Recent advances and challenges in the description of nuclear reactions at the limit of stability
05 Mar 2018 to 09 Mar 2018

# Alpha clustering and pairing correlation in heavy nuclei and two-neutron transfer reaction


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**Outline:**

- Theoretical description of alpha and proton decays and the formation amplitude
- Clustering, di-nucleon correlation, and pairing gap
- Systematics of alpha formation amplitudes

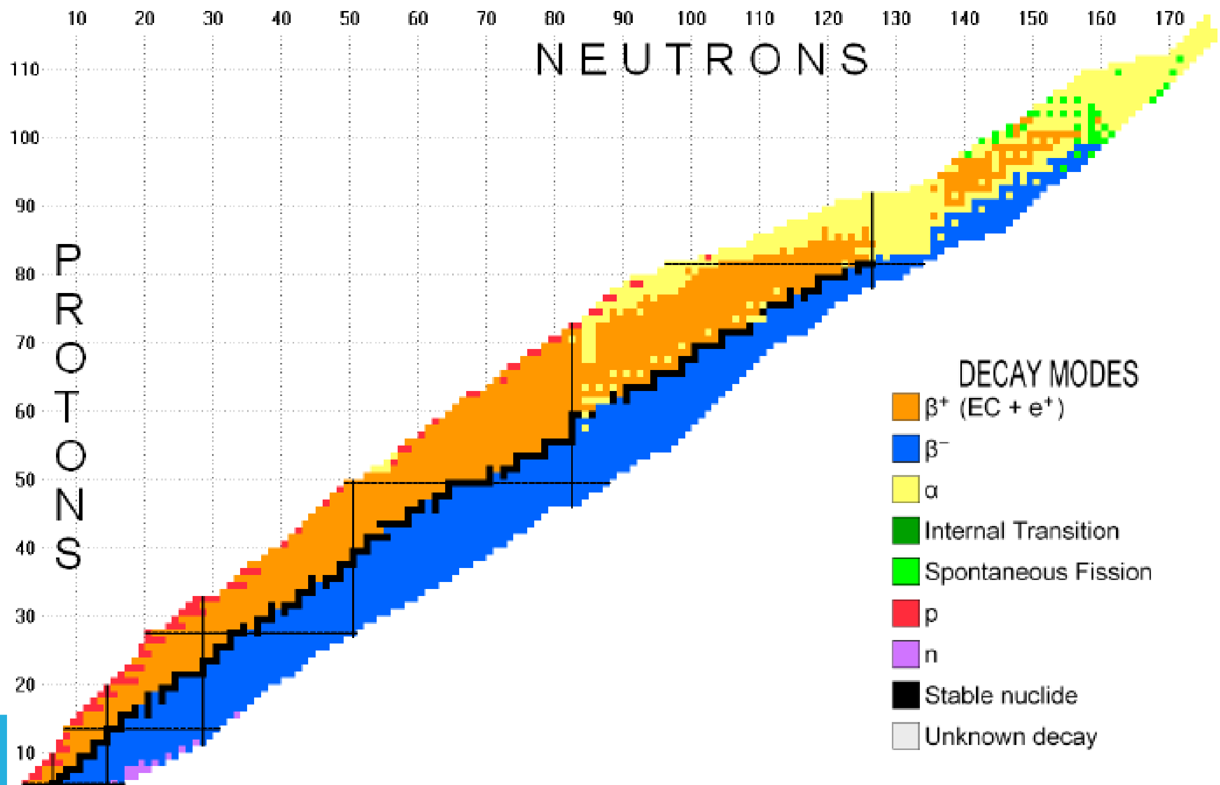
# 120 years of radioactivity studies

- **α radioactivity** ~ 400 events observed in A > 150 nuclei;
  - **α decay of N≈Z nuclei (2000s-)**
  - **one of the oldest subjects and long history of success**

- **Heavier cluster decays** 11 events observed in trans-lead nuclei

*$^{223}$Ra($^{14}$C), Rose and Jones, Nature 307, 245 (1984);*

- **Proton decay** more than 40 events observed in the rare-earth region.




G. Audi et al. / Nuclear Physics A 729 (2003) 3-

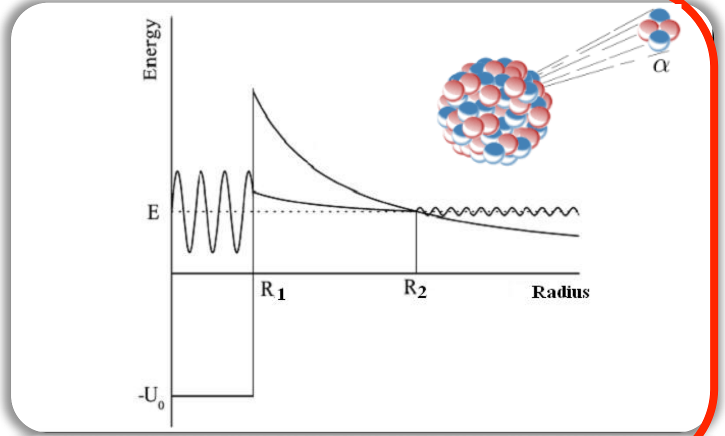
- **New decay modes**
  - **Di-proton decay**
  - **Neutron decay**
  - **$^{12}$C cluster decay**

# Quantum tunneling interpretation

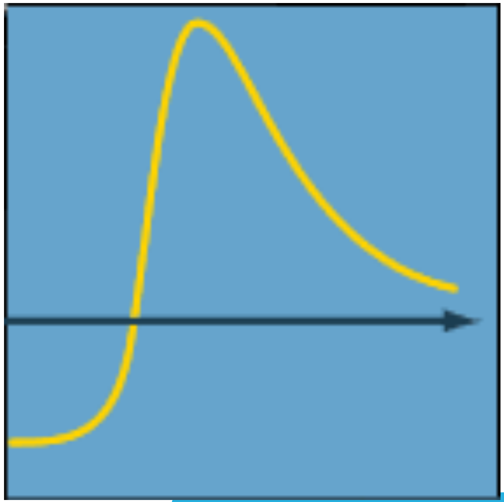
G. Gamow, Z. Phys. 51, 204 (1928).

R. W. Gurney and E. U. Condon, Nature 122, 439 (1928).

$$P = \exp\left\{-2\int_{R_1}^{R_2}\sqrt{\frac{2\mu}{\hbar^2}|V_C(r) - Q_\alpha|}\mathrm{d}r\right\},$$




"Standard" picture of the alpha decay process



$$\lambda = \ln 2/T = \nu S P_s$$

- **The alpha particle is not a basic constitute of the atomic nucleus**
  - **Alpha particle model of atomic nucleus before the neutron was discovered**
  - **Light nuclei may exhibit profound alpha clustering structure**
- **The Gamow theory does not carry nuclear structure information**
- **Spectroscopic factor is not an observable**

# Microscopic description of alpha decay

R.G. Thomas, Prog. Theor. Phys. **12**, 253 (1954).

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_c} = \frac{\ln 2}{\nu} \left| \frac{H_l^+(\chi, \rho)}{R F_c(R)} \right|^2,$$

$\nu$ is the outgoing velocity of the emitted particle

$F_c(R)$ is the formation amplitude

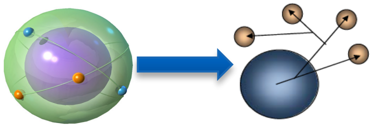
$H_l^+$ is the Coulomb-Hankel function The penetrability is proportional to $|H_l^+(\chi, \rho)|^{-2}$.

*R is the distance between the center of mass of the cluster and daughter nucleus which divides the decay process into an internal region and* complementary external region.

F(R) describes the formation amplitude of the alpha particle inside the nucleus

Yes, it depend on the radius.

$$m \rightarrow d + \alpha$$



$$\mathcal{F}_l(R) = \int d\mathbf{R} d\xi_d d\xi_\alpha [\Psi(\xi_d)\phi(\xi_\alpha)Y_l(\mathbf{R})]^*_{J_m M_m} \Psi_m(\xi_d, \xi_\alpha, \mathbf{R}),$$

**Shell Model**

H.J. Mang, PR 119,1069 (1960); I. Tonozuka, A. Arima, NPA 323, 45 (1979).

**BCS approach**

HJ Mang and JO Rasmussen, Mat. Fys. Medd. Dan. Vid. Selsk. (1962)

DS Delion, A. Insolia and RJ Liotta, PRC46, 884(1992).

# Proton decay formation amplitude

Overlap at a given Radius

$$\mathcal{F}_l(R) = \int d\mathbf{R}\, d\xi_d [\Psi(\xi_d)\xi_p Y_l(\mathbf{R})]^*_{J_m M_m} \Psi_m(\xi_d, \xi_p, \mathbf{R}),$$

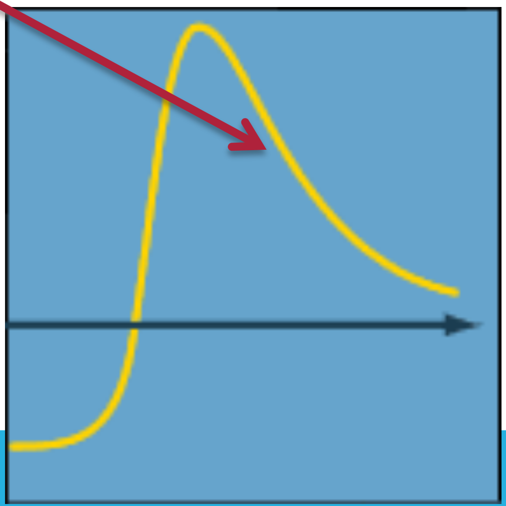
that $\mathcal{F}_l(R)$ would indeed be the wave function of the outgoing particle $\psi_p(R)$ if the mother nucleus would behave at the point $R$ as

$$\Psi_m(\xi_d, \xi_p, \mathbf{R}) = [\Psi(\xi_d)\xi_p \psi_p(R) Y_l(\mathbf{R})]_{J_m M_m}. \qquad (3)$$

# Spectroscopic factor vs the formation amplitude

$$S_{f,j,i} = \frac{|< \Psi_f^{A\pm1} J_f \| a_j^{\pm} \| \Psi_i^A J_i >|^2}{2J_i + 1}$$

**integration over the whole space;** One usually assume that the mother, daughter and the emitted particle share the same single-particle wave function



# Proton decay formation amplitude

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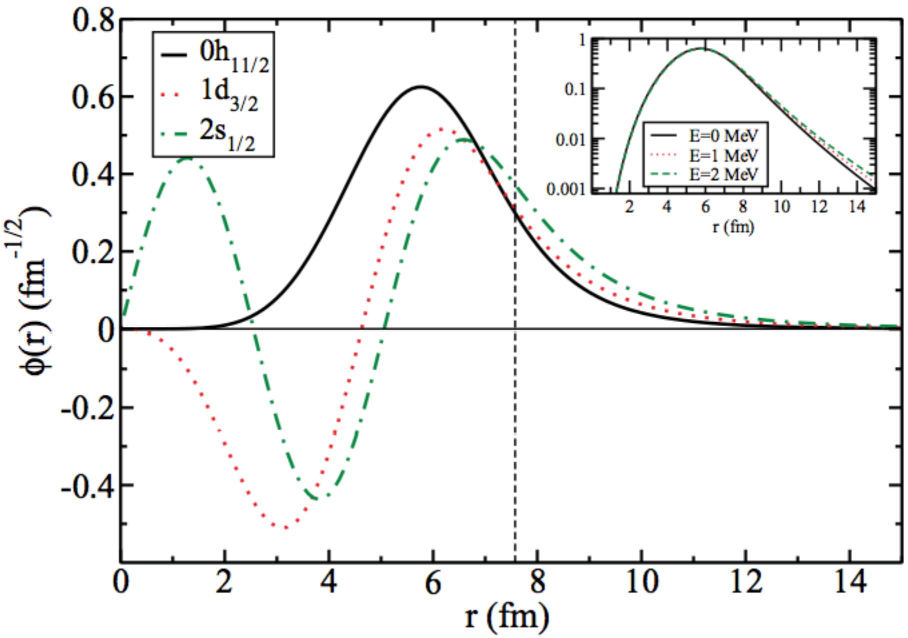


C. Qi, D.S. Delion, R.J. Liotta, and R. Wyss, 85, 011303(R) (2012)

# Microscopic description of alpha decay

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$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_c} = \frac{\ln 2}{\nu} \left| \frac{H_l^+(\chi, \rho)}{RF_c(R)} \right|^2 ,$$

$\nu$ is the outgoing velocity of the emitted particle

$F_c(R)$ is the formation amplitude

$H_l^+$ is the Coulomb-Hankel function The penetrability is proportional to $|H_l^+(\chi, \rho)|^{-2}$.

*R is the distance between the center of mass of the cluster and daughter nucleus which divides the decay process into an internal region and* complementary external region.

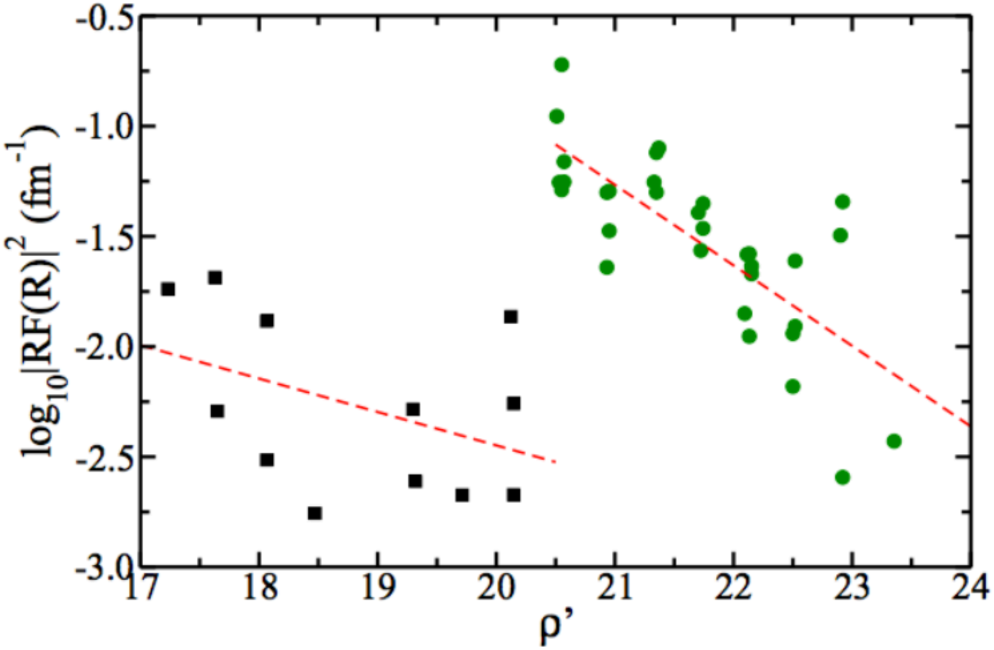
The Coulomb function is 'well' understood from the Gamow theory.

$$H_0^+(\chi, \rho) \approx (\cot \beta)^{1/2} \exp[\chi(\beta - \sin \beta \cos \beta)] ,$$

$\chi = 2Z_c Z_d e^2 / \hbar v$ $\quad$ $\rho = \mu v R / \hbar$ $\quad$ $\cos^2 \beta = \frac{\rho}{\chi} = \frac{Q_c R}{e^2 Z_c Z_d}.$

# Proton formation probability from experimental data

$$\log|RF(R)|^{-2} = \log T_{1/2}^{\text{Expt.}} - \log\left[\frac{\ln 2}{\nu}|H_l^+(\chi,\rho)|^2\right],$$




$$\rho' = \sqrt{AZ_pZ_d(A_d^{1/3}+A_p^{1/3})},\ A = A_dA_p/(A_d+A_p)$$

A transition happens around $^{145}$Tm

# Formation vs 'u' for spherical proton emitters

$$F_l(R) \approx u\varphi_l(R)$$

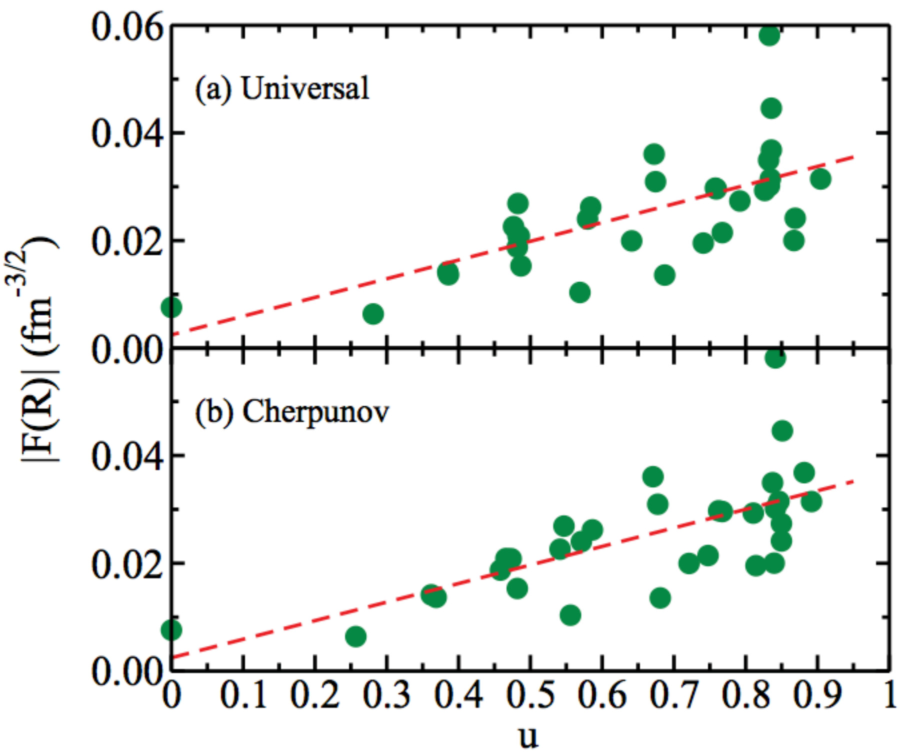



FIG. 4. (Color online) The formation amplitudes $|F_l(R)|$ extracted from experimental data for proton decays of nuclei $N \geqslant 75$ ($Z > 67$) as a function of $u$ calculated from BCS calculations using for the Woods-Saxon mean field the universal parameters [23] (upper) and the Cherpunov parameters [24] (lower).

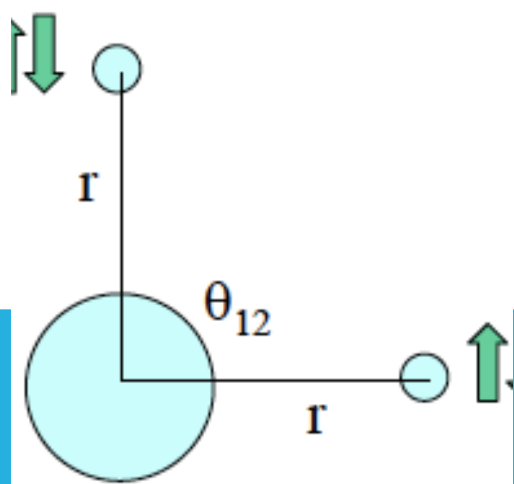
# Simple alpha-emitter examples: $^{212}$Po vs $^{210}$Po

$$|^{212}\mathrm{Po}(\alpha_4)\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\alpha_4)|^{210}\mathrm{Pb}(\alpha_2)\otimes^{210}\mathrm{Po}(\beta_2)\rangle$$

**If we neglect the proton-neutron interaction between the four valence nucleons (Or pn interaction only being considered at the mean field level)**

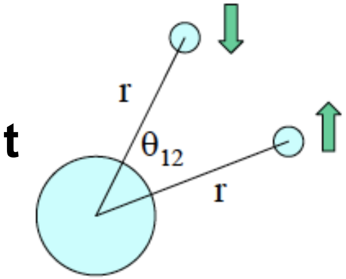
$$|^{212}\mathrm{Po}(\alpha,\mathrm{g.s.})\rangle = |^{210}\mathrm{Pb}(2\nu,\mathrm{g.s.})\otimes|^{210}\mathrm{Po}(2\pi,\mathrm{g.s.})\rangle,$$
$$|^{210}\mathrm{Po}(\alpha,\mathrm{g.s.})\rangle = |^{206}\mathrm{Pb}(2\nu^{-1},\mathrm{g.s.})\otimes|^{210}\mathrm{Po}(2\pi,\mathrm{g.s.})\rangle.$$

$$\mathscr{F}_\alpha(R;^{212}\mathrm{Po(gs)}) = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi(\mathbf{r_1r_2};^{210}\mathrm{Pb(gs)})\Psi(\mathbf{r_3r_4};^{210}\mathrm{Po(gs)}),$$
$$\mathscr{F}_\alpha(R;^{210}\mathrm{Po(gs)}) = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi(\mathbf{r_1r_2};^{206}\mathrm{Pb(gs)})\Psi(\mathbf{r_3r_4};^{210}\mathrm{Po(gs)}).$$




Two-body clustering

# Two-body clustering




**Configuration mixing from higher lying orbits is important for clustering at the surface**

**Pair correlations result in a constructive interference of formation amplitudes**

$$\Psi_2(\mathbf{r_1},\mathbf{r_2}) = (\chi_1\chi_2)_0\Phi_2(r_1,r_2,\theta_{12}) = (\chi_1\chi_2)_0\frac{1}{4\pi}\sum_p\sqrt{\frac{2j_p+1}{2}}X_p\phi_p(r_1)\phi_p(r_2)P_{l_p}(\cos\theta_{12}),$$

$r_1$=9fm

$^{210}$Pb

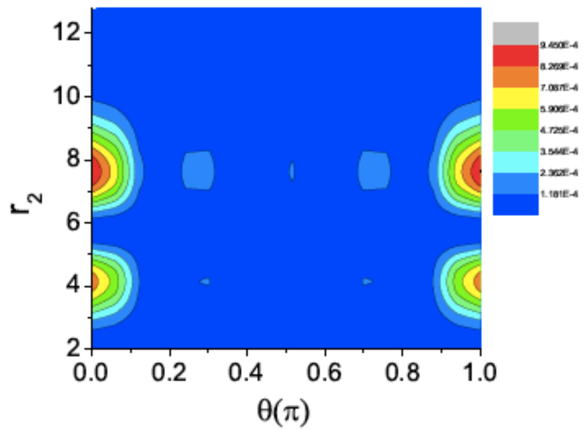
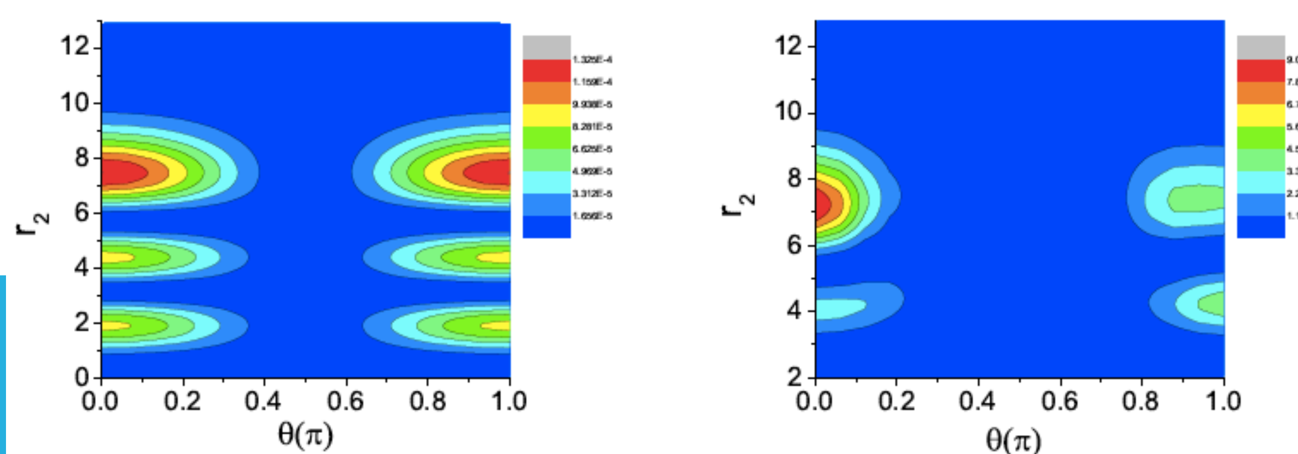


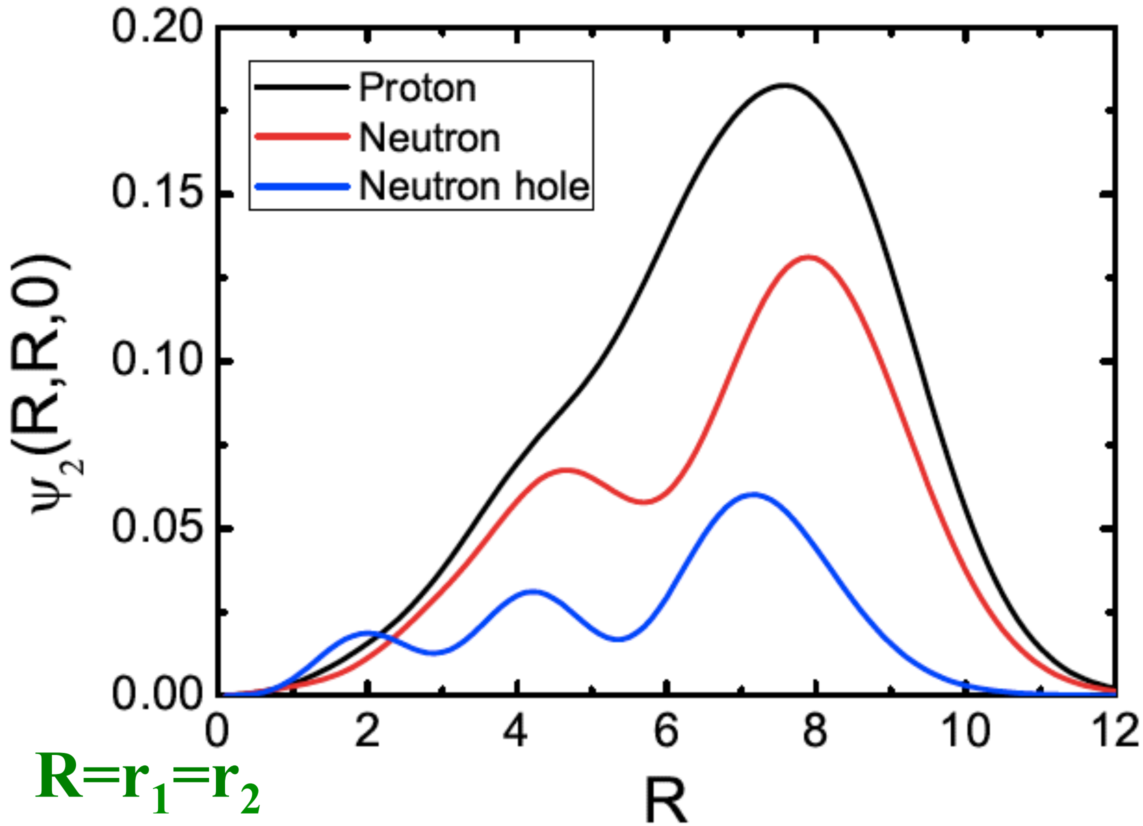

FIG. 10: (color online). The square of the two-neutron wave function $|\Psi_{2\nu}(r_1,r_2,\theta)|^2$ with $r_1 = 9$ fm as a function of $r_2$ and $\theta$. Left: the leading configuration; Right: 4 major shells

$^{206}$Pb

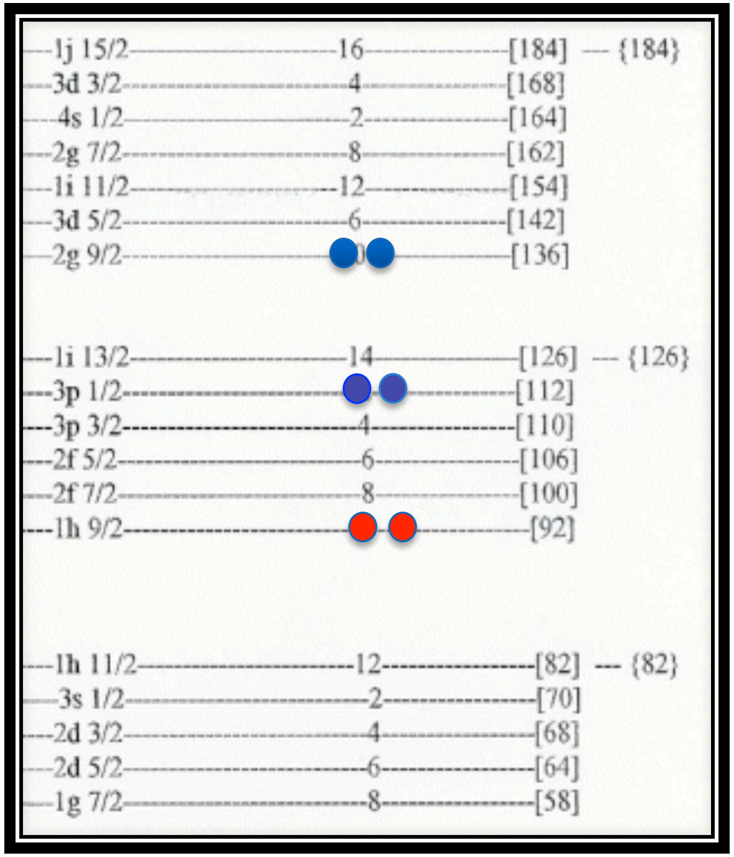


# Two-body clustering

$$\Psi_2(\mathbf{r_1},\mathbf{r_2}) = (\chi_1\chi_2)_0\Phi_2(r_1,r_2,\theta_{12}) = (\chi_1\chi_2)_0\frac{1}{4\pi}\sum_p\sqrt{\frac{2j_p+1}{2}}X_p\phi_p(r_1)\phi_p(r_2)P_{l_p}(\cos\theta_{12}),$$



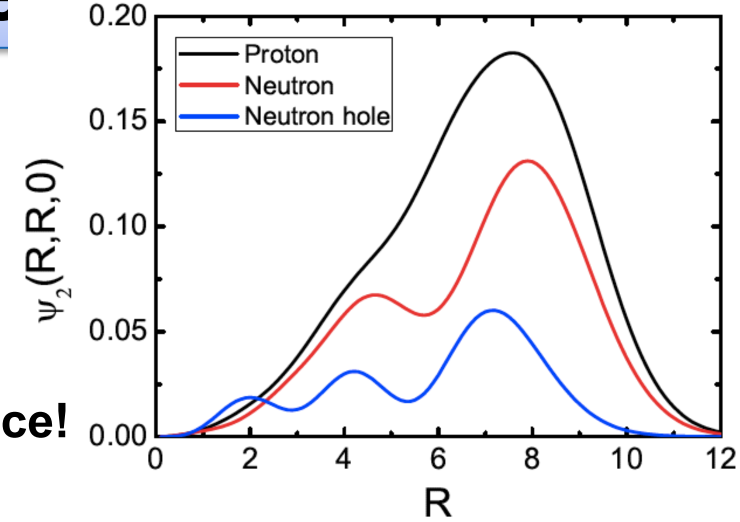

**R=r$_1$=r$_2$**



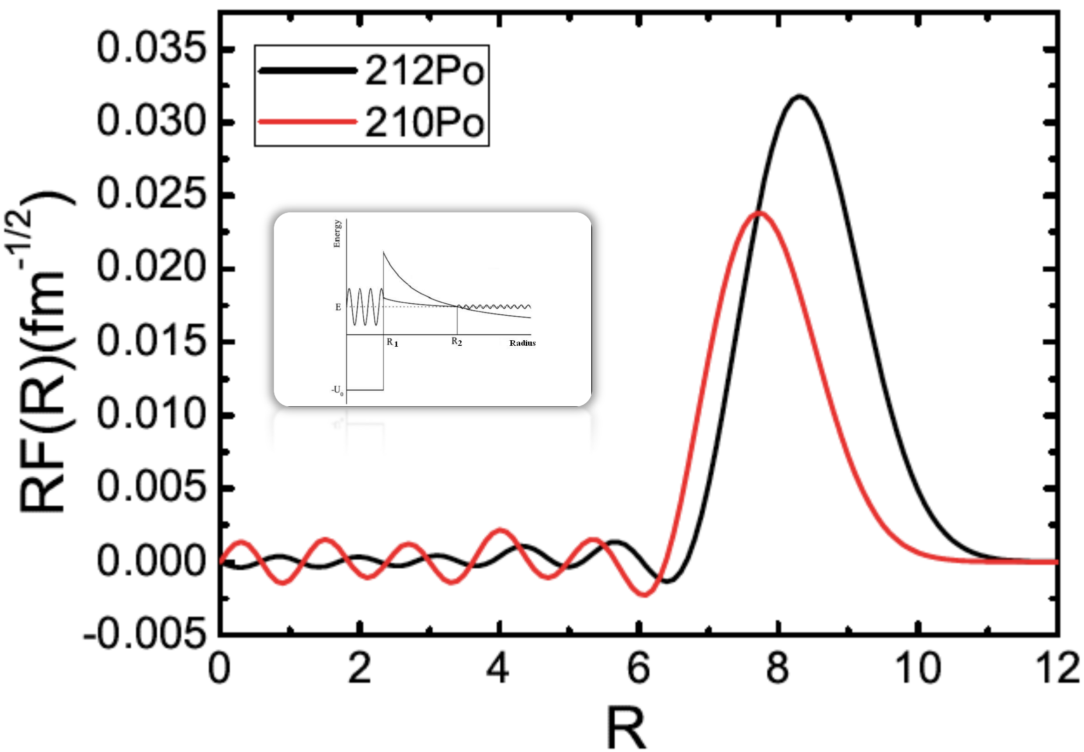

- The two-body wave functions are indeed strongly enhanced at the nuclear surface;
- The enhancement is much weaker in $^{206}$Pb(gs) than that in $^{210}$Pb(gs)
  - *Relatively small number of configurations in the hole-hole case;*
  - *$p_{1/2}$ dominance in $^{206}$Pb(gs);*
  - *Radial wave functions of hole states less extended.*

# Alpha formation amplitude

$$F_\alpha(R) = \sqrt{\frac{1}{4\pi}} \int dr_\alpha \phi_\alpha(r_\alpha) \Psi_{2\pi}(\mathbf{r_1}, \mathbf{r_2}) \Psi_{2\nu}(\mathbf{r_3}, \mathbf{r_4}),$$




**Alpha clustering is prominent (only) at the surface!**

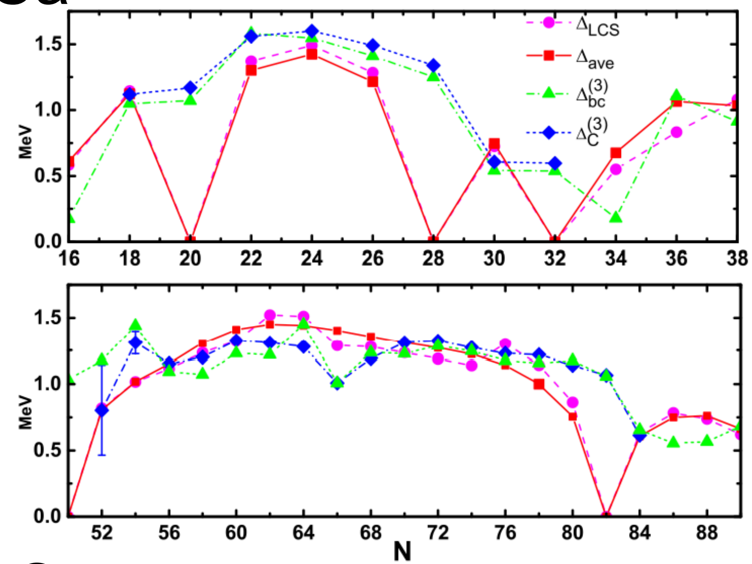



- **Alpha particle is formed on the nuclear surface;**
- **The clustering induced by the pairing mode is inhibited if the configuration space does not allow a proper manifestation of the pairing collectivity.**

CQ et al., Phys. Rev. C 81, 064319 (2010).

## Pairing gap

Ca




Sn

## Two-body wave function from HFB

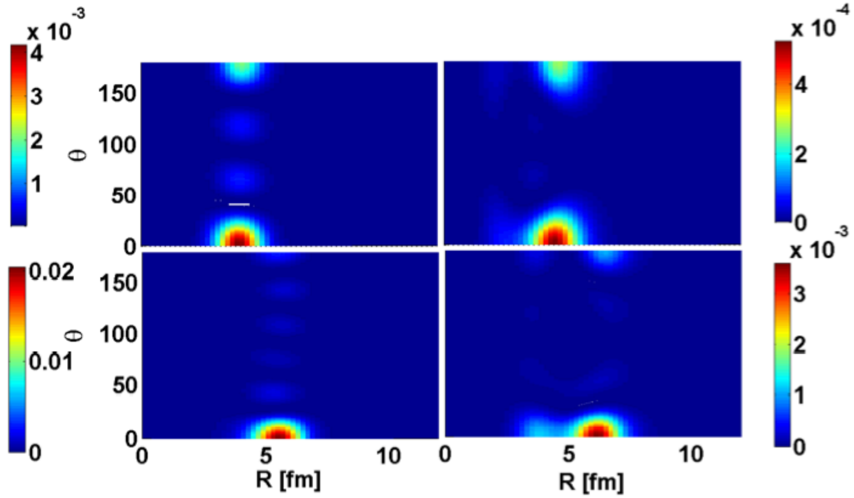
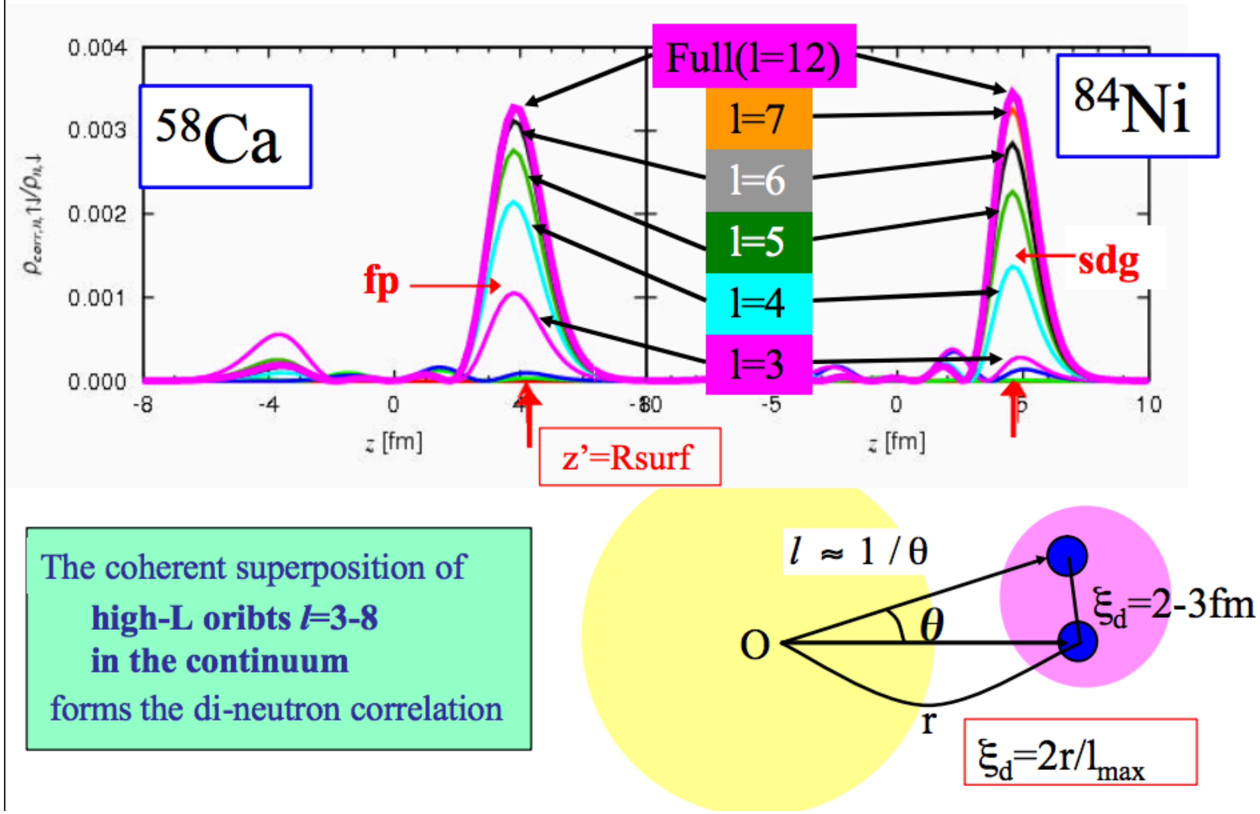



Figure 5: Upper: The two-neutron correlation plots for $^{46}$Ca (left) and $^{54}$Ca (right). Lower: Same as upper but for $^{128}$Sn (left, 4 holes) and $^{136}$Sn (right, 4 particles). Notice that the scale is different.

S. Changizi, CQ, Phys. Rev. C 91, 024305 (2015); Nucl. Phys. A 940, 210 (2015)

# Di-neutron correlation and high-L orbits




http://pro.ganil-spiral2.eu/events/seminars/files/m.-matsuo

# Two neutron clustering in $^6$He

Di-neutron: Two neutrons locate together outside the core
Cigar: Two neutrons in opposite directions

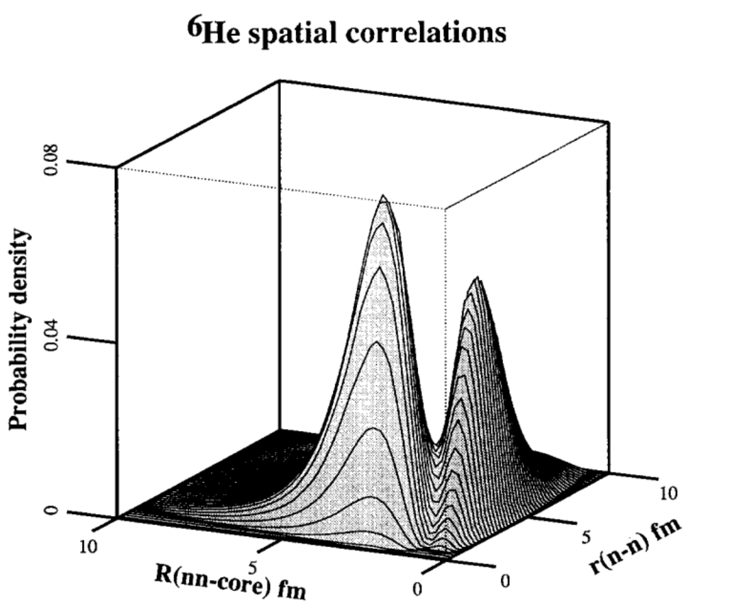
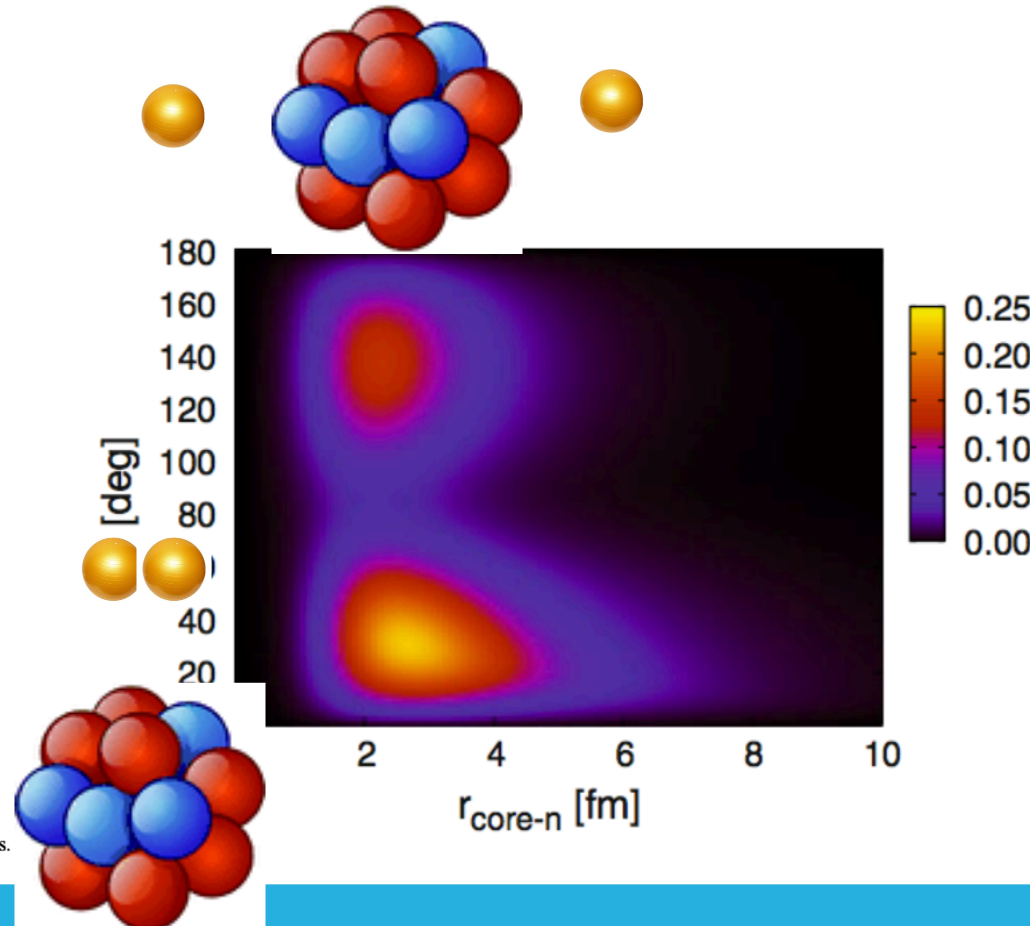



Fig. 4. Correlation density plot for the ground state of $^6$He in the (nn) and (nn)α variables.




M.V. Zhukov, B.V. Danilin, D.V. Fedorov, J.M. Bang, I.J. Thompson, J.S. Vaagen, Phys. Rep. 231, 151 (1993)

Kiyomi Ikeda, Takayuki Myo, Kiyoshi Kato, Hiroshi Toki, Lect.Notes Phys.818:165-221,2010

For two particles in a non-degenerate system with a constant pairing, the energy can be evaluated through the well known relation,

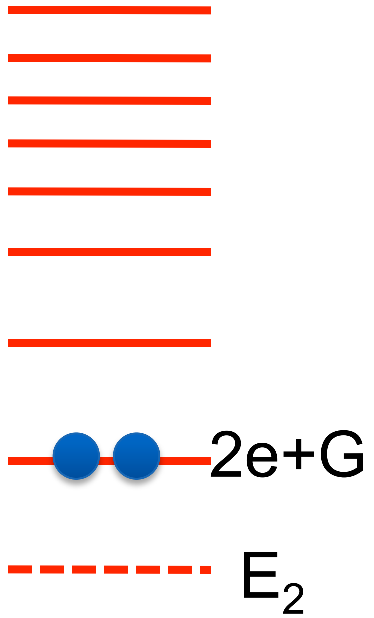
$$G \sum_i \frac{2j_i + 1}{2\varepsilon_i - E_2} = 2. \tag{10}$$

The corresponding wave function amplitudes are given by

$$X_i = N_n \frac{2j + 1}{2\varepsilon_i - E_2} \tag{11}$$

The correlation energy induced by the monopole pairing corresponds to the difference

$$\Delta = \varepsilon_\delta - \frac{1}{2} E_2, \tag{12}$$




where $\delta$ denotes the lowest orbital. As the gap $\Delta$ increases the amplitude $X_i$ becomes more dispersed, resulting in stronger two-particle correlation.

# Alpha formation probability from experiments

$$\log|RF(R)|^{-2} = \log T_{1/2}^{\text{Expt.}} - \log\left[\frac{\ln 2}{\nu}|H_0^+(\chi,\rho)|^2\right],$$

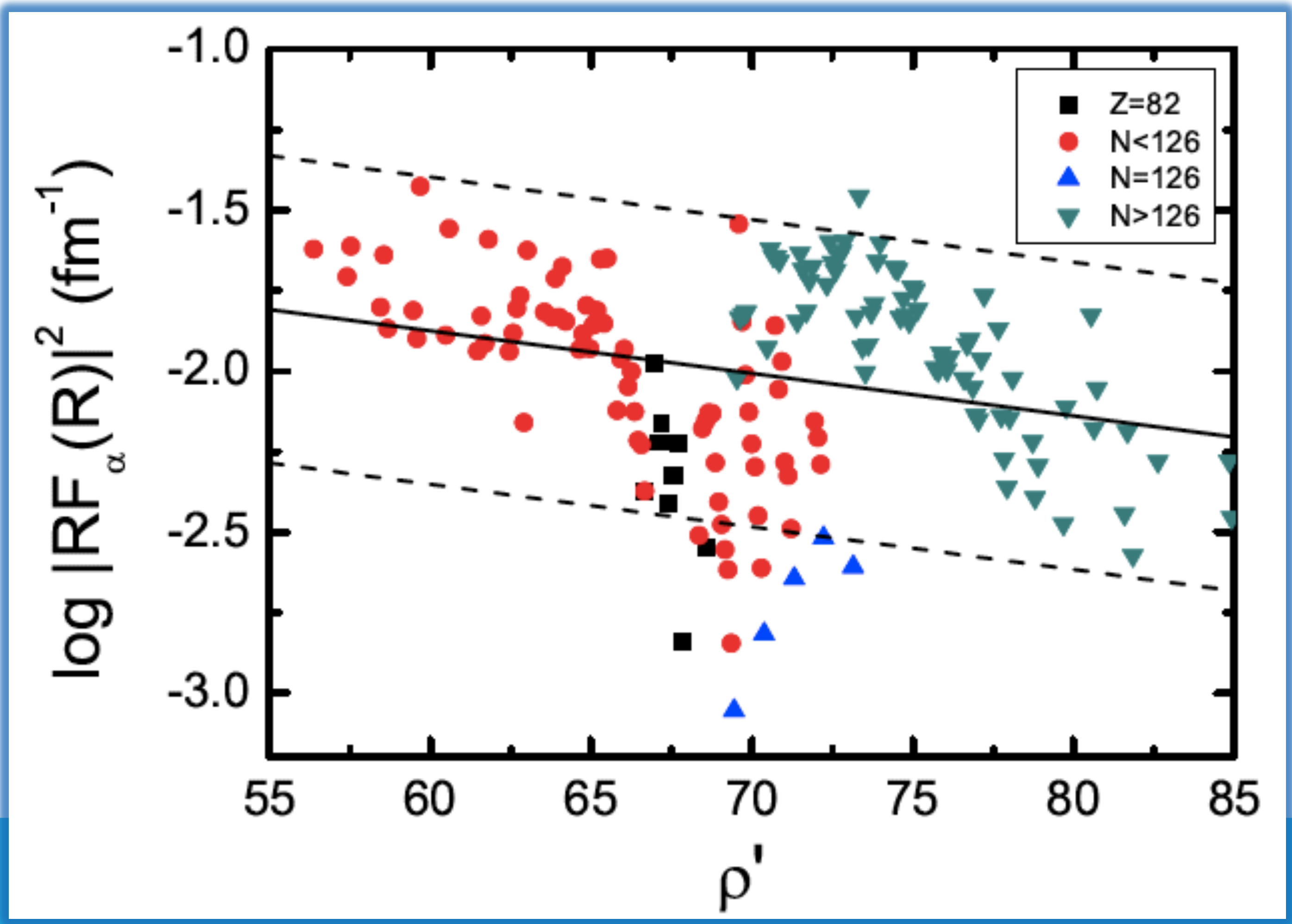
*R should be large enough that the nuclear interaction is negligible, i.e., at the nuclear surface.*

$R=1.2(A_d^{1/3}+A_c^{1/3})$

$$\chi' = \frac{\hbar}{e^2\sqrt{2M}}\chi = Z_cZ_d\sqrt{\frac{A}{Q_c}},$$

$$\rho' = \frac{\hbar}{\sqrt{2MR_0e^2}}(\rho\chi)^{1/2}$$

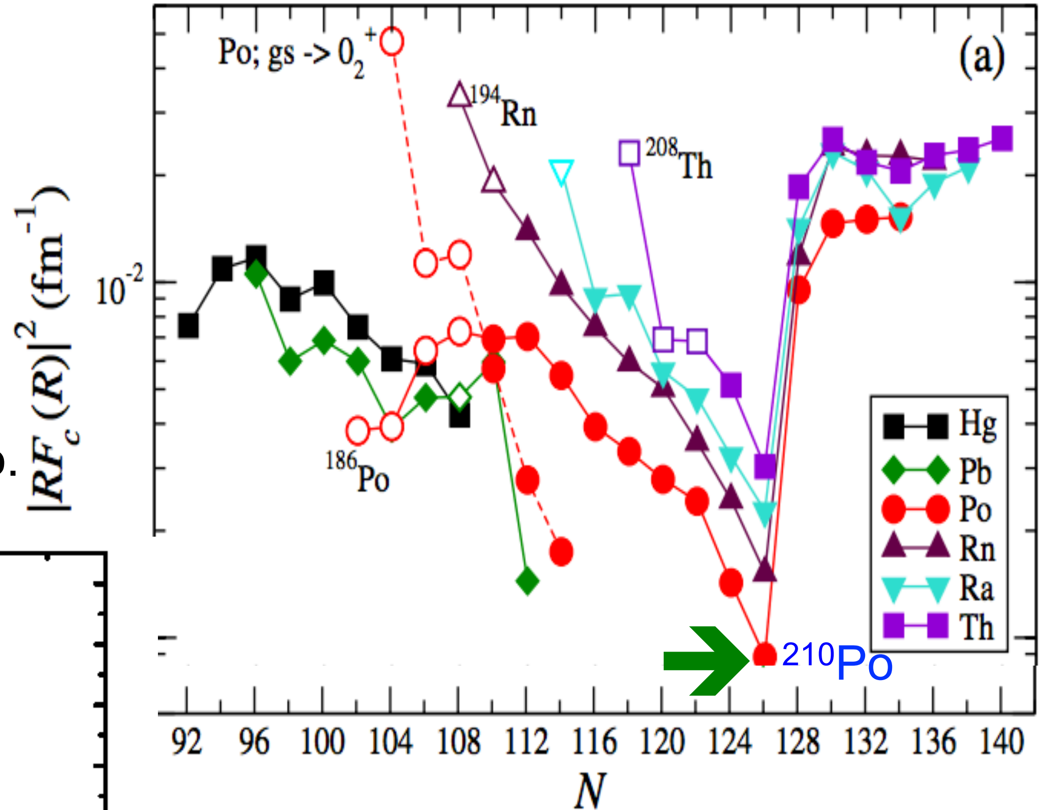
$$= \sqrt{AZ_cZ_d(A_d^{1/3}+A_c^{1/3})},$$

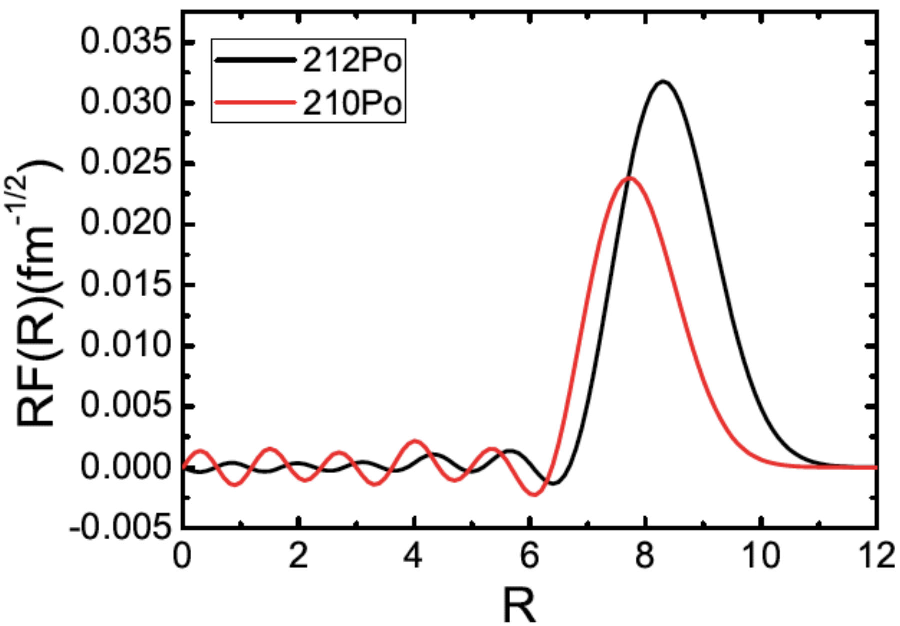



CQ et al, Phys.Rev.C80,044326 (2009); 81,064319 (2010).

# Sudden change *at N = 126*

- a division occurs between decays corresponding to *N <126 and N >126;*
- The case that shows the most significant hindrance corresponds to the α decay of the nucleus $^{210}$Po, one order of magnitude smaller than that of $^{212}$Po.



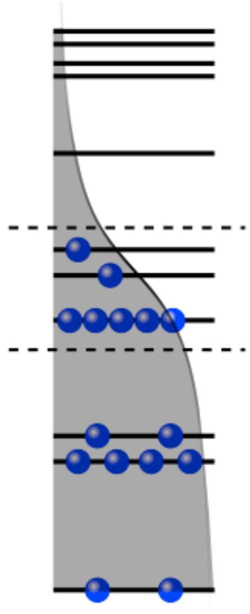



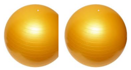

- **The clustering induced by the pairing mode is inhibited if the configuration space does not allow a proper manifestation of the pairing collectivity.**

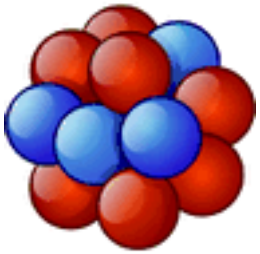
Larger pairing energy => Enhanced two-particle clustering at the nuclear surface

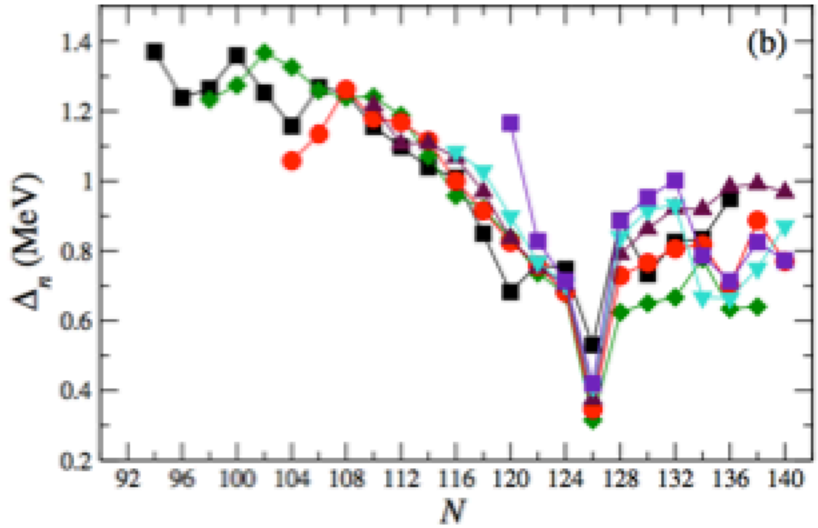
$$\Delta_n(Z, N) = \frac{1}{2}[B(Z, N) + B(Z, N-2) - 2B(Z, N-1)].$$

$$\Delta = G\sum_k u_k v_k,$$





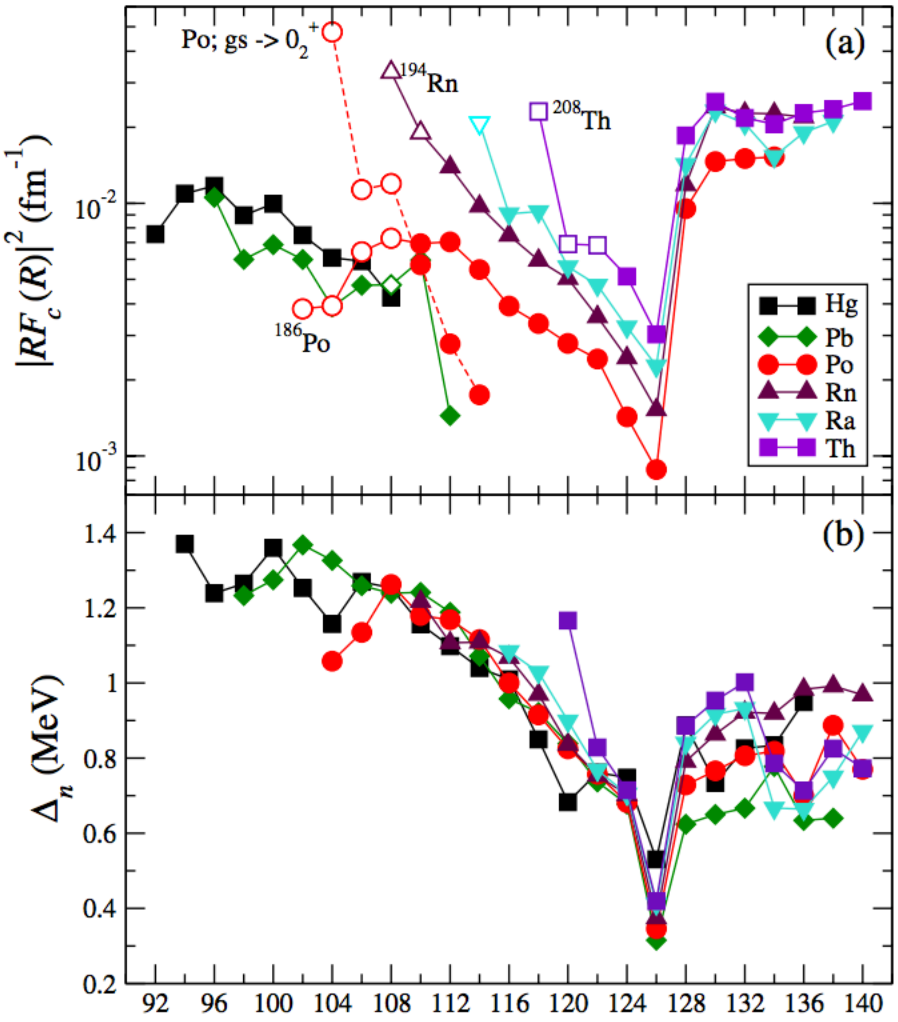




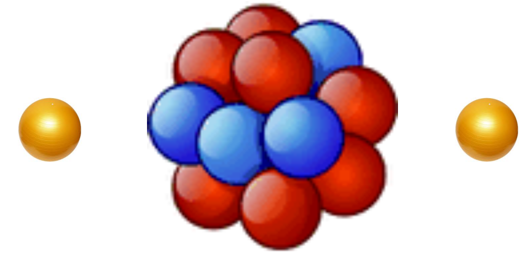

**Enhanced contribution due to Strong pairing**

# Pairing gap and the alpha formation

Larger pairing energy => Enhanced two-particle clustering at the nuclear surface

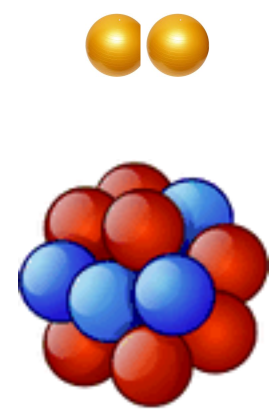



$$\Delta_n(Z,N) = \frac{1}{2}[B(Z,N) + B(Z,N-2) - 2B(Z,N-1)].$$



**No Pairing**

$$\Delta = G \sum_i u_i v_i,$$



**’Strong’ pairing**

A.N. Andreyev CQ et al., PhysRevLett.110.242502 (2013).

# Binding energy and odd-even staggering in Pb isotopes

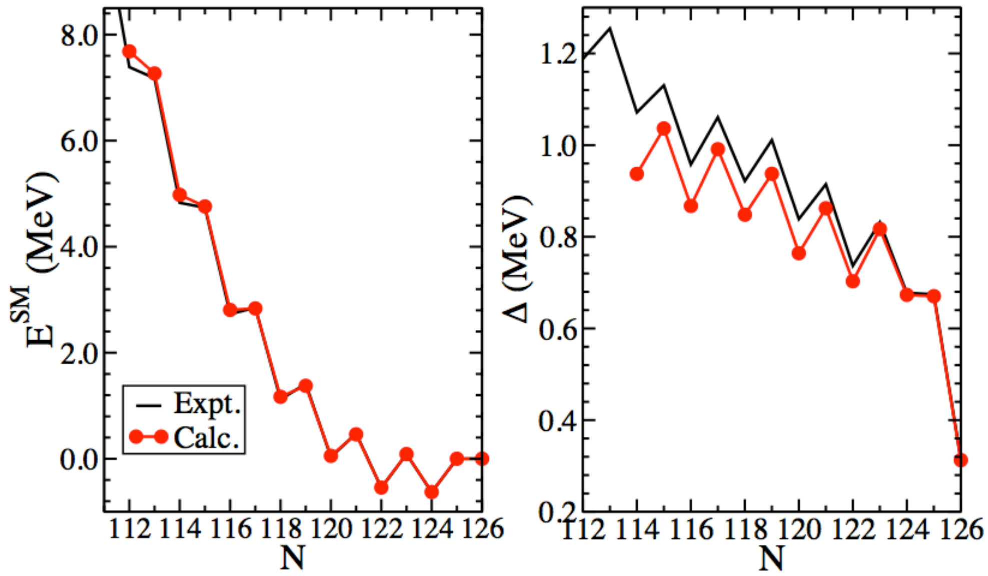


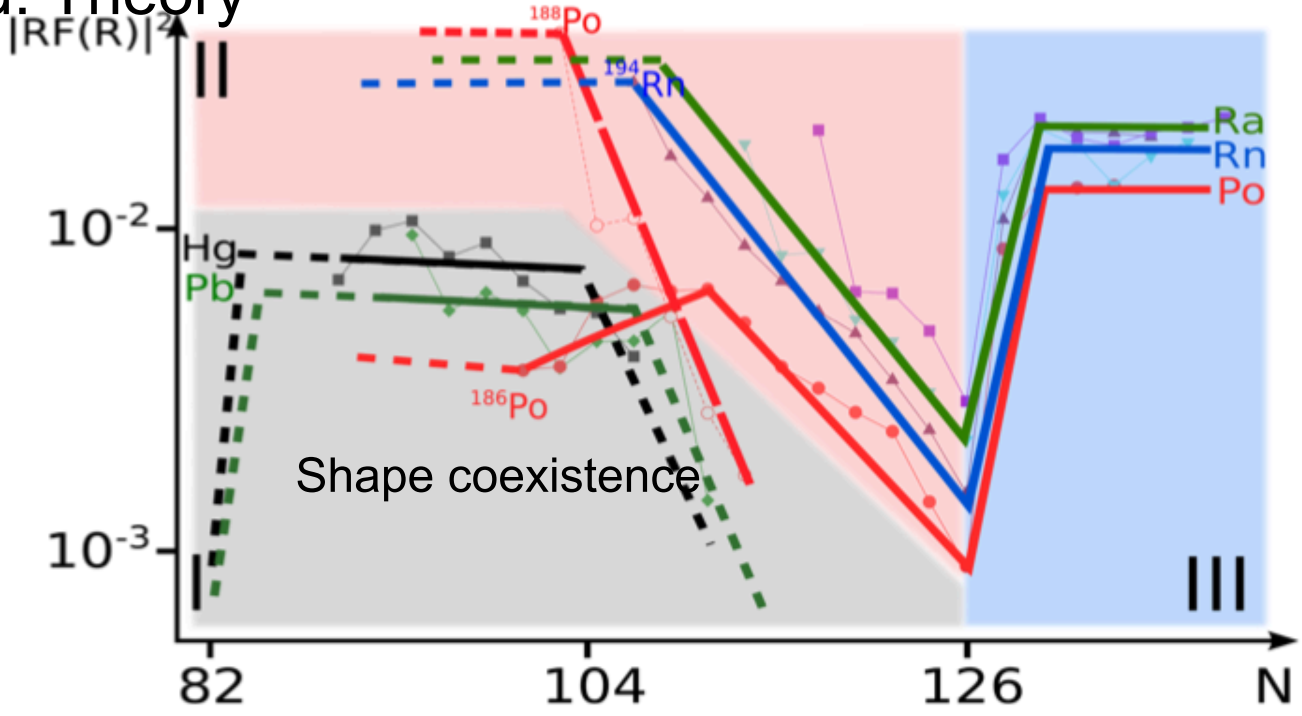

FIG. 9. (color online) Left: Experimental [80] and calculated shell-model correlation energies as a function of neutron number; Right: The empirical pairing gaps as extracted according to Eq. (5).

$$E_i^{\mathrm{cal}} = C + N\varepsilon_0 + \frac{N(N-1)}{2}V_m + \langle \Psi_I|H|\Psi_I\rangle,$$

CQ, LY Jia, GJ Fu, Phys. Rev. C 94, 014312 (2016)

Where the alpha formation saturate?

Solid: Observed
Dashed: Theory




C. Qi, A.N. Andreyev, M. Huyse, R.J. Liotta, P. Van Duppen, R. Wyss Phys. Lett. B 734, 203 (2014)

# Probing shape coexistence by α decays to $0^+$ states

D. S. Delion, R. J. Liotta, Chong Qi, and R. Wyss
Phys. Rev. C **90**, 061303(R) – Published 16 December 2014

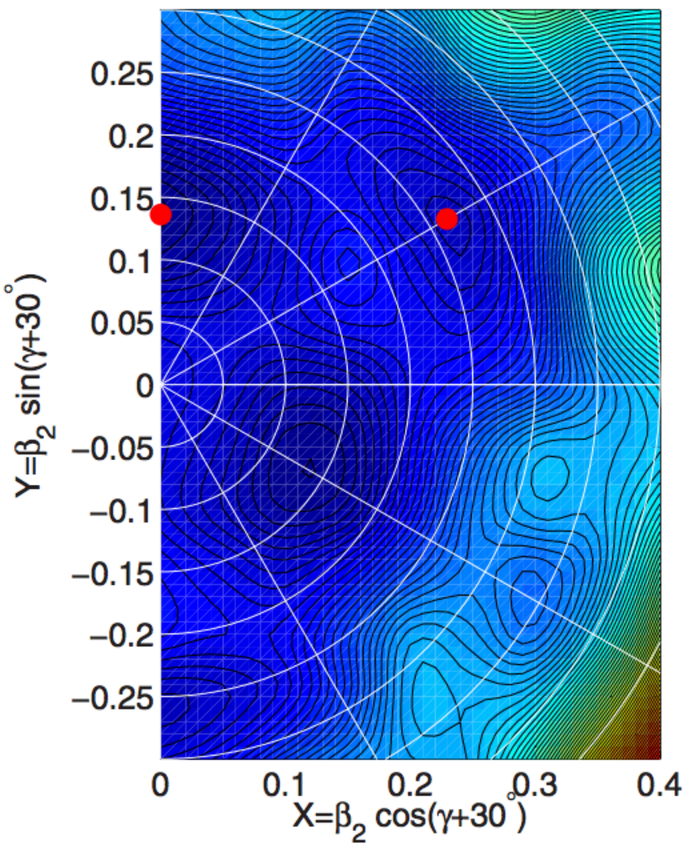



FIG. 1. (Color online) PES of $^{182}$Hg. Local minima for the

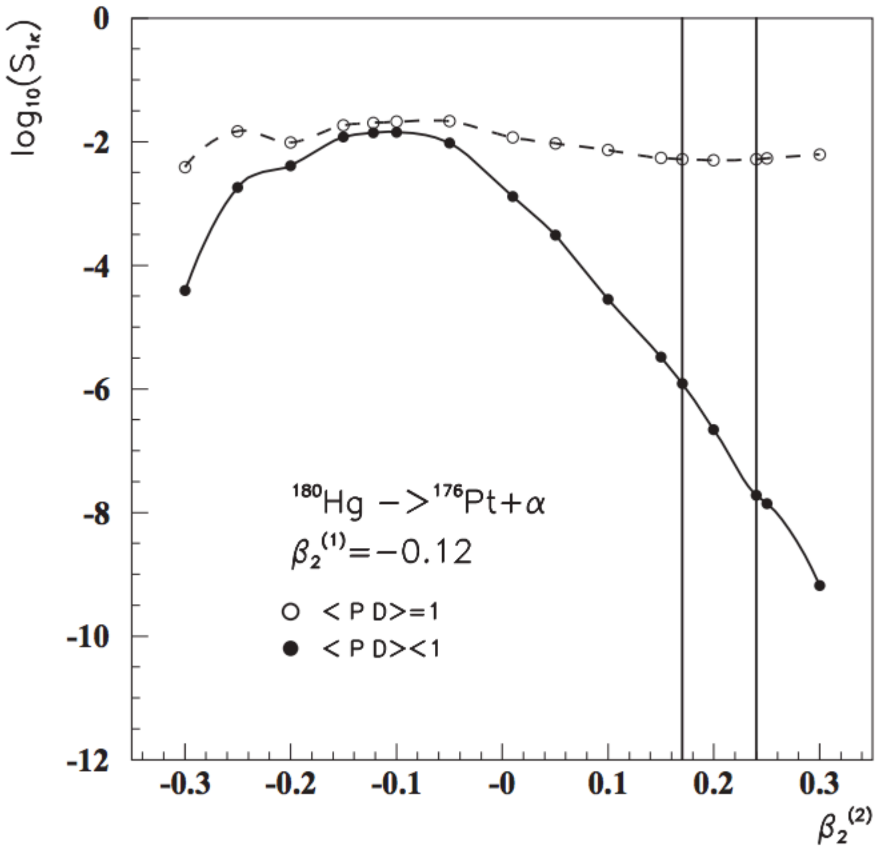
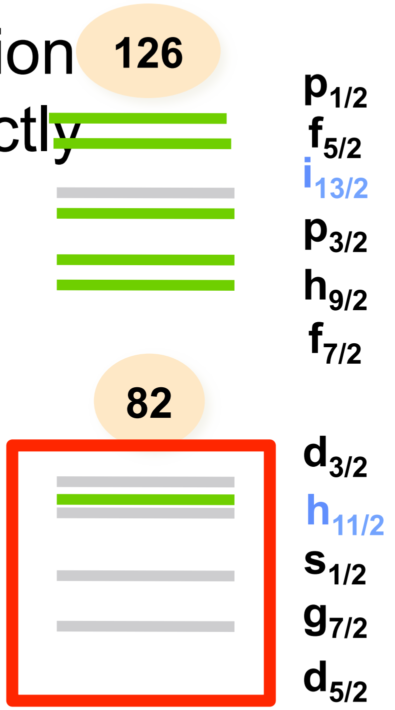
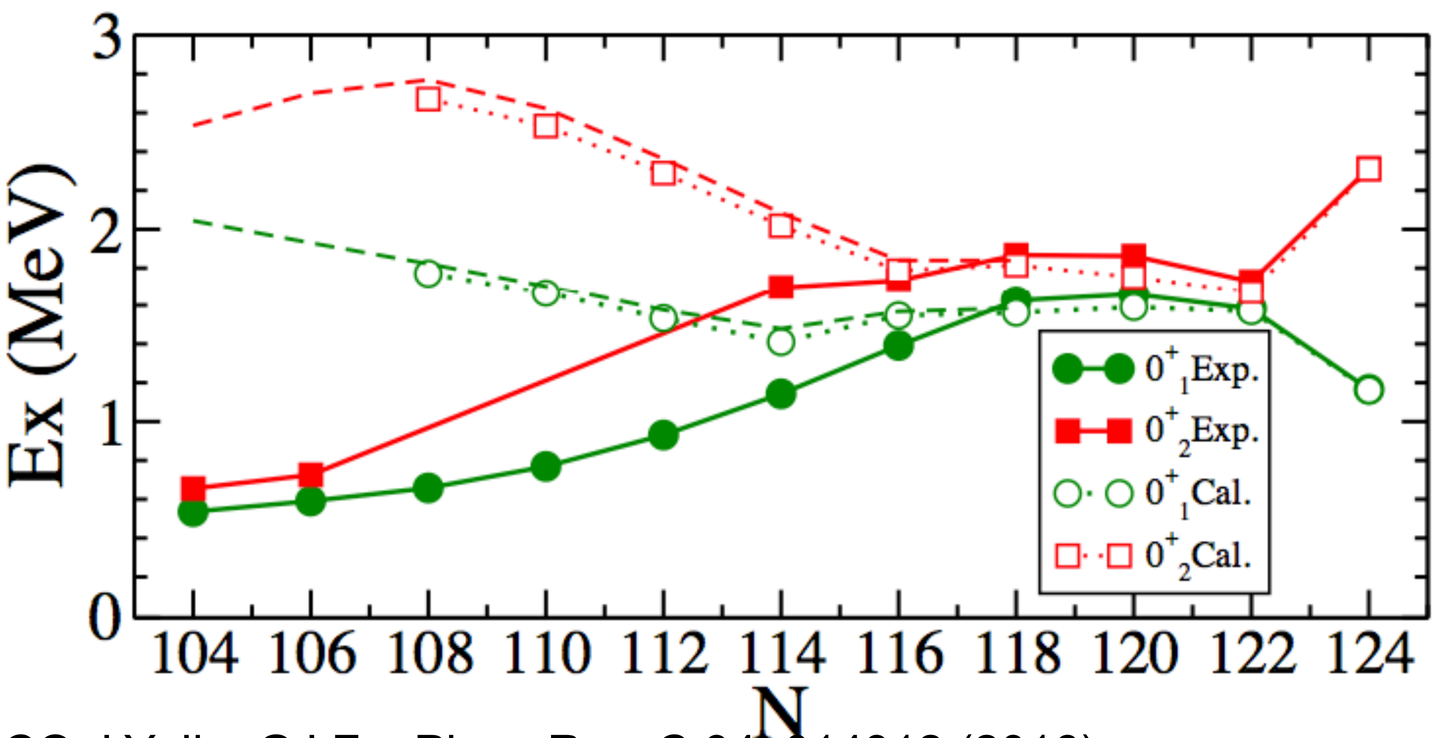


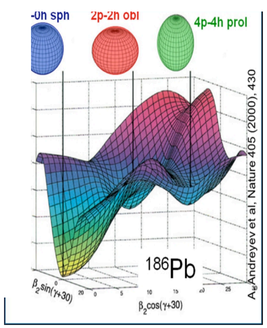

FIG. 2. Spectroscopic factor for the transition $^{180}$Hg(gs) → $^{176}$Pt($0_2^+$) + $\alpha$ between BCS states with $k = 1$ in Eq. (4). The

# ➢ The excited 0+ states in Pb isotopes

- ➢ Large-scale shell model calculations still **not sufficient** for those deformed 0+ states (and too complex for alpha clustering calculations in general);
- ➢ Now we have shell-model-like diagonalization in the seniority-zero space as a way to solve exactly the pairing Hamitlonian



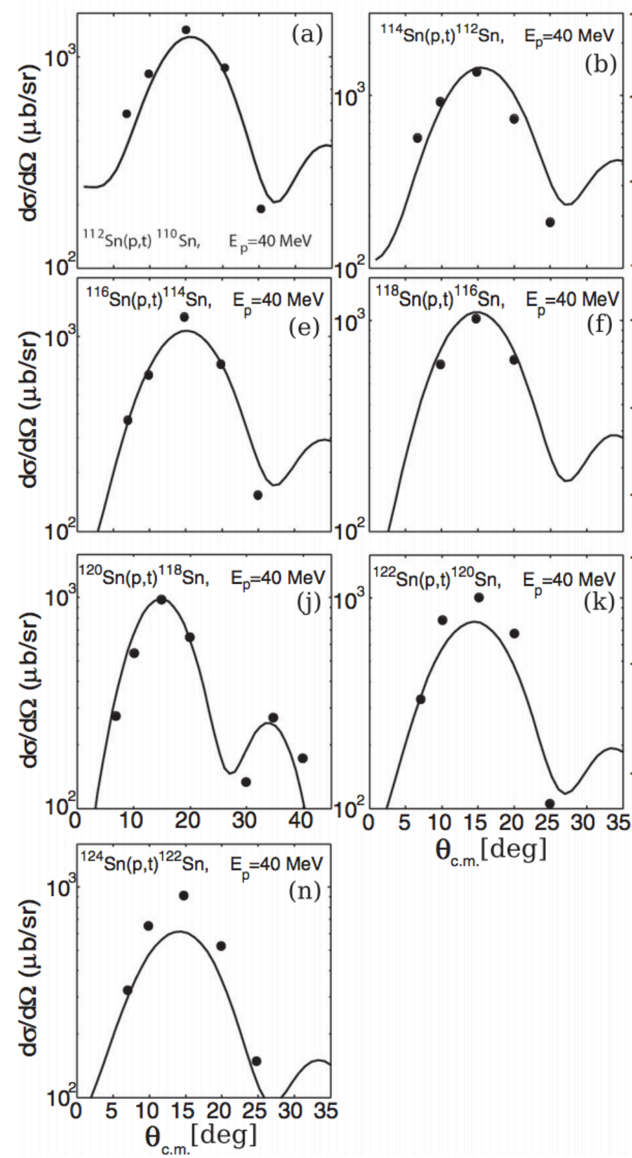




CQ, LY Jia, GJ Fu, Phys. Rev. C 94, 014312 (2016)

# Pair transfer in nuclei

- Two particle transfer reactions like (t,p) or ($^3$He,p) may provide an specific tool to probe pairing correlations.
- Pair correlations result in a constructive interference of reaction amplitudes and give a strongly enhanced two-nucleon transfer




IG. 7. Predicted absolute differential $^{A+2}$Sn$(p,t)^A$Sn(gs) cross sec

G. Potel, A. Idini, F. Barranco, E. Vigezzi, and R. A. Broglia Phys. Rev. C 87, 054321 (2013)

## Seniority

$$\langle N+2, v, \alpha | P^\dagger | N, v, \alpha \rangle = \tfrac{1}{2}\sqrt{(2\Omega - N - v)(N - v + 2)},$$

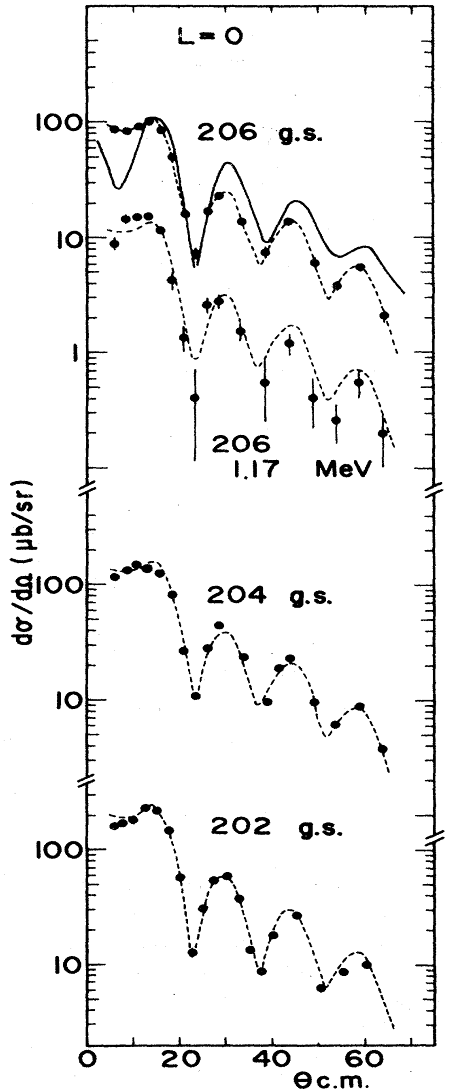
$$\langle N-2, v, \alpha | P | N, v, \alpha \rangle = \tfrac{1}{2}\sqrt{(N - v)(2\Omega - N - v + 2)}$$

## BCS

$$\langle \mathrm{BCS} | P^\dagger | \mathrm{BCS} \rangle = \frac{\Delta}{G}.$$

$$\Delta = G \sum_i u_i v_i,$$

# cross sections of (p,t) reactions on Pb isotopes



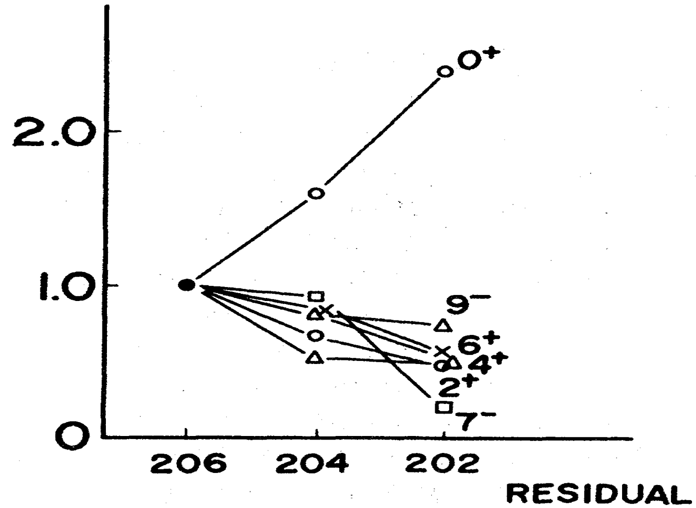
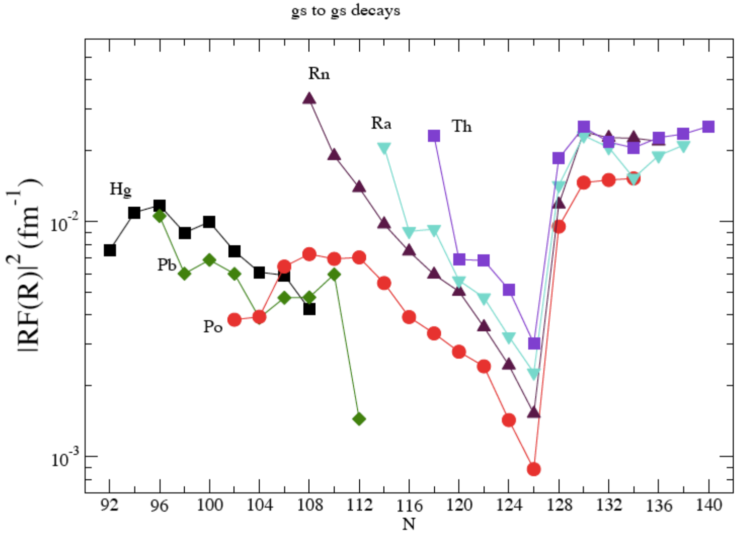




FIG. 13. Isotope dependence of triton strengths leading to the lowest state of $J^\pi = 0^+, 2^+, 4^+, 6^+, 7^-$, and $9^-$.




M. Takahashi, PRC27,1454(1983)

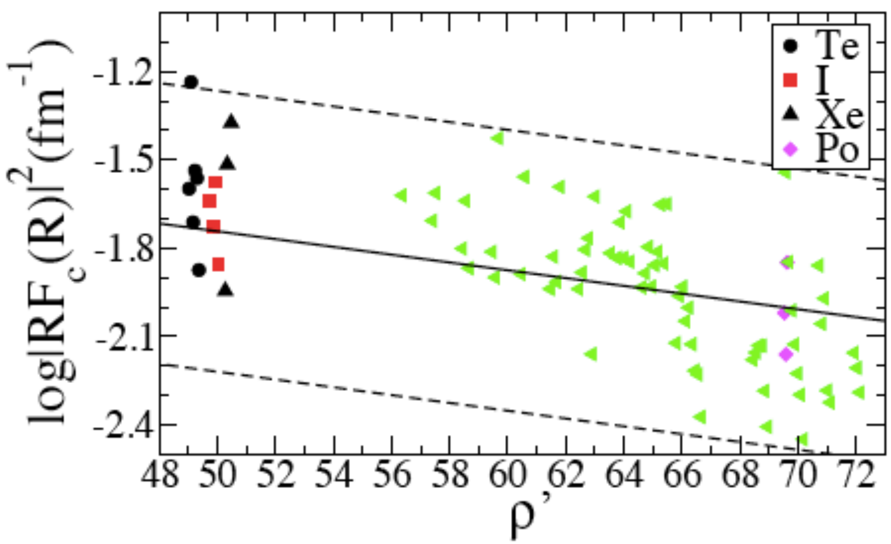
# ‘Superallowed’ alpha decay around N=Z nuclei

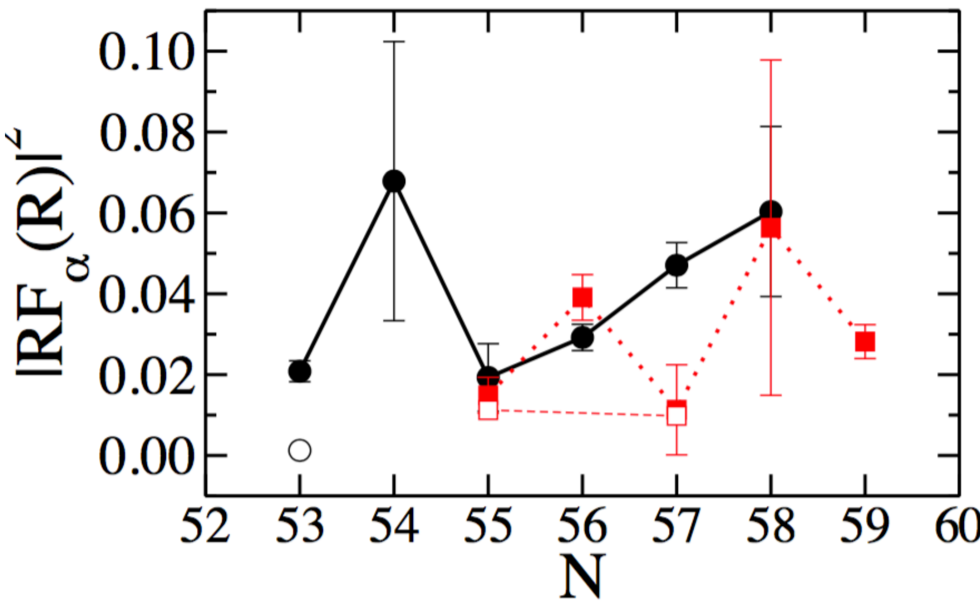
PRL **97**, 082501 (2006) PHYSICAL REVIEW LETTERS week ending 25 AUGUST 2006

## Discovery of $^{109}$Xe and $^{105}$Te: Superallowed $\alpha$ Decay near Doubly Magic $^{100}$Sn

S. N. Liddick,[1] R. Grzywacz,[2,3] C. Mazzocchi,[2] R. D. Page,[4] K. P. Rykaczewski,[3] J. C. Batchelder,[1] C. R. Bingham,[2,3] I. G. Darby,[4] G. Drafta,[2] C. Goodin,[5] C. J. Gross,[3] J. H. Hamilton,[5] A. A. Hecht,[6] J. K. Hwang,[5] S. Ilyushkin,[7] D. T. Joss,[4] A. Korgul,[2,5,8,9] W. Królas,[9,10] K. Lagergren,[9] K. Li,[5] M. N. Tantawy,[2] J. Thomson,[4] and J. A. Winger[1,7,9]

## Alpha formation properties in N~Z nuclei

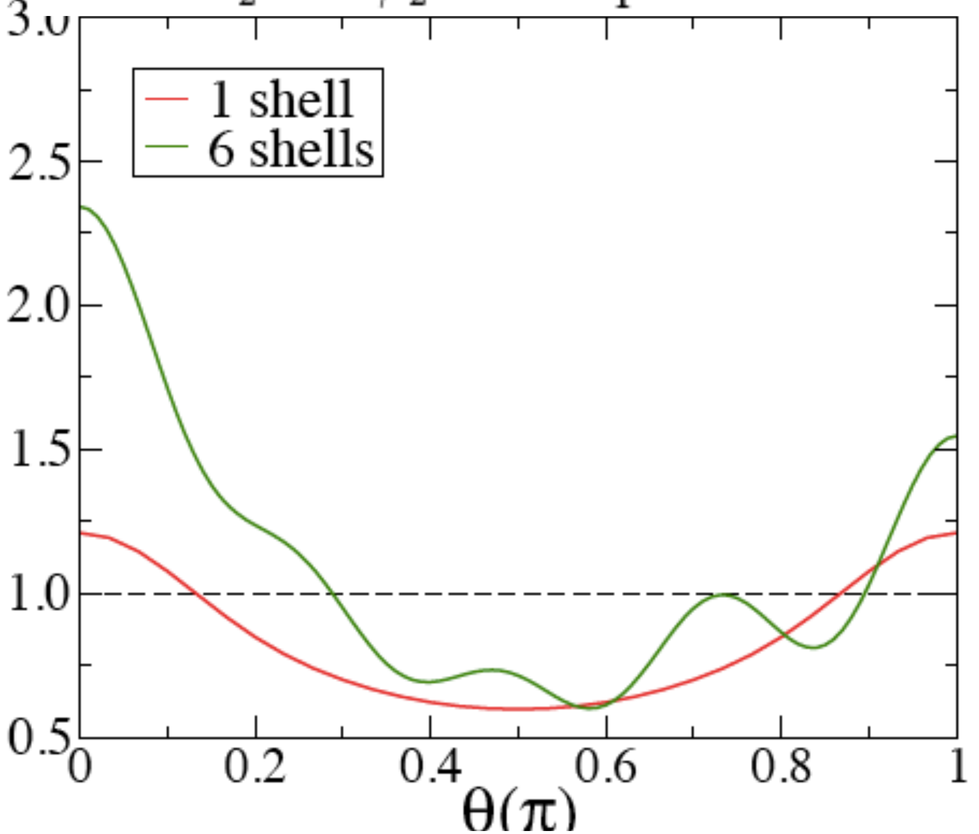


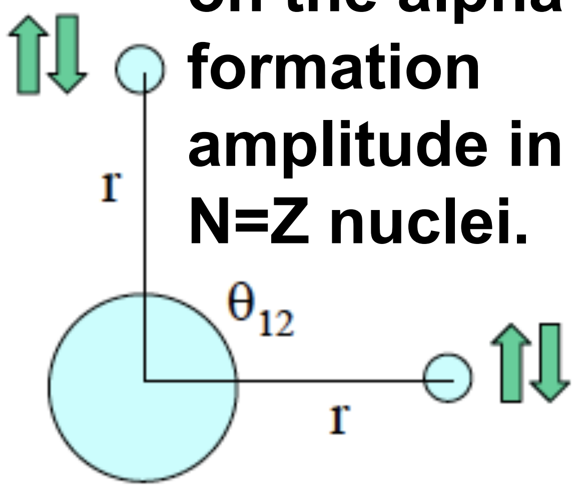
The four-body (alpha) wave function can be written as

$$|\gamma_4\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\gamma_4)|\alpha_2 \otimes \beta_2\rangle,$$

where $\alpha_2$ and $\beta_2$ denote proton and neutron wave functions, respectively.

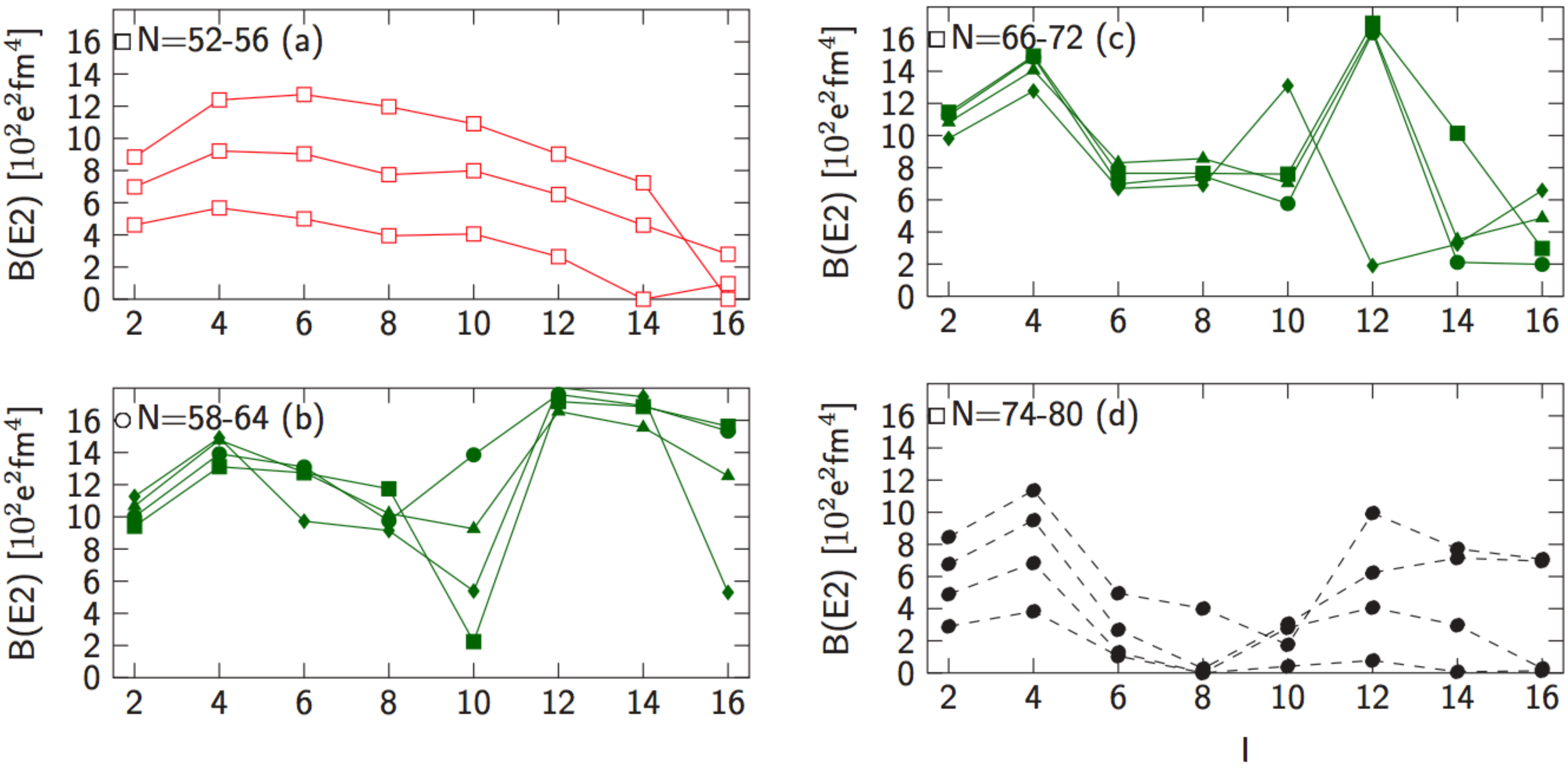



Relative angular distribution of four-particle wave function with (solid lines) and without (dashed line) neutron-proton interactions.




**Shell model calculations on the alpha formation amplitude in N=Z nuclei.**

# E2 transition properties of Te isotopes




CQ, Phys. Rev. C 94, 034310 (2016)

## Summary

- Microscopic studies of the alpha and proton decays
- Alpha clustering and nuclear pairing
- The extraction of alpha and proton formation amplitudes from experimental data
- Abrupt changes around the N=126 shell closure and effect of pairing collectivity;
- Influence of the neutron-proton correlation on alpha formation

# Thank you!

**Collaborators:**

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A.N. Andreyev (York), M. Huyse, P. Van Duppen (KU Leuven, Belgium)