



LUND UNIVERSITY



Optical potentials and knockout
reactions from chiral interactions

Andrea Idini

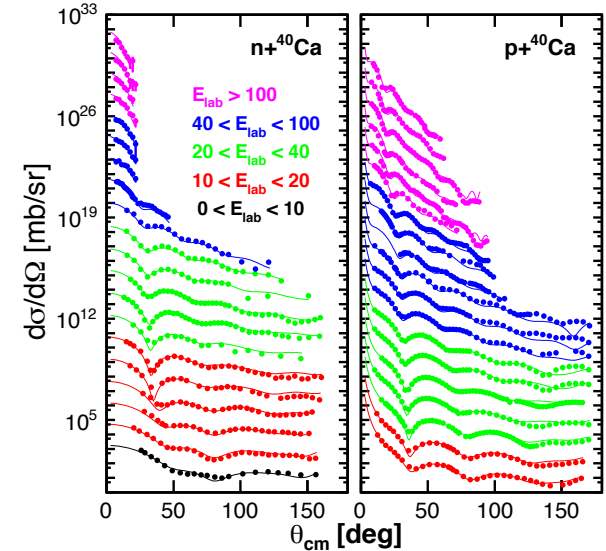
**“Recent advances and challenges in the description
of nuclear reactions at the limit of stability”**

Andrea Idini

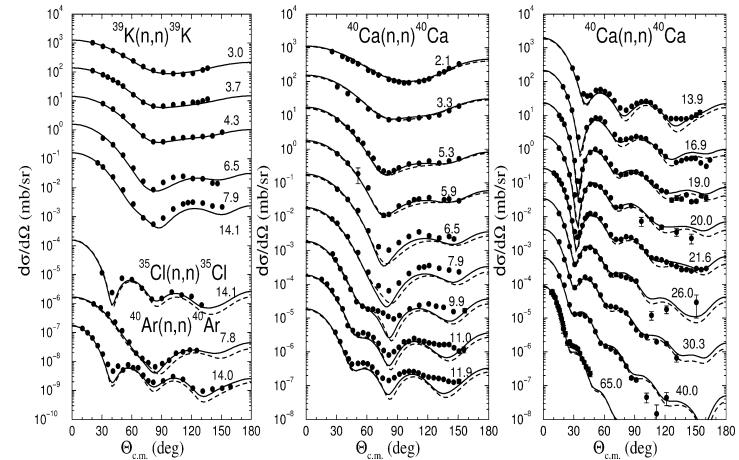
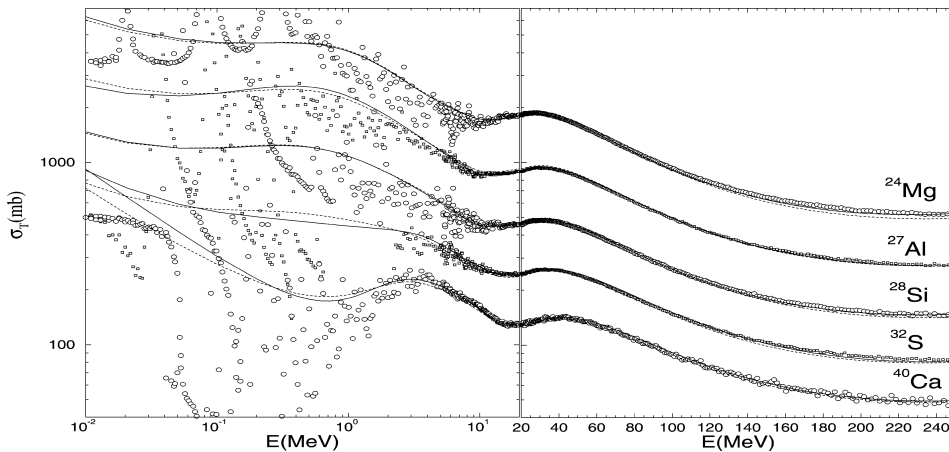
ECT*, 5-9 March 2018

Why optical potentials?

- Optical potentials **reduce many-body complexity** decoupling structure contribution and reactions dynamics.
- Often fitted on elastic scattering data (locally or globally)
- A microscopic model is difficult but worth it



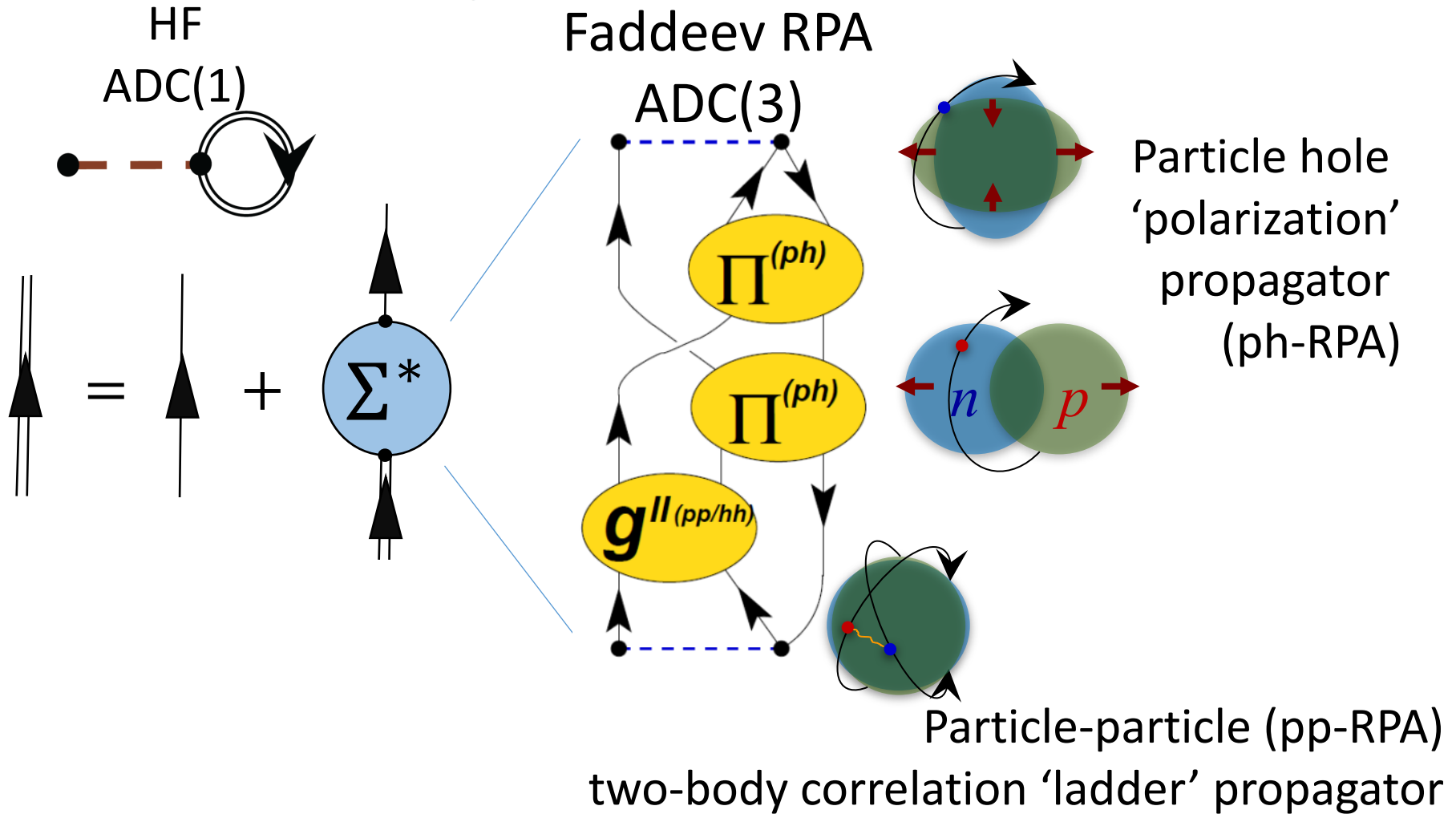
Dickhoff, Charity, Mahzoon, JPG44, 033001 (2017)



Koning, Delaroche, NPA713, 231 (2002)

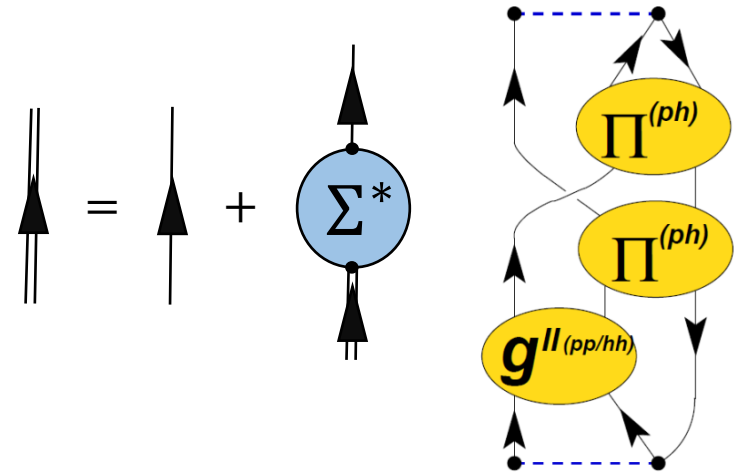
Dyson Equation

$$g_{\alpha\beta}(\omega) = g_{\alpha\beta}^0(\omega) + \sum_{\gamma\delta} g_{\alpha\gamma}^0(\omega) \Sigma_{\gamma\delta}^*(\omega) g_{\delta\beta}(\omega)$$



Källén–Lehmann spectral representation

$$H(A) = T - T_{c.m.}(A + 1) + V + W$$

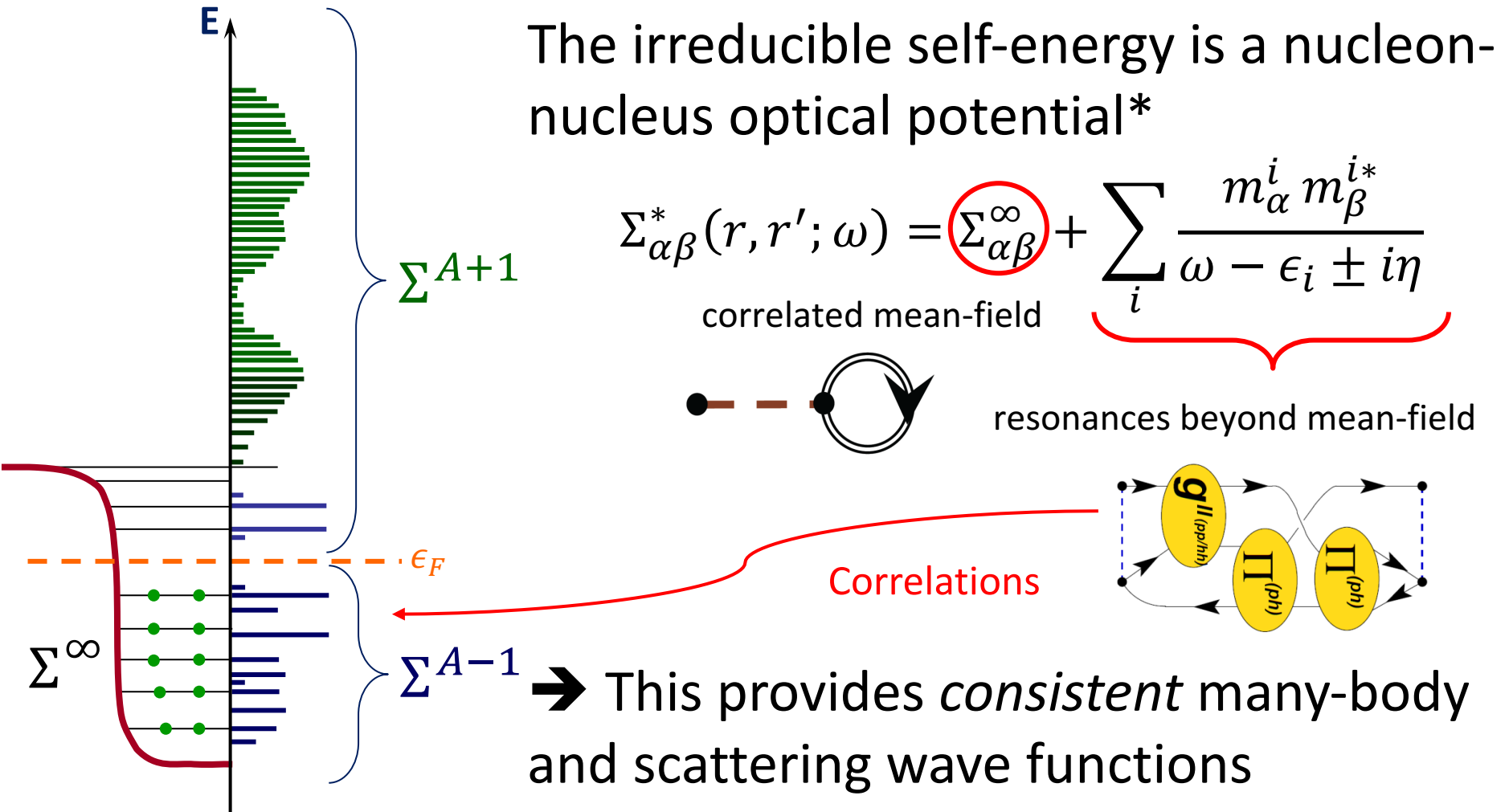


$$g_{\alpha,\beta}(E, \Gamma) = \sum_n \frac{\langle \Psi_0^A | c_\alpha | \Psi_n^{A+1} \rangle \langle \Psi_n^{A+1} | c_\beta^\dagger | \Psi_0^A \rangle}{E - E_n^{A+1} + E_0^A + i\Gamma} + \sum_i \frac{\langle \psi_0^A | c_\alpha^\dagger | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | c_\beta | \Psi_0^A \rangle}{E - E_0^A + E_i^{A-1} - i\Gamma},$$

Overlaps of
A+1 and A-1 states

Excited states calculated from Dyson equation

Nucleon elastic scattering



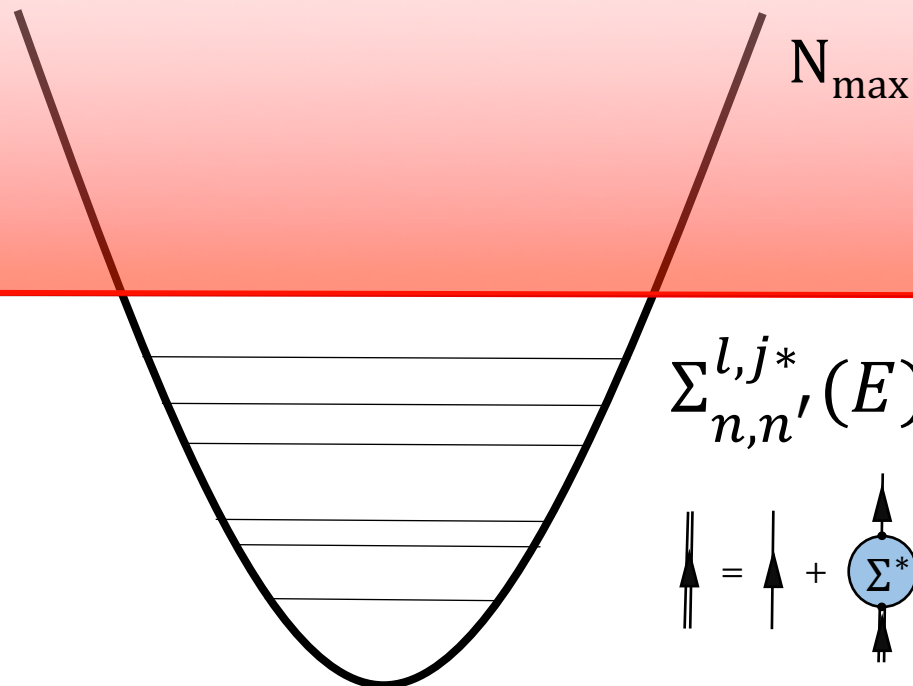
*Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991), Escher & Jennings PRC66:034313 (2002)

- Solve Dyson equation in HO Space, find $\Sigma_{n,n'}^{l,j*}(E)$



- diagonalize in full continuum momentum space $\Sigma^{l,j*}(k, k', E)$

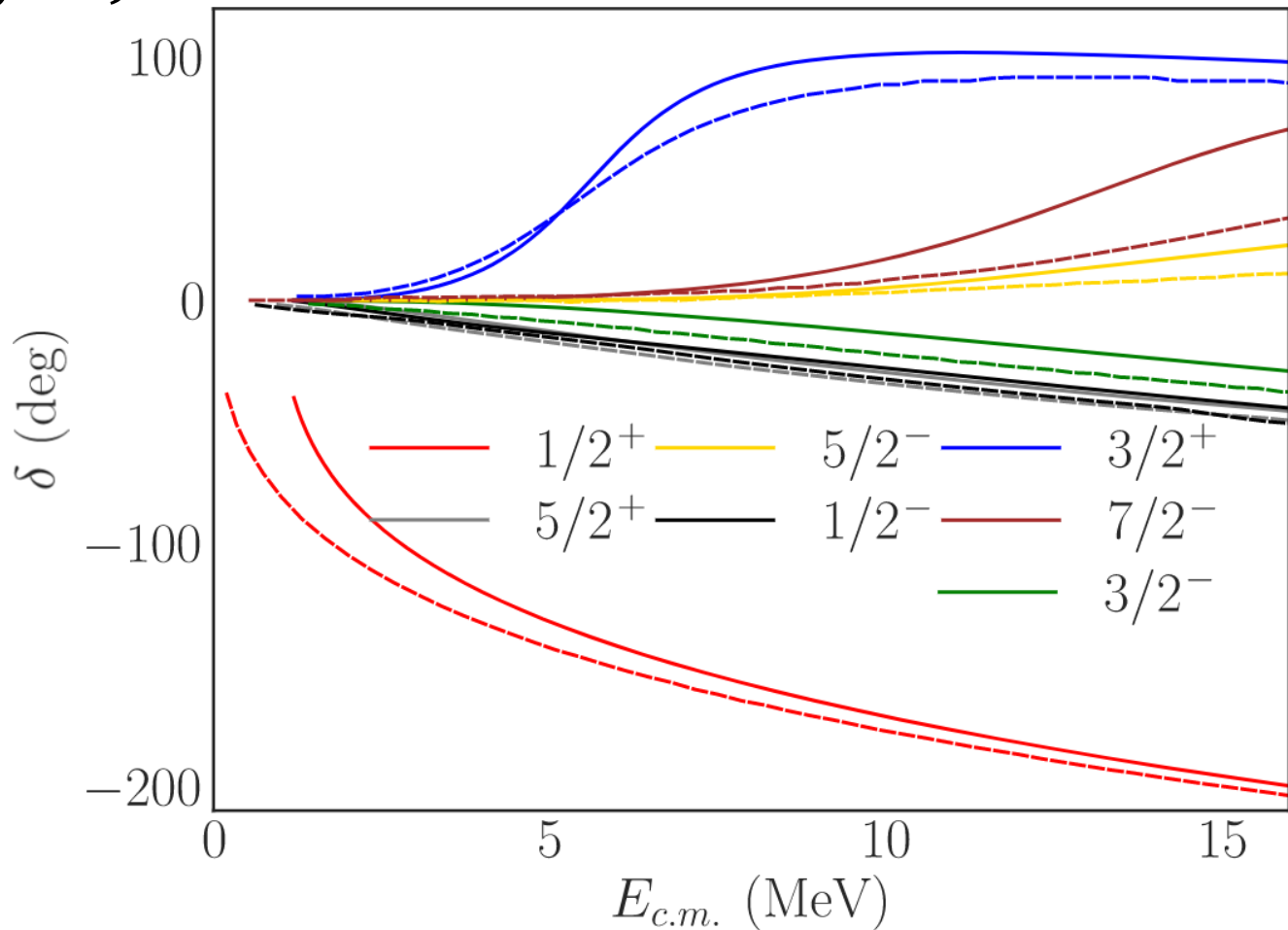
$$\frac{k^2}{2m} \psi_{l,j}(k) + \int dk' k'^2 \left(\Sigma^{l,j*}(k, k', E) \right) \psi_{l,j}(k') = E \psi_{l,j}(k)$$



RESULTS

SRG-N³LO, $\Lambda = 2.66 \text{ fm}^{-1}$

$n + {}^{16}\text{O} (g.s.)$

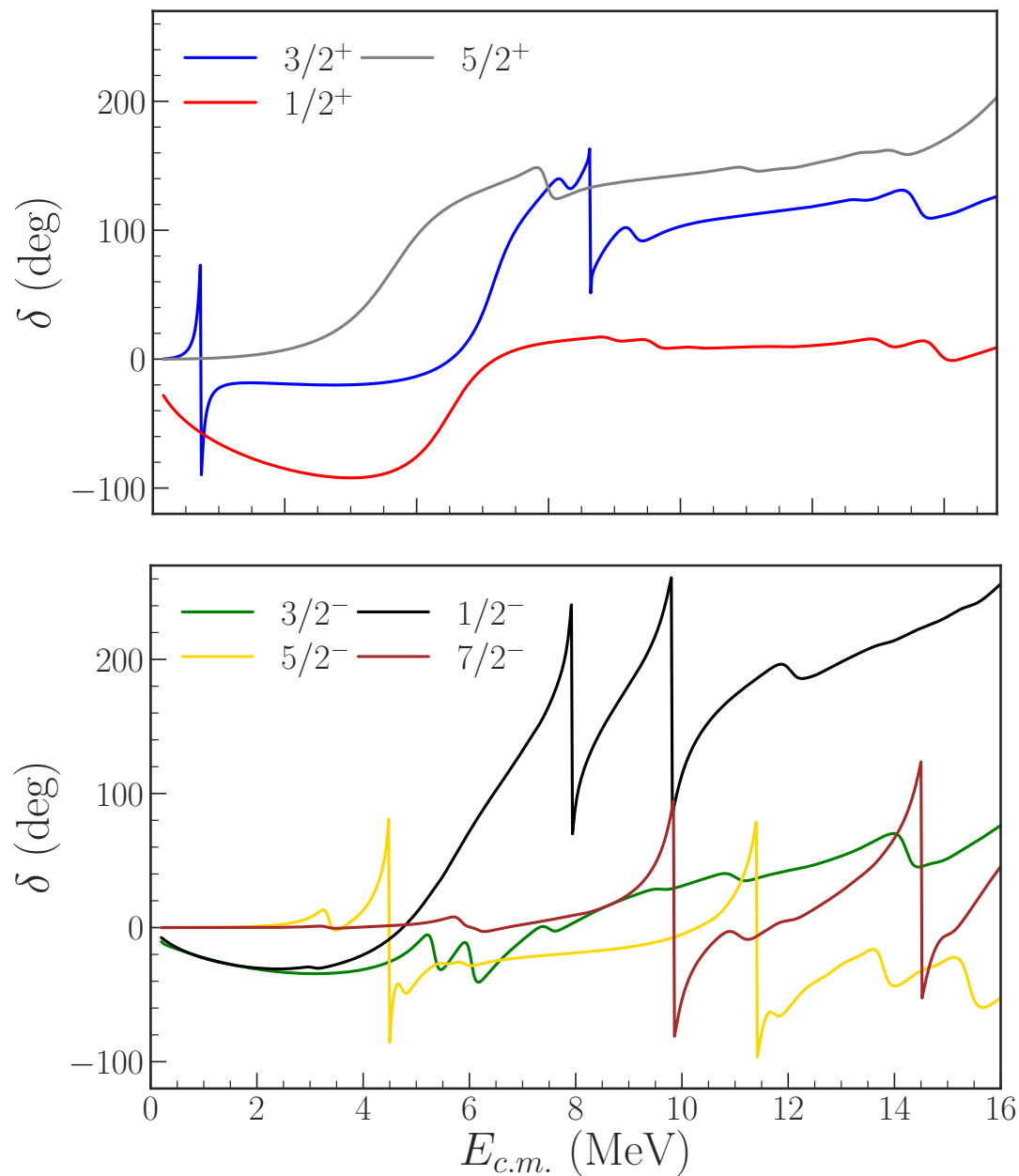
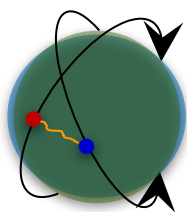
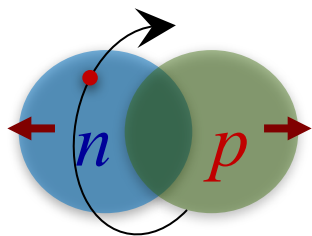
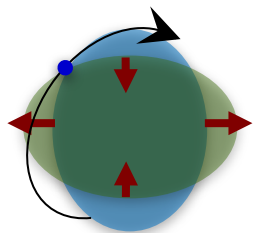


--- Navr̀atil, Roth, Quaglioni,
PRC82, 034609 (2010)

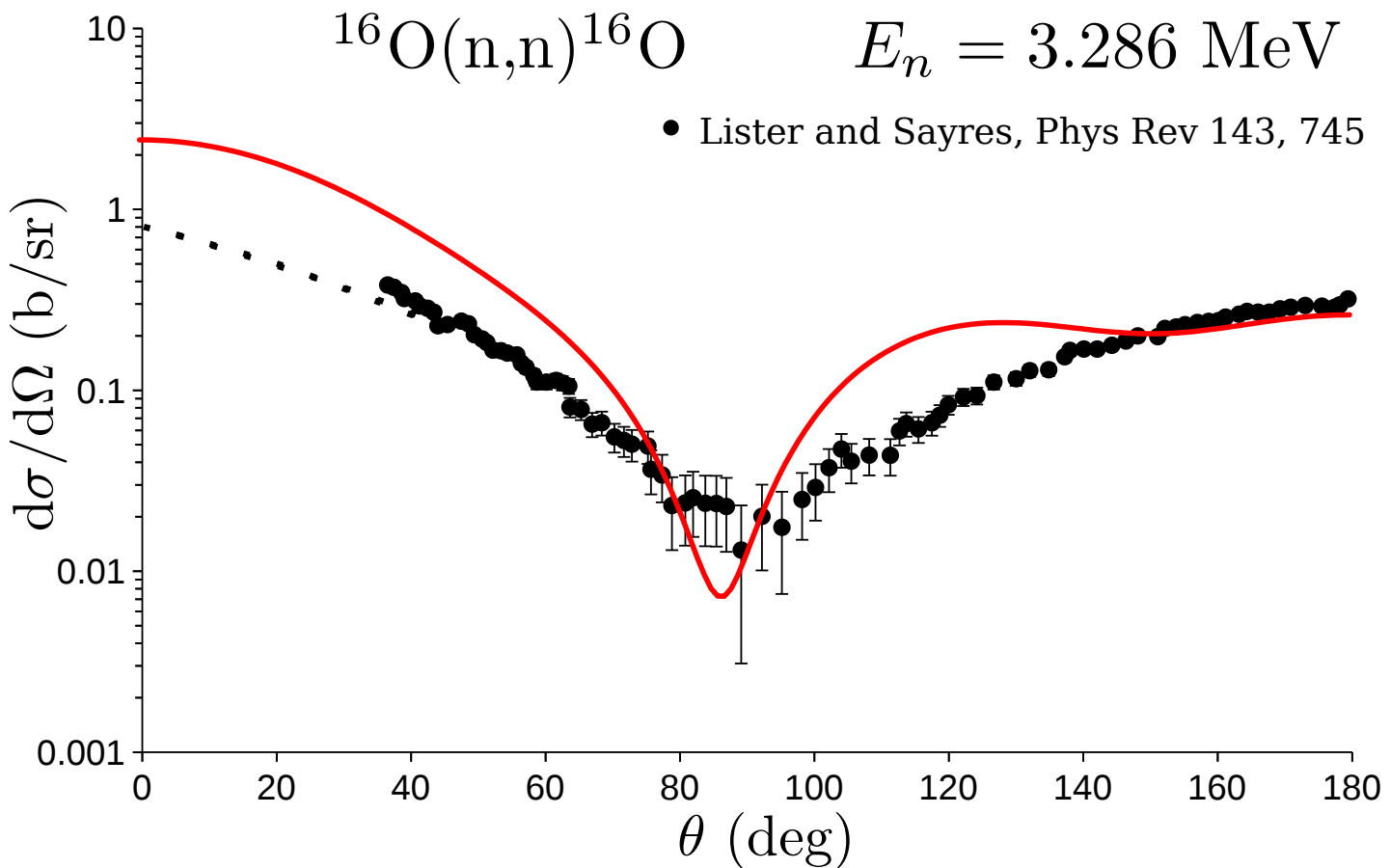
— Σ^∞

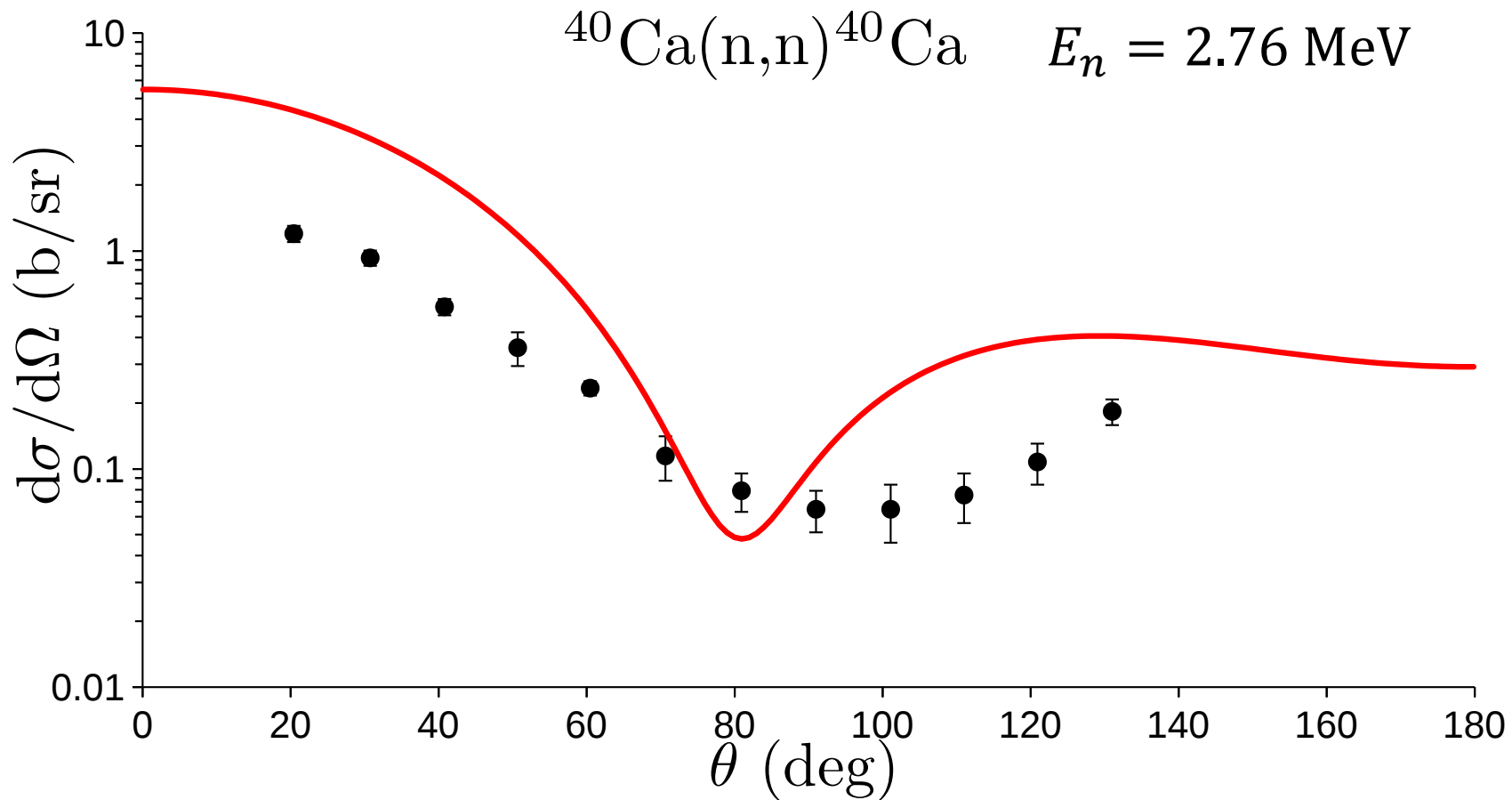
NNLO_{sat}

$n + {}^{16}\text{O}$ (*g.s. + exc*)



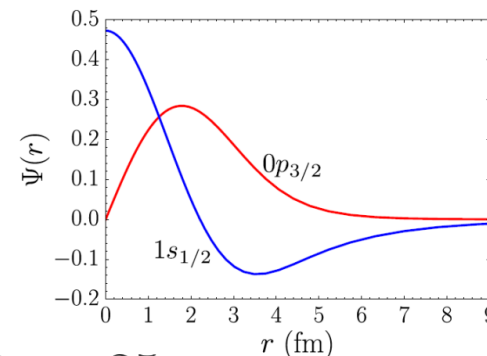
Using the ab initio optical potential for neutron elastic scattering on Oxygen



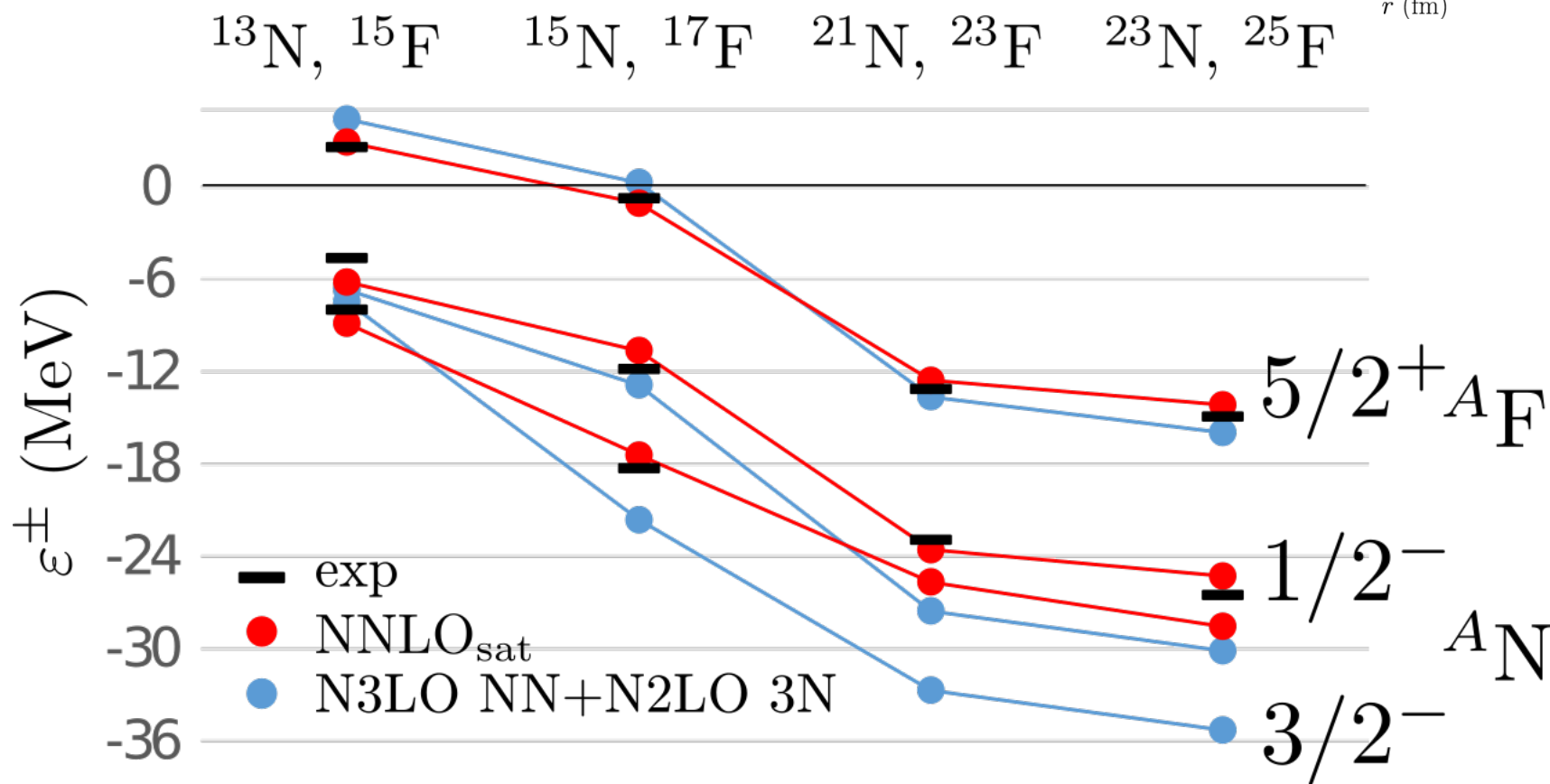


Overlap function

$$\Psi_i(r) = \sqrt{A} \int \prod_{i=1}^A dr_i \Phi_{(A-1)}^+(r_1, \dots, r_{A-1}) \Phi_{(A)}^+(r_1, \dots, r_A)$$



Proton particle-hole gap

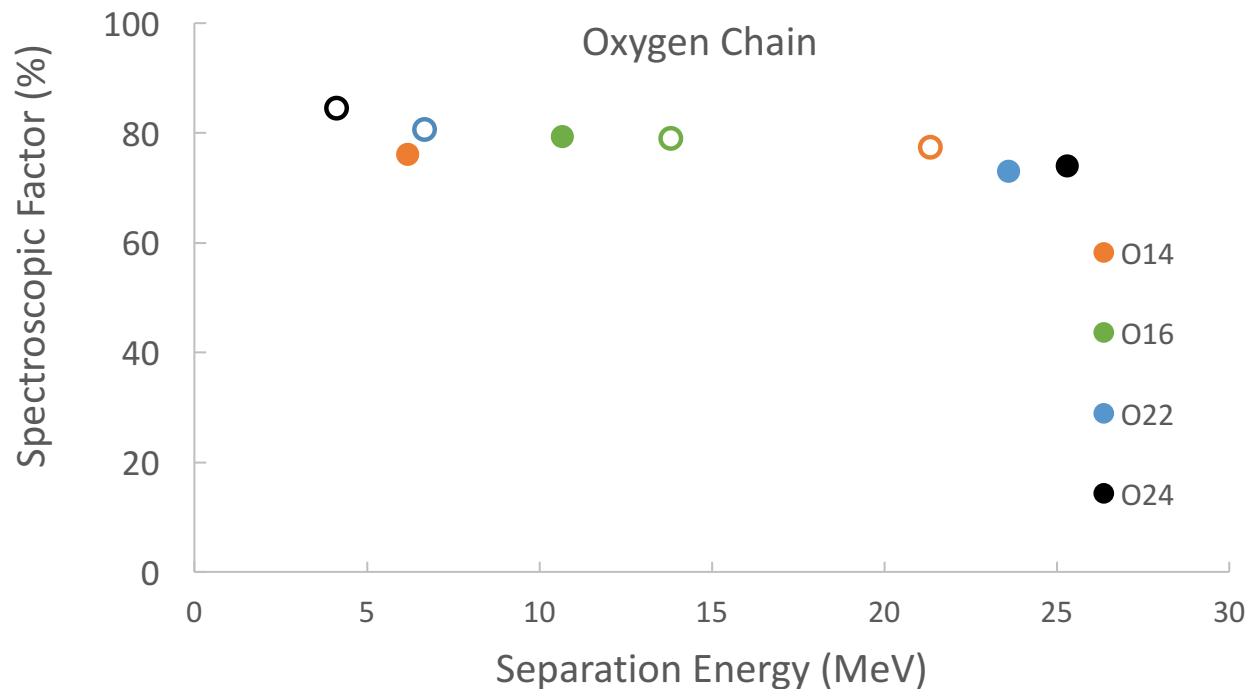


EM results from A. Cipollone PRC92, 014306 (2015)

Knockout Spectroscopic Factors

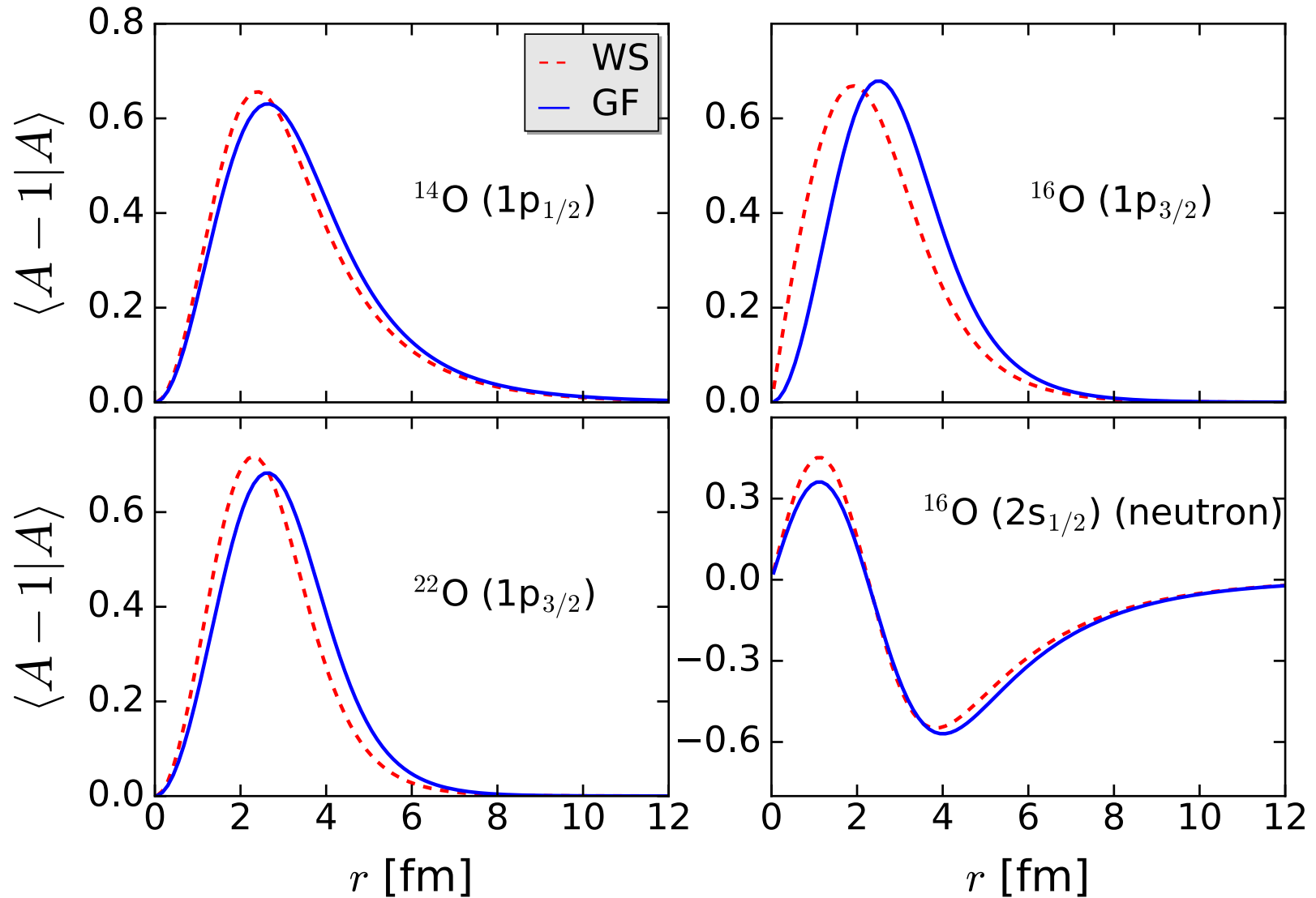
$$\frac{k^2}{2m} \psi_{l,j}(k) + \int dk' k'^2 \left(\Sigma^{l,j*}(k, k', E) \right) \psi_{l,j}(k') = E \psi_{l,j}(k)$$

$$SF = \left| \left\langle \Phi_n^{(A-1)} \left| \Phi_{g.s.}^A \right. \right\rangle \right|^2 \quad \text{Calculated from overlap wavefunctions}$$



open circles neutrons, closed protons

Overlap wavefunctions



Collaboration with C. Bertulani

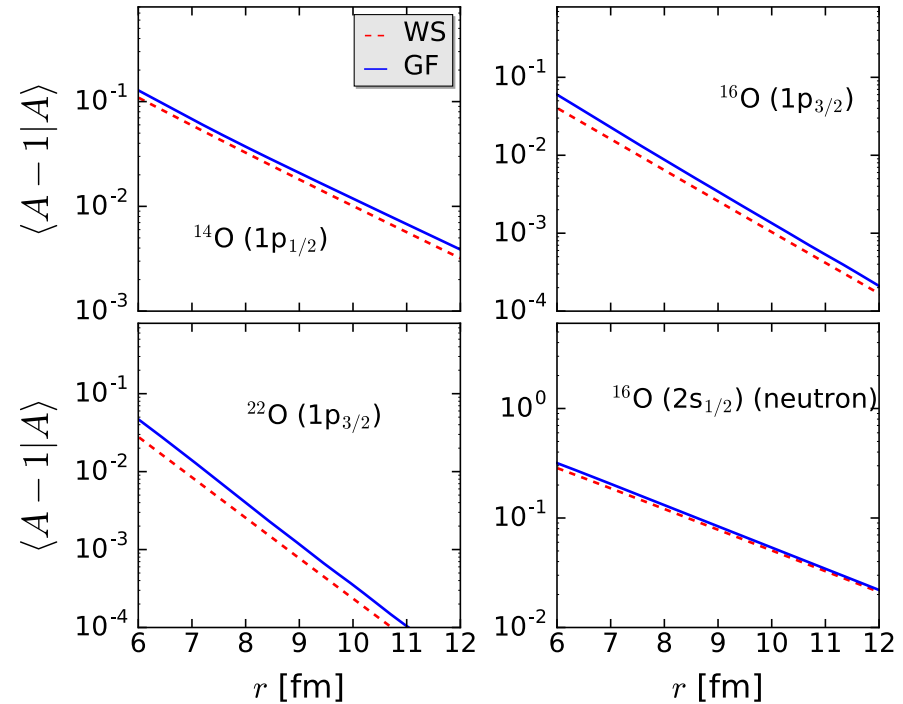
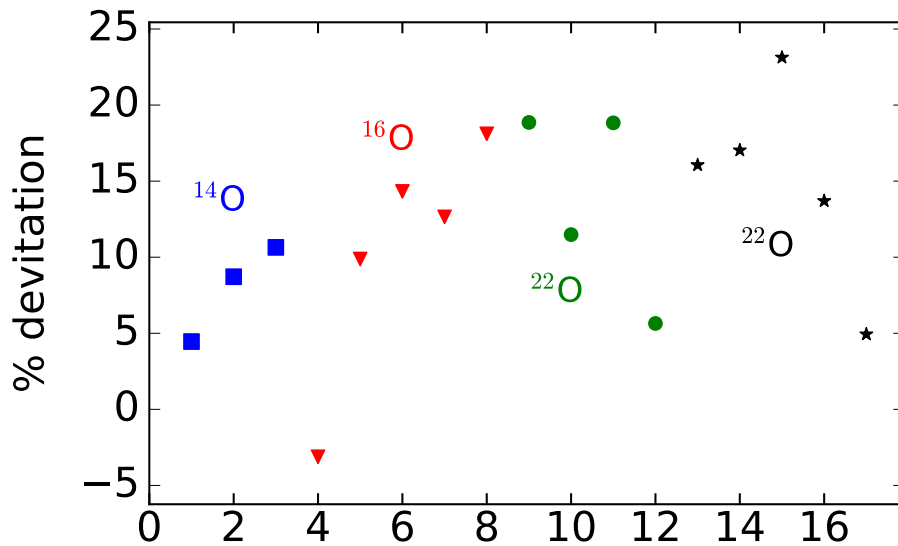
Nucleus (state)	E_B [MeV]	$\langle r^2 \rangle_{WS}^{1/2}$ [fm]	$\langle r^2 \rangle_{GF}^{1/2}$ [fm]	C_{WS} [fm $^{-1/2}$]	C_{GF} [fm $^{-1/2}$]	σ_{qf}^{WS} [mb]	σ_{qf}^{GF} [mb]	σ_{kn}^{WS} [mb]	σ_{kn}^{GF} [mb]	$C^2 S_{GF}$
^{14}O ($\pi 1p_{3/2}$)	8.877	2.856	2.961	6.785	7.172	27.38	28.60	27.19	27.42	0.548

5%

<1%

Deviation of quasifree (p, pn)
cross section calculation
for different wavefunctions

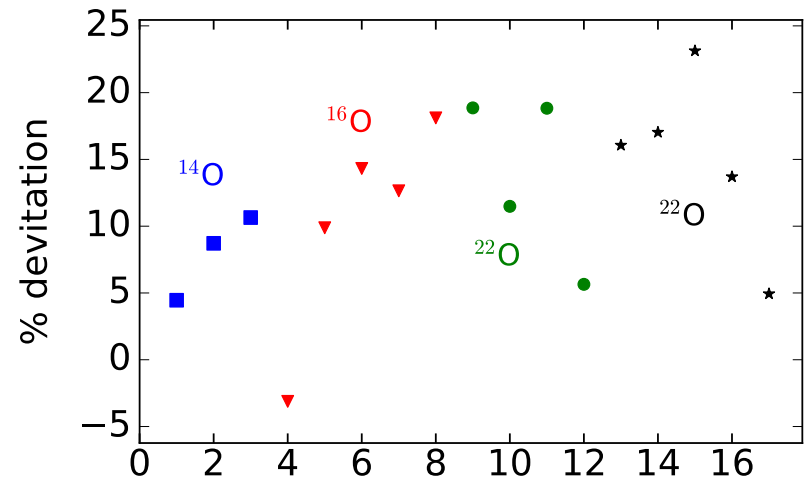
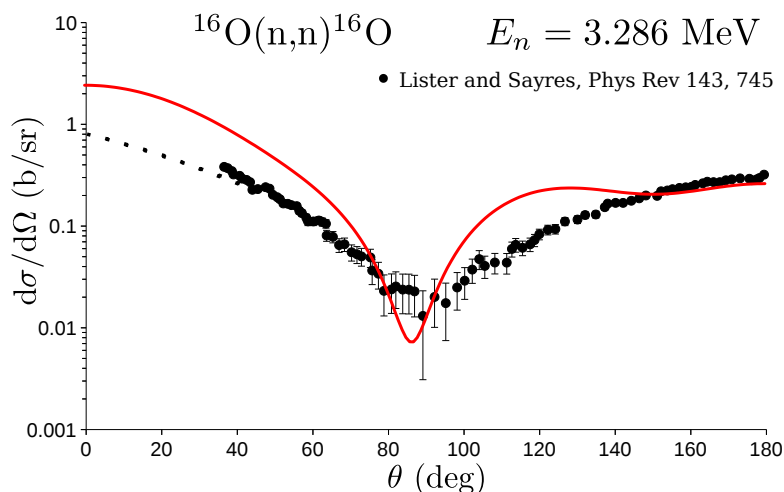
$$(\sigma_{GF} - \sigma_{WS})/\sigma_{WS}$$

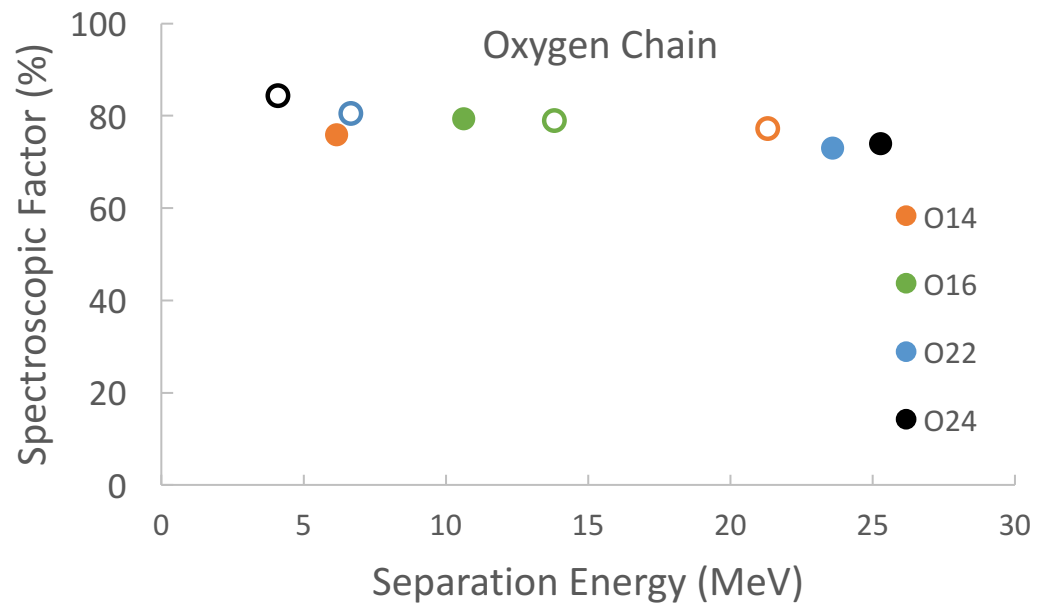


Collaboration with C. Bertulani

Conclusions and Perspectives

- We are developing an interesting tool to study nuclear reactions effectively.
We have defined a non-local generalized optical potential corresponding to nuclear self energy.
- Spectroscopic Factors from ab-initio overlap wavefunctions differ from effective wood saxon. These do not seem to depend much on proton-neutron asymmetry



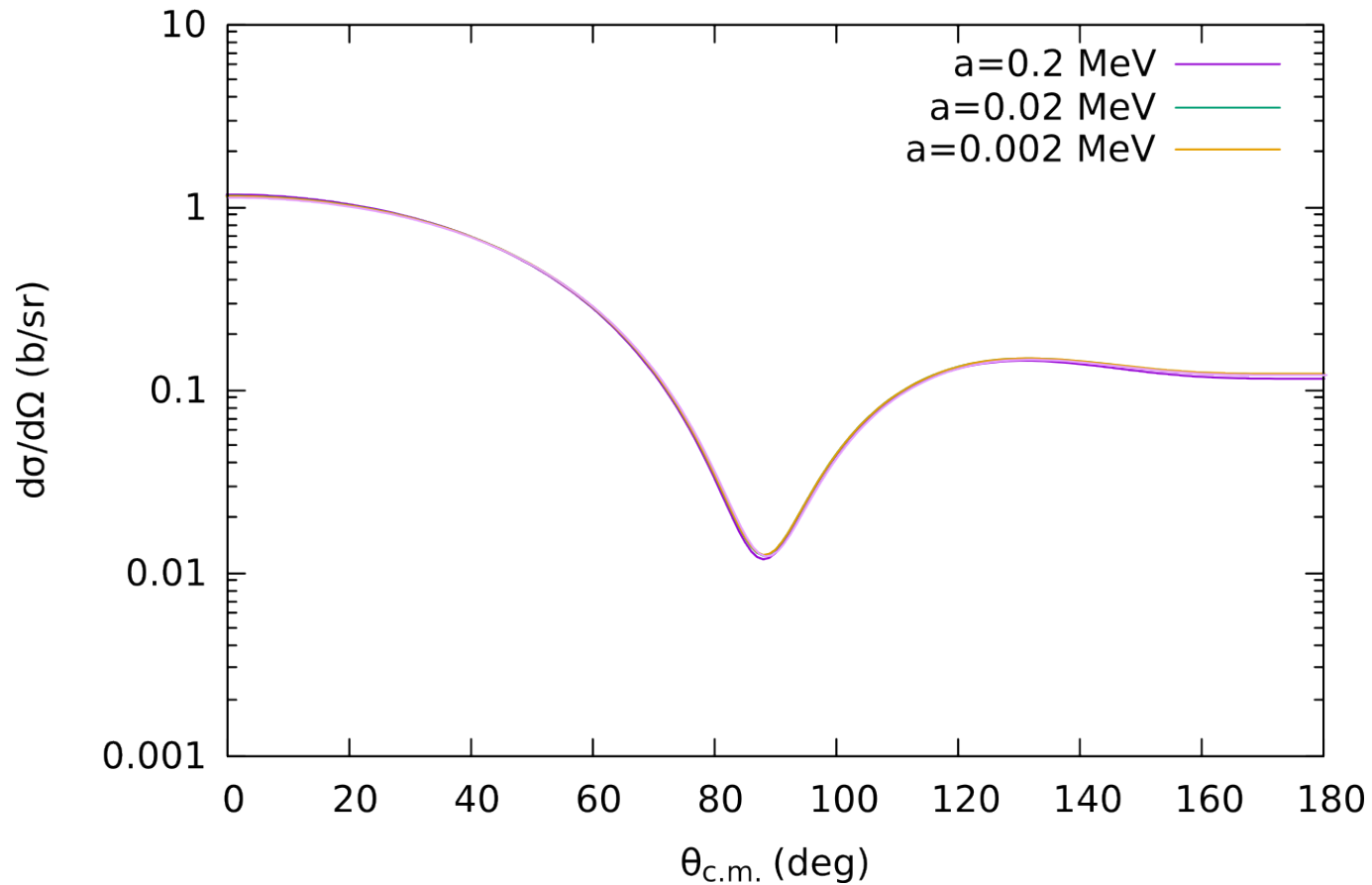


«Imaginary» Parameter

$$\Gamma(E) = \frac{1}{\pi} \frac{a (E - E_F)^2}{(E - E_F)^2 - b^2}$$

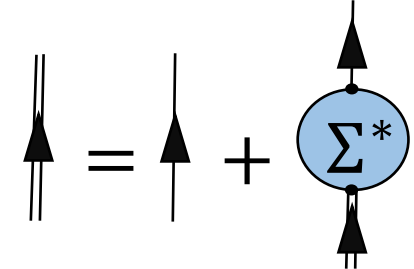
$$b = 22.36 \text{ MeV}$$

$^{16}\text{O}(n,n)^{16}\text{O}$ $E_n = 3.286 \text{ MeV}$



Why Green's Functions?

Dyson Equation

$$g_{\alpha\beta}(\omega) = g_{\alpha\beta}^0(\omega) + \sum_{\gamma\delta} g_{\alpha\gamma}^0(\omega) \Sigma_{\gamma\delta}^*(\omega) g_{\delta\beta}(\omega)$$


Equation of motion

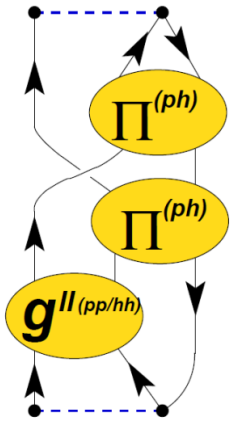
$$\left(E + \frac{\hbar^2}{2m} \nabla_r^2 \right) G(\mathbf{r}, \mathbf{r}'; E) - \int d\mathbf{r}'' \Sigma(\mathbf{r}, \mathbf{r}''; E) G(\mathbf{r}'', \mathbf{r}'; E) = \delta(\mathbf{r} - \mathbf{r}')$$

Corresponding Hamiltonian

$$\mathcal{H}_{\mathcal{M}}(\mathbf{r}, \mathbf{r}') = -\frac{\hbar^2}{2m} \nabla_r^2 \delta(\mathbf{r} - \mathbf{r}') + \Sigma(\mathbf{r}, \mathbf{r}'; E + i\epsilon)$$

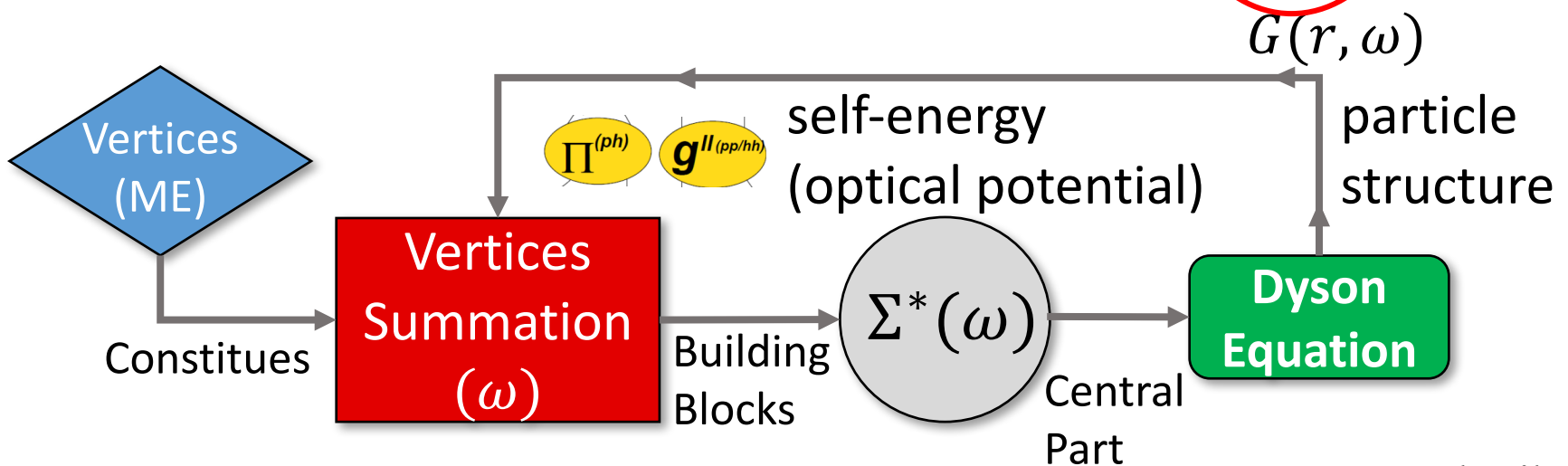
Σ corresponds to the Feshbach's generalized optical potential

$$\Sigma_{\alpha\beta}^*(\omega) = \Sigma_{\alpha\beta}^{(\infty)} + \sum_{i,j} M_{\alpha,i}^\dagger \left[\frac{1}{\omega - (\mathbf{K}^> + \mathbf{C}) + i\eta} \right]_{i,j} M_{j,\beta} + \sum_{r,s} N_{\alpha,r} \left[\frac{1}{\omega - (\mathbf{K}^< + \mathbf{D}) - i\eta} \right]_{r,s} N_{s,\beta}^\dagger$$



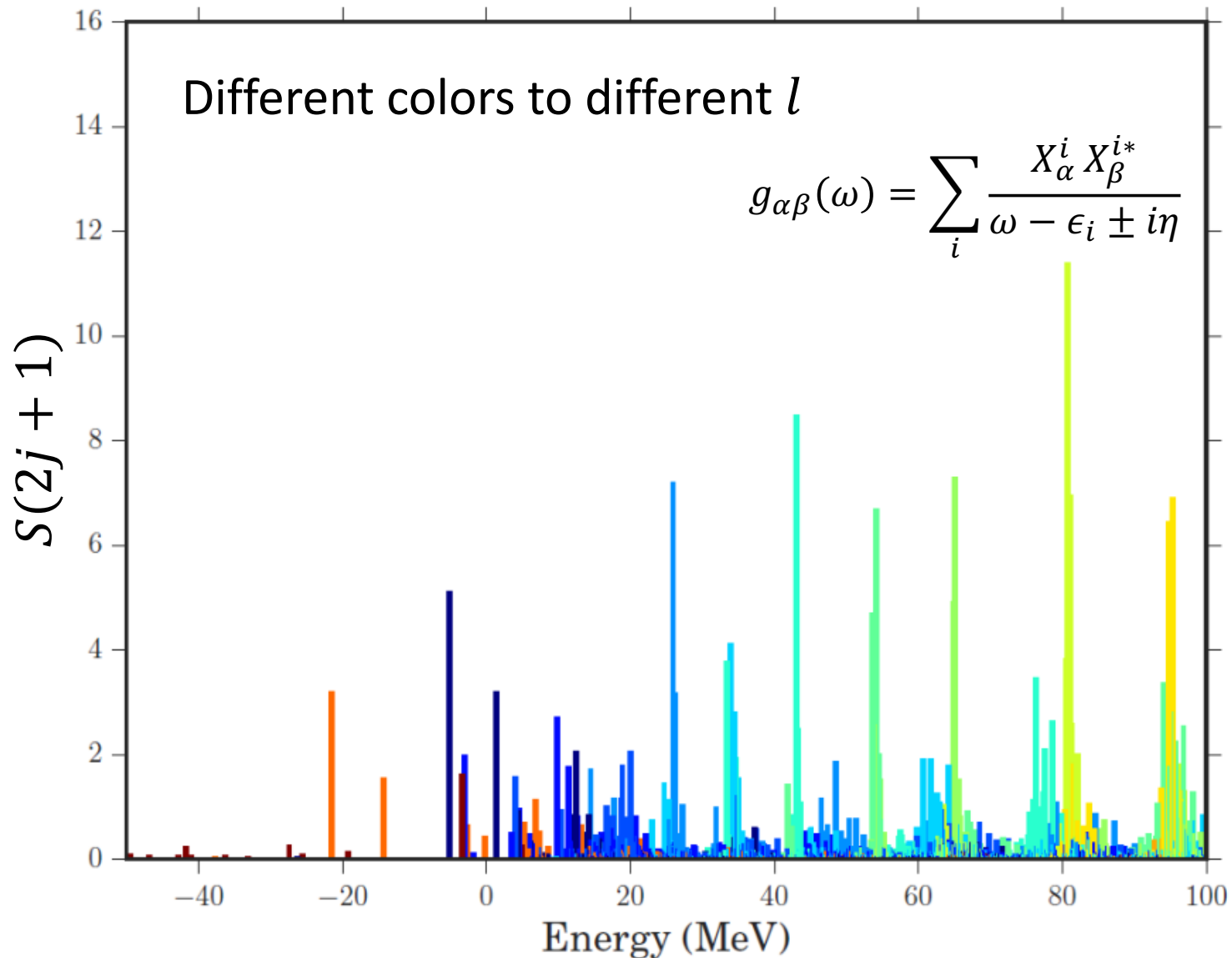
$$\begin{pmatrix} \hat{T} + \Sigma^{(\infty)} & M^\dagger & N \\ M & E^> + C & \\ N^\dagger & & E^< + D \end{pmatrix} \begin{pmatrix} z^i \\ w^i \\ v^i \end{pmatrix} = \begin{pmatrix} z^i \\ w^i \\ v^i \end{pmatrix} \varepsilon_i$$

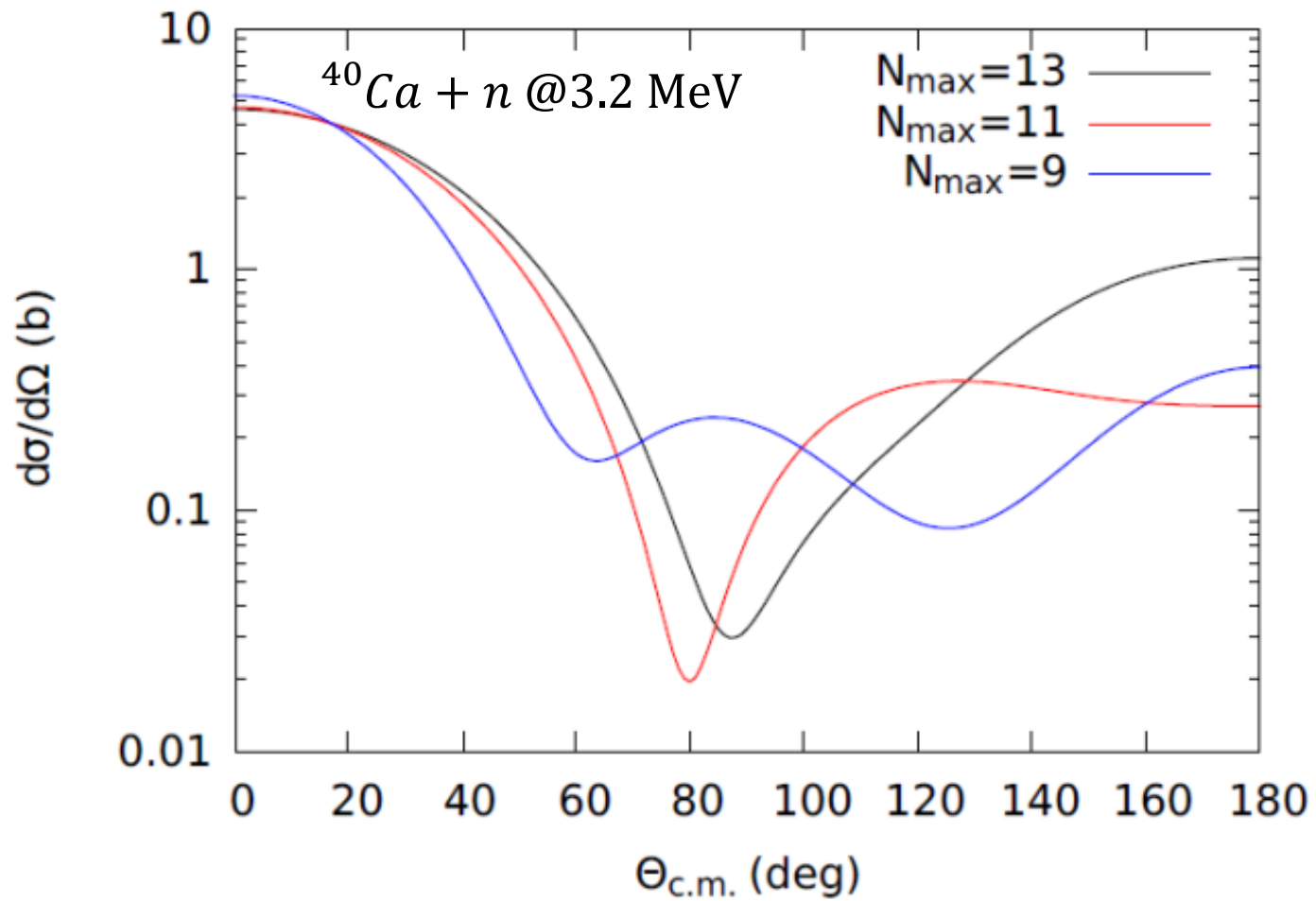
Unknown



More details in

^{16}O neutron propagator





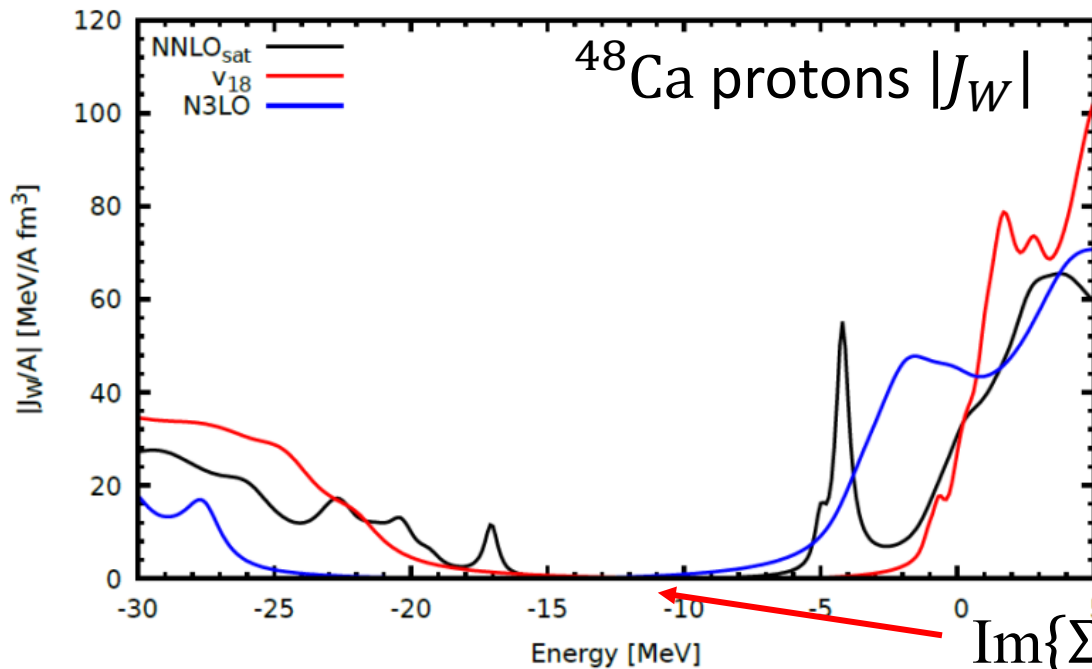
Volume integrals

$$J_W^\ell(E) = 4\pi \int dr r^2 \int dr' r'^2 \text{Im} \Sigma_0^\ell(r, r', E)$$

Non local potential

$$J_V^\ell(E) = 4\pi \int dr r^2 \int dr' r'^2 \text{Re} \Sigma_0^\ell(r, r'; E).$$

$$\tilde{\Sigma}_{n_a, n_b}^{\ell j}(E) = \sum_r \frac{m_{n_a}^r m_{n_b}^r}{E - \varepsilon_r \pm i\eta}$$



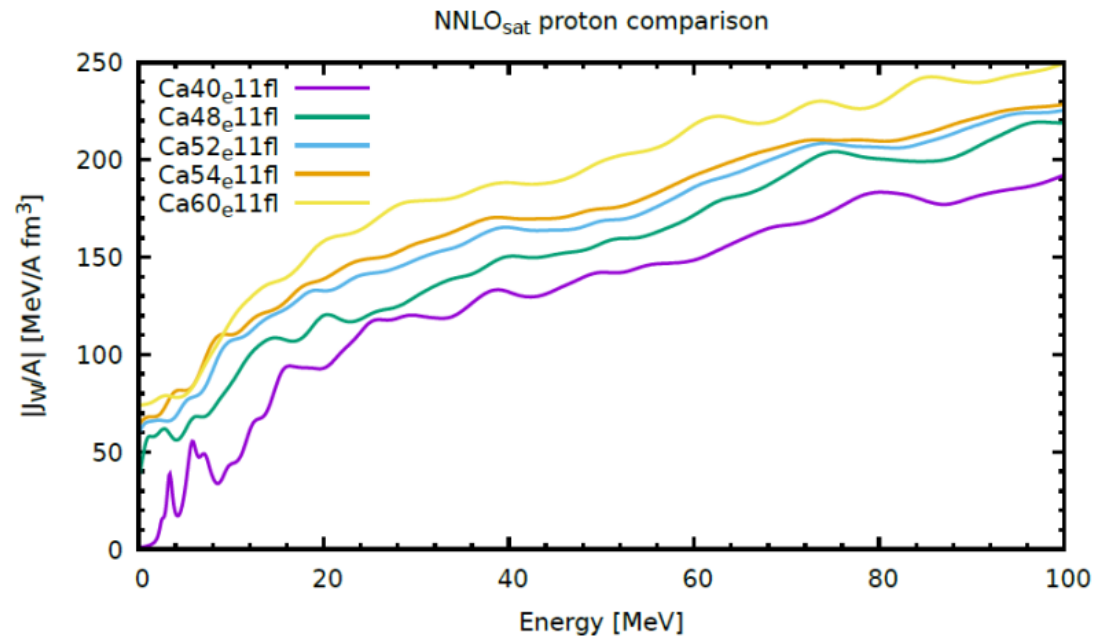
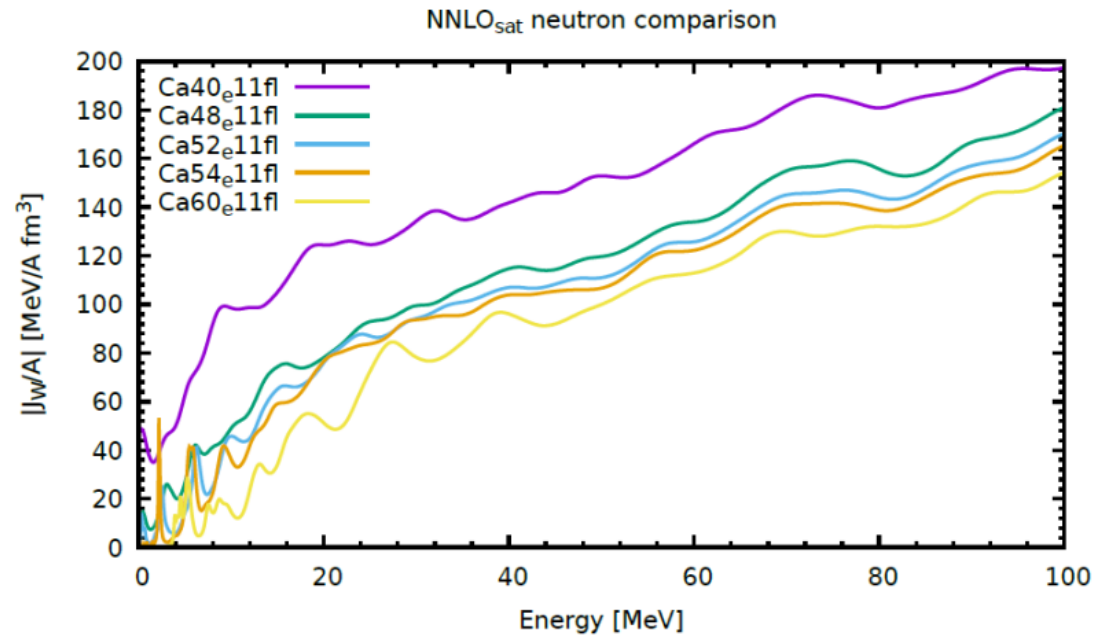
different Fermi energies and particle-hole gap for different interactions

$$\text{Im}\{\Sigma(\epsilon_F)\} = 0$$

S. Waldecker et al. PRC**84**, 034616(2011)

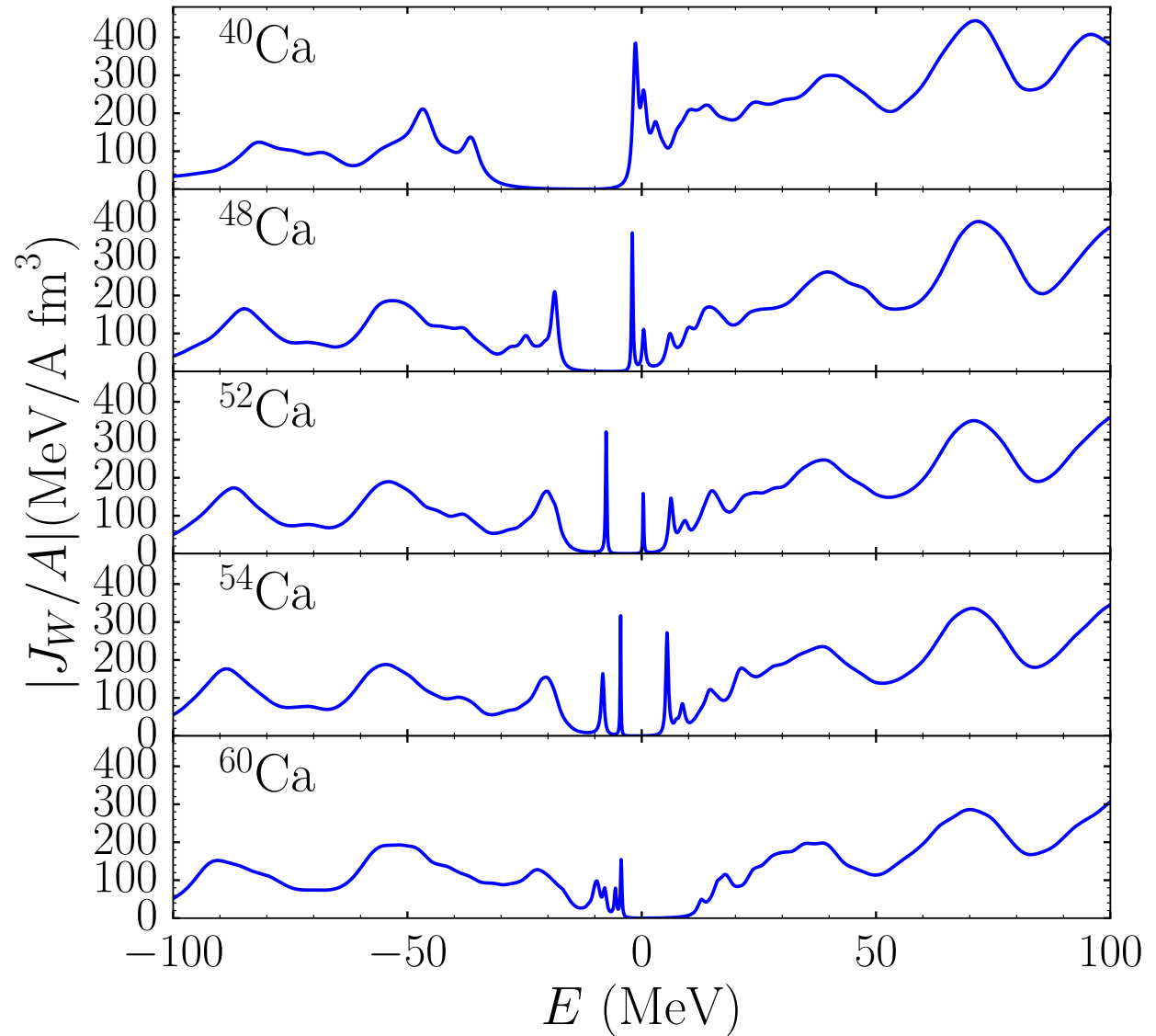
Ca isotopes

neutron and proton
volume integrals of
self energies.



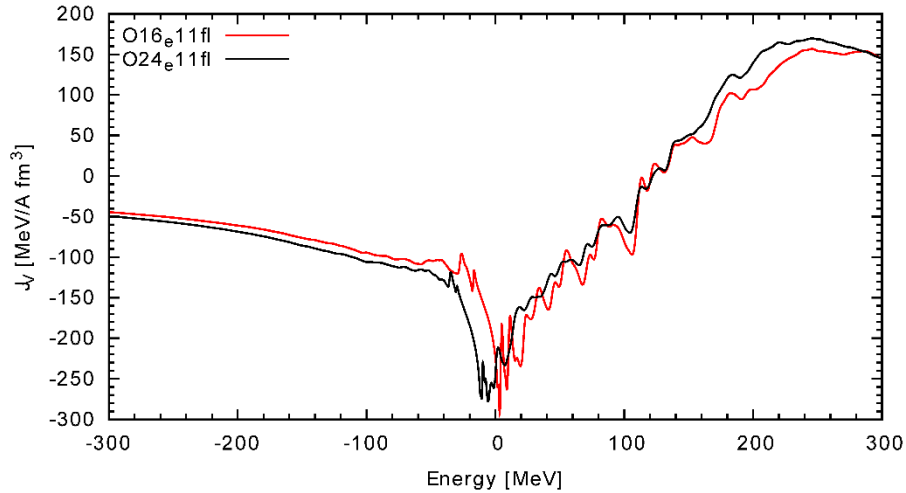
Ca isotopes

neutron volume
integrals of self
energies.

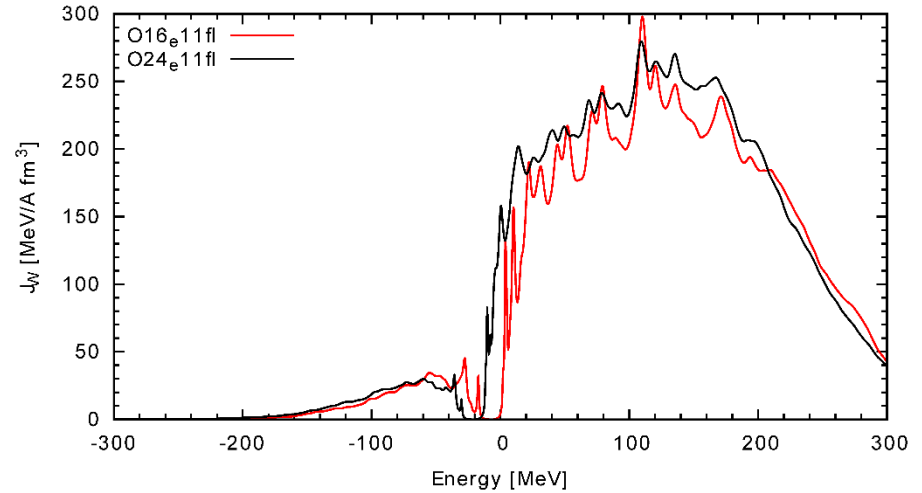


^{16}O and ^{24}O

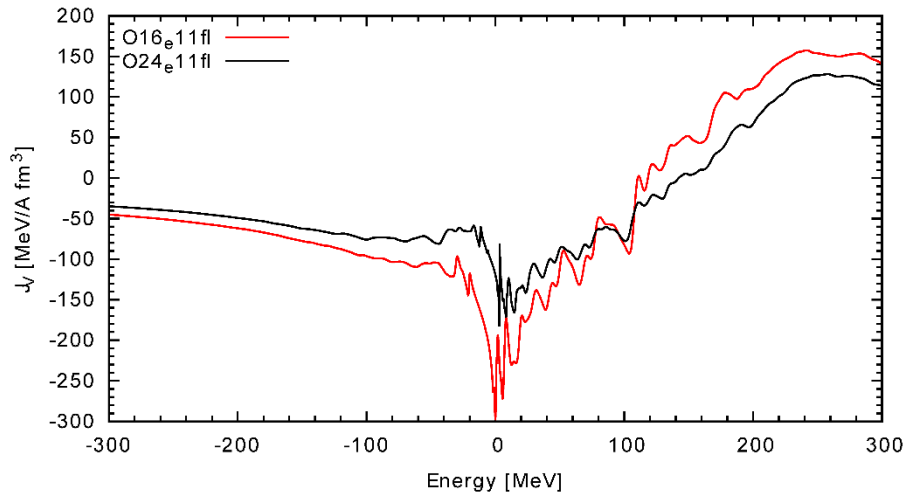
NNLO_{sat} proton comparison



NNLO_{sat} proton comparison



NNLO_{sat} neutron comparison



NNLO_{sat} neutron comparison

