>He
               E = -7.33 MeV
               point-proton radius = 1.82 fm
       4He
               E= -26.60(1) MeV
               point-proton radius = 1.475(5) fm
N<sup>4</sup>LO500 NN + N<sup>2</sup>LO 3N<sub>In1</sub>
       3N fitted to {}^{3}\text{H}\beta decay and binding energy
       c_1= -0.73, c_3= -3.38, c_4= 1.69, \Lambda_{loc}= 650 MeV, \Lambda_{nonloc}= 500 MeV, c_D= -1.8, c_E= -0.31
       ■ <sup>3</sup>H
               E = -8.48 \text{ MeV} < V_{3N2\pi} >= -0.54 \text{ MeV} < V_{3ND} >= -0.32 \text{ MeV} < V_{3NE} >= 0.40 \text{ MeV}
               point-proton radius = 1.60 \text{ fm}
       <sup>3</sup>He
               E = -7.73 \text{ MeV}
               point-proton radius = 1.78 fm
            <sup>4</sup>He
       E = -28.25(2) \text{ MeV} \langle V_{3N2pi} \rangle = -2.16(5) \text{ MeV} \langle V_{3ND} \rangle = -2.22(5) \text{ MeV} \langle V_{3NE} \rangle = 2.41(5) \text{ MeV}
               point-proton radius = 1.46(1) fm
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Applications to β decays in p-shell nuclei and beyond

- Does inclusion of the MEC explain g_A quenching?
- In light nuclei correlations present in *ab initio* (NCSM) wave functions explain almost all of the quenching compared to the standard shell model
 - MEC inclusion overall improves agreement with experiment
- The effect of the MEC inclusion is greater in heavier nuclei
- SRG evolved matrix elements used in coupled-cluster and IM-SRG calculations (up to ¹⁰⁰Sn)



Hollow symbols – GT Filled symbols – GT+MEC Both Hamiltonian and operators SRG evolved



Original N³LO NN + 3N(400,500) vs. N²LO_{sat} vs. new N³LO NN + 3N_{Inl}



- Local: chiral N³LO NN + N²LO 3N(500)
 - c_D=-0.2 c_E=-0.205 (³H E_{gs}=-8.48 MeV)
 - 4He

<H>=-28.4939 <V3b_2pi>= -5.8819 <V3b_D>= -0.2206 <V3b_E>= 1.2665

- Local: chiral N³LO NN + N²LO 3N(400)
 - $c_D = -0.2$ $c_E = +0.098$ (³H E_{gs} = -8.32 MeV)
 - 4He

<H>=-28.2839 <V3b_2pi>= -2.7173 <V3b_D>= -0.2801 <V3b_E>= -0.6630

- Non-local: chiral N²LO_{sat} NN + 3N
 - c_D=+0.8168 c_E=-0.0396 (³H E_{gs}=-8.53 MeV)
 ⁴He

<H>=-28.4596 <V3b_2pi>= -4.7260 <V3b_D>= 1.3897 <V3b_E>= 0.4174

Local/Non-local: chiral N³LO NN + N²LO 3N_{Inl}

•
$$c_{D}$$
=+0.7 c_{E} =-0.06 (³H E_{gs}=-8.44 MeV)

<H>=-28.2530 <V3b_2pi>= -4.8124 <V3b_D>= 0.7414 <V3b_E>= 0.4255



Leading terms of the chiral NNN force and axial currents



Chiral EFT provides a link between the medium-range ($c_{\rm D}$ term) NNN force and the meson-exchange current appearing in nuclear beta decay





Dawning of the N = 32 Shell Closure Seen through Precision Mass Measurements of Neutron-Rich Titanium Isotopes

E. Leistenschneider,^{1,2,*} M. P. Reiter,^{1,3} S. Ayet San Andrés,^{3,4} B. Kootte,^{1,5} J. D. Holt,¹ P. Navrátil,¹ C. Babcock,¹
C. Barbieri,⁶ B. R. Barquest,¹ J. Bergmann,³ J. Bollig,^{1,7} T. Brunner,^{1,8} E. Dunling,^{1,9} A. Finlay,^{1,2} H. Geissel,^{3,4} L. Graham,¹
F. Greiner,³ H. Hergert,¹⁰ C. Hornung,³ C. Jesch,³ R. Klawitter,^{1,11} Y. Lan,^{1,2} D. Lascar,^{1,†} K. G. Leach,¹² W. Lippert,³
J. E. McKay,^{1,13} S. F. Paul,^{1,7} A. Schwenk,^{11,14,15} D. Short,^{1,16} J. Simonis,¹⁷ V. Somà,¹⁸ R. Steinbrügge,¹ S. R. Stroberg,^{1,19}
R. Thompson,²⁰ M. E. Wieser,²⁰ C. Will,³ M. Yavor,²¹ C. Andreoiu,¹⁶ T. Dickel,^{3,4} I. Dillmann,^{1,13} G. Gwinner,⁵
W. R. Plaß,^{3,4} C. Scheidenberger,^{3,4} A. A. Kwiatkowski,^{1,13} and J. Dilling^{1,2}

TRIUMF TITAN Penning trap mass measurements & several ab initio calculations

New N³LO NN + 3N_{InI} applied to Ti isotopes





Hollow symbols – GT Filled symbols – GT+MEC



Chiral NNN interaction with N⁴LO contacts

PHYSICAL REVIEW C 84, 014001 (2011)

Subleading contributions to the three-nucleon contact interaction

L. Girlanda,¹ A. Kievsky,² and M. Viviani² ¹Dipartimento di Fisica, Università del Salento, and INFN Sezione di Lecce, Via Arnesano, I-73100 Lecce, Italy ²INFN, Sezione di Pisa, Largo Bruno Pontecorvo 3, I-56127 Pisa, Italy



Progress in the quest for a realistic three-nucleon force

Luca Girlanda*

Department of Mathematics and Physics, University of Salento, and INFN Lecce, Italy E-mail: luca.girlanda@le.infn.it

Alejandro Kievsky INFN Pisa, Italy E-mail: alejandro.kievsky@pi.infn.it

Michele Viviani INFN Pisa, Italy

INFN Pisa, Italy	
E-mail: michele.viviani@pi.infn.it	

Laura Elisa Marcucci

Department of Physics, University of Pisa, and INFN Pisa, Italy E-mail: laura.elisa.marcucci@unipi.it

- Ten contact terms
- Include spin-orbit contribution that helps to fix the famous A_v puzzle in p-d scattering

$$V^{(2)} = \sum_{i \neq j \neq k} \left(E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik})$$

$$+ (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik})$$

$$+ (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\mathbf{L} \cdot \boldsymbol{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik})$$

$$+ (E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})$$

• I used the E_7 (and E_8) terms

Chiral NNN interaction with N⁴LO contacts



120

90

60

30

 $\delta \, \, [deg]$

 $^2P_{3}$,

 ${}^{2}P_{1/2}$

 ${}^{2}D_{3}{}_{/_{2}}$

× ×.

8

10

16

NN+3N(500)



Conclusions and outlook

- Ab initio calculations of beta decays with SRG evolved chiral forces:
 SRG evolution of the transition operators essential
- Role of 2-body currents small in light nuclei
 - The same seen in the GFMC calculations
- We find a systematic improvement when consistent forces and currents are included
- Corrected c_D term in N³LO NN + 3N leads to a substantially better description of medium mass nuclei
- SRG evolved transition operators provided for VS-IMSRG and CCM medium mass nuclei calculations

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Thank you! Merci!

