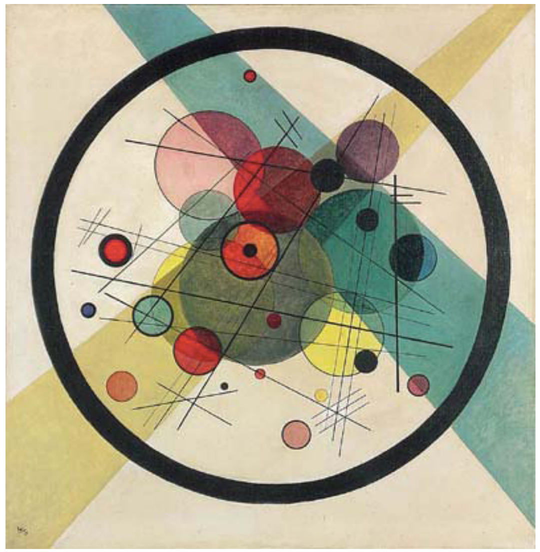
Two-particle transfer reactions:
a key tool for the
study of phase transitions in nuclei

Andrea Vitturi (Padova)



Trento, ECT*, March 8, 2018

The phenomenon of quantum phase transition in nuclei can take place in different situations depending on control parameters as the excitation energy (i.e. the temperature in a thermodynamical framework) or the angular momentum. But equally important are the transitions taking place for the ground states along a chain of isotopes (or isotones), where the control parameter is the number of neutrons (or protons).

Order parameters systematically used in these cases are, in the case of even-even nuclei, the energy of the first 2+ state, the ratio *E4/E2* and the magnitude of the electromagnetic E2 transition connecting ground state and the first excited 2+ state.

Basic point to discuss: how the nuclear behavior of the pairing degree of freedom can provide an additional and complementary clear-cut signature of the occurrence of the phase transition in nuclear systems.

This dynamical source of information should be complementary (but as important) to the one associated to other properties (as energy spectra or electromagnetic transition rates, for example)

The main road to use dynamics to study pairing effects along phase transitions is clearly provided by the study of those processes where a pair of particles is involved, e.g. transferred from/to another nucleus (two-particle transfer) or ejected into the continuum (two-particle break-up or
two-particle knock-out). Clearly the probabilities for such processes must be influenced by the particle-particle correlations, but these will depend on the specific "phase" of the system. So they will be sensitive to any change in the status of the system, for example along an isotope chain.

The essential quantity to characterize the system from the pairing point of view is given by the "pairing response", namely all the T_0 values of the square of the matrix element of the pair creation (or removal) operator

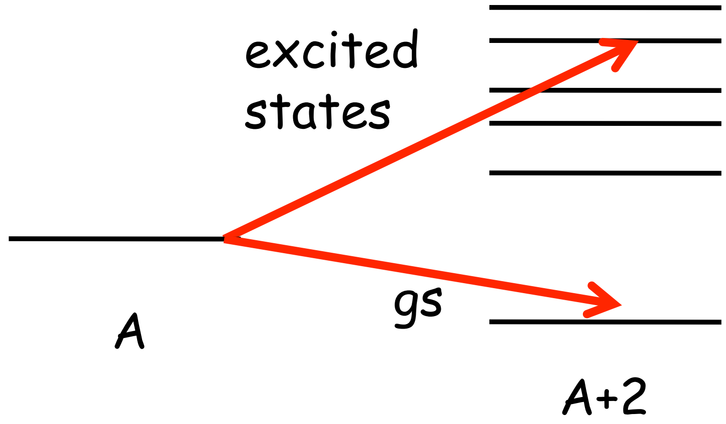
$$P^+ = \sum_j [a^+_j a^+_j]_{00} \qquad \text{(and similarly for } P^-\text{)}$$

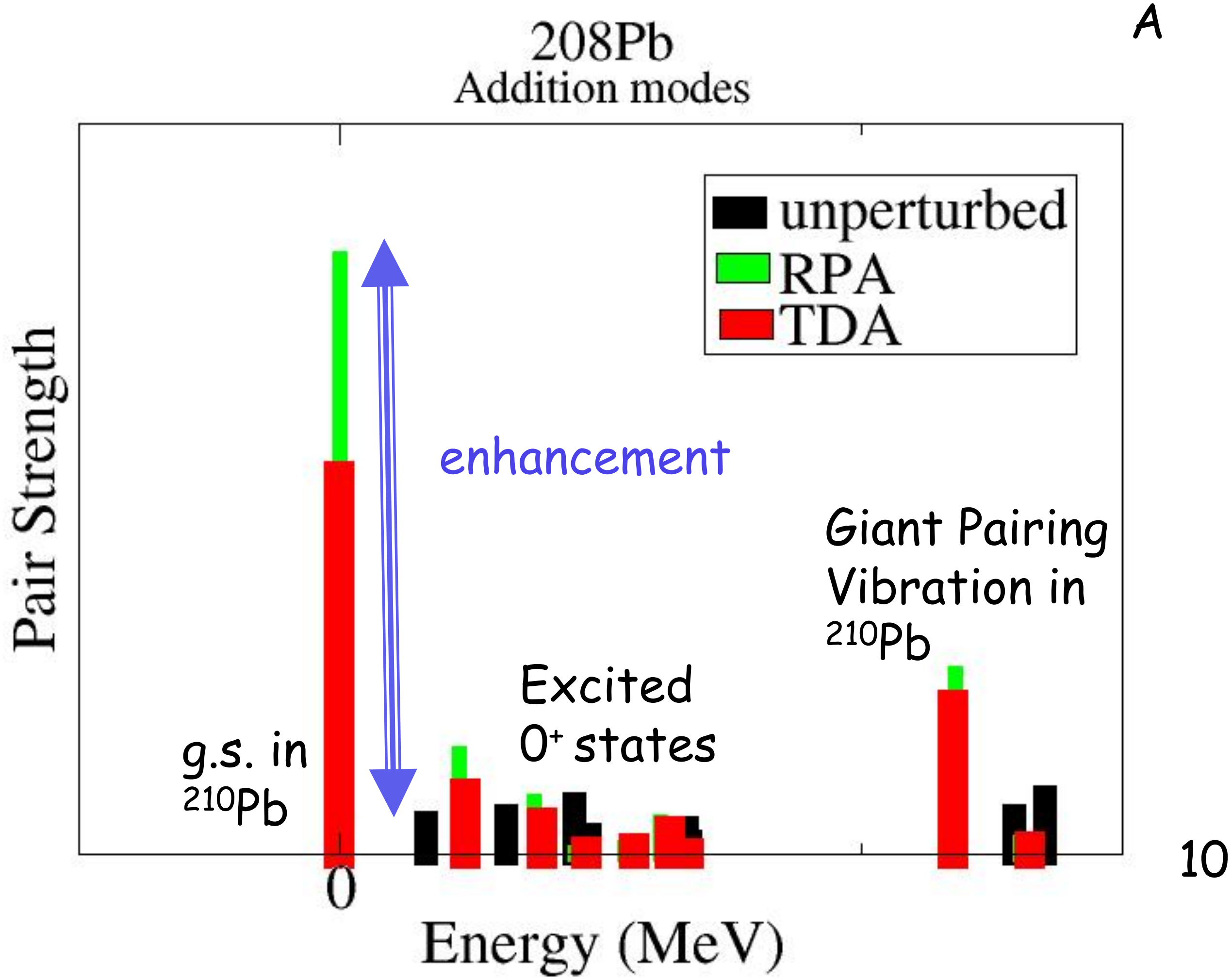
connecting the ground state of nucleus N with all 0+ states of nucleus A+2 (or A-2). It is often assumed that the cross section for two-particle transfer just scale with T_0.

The traditional way to define and measure the collectivity of pairing modes is to compare with single-particle pair transition densities and matrix elements to define some "pairing" single-particle units and therefore "pairing" enhancement factors.

Obs: We discuss here monopole T=1 pairing modes, i.e. 0+states, but similar arguments would apply to T=0 nutron-proton pairs.

Typical "pairing" response



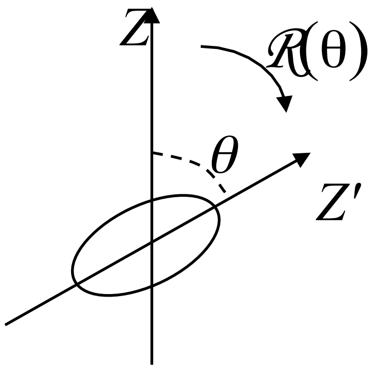
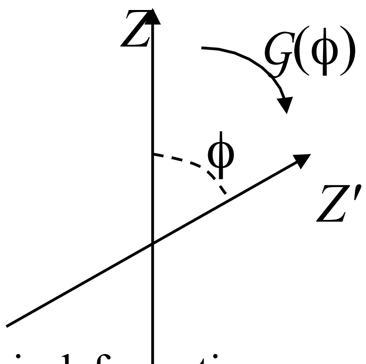


Sofia, Dasso, Vitturi

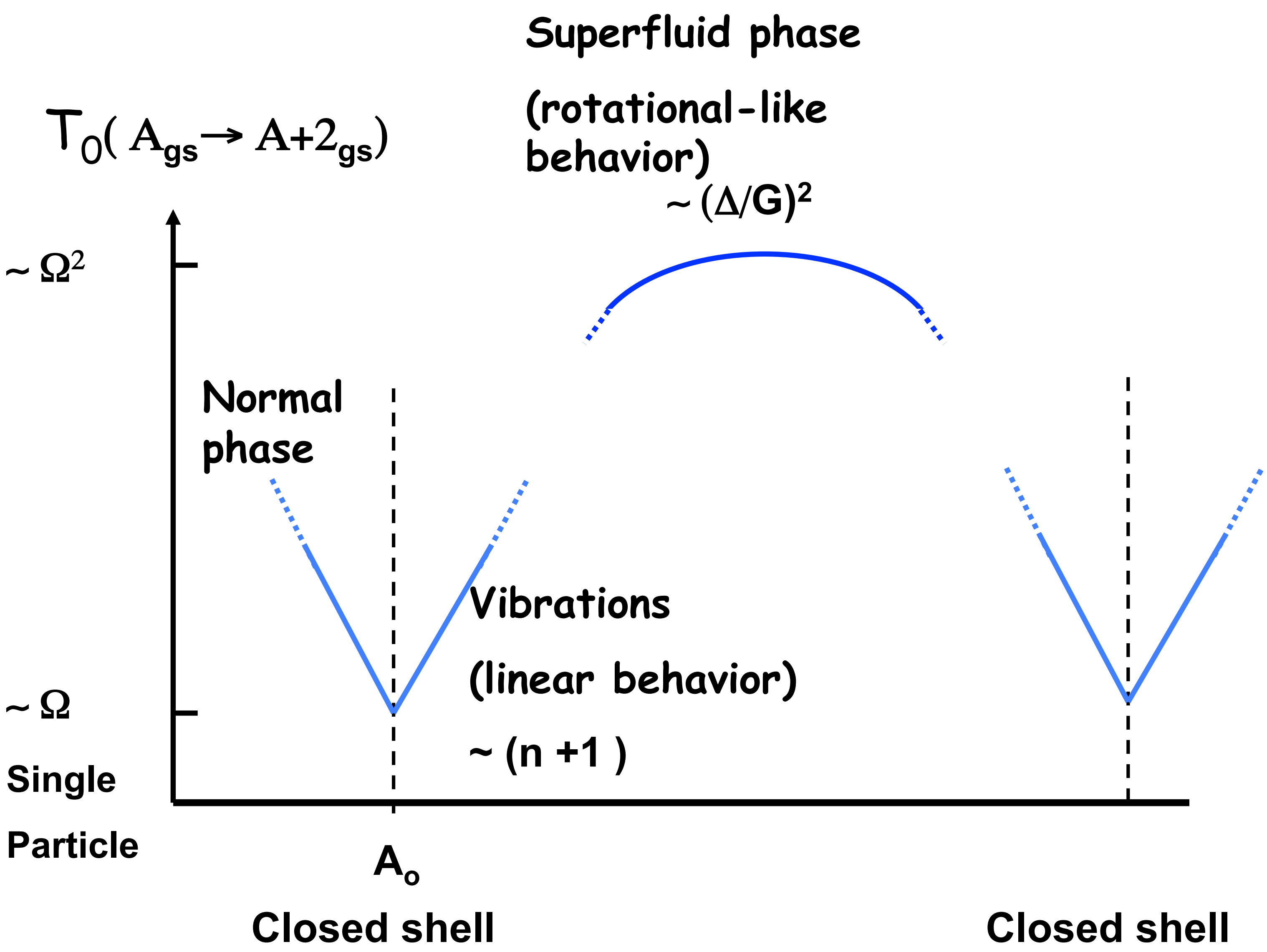
The pairing response is characterized by the pairing phase (normal or superfluid) and by the shape phase (e.g. spherical or deformed). Therefore it will be a clear signature of phase transitions (in addition to the standard signatures, as E_4/E_2, B(E2), etc) in both the

shape degree of freedom

pairing degree of freedom

Shape Transitions	Pairing Transitions
$\mathcal{R}(\theta)=\exp(-iI\theta)$	$\mathcal{G}(\phi)=\exp(-i\mathcal{N}\phi)$
Angular Momentum, I	Particle Number, $\mathcal{N}$
	
β, γ, Euler angles θ	Pair deformation, α Gauge angle, ϕ
Violation of spherical symmetry	Violation of particle number
Physical space	Abstract "gauge" space

Phase transition from "normal" to "superfluid" phases: characteristic behavior of the pair transfer matrix element



OBS: Similar behavior as a function of temperature or angular momentum

PHYSICAL REVIEW C **85**, 034317 (2012)

Pair-transfer probability in open- and closed-shell Sn isotopes

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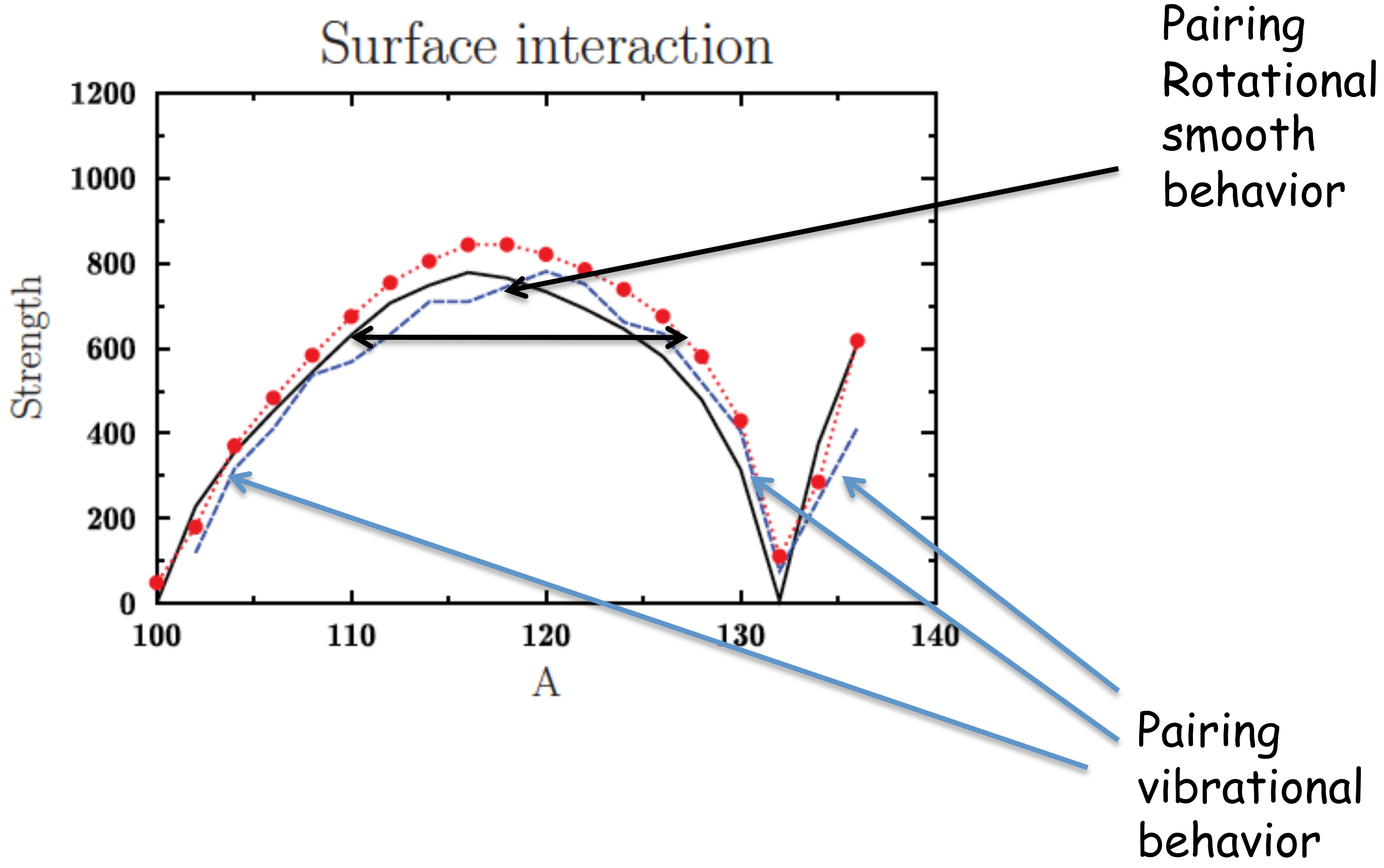
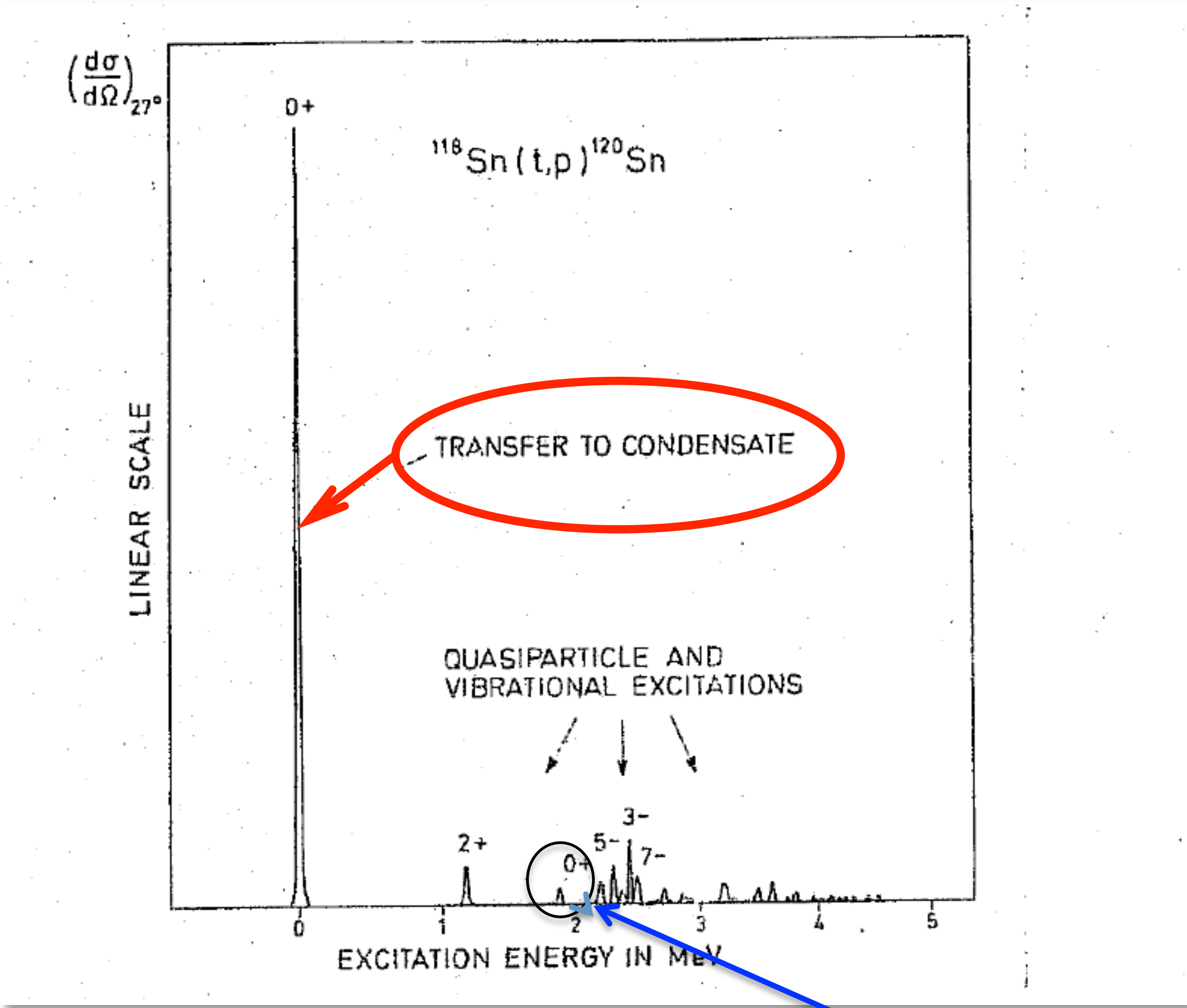


An example of a “superfluid” nucleus (pairing rotations), which shows a characteristic pairing response



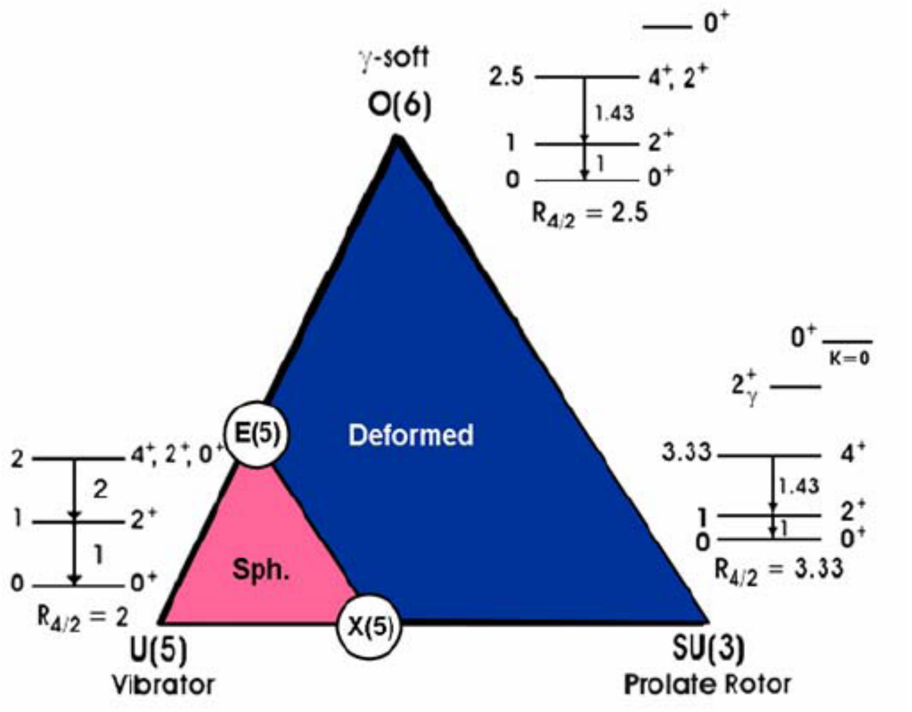
Practically **all** pairing strength goes to the ground state

J.H.Bjerregaard *et al.* NPA 110 **1** (1968)

excited 0+ state

In a similar way pair-transfer probabilities show characteristic behaviors in correspondence of shape phase transitions

For simplicity we move within the framework of the Interacting Boson Model, but the results are similar within other microscopic models



The IBM does not explicitly use the fermion degrees of freedom. From mapping procedure the "form" of the two-particle addition operator Is simply assumed as s^+, neglecting higher-order terms, as s^+s^+s or $[d^+d^+]_0s$ or $[d^+s^+d]_0$ etc ……

OBS: See OAI mapping

Within the IBM the transition from sphericity to axial symmetry can be obtained in even-even nuclei within a hamiltonian that move from U(5) to SU(3)

$$
\begin{aligned}
H^B &= (1-x)n_d - \frac{x}{4N_B} Q_B.Q_B \\
&= (1-x)\ C_1(U^B 5) \\
&- \frac{x}{8N_B}\left[\frac{3}{2}\ C_2(SU^B 3) - \frac{3}{8}\ C_2(O^B 3)\right]
\end{aligned}
$$

with the boson quadrupole operator

$$
Q_B = (s^\dagger \times \tilde{d})^{(2)} + (d^\dagger \times \tilde{s})^{(2)} - \frac{\sqrt{7}}{2}(d^\dagger \times \tilde{d})^{(2)}
$$

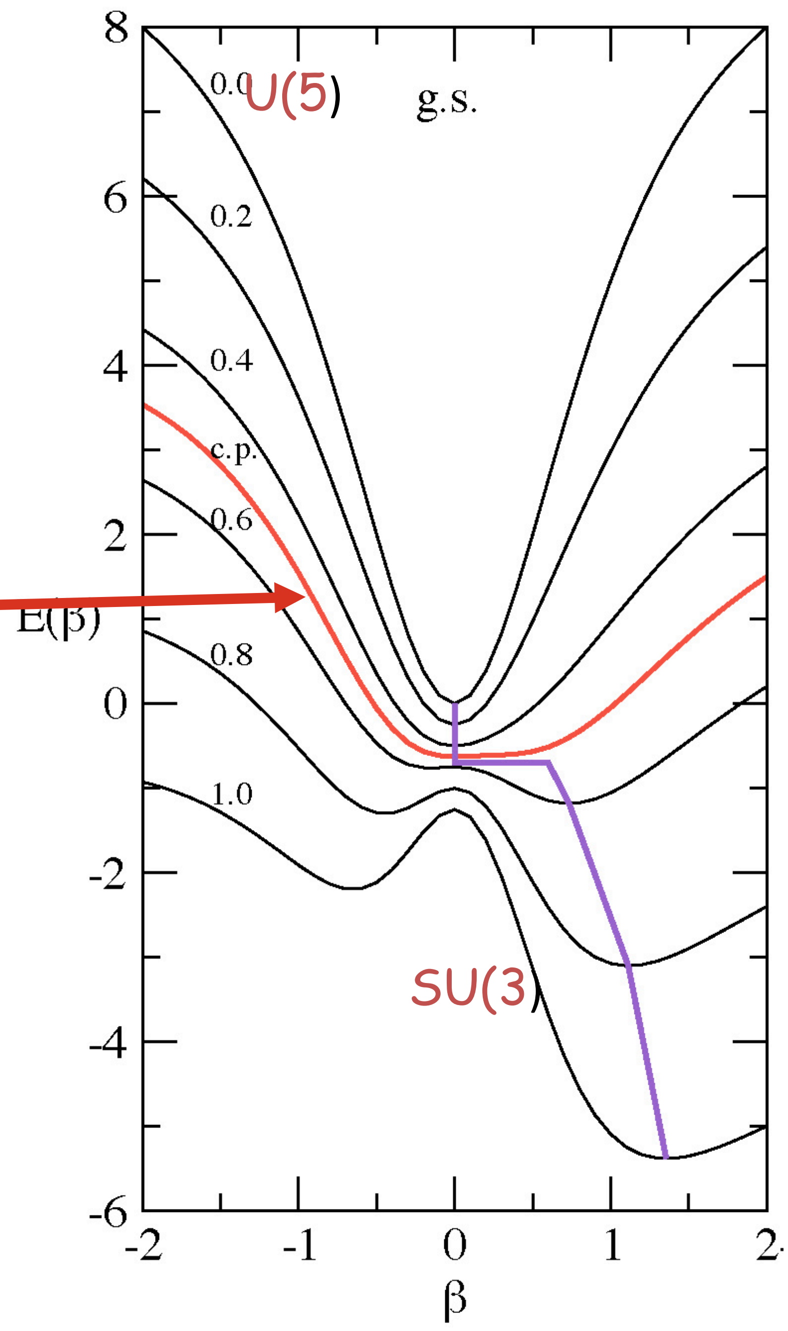
Energy surfaces E(β,γ=0)

U(5) to SU(3) transition
(varying the value of x)

critical point

(first-order
phase transition)

x_{crit}=16N/(34N-27)
≈ 16/34 ≈ 0.5



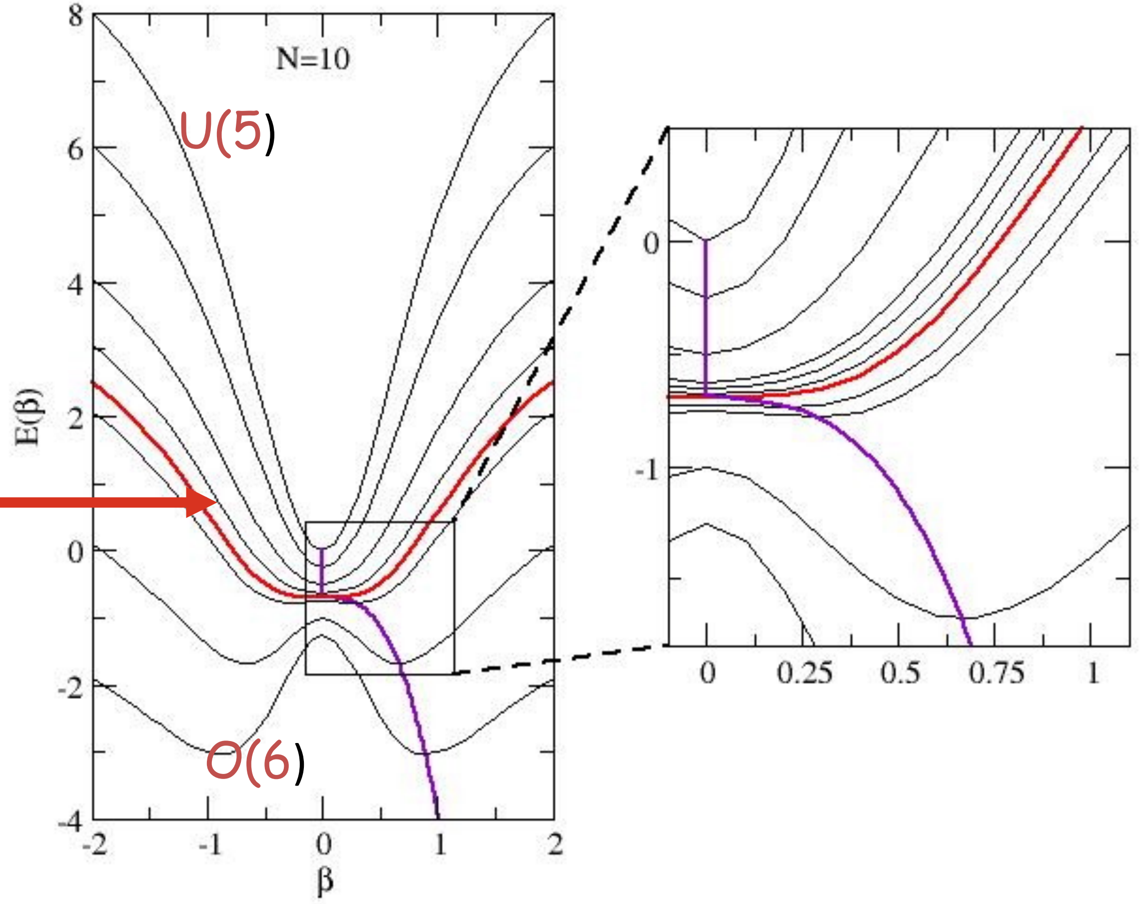
Alonso, Arias, Fortunato, Vitturi

Energy surfaces E(β)   γ-independent for any value of x

U(5) to O(6) transition
(varying the value of x)

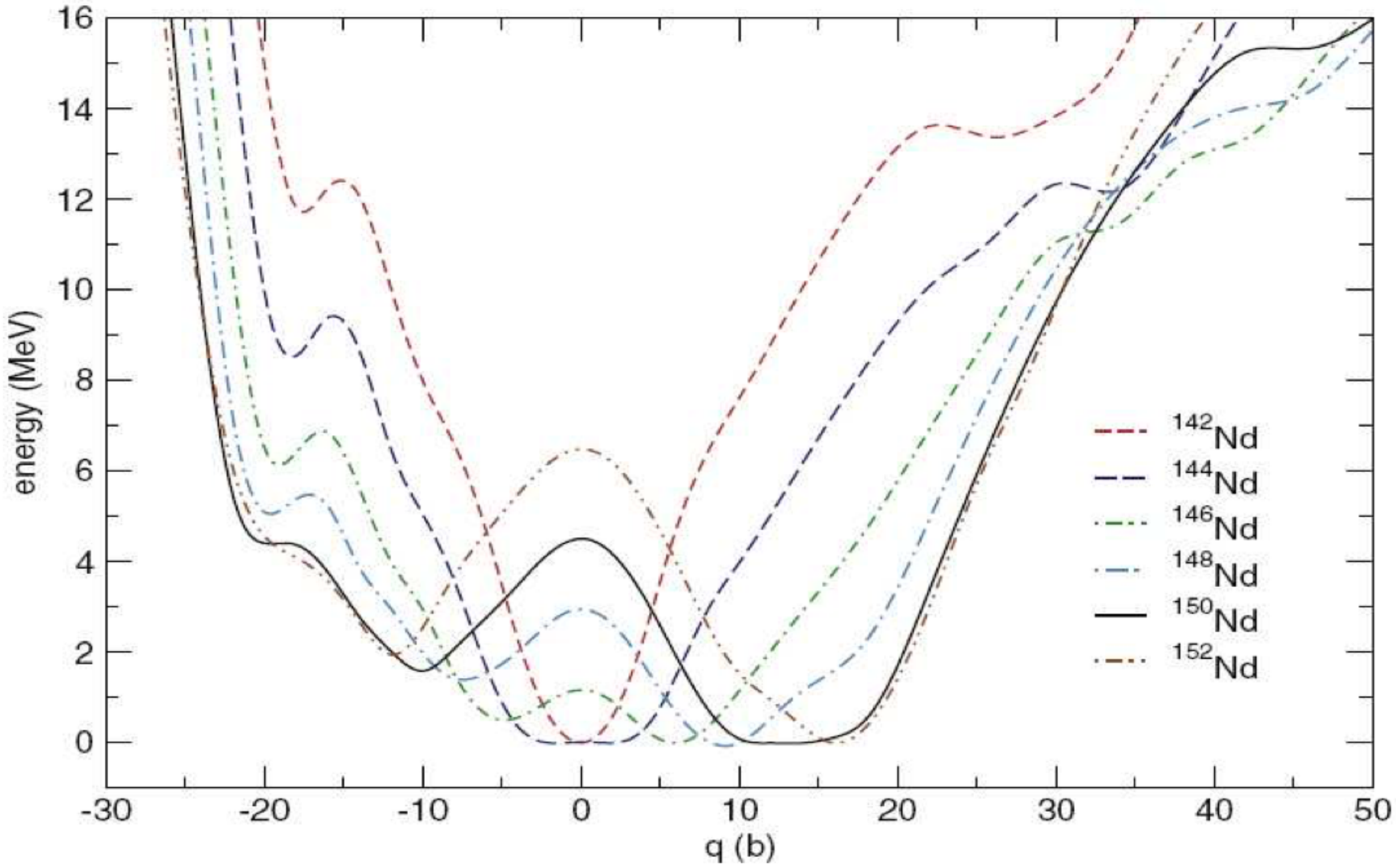
critical point

(second-order
phase transition)



Alonso, Arias, Fortunato, Vitturi

Spherical to deformed transition (microscopic derivation)



Relativistic mean field (Nisick et al.)

Stimulating problem: how the phase transition occurs in the neighbor odd nuclei (phase transition in systems that are a mixture of bosons and fermions)

The corresponding boson fermion (IBFM) Hamiltonian is written as parametrized in the usual way **[U(5)→SU(3)]** :

$$H = H_B + H_F + V_{BF}$$

$$H = (1-c)\hat{n}_d - \frac{c}{4N_B}\hat{Q}_{BF}\cdot\hat{Q}_{BF}$$

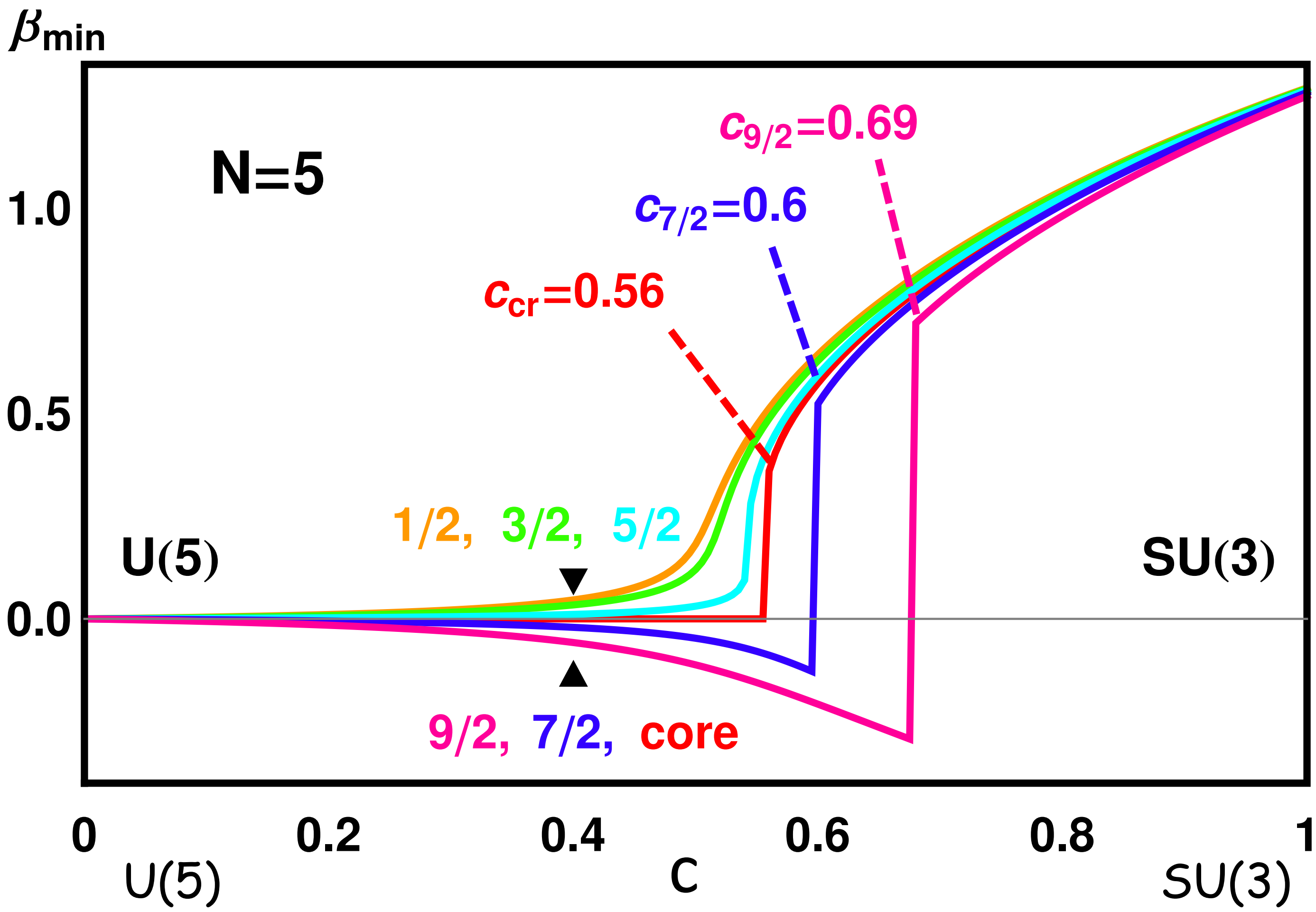
$$\hat{Q}_{BF} = \hat{Q}_B + \hat{q}_F$$

$$Q_B = (s^\dagger \times \tilde{d})^{(2)} + (d^\dagger \times \tilde{s})^{(2)} - \frac{\sqrt{7}}{2}(d^\dagger \times \tilde{d})^{(2)} \qquad \hat{q}_F = t_j(a_j^\dagger \times \tilde{a}_j)^{(2)}$$

Example: just a single-particle orbital j=9/2 coupled to the boson core

Boyukata, Alonso, Arias, Fortunato, Vitturi

Single-j shell (j=9/2)



The phase transition is in this case strongly perturbed: the odd system shows coexistence of states where the phase transition is anticipated or delayed with respect to the even nuclei

Weakening of the N=40 shell. ^{67}Co between spherical (?) ^{68}Ni and deformed ^{66}Fe

OBS Quantum phase transitions in odd systems

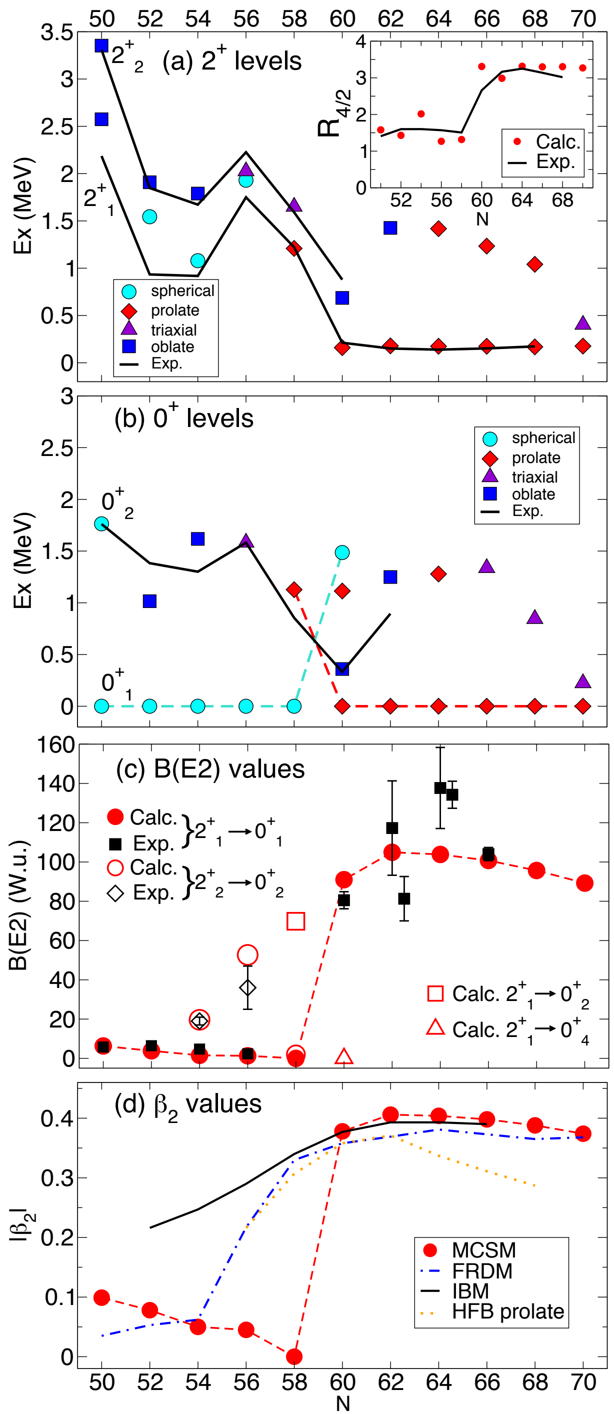
^{67}Co

Spherical ?

	N=35	36	37	38	39	40		
	64Cu 12.701 H ϵ: 61.50% β-: 38.50%	65Cu STABLE 30.85%	66Cu 5.120 M β-: 100.00%	67Cu 61.83 H β-: 100.00%	68Cu 30.9 S β-: 100.00%	69Cu 2.85 M β-: 100.00%	70Cu 44.5 S β-: 100.00%	71Cu 19.4 S β-: 100.00%
Ni	63Ni 101.2 Y β-: 100.00%	64Ni STABLE 0.9255%	65Ni 2.5175 H β-: 100.00%	66Ni 54.6 H β-: 100.00%	67Ni 21 S β-: 100.00%	68Ni 29 S β-: 100.00%	69Ni 11.2 S β-: 100.00%	70Ni 6.0 S β-: 100.00%
Co	62Co 1.50 M β-: 100.00%	63Co 27.4 S β-: 100.00%	64Co 0.30 S β-: 100.00%	65Co 1.16 S β-: 100.00%	66Co 0.20 S β-: 100.00%	67Co 0.425 S β-: 100.00%	68Co 0.199 S β-: 100.00%	69Co 229 MS β-: 100.00%
Fe	61Fe 5.98 M β-: 100.00%	62Fe 68 S β-: 100.00%	63Fe 6.1 S β-: 100.00%	64Fe 2.0 S β-: 100.00%	65Fe 0.81 S β-: 100.00%	66Fe 440 MS β-: 100.00%	67Fe 0.40 S β-: 100.00%	68Fe 180 MS β-: 100.00%

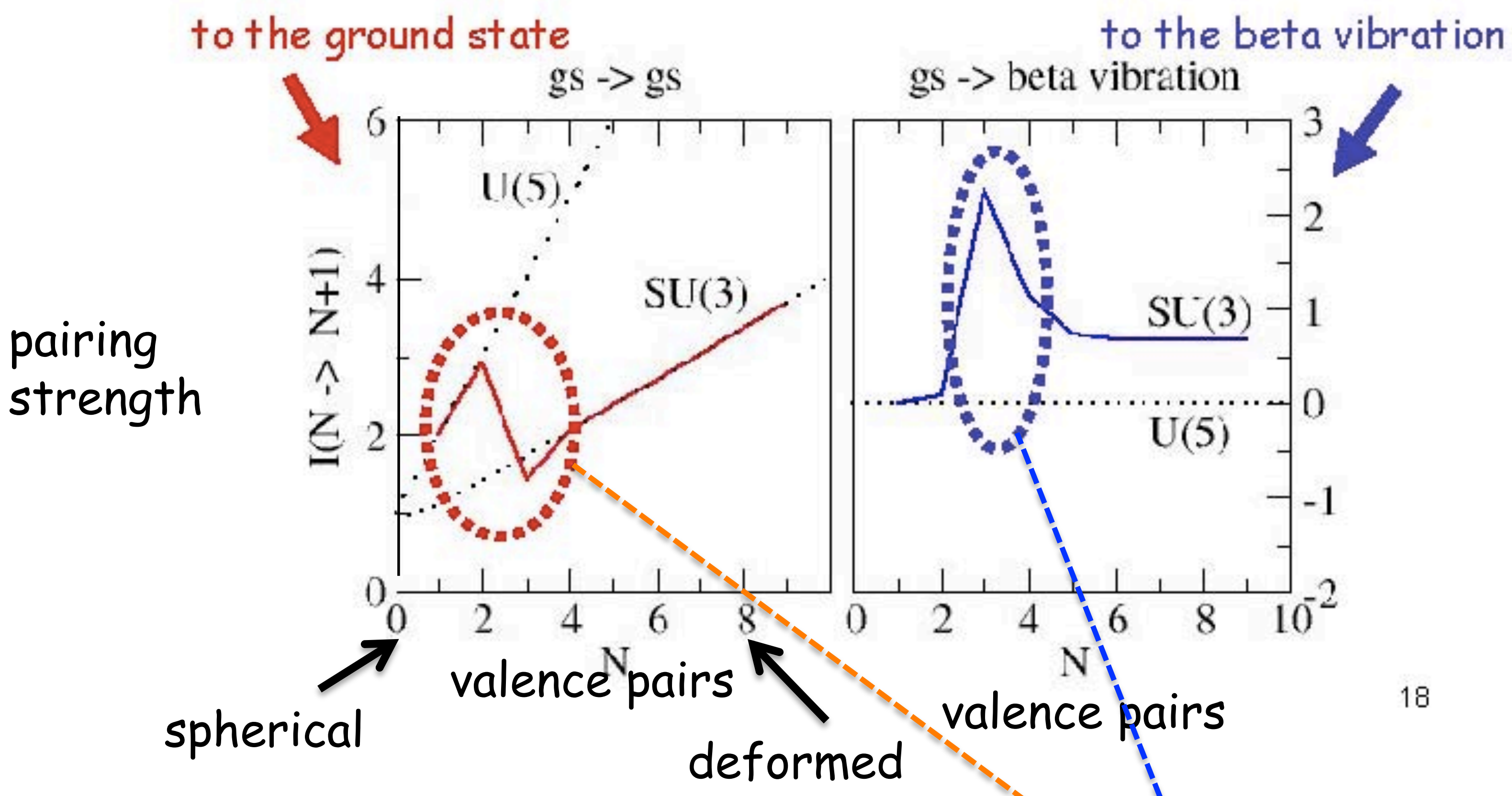
Deformed ?

How to single out phase transitions?



And what about two-particle transfer cross sections as possible signatures for shape phase transitions?

Example: L=0 pair transfer in a phase transition from spherical to axial deformation (from U(5) to SU(3) in algebraic language)



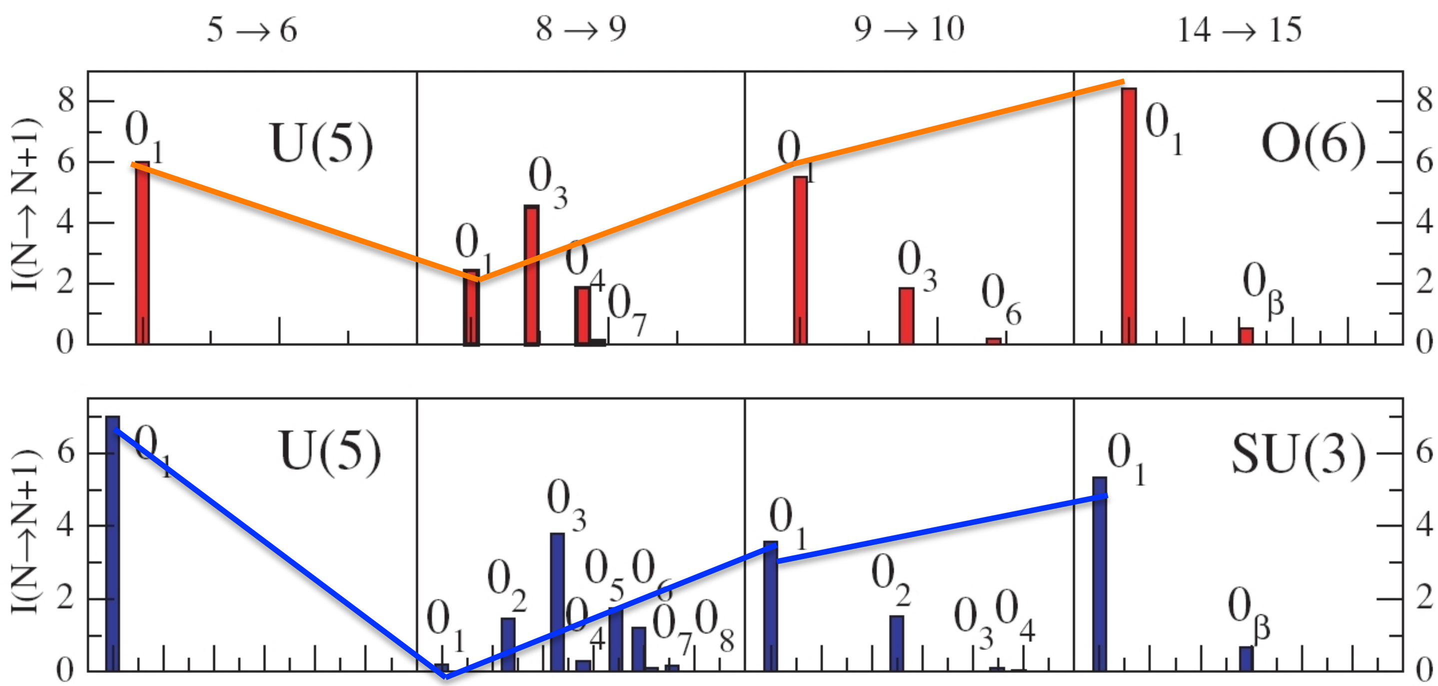
There is a clear signal at the phase transition

Obs: fragmentation of the pairing strength in correspondence to phase transitions along an isotope chain (in this case chosen to take place at N=8)

Number of valence pairs

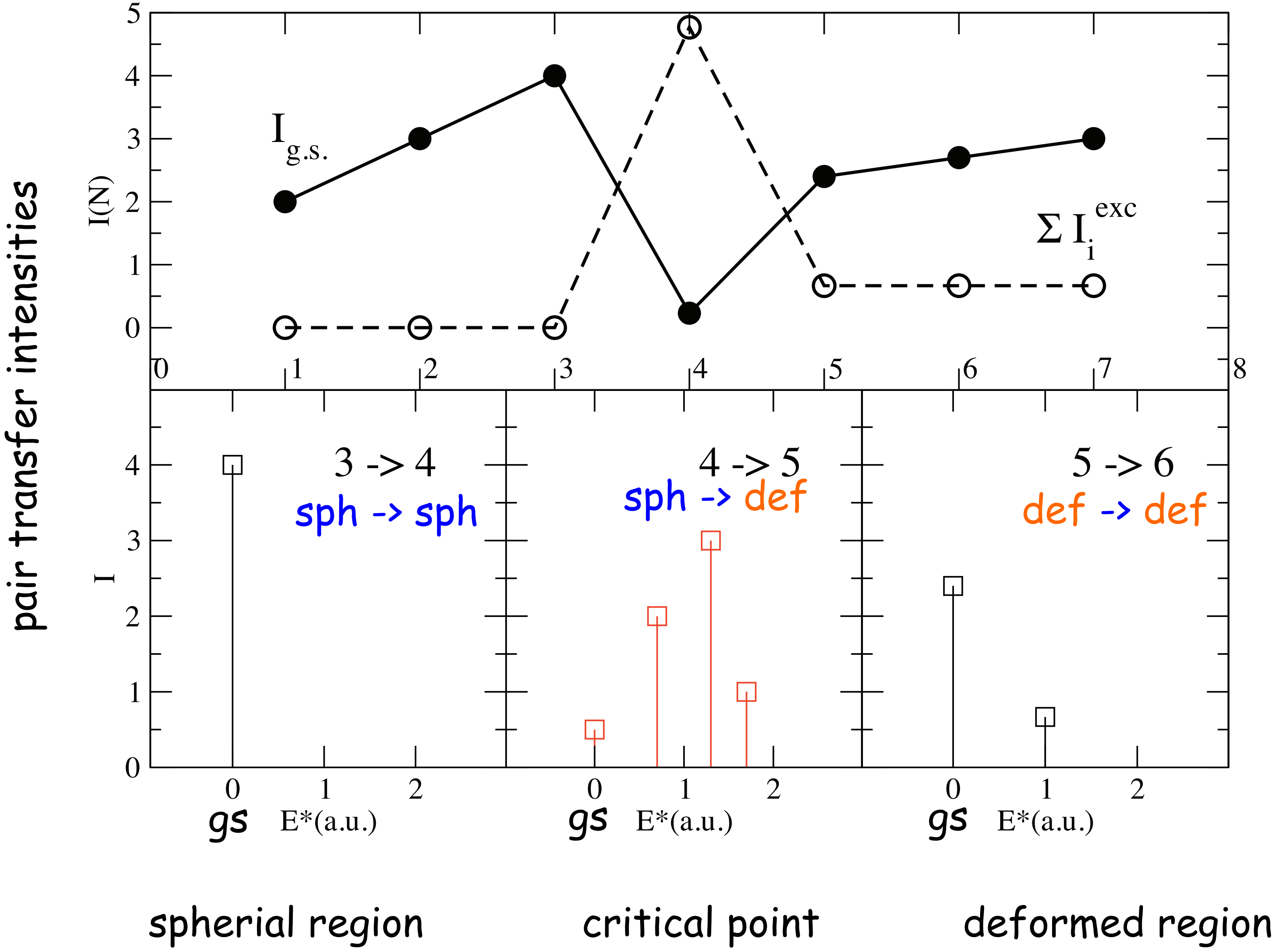
U(5) -> O(6)
2nd order

U(5) -> SU(3)
1st order



E_x E_x E_x E_x

fragmentation of the pairing strength



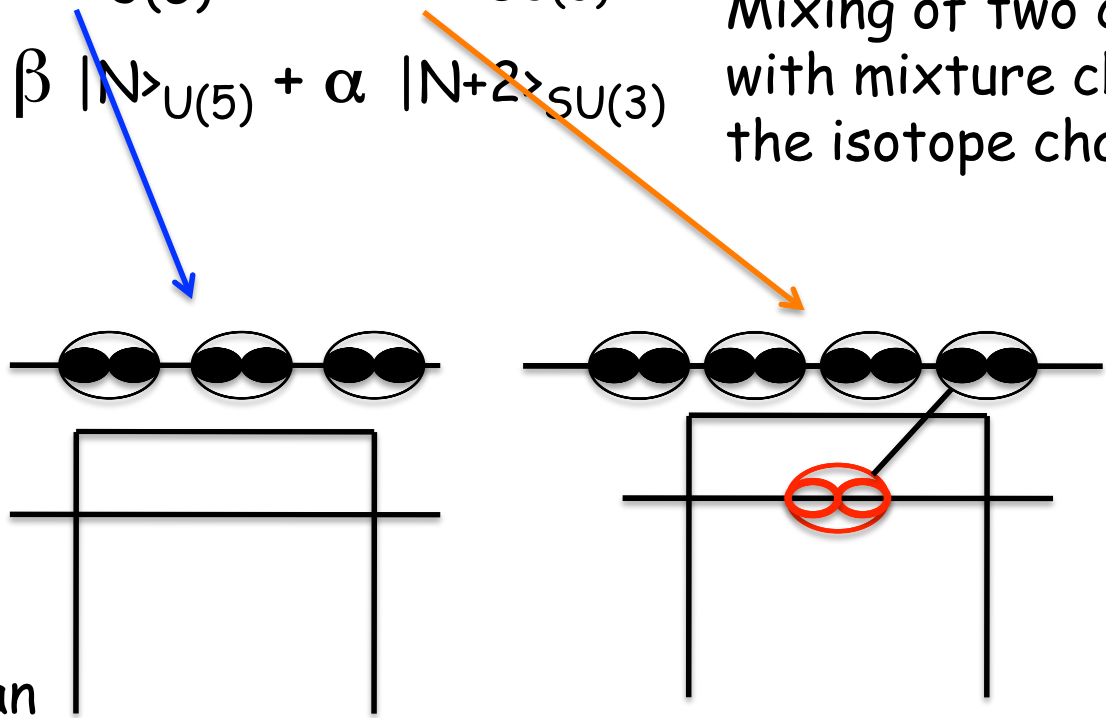
pair transfer intensities
I(N)
$I_{g.s.}$
ΣI_i^{exc}
3 -> 4
sph -> sph
4 -> 5
sph -> def
5 -> 6
def -> def
I
gs
E*(a.u.)
spherial region
critical point
deformed region

Another scenario of phase transition:
shape co-existence, for example of a sherical and a deformed state within the same nucleus

$$|0^+_{gs}, N\rangle = \alpha\ |N\rangle_{U(5)} + \beta\ |N+2\rangle_{SU(3)}$$

$$|0^+_{exc}, N\rangle = -\ \beta\ |N\rangle_{U(5)} + \alpha\ |N+2\rangle_{SU(3)}$$

Mixing of two configurations, with mixture changing along the isotope chain



N particle pairs

0 hole pairs

U(5) hamiltonian (spherical)

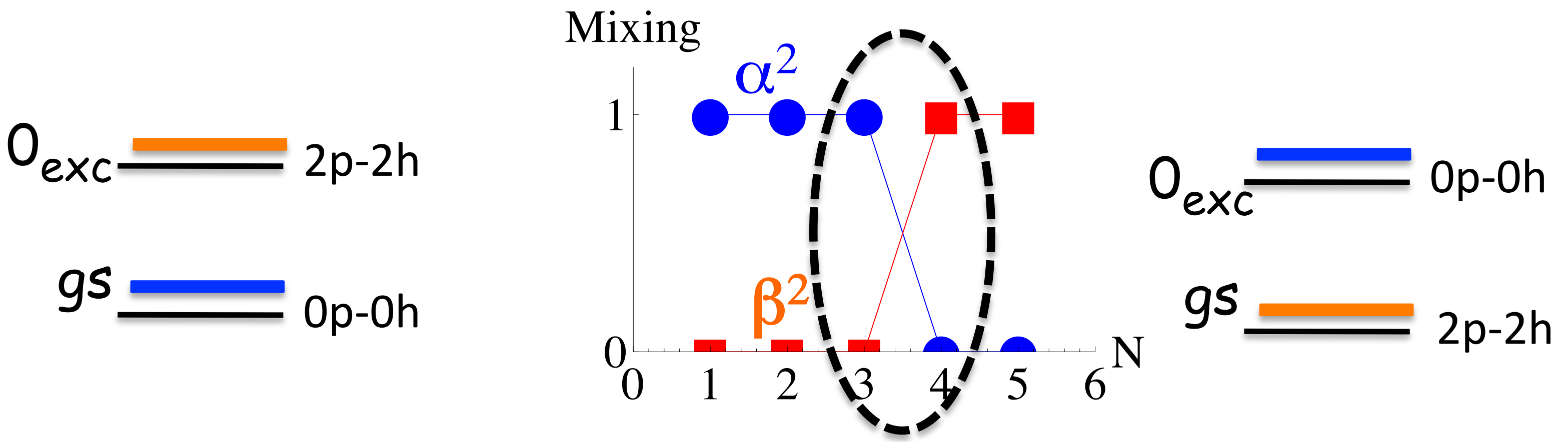
$|N\rangle_{U(5)}$

$|N+2\rangle_{SU(3)}$

N+1 particle pairs,
1 hole pair (2p-2h exc):
total N+2 pairs

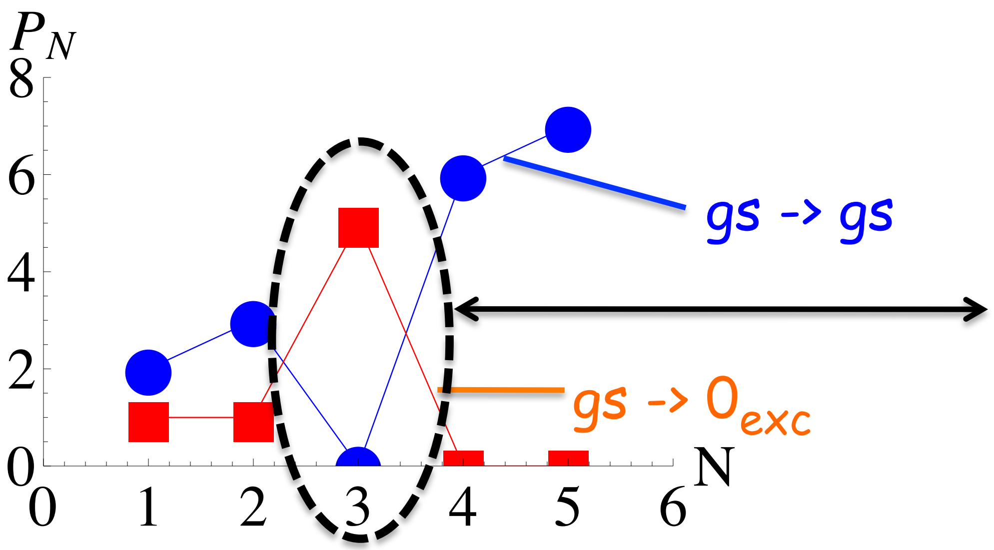
SU(3) hamiltonian (deformed)

A simple model: along the isotope chain a sharp inversion of the structure

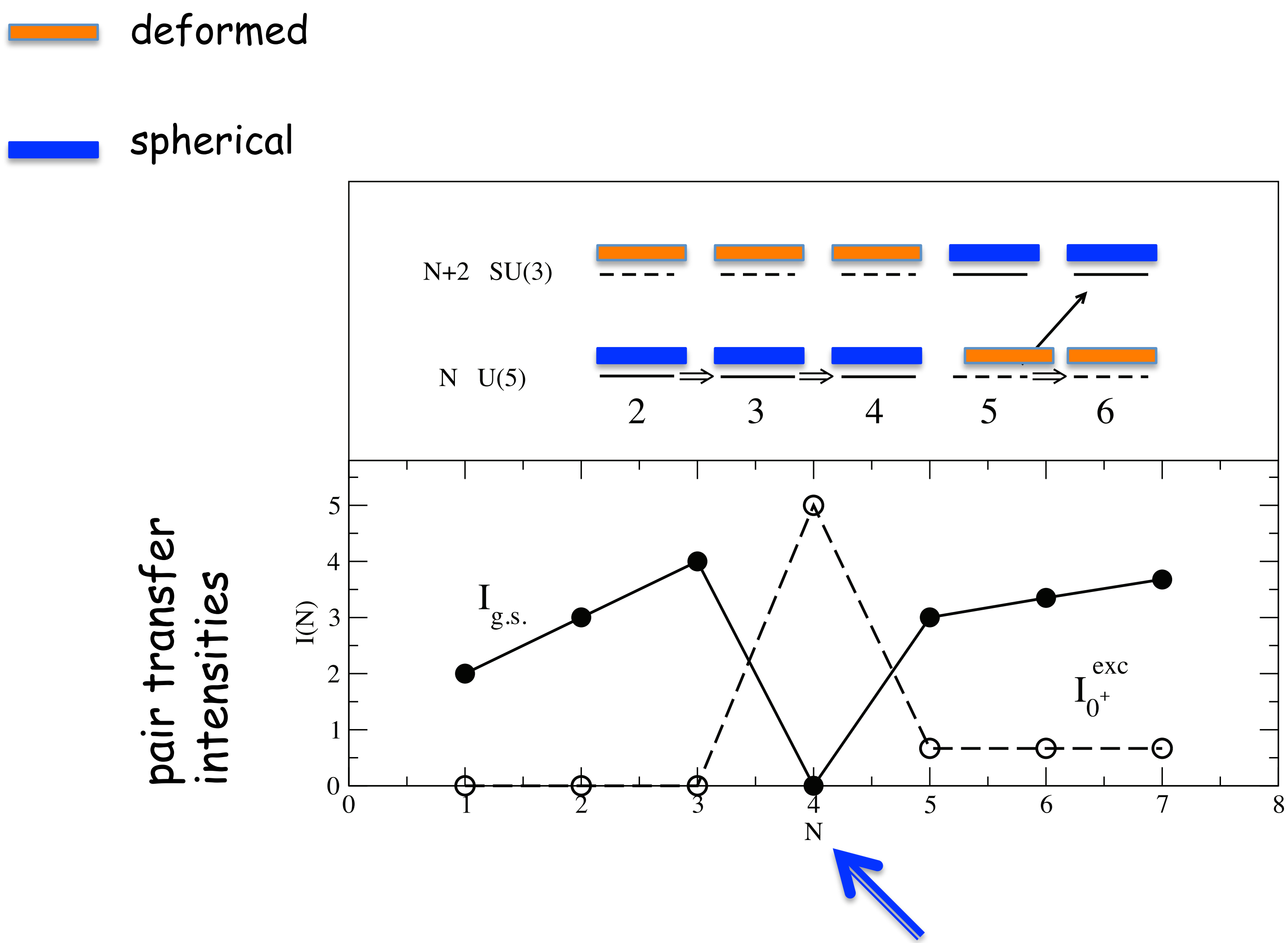


Transfer operator in now more complex: S^+ + S (one can create a particle pair or destroy a hole pair)

Pair creation probability N -> N+2

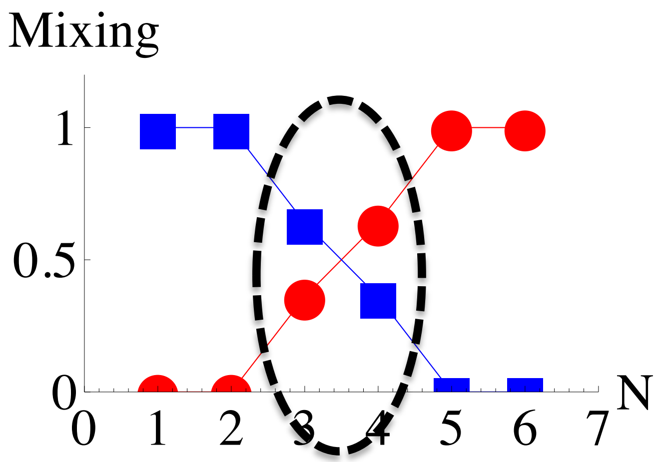


There is a clear signal at the phase transition



As in the previous situation a clear discontinuity appears at the critical point. However, at variance with the previous case, the pair strength is always practically concentrated in a single state, without the fragmentation illustrated in the previous case

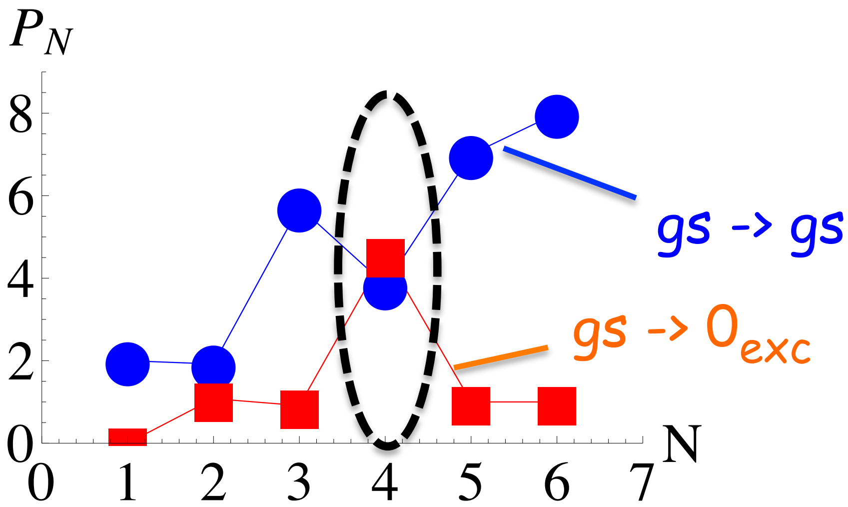
Another case: shape-coexistence with a smoother transition



OBS: Cf. E0 transitions between the two 0+ states

Transfer operator for pair removal : s + s^{+} (one can destroy a particle pair or create a hole pair)

Pair removal probability N <- N+2



So far we have considered matrix elements of the pair operator: but what about pair transfer cross sections?

Unfortunately, at variance, for example, from low-energy one-step Coulomb excitation, where the excitation probability is directly proportional to the B(Eλ) values, the reaction mechanism associated with pair transfer is rather complicated and the possibility of extracting spectroscopic information on the pairing field is not obvious. The situation is actually more complicated even with respect to other processes (as inelastic nuclear excitation) that may need to be treated microscopically, but where the reaction mechanism is somehow well established.

We expect an correlation between cross sections and square of the pair operator. But if the qualitative behavior may be clear, the quantitative aspects require a proper treatment of the reaction mechanism. All approaches, ranging from macroscopic to semi-microscopic and to fully microscopic, try to reduce the actual complexity of the problem, which is a four-body scattering (the two cores plus the two transferred particles), to more tractable frameworks.

Two models are most popular:
A, Successive single-particle transfer
B. Cluster transfer

A

Sequential two-step process: each step transfers one particle

Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in (a-1) and (A+1) nuclei, due to the correlations in initial and final wave functions

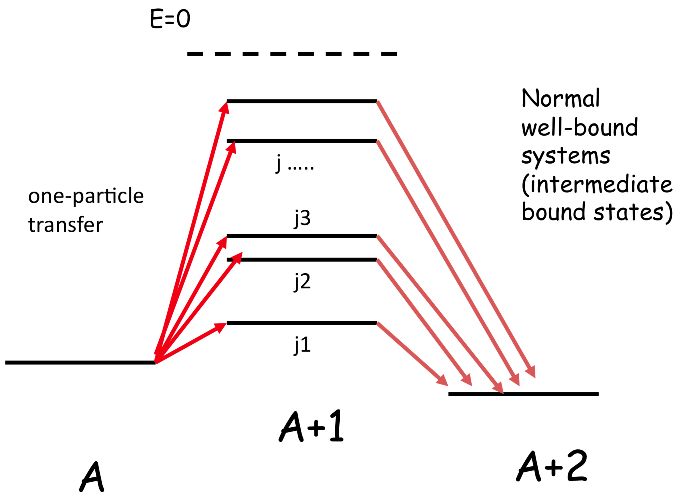
Basic idea: dominance of mean field, which provides the framework for defining the single-particle content of the correlated wave functions

Expansion to second-order in the transfer potential

Simultaneus	+	Sequential	+	not-orthogonality
(first-order)		(second-order)		(second-order)

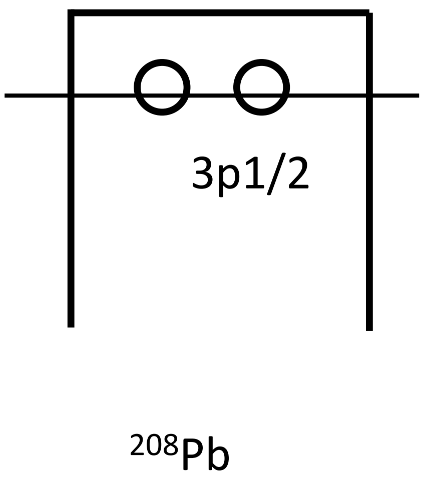
this is not the cluster contribution

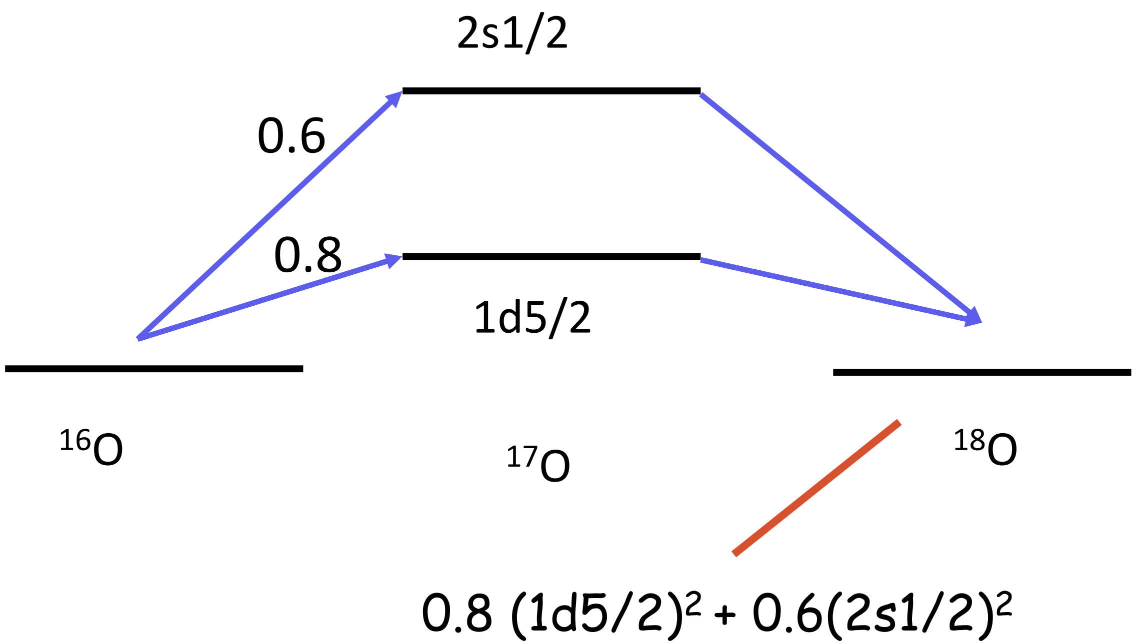
these two terms may approximately cancel each other



Example with just two components

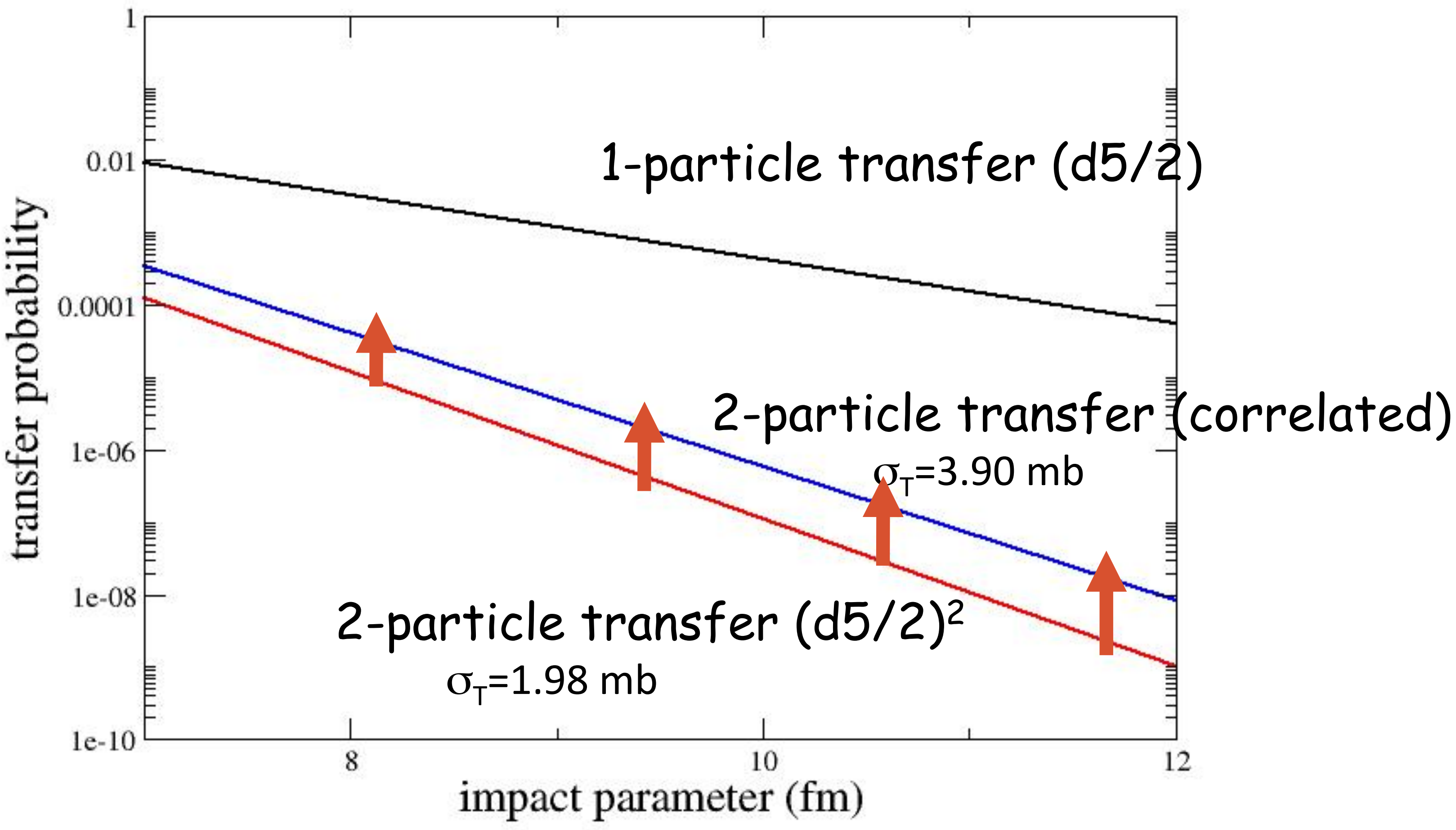
$^{208}Pb(^{16}O,^{18}O)^{206}Pb$





Example of calculation

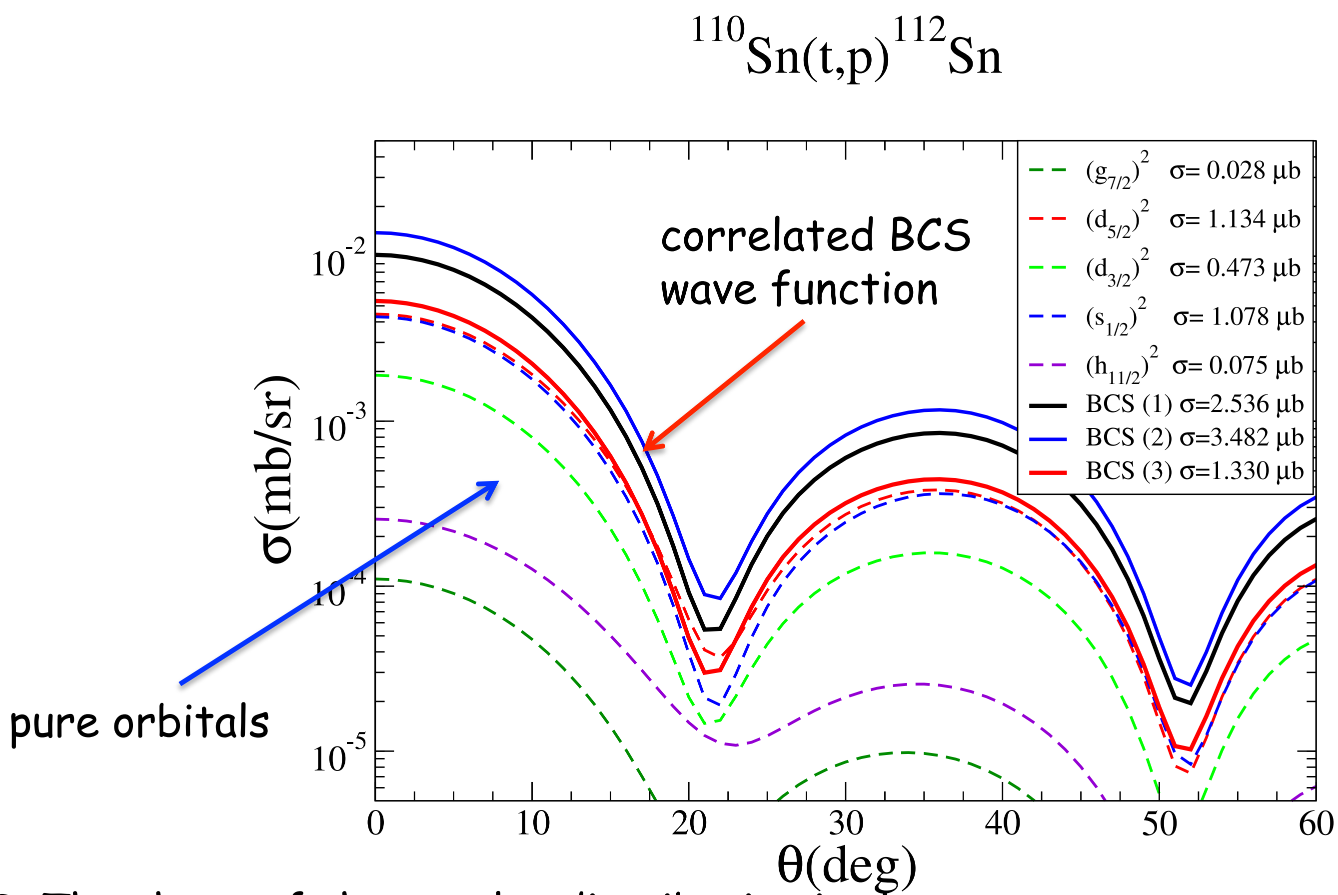
$^{208}Pb(^{16}O,^{17,18}O)^{207,206}Pb$



In this case the pairing enhancement factor in the cross section is about a factor 2

Effect of kinematical conditions

The transfer probabilities vary strongly with the involved orbital. In addition whether the final wave function only involves a “pure” orbital, or whether it is correlated



OBS: The shape of the angular distribution is the same, being associated with the L=0 transfer

Due to kinematical conditions, the final two-particle cross sections depend strongly on the microscopy and the involved single-particle orbits, and not simply on the global "pairing strength" (measured for example from the gap Δ in BCS systems)

Example: ^{110}Sn (t,p) ^{112}Sn (gs)

Three different BCS wave functions characterized by the same value of Δ (1.2 MeV) yields different cross sections

	BCS 1		BCS 2		BCS 3	
	ϵ_i(MeV)	B_i	ϵ_i(MeV)	B_i	ϵ_i(MeV)	B_i
$0g_{7/2}$	-0.027	0.75	-0.027	1.15	-2.027	0.64
$1d_{5/2}$	0.882	1.13	-0.118	0.57	0.882	1.02
$2s_{1/2}$	1.330	0.53	-0.670	0.33	1.330	0.59
$0h_{11/2}$	2.507	0.79	4.507	0.61	5.507	0.46
$2d_{3/2}$	2.905	0.39	2.905	0.26	2.905	0.27
σ (mb)	2.5		3.4		1.3	

We consider the same case as before, i.e. the transfer of two neutrons from ^{110}Sn to ^{112}Sn (0+; *gs*) using the reactions

(14C,12C) or (18O,16O)

In addition to the information on the target, we need now to specify on which orbit the particle are transferred in the projectile

In the (14C,12C) the two neutrons are assumed to be picked-up from the p1/2 shell.

In the (18O,16O) from the pure d5/2 shell, or from a combination of $(d5/2)^2$ and $(s1/2)^2$

If we consider the same case as before, i.e. the transfer of two neutrons from ^{110}Sn to ^{112}Sn (0+; gs), but using different reactions, e.g. (14C,12C) or (18O,16O) the ranking of the cross sections associated to the different orbitals changes.

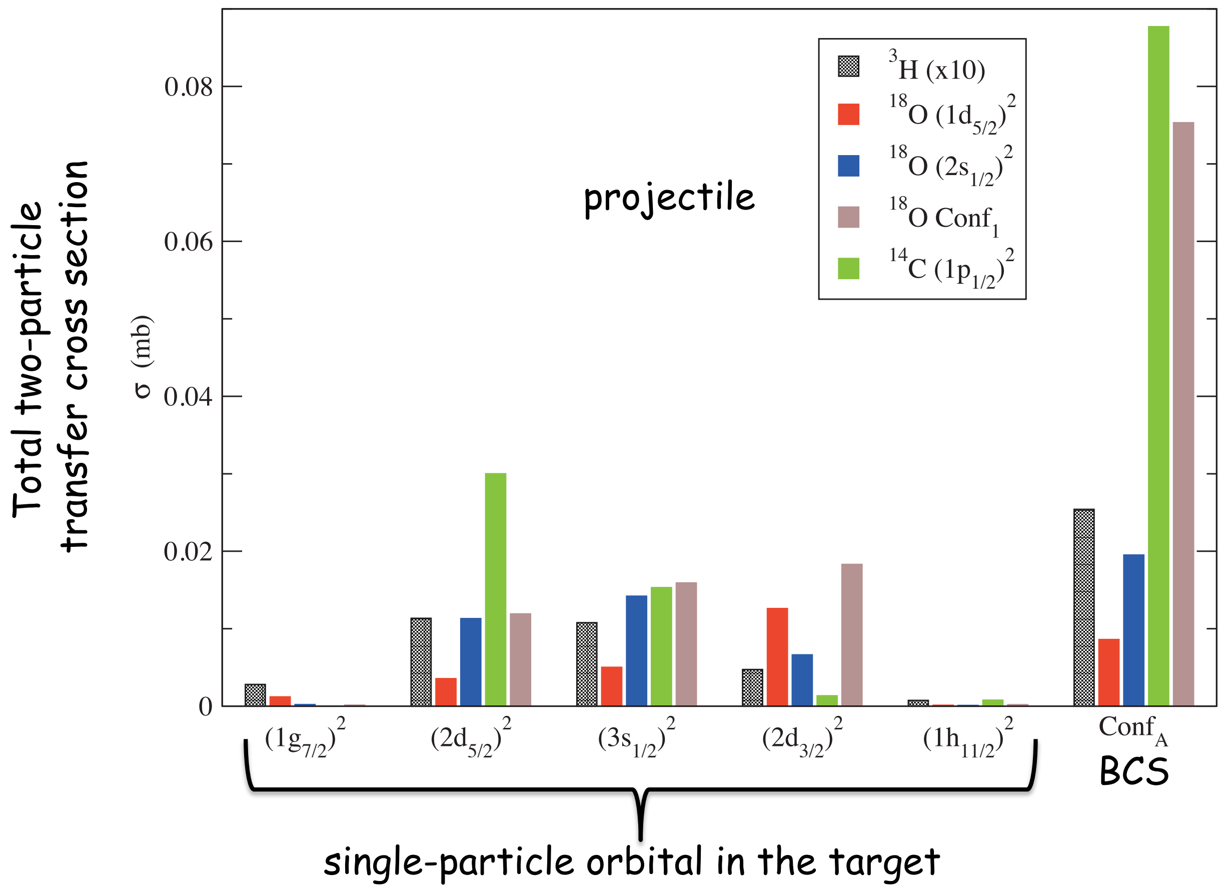
projectile

single-particle orbital in the target

^{112}Sn	(t,p)	(^{14}C,^{12}C) $(0p_{1/2})^2$	(^{14}C,^{12}C) $(1s_{1/2})^2$	(^{14}C,^{12}C) $(0p_{3/2})^2$	(^{14}C,^{12}C) $(0d_{5/2})^2$	(^{18}O,^{16}O) $(0d_{5/2})^2$	(^{18}O,^{16}O) $(1s_{1/2})^2$	(^{18}O,^{16}O) Conf$_1$
$(0g_{7/2})^2$	2.80E-5	1.73E-5	1.19E-4	7.09E-4	9.00E-4	1.19E-3	2.01E-4	1.24E-3
$(1d_{5/2})^2$	1.13E-3	3.00E-2	4.71E-3	5.54E-3	1.18E-3	3.55E-3	1.13E-2	1.19E-2
$(2s_{1/2})^2$	1.08E-3	1.53E-2	5.38E-3	7.05E-3	1.16E-3	5.02E-3	1.42E-2	1.59E-2
$(1d_{3/2})^2$	4.73E-4	1.34E-3	2.79E-3	9.87E-3	4.14E-3	1.26E-2	6.62E-3	1.83E-2
$(0h_{11/2})^2$	7.50E-5	7.77E-4	5.29E-5	1.05E-4	7.65E-5	1.10E-4	9.06E-5	1.88E-4
Conf$_A$	2.54E-3	8.77E-2	2.26E-2	3.77E-2	1.21E-2	8.60E-3	1.95E-2	7.53E-2

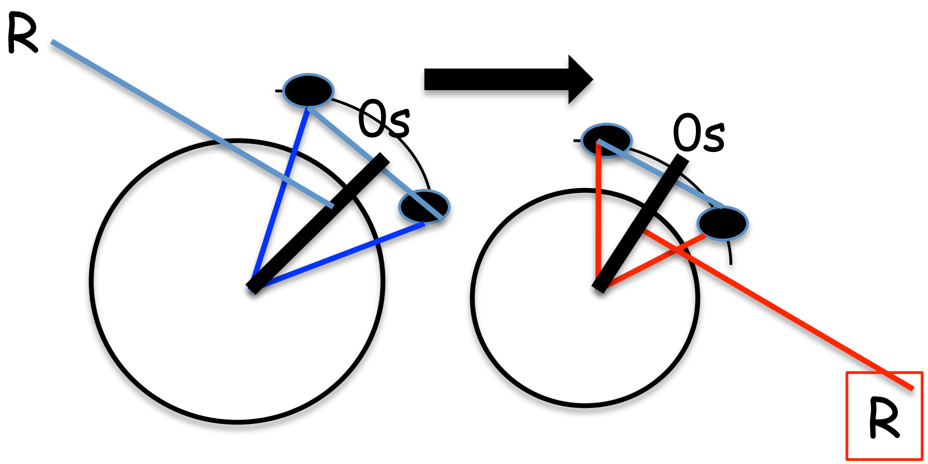
BCS

Same results shown as histograms



B

Cluster-transfer model (suggested by the close radial correlation of the pairs)



Initial and final cluster wave functions are obtained by taking the overlap between the two-particle wave functions and a 0s wave function for the relative motion

Also in this case the resulting cross section depends on the specific single-particle orbitals (via the Talmi-Moshinsky brackets), but the dependence is different from the one associated with the sequential transfer (!!!)

The preference to either model may depend on the colliding systems and on kinematical conditions.

The proper approach will depend on the competition between the two colliding single-particle mean fields and the residual two-body interaction (for relatively weak interaction the mean fields will prevail, while in the other extreme of infinite pairing correlation the cluster structure will take over).

Let us not forget Q-value effects

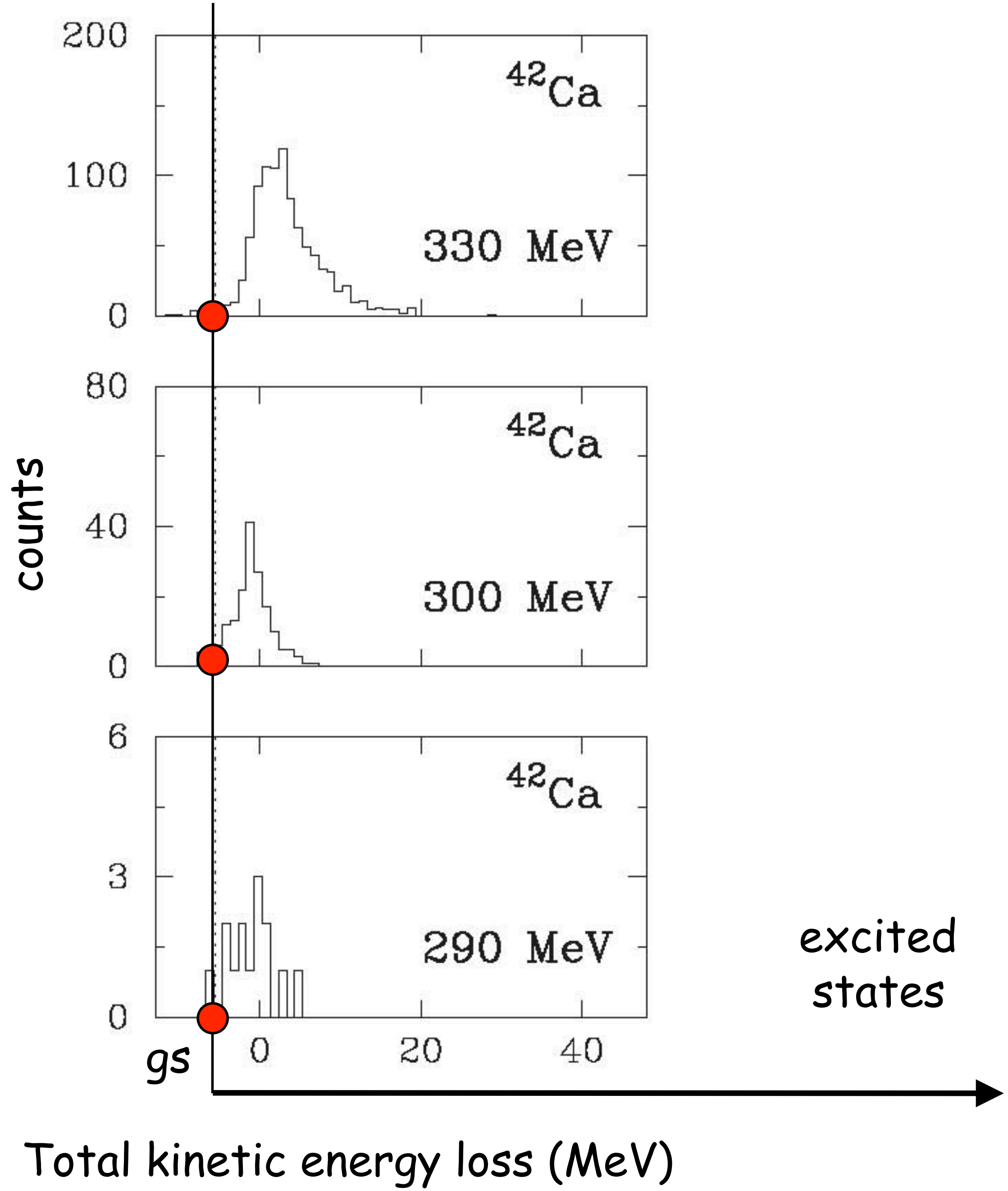
Keeping fixed any other parameter, the probability for populating a definite final channel depends on the Q-value of the reaction. The dependence (in first approximation a gaussian distribution centered in the optimum Q-value) is very strong in the case of heavy-ion induced reactions, weaker in the case of light ions.

The optimum Q-value depends on the angular momentum transfer and on the charge of the transferred particles. In the specific case of L=0 two-neutron transfer, the optimal Q-value is zero.

Experimental evidence

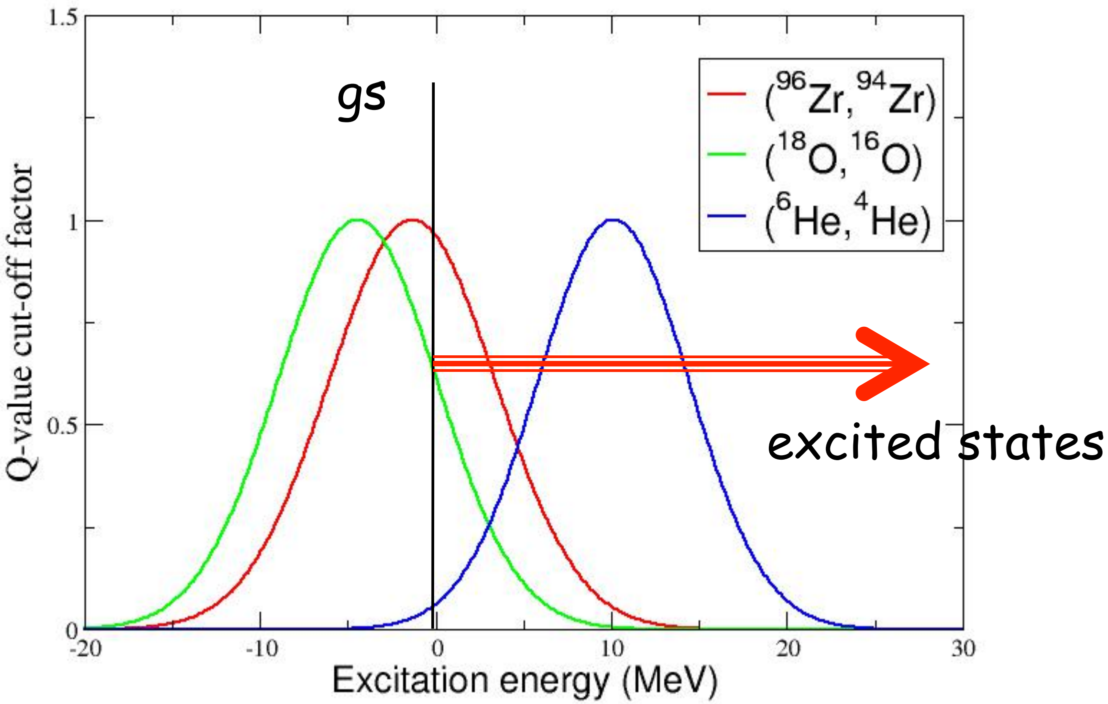
$^{96}Zr+^{40}Ca$

Selecting final ^{42}Ca mass partition

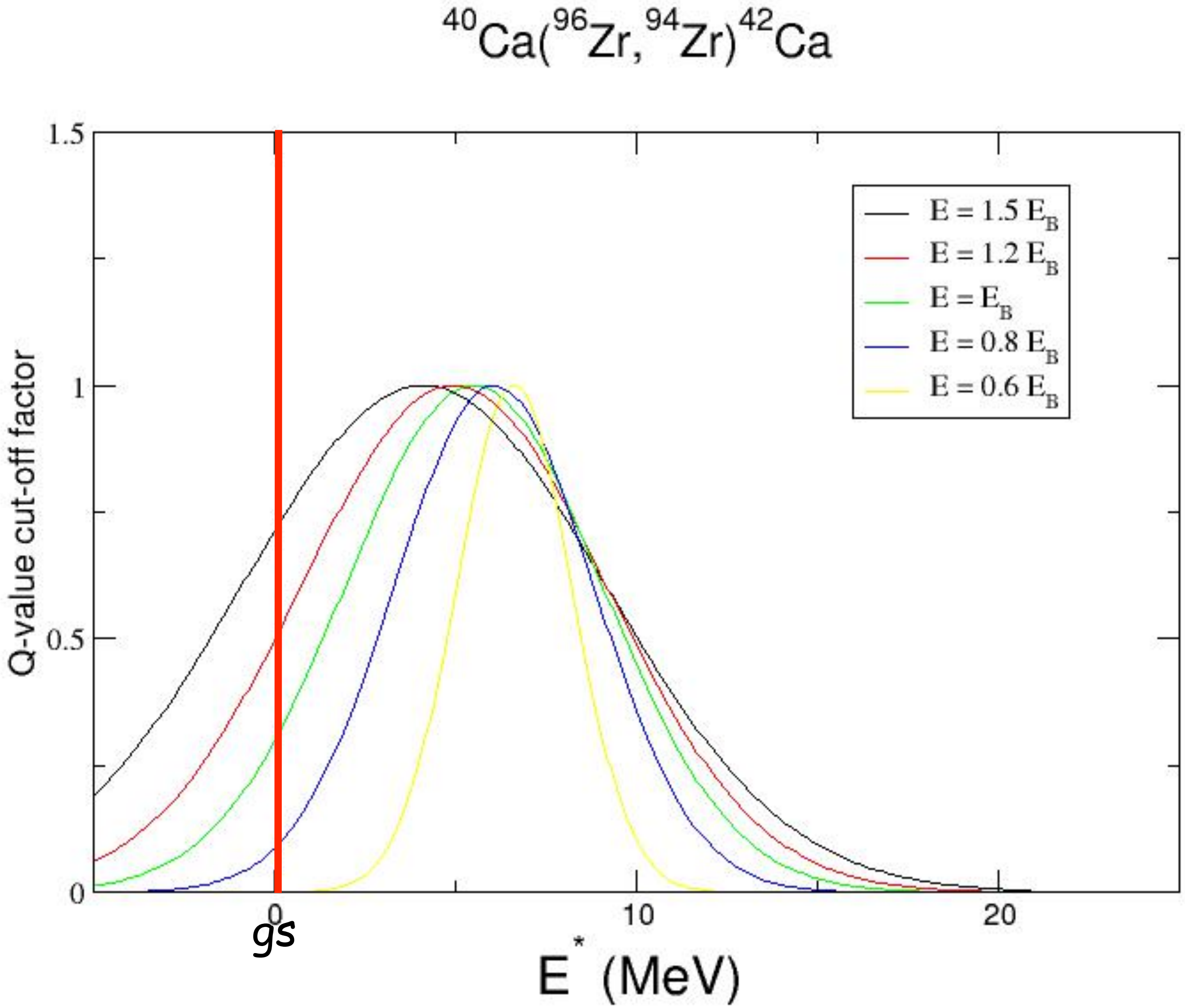


Playing with different combinations of projectile/target (having different Q_{gg}-value) one can favour different energy windows

Example: Target ^{208}Pb Final ^{210}Pb (at bombarding energy E_{cm} = 1.2 $E_{barrier}$)



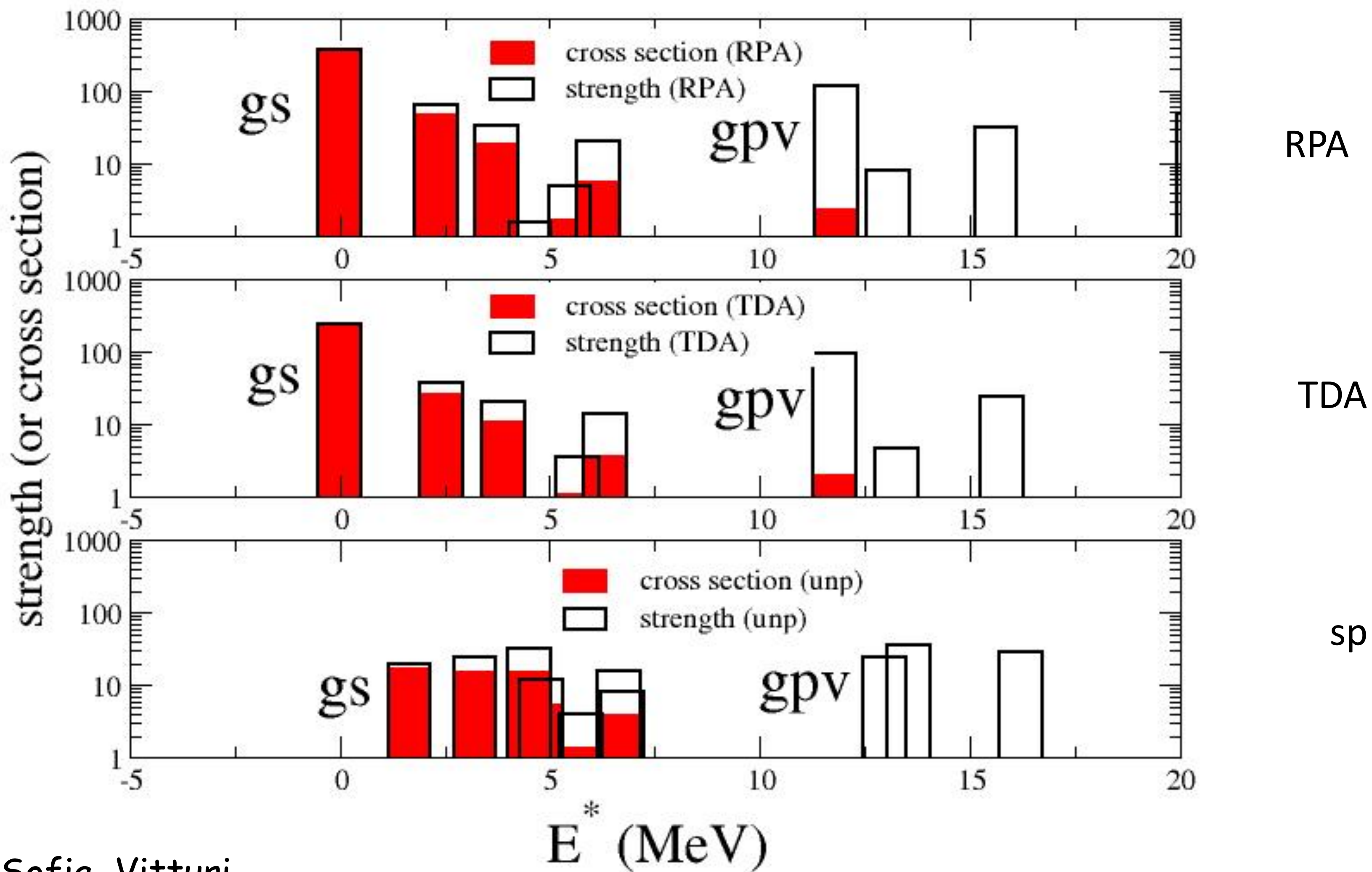
The width of the Q-value window increases
with the bombarding energy



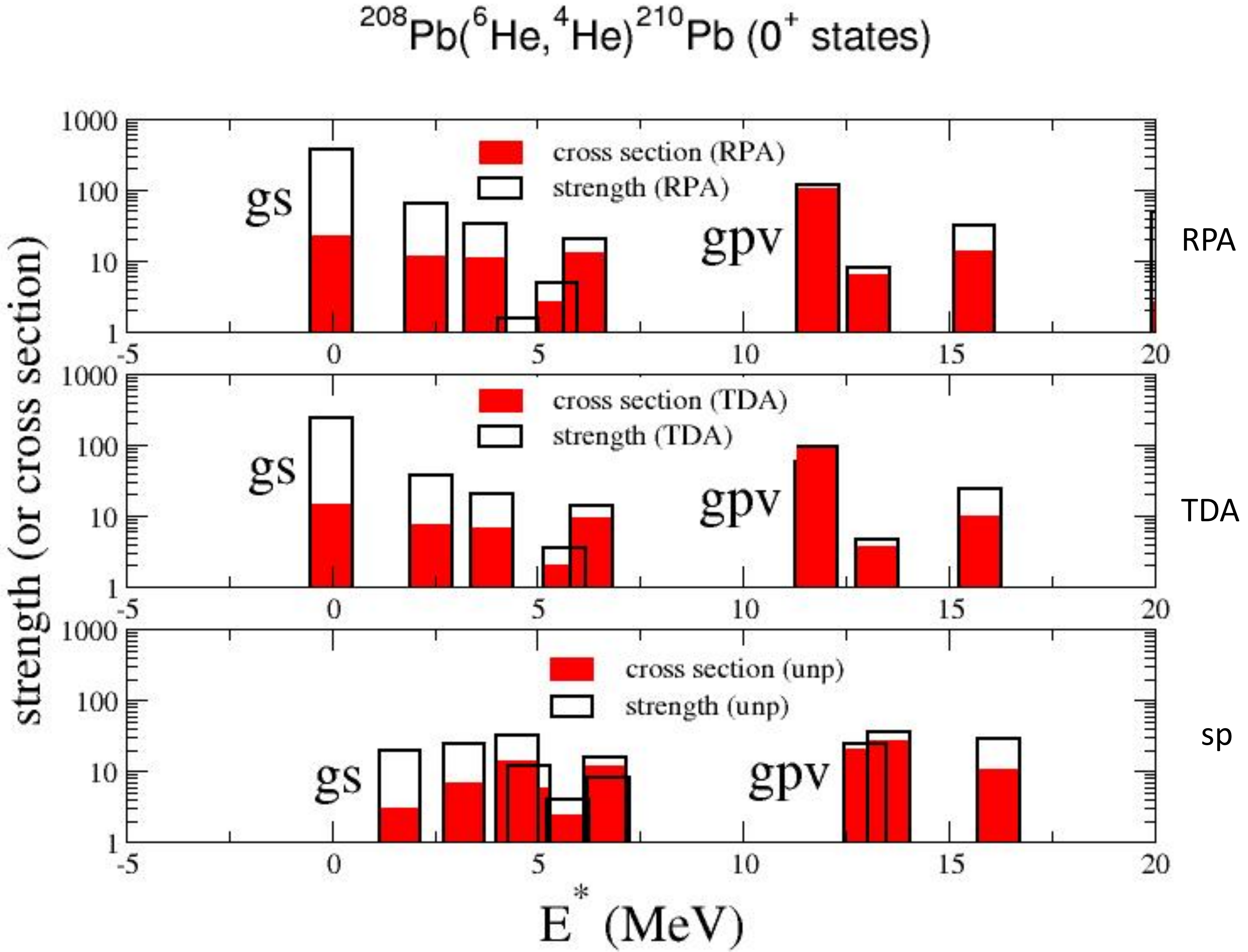
The pairing strength is therefore modulated by the Q-value cut-off to yield the final two-particle cross section

The pairing strength is therefore modulated by the Q-value cut-off to yield the final two-particle cross section

$^{208}Pb(^{18}O,^{16}O)^{210}Pb$ (0^+ states)



Dasso, Sofia, Vitturi



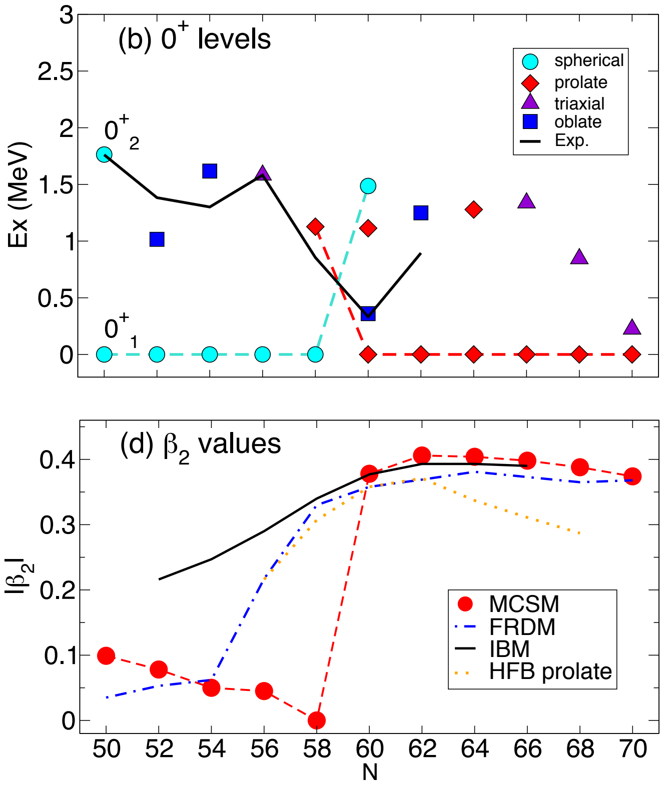
208Pb(6He,4He)210Pb (0+ states)
cross section (RPA)
strength (RPA)
gs
gpv
cross section (TDA)
strength (TDA)
gs
gpv
cross section (unp)
strength (unp)
gs
gpv
strength (or cross section)
E* (MeV)
1000
100
10
1
-5
0
5
10
15
20
RPA
TDA
sp

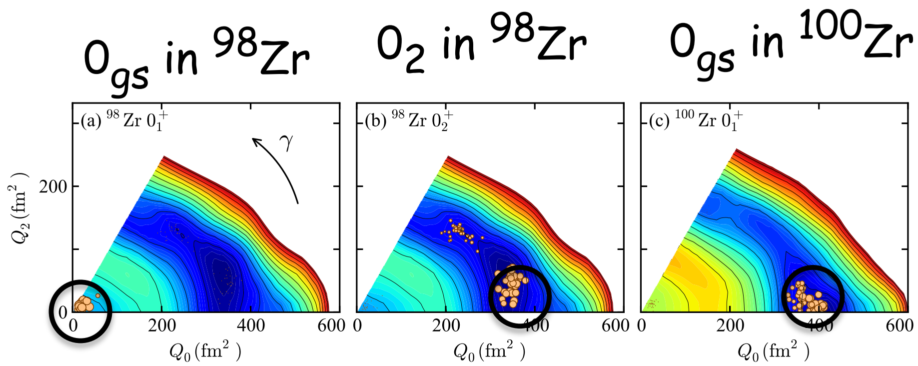
Two cases in more details (with full microscopic wave functions):

1. Shape phase transition in Zr isotopes

2. Possible breaking of shell closure in 32Mg (N=20)

First example: Shape phase transition in Zr isotopes between N=58 and 60





similar shape

Quantum Phase Transition in the Shape of Zr isotopes

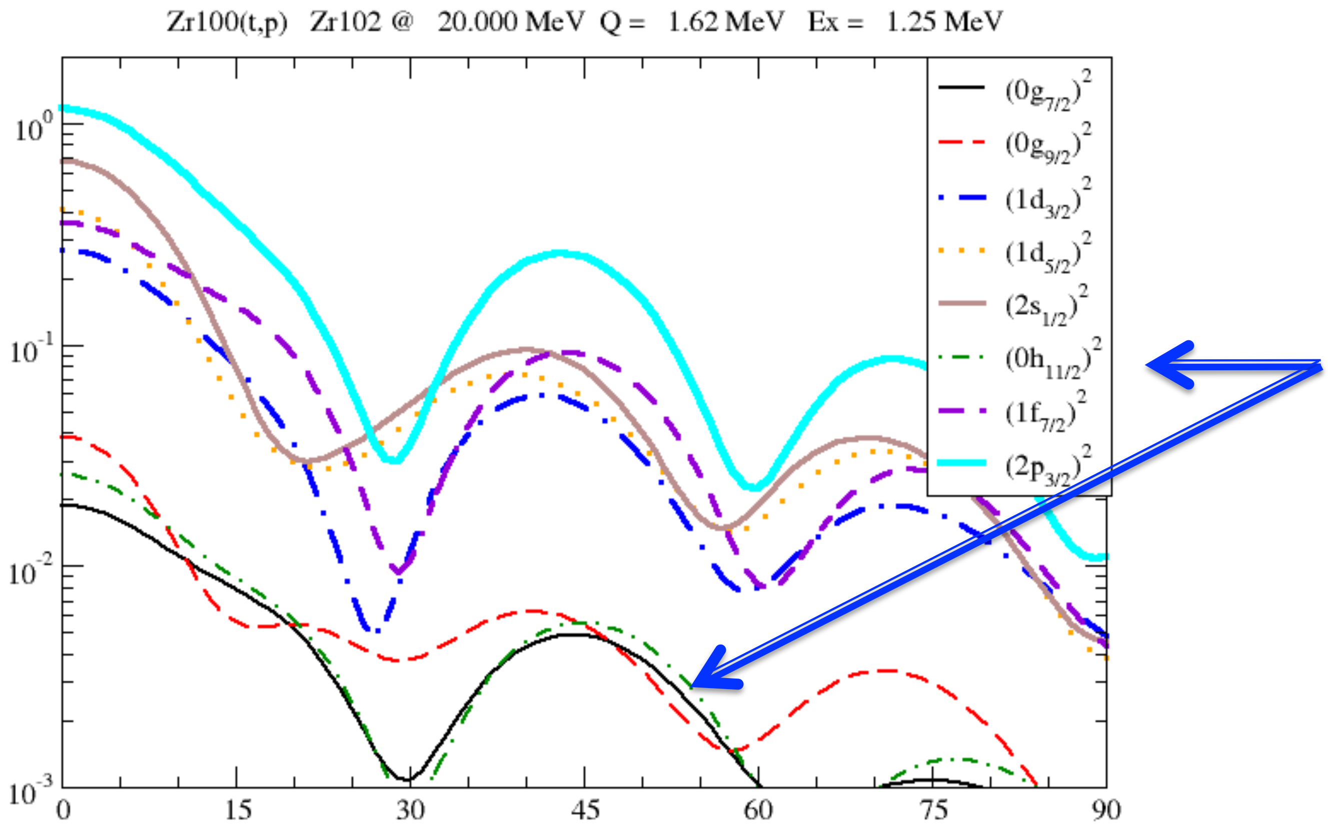
Tomoaki Togashi[1], Yusuke Tsunoda[1], Takaharu Otsuka[1,2,3,4] and Noritaka Shimizu[1]

relevant 2-particle spectroscopic amplitudes

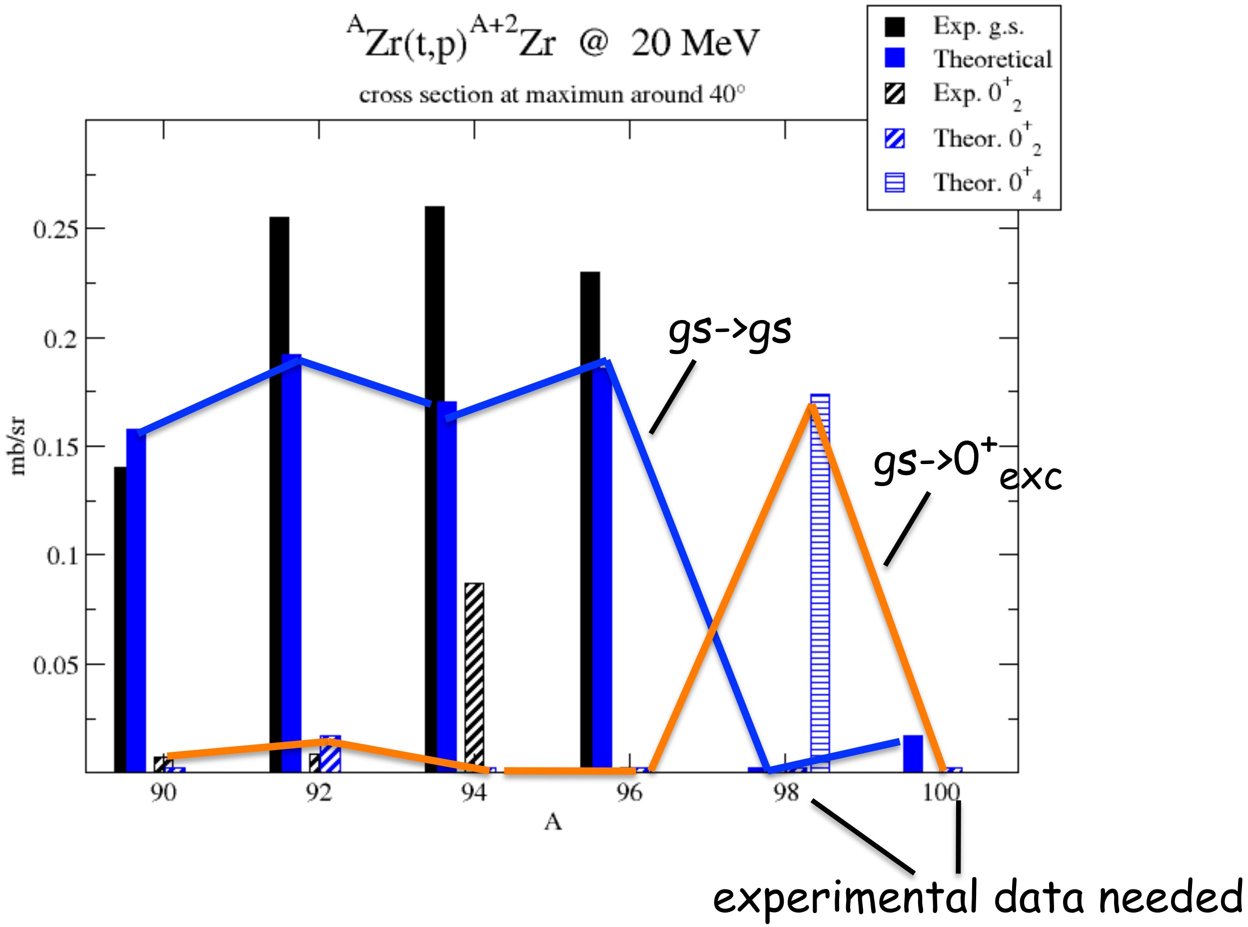
	90>92gs	92>94gs	94>96gs	96>98gs	98>100gs	98>100 (0^+_4)	100>102gs
d5/2	**0.74**	**0.86**	**0.86**	0.13	0.0	0.16	0.08
S1/2	0.10	0.08	0.10	**0.90**	0.0	0.16	0.05
d3/2	0.13	0.18	0.16	0.07	0.0	**0.90**	0.04
h11/2	0.22	0.20	0.19	0.08	0.0	0.14	**0.55**

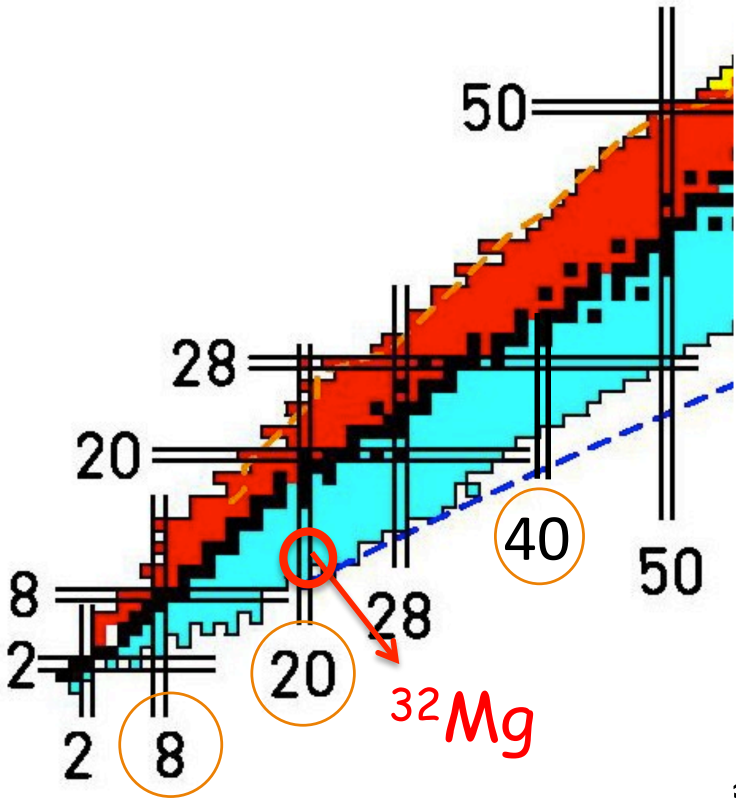
$d_{5/2}$ $s_{1/2}$ $d_{3/2}$ $h_{11/2}$

Cross sections for pure configurations

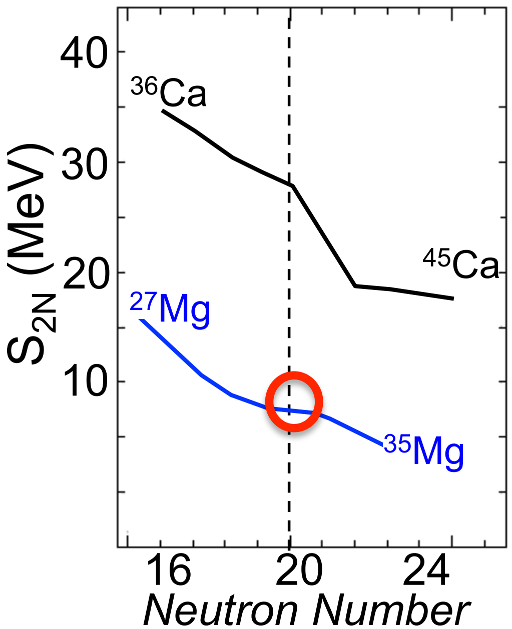


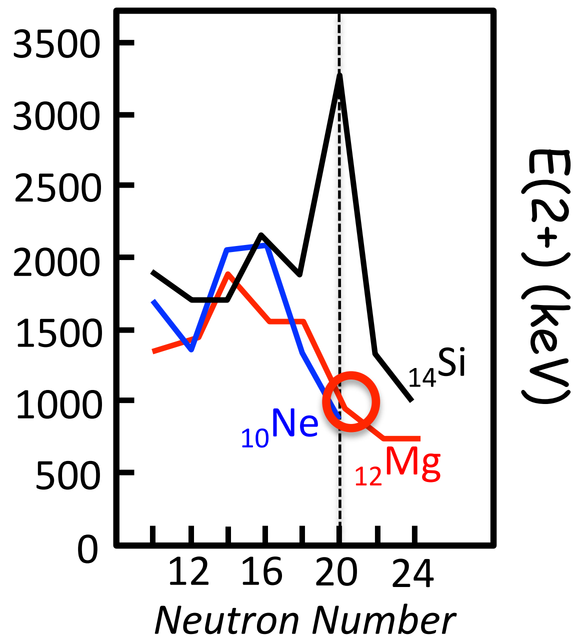
Calculation of two-particle transfer reactions using:
sequential model for the reaction mechanism
one- and two-particle spectroscopic amplitudes from the Tokyo group





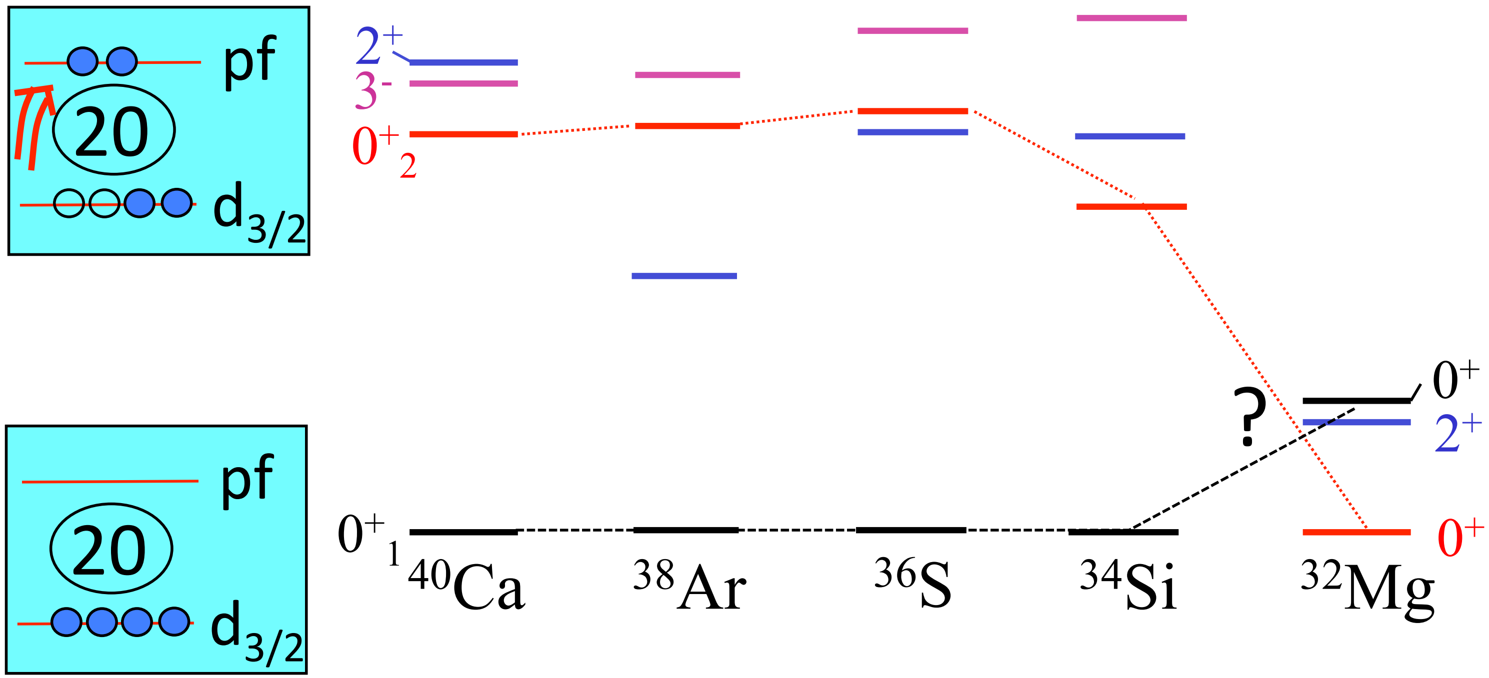
Second example:
the case of the N=20 shell and ^{32}Mg





The N=20 shell seems to be washed out for Z<14

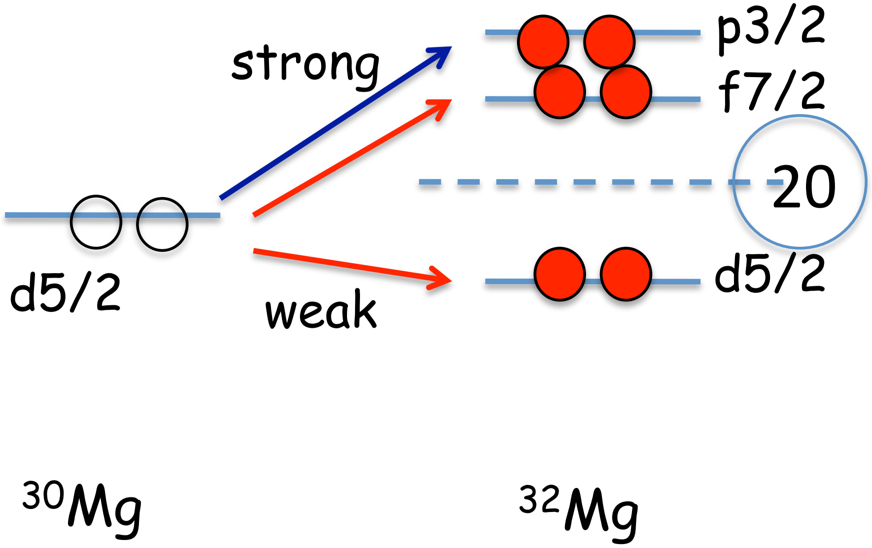
Evolution of 0+ states in N=20 isotones

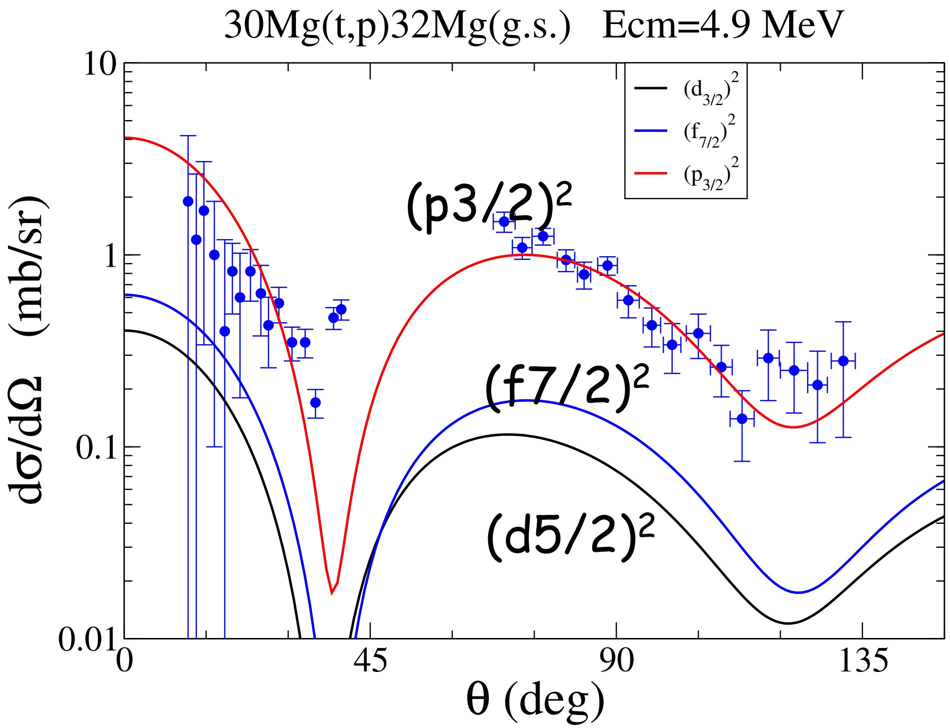


Inversion of shape (spherical and deformed)? Mixing of 0p-0h with 2p-2h?

Microscopic calculation of (t,p) cross section

Pure configurations



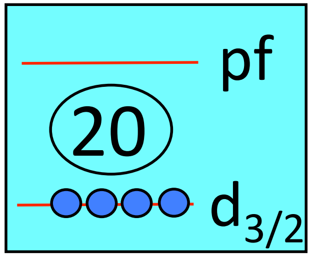


OBS: The relative population for single particle transfer may be reversed for other reactions, as (18O,16O), (14C,12C) etc

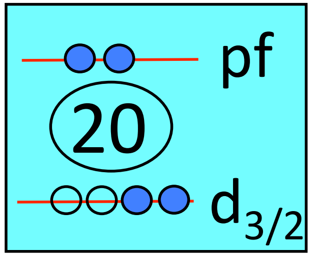
In the "standard" single-particle sequence with shell closure at N=20 the transition to the ground state will involve the transfer of two particles in the (d5/2) shell, while the transition to the excited 0+ (2p-2h) involves the (p3/2) or (f7/2). The findings of (t,p) reaction are not compatible with shell closure at N=20.

The possible vanishing of the N=20 shell will generate an inversion (or at least a mixing) of the 0p-0h state with the 2p-2h state. As a first simple model we assume

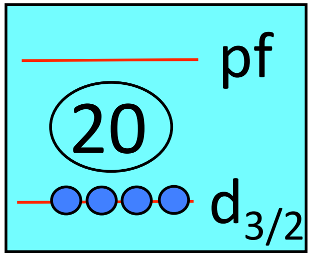
Ground state = α



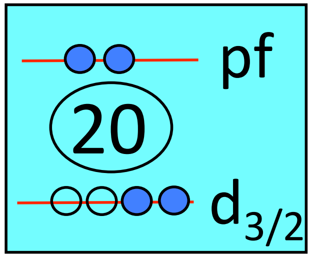
$+ \beta$



Excited state = $-\beta$

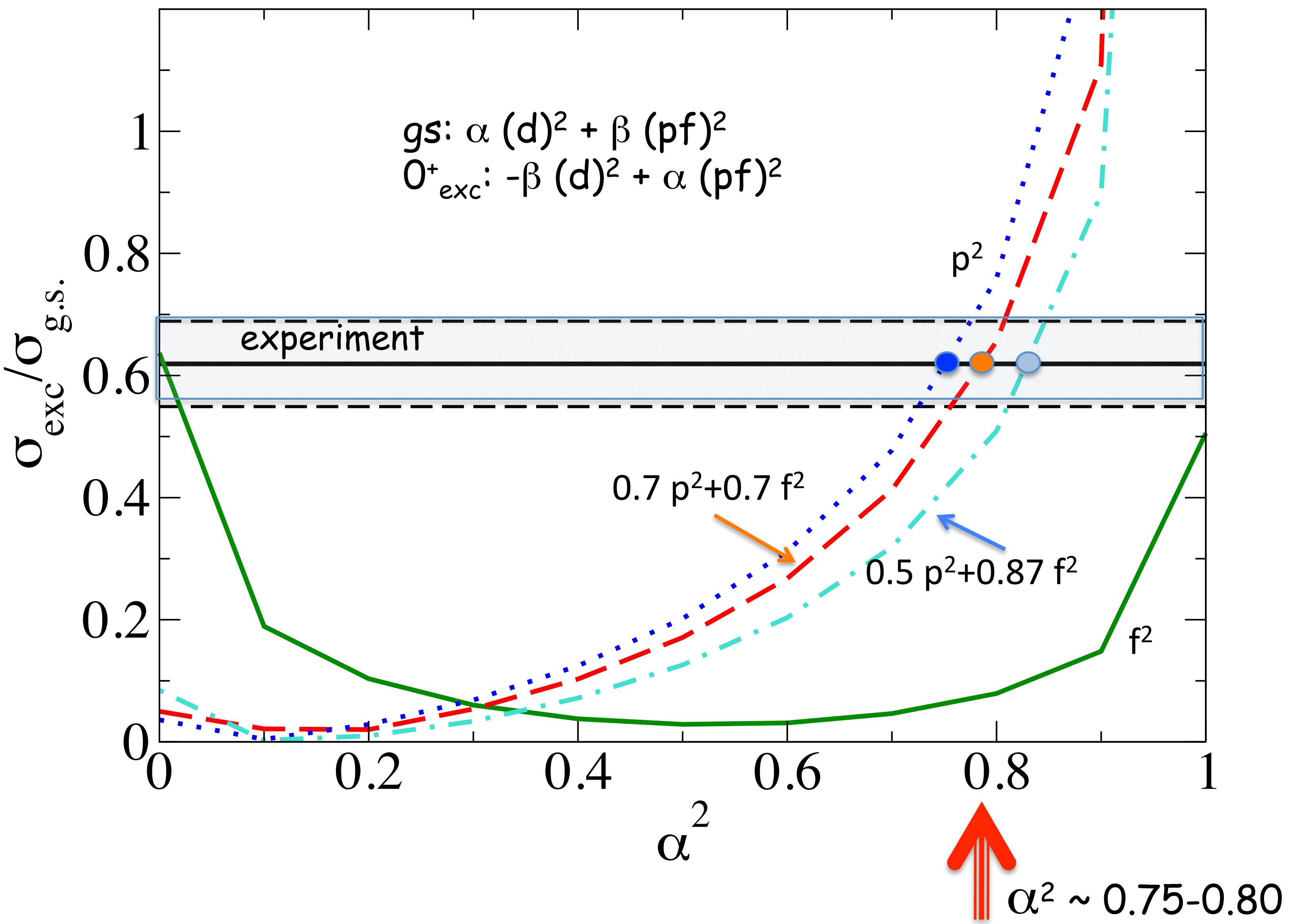


$+ \alpha$



and determine the mixing coefficient α from fitting (t,p) cross section ratio

$^{30}Mg(t,p)^{32}Mg$



OBS: The $(pf)^2$ pair will be a combination of $(f7/2)^2$ and $(p3/2)^2$ components

Conclusions:

Pairing response (tested in two-particle transfer reactions but also in other dynamical processes involving pairs of particles) gives strong constrains on nuclear wave functions. The effect is amplified in correspondence of critical situations associated with shape phase transitions, with “abnormal” population of excited 0+ states and weakening of the ground state transition.

Further data on two-particle transfer reactions are definitely needed

QPTn9

Padova (Italy)
22-25 May
2018

9th International Workshop on Quantum Phase Transitions in Nuclei and Many-body Systems

Con il Patrocinio del Comune di Padova

S

Scientific Programme:

- Experimental Signatures & Spectroscopic Data
- Quantum Phase Transitions in Nuclei
- Transitional Nuclei & Critical Point Symmetries
- Shape Coexistence, Shell Evolution
- Excited States Phase Transitions
- Phase Transitions in Atomic, Molecular and other domains

V

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