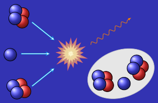
Dynamics of ^{9}Be in a three-cluster model



Manuela Rodríguez-Gallardo
Jesús Casal and José M. Arias

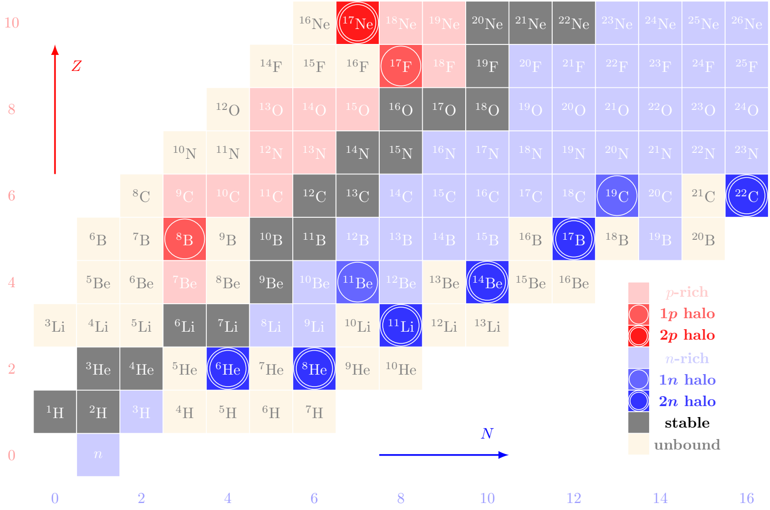
Universidad de Sevilla

Trento, 8 March 2018

Outline

- Motivation: weakly-bound systems
  - The Borromean nucleus ^{9}Be$(\alpha + \alpha + n)$
- Pseudo-State methods to obtain the structure:
  - The analytical transformed harmonic oscillator (ATHO) method
- What can we study for ^{9}Be?
  - The reaction rate for the radiative capture $\alpha(\alpha n, \gamma)^9$Be at T of astrophysical interest
  - Direct reactions induced by ^{9}Be on a stable target using the 4b-CDCC formalism:
    - ^{9}Be+^{208}Pb at 44 and 38 MeV
    - ^{9}Be+^{120}Sn at 27, 28, 29.5 and 31 MeV
- Summary and conclusions

Motivation


Z
N
^{16}Ne ^{17}Ne ^{18}Ne ^{19}Ne ^{20}Ne ^{21}Ne ^{22}Ne ^{23}Ne ^{24}Ne ^{25}Ne ^{26}Ne
^{14}F ^{15}F ^{16}F ^{17}F ^{18}F ^{19}F ^{20}F ^{21}F ^{22}F ^{23}F ^{24}F ^{25}F
^{12}O ^{13}O ^{14}O ^{15}O ^{16}O ^{17}O ^{18}O ^{19}O ^{20}O ^{21}O ^{22}O ^{23}O ^{24}O
^{10}N ^{11}N ^{12}N ^{13}N ^{14}N ^{15}N ^{16}N ^{17}N ^{18}N ^{19}N ^{20}N ^{21}N ^{22}N ^{23}N
^{8}C ^{9}C ^{10}C ^{11}C ^{12}C ^{13}C ^{14}C ^{15}C ^{16}C ^{17}C ^{18}C ^{19}C ^{20}C ^{21}C ^{22}C
^{6}B ^{7}B ^{8}B ^{9}B ^{10}B ^{11}B ^{12}B ^{13}B ^{14}B ^{15}B ^{16}B ^{17}B ^{18}B ^{19}B ^{20}B
^{5}Be ^{6}Be ^{7}Be ^{8}Be ^{9}Be ^{10}Be ^{11}Be ^{12}Be ^{13}Be ^{14}Be ^{15}Be ^{16}Be
^{3}Li ^{4}Li ^{5}Li ^{6}Li ^{7}Li ^{8}Li ^{9}Li ^{10}Li ^{11}Li ^{12}Li ^{13}Li
^{3}He ^{4}He ^{5}He ^{6}He ^{7}He ^{8}He ^{9}He ^{10}He
^{1}H ^{2}H ^{3}H ^{4}H ^{5}H ^{6}H ^{7}H
n
10 8 6 4 2 0
0 2 4 6 8 10 12 14 16
p-rich
$1p$ halo
$2p$ halo
n-rich
$1n$ halo
$2n$ halo
stable
unbound

Motivation

Dynamics of ^{9}Be in a three-cluster model

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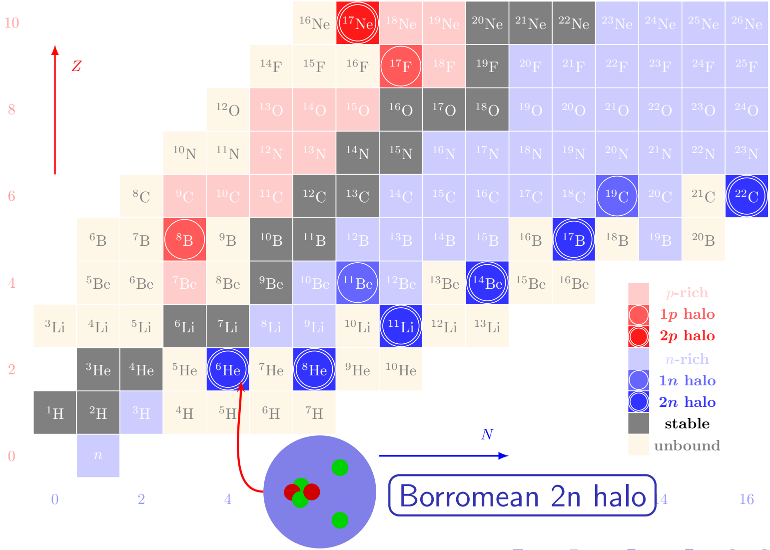
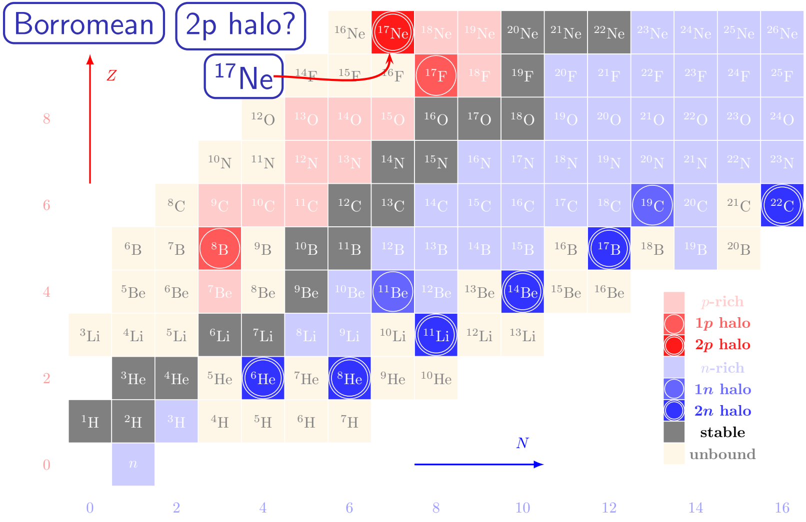
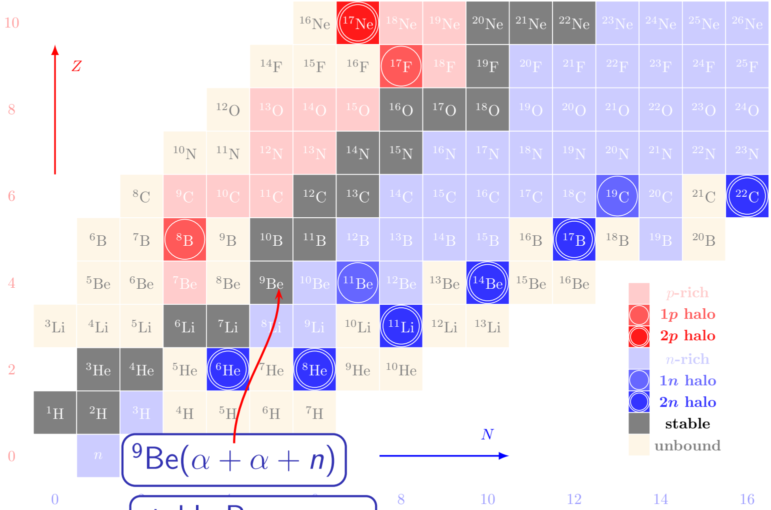


Motivation



Borromean
2p halo?
^{17}Ne
Z
N
p-rich
1p halo
2p halo
n-rich
1n halo
2n halo
stable
unbound

Motivation



^{9}Be($\alpha + \alpha + n$)

stable Borromean

Motivation

Dynamics of ^{9}Be in a three-cluster model

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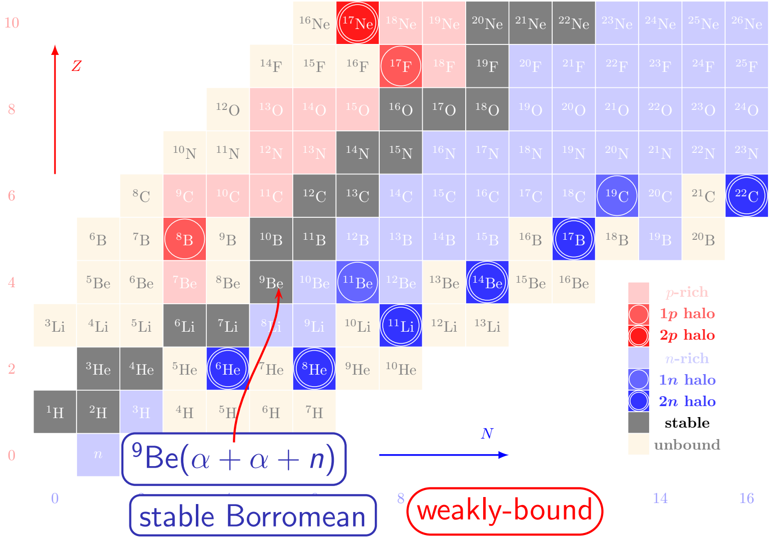
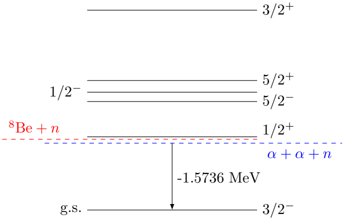
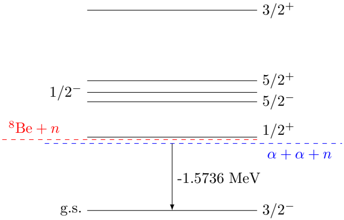


Why studying ^{9}Be?

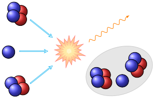


- ^{9}Be is stable but has a small separation energy
- Breakup effects are expected to be important in reactions induced by this nucleus

Why studying ^{9}Be?

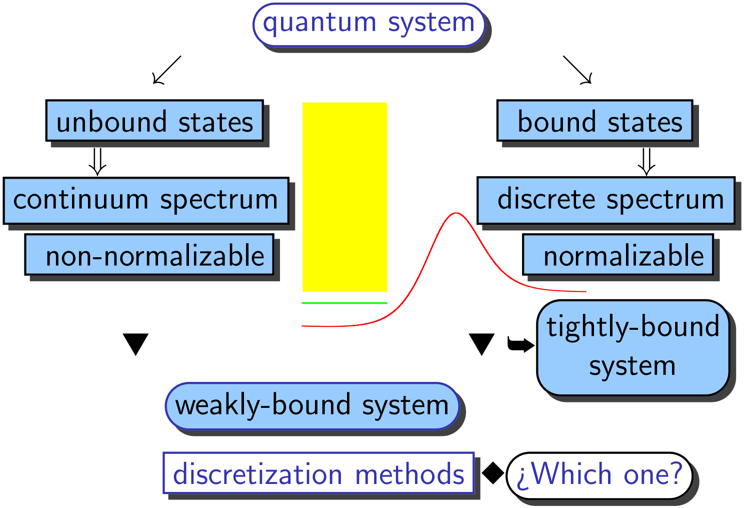


- ^{9}Be is stable but has a small separation energy
- Breakup effects are expected to be important in reactions induced by this nucleus
- $\alpha(\alpha n, \gamma)^9$Be followed by ^{9}Be$(\alpha, n)^{12}$C may provide an alternative path to the triple-alpha process



- This process has been linked to the r-process in type II supernovae [*Langanke & Wiescher, Rep. Prog. Phys. 64 (2001) 1657*]

Weakly-bound systems

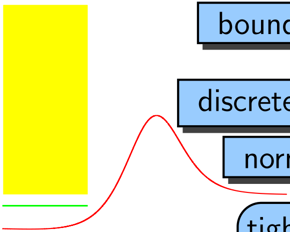


Weakly-bound systems

quantum system

↙ ↘

unbound states	bound states
⇓	⇓
continuum spectrum	discrete spectrum
non-normalizable	normalizable



▼ ▼ ➥ tightly-bound system

weakly-bound system

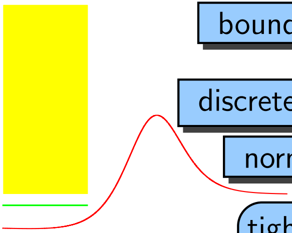
discretization methods ◆ PS methods

Weakly-bound systems

quantum system

↙ ↘

unbound states ⇓ continuum spectrum — non-normalizable



bound states ⇓ discrete spectrum — normalizable

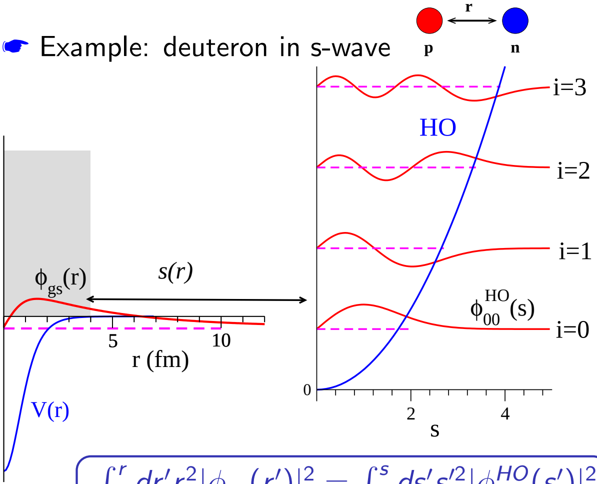
▼ ▼ ➥ tightly-bound system

weakly-bound system

discretization methods ◆ THO

THO method

☛ Example: deuteron in s-wave

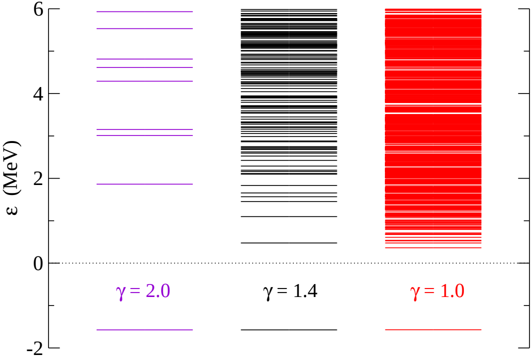


$$\int_0^r dr' r^2 |\phi_{gs}(r')|^2 = \int_0^s ds' s'^2 |\phi_{00}^{HO}(s')|^2$$

Analytical THO (ATHO)

$$s(r) = \sqrt{\frac{1}{2b}} \left[\left(\frac{1}{r}\right)^m + \left(\frac{1}{\gamma\sqrt{r}}\right)^m \right]^{-\frac{1}{m}}$$

- Easier to implement
- Flexibility in PSs distribution as function of γ/b



$m = 4$
$b = 0.7$ fm

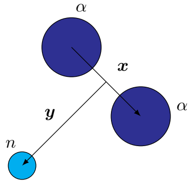
But, what about three-body systems?

- The states of the system are expanded in Hyperspherical Harmonics (HH)

$$\phi_{nj\mu}(\rho,\Omega) = \sum_\beta R_{n\beta j}(\rho)\mathcal{Y}_{\beta j\mu}(\Omega_5)$$

$$\beta \equiv K, \ell_x, \ell_y, \ell, S_x, j_{ab}$$

$$\rho^2 \equiv x^2 + y^2$$
$$\tan\alpha = x/y$$



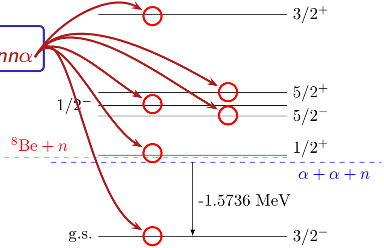
- The hyperradial functions $R_{n\beta j}(\rho)$ are obtained with the THO method

- The H is diagonalised in a THO basis with $i = 0, \ldots, i_{max}$ functions in each channel β

Application to ^{9}Be

$$\widehat{H}(\rho,\Omega) = \widehat{T}(\rho,\Omega) + \widehat{V}(\rho,\Omega)$$

$$V = V_{n\alpha} + V_{n\alpha} + V_{\alpha\alpha} + V_{nn\alpha}$$



- $V_{n\alpha}$: GPT Phys. Lett. B32 (1070) 591
- $V_{\alpha\alpha}$: Ali-Bodmer Nucl. Phys. 80 (1966) 99
- Pauli forbidden states: repulsive central $V_{n\alpha}$ in s-wave

Energy spectrum

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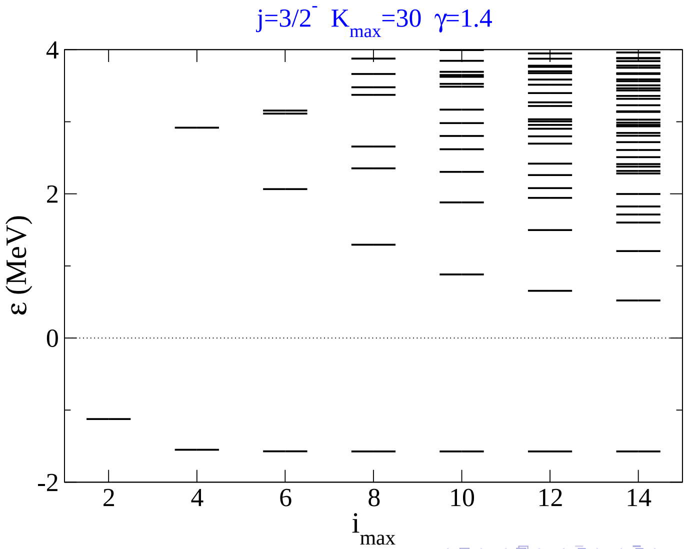
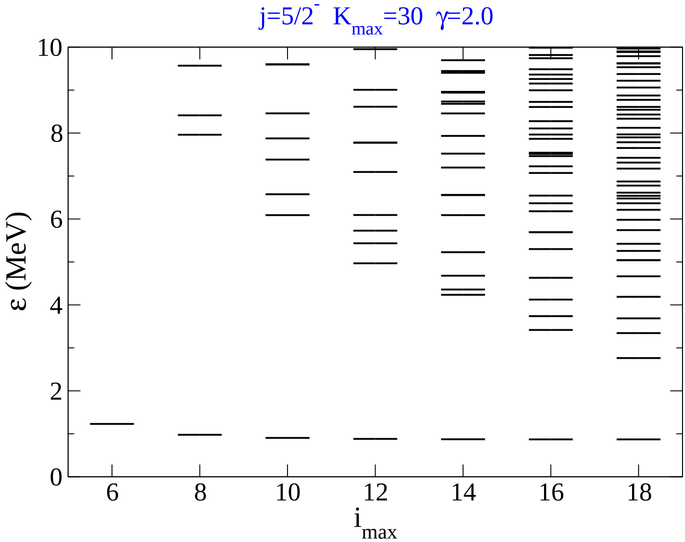


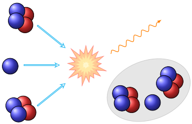
Energy spectrum

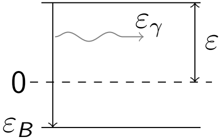


Radiative capture reaction rate

- The energy-averaged reaction rate for the radiative capture of 3 particles (abc) into a bound nucleus A, $a + b + c \rightarrow A + \gamma$, is given as function of T:

$$\langle R_{abc}(\varepsilon)\rangle(T) = \mathcal{N}\, (k_B T)^{-3} \int_0^{\infty} \varepsilon_\gamma^2 \sigma_\gamma(\varepsilon_\gamma) e^{\frac{-\varepsilon}{k_B T}} d\varepsilon$$

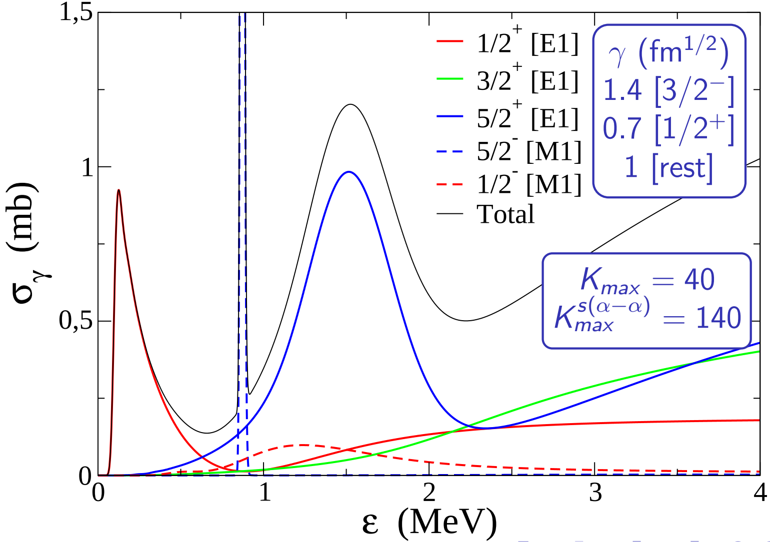




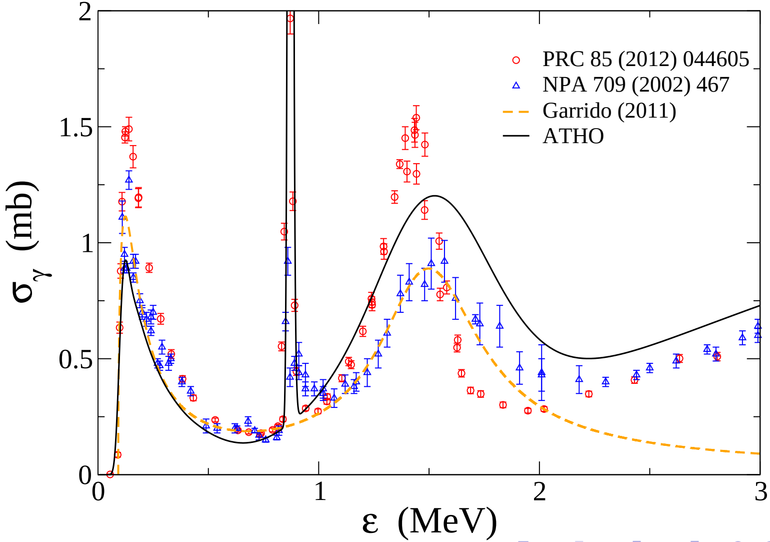
- The photodissociation cross section σ_γ can be expanded into electric and magnetic multipoles ($\mathcal{O}\lambda$):

$$\sigma_\gamma^{(\mathcal{O}\lambda)}(\varepsilon_\gamma) = \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \left(\frac{\varepsilon_\gamma}{\hbar c}\right)^{2\lambda-1} \frac{dB(\mathcal{O}\lambda)}{d\varepsilon}$$

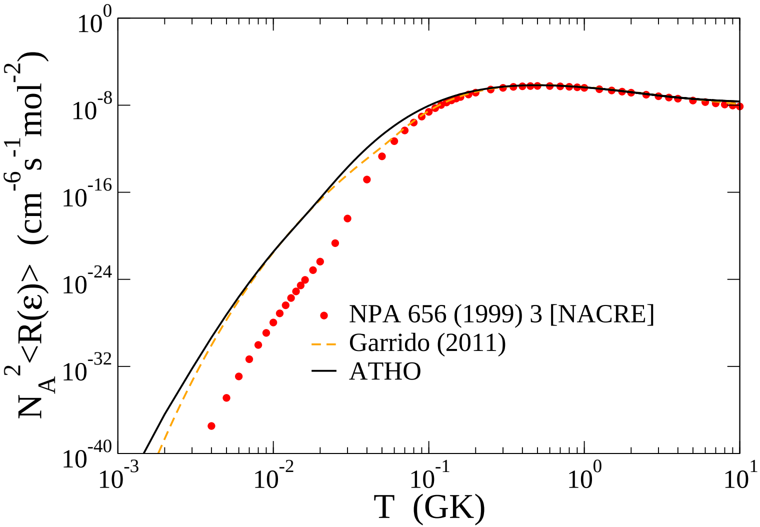
Photodissociation cross section for ^{9}Be$+\gamma \rightarrow \alpha + \alpha + n$



Photodissociation cross section for $^9\mathrm{Be}+\gamma \rightarrow \alpha + \alpha + n$

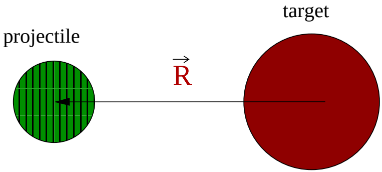


Reaction rate for $\alpha(\alpha n, \gamma)^9\mathrm{Be}$

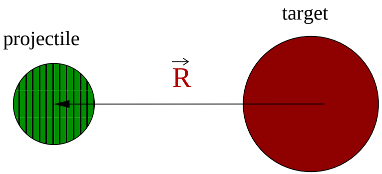


J. Casal et al., Phys. Rev. C 90 (2014) 044304

Reactions induced by ^{9}Be: CDCC formalism

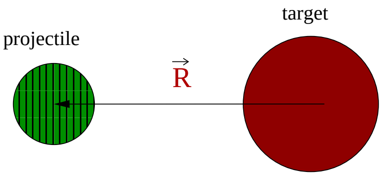


Reactions induced by ^{9}Be: CDCC formalism



$$\Psi_J^M(\vec{R},\xi) = \sum \phi_{jn}^{\mu}(\xi)\langle LM_L j\mu|JM\rangle \frac{i^L}{R} Y_L^{M_L}(\widehat{R}) f_{Lnj}^J(R)$$

Reactions induced by ^{9}Be: CDCC formalism

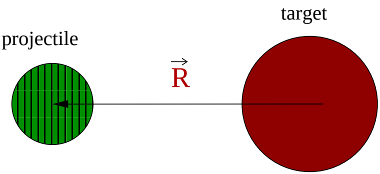


$$\Psi_J^M(\vec{R},\xi) = \sum \phi_{jn}^{\mu}(\xi)\langle L M_L j \mu | J M\rangle \frac{i^L}{R} Y_L^{M_L}(\widehat{R}) f_{Lnj}^J(R)$$

$\Uparrow$

1, 2, 3 $\cdots$ n particles

Reactions induced by ^{9}Be: CDCC formalism



$$\Psi_J^M(\vec{R},\xi) = \sum \phi_{jn}^{\mu}(\xi)\langle L M_L j \mu | J M\rangle \frac{i^L}{R} Y_L^{M_L}(\widehat{R}) f_{Lnj}^J(R)$$

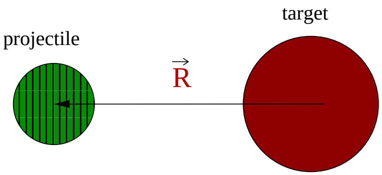
Coupled-channels system

$$\left[-\frac{\hbar^2}{2m_r}\left(\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2}\right) + \varepsilon_{nj} - E\right] f_{Lnj}^J(R) + \sum_{L'n'j'} i^{L'-L} V_{Lnj,L'n'j'}^J(R) f_{L'n'j'}^J(R) = 0$$

$\Uparrow$

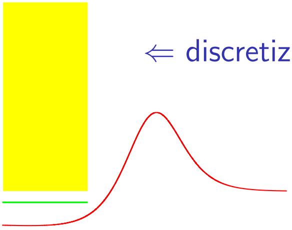
1, 2, $3 \cdots n$ particles

Reactions induced by ^{9}Be: CDCC formalism



$$\Psi_J^M(\vec{R},\xi) = \sum \phi_{jn}^{\mu}(\xi)\langle L M_L j \mu | J M\rangle \frac{i^L}{R} Y_L^{M_L}(\widehat{R}) f_{Lnj}^{J}(R)$$

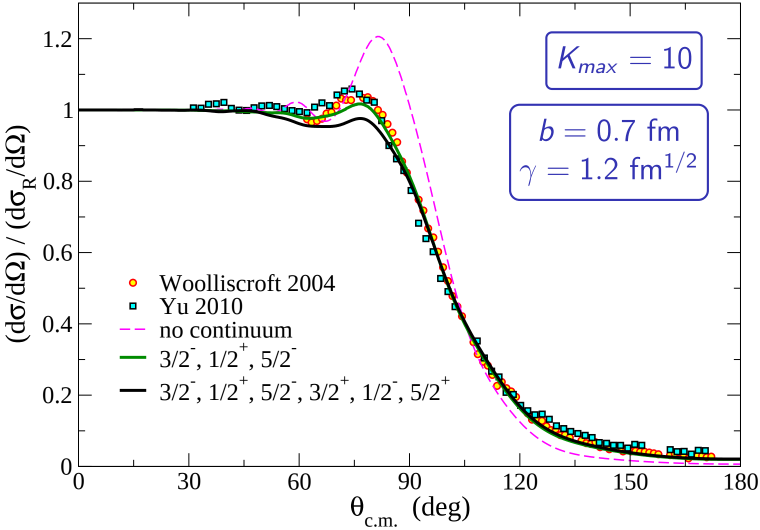
$\phi_n^{j\mu}(\xi)$?



$\Leftarrow$ discretization methods

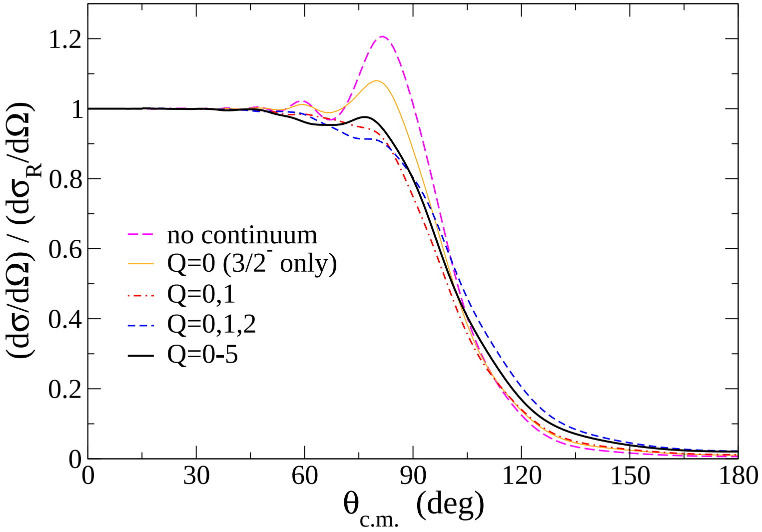
ATHO

^{9}Be+^{208}Pb at 44MeV

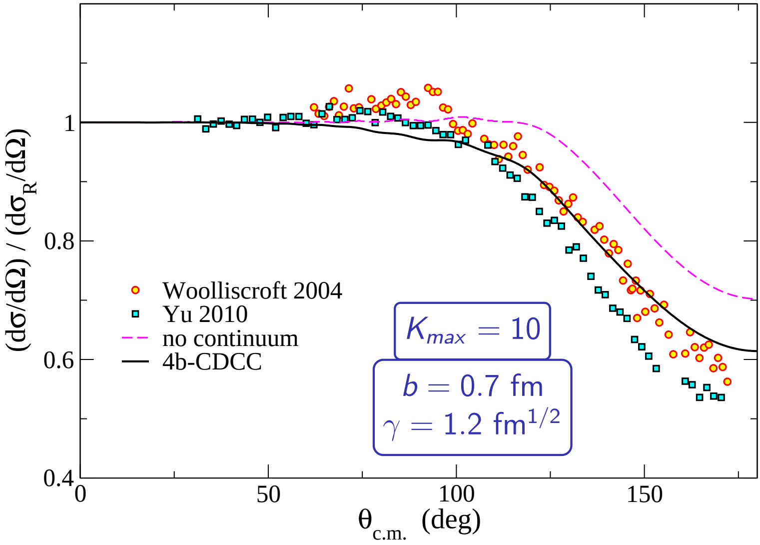


J. Casal et al., Phys. Rev. C 92 (2015) 054611

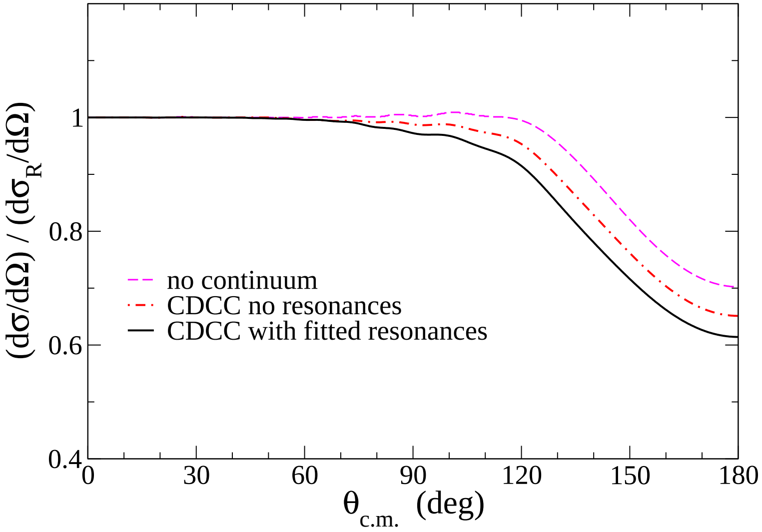
$^9Be+^{208}Pb$ at 44MeV: multipoles



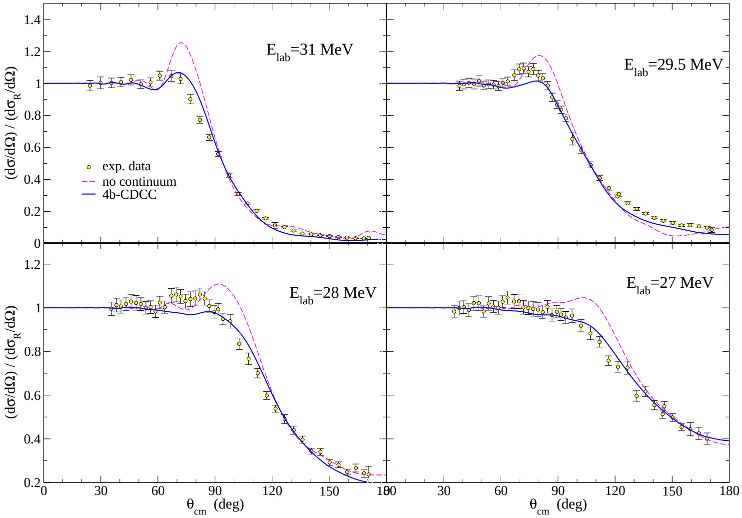
^{9}Be+^{208}Pb at 38MeV



$^{9}Be+^{208}Pb$ at 38MeV: resonances

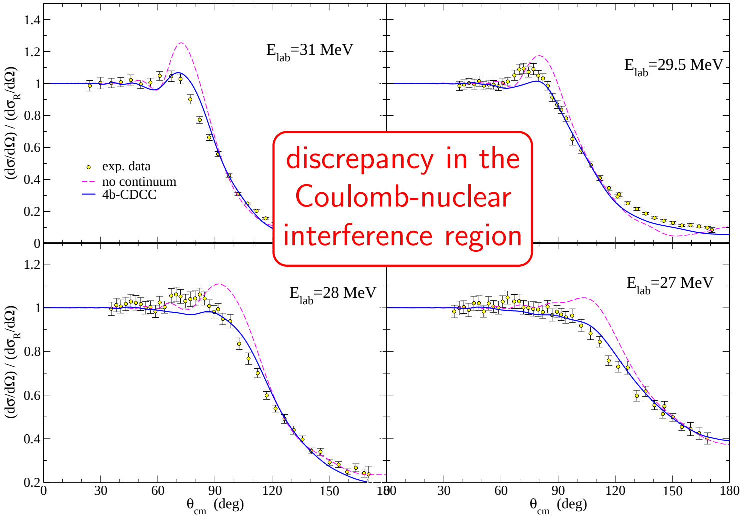


^{9}Be+^{120}Sn at TANDAR (Argentina)



Arazi et al., submitted to PRC (2017)

^{9}Be+^{120}Sn at TANDAR (Argentina)



Arazi et al., submitted to PRC (2017)

$^{9}Be+^{120}Sn$ at TANDAR (Argentina)

New experiment at TANDAR laboratory (Buenos Aires, Argentina) in 2018/19

Simultaneous measurements for elastic and exclusive breakup 2 alphas in coincidence

Collaboration theory-experiment international Argentina-Brasil-España

Summary and conclusions

- We have studied the Borromean nucleus ^{9}Be($\alpha + \alpha + n$) in a 3-body model
- We have used the ATHO method to obtain the states of ^{9}Be in this model
- We reproduce very well the σ_γ measured for ^{9}Be
- We have estimated the reaction rate for $\alpha(\alpha n, \gamma)^9$Be for the T of astrophysical interest
  - We find an important increase at the low-T region
- We have applied the 4b-CDCC formalism to ^{9}Be+^{208}Pb,^{120}Sn at E around the Coulomb barrier
  - The 4b-CDCC reproduces quite well the exp. data in general but there is a discrepancy in the C-N interference region
  - A new experiment (elastic+bu) is underway

In addition... Radiative capture reaction rate from inclusive breakup measurements

Direct relation at first order for $t = \frac{1}{k_B T}$

$$\langle R_{abc} \rangle (T) = \mathcal{C} \; t^3 e^{|\varepsilon_B| t} \frac{d^2}{dt^2} \left(\frac{1}{t^2} P_r(t) \right)$$

$t = f(E_{lab}, \theta)$: collision time

In addition... Radiative capture reaction rate from inclusive breakup measurements

Direct relation at first order for $t = \frac{1}{k_B T}$

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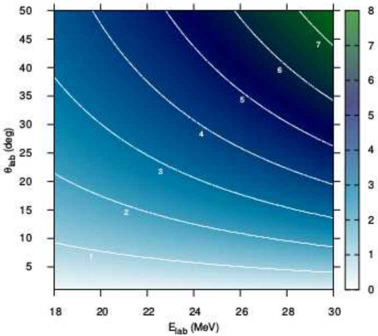
$t = f(E_{lab}, \theta)$: collision time

J. Casal et al., Phys. Rev. C 93 (2016) 041602(R)

^{6}He+^{208}Pb



In addition... Radiative capture reaction rate from inclusive breakup measurements

PAC Date :	EXP # (Do not fill in):
(PAC Date)	**E**

LETTER OF INTENT

Title: Determining the astrophysical three-body radiative capture reaction rate for ^{4}He(2n,γ)^{6}He from inclusive Coulomb break-up measurements

Spokesperson: A. M. Sánchez-Benítez
Address: Dpto. Ciencias Integradas. Fac. CC. EE. Avd. Fuerzas Armadas s/n CP 21007 Huelva, Spain.

Phone: +34 959219799	Fax:	Email: angel.sanchez@dfaie.uhu.es

Backup Spokesperson: J. P. Fernández-García
Address:

Phone:	Fax:	Email: jpfernandez@us.es

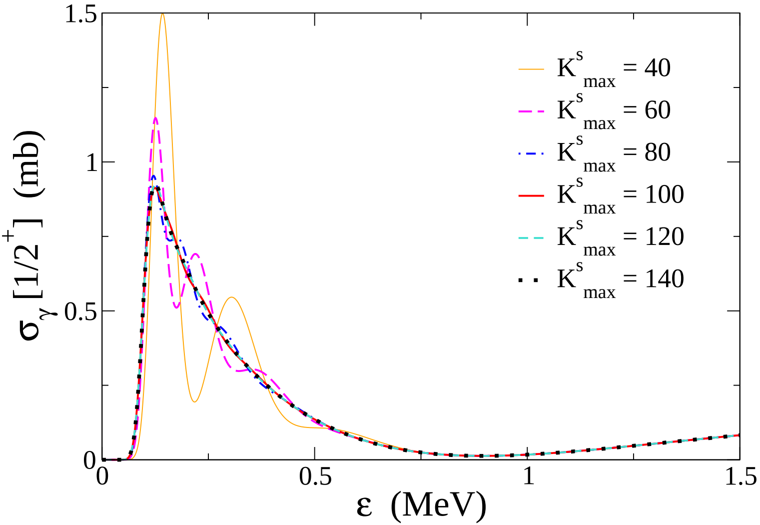
GANIL Scientific Coordinator: **A. Chbihi**

Collaboration : Participant names, institutions, and indicate students (S), and post-doctoral fellows (PDF):

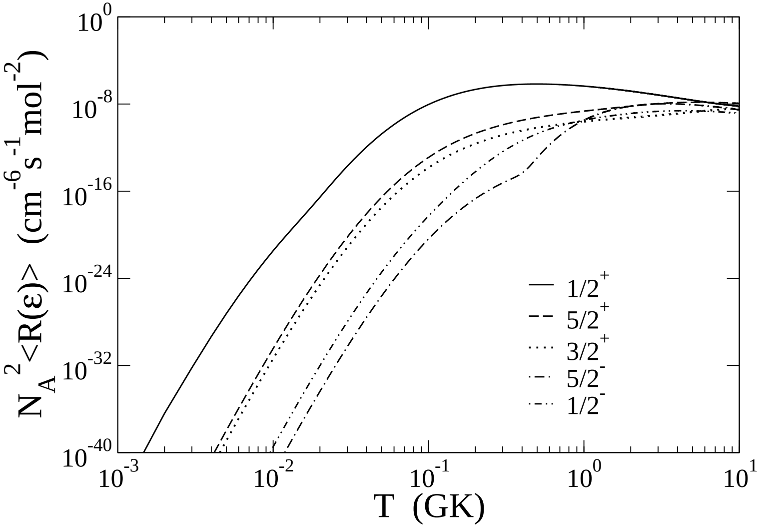
University of Huelva, A. M. Sánchez-Benítez, I. Martel, J. A. Dueñas, J. E. García-Ramos, F. Pérez-Bernal
University of Seville M. Rodríguez-Gallardo, J. M. Arias, J. P. Fernández-García, J. Gómez-Camacho, B. Fernández (PDF), F. J. Ferrer (PDF)
ECT*, J. Casal (PDF)
CSIC-Madrid, Spain M. J. G. Borge, O. Tengblad, J. D. Oveja (S), S. Viñals (S)
Instituto de Física-UNAM, Mexico L. Acosta, E. Chávez
Laboratori Nazionali del Sud, INFN, Italy A. Di Pietro, P. Figuera
University of Aarhus, Denmark H. Fynbo
LIP-Lisboa, Portugal D. Galaviz, L. Peralta, P. Teubig (S)
GANIL, France: F. de Oliveira

Previous experiment: ^{6}Li+^{27}Al

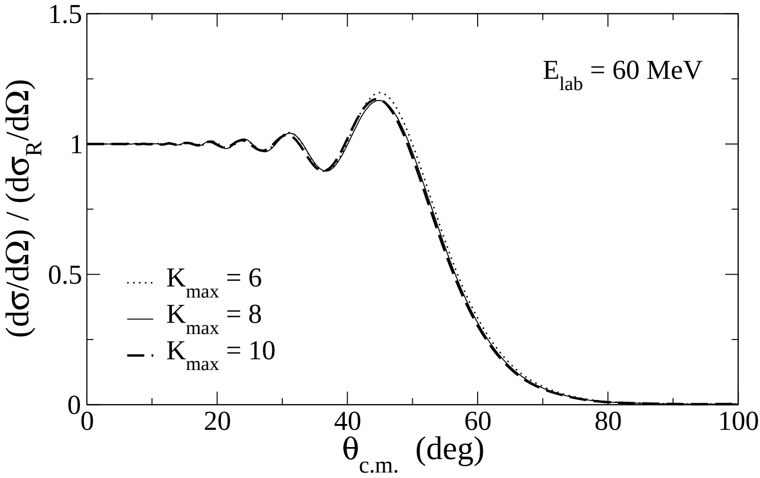
Photodissociation cross section: convergence in s-wave



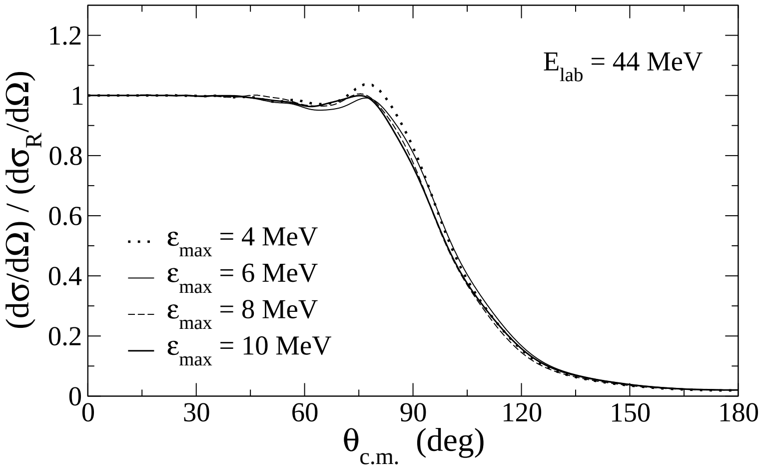
Radiative reaction rate: j^π contributions



$^{9}Be+^{208}Pb$@60MeV: convergence in K_{max}



^{9}Be+^{208}Pb@44MeV: convergence in ε_{max}



$^9Be+^{208}Pb$@44MeV: convergence in i_{max}