

Investigation of transfer to the bound states and resonance of ^{11}Be via the $^{10}\text{Be}(d,p)$ reaction using the ADWA method

A Spectroscopic Study of Halo Nucleus ^{11}Be

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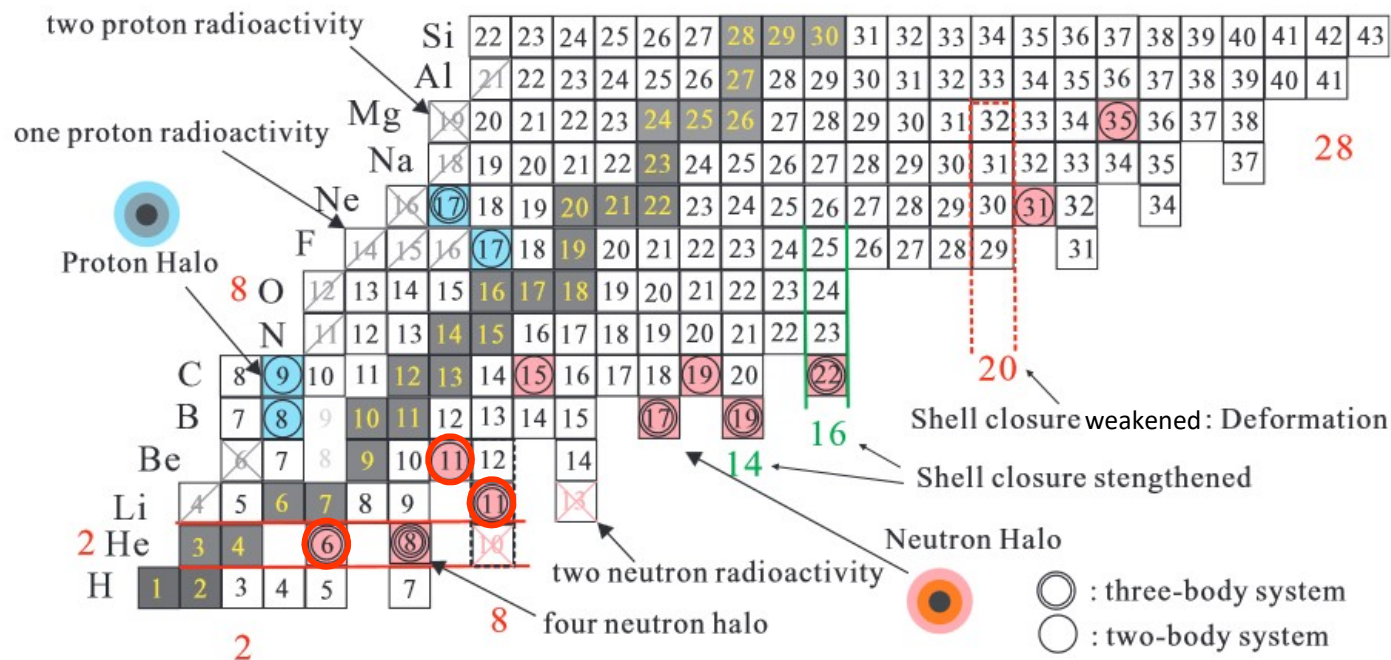
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Physics Motivation

• Halo nuclei

(Pictures are taken from WIKIMEDIA and other websites)



${}^6\text{He}$ — first halo produced

${}^{11}\text{Be}$ — one neutron halo

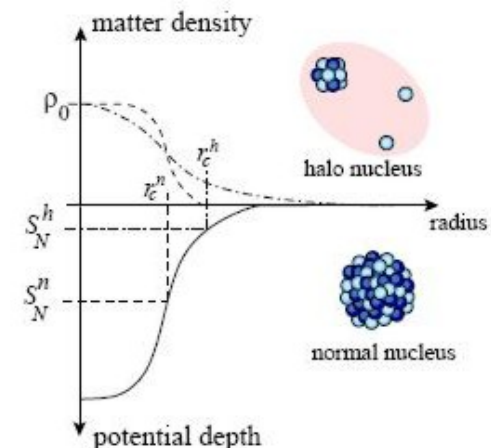
${}^{11}\text{Li}$ — famous Borromean nucleus

How to study halo nuclei ?

What is the property of this “halo” ?

→ **Transfer reaction**, elastic scattering, break up, ...

→ Spectroscopic factor, **ANC**, ...



Features:

- i) weakly bound
- ii) large size
- iii) short lifetime
- ...

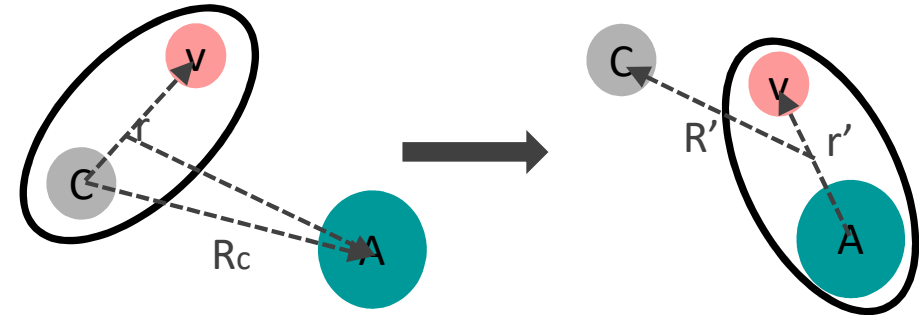
Outline

- **Theoretical framework of transfer reaction**
 - What is transfer reaction
 - Theoretical approximation
- **ADWA calculation of $^{10}\text{Be}(d,p)^{11}\text{Be}$**
 - Influence of the description of n- ^{10}Be bound state
 - ANC extraction from the peripheral part
- **Conclusion and prospects**

Theoretical Framework

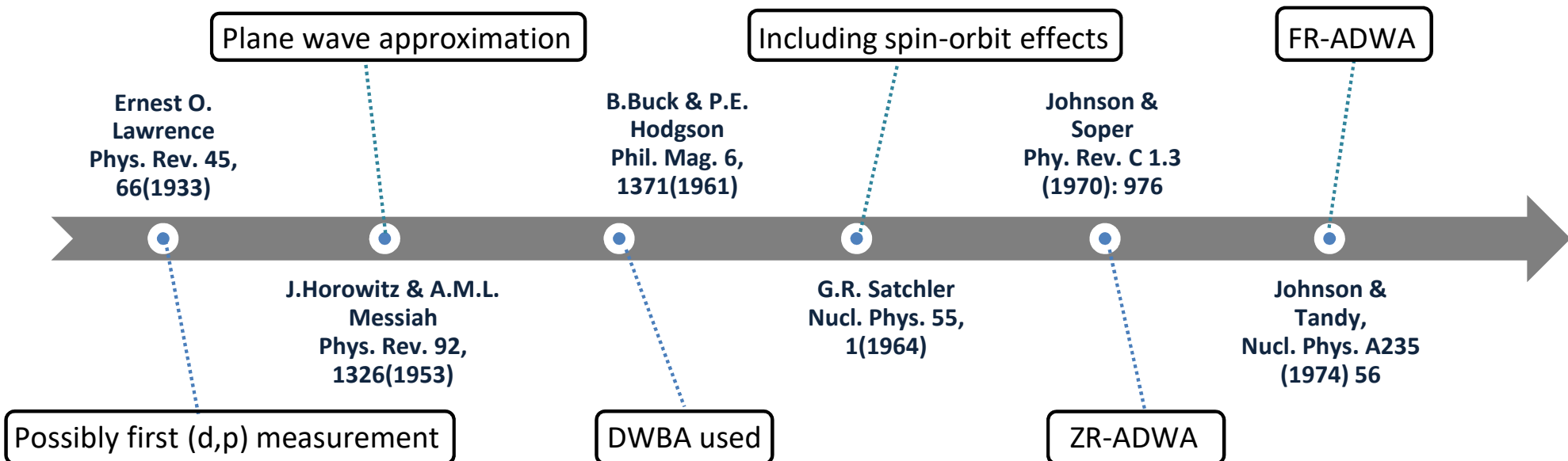
Transfer reaction

- Transfer up to several nucleons between the projectile and target
- A powerful tool to selectively populate states with a strong single-particle character
- (d,p) reaction



Some history of deuteron stripping reaction

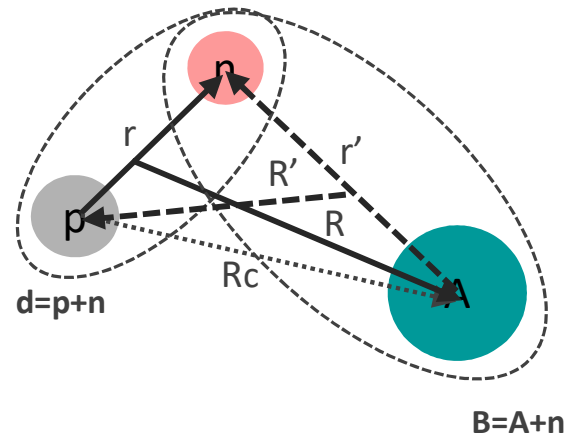
Ref: Pang's seminar, ect Trento, 2016



Theoretical Framework

(d,p) reaction

- Mathematical description



Transition amplitude (post form)

$$T(pB, dA) = \langle \Phi^{(-)}(R', r') | V_{post} | \Psi^{(+)}(R, r) \rangle$$

Interaction term

$$V_{post} = V_d(r) + U_{pA}(R_c) - U_{pB}(R')$$

Differential cross section

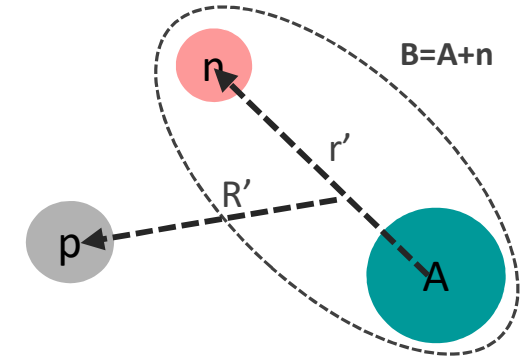
$$\frac{d\sigma}{d\Omega} = \frac{\mu_{dA}\mu_{pB}}{(2\pi\hbar^2)^2} \frac{k_p}{k_d} |T(pB, dA)|^2$$

Theoretical Framework

Final state $\Phi^{(-)}$

$$\Phi^{(-)} = \phi_B \chi_{pB}(R') \quad \text{with } \phi_B = [S_{nA}^B]^{1/2} \phi_A \varphi_{nA}(r') + \phi_B^C$$

$$\xrightarrow{\text{two-body approx}} [S_{nA}^B]^{1/2} \varphi_{nA}(r') \chi_{pB}(R')$$



- Bound-state wave function

$$\varphi_{nA}(r') \xrightarrow{r' \rightarrow \infty} b_{nlj} W_{-\eta, l + \frac{1}{2}}(2kr')/r' \xrightarrow{l=0} b_{nlj} \exp(-kr')/r', \text{ in which } b_{nlj} \text{ is the single-particle ANC (SPANC)}$$

- Overlap function

$$[S_{nA}^B]^{1/2} \varphi_{nA} = I_{nA}^B(r') \xrightarrow{r' \rightarrow \infty} C_{lj} W_{-\eta, l + \frac{1}{2}}(2kr')/r, \text{ in which } C_{lj} \text{ is the ANC}$$

In the single-particle approach, the spectroscopic factor $S \approx S_{nA}^B$, which is always obtained by $S \left(\frac{d\sigma}{d\Omega} \right)^{th} = \left(\frac{d\sigma}{d\Omega} \right)^{exp}$

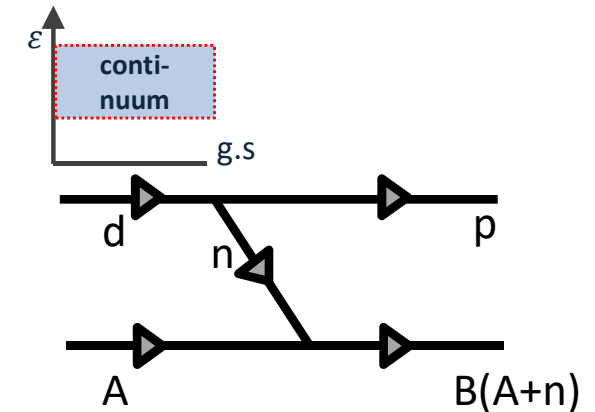
- Relationship between

$$C_{lj}^2 = S_{nA}^B b_{nlj}^2$$

Theoretical Framework

3-body solution $\Psi^{(+)}$

$$\Psi^{(+)}(r, R) = \phi_{pn}(r)\chi_{dA}^{(+)}(R) + \int dk \phi_k(\varepsilon_k, r)\chi_k^{(+)}(\varepsilon_k, R)$$



	DWBA	ADWA	CDCC
•	$\Psi^{(+)}(r, R) \approx \phi_{pn}(r)\chi_{dA}^{(+)}(R)$	Adiabatic approximation: Replacing all the continuum states by one state	discretize continuum into bin states $\Psi^{(+)}(r, R) = \sum_{i=0} \phi_{pn,i}(r)\chi_{dA,i}^{(+)}(R)$
	U_{dA} : Omit all except elastic part in the 3-body wave function	Effective d-A interaction (zero-range) $U_{dA} = U_{nA} + U_{pA}$ <small>Phys. Rev. C 1, 976 (1970).</small>	Three-body equation turned into Coupled-Channel equations

- Connection with the Faddeev formalism

Theoretical Framework

- Comparison --- ADWA versus Other methods

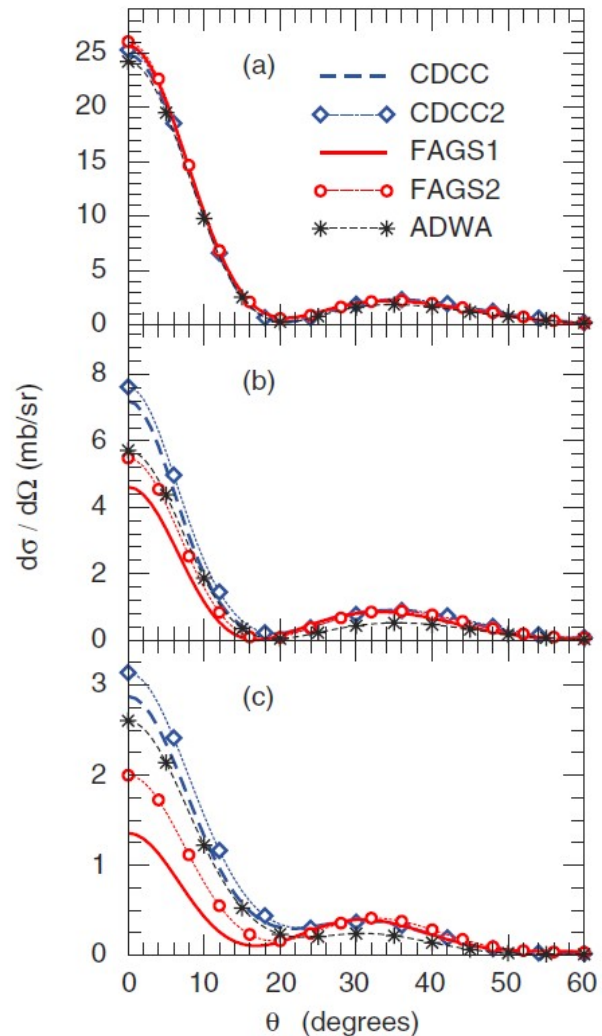


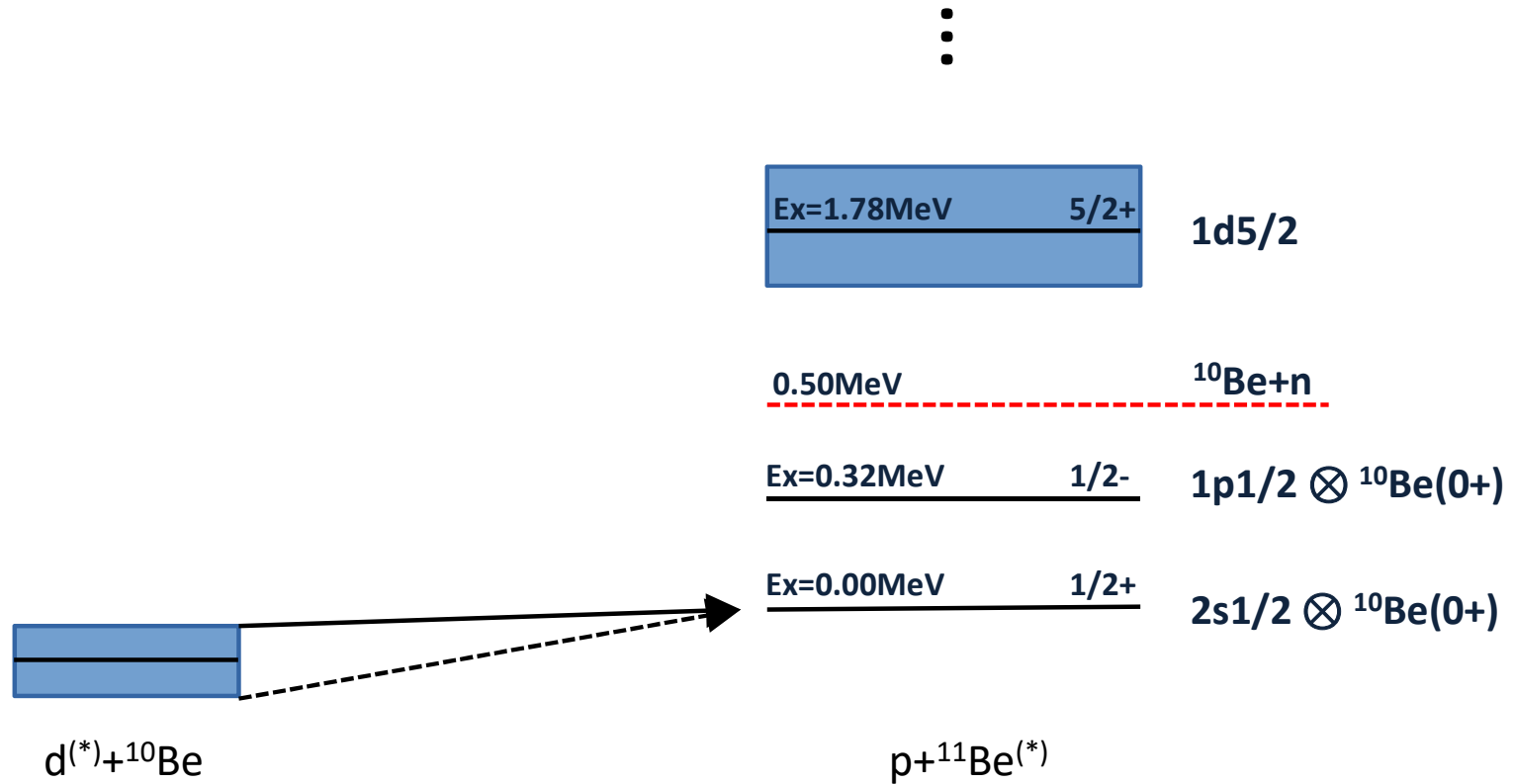
Fig. $^{10}\text{Be}(d,p)^{11}\text{Be}$ computed with Faddeev, CDCC and ADWA:
 (a) $E_d = 21.4$ MeV, (b) 40.9 MeV, and (c) 71 MeV

Ref: Upadhyay PRC 85, 054621 (2012)

- For this reaction at low energy, ADWA is in good agreement with the CDCC and the Faddeev-type results.
- For the reactions on ^{10}Be , ADWA performs just as well or even better than CDCC.

ADWA Calculation

- $^{10}\text{Be}(d,p)^{11}\text{Be}$



ADWA model

- **Fresco**: program developed by Ian Thompson to perform reaction calculations

Calculation of $^{10}\text{Be}(d,p)^{11}\text{Be}$

● $^{10}\text{Be}(d,p)^{11}\text{Be}$ (g.s) at different energies

Potential choice

n - ^{10}Be : Woods-Saxon form

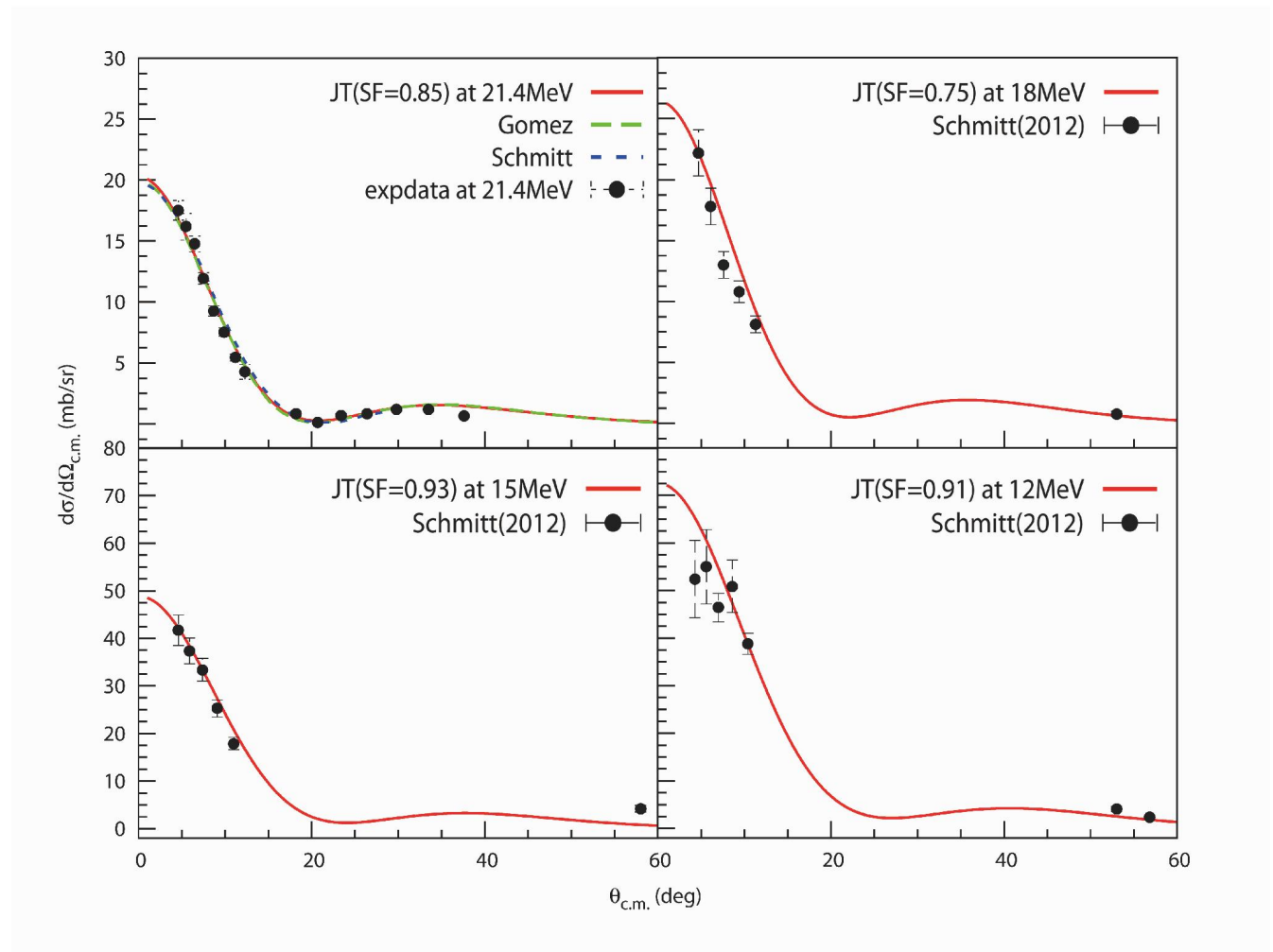
d - ^{10}Be : Johnson & Tandy (CH89)

n - p : Reid soft core

Rest : CH89

Ref: Schmitt et al, PRL 108,192701 (2012)
Gomez et al, PRC 92,014613 (2015)

- ◇ Successfully reproduce the calculation results and good agreement with experimental data obtained



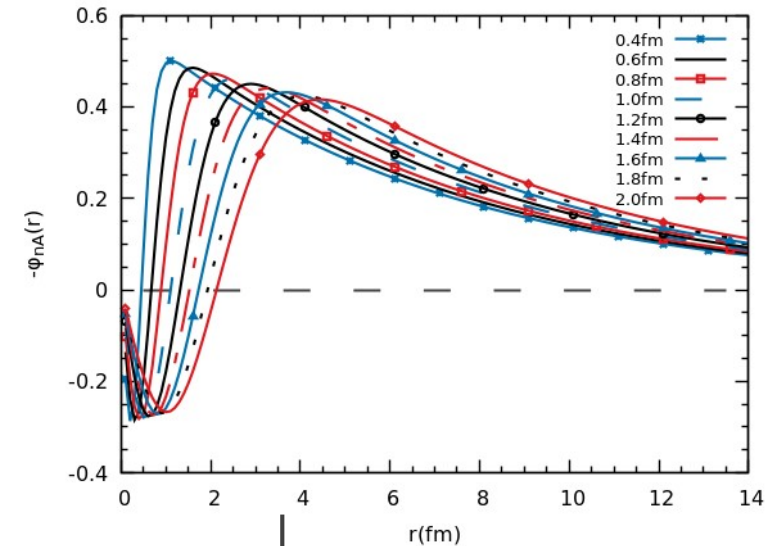
- ◇ What the reaction is sensitive to with respect to the description of the halo nucleus ^{11}Be ?

Calculation of $^{10}\text{Be}(d,p)^{11}\text{Be}$ (g.s)

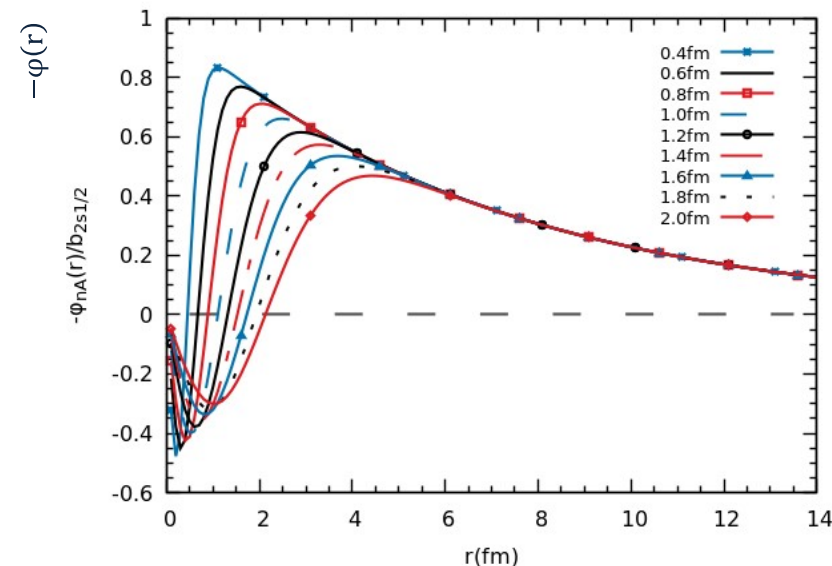
- **Description of $n\text{-}^{10}\text{Be}$: $2s_{1/2} \otimes ^{10}\text{Be}(0^+)$**
- Nine sets of Gaussian potentials developed to help study the peripheral characteristics of the reaction

$$V(r) = -V_0 \cdot \exp\left(\frac{-r^2}{2r_0^2}\right)$$

	r_0 (fm)	V_0 (MeV)	$b_{2s_{1/2}}$
V1	0.4	1314.6	0.601
V2	0.6	592.3	0.632
V3	0.8	337.8	0.664
V4	1.0	219.2	0.697
V5	1.2	154.4	0.732
V6	1.4	115.1	0.769
V7	1.6	89.3	0.807
V8	1.8	71.6	0.846
V9	2.0	58.8	0.888



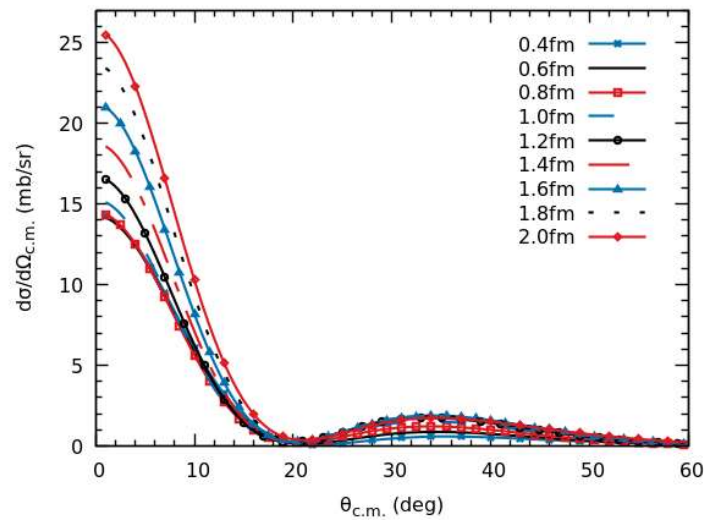
Scaling by b_{nlj}



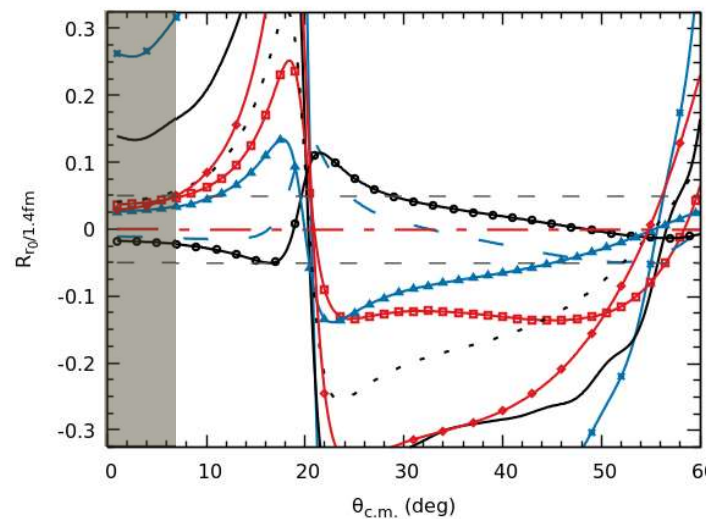
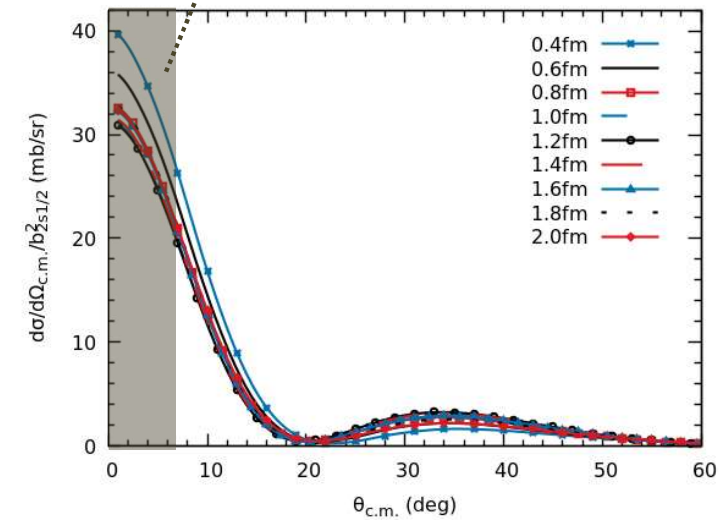
Calculation of $^{10}\text{Be}(d,p)^{11}\text{Be}$ (g.s)

- $E_d = 21.4\text{MeV}$

Peripheral part: 0-7deg except 0.4, 0.6fm



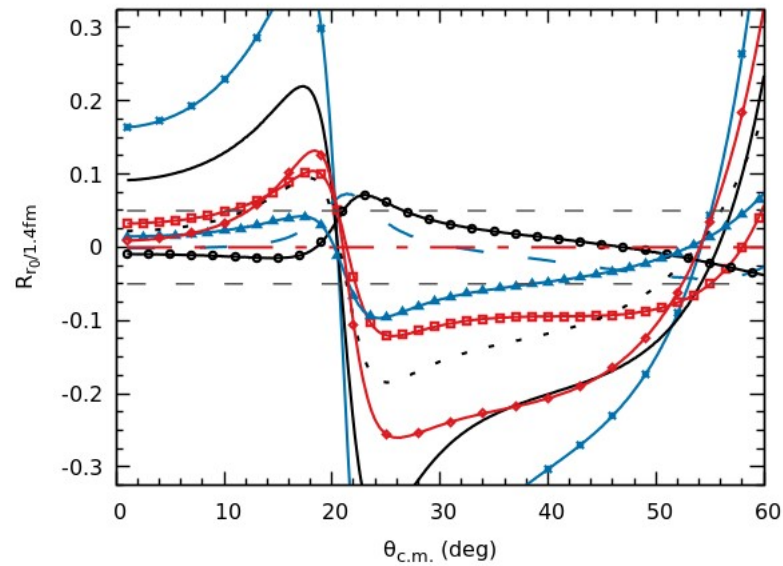
Scaling by b_{nlj}^2



Relative difference

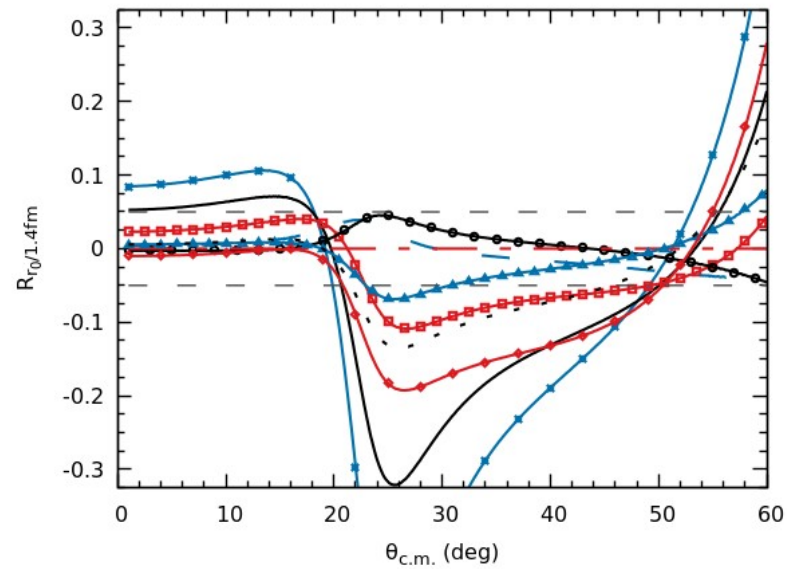
Calculation of $^{10}\text{Be}(d,p)^{11}\text{Be}$ (g.s)

- $E_d = 18\text{MeV}$



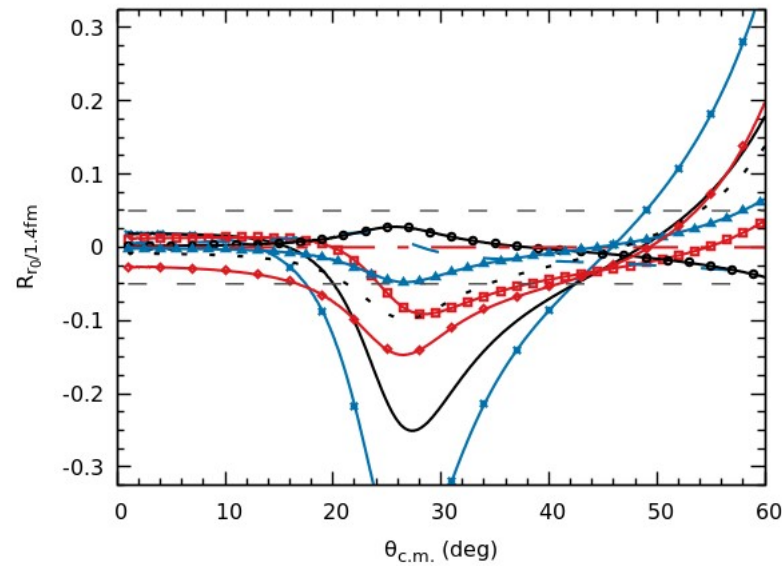
Calculation of $^{10}\text{Be}(d,p)^{11}\text{Be}$ (g.s)

- $E_d = 15\text{MeV}$



Calculation of $^{10}\text{Be}(d,p)^{11}\text{Be}$ (g.s)

- $E_d = 12\text{MeV}$



Lowering the energy, the reaction becomes more and more peripheral, mostly at forward angles.

ANC extraction from peripheral part

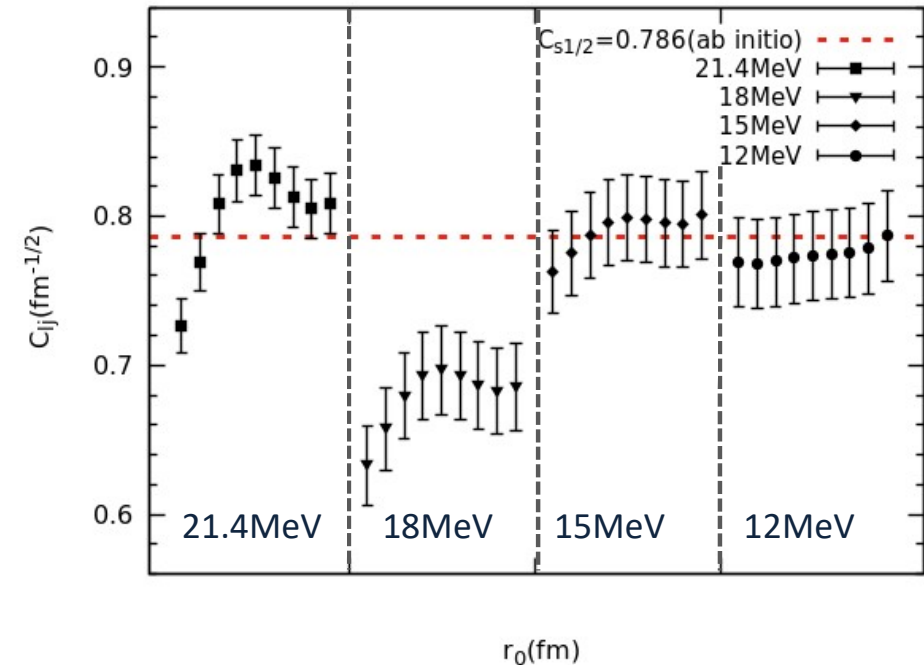
- Extract the ANCs with the experimental data at peripheral part

χ^2 analysis

$$\chi^2 = \sum_i \frac{\left(C_{lj}^2 \cdot \left(\frac{d\sigma}{d\Omega} \right)_i^{th} / b_{nlj}^2 - \left(\frac{d\sigma}{d\Omega} \right)_i^{exp} \right)^2}{\delta_i^2}$$

i represent all the data points in the peripheral region

C_{lj} is the ANC obtained by minimizing the χ^2



Conclusion

- The peripheral area of this transfer reaction is always found **at forward angles**;
- When **the incident energy decreases**, the scaling by b_{nlj}^2 works better which means the reaction exhibits a more pronounced peripheral property;
- The ANC obtained for the g.s of ^{11}Be ($C_{lj} = 0.785^{+0.029}_{-0.030} \text{fm}^{-1/2}$) shows perfect agreement with the one given by ab initio calculations ($0.786 \text{fm}^{-1/2}$).

Ref: PRL 117, 242501 (2016)

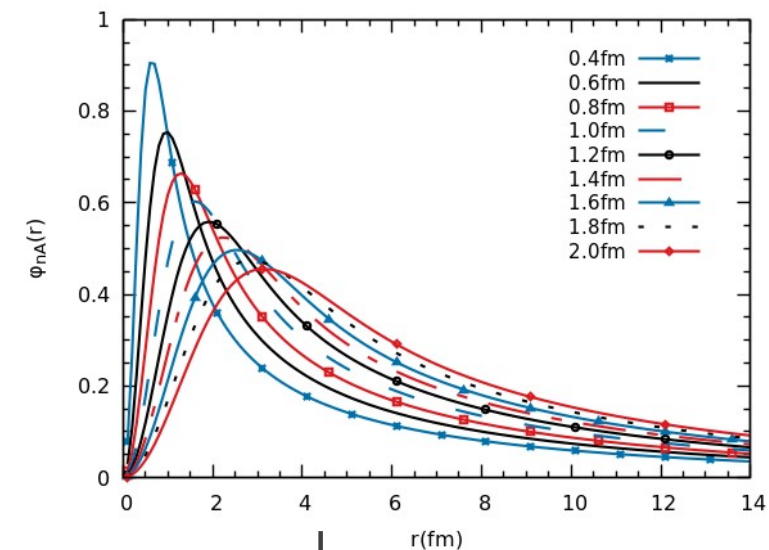
Calculation of $^{10}\text{Be}(d,p)^{11}\text{Be}$ (ex.s)

- Description of $n\text{-}^{10}\text{Be}$: $1p_{1/2} \otimes ^{10}\text{Be}(0^+)$

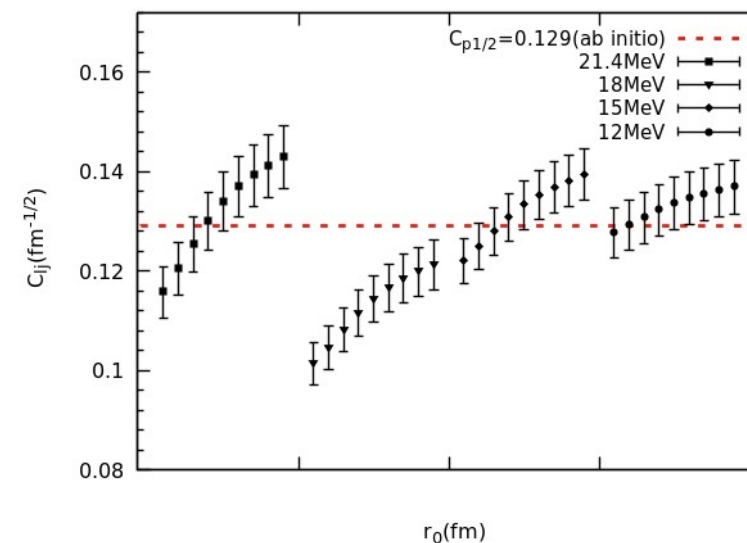
- Similar method used to study the excited state of ^{11}Be

	r_o (fm)	V_o (MeV)	$b_{1p_{1/2}}$
V1	0.4	869.4	0.068
V2	0.6	387.3	0.085
V3	0.8	218.4	0.100
V4	1.0	140.2	0.114
V5	1.2	97.7	0.127
V6	1.4	72.1	0.140
V7	1.6	55.4	0.152
V8	1.8	44.0	0.165
V9	2.0	35.8	0.177

- The ANC obtained for the ex.s of ^{11}Be is $0.136^{+0.005}_{-0.005}\text{fm}^{-1/2}$ while the ab initio method gives $0.129\text{fm}^{-1/2}$.



ANC extracted



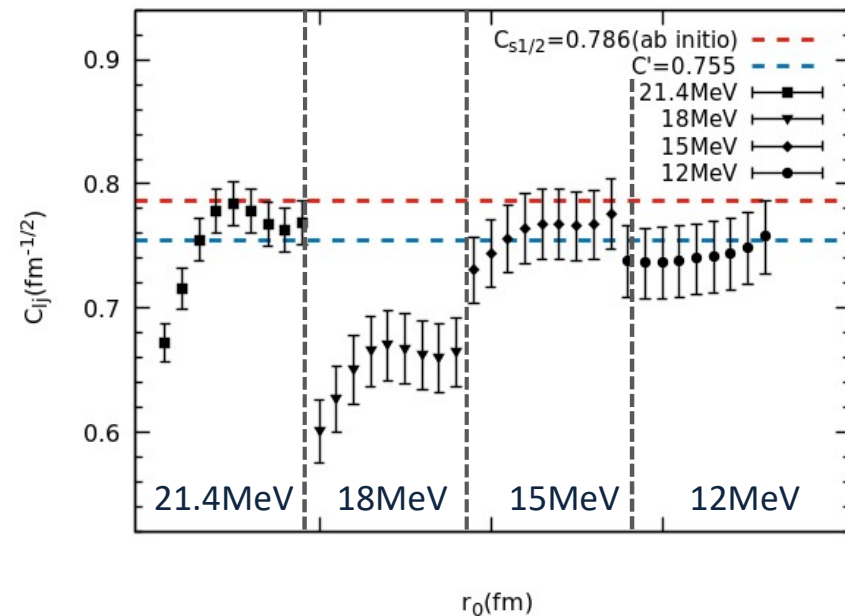
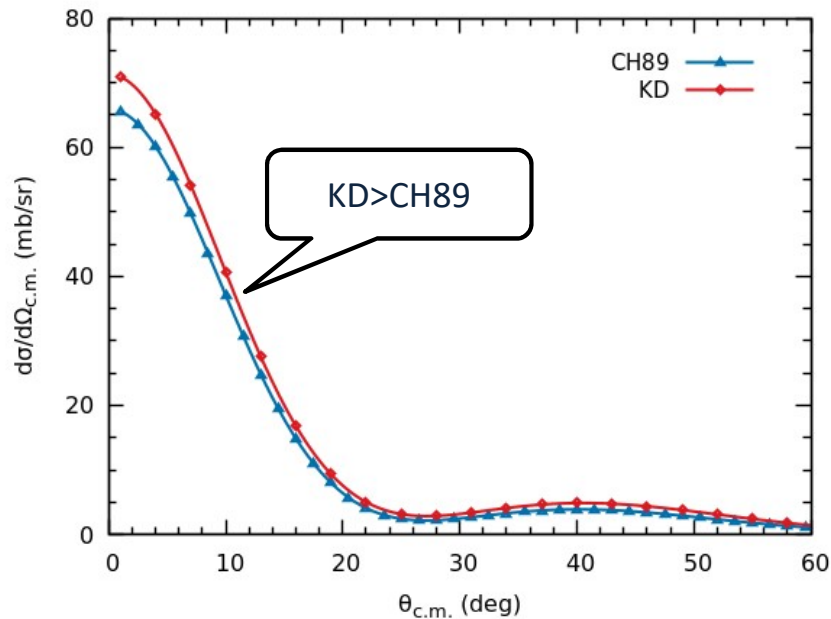
ANC extraction from peripheral part

- **Optical potentials for the entrance channel $d^{-10}\text{Be}$**

- a. Johnson & Tandy (CH89) pot

- b. Johnson & Tandy (KD) pot

A example at 12MeV



Conclusion

- ANC extraction is sensitive to the optical potential choice.

Transfer to the first resonance of ^{11}Be

- Description of $n\text{-}^{10}\text{Be}$: $1d_{5/2} \otimes ^{10}\text{Be}(0^+)$

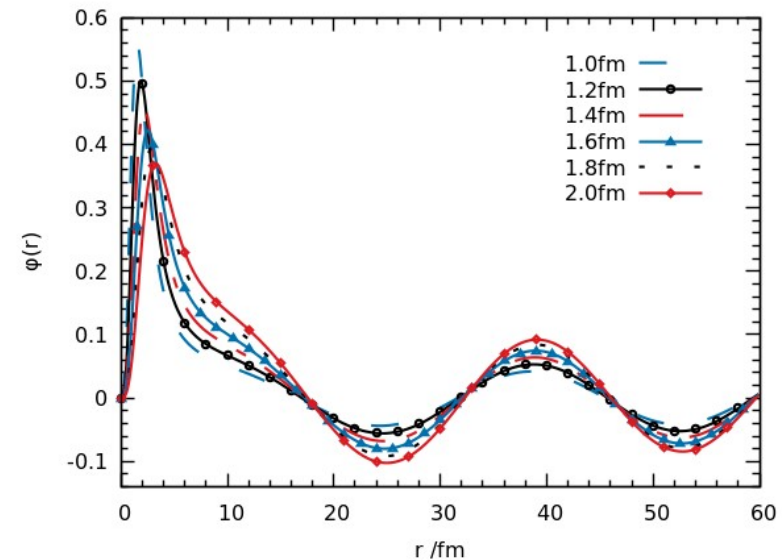
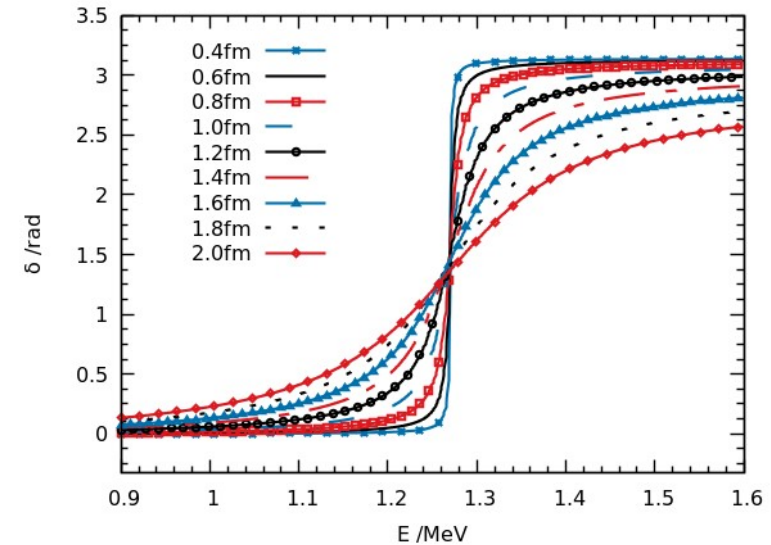
- Bin description for the overlap function

$$\phi(r) = \sqrt{\frac{2}{\pi N_p}} \int_{k_{p-1}}^{k_p} g_p(k) u_k(r) dk$$

- Relation with ANC (PRC 59.6 (1999): 3418)

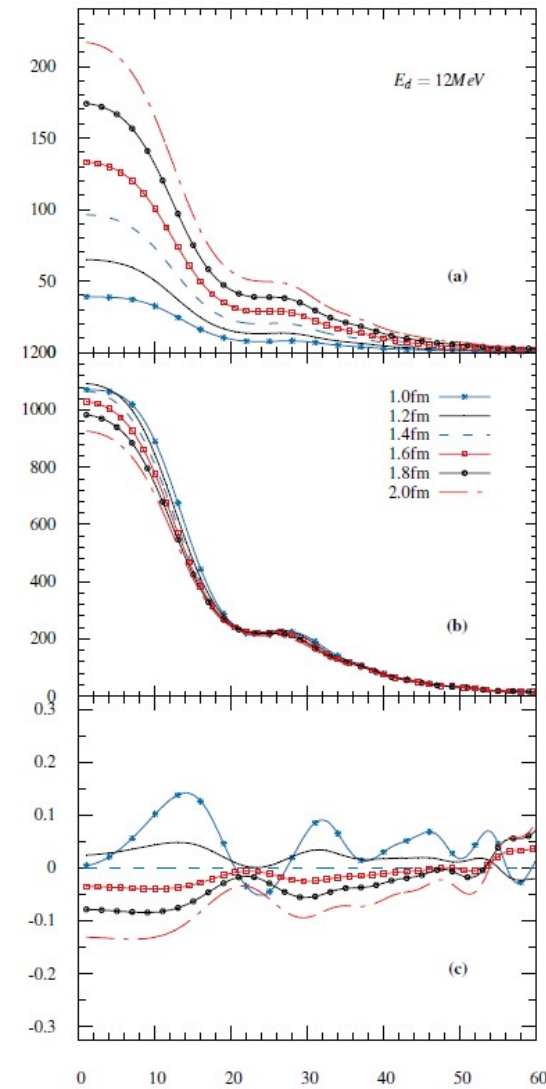
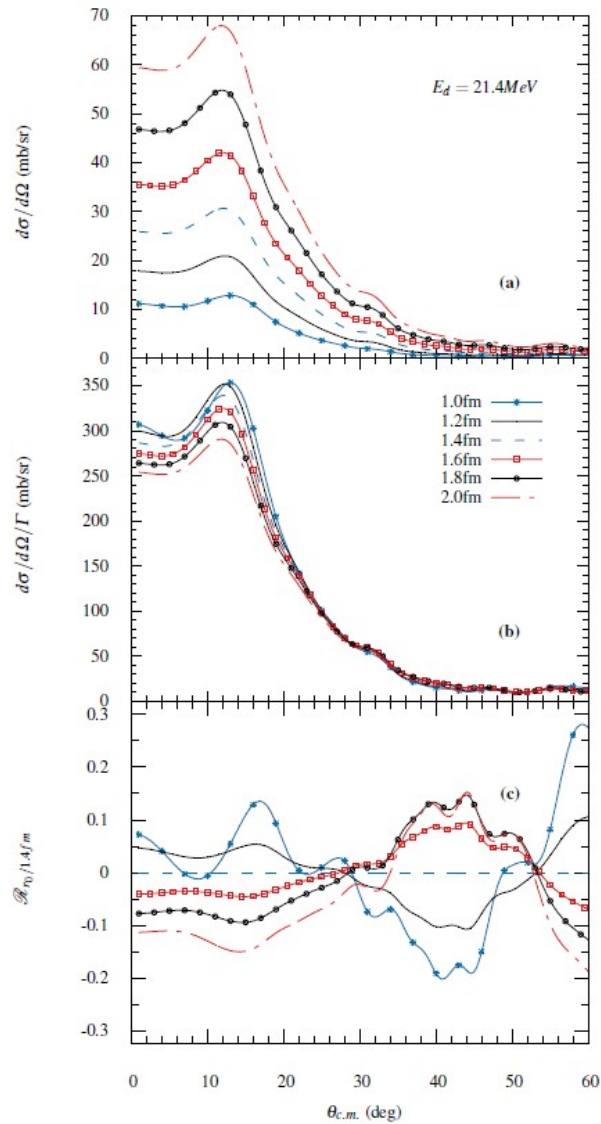
$$C_{lj}^2 \propto \Gamma$$

	r_o (fm)	V_o (MeV)	Γ (MeV)
V1	1.0	303.86	0.0364
V2	1.2	209.29	0.0595
V3	1.4	152.22	0.0904
V4	1.6	115.14	0.1294
V5	1.8	89.67	0.1771
V6	2.0	71.42	0.2340



Variation with different energies

- Transfer to the first resonance of ^{11}Be



Conclusion and prospects

Conclusion

- Brief review of the theoretical framework of transfer reaction
- ADWA calculation performed for $^{10}\text{Be}(d,p)^{11}\text{Be}$
- Spectroscopic study of this reaction
 - Influence of the description of n- ^{10}Be bound state
 - ANC extraction from the peripheral part
- Investigation at lower energies and forward angles for transfer reaction can ensure us the peripherality of the reaction and is the best way to obtain a reliable ANC from experimental data
 - When the incident energy decreases, the scaling by b_{nlj}^2 works better
 - The peripheral area of this transfer reaction is always found at forward angles
- The role of the γ width in the resonance can be analogous to the effect of the square of the ANC on bound states.

Thanks for your attention!