Electromagnetic nuclear responses

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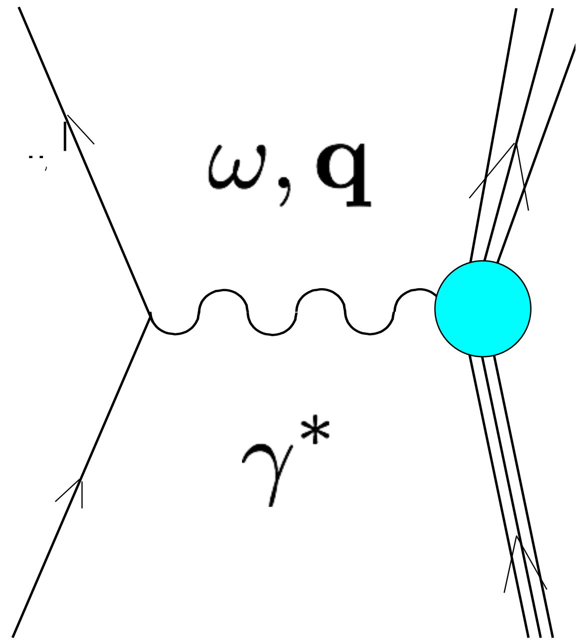
Johannes Gutenberg Universität Mainz and TRIUMF

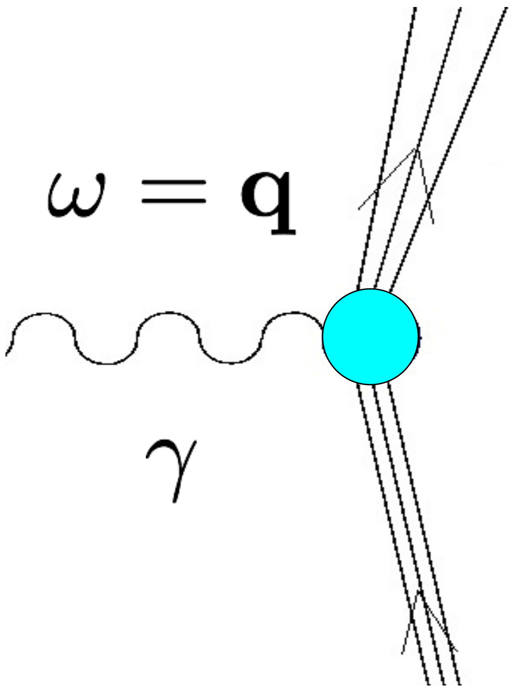
April 24th, 2018

ECT* Workshop on "Exploring the role of electroweak currents in atomic nuclei"

Electromagnetic probes

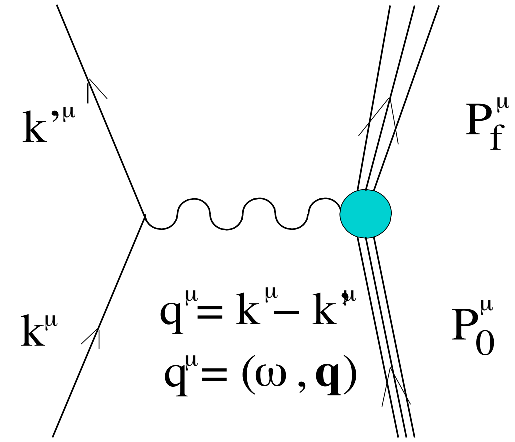


Electron scattering



Photoabsorption

Inclusive electron scattering

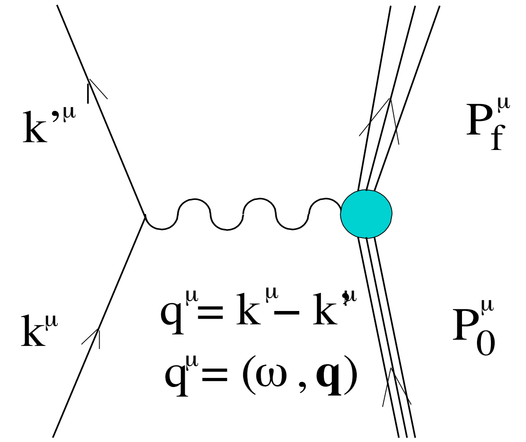


$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega,\mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega,\mathbf{q}) \right]$$

with $Q^2 = -q_\mu^2 = \mathbf{q}^2 - \omega^2$ and θ scattering angle

and σ_M Mott cross section

Inclusive electron scattering



$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega,\mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega,\mathbf{q}) \right]$$

$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

charge operator

$$R_T(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,J_T(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

current operator

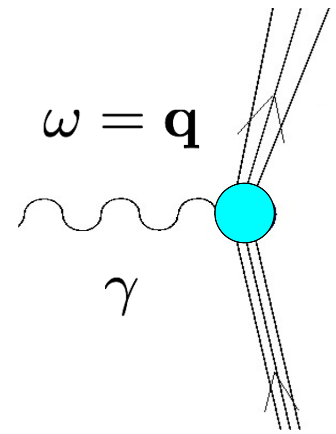
$$\rho = \rho_{(1)} + \rho_{(2)} + \dots = \sum_i^A \rho_i + \sum_{i<j}^A \rho_{ij} + \dots$$

2BC at N3LO

$$\mathbf{J} = \mathbf{J}_{(1)} + \mathbf{J}_{(2)} + \dots = \sum_i^A \mathbf{J}_i + \sum_{i<j}^A \mathbf{J}_{ij} + \dots$$

2BC at NLO

Photoabsorption



$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) \overset{\omega = \mathbf{q}}{R_T(\omega, \mathbf{q})} \right]$$

Photoabsorption

$$R_T(\omega=\mathbf{q}) \rightarrow |\langle\Psi_f|J_T(q)|\Psi_0\rangle|^2 = \sum_{\lambda=\pm1} |\langle\Psi_f|J_\lambda(q)|\Psi_0\rangle|^2$$

Multipole expansion

$$J_\lambda(q) \longrightarrow T^{el}_{J\pm1} = -\frac{1}{4\pi}\int d\hat{q}'\ \sqrt{\frac{J+1}{J}}\,\hat{\mathbf{q}}'\cdot\mathbf{J}(\mathbf{q}')\ Y^J_\mu(\hat{q}') + \dots$$

Siegert theorem: using continuity equation

$$T^{el}_{J\mu}(q) = -\frac{1}{4\pi}\sqrt{\frac{J+1}{J}}\frac{\omega}{q}\int d\hat{q}'\rho(\mathbf{q}')Y^J_\mu(\hat{q}') + \dots \leftarrow \quad C_{J\mu}$$

Coulomb Multipole

$$C_{1\pm1} \rightarrow Y^1(\hat{r})j(qr) \overset{\text{low q}}{\rightarrow} Y^1(\hat{r})\ qr \ \rightarrow \omega\mathbf{r}$$

Dipole operator

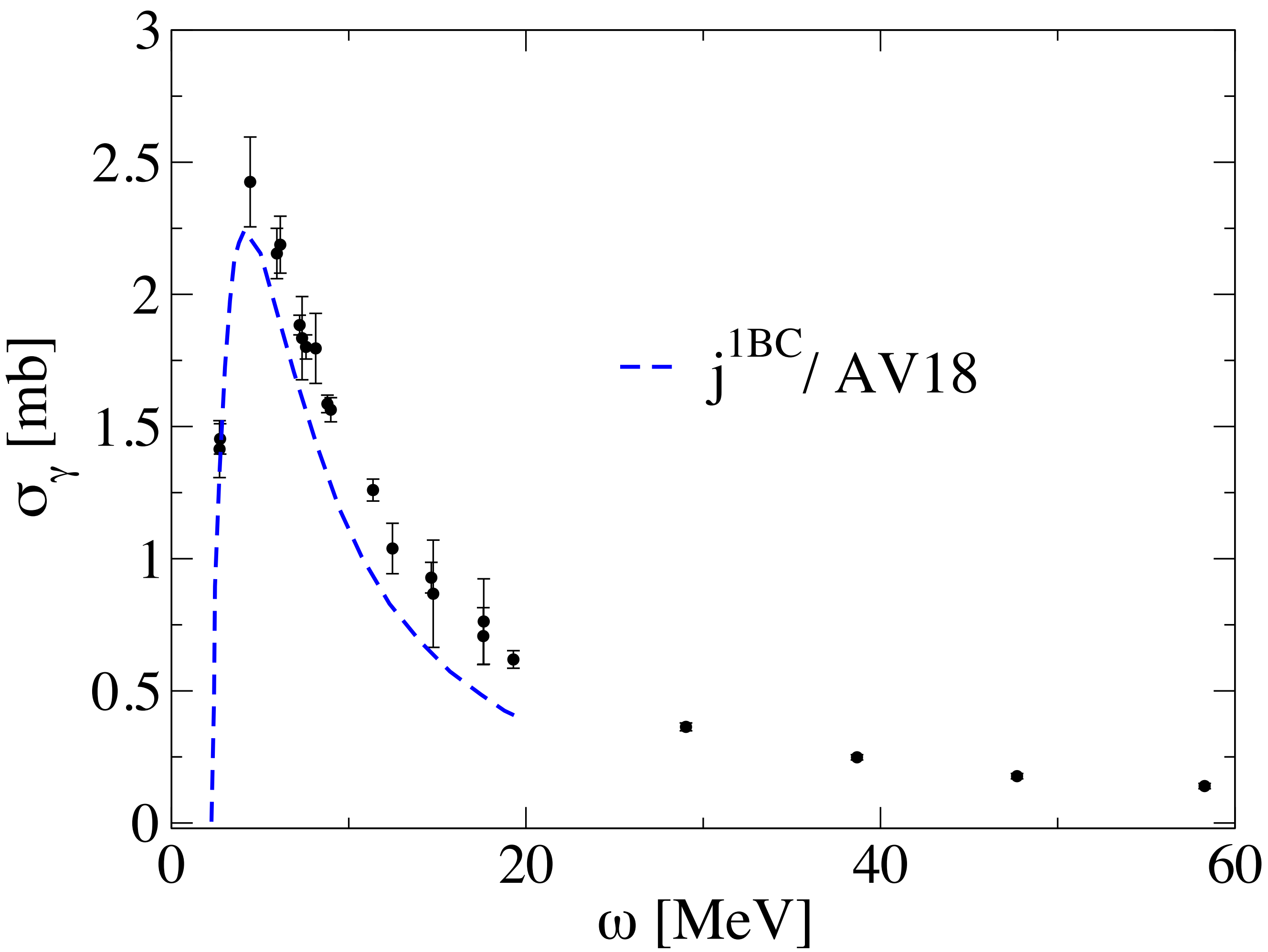
Need to calculate the response to a dipole operator, which is a one-body operator

2BC are implicitly (via continuity equation) included

Classical Example ^2H

From S. Bacca and S. Pastore, J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014).

Work by Pisa and Trento groups



$$\sum_{\lambda=\pm 1} |\langle \Psi_f | J_\lambda(q) | \Psi_0 \rangle|^2$$

1BC

Using the one-body current only it is not enough to explain data

Classical Example ^{2}H

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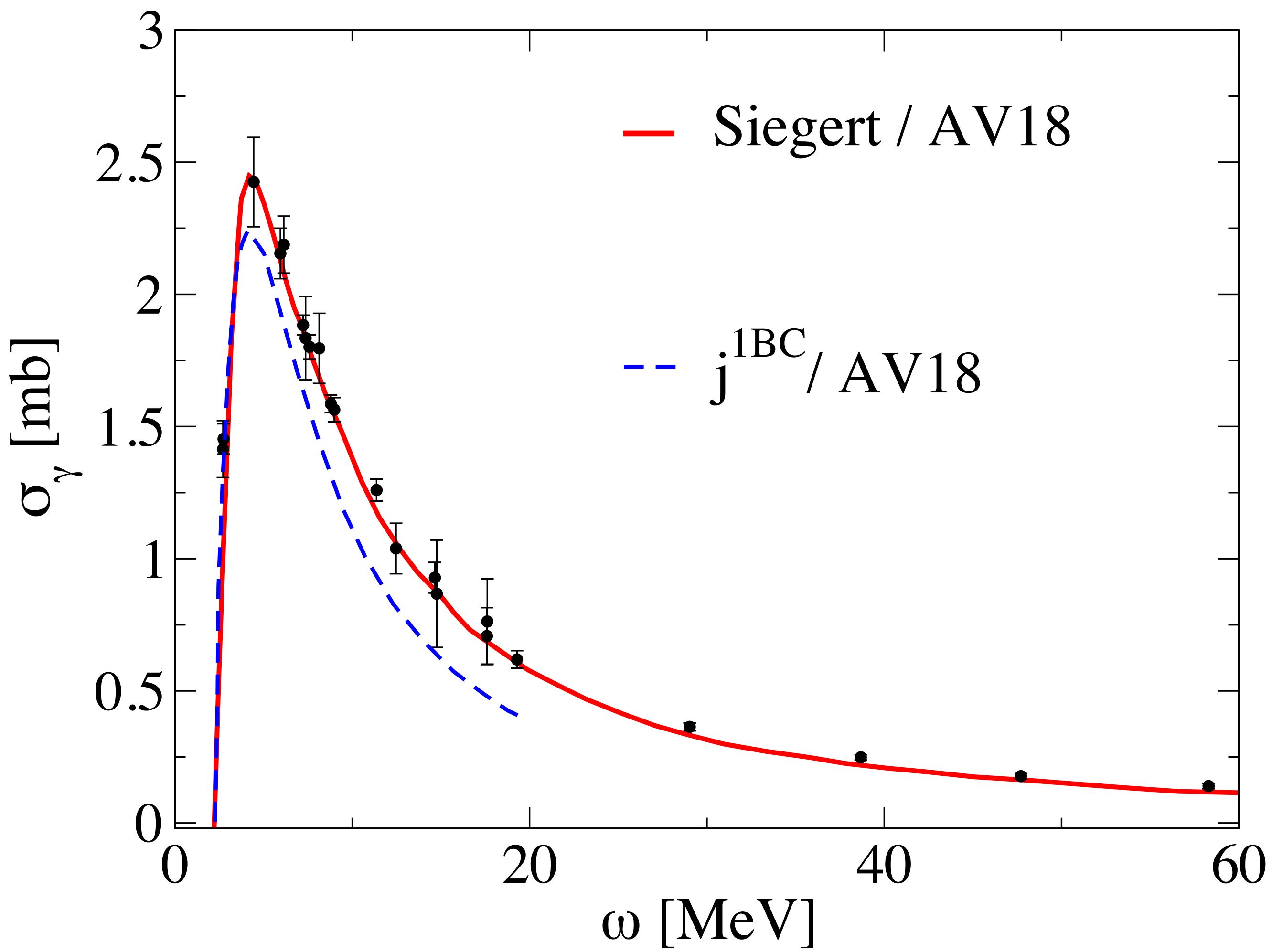

$$|\langle \Psi_f | \, D \, | \Psi_0 \rangle|^2$$

Siegert

$$\sum_{\lambda=\pm1} |\langle \Psi_f | J_\lambda(q) | \Psi_0 \rangle|^2$$

1BC

Using the one-body current only it is not enough to explain data

The Siegert operator explains data

Classical Example ^{2}H

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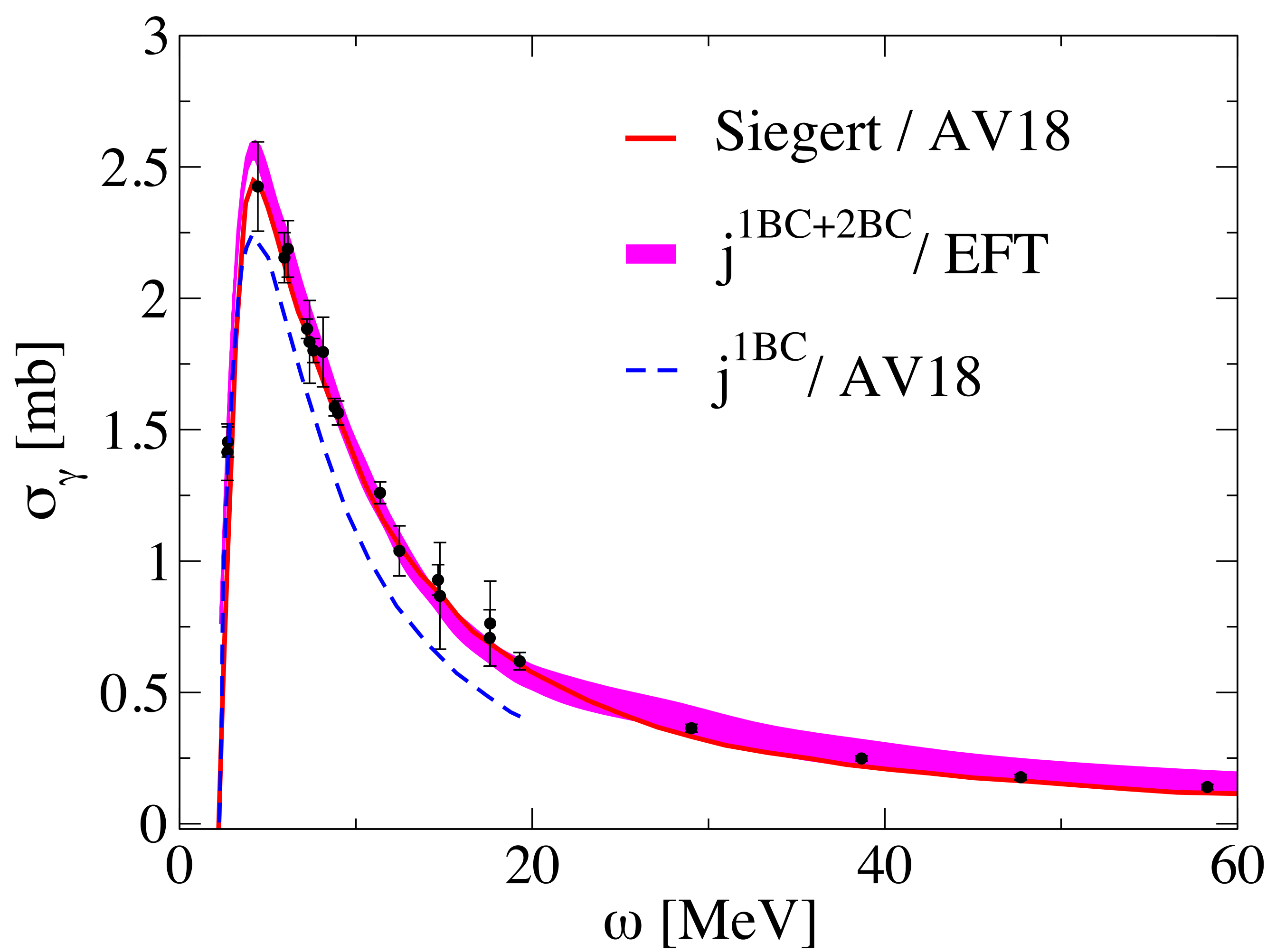


$$|\langle \Psi_f | D | \Psi_0 \rangle|^2$$

Siegert

$$\sum_{\lambda=\pm 1} |\langle \Psi_f | J_\lambda(q) | \Psi_0 \rangle|^2$$

1BC

1BC+2BC

Using the one-body current only it is not enough to explain data

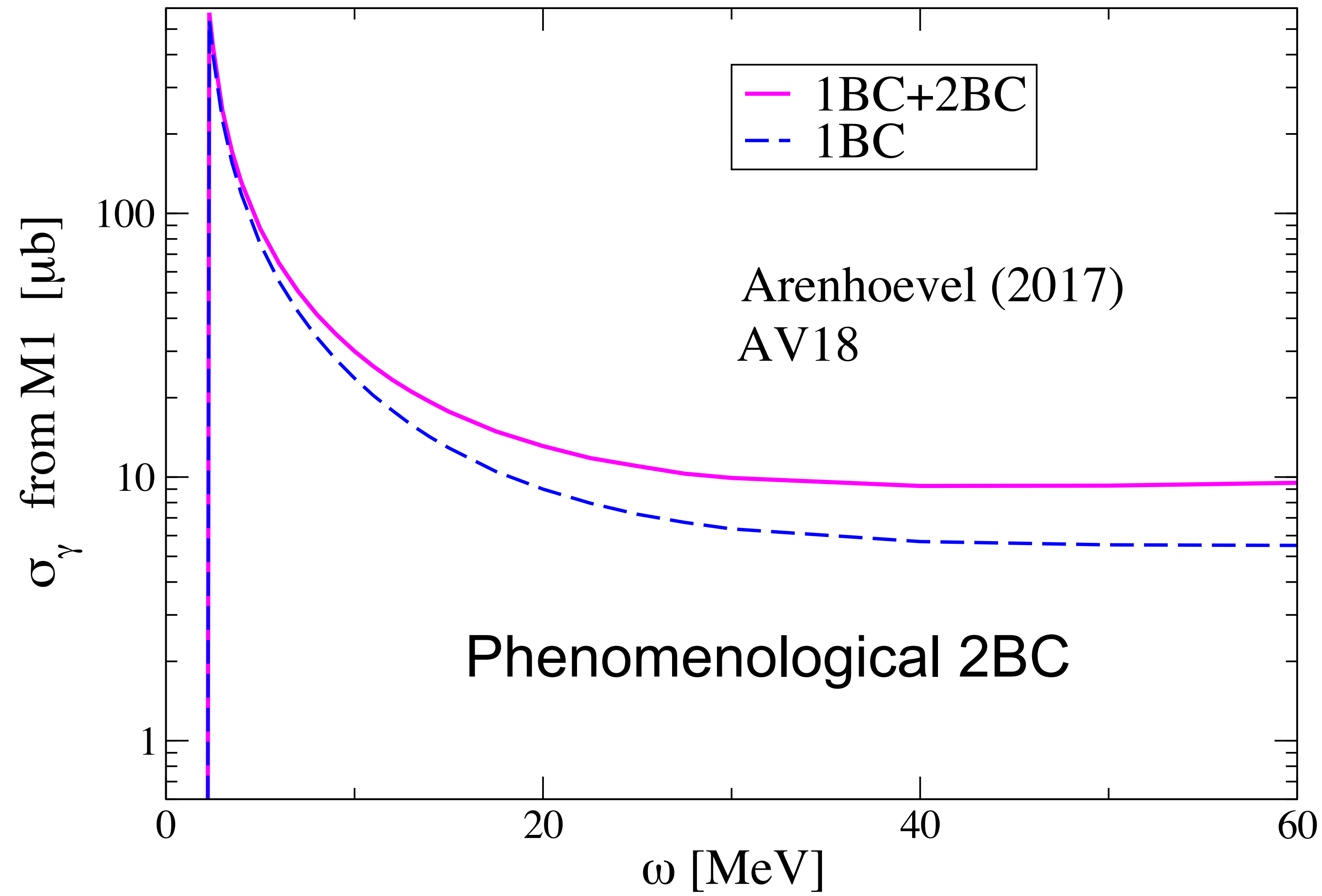
The Siegert operator explains data and agrees with full 1BC+2BC calculation

Magnetic Response ^2H

$$\overset{M1}{R}(\omega) = \frac{1}{2J_0+1} \sum\!\!\!\!\!\!\int_f \left|\langle \Psi_f || \boldsymbol{\mu} || \Psi_0 \rangle\right|^2 \delta\left(E_f - E_0 - \omega\right)$$

There is no Siegert theorem for magnetic multipoles. 2BC have to be calculated explicitly.

$$\boldsymbol{\mu} = \sum_i \boldsymbol{\mu}_i + \sum_{i<j} \boldsymbol{\mu}_{ij}$$



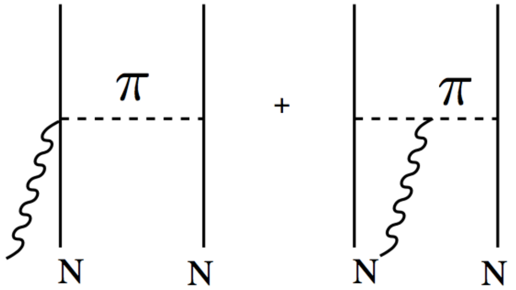
Magnetic Response ^{2}H

In chiral EFT Hernandez, Bacca, Wendt, PoS BORMIO2017 (2017), C17-01-23

Magnetic sum-rules in ^{2}H $\quad m_n = \int_0^\infty d\omega\, \omega^n \overset{\text{M1}}{R}(\omega)$

$$\boldsymbol{\mu}_i^{\text{LO}} = \mu_N \left[\left(\frac{\mu^S + \mu^V \tau_i^3}{2} \right) \boldsymbol{\sigma}_i + \left(\frac{1+\tau_i^3}{2} \right) \boldsymbol{\ell}_i \right]$$

$$\boldsymbol{\mu}_{ij}^{\text{NLO}} = -\frac{e g_A^2 m}{8\pi F_\pi^2} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^3 \left[\left(1 + \frac{1}{mr} \right) ((\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \right] e^{-mr}$$



	m_{-1}	m_0	
LO	14.0 fm^3	0.245 fm^2	1-body
LO+ NLO	15.1 fm^3	0.277 fm^2	1-body+2-body
2BC effect	8%	13%	

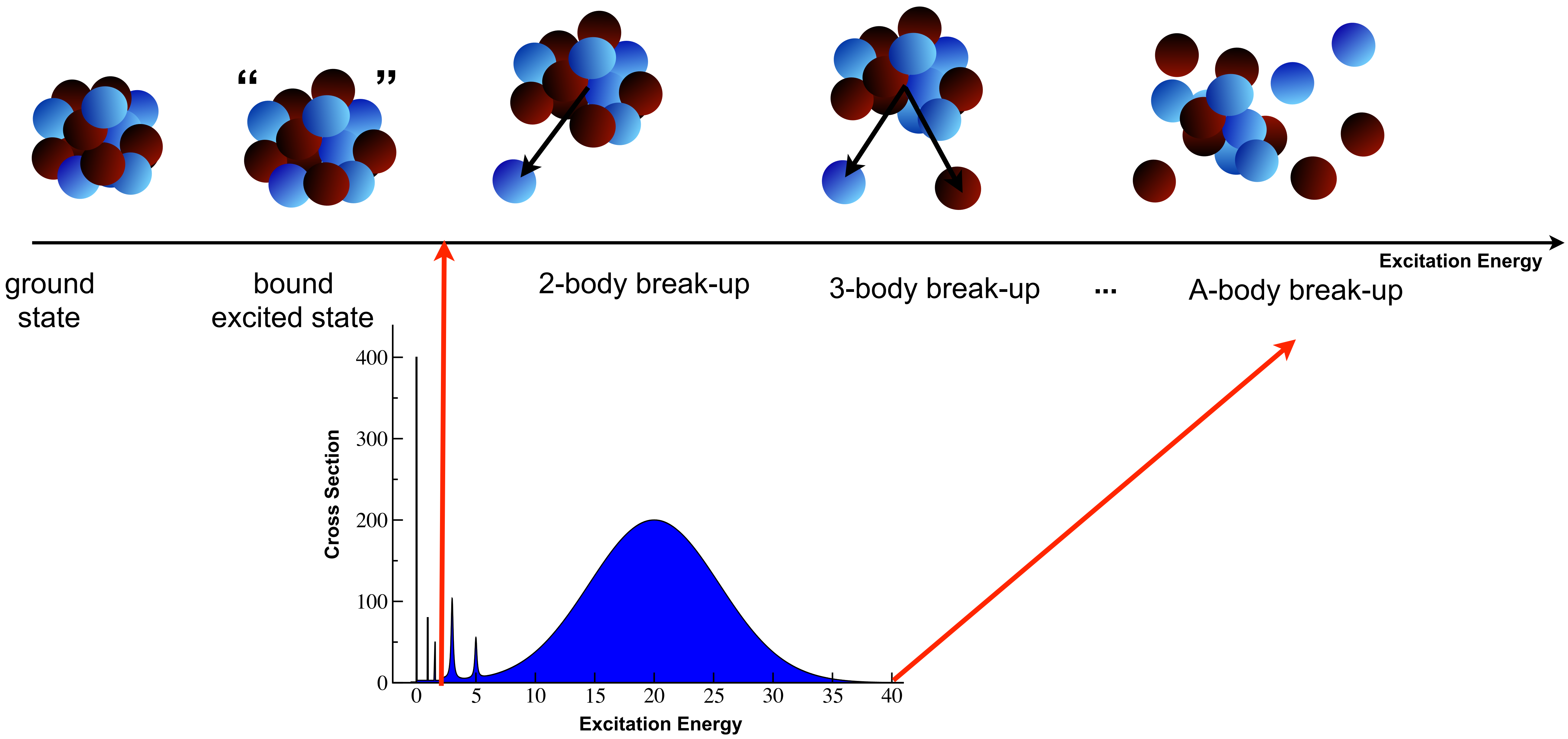
2BC in precision physics → Muonic Atoms, talk by Nir Barnea

What about other more complex nuclear systems?

Continuum problem

$$R(\omega) = \sum\!\!\!\!\!\!\int_f \left| \left\langle \psi_f \middle| \Theta \middle| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

Depending on E_f, many channels may be involved



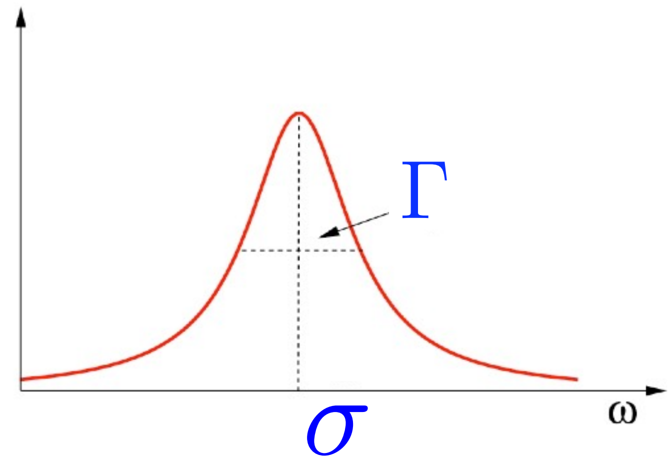
How do we address it?

LIT Lorentz Integral Transform

A method that allows to circumvent the continuum problem by reducing it to the solution of a bound-state-like equation

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

Trento



$$L(\sigma,\Gamma) = \frac{\Gamma}{\pi}\int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \langle\tilde{\psi}|\tilde{\psi}\rangle < \infty$$

$$(H - E_0 - \sigma + i\Gamma)\,|\,\tilde{\psi}\rangle = \Theta\,|\,\psi_0\rangle$$

Schrödinger-like equation
bound-state-like

It has been solved with hyperspherical harmonics, no-core shell-model.
S-shell nuclei and selected p-shell nuclei have been addressed

Example where 2BC where studied

^{3}He → Study of $R_T(\omega, \mathbf{q})$

Inelastic Electron Scattering

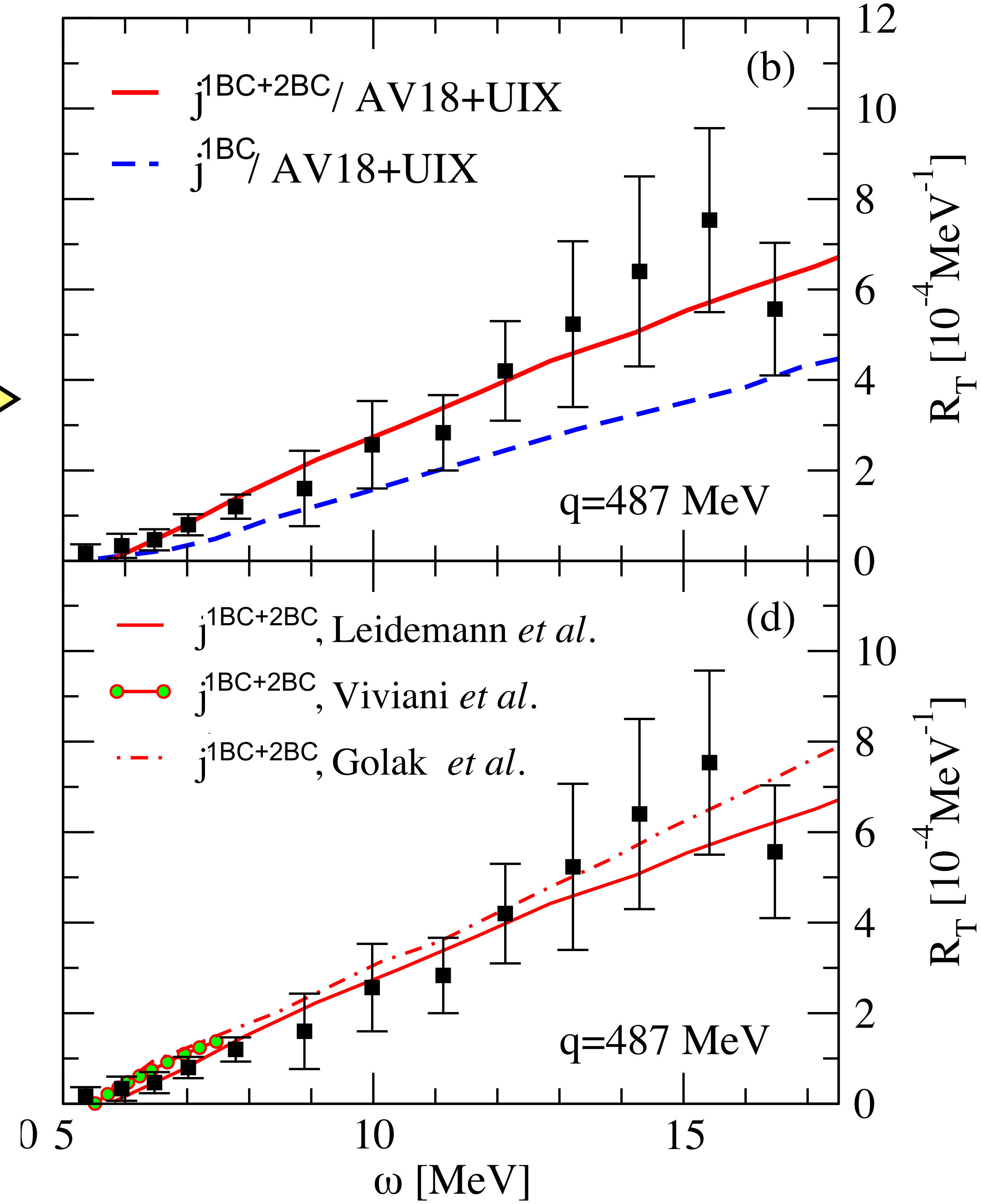
From S. Bacca and S. Pastore, J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014).

Phenomenological 2BC

Work by Pisa, Cracow and Trento groups

2BC enhance strength by factor 2

Different calculations with 2BC are within experimental error bars



What about heavier nuclei?

First we need to develop a method that is capable of calculating response functions for medium-mass nuclei

Many-body formulation of LIT

LIT Lorentz Integral Transform

A method that allows to circumvent the continuum problem by reducing it to the solution of a bound-state-like equation

$$(H - E_0 - \sigma + i\Gamma) \mid \tilde{\psi}\rangle = \Theta \mid \psi_0\rangle$$

+

CC Coupled-cluster theory

Accurate many-body theory with mild polynomial scaling in mass number

=

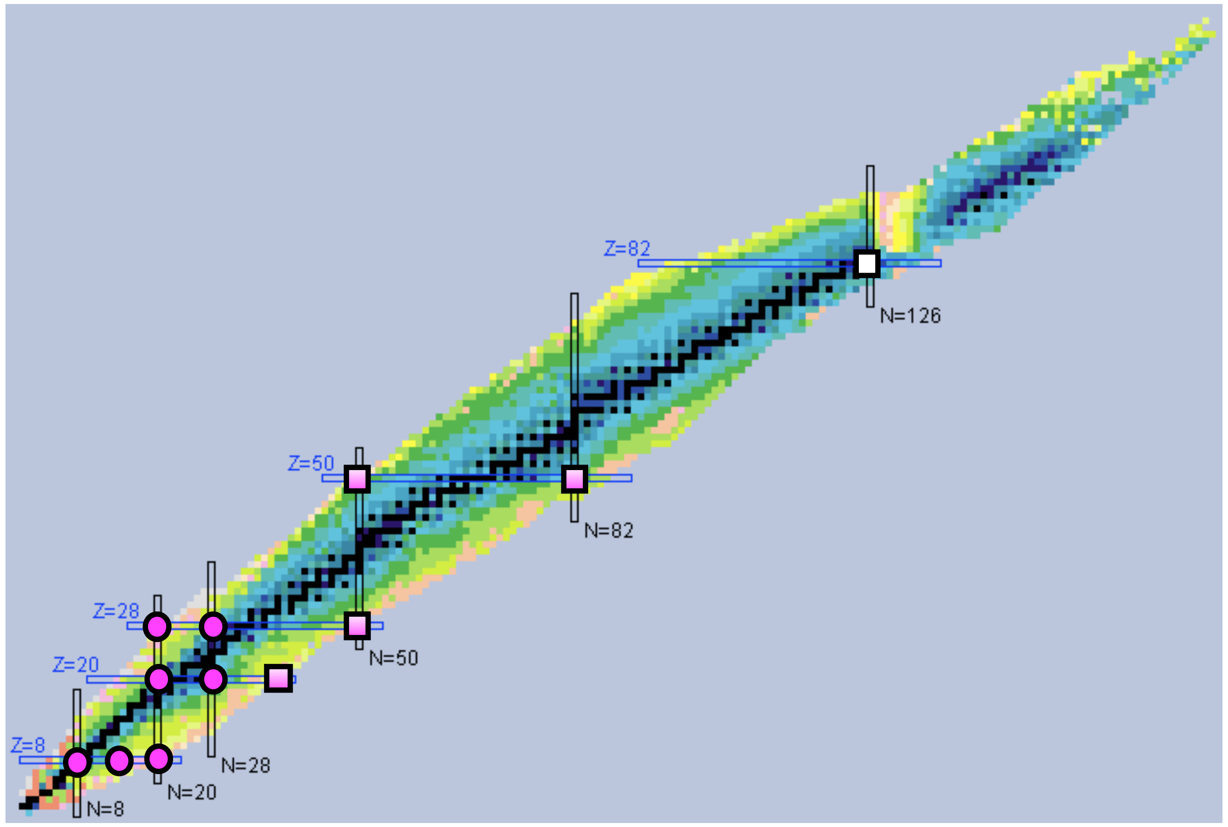
LIT-CC

An approach to many-body break-up induced reactions with a proper accounting of the continuum

Coupled-cluster theory

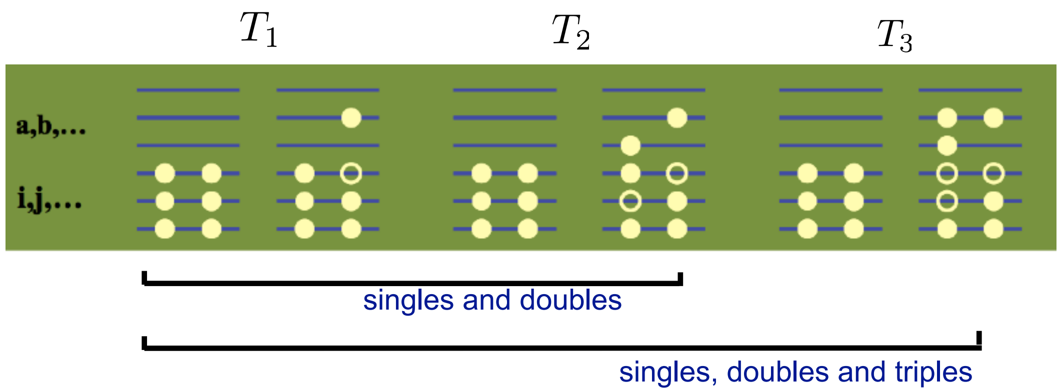
See talk by G.Hagen

Many-body method that can extend the frontiers of ab-initio calculations to heavier and neutron nuclei



$$|\psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle$$

$T = \sum T_{(A)}$ cluster expansion



Can we calculate electromagnetic break-up reactions?

S.B. *et al.*, Phys. Rev. Lett. **111**, 122502 (2013)

$$(\bar{H} - E_0 - \sigma + i\Gamma)|\tilde{\Psi}_R\rangle = \bar{\Theta}|\Phi_0\rangle$$

$$\bar{H} = e^{-T} H e^T$$

$$\bar{\Theta} = e^{-T} \Theta e^T$$

$$|\tilde{\Psi}_R\rangle = \hat{R}|\Phi_0\rangle$$

First implementation with singles and doubles

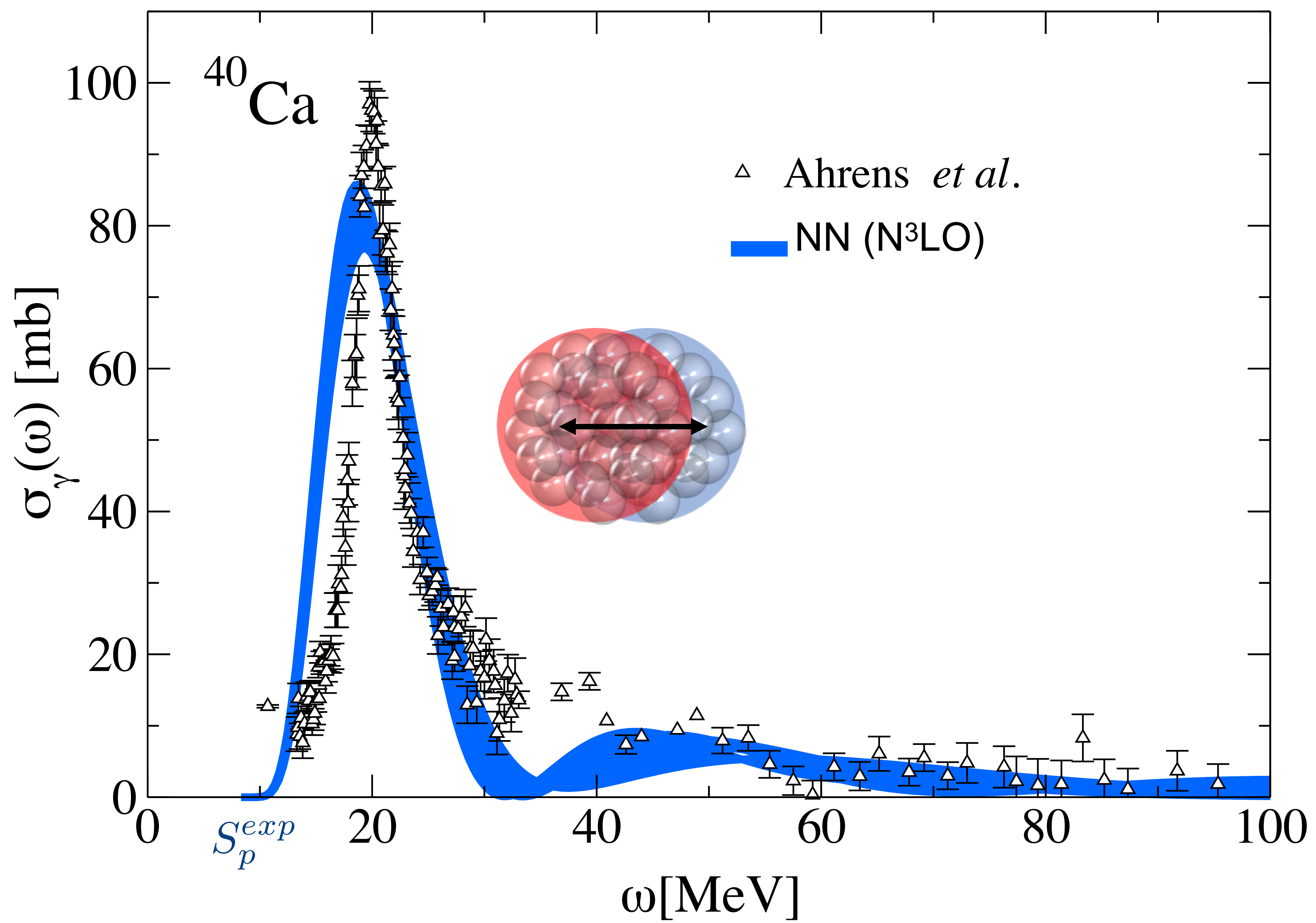
$T = T_1 + T_2$ and same for Λ

$R = R_0 + R_1 + R_2$ and same for L

Photo-absorption

Using the Siegert theorem

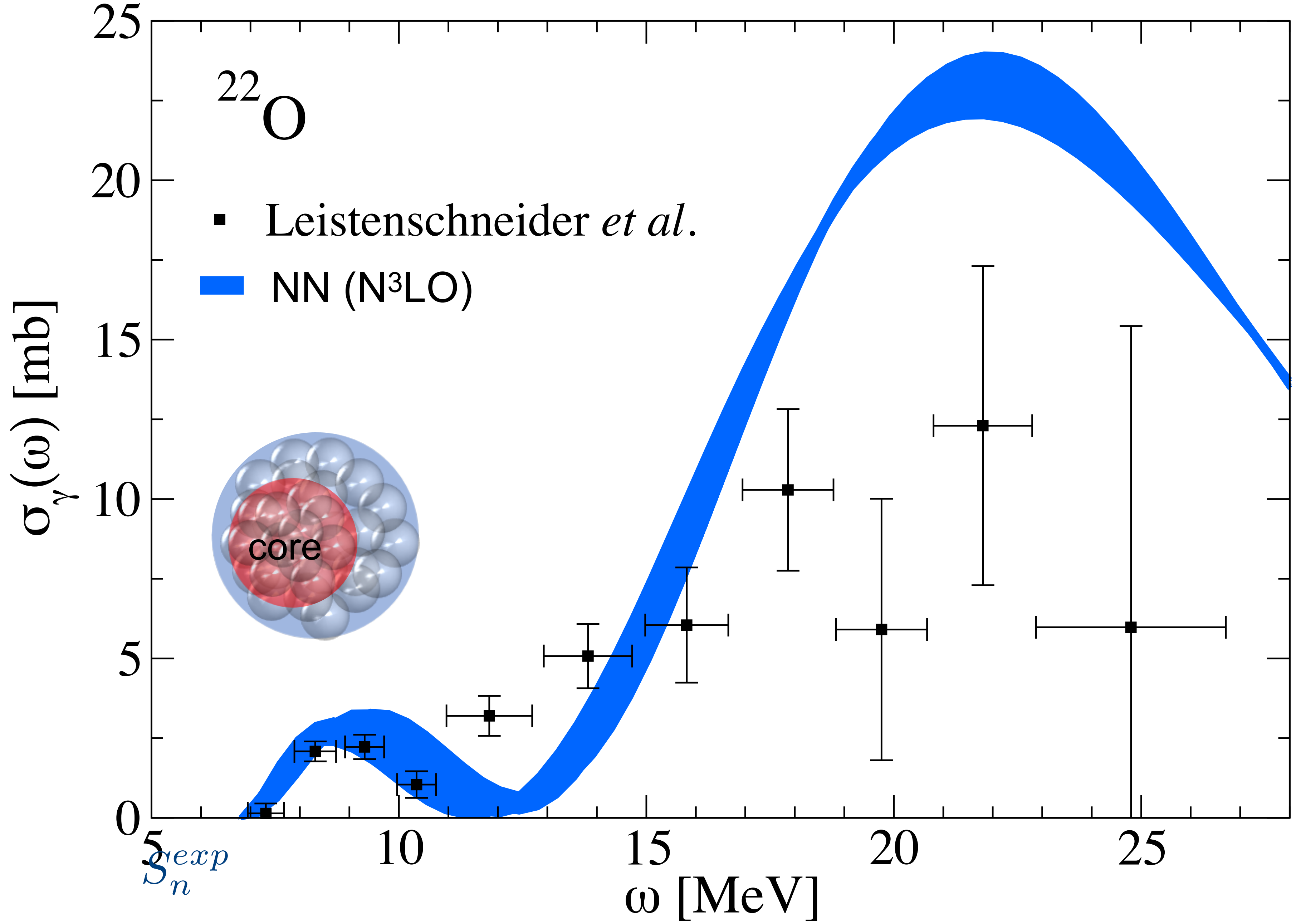
SB et al., PRC **90**, 064619 (2014)



Neutron-rich nuclei

Using the Siegert theorem

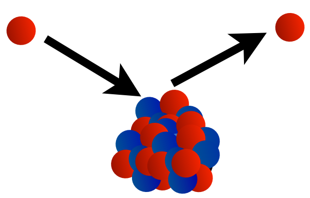
SB et al., PRC **90**, 064619 (2014)

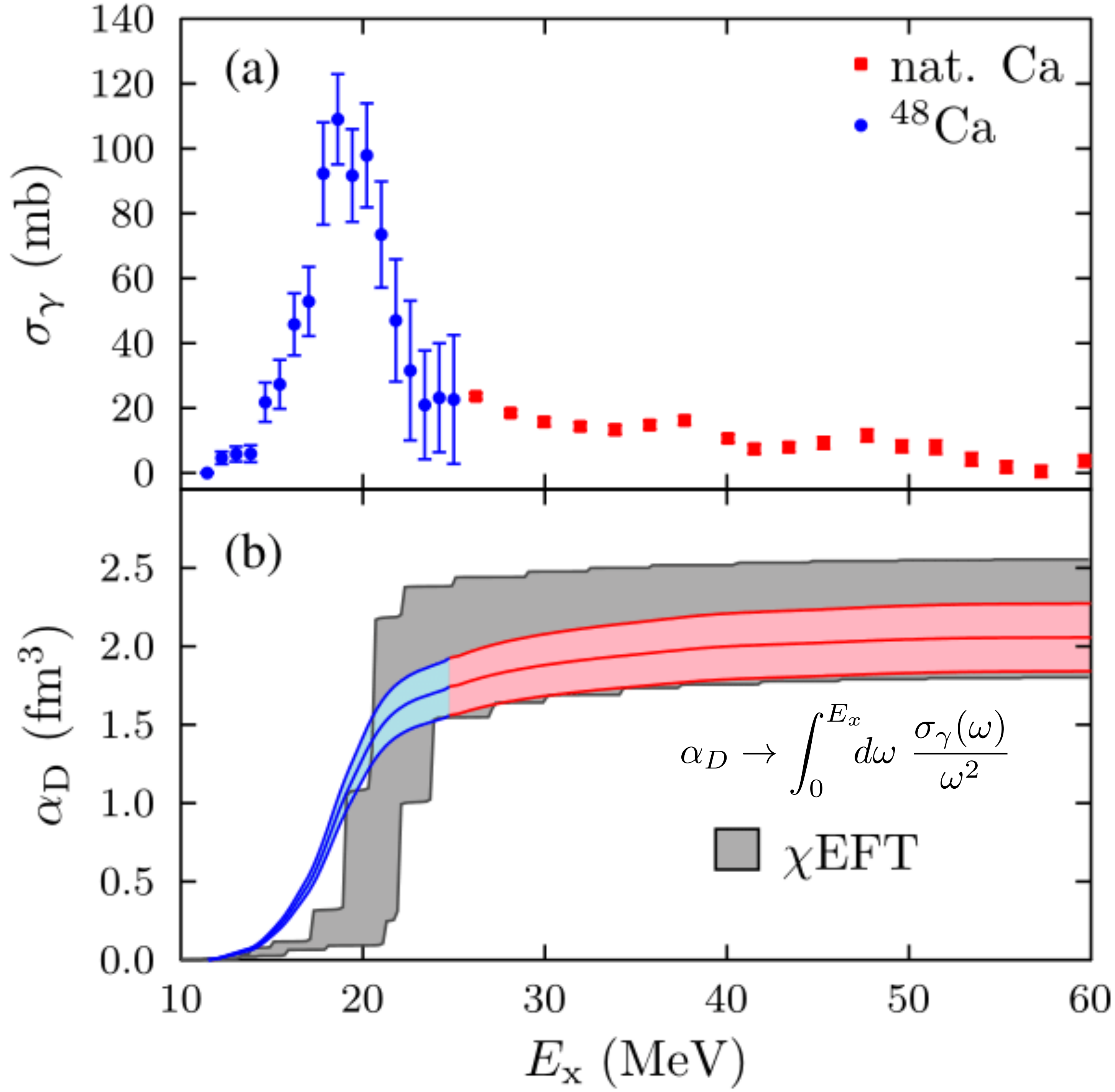


Running polarizability

Data by the Osaka-Darmstadt collaboration from (p,p')

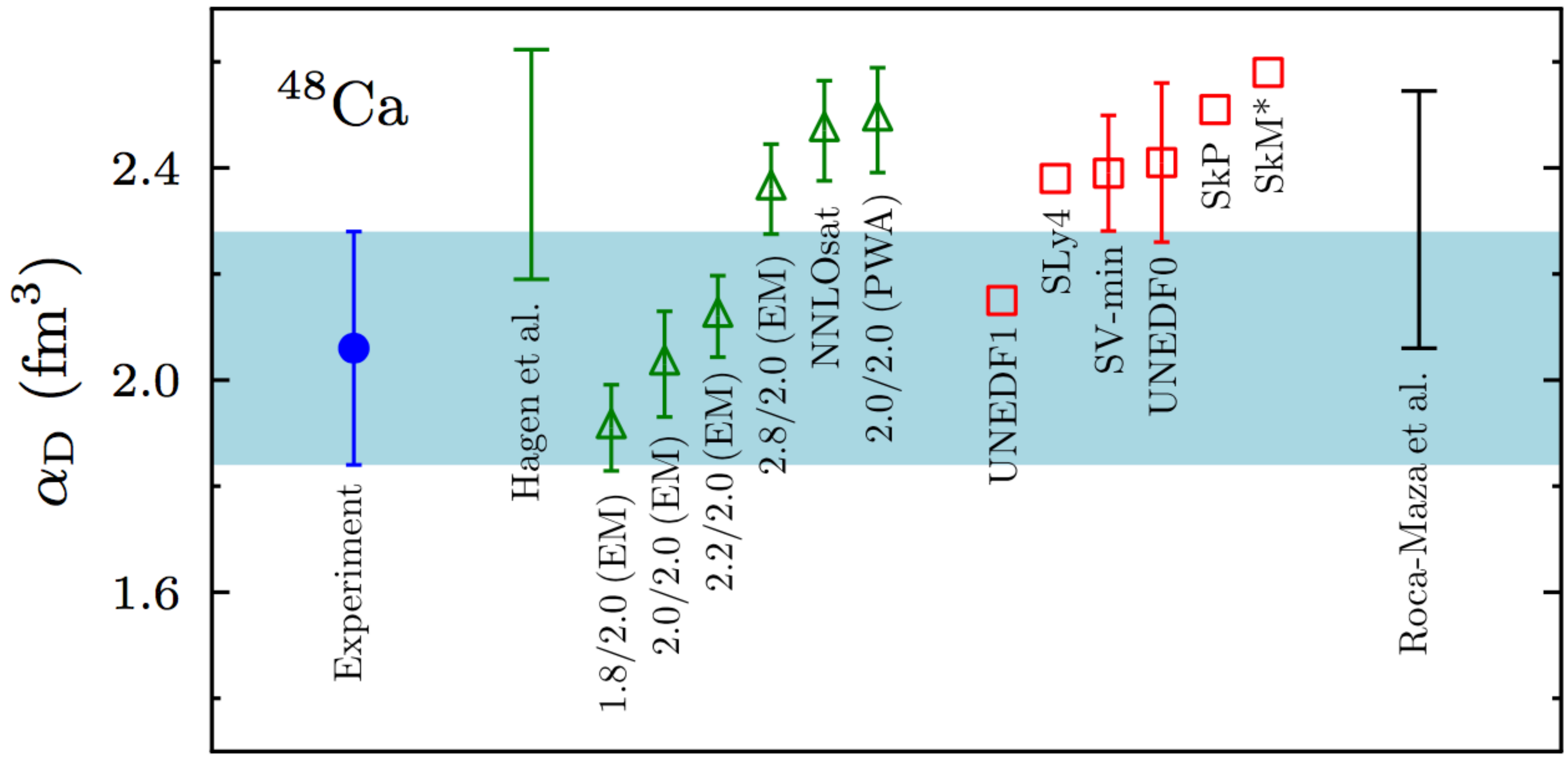
J.Birkhan, *et al.*, Phys. Rev. Lett. **118,** 252501 (2017)





^{48}Ca polarizability summary

J.Birkhan, *et al.*, Phys. Rev. Lett. **118,** 252501 (2017)



Coupled-cluster theory tends to overestimate the experimental value

Can we improve the theoretical prediction?

Adding triples

Full triples are prohibitive

We will use linearized triples for ground state and excited states $T_3 = f(T_1, T_2)$

Similarity transformed operator

M. Miorelli, PhD Thesis (2017)
M. Miorelli *et al.*, arXiv:1804.01718

$$\bar{\Theta}_N = e^{-T}\Theta_N e^T$$

$$\bar{\Theta}_N = \left[\Theta_N e^{T_1+T_2+T_3}\right]_C = \bar{\Theta}_N^D + \left[\Theta_N\left(\frac{T_2^2}{2} + T_3 + T_1 T_3\right)\right]_C$$

$$\simeq \bar{\Theta}_N^D + \left[\Theta_N\left(\frac{T_2^2}{2}\right)\right]_C$$

$$\simeq \bar{\Theta}_N^D$$

^{4}He	^{16}O
m_0[fm]	
0.951	4.87
0.950	4.92
0.949	4.90

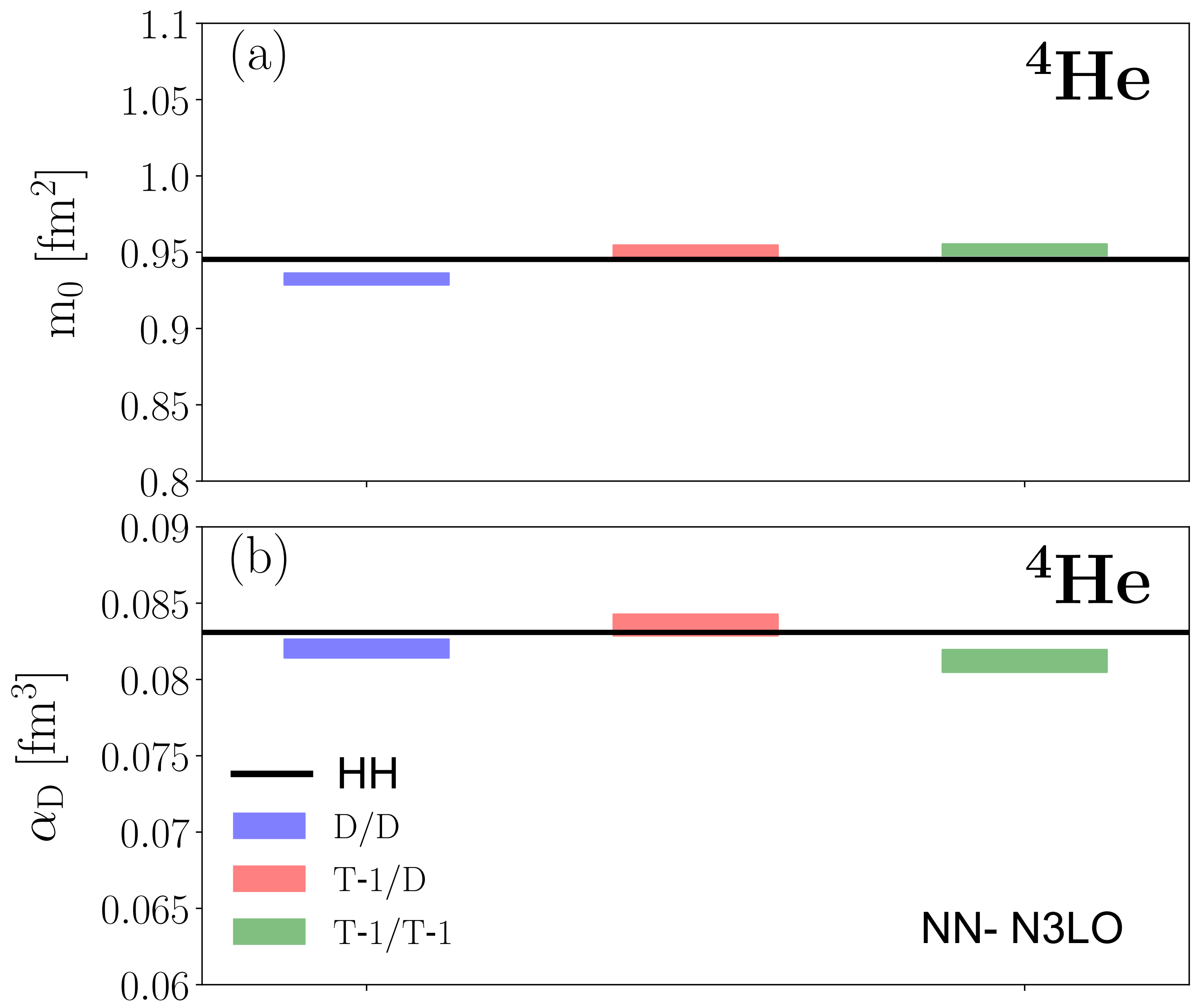
By using only $\bar{\Theta}_N^D$ you are missing 0.2 - 0.6% of the strength only 

Much simpler and the only feasible calculation in heavy nuclei

Benchmark

M. Miorelli *et al.*, arXiv:1804.01718

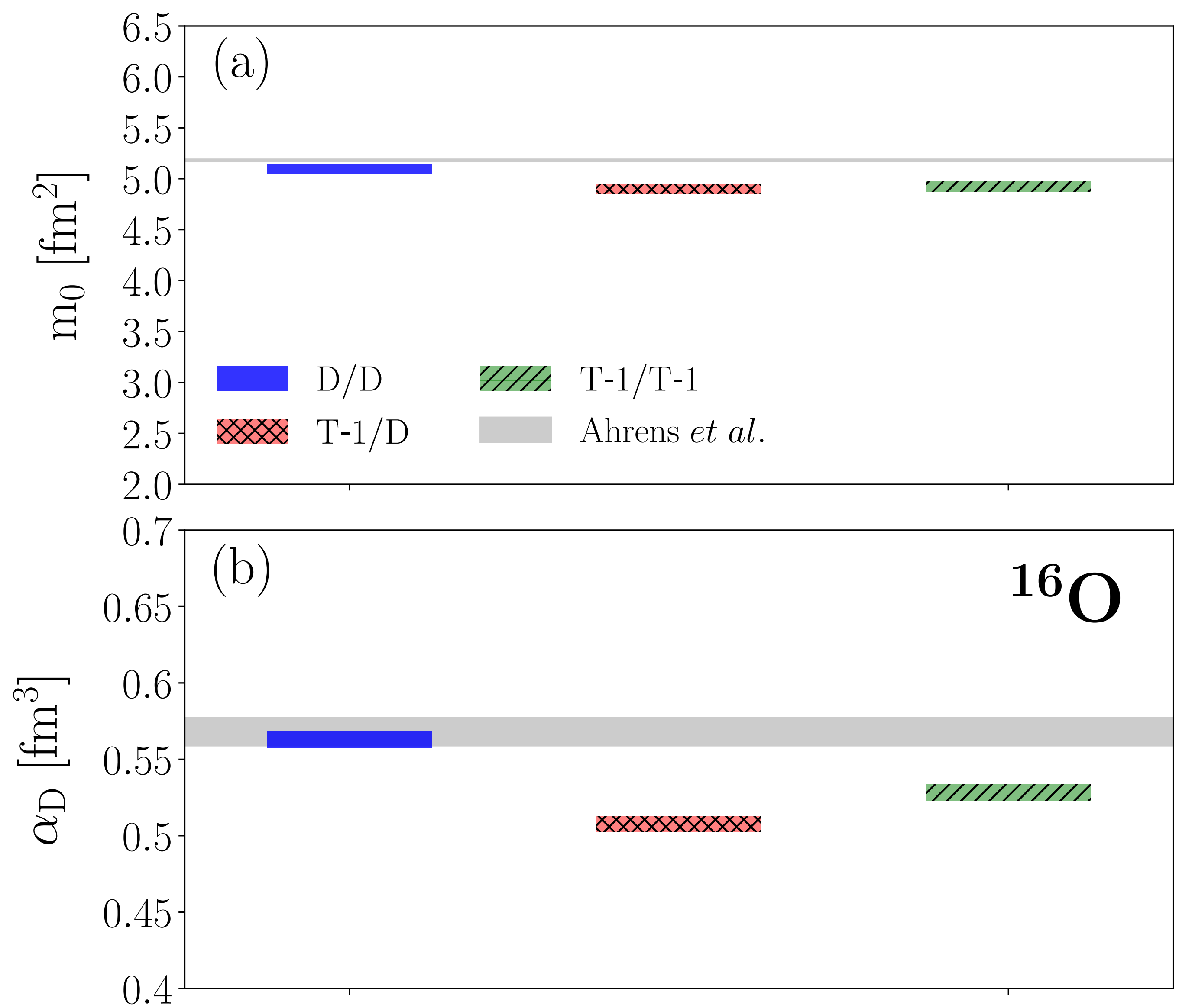
Hypersperical harmonics (HH) contain all correlations (up to quadruples)

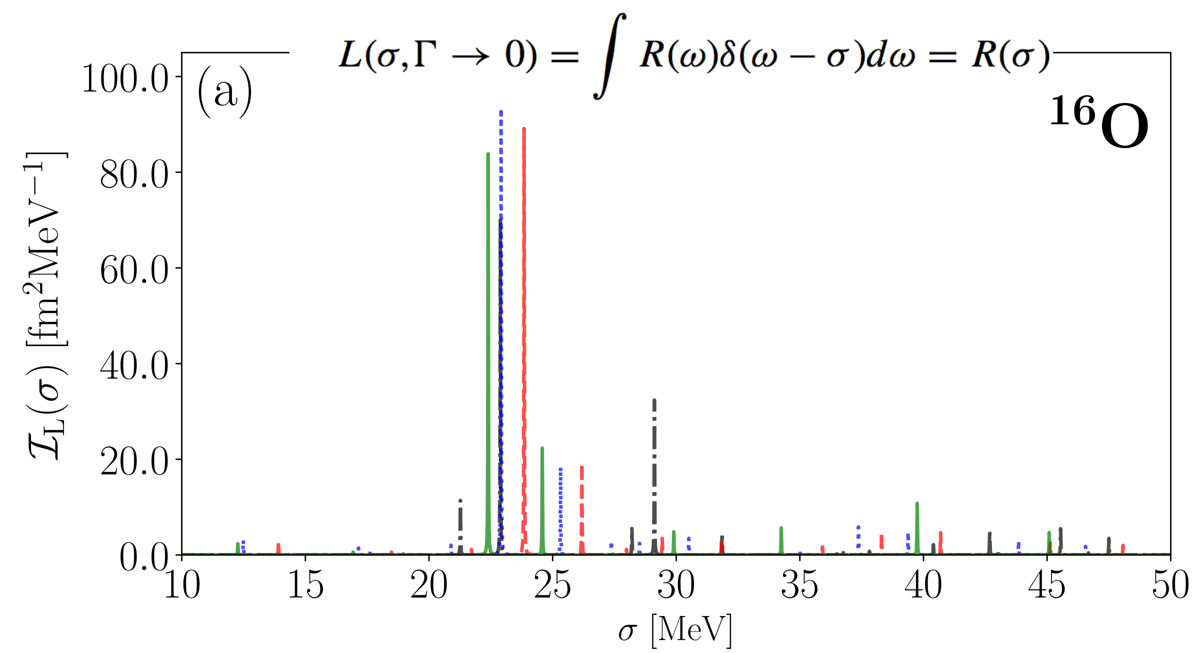


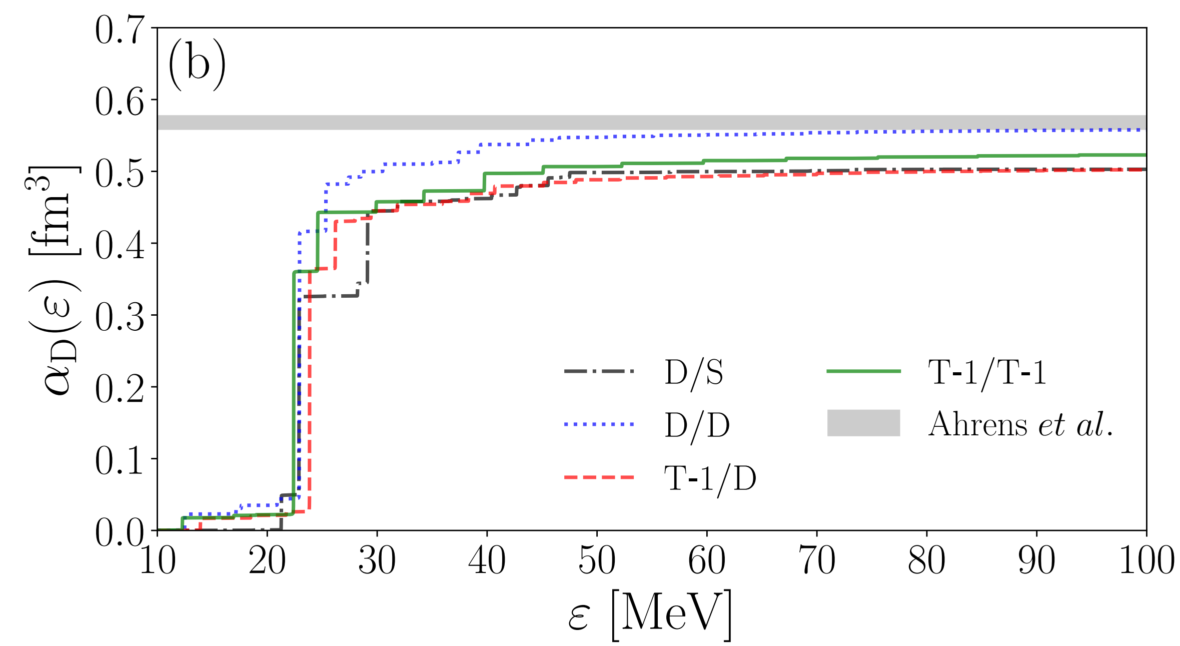
Heavier Nuclei

M. Miorelli *et al.*, arXiv:1804.01718   N2LOsat

Experimental data from photoabsorption cross sections





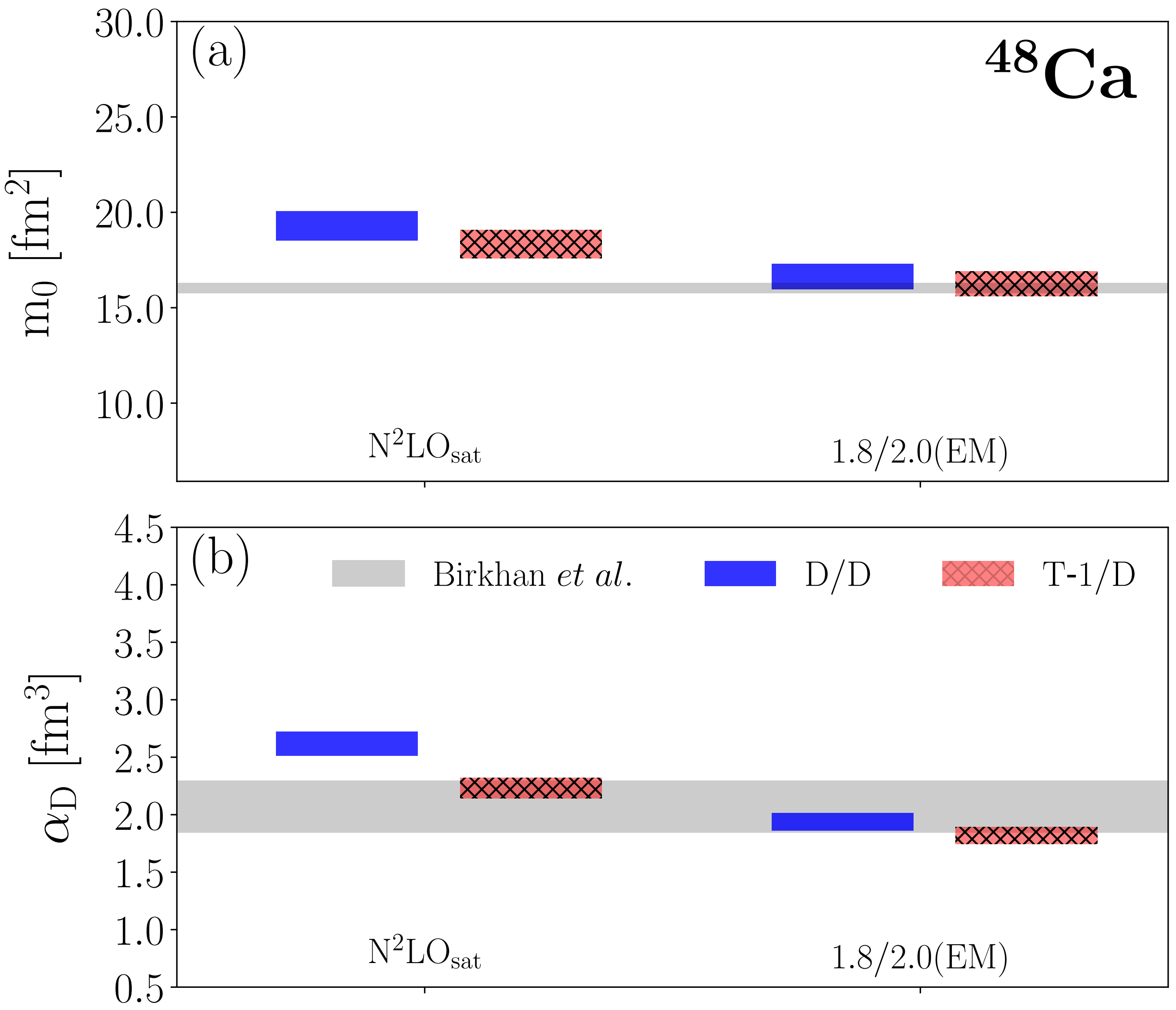


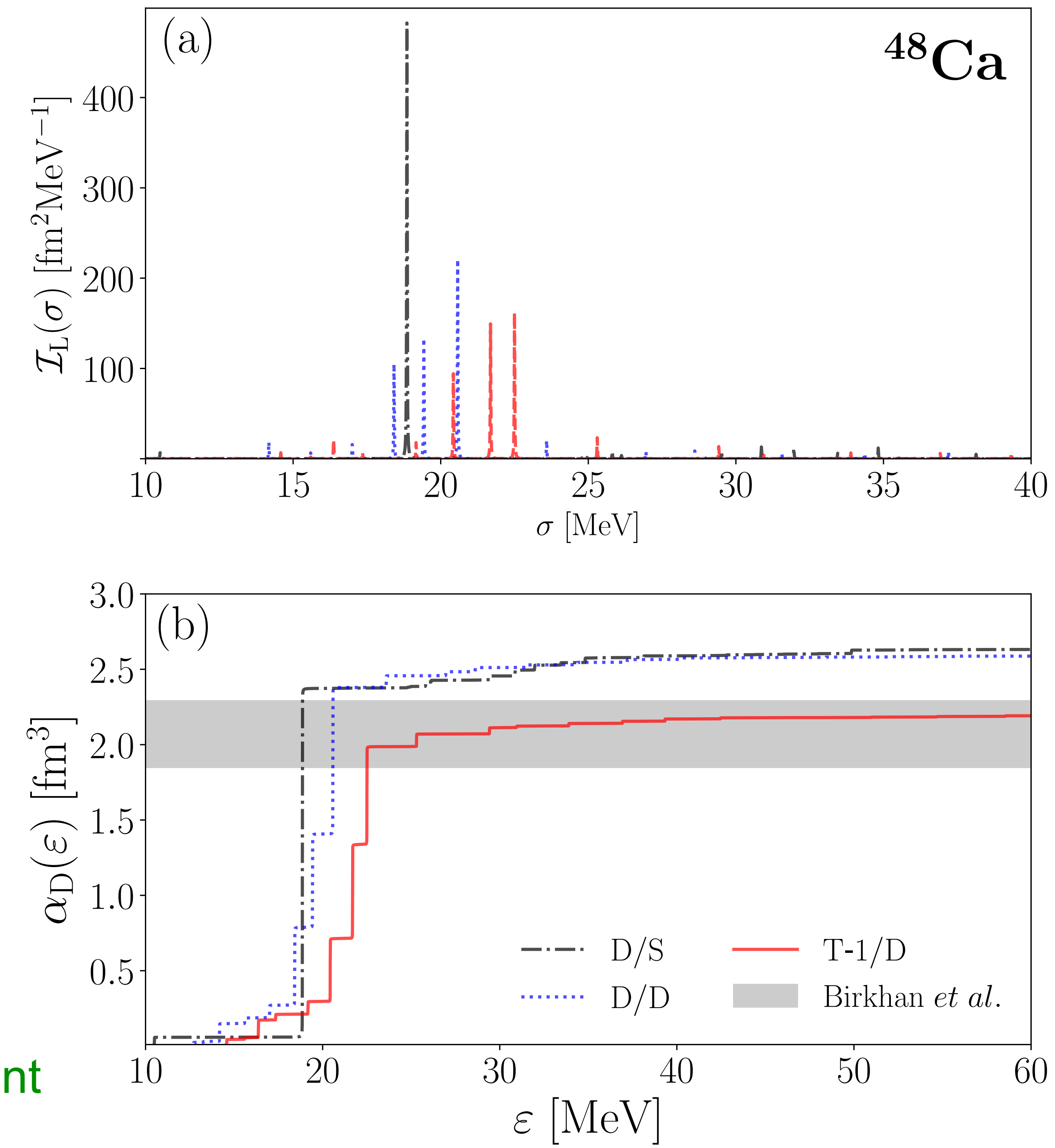
Barbieri et al., arXiv:1711.04698 SCGF approach obtains 0.50 fm³ comparable to D/S giving 0.502 fm³

Revisiting ^{48}Ca

M. Miorelli *et al.*, arXiv:1804.01718

Experimental data from (p,p’) scattering





Triples improve the comparison with experiment

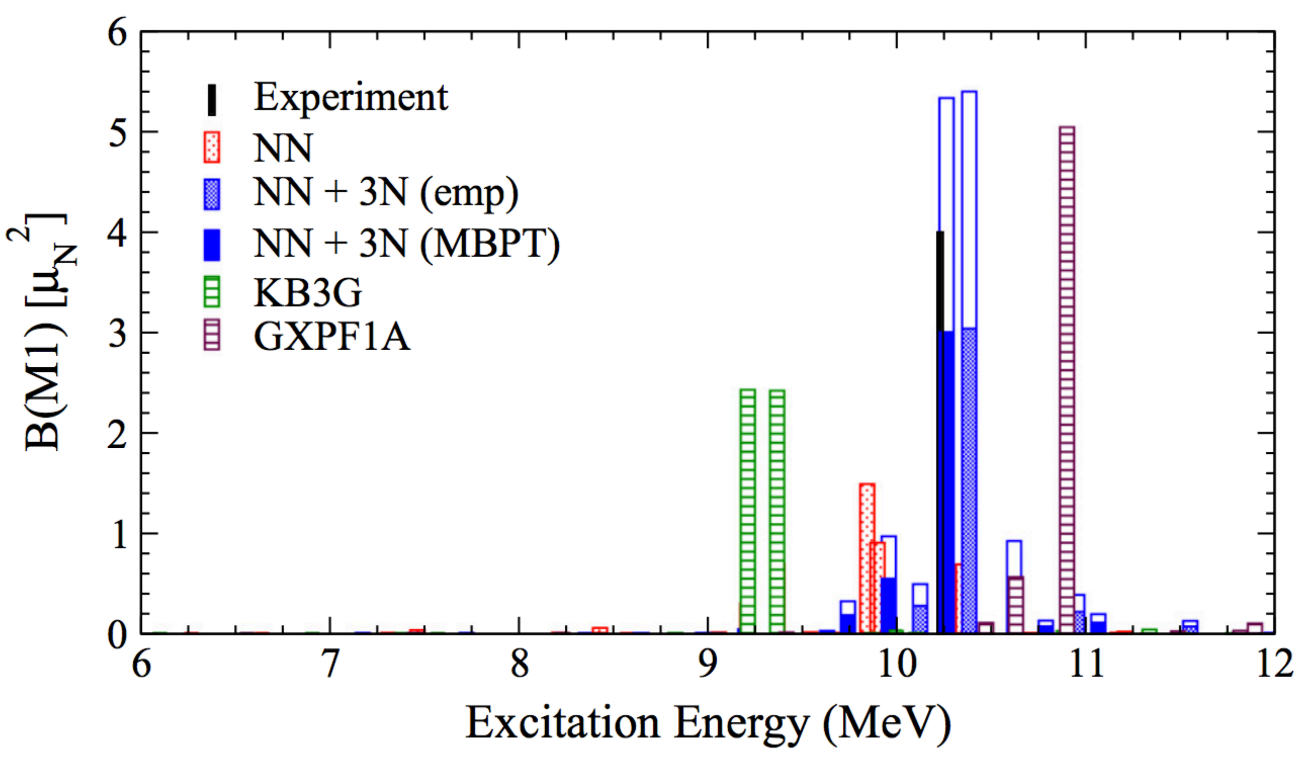
Future plans

Address magnetic transitions in ^{48}Ca

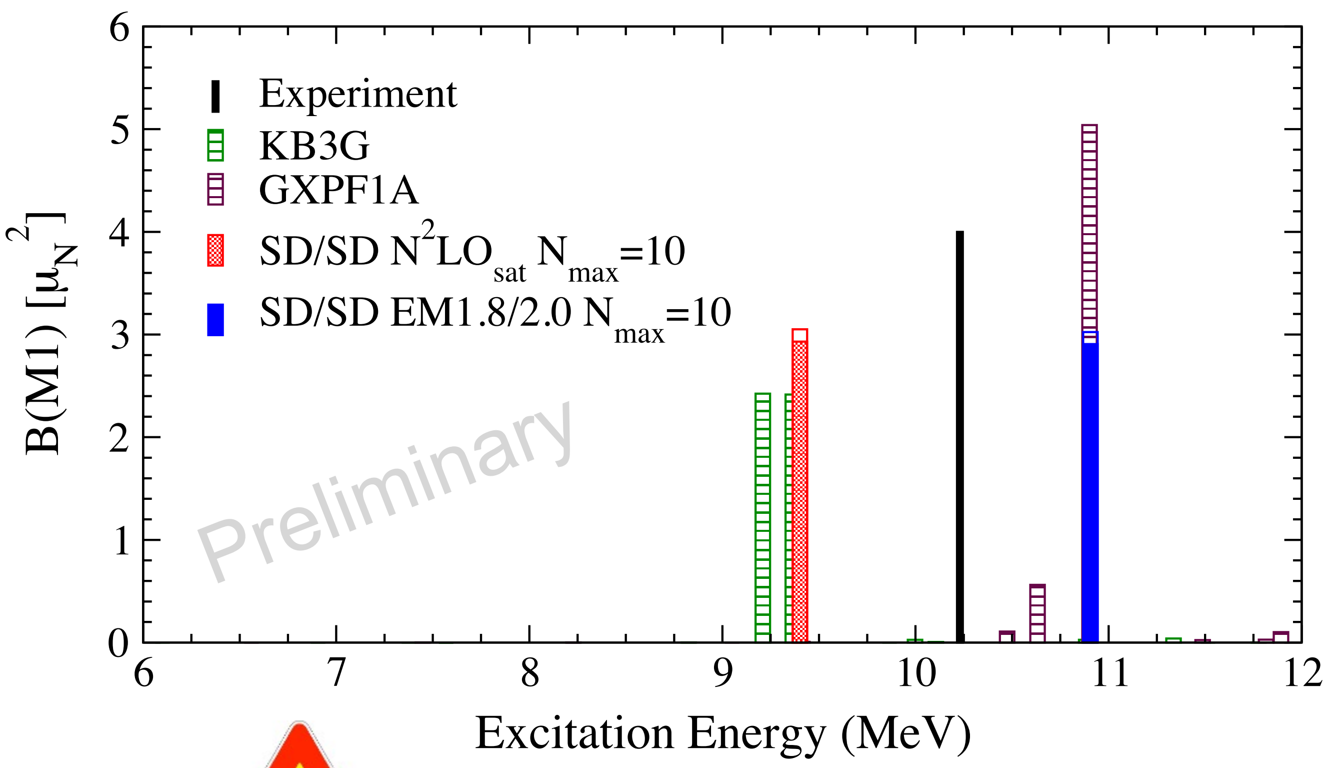
with J. Simonis, O.J.Herndandez, G. Hagen, J. Holt et al.

Goal: Coupled-cluster and IM-SRG with 1BC+2BC

Holt et al, Phys. Rev. C 90, 024312 (2014)



Coupled-cluster singles and doubles



Outlook

- **Electromagnetic nuclear response are rich dynamical observables to study the effect of two-body currents.**
  An implicit inclusion via the Siegert theorem is sufficient at low energy/momentum.

- **Many-body study in coupled-cluster theory:**
  Corrections beyond D in the similarity transformed operator are negligible.
  The T-1 in the ground-state are most important.

- **In the future we plan to address electron-nucleus and neutrino-nucleus scattering**
  B. Acharya

Thanks to all my collaborators

B. Acharya, N. Barnea, O.J. Hednandez, G. Hagen, J. Holt, W. Leidemann,
M. Miorelli, J. Simonis, G. Orlandini, T. Papenbrock, S. Pastore, A. Schwenk, K. Wendt, et al.

Thanks for your attention!