Description of high energy breakup of ^{11}Be at GSI

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Outline

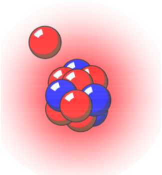
Introduction

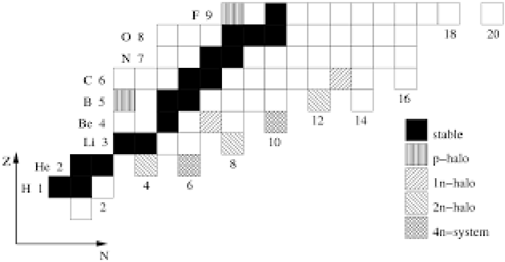
Reaction model

Results

Conclusions

The study of ^{11}Be breakup

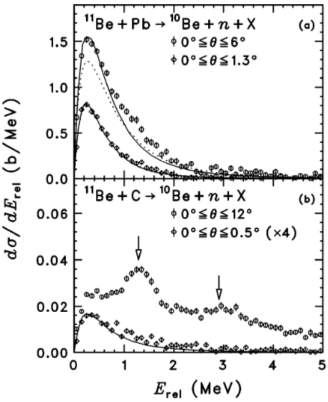




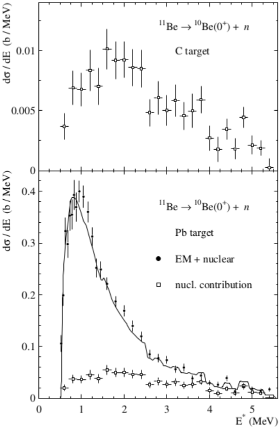
One-neutron halo nucleus
$^{11}\mathrm{Be} = {}^{10}\mathrm{Be}+\mathrm{n}$

Results for ^{11}Be+^{208}Pb and ^{11}Be+^{12}C

@ ∼70A MeV (RIKEN)
Fukada *et al.*, PRC **70** (2004)



@ 520A MeV (GSI)
Palit *et al.*, PRC **68** (2003)



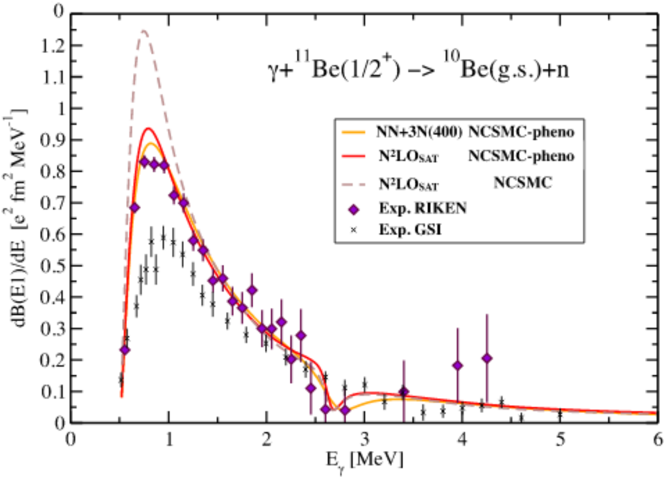
...an open problem!

The electric dipole response $\frac{dB(E1)}{dE}$ of ^{11}Be

extracted from GSI data differs from RIKEN results

...an open problem!

Calci, Navrátil, Roth, Dohet-Eraly, Quaglioni and Hupin, PRL **117** (2016)



Our proposal

We develop an Eikonal model to describe GSI data

Capel, Baye and Suzuki, PRC **78** (2008)

...and we include relativistic corrections!

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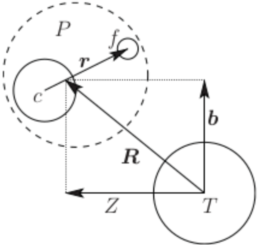
The initial conditions
in the laboratory frame of reference (LAB)

- target T is at rest
  ^{208}Pb or ^{12}C
- projectile P composed by an inert core plus a fragment $C + f$:
  ^{11}Be$\rightarrow$^{10}Be+n
- bombarding energy $E_{LAB} = 520$MeV/nucleon
  - $\rightarrow$ P has constant velocity
  - $\rightarrow$ P follows a straight line

$\Rightarrow$ ideal conditions to apply the Eikonal model!



The Eikonal model

in the frame of reference of the P-T center of momentum (CM)

We should start with a Klein-Gordon equation

because of high energy regime

but following the kinematical prescriptions proposed by Satchler

we are able to reduce the Klein-Gordon to a Schroedinger equation

Satchler, *Nucl. Phys. A* **540** (1992)

The Eikonal model

in the frame of reference of the P-T center of momentum (CM)

In the T-P CM system Klein-Gordon equation is

$$\left[(\hbar c)^2\nabla^2 + (\hbar c k)^2 - 2EV_{PT}\right]\Psi = 0$$

- $\hbar k$ is the relativistic momentum of P in CM
- $E = (M_P M_T c^2)/(M_P + M_T)$ reduced energy
- $M_P c^2$ and $M_T c^2$ are P and T total energies in the CM
- $M_P = \gamma_P m_P$ and $M_T c^2 = \gamma_T m_T$ are the relativistic masses

The Eikonal model

in the frame of reference of the P-T center of momentum (CM)

So KG equation reduces to a Schroedinger equation
for the scattering of two nuclei of masses M_P and M_T
and a CM kinetic energy $E_{CM} = (\hbar k)^2/2\mu$
where $\mu = E/c^2$ plays the role of reduced mass

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(\mathbf{R},\mathbf{r})\right]\Psi(\mathbf{R},\mathbf{r}) = E_{CM}\Psi(\mathbf{R},\mathbf{r})$$

$\Rightarrow$ one can solve the usual nonrelativistic model
provided one uses these kinematics prescriptions

P is initially bound in its ground state $\Phi_{l_0 j_0 m_0}$ of energy E_0

The Eikonal approximations

Adiabatic approximation:
the collision occurs in a very brief time
and the internal P coordinates are frozen during reaction

The Eikonal approximations

In the Eikonal description the wavefunction is factorized as:

$$\Psi(\mathbf{b}, z, \mathbf{r}) = e^{ikz}\hat{\Psi}(\mathbf{b}, z, \mathbf{r})$$

given $\nabla^2\Psi = -k^2 e^{ikz}\hat{\Psi} + 2ike^{ikz}\frac{\partial}{\partial z}\hat{\Psi} + e^{ikz}\nabla^2\hat{\Psi}$ we obtain

$$\Rightarrow \left[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{\hbar k}{\mu} i\hbar\frac{\partial}{\partial z} + V(\mathbf{R}, \mathbf{r})\right]\hat{\Psi}(\mathbf{b}, z, \mathbf{r}) = 0$$

since $\hat{\Psi}$ is expected to vary weakly in $\mathbf{R}$
$\rightarrow$ we assume ∇^2 negligible with respect to $\frac{\partial}{\partial z}$

$$i\hbar v\frac{\partial}{\partial_z}\hat{\Psi}(\mathbf{b}, z, \mathbf{r}) = V_{PT}(\mathbf{R}, \mathbf{r})\hat{\Psi}(\mathbf{b}, z, \mathbf{r})$$

$v = \hbar k/\mu$ relative P-T velocity

The solution for a one-neutron halo nucleus

So we obtain the Eikonal expression

$$\hat{\Psi}(\mathbf{b}, z, \mathbf{r}) = e^{i\chi(\mathbf{b},\mathbf{s})}\Phi_{l_0 j_0 m_0}(E_0, \mathbf{r})$$

where the Eikonal phase could be divided
into its nuclear and Coulomb contributions:

$$\chi(\mathbf{b}, \mathbf{s}) = \chi^N(\mathbf{b}, \mathbf{s}) + \chi^C(\mathbf{b}, \mathbf{s}) + \chi^C_{PT}(b)$$

The nuclear interaction

The nuclear interaction is usually calculated using optical potentials

$$\chi^N = -\frac{1}{\hbar v}\int_{-\infty}^{z} V_{CT}(\mathbf{b}, z', \mathbf{r}) + V_{fT}(\mathbf{b}, z', \mathbf{r}) dz'$$

no data to fit the optical pot parameters at high energies

$\Rightarrow$ we can not use an optical potential

The nuclear interaction

We apply the optical limit approximation of Glauber theory

$$\chi^{N}_{OLA}(\mathbf{b}) = i \int \int \rho_T(\mathbf{r'})\rho_X(\mathbf{r''})\Gamma_{NN}(\mathbf{b} - \mathbf{s'} + \mathbf{s''}) d\mathbf{r''} d\mathbf{r'}$$

- $\rho(\mathbf{r})$ is neutron or proton Fermi density
- $\Gamma_{NN}(\mathbf{b}) = \frac{1-i\alpha_{NN}}{4\pi\beta_{NN}}\sigma^{tot}_{NN} e^{-\frac{b^2}{2\beta_{NN}}}$ is a profile function that correspond to effective nucleon-nucleon interaction
  - σ^{tot}_{NN} total cross section for the NN collision
  - α_{NN} ratio of real to imag. part of the NN-scattering amplitude
  - β_{NN} slope of NN elastic differential cross section

Horiuchi, Suzuki, Capel and Baye, PRC **81** (2010)

The Coulomb phase corrections

The Eikonal solution is valid for short-range potential
we have to deal with Coulomb phase divergence

Margueron, Bonaccorso and Brink, *Nucl. Phys. A* **720** (2003)

The Coulomb phase corrections

- Rutherford scattering between the P center-of-mass and the T

$$\chi^C_{PT} = -\eta \int_{-\infty}^{+\infty} \frac{dz}{R}$$

- Coulomb tidal force

$$\chi^C = -\eta \int_{-\infty}^{\infty} \left(\frac{1}{|\mathbf{R} - \frac{m_f}{m_P}\mathbf{r}|} - \frac{1}{R} \right) dz$$

The Coulomb phase corrections

- Rutherford scattering between the P center-of-mass and the T
  $\rightarrow$ truncated to avoid divergence

$$\chi^C_{PT} = -\eta \int_{-z_{max}}^{z_{max}} \frac{dz}{R} \sim 2\eta \ln\left(\frac{b}{2z_{max}}\right)$$

- Coulomb tidal force

$$\chi^C = -\eta \int_{-\infty}^{+\infty} \left(\frac{1}{|\mathbf{R} - \frac{m_f}{m_P}\mathbf{r}|} - \frac{1}{R} \right) dz$$

The Coulomb phase corrections

- Rutherford scattering between the P center-of-mass and the T

$$\chi_{PT}^{C} \sim 2\eta \ln\left(\frac{b}{2z_{max}}\right)$$

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The Coulomb phase corrections

- Rutherford scattering between the P center-of-mass and the T

$$\chi^C_{PT} \sim 2\eta \ln\left(\frac{b}{2z_{max}}\right)$$

- Coulomb tidal force

$$\chi^C = \eta \ln\left(1 - 2\frac{m_f}{m_P}\frac{\mathbf{b}\cdot\mathbf{s}}{b} + \frac{m_f^2}{m_P^2}\frac{s^2}{b^2}\right)$$

→Divergence due to the slow decrease of χ^C in b:
when integrating over b, the $1/b$ behavior of term $i\chi^C$
$e^{i\chi^C} = 1 + i\chi^C - \frac{1}{2}(\chi^C)^2 + \ldots$

The Coulomb phase corrections

- Rutherford scattering between the P center-of-mass and the T

$$\chi^C_{PT} \sim 2\eta \ln\left(\frac{b}{2z_{max}}\right)$$

- Coulomb tidal force: $\Rightarrow$ we make the replacement

$$e^{i\chi} = e^{i\chi^N}(e^{i\chi^C} - i\chi^C + i\chi^{FO})e^{i\chi^C_{PT}}$$

first order term of the perturbation theory

$$\chi^{FO} = -\eta\int_{-\infty}^{\infty} e^{i\omega z/v}\left(\frac{1}{|\mathbf{R} - \frac{m_f}{m_P}\mathbf{r}|} - \frac{1}{R}\right)dz$$

where $\omega = (E - E_0)/\hbar$, and E C-f relative energy after dissociation

Changing frame of reference

from the P-T CM frame to P rest frame

The equations we use for the dynamics should be Lorentz invariant this is true if if $V_{PT(\mathbf{b},z,\mathbf{r})}$ is Lorentz invariant

$\Rightarrow$ it should transform as the time-like component of a Lorentz four-vector

$$V_{PT(\mathbf{b},z,\mathbf{r})} \to \gamma V_{PT(\mathbf{b},\gamma z,\mathbf{r})}$$

where $\gamma = \left(1 - w^2/c^2\right)^{-1/2}$
and w the P velocity in P-T CM frame

Winther and Alder, *Nucl. Phys. A* **319** (1979)
Bertulani, *Phys. Rev. Lett.* **94** (2005)
Ogata and Bertulani, *Progr. Theor. Phys.* **123** (2010)

Changing frame of reference

from the P-T CM frame to P rest frame

$$V_{PT(\mathbf{b},z,\mathbf{r})} \to \gamma V_{PT(\mathbf{b},\gamma z,\mathbf{r})}$$

where $\gamma = \left(1 - w^2/c^2\right)^{-1/2}$
and w the P velocity in P-T CM frame

This transformation is

- well established for electromagnetic field
- just a conjecture for the nuclear interaction

Winther and Alder, *Nucl. Phys. A* **319** (1979)
Bertulani, *Phys. Rev. Lett.* **94** (2005)
Ogata and Bertulani, *Progr. Theor. Phys.* **123** (2010)

Changing frame of reference

from the P-T CM frame to P rest frame

Let's apply the Lorentz boost:

- Nuclear phase χ^N and Coulomb phases χ^C_{PT} and χ^C are already Lorentz invariant in our model:
  no changes under the transformation $V(z) = \gamma V(\gamma z)$
- The phase χ^{FO} is not Lorentz invariant:
  $\chi^{FO} = -\eta \int_{-\infty}^{\infty} e^{i\omega z/\gamma v} \left(\frac{1}{|\mathbf{R} - \frac{m_f}{m_P}\mathbf{r}|} - \frac{1}{R} \right) dz$
  consistent with Winther and Alder's
  relativistic Coulomb excitation result

The breakup cross section

So the breakup amplitude is

$$S^{m_0}_{kljm}(b) \sim \langle \Phi_{ljm}(E)| e^{i\chi^N} (e^{i\chi^C} - i\chi^C + i\chi^{FO}) e^{i\chi^C_{PT}} |\Phi_{l_0 j_0 m_0}(E_0)\rangle$$

Breakup cross section
as a function of C-f relative energy E after dissociation

$$\frac{d\sigma_{bu}}{dE} = \frac{4\mu_{cf}}{\hbar^2 K} \frac{1}{2j_0+1} \sum_{m_0} \sum_{ljm} \int_0^\infty bdb |S^{m_0}_{kljm}(b)|^2$$

where μ_{cf} and K
are C-f reduced mass and momentum in P rest frame
no relativistic effects considered here

The breakup cross section

So the breakup amplitude is

$$S^{m_0}_{kljm}(b) \sim \langle \Phi_{ljm}(E) | e^{i\chi^N} (e^{i\chi^C} - i\chi^C + i\chi^{FO}) e^{i\chi^C_{PT}} | \Phi_{l_0 j_0 m_0}(E_0) \rangle$$

Breakup cross section
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where μ_{cf} and K
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no relativistic effects considered here

The ^{11}Be internal structure

We use a halo EFT model
i.e. gaussian potentials with differenth widths
adjusted on same properties
(binding energy and ANC)
of *ab initio* model
Calci, Navrátil, Roth, Dohet-Eraly, Quaglioni and Hupin, PRL **117** (2016)
which gives the parity inversion of ^{11}Be g.s.

we include only bound states, not resonances!

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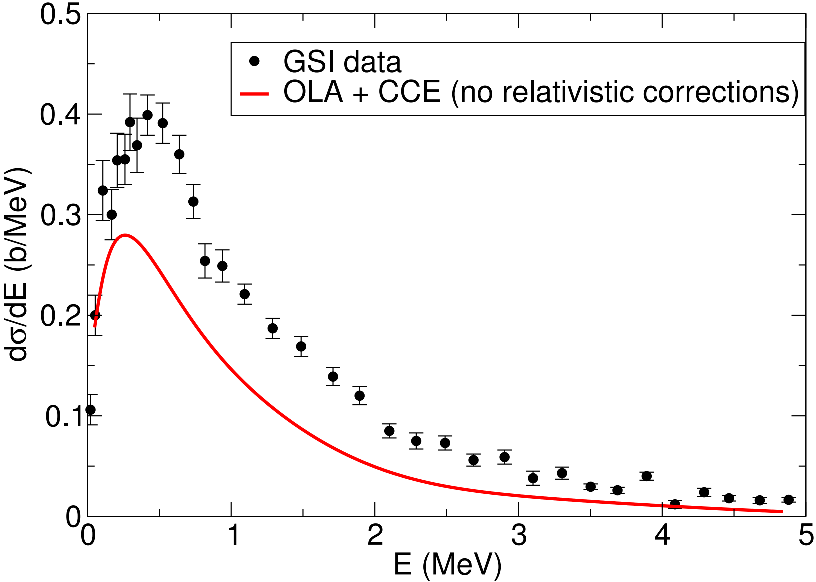
^{11}Be model tested at GSI energy

Let's look at GSI breakup cross section

$^{11}Be + ^{208}Pb$ @ 520 AMeV



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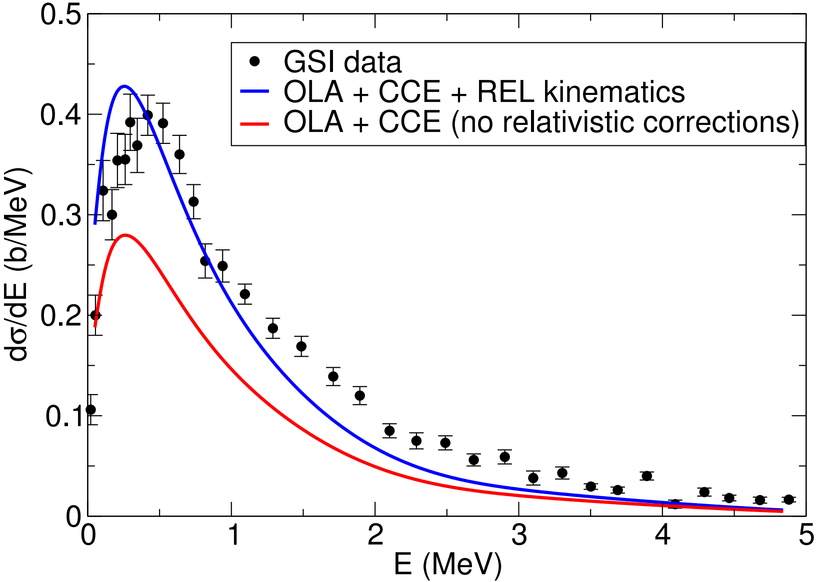
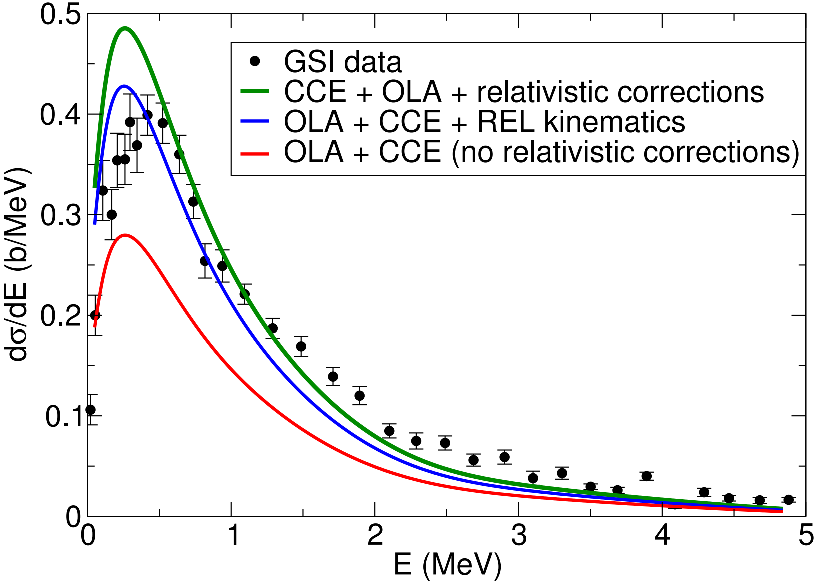


$^{11}Be + ^{208}Pb$ @ 520 AMeV



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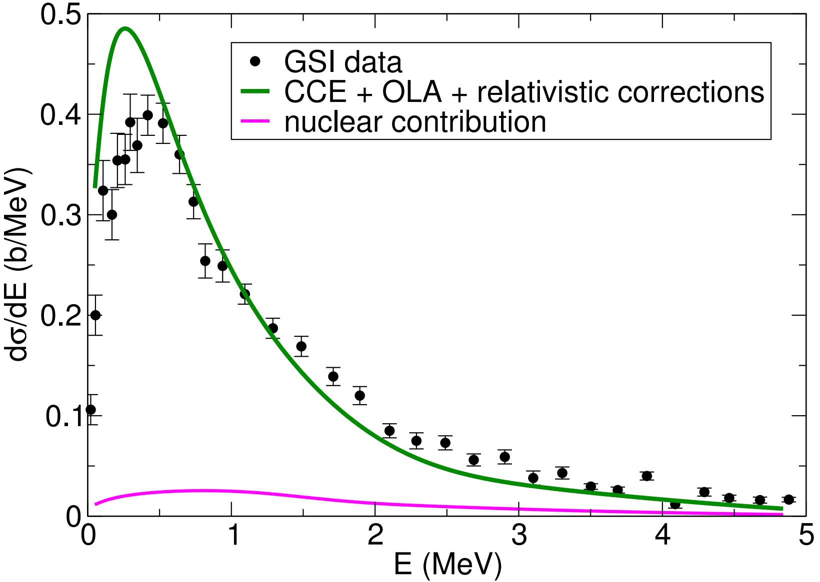
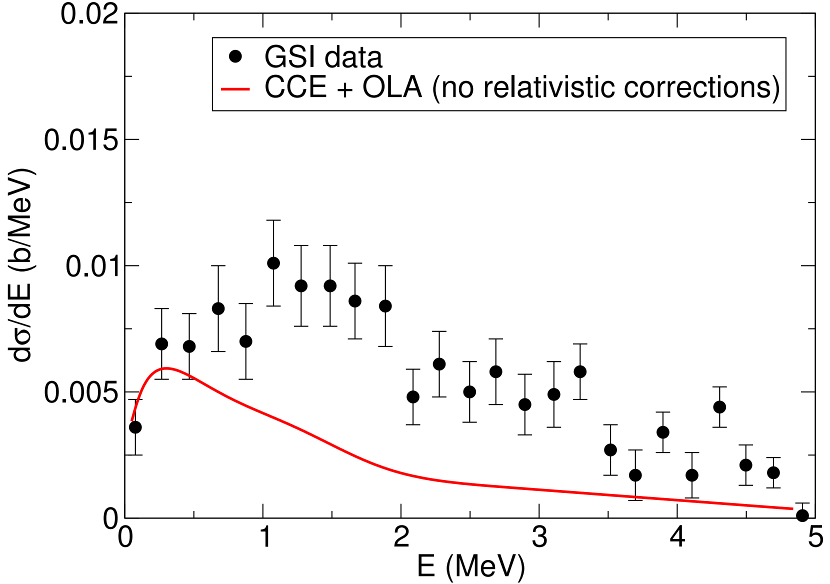
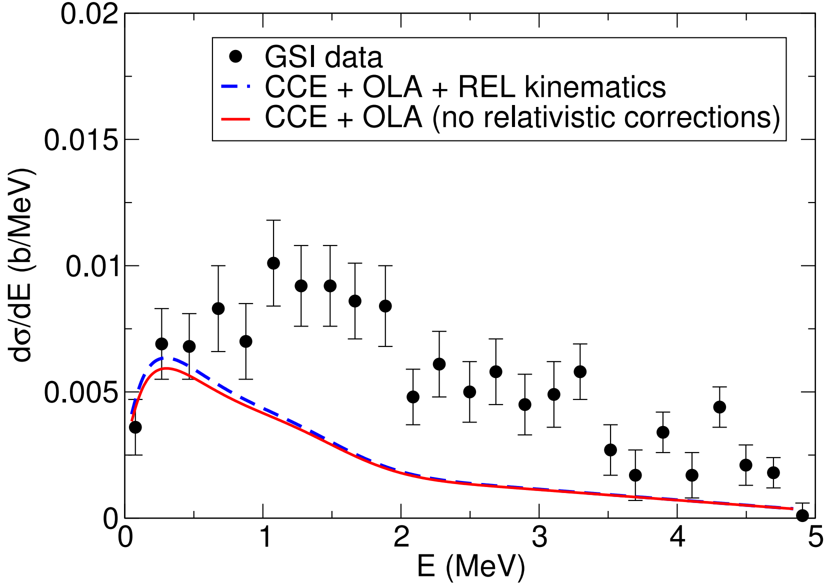


^{11}Be + ^{12}C @ 520 AMeV



$^{11}Be + ^{12}C$ @ 520 AMeV



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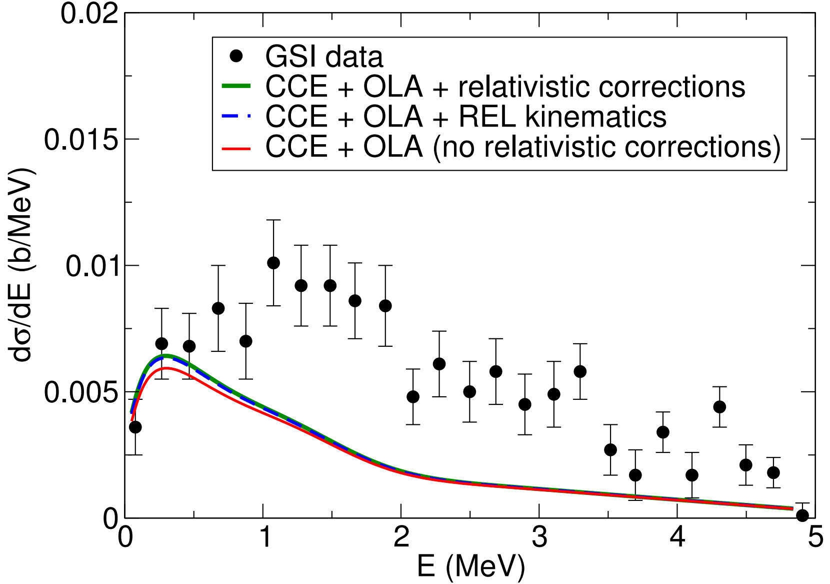
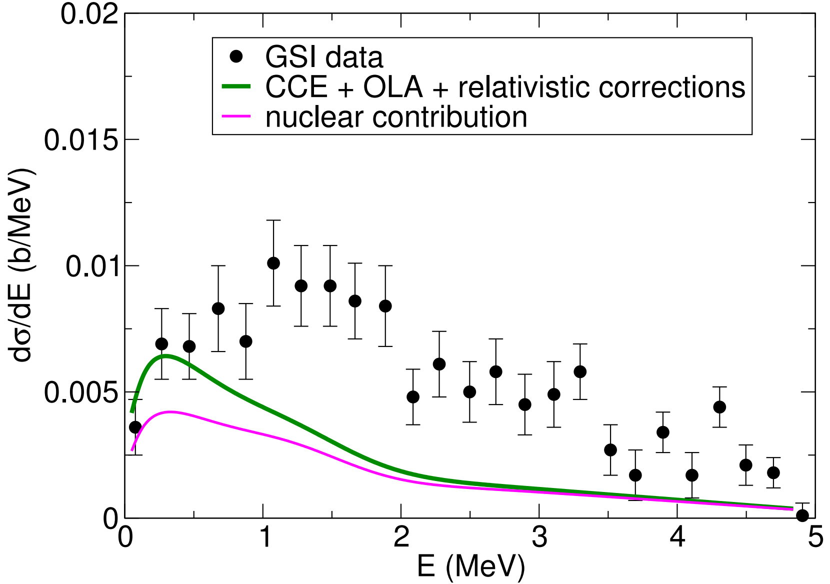


^{11}Be + ^{12}C @ 520 AMeV

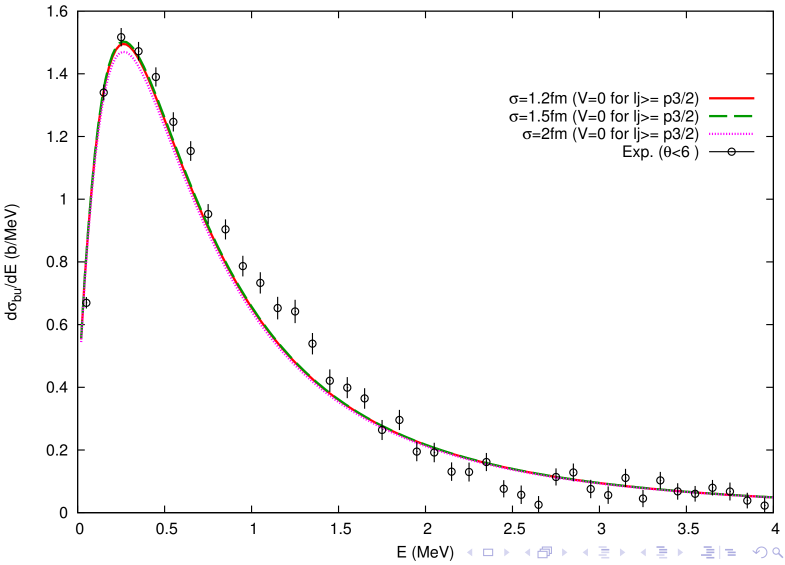


^{11}Be model tested at RIKEN energy

Let's look now at RIKEN breakup cross section described with the same ^{11}Be structure

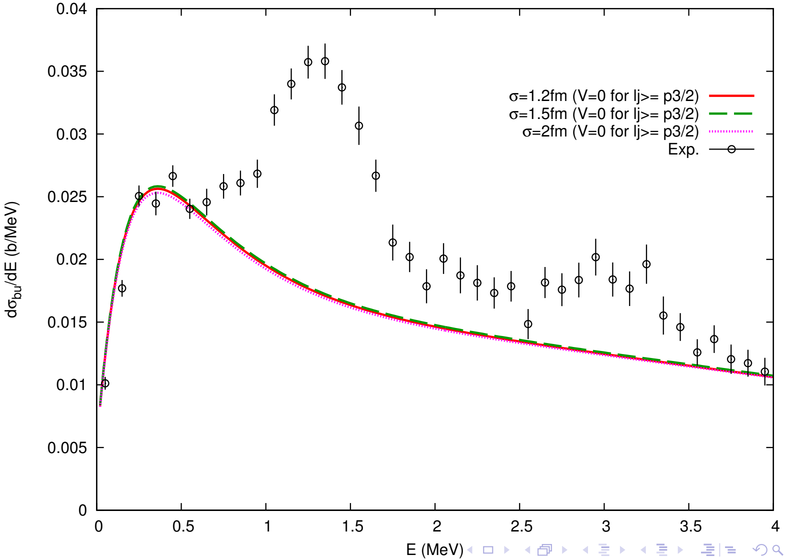
^{11}Be model tested at RIKEN energy

^{11}Be + ^{208}Pb @ 69 AMeV



^{11}Be model tested at RIKEN energy

^{11}Be + ^{12}C @ 67 AMeV



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We have developed an *Eikonal model* with *Coulomb corrections* which takes into account *relativistic kinematics and dynamics* to describe GSI data at 520 MeV/nucleon

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We found that

- the model reproduces quite well GSI data
- the same ^{11}Be internal structure model permits to describe RIKEN results

Conclusions

We have developed an *Eikonal model* with *Coulomb corrections* which takes into account *relativistic kinematics and dynamics* to describe GSI data at 520 MeV/nucleon

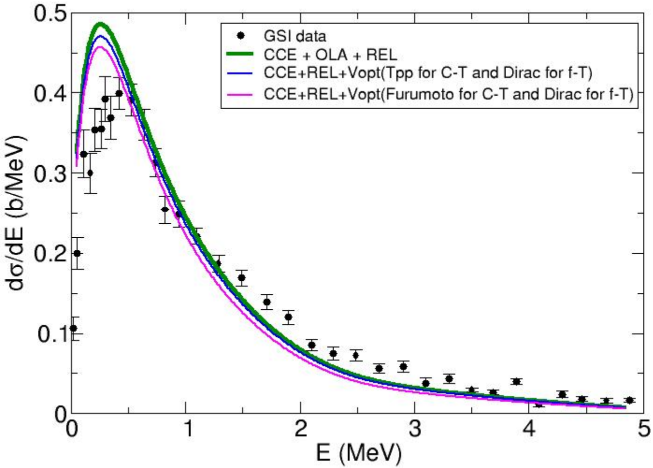
We found that

- the model reproduces quite well GSI data
- the same ^{11}Be internal structure model permits to describe RIKEN results

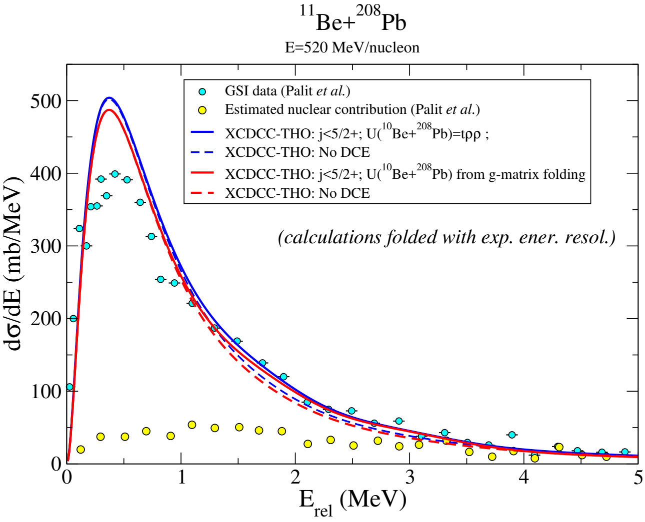
...does this solve the problem?!

Appendix

Response to different nuclear potentials



XCDCC description of GSI cross section



Satchler kinematical presctiptions

In the T-P CM system Klein-Gordon equation is

$$\left[(\hbar c)^2\nabla^2 + (\hbar ck)^2 - 2EV_{PT}\right]\Psi = 0$$

- $\hbar k$ is the relativistically correct CM momentum of P
- $E = (M_P M_T c^2)/(M_P + M_T) \rightarrow$ reduced energy function of P and T total energies in the CM frame: $M_P c^2$ and $M_T c^2$
- $M_P = \gamma_P m_P$ is the corrected projectile mass
  - $\gamma_P = \frac{x+\gamma_L}{\sqrt{1+x^2+2x\gamma_L}}, \quad x = m_P/m_T, \quad \gamma_L = 1 + (E_{LAB}/m_P c^2)$
  - E_{LAB} is the projectile bombarding energy in the LAB system
- same for M_T
- $\mu = E/c^2 = M_P M_T/(M_P + M_T) \rightarrow$ reduced "mass"
- $\Rightarrow K = \frac{m_P c}{\hbar}\sqrt{\gamma_P^2 - 1}$