ON the FOUNDATION of the TROJAN HORSE and the SURROGATE METHODS

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Theories of Inclusive Non-Elastic Breakup and Incomplete Fusion of Weakly Bound Two-Cluster Projectiles

1) Trojan Horse Method: Specific Direct Reaction Induced by a Tertiary Beam!
2) Surrogate Reaction Method Relies on Specific Compound Nuclear Reaction Induced by Tertiary Beam

.Introduction

.Inclusive Breakup Cross Section

.The Austern Formula

.Recent calculation, of (d,p) reactions

.Recent extensions:

Inclusive Breakup of 3-Fragment Projectiles

.Inclusive Breakup of Borromean Nuclei

.Conclusions

The inclusive breakup of cluster type nuclei is an important source of information about the interacting sub-system. The spectrum of the observed fragment, b, in reactions of the type,

$$a + A \dashrightarrow b + (x + A),$$

furnishes the reaction cross section of the (x + A) subsystem, which is otherwise not available from a primary x + A reaction. This is the spirit of the Surrogate method, which is applied in the (d, p) reaction to get the (n + A) "capture" cross section.

The Inclusive Breakup Cross section

Use the spectator model where the observed fragment, b, suffers only elastic scattering and does not otherwise interact with the target. The spectrum of b was derived by several people and is

$$d^2\sigma/d\Omega_b dE_b = d^2\sigma^{(EB)}/d\Omega_b dE_b + d^2\sigma^{(NEBP)}/d\Omega_b dE_b$$

The first term is the elastic breakup (EB) given by,

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- $d^2\sigma^{(EB)}/d\Omega_b dE_b = (2\pi/hv_a)$
- $\Sigma_{(kx)}\ |<\chi^{(-)}{}_b\,\chi^{(-)}{}_x\ |V_{xb}|\Psi^{(+)}{}_{(3B)}>|^2\,\delta(E_a-E_x)$
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- while the second one is the non-elastic breakup (NEB) cross section. We shall concentrate our attention on the NEB cross section. For this purpose we turn now to briefly review what has become known as the Austern formula.

$$d^2\sigma^{(NEB)}/d\Omega_b dE_b =$$
$$(2/hv_a)\ \rho_b(E_b)$$
$$\int dr_x\ |(\chi^{(-)}{}_b|\Psi^{(+)}{}_{(3B)}>(r_x)|^2\ W_x(r_x)$$

where $\Psi^{(+)}{}_{(3B)}$ is the exact three-body, b + x + A, scattering wave function times the intrinsic wf of the projectile, $\chi^{(-)}{}_b$ is the distorted wave of b, and $E_a = E_b + E_x$,

- $[E_x - K_x - K_b - U_x - U_b - V_{xb}]\ \Psi^{(+)}{}_{(3B)} = 0$

$$[E_b - K_b - U^{\dagger}{}_b]\,\chi^{(-)}{}_b = 0,$$

$\rho_b(E_b)$ is the density of states of b

$W_x(r_x)$ is the imaginary part of the optical potential of x, U_x

Recent calculation of (d,p) reactions

The following papers used DWBA approximation for $\Psi^{(+)}_{(3B)}$ along the lines of IAV (Phys.Rev C. 32, 431(1985)), UT(Phys. Rev. C 24, 1348 (1981)), and HM (Nucl. Phys. A, 445, 124 (1985):

-G. Potel, F. M. Nunes, and I. J. Thompson, Phys. Rev. C 92, 034611 (2015). Used UT (Prior)

-J. Lei and A. M. Moro, Phys. Rev. C 92, 044616 (2015); J. Lei and A. M. Moro, C 92, 061602(R)(2015). Used IAV (Post) and compared with UT and HM

-B.V. Carlson, R. Capote, M. Sin, Few-Body Syst. 57, 307 (2016).

Used IAV (Post, zero range)

They calculated the spectrum of the protons in the (d, p) reaction and extracted the neutron "capture" cross section using the IAV expression(DWBA version of the Austern formula). These three DWBA-based theories are related:

$$(\chi^{(-)}{}_b|\Psi^{(+)}{}_{(IAV)}> = (\chi^{(-)}{}_b|\Psi^{(+)}{}_{(UT)}> + (\chi^{(-)}{}_b|\Psi^{(+}{}_{(HM)}>$$

where, $(\chi^{(-)}{}_b\ |\Psi^{(+)}{}_{(IAV)}> = G_x(\chi^{(-)}{}_b\ |V_{xb}|\chi^{(+)}{}_a\phi_a >,$

, $(\chi^{(-)}{}_b\ |\Psi^{(+)}{}_{(UT)}> = G_x \cdot (\chi^{(-)}{}_b|[U_x + U_b - U_a]|\chi^{(+)}{}_a\phi_a >,$

$$(\chi^{(-)}_b \,|\Psi^{(+)}_{(HM)}> = (\chi^{(-)}_b \,|\chi^{(+)}_a \phi_a >$$

G_x is the optical Green's function of the interacting fragment x,

$$G_x = [E_x - K_x - U_x + i\varepsilon]^{-1}$$

We must mention that the extracted "capture" cross section,

$$\sigma_{(xA)} = (k_x/E_x) \int dr_x \, |(\chi^{(-)}_b|\Psi^{(+)}_{(IAV)}>(r_x)|^2 \, W_x(r_x)$$

is the total reaction cross section of x.

- This cross section can be decomposed into a Direct D, piece plus a compound nucleus, CN piece. Thus,
- $\sigma_{(xA)} = (k_x/E_x)\int dr_x \,|(\chi^{(-)}_b|\Psi^{(+)}_{(IAV)}>(r_x)^2\, W_x(r_x)$
- With
- $W_x(r_x) = W^D{}_x(r_x) + W^{CN}{}_x(r_x)$
- **and accordingly**
- $\sigma_{(xA)} = \sigma^D{}_{(xA)} + \sigma^{CN}{}_{(xA)}$

- We have shown in Phys. Lett. B776 (2018) 217
- (Bertulani, Hussein, Typel) that the direct piece in the DWBA framework is a one-step reaction and coincides with the Trojan Horse cross section when the a + A $\rightarrow$ y + B reaction is considered.
- We have argued in EPJA 53 (2017)53 (Hussein)
- That the CN cross section is the SM one, a genuine two-step nuclear reaction.

A brief history

Optical Background Respresentation

Modification of Hauser-Feshbach calculations by direct-reaction channel coupling

Annals of Physics, Volume 75, Issue 1, January 1973, Pages 156-170

Mitsuji Kawai, A.K Kerman, K.W McVoy·

Fluctuations in two-step reactions through doorways

Annals of Physics, Volume 122, Issue 1, 15 September 1979, Pages 197-216

A.K Kerman, K.W McVoy

Baur et al., DWBA, Post

- **The break-up of the deuteron and stripping to unbound states**
- *Physics Reports, Volume 25, Issue 4, June 1976, Pages 293-358*
- G. Baur, D. Trautmann
-
- **Fragmentation processes in nuclear reactions**
- *Physics Reports, Volume 111, Issue 5, September 1984, Pages 333-371*
- G. Baur, F. Rösel, D. Trautmann, R. Shyam

Udagawa& Tamura (UT), DWBA, Prior

- Derivation of breakup-fusion cross sections from the optical theorem
- *T. Udagawa and T. Tamura*
- *Phys. Rev. C 24, 1348 (1981) - Published 1 September 1981*
- Formulation of elastic and inelastic breakup-fusion reactions
- *T. Udagawa and T. Tamura*
- *Phys. Rev. C 33, 494 (1986) - Published 1 February 1986*
- Exact and approximate sum rules for inclusive breakup reactions
- *T. Udagawa, T. Tamura, and R. C. Mastroleo*
- *Phys. Rev. C 37, 2261 (1988) - Published 1 May 1988*

Ichimura, Austern, Vincent (IAV), DWBA, Post

- Equivalence of post and prior sum rules for inclusive breakup reactions
- ***M. Ichimura, N. Austern, and C. M. Vincent***
- ***Phys. Rev. C 32, 431 (1985)***
- Comparison of approximate formalisms for inclusive breakup reactions
- ***M. Ichimura, N. Austern, and C. M. Vincent***
- ***Phys. Rev. C 34, 2326 (1986)***
- *Approximate sum rules for inclusive breakup reactions*
- ***M. Ichimura, N. Austern, and C. M. Vincent***
- ***Phys. Rev. C 37, 2264 (1988)***
-

Hussein&McVoy (HM), DWBA, Post

- **Inclusive projectile fragmentation in the spectator model**
- *Nuclear Physics A, 445, (1985), 124-139*
- M.S. Hussein, K.W. McVoy
- **Glauber calculation of heavy-ion and light-ion inclusive break-up cross sections**
- *Nuclear Physics A, Volume 491, (1989), 468-476*
- M.S. Hussein, R.C. Mastroléo
- **Inclusive annihilation of antiprotons on deuterium**
- T. Frederico, B. V. Carlson, R. C. Mastroleo, Lauro Tomio, and M. S. Hussein
- Phys. Rev. C 42, 138 (1990)

CDCC, Austern et al.

- **Continuum-discretized coupled-channels calculations for three-body models of deuteron-nucleus reactions**

  Physics Reports, 154, (1987), *125-204*
- N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. Rawitscher, M. Yahiro

Comparison, IAV, UT and HM

- **Faddeev and DWBA description of inclusive break-up and incomplete fusion reactions**
- *Nuclear Physics A, Volume 511, (1990), 269-278*
- M.S. Hussein, T. Frederico, R.C. Mastroleo
- **Relation among theories of inclusive breakup reactions**
- Munetake Ichimura
- Phys. Rev. C 41, 834 (1990)

Within DWBA,

$$(\chi^{(-)}_{b}|\Psi^{(+)}_{(IAV)}> = (\chi^{(-)}_{b}|\Psi^{(+)}_{(UT)}> + (\chi^{(-)}_{b}|\Psi^{(+}_{(HM)}>$$

Inclusive Breakup of Three-Fragment Projectiles

- For a three-fragments, clustered projectiles, the inclusive breakup cross section has been recently derived. Calling the projectile
- $a = b + x_1 + x_2$, the spectrum of b
- in the reaction $a + A \rightarrow b + (x_1 + x_2 + A)$ is :

These fragments interacts with each other through $V_{x1\ x2}$ as they scatter off the target through the optical potentials

U_{x1} and U_{x2}

$$[E_{(x1\ +\ x2)} - K_{x1} - K_{x2} - U_{x1} - U_{x2} - V_{x1x2}]\ \psi^{(+)}{}_{(x1+x2)} = 0.$$

•

$$\mathbf{d^2\sigma/d\Omega_b dE_b = d^2\sigma^{(EB,4B)}/d\Omega_b dE_b +}$$

$$\mathbf{+\ d^2\sigma^{(NEB,4B)}/d\Omega_b dE_b}$$

The 4B elastic breakup cross section,

- $d^2\sigma^{(EB,4B)}/d\Omega_b dE_b = (2\pi/hv_a)$
- $\Sigma_{(kx1,\ kx2)}\ |<\chi^{(-)}{}_b\ \psi_X{}^{(-)}\ |$
- $[V_{x1,b} + V_{x2,b}\ ]\,|\Psi^{(+)}{}_{(4B)}>|^2$
- $\delta(E_a - E_{x1} - E_{x2})$
- where $\psi^{(-)}{}_X \equiv \psi^{(-)}{}_{(x1+x2)}$ is the 3B scattering wf of the unobserved fragments.

We can use the following approximation :

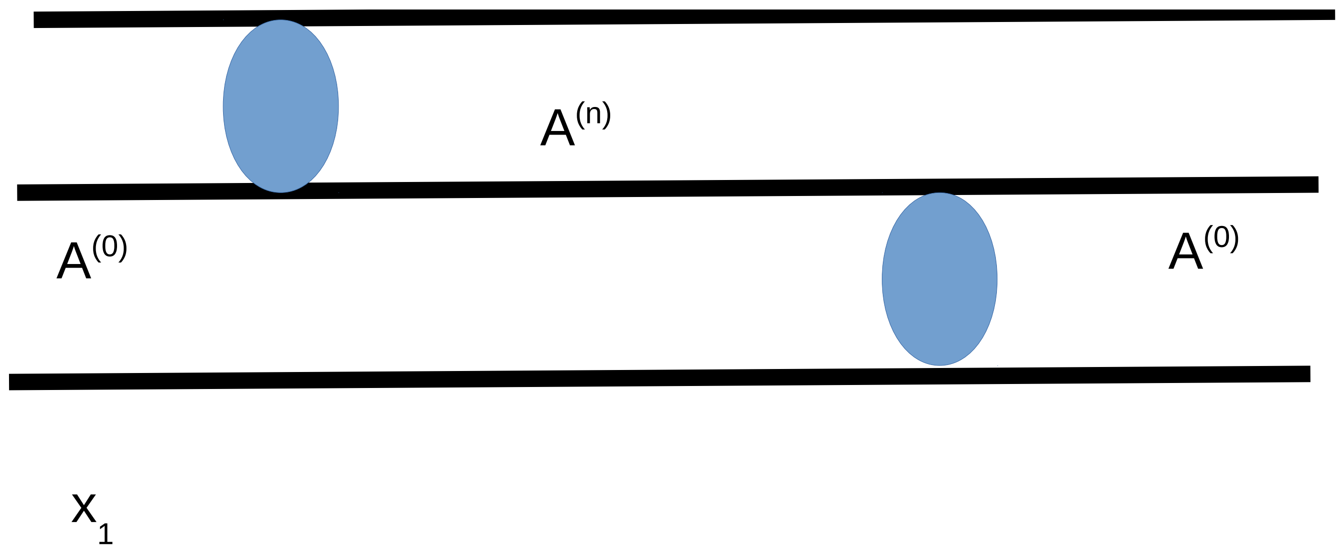
- **Describe the center of mass of the two fragments with a distorted wave, $\chi_X^{(-)}$,while the relative motion is described by ϕ_x ;**
- **$\psi^{(-)}{}_{(x1+x2)} \approx \phi_x \chi_X^{(-)}$. The function ϕ_x contains the two fragments t-matrix, $t_{x1x2}(e_{x1x2})$.**

The NEBP is derived to be

$$d^2\sigma^{(NEB,4B)}/d\Omega_b dE_b = (2/hv_a)\, \rho_b(E_b)$$
$$\int dr_{x1}\, dr_{x2} |(\chi^{(-)}{}_b|<\Psi^{(+)}{}_{(4B)}>|^2 \, .$$
$$.[W_{x1}(r_{x1}) + W_{x2}(r_{x2}) + W_{3B}(r_{x1}, r_{x2})]$$

- The new conspicuous feature is the three-body absorptive potential, $W_{3B}(r_{x1}, r_{x2})$. It accounts for flux loss in the $x_1 + x_2 + A$ subsystem

due to excitation of the target by one fragment, say, x_1, and its subsequent de-excitation by the other fragment x_2 .



$A^{(n)}$
$A^{(0)}$
$A^{(0)}$
x_1

The calculation of the total "reaction cross section" of the three-body system

$x_1 + x_2 + A$, namely,

$\sigma(R, (x_1 + x_2)A) = F(k_{x1}, k_{x2}) \int dr_{x1}\ dr_{x2}$
$\cdot |(\chi^{(-)}{}_b | < \Psi^{(+)}{}_{(4B)} > |^2\ [W_{x1}(r_{x1}) +$
$W_{x2}(r_{x2}) + W_{3B}(r_{x1}, r_{x2})].$

Can it be done (the first two terms) by integrating the three- body NEB cross section over energy and solid angle?

This amounts to getting the yield of x_2 and that of x_1 . To be investigated!

The three-body absorption term related to $W_{3B}(r_{x1},r_{x2})$ needs a theory!

Inclusive Breakup of Borromean Nuclei

- The cases of ^{6}He, ^{11}Li, ^{14}Be, ^{22}C are of interest, as the three-body system has the interesting property of not having any of its two-body subsystems bound on their own. Accordingly two-body correlations are very important.

These correlations are clearly felt in the:

1) EB cross section through the two neutron scattering wave function, $\psi^{(-)}_{(2n)}$, and through

2) the three-body absorption in the NEB cross section, represented by W_{3B}, where 3B stands here for the n + n + A system.

Conclusions

- The inclusive breakup cross section contains invaluable information about the reaction cross section of the two-body subsystem in the case of two-fragments clustered projectile.
- The Surrogate method as applied to (d, p) reactions uses the theory to obtain the reaction cross section of the n+A subsystem.

The new theory of 4B inclusive breakup in the case of three-fragments clustered projectiles

- will allow the extension of the Surrogate method, to reactions of the type (t, p),
- and the application to the breakup of two-neutron halo, Borromean, Nuclei.

Thank You!