Electromagnetic reactions from few to many-body systems

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INFN

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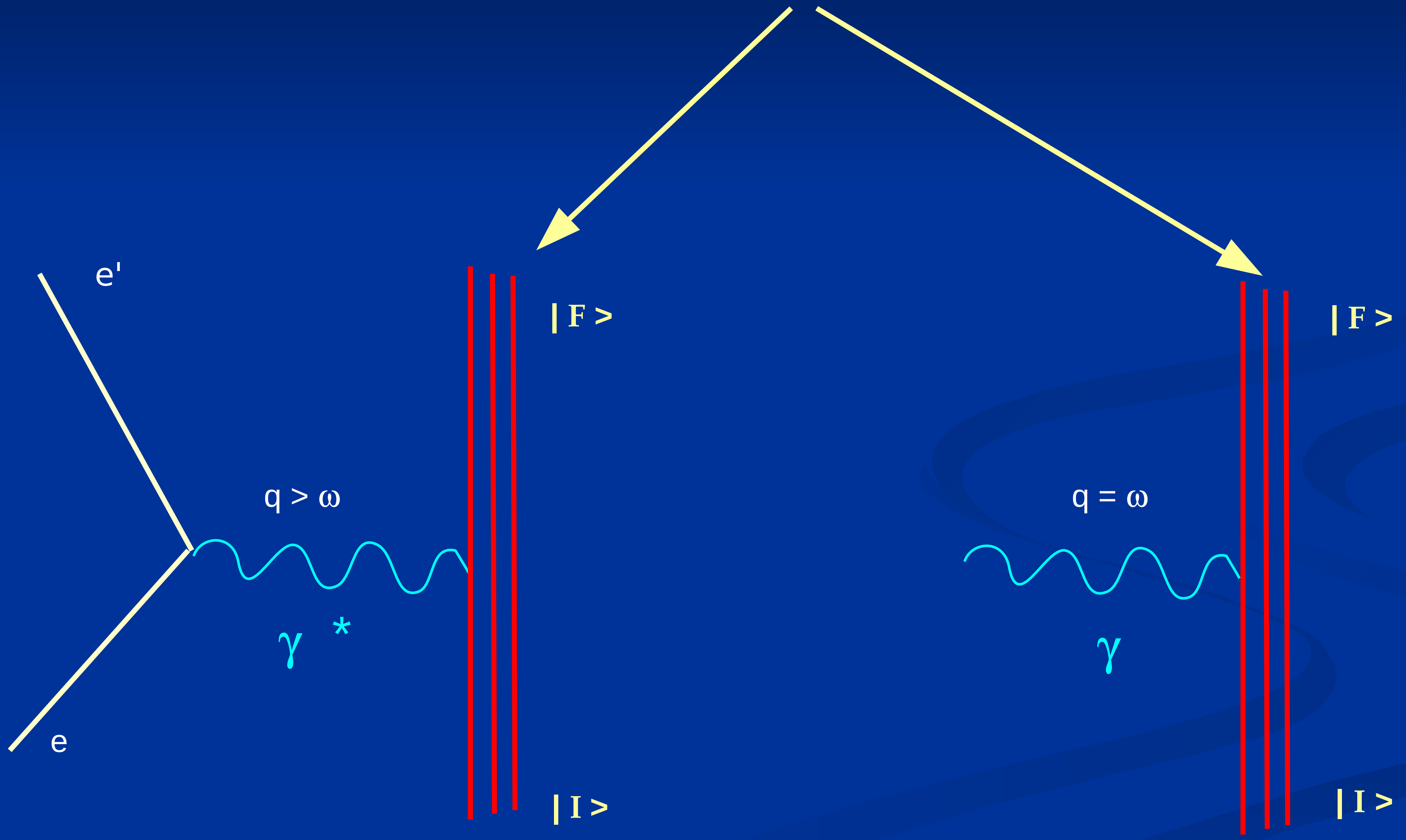
Physics of e.m. Interactions with Nuclei



The e.m. interaction is **perturbative** compared to the nuclear **strong** interaction

$$H = H_N + V_{em}$$

Therefore the reaction cross sections are proportional to

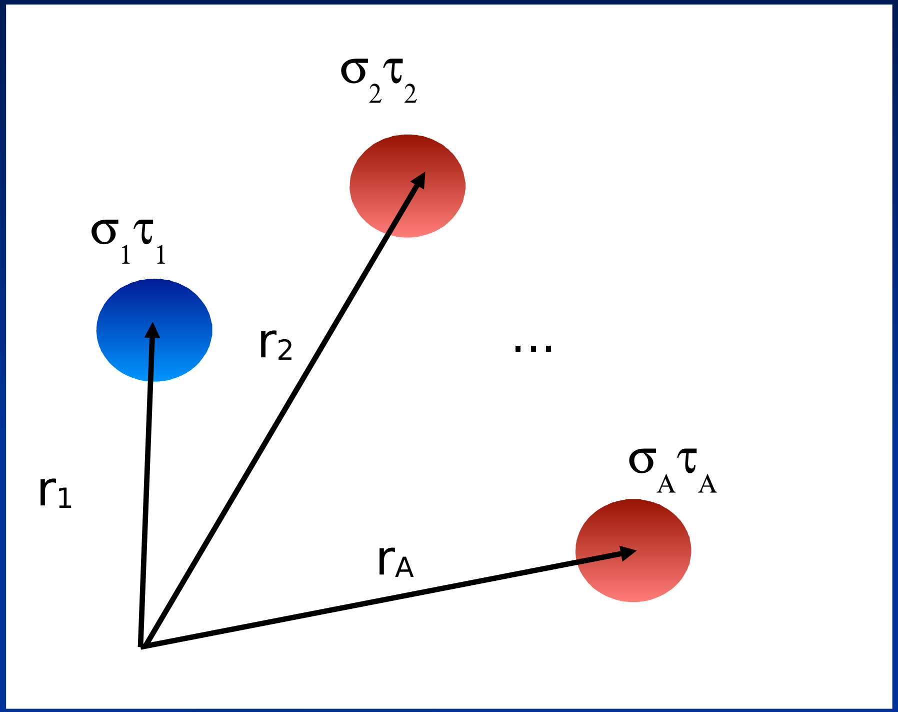
$$\sigma \sim | < F \,|\, J_\mu \,|\, I > |^{\,2}$$

- **| F > and | I > are eigenstates of H_N**
- **| I > is a bound state (g.s.) ,**
- **| F > can be a bound or a continuum (scattering) state**

- **J_μ is the nuclear current**

The ab-initio approach

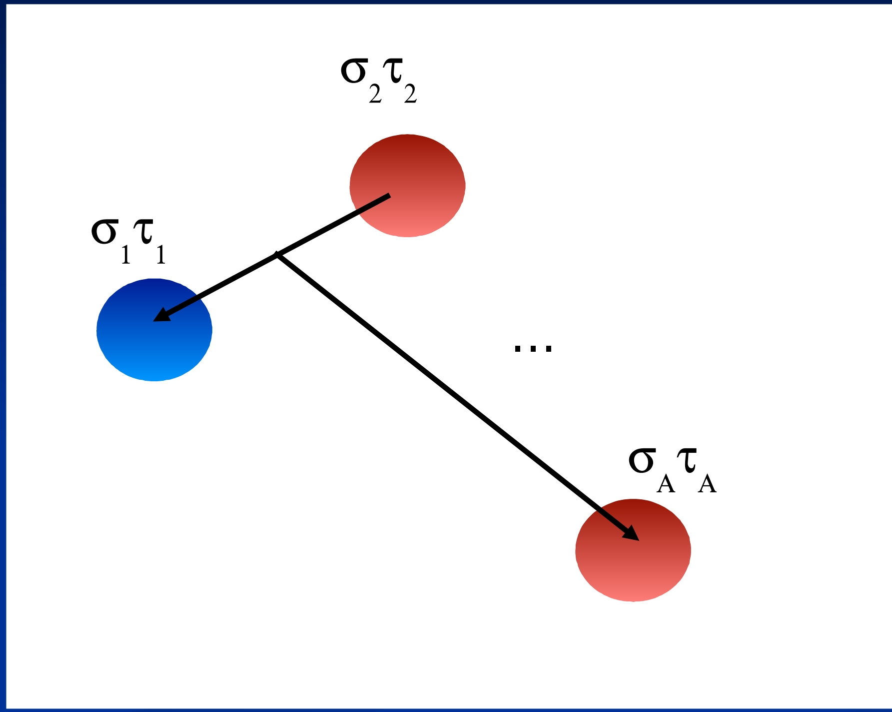
- **Start from neutrons and protons as building blocks (positions, spins, isospins)**



The ab-initio approach

- **Start from neutrons and protons as building blocks (positions, spins, isospins)**

- **Subtract the cm motion and remain with relative coordinates**



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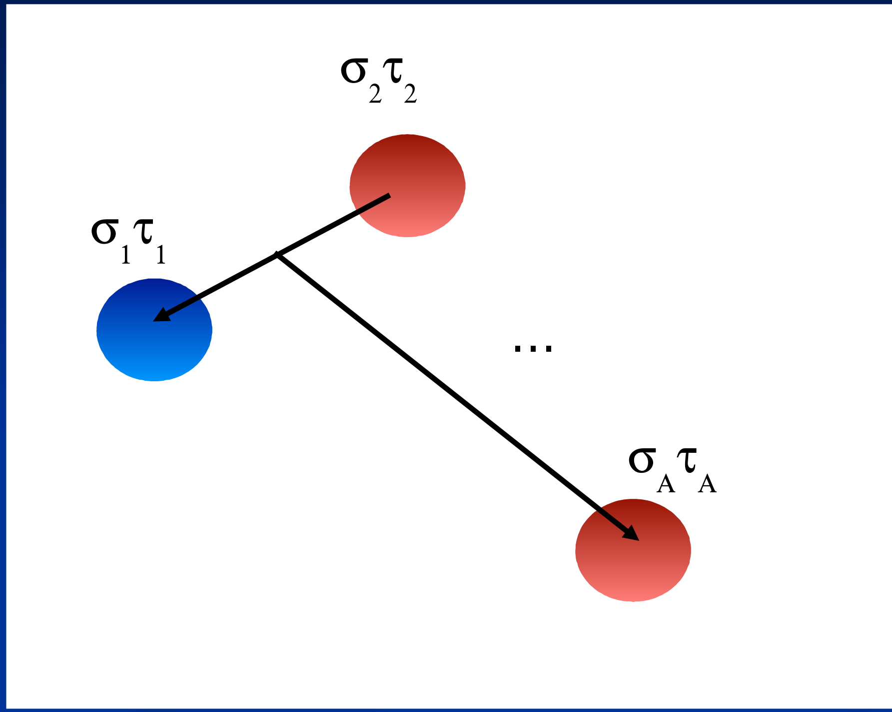

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- Solve the non-relativistic quantum mechanical problem of A-interacting nucleons

The ab-initio approach

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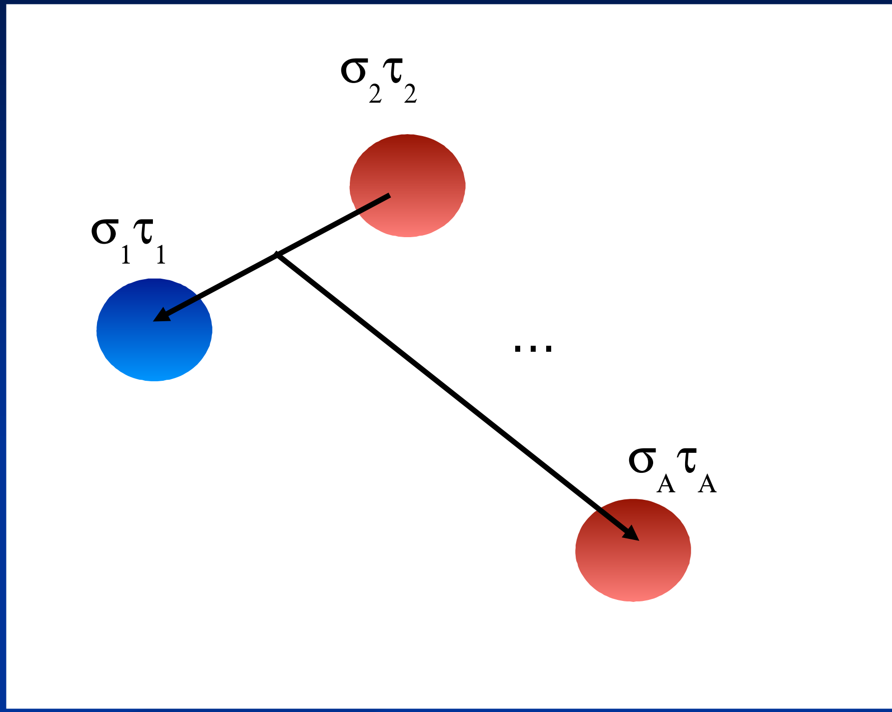
- Find numerical solutions with “no approximations” or “controllable approximations” (error bars)



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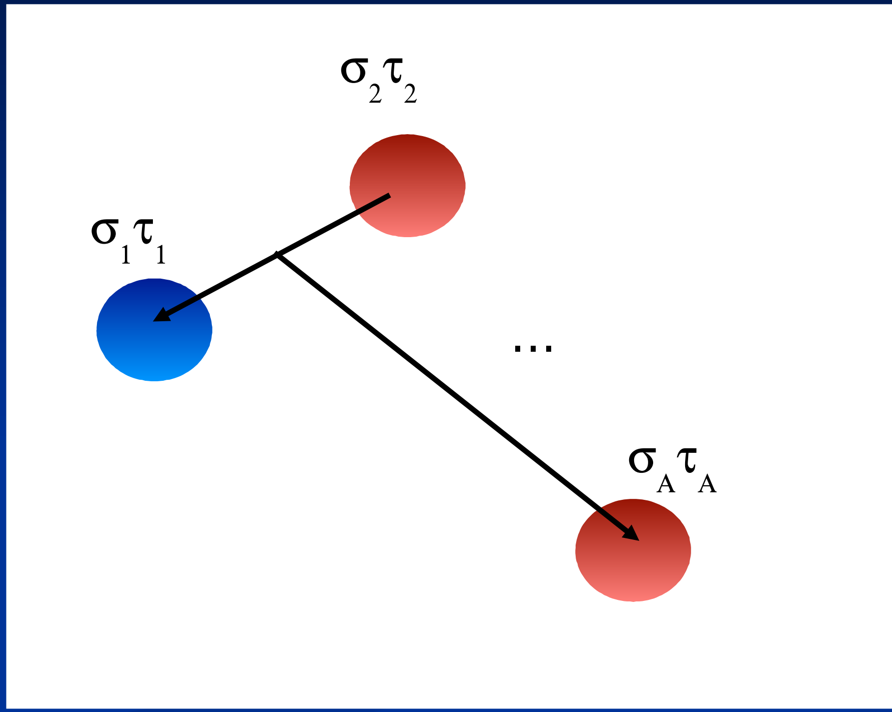

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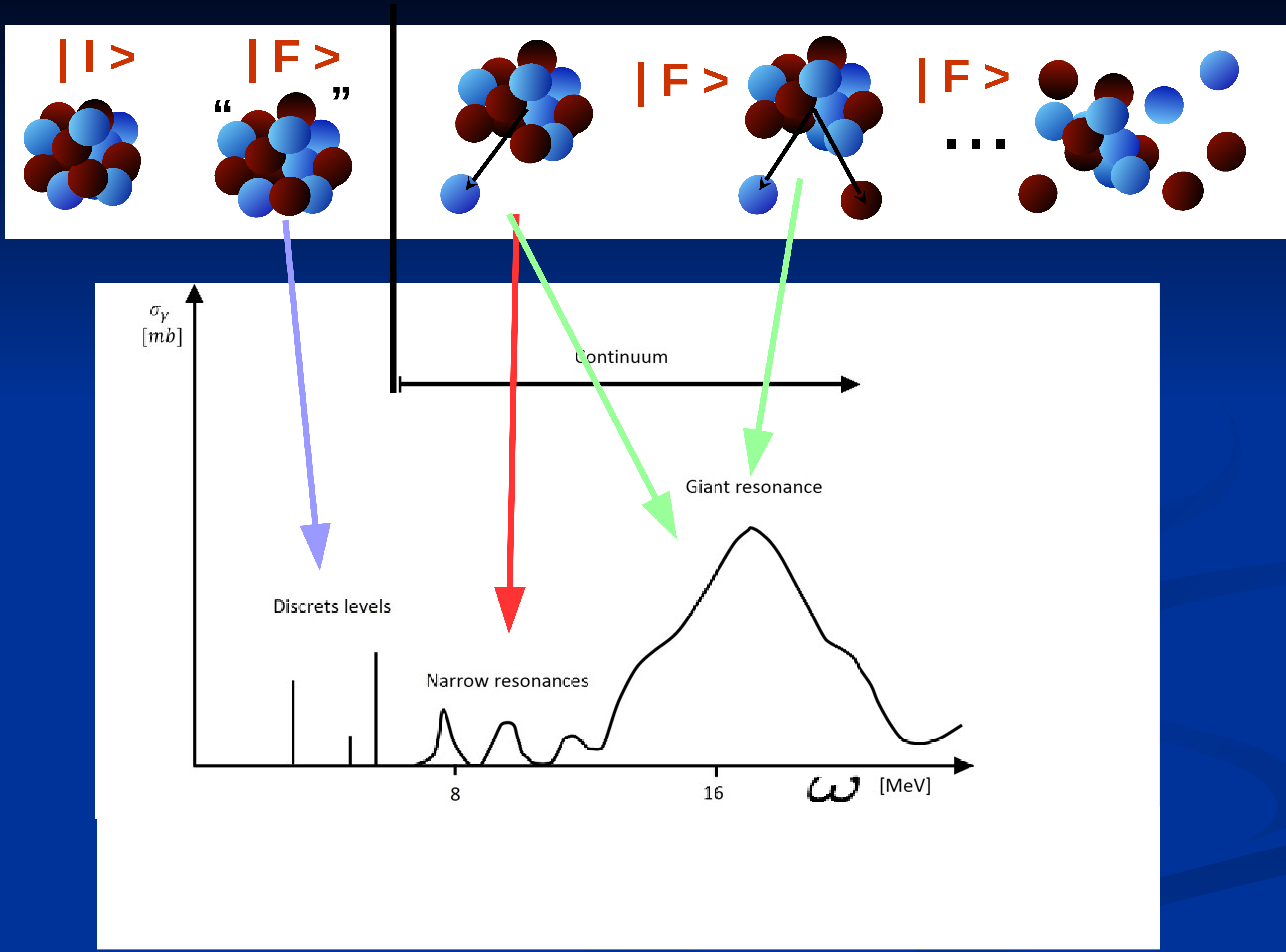
- Calculate low-energy observables and compare with experiment to test nuclear forces and provide predictions for future experiments and microscopic interpretations for older experiments

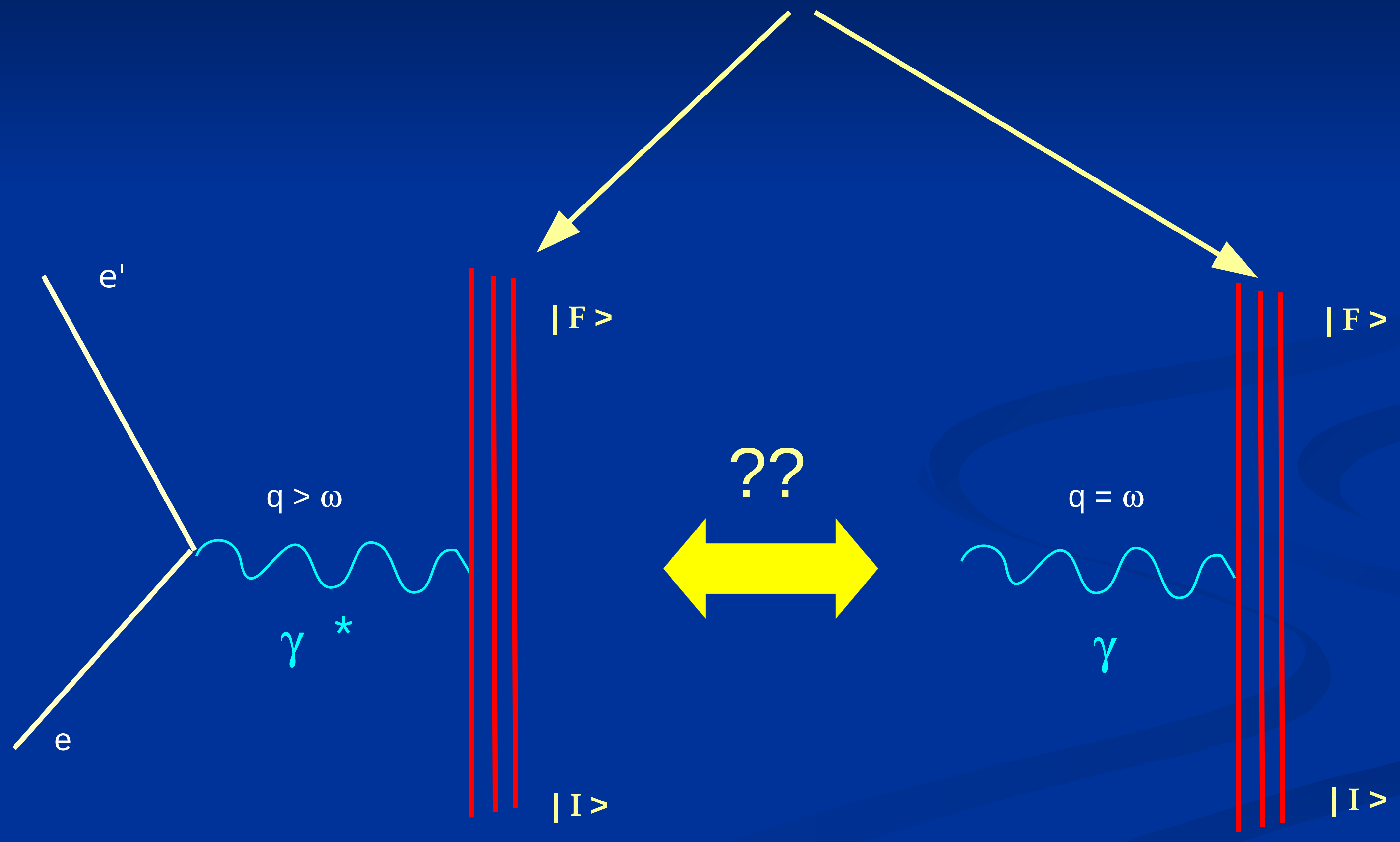
For the “ab-initio” program, as previously defined, the most interesting reactions are those at low-energy (up to ~ 50 - (100??) MeV)

| I > is in general a ground state

| F > can be a bound or a continuum (scattering) state


| I >
| F >
“ ”
| F >
| F >
...
σγ
[mb]
Continuum
Giant resonance
Discrets levels
Narrow resonances
8
16
ω
[MeV]

Physics of e.m. Interactions with Nuclei



$$\sigma \sim |\langle F | J_\mu | I \rangle|^2 \quad \text{where} \quad J_\mu = (\rho, \vec{J})$$

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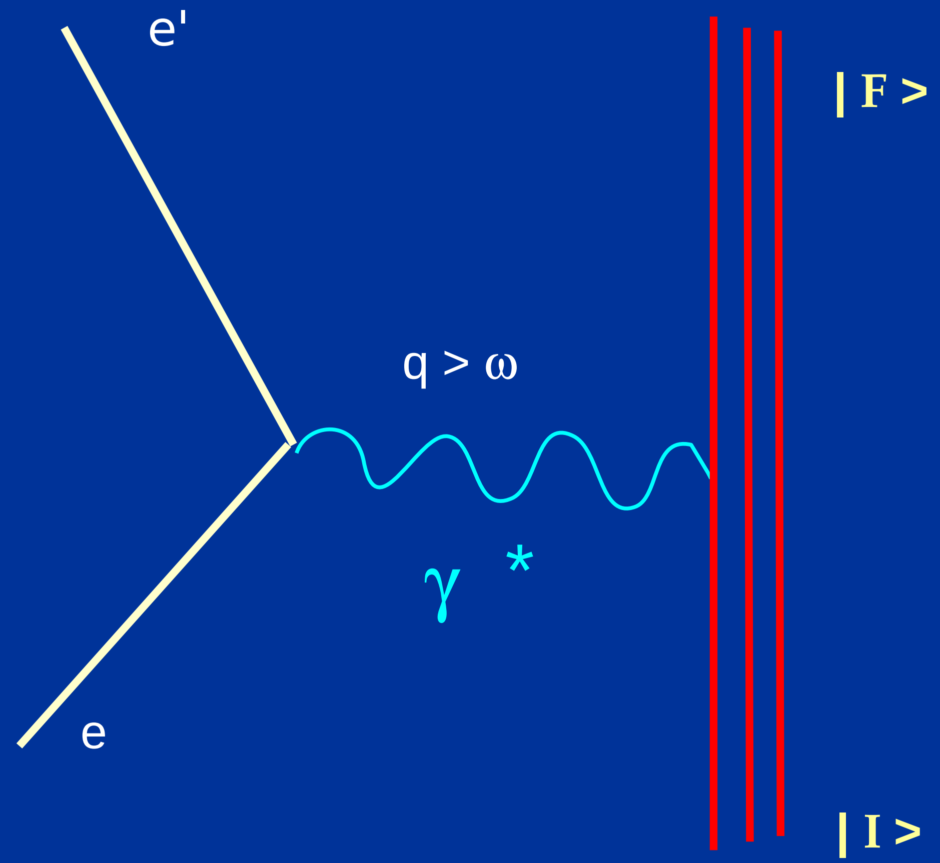
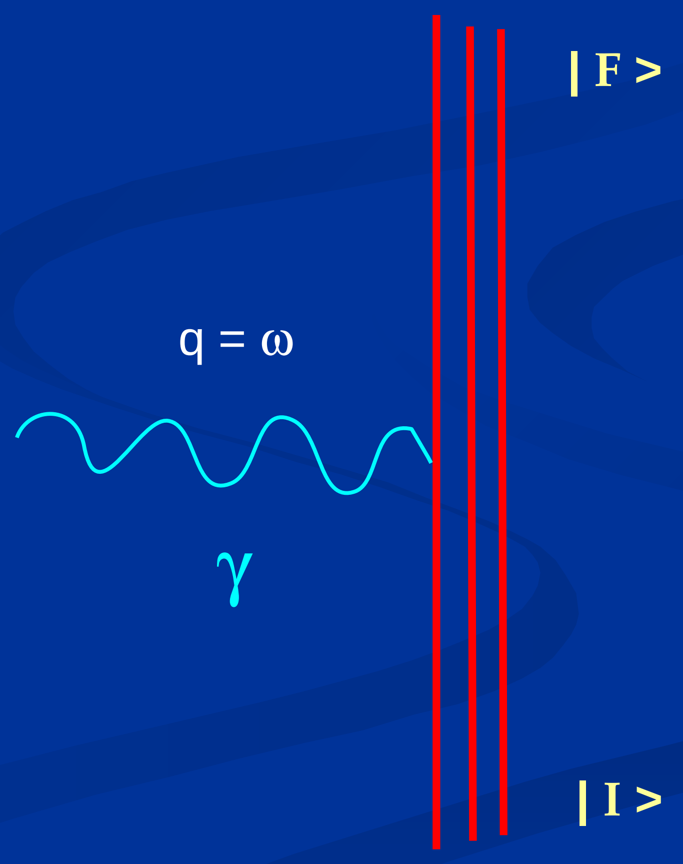
$$J_\mu = (\rho, J_T)$$



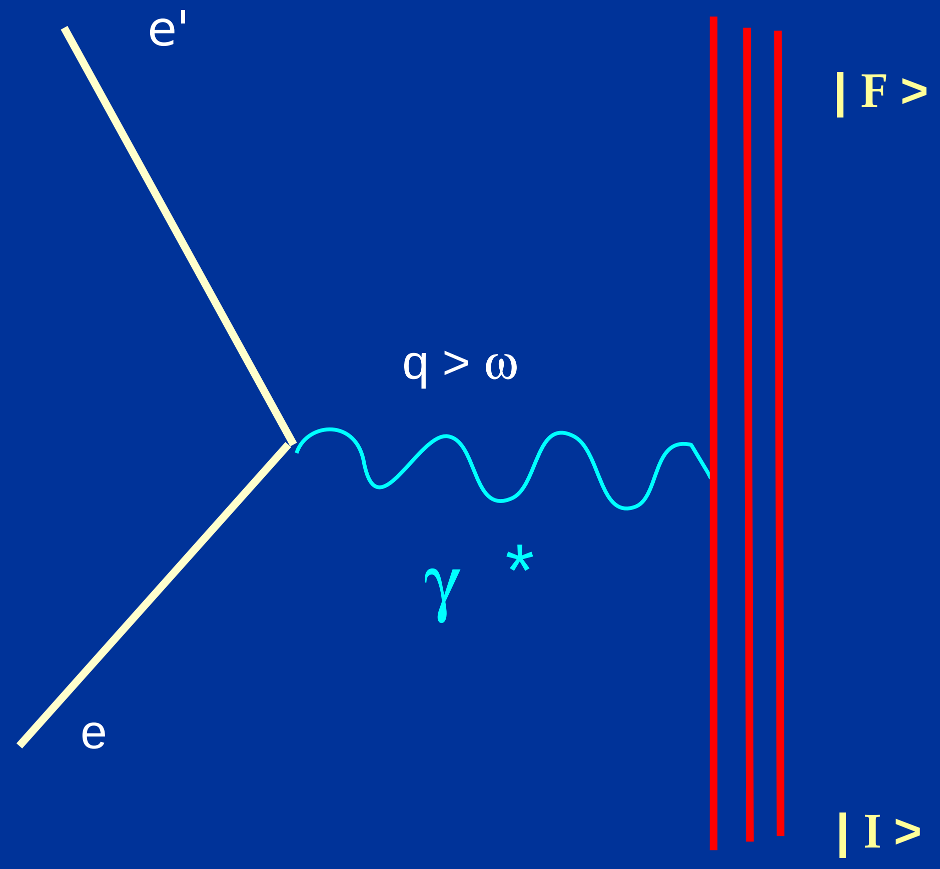


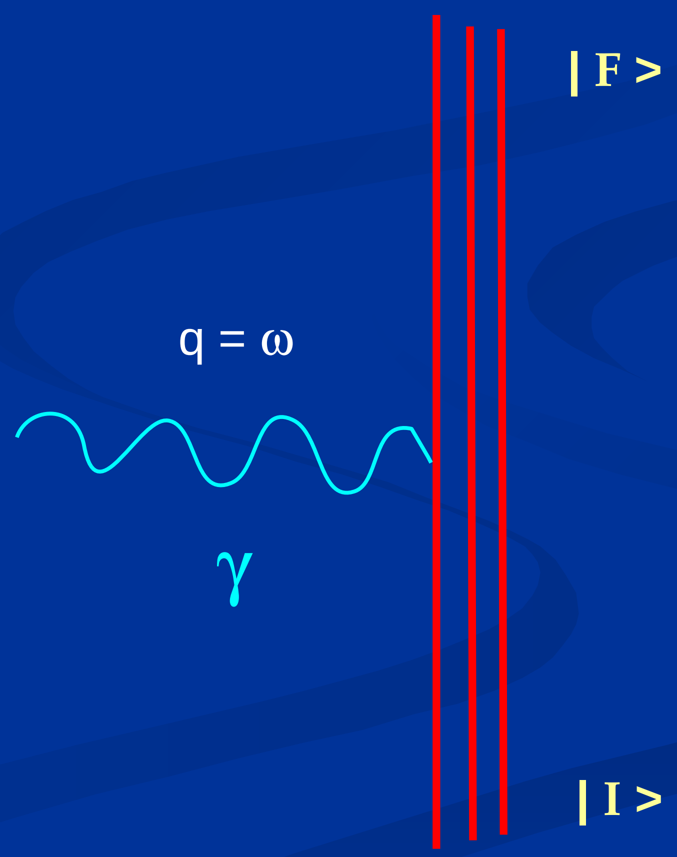
However

For q (and $\omega < q$) "small"

(q R –-> 1)

$$\langle F \,|\, \rho \sim D \,|\, I \rangle \;\cong\; \langle F \,|\, J_T \sim E1 \,|\, I \rangle$$

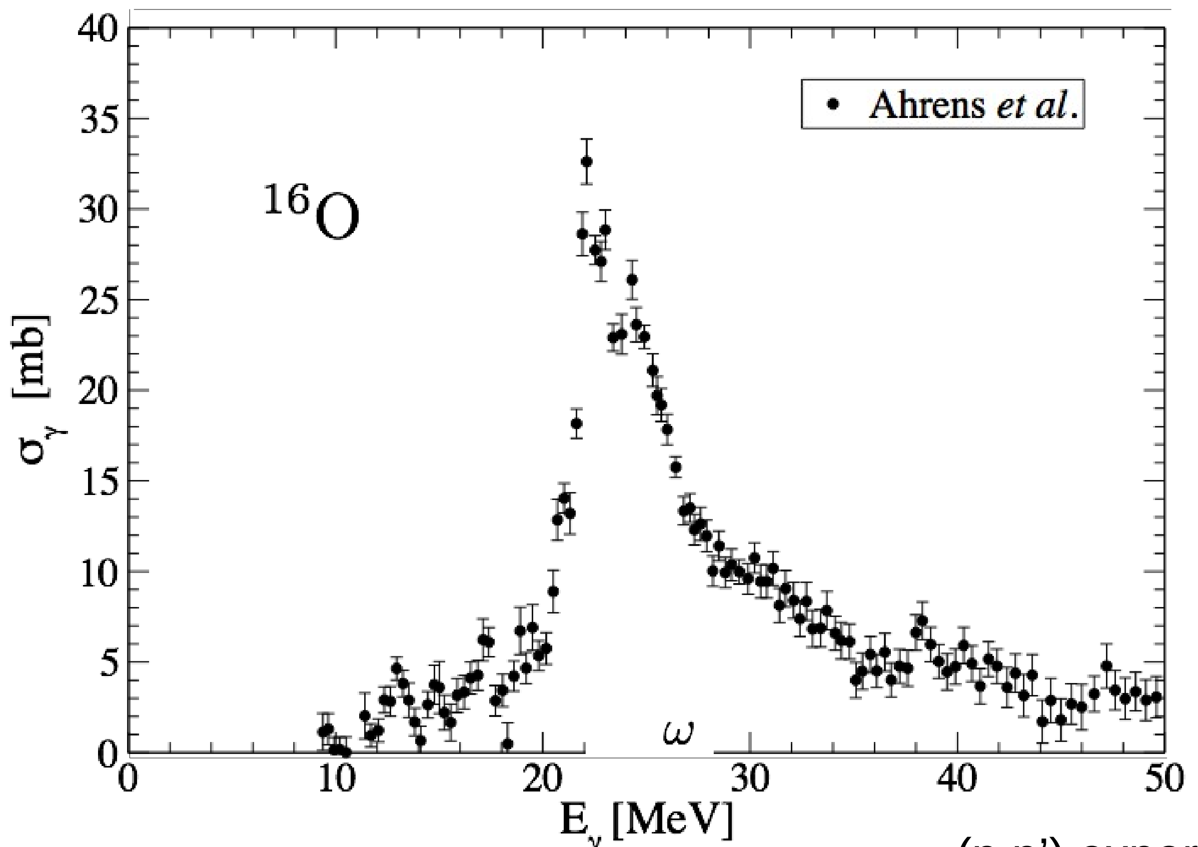




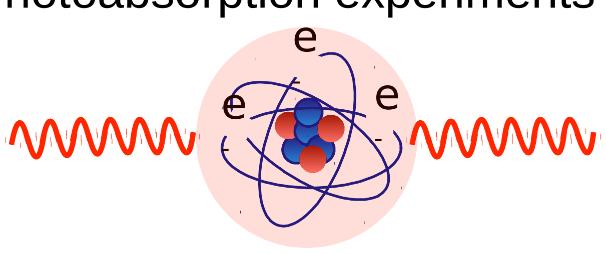
Experimental status

Stable Nuclei

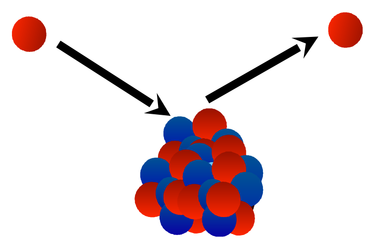
We have data on ~180 stable nuclei
"Giant dipole resonances"



Photoabsorption experiments

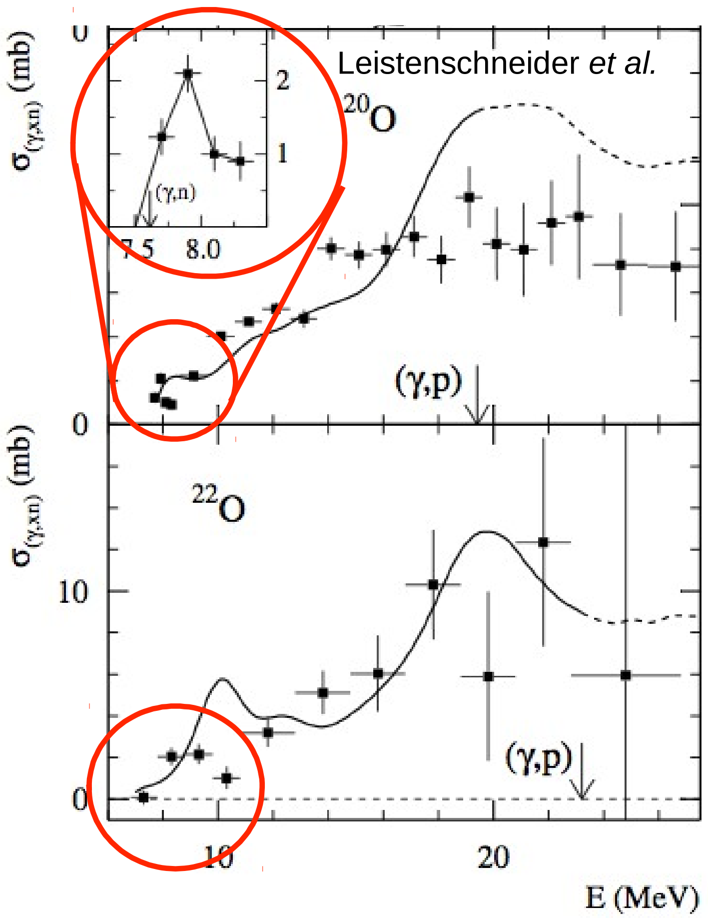


(p,p') experiments

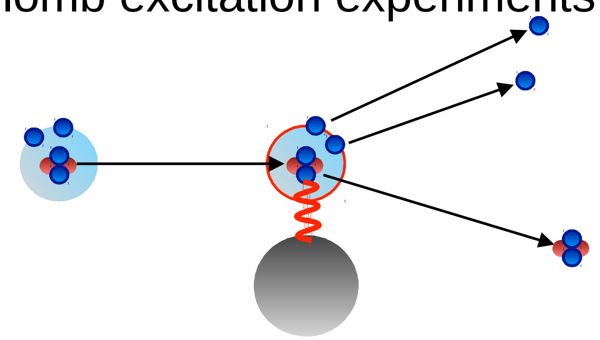


Unstable Nuclei

Few data
"pigmy dipole resonances"



Coulomb excitation experiments



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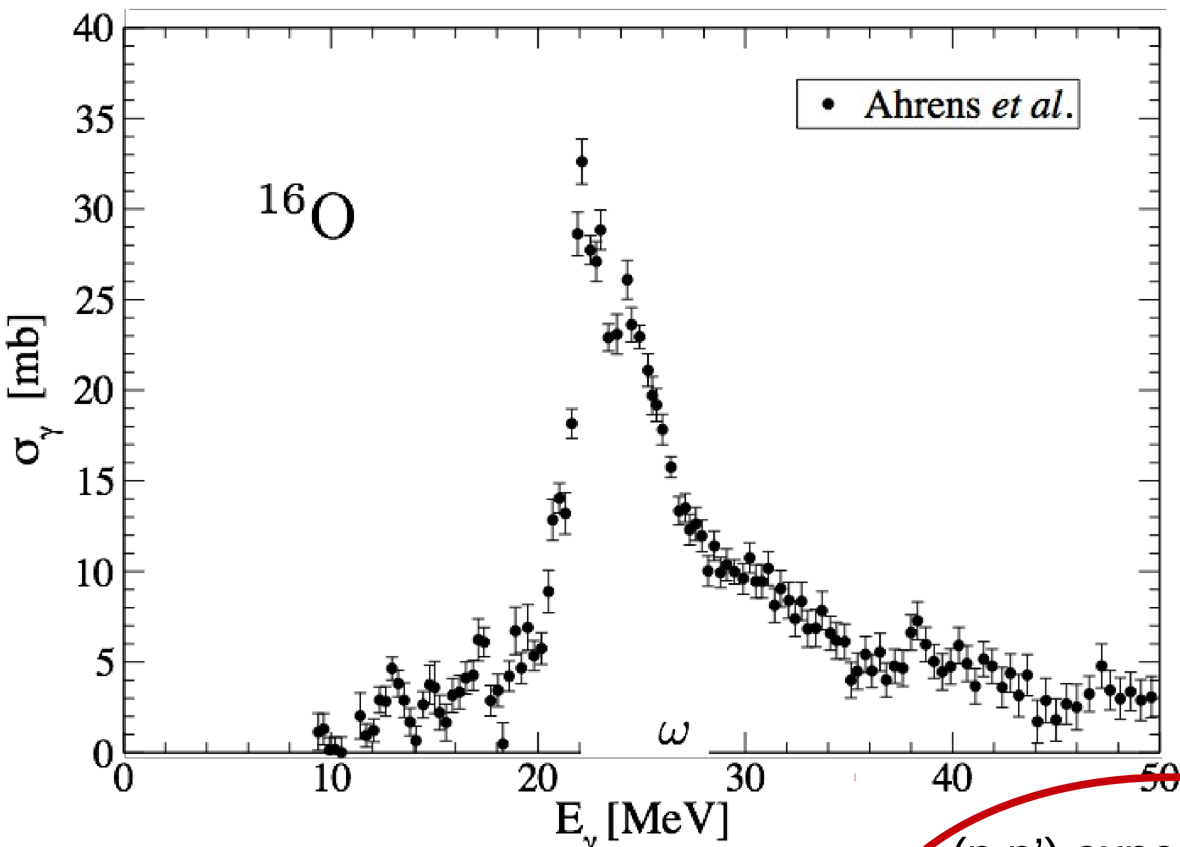

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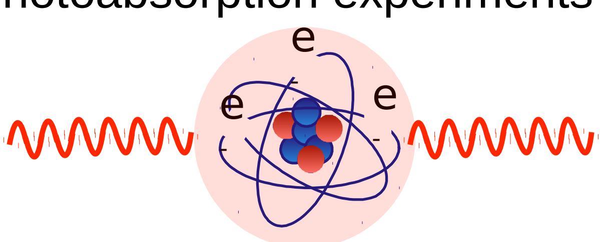

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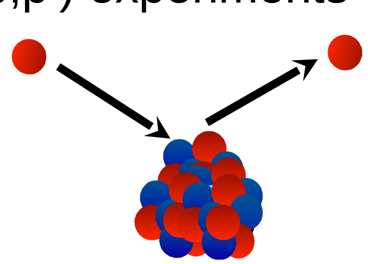

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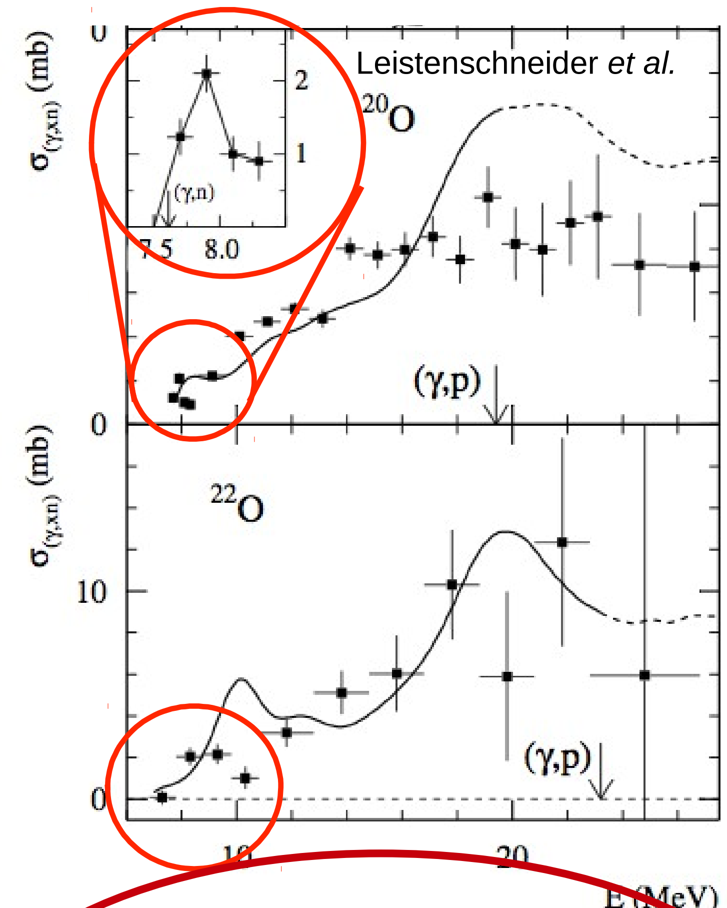

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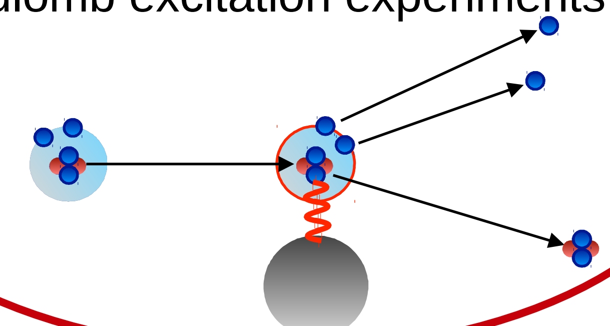

dipole strength “**collective** interpretation”

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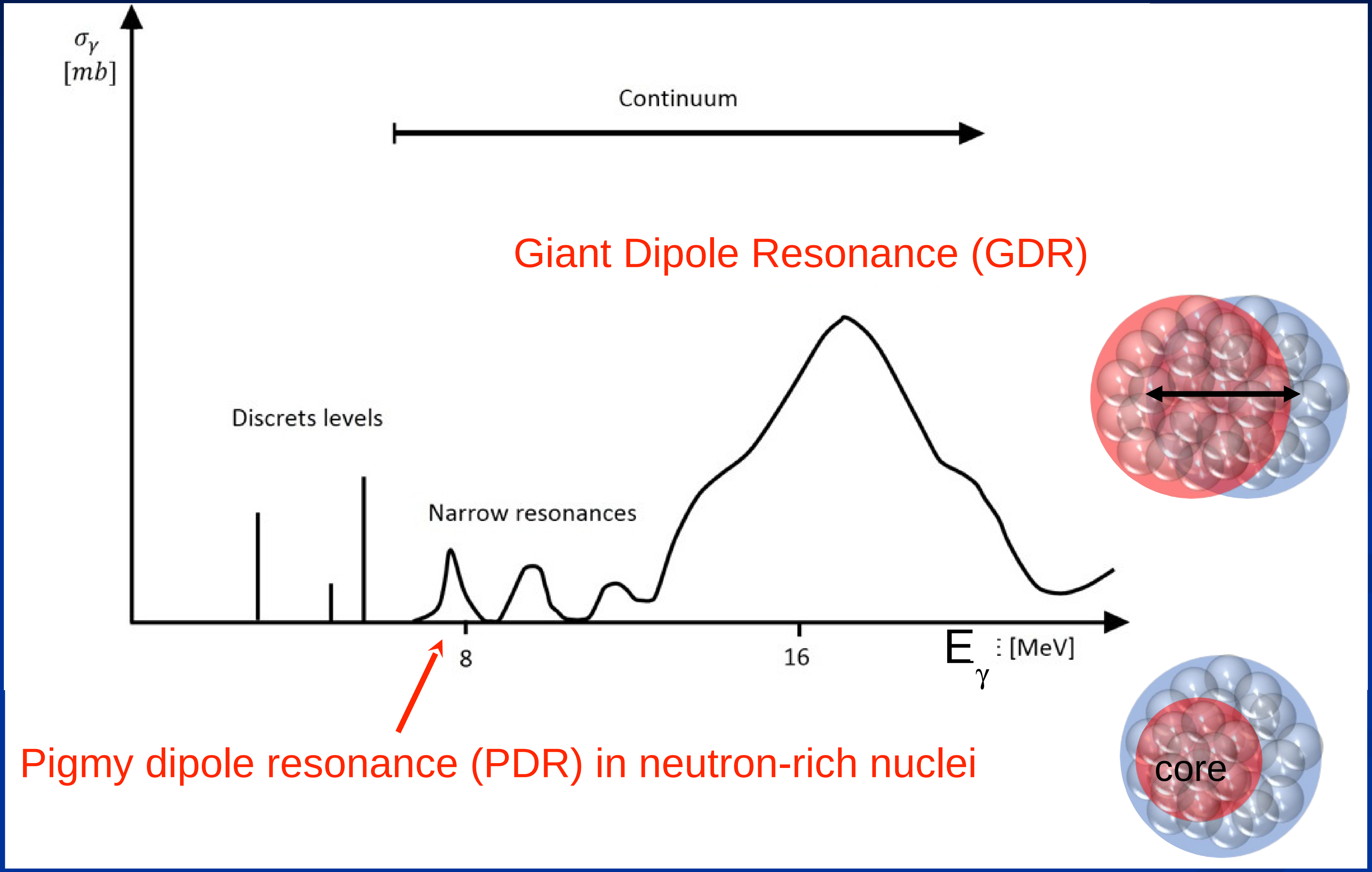


Do we see the emergence of collective modes from first principle calculations?

Reactions to continuum

Framework:

- Energies in the non-relativistic regime
  → Non-Relativistic Quantum Mechanics
  (including ***Translation*** *,* ***Galileian*** *,* ***Rotational*** *invariances)*
  $[H, \mathbf{P}_{cm}]=0$ $[H, \mathbf{R}_{cm}]=0$ $[H, \mathbf{J}]=0$
- Degrees of freedom: total A nucleons
  ("microscopic" model)
- $H = T + \mathbf{V}$

$$\mathbf{V} = \Sigma_{ij}\, v_{ij} + (\, \Sigma_{ijk}\, v_{ijk} + \ldots)$$

Reactions to continuum

perturbative (electro-weak)

- **First order** perturbation theory *(Fermi-Golden Rule)*
- **Linear** Response theory

$$\gamma^{(*)} + b \dashrightarrow b^*$$

$$\sigma(\omega) \sim \sum_n |\langle n | \Theta | 0 \rangle|^2 \quad \delta(\omega - E_n + E_0)$$

$$H | n \rangle = E_n | n \rangle$$

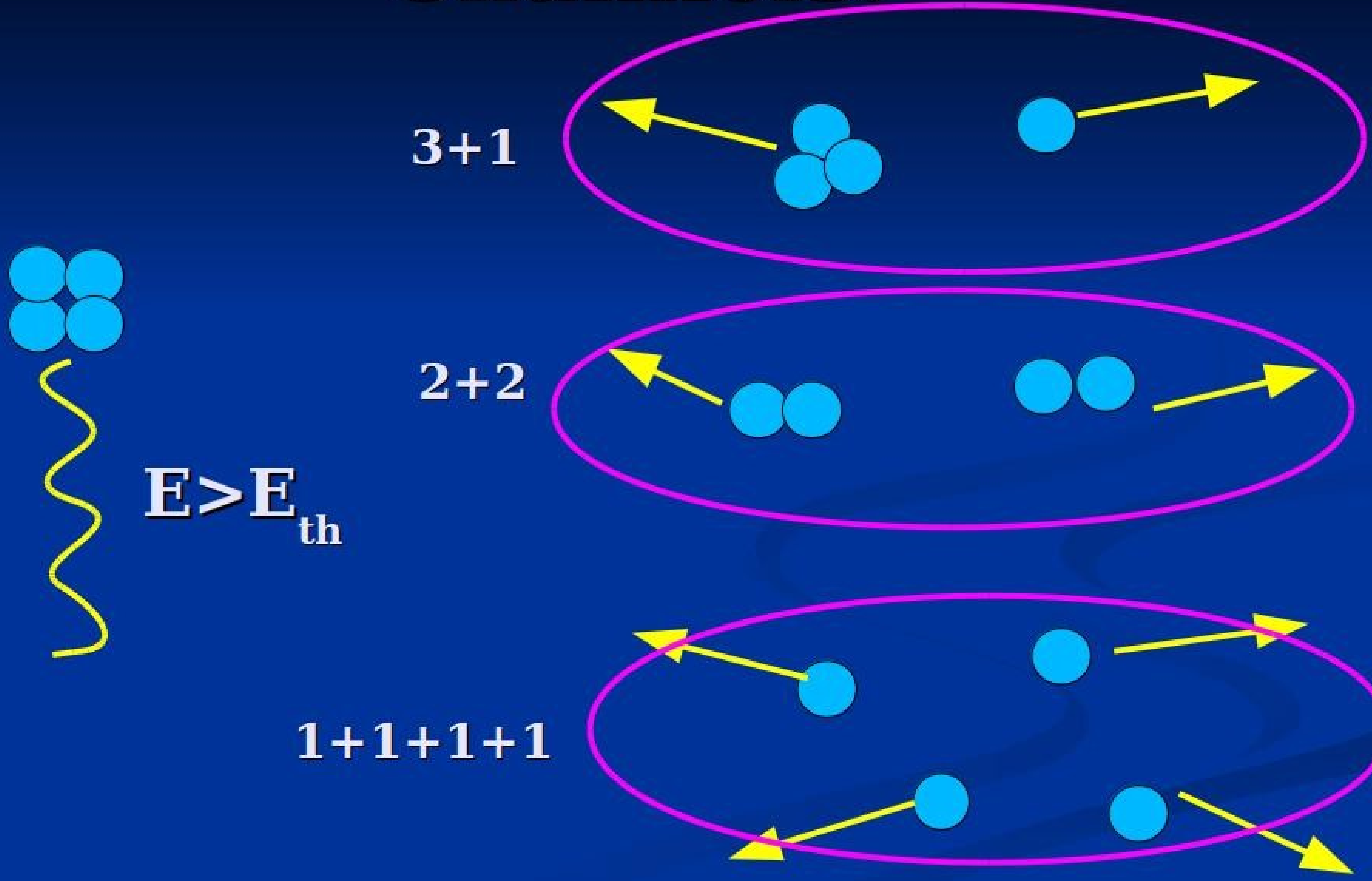
Reactions to continuum

PERTURBATIVE INCLUSIVE

$$S(\omega) = \sum_n |\langle n | \Theta | 0 \rangle|^2 \, \delta(\omega - E_n + E_0)$$

$S(\omega)$ represents the crucial quantity
Requires the solution of both
the bound and continuum A-body problem

Channels:



Integral transform (IT)

$$\Phi(\sigma) = \int d\omega \; K(\omega,\sigma) \, S(\omega)$$

One **IS NOT** able to calculate $S(\omega)$
(the quantity of direct physical meaning)
but **IS** able to calculate $\Phi(\sigma)$

In order to obtain $S(\omega)$ one needs to invert the transform
Problem:
Sometimes the “inversion” of $\Phi(\sigma)$ may be problematic

$$S(\omega) = \sum_n |\langle n|\Theta|0\rangle|^2 \, \delta(\omega - E_n + E_0)$$

$$\Phi(\sigma) = \int S(\omega) \, K(\omega,\sigma) \, d\omega =$$

1) integrate in **dω** using delta function

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1) integrate in **dω** using delta function

$$\Phi(\sigma) = \sum_n K(E_n - E_0, \sigma) < 0 \,|\, \Theta^+ \,|\, n > < n \,|\, \Theta \,|\, 0 >$$

$$S(\omega) = \sum\!\!\!\!\!\!\int_n \left|\langle n|\Theta|0\rangle\right|^2 \delta(\omega - E_n + E_0)$$

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$$= \sum_n \langle 0 | \Theta^+ K(H - E_0, \sigma) | n \rangle \langle n | \Theta | 0 \rangle$$

$$S(\omega) = \sum\!\!\!\!\!\!\int_n |\langle n|\Theta|0\rangle|^2 \, \delta(\omega - E_n + E_0)$$

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$$= \Sigma_n < 0 \,|\, \Theta^+ K(H - E_0, \sigma) \,|\, n > < n \,|\, \Theta \,|\, 0 >$$

2) Use $\Sigma_n | n > < n | = I$

$$\Phi(\sigma) = \langle 0 | \, \Theta^+ K(H - E_0, \sigma) \, \Theta | 0 \rangle$$

$$\Phi(\sigma) = \int S(\omega)\, K(\omega,\sigma)\, d\omega =$$

$$\langle 0|\, \Theta^{+}\, K(H-E_0,\sigma)\, \Theta\,|0\rangle$$

The calculation of **ANY** transform seems to require, **in principle**, only the knowledge of the ground state!

However,

$K(H-E_0,\sigma)$ can be quite a complicate operator.

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So, which kernel is suitable for calculation of this?

$$\Phi(\sigma) = \langle 0|\, \Theta^{+}\, K(H-E_0,\sigma)\, \Theta|0\rangle$$

a **“good” Kernel** has to satisfy **two** requirements

1) one must be able to calculate the integral transform

2) one must be able to invert the transform minimizing uncertainties

Which is the best kernel?

The δ-function!

What would be the "perfect" Kernel?

the delta-function!

in fact

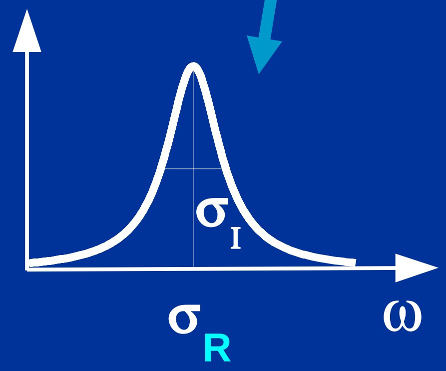
$$\Phi(\sigma) = S(\sigma) = \int \delta(\omega - \sigma)\, S(\omega)\, d\omega$$

… but what about a representation of the δ-function?

The Lorentzian kernel:

σ complex! $\sigma = \sigma_R + i\sigma_I$

$$K(\omega, \sigma) = C\,(\omega - \sigma)^{-1}\,(\omega + \sigma^*)^{-1}$$



It is a representation of the δ-Function

$$\Phi(\sigma_R, \sigma_I) = C \int [(\omega - \sigma_R)^2 + \sigma_I^2]^{-1}\, S(\omega)\, d\omega$$

Illustration of requirement N.1: one can calculate the integral transform

Remember!

$$\Phi(\sigma) = \int S(\omega)\, K(\omega,\sigma)\, d\omega =$$

$$\langle 0|\, \Theta^{+}\, K(H-E_0,\sigma)\, \Theta |0\rangle$$

$$K(\omega,\sigma)$$

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$$\langle 0|\,\Theta^+ (H\text{-}E_0 - \sigma)^{-1} (H\text{-}E_0 - \sigma^*)^{-1}\,\Theta|0\rangle = \langle \tilde{\Psi} | \tilde{\Psi} \rangle$$

main point of the LIT :

Schrödinger-like equation with a source

$$(H - E_0 - \sigma_R - i\,\sigma_I)\,|\tilde{\Psi}> = \Theta|0>$$

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$$(H - E_0 - \sigma_R - i\,\sigma_I) | \tilde{\Psi} > = \Theta | 0 >$$

Theorem:

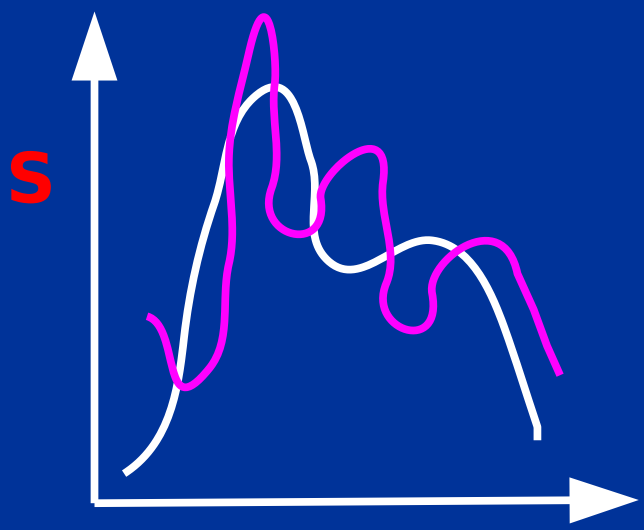
The $\tilde{\Psi}$ solution is unique and has ***bound state*** asymptotic

conditions → one can apply ***bound state methods***

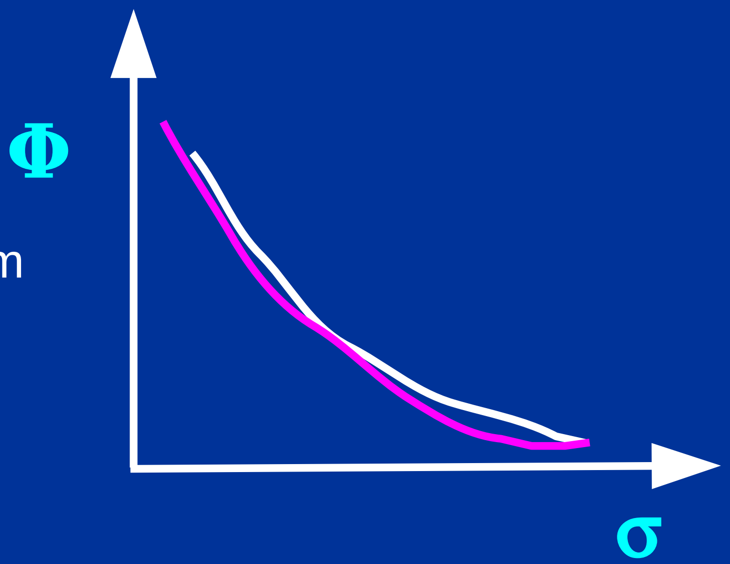
Illustration of requirement
N.2: one can invert the integral transform minimizing uncertainties

Illustration of the problem of inversion:

Suppose that $K(H-E_0,\sigma)=e^{-(H-E_0)\sigma}$



Laplace transform



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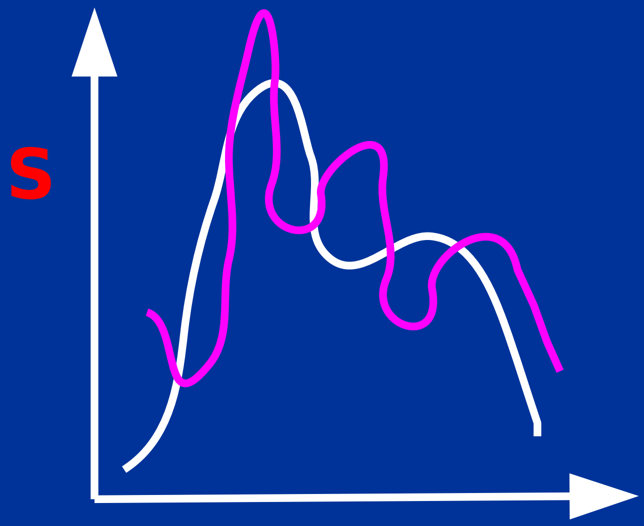

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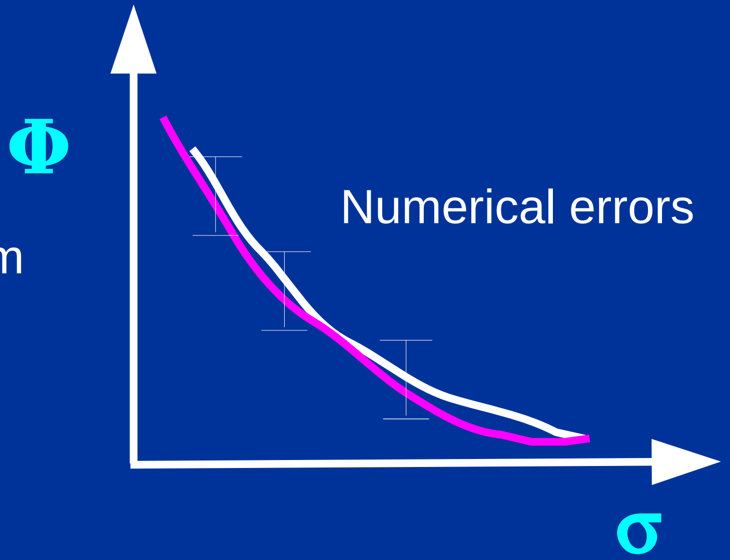

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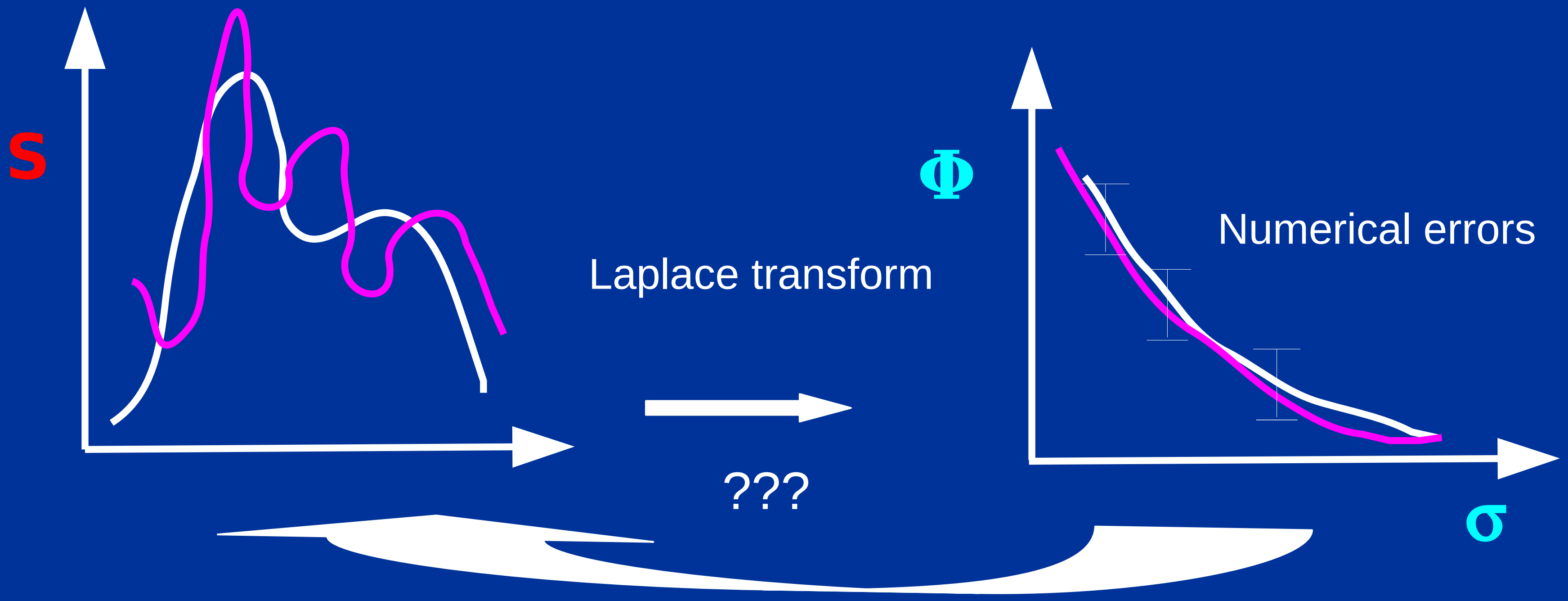

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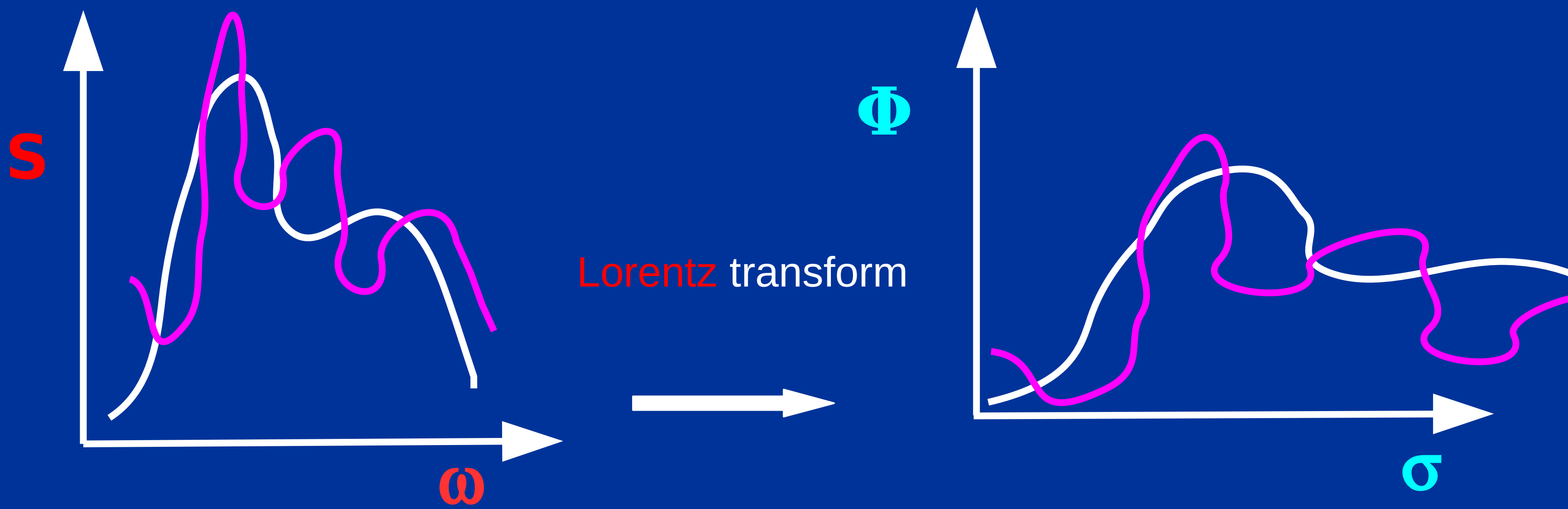

blurred, but still distinguishable

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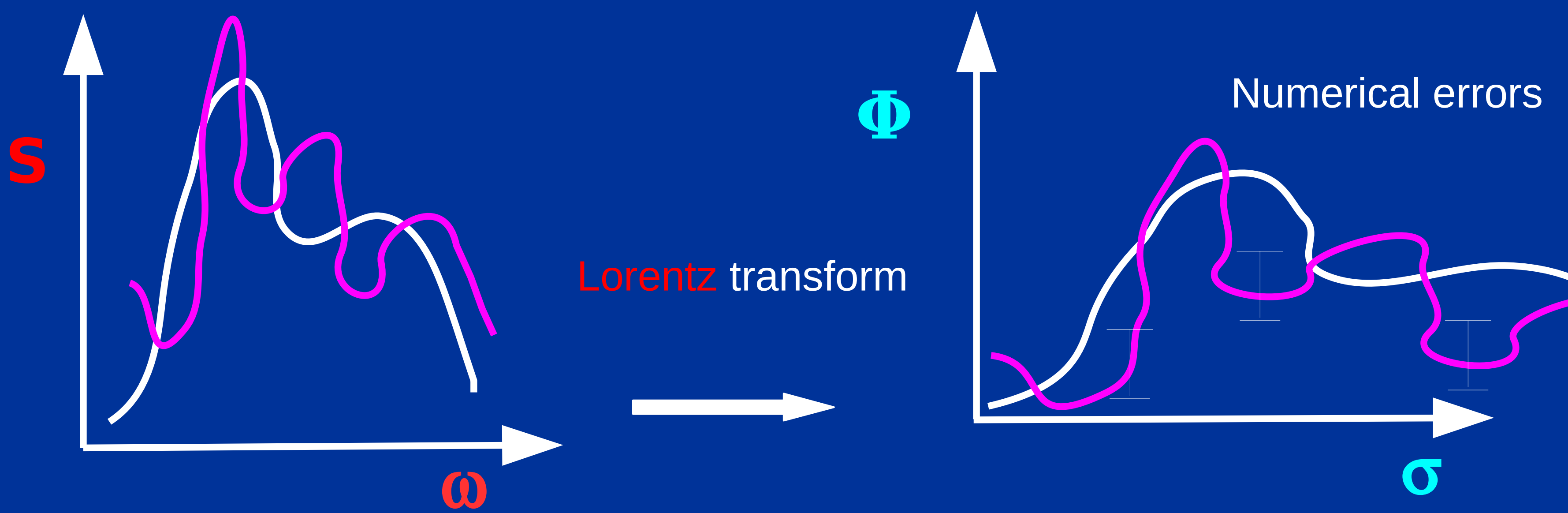
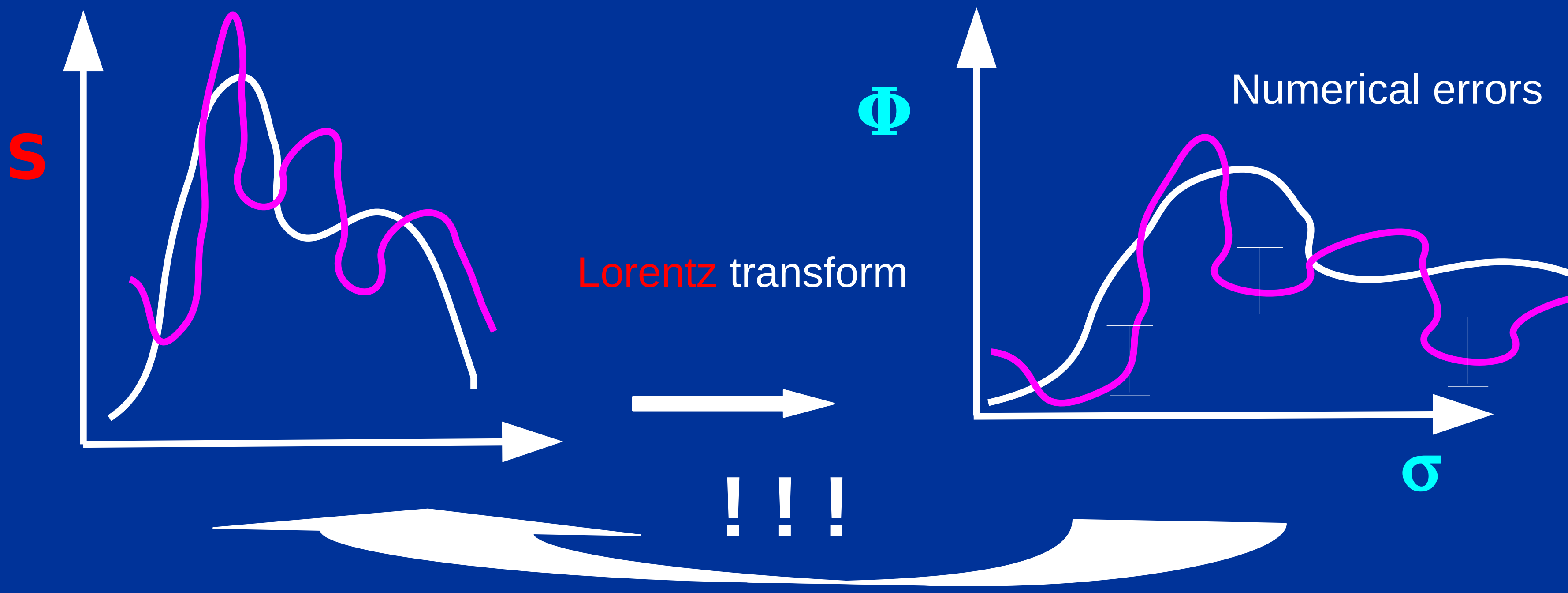


blurred, but still distinguishable also with errors!

How can one easily understand why the inversion is **much less** problematic if

$K(H-E_0,\sigma)$= lorentzian

Inversion: e.g. “regularization method” at fixed width



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The $\tilde{\Psi}$ solution is unique and has ***bound state*** asymptotic conditions

→ **one can apply *bound state methods***

bound state methods:

$$(H - E_0 - \sigma_R - i\,\sigma_I) \,|\, \tilde{\Psi} > \, = \Theta|0>$$

Represent $H, \;\; |\,\tilde{\Psi}>, \;\; \Theta|0>$ ***on a complete b.s. basis***

and invert the linear problem

A very efficient basis for few-body systems:

Hyperspherical Harmonics (HH)

[generalization to Spherical Harmonics Y_{lm}

to a 3(A-1) dimensional space]

Photodisintegration of ^{4}He

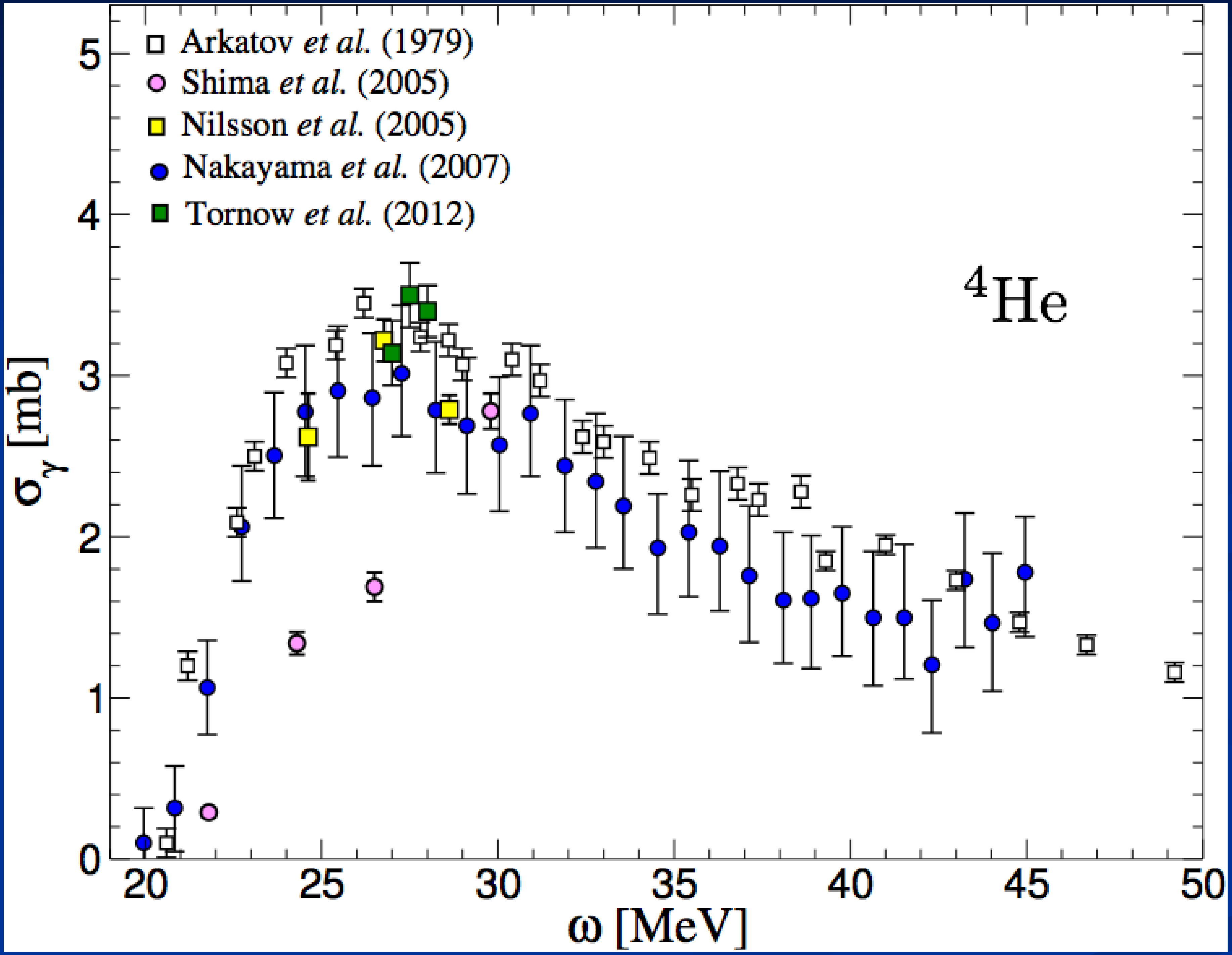


Figure from Bacca and Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

Photodisintegration of ^{4}He

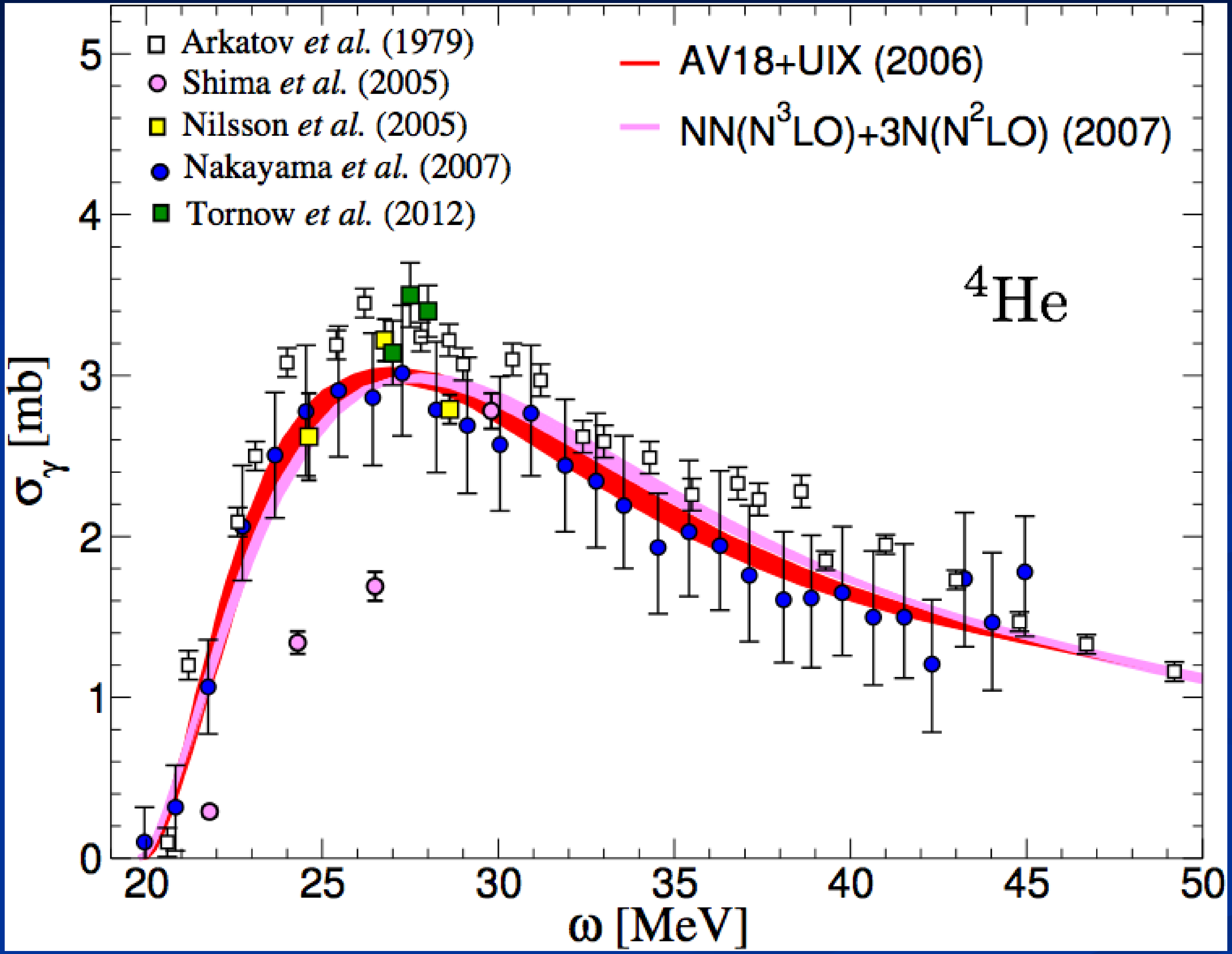


Figure from Bacca and Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

What about many-body systems?

LIT Lorentz Integral Transform

A method that allows to circumvent the continuum problem by reducing it to the solution of a bound-state-like equation

+

CC Coupled-cluster theory

Accurate many-body theory with mild polynomial scaling in mass number

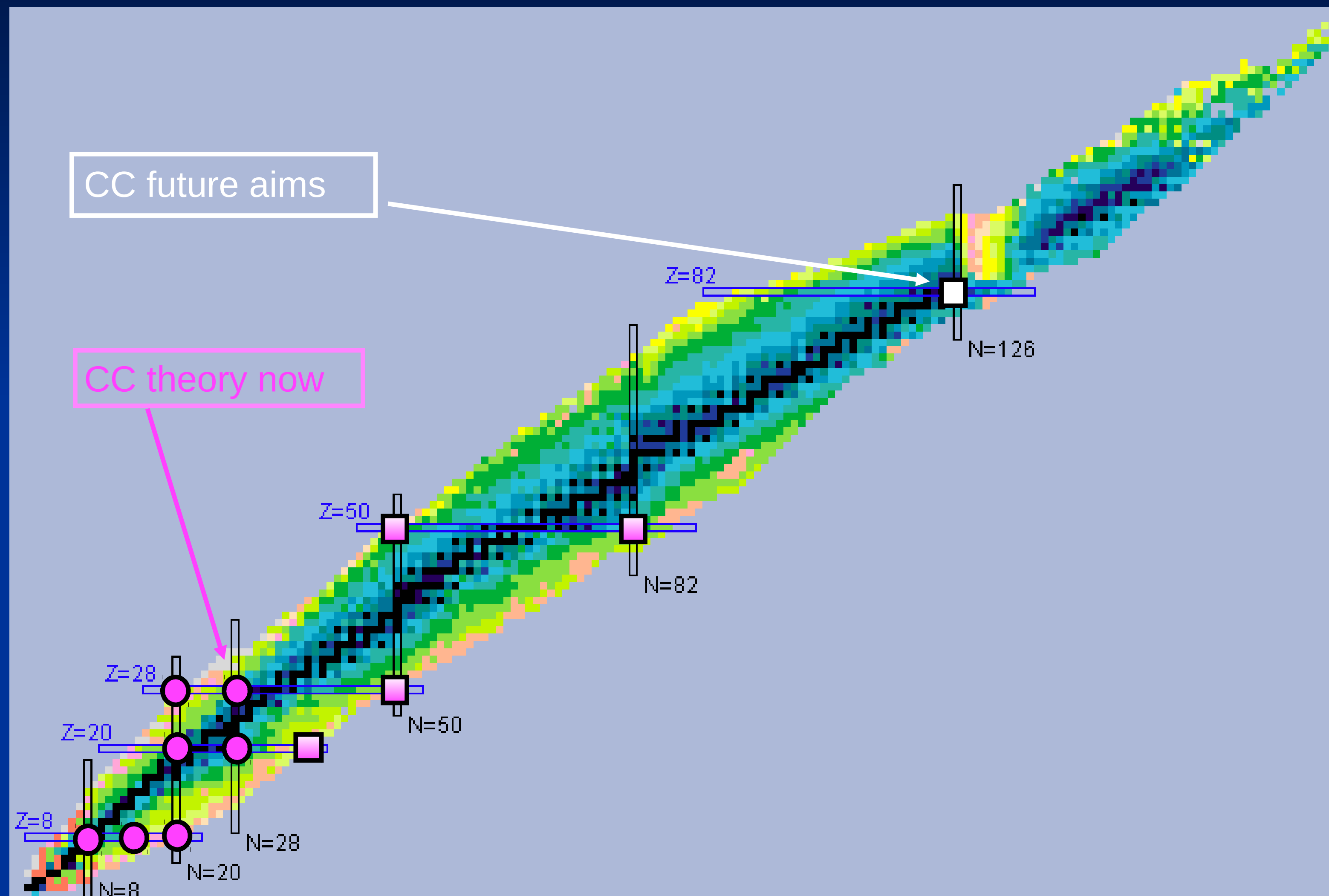
=

LIT-CC

An approach to many-body break-up induced reactions with a proper accounting of the continuum

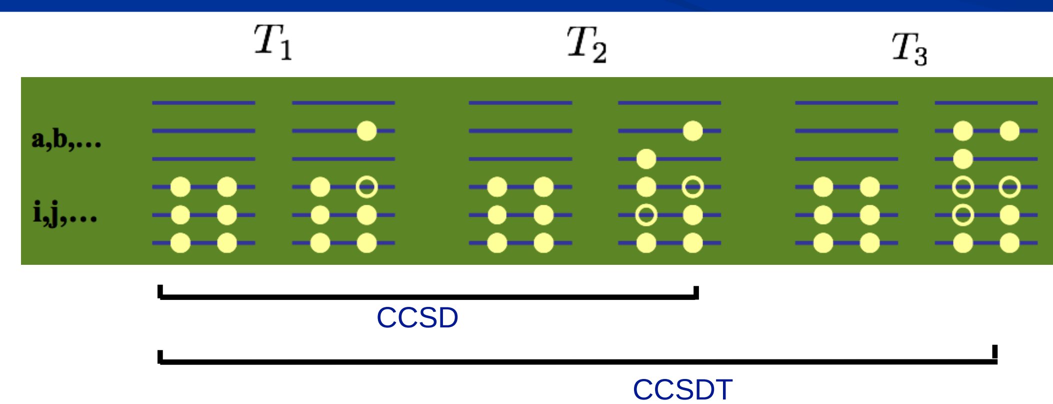
Coupled-cluster theory

Many-body method that can extend the frontiers of ab-initio calculations to heavier and neutron nuclei



$$|\psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle$$

$$T = \sum T_{(A)}$$ cluster expansion



Coupled-cluster theory formulation of LIT

Phys. Rev. Lett. **111**, 122502 (2013)

$$(\bar{H} - E_0 - \sigma + i\Gamma)|\tilde{\Psi}_R\rangle = \bar{\Theta}|\Phi_0\rangle$$

$$\bar{H} = e^{-T} H e^{T}$$

$$\bar{\Theta} = e^{-T} \Theta e^{T}$$

$$|\tilde{\Psi}_R\rangle = \hat{R}|\Phi_0\rangle$$

Results with implementation at CCSD level

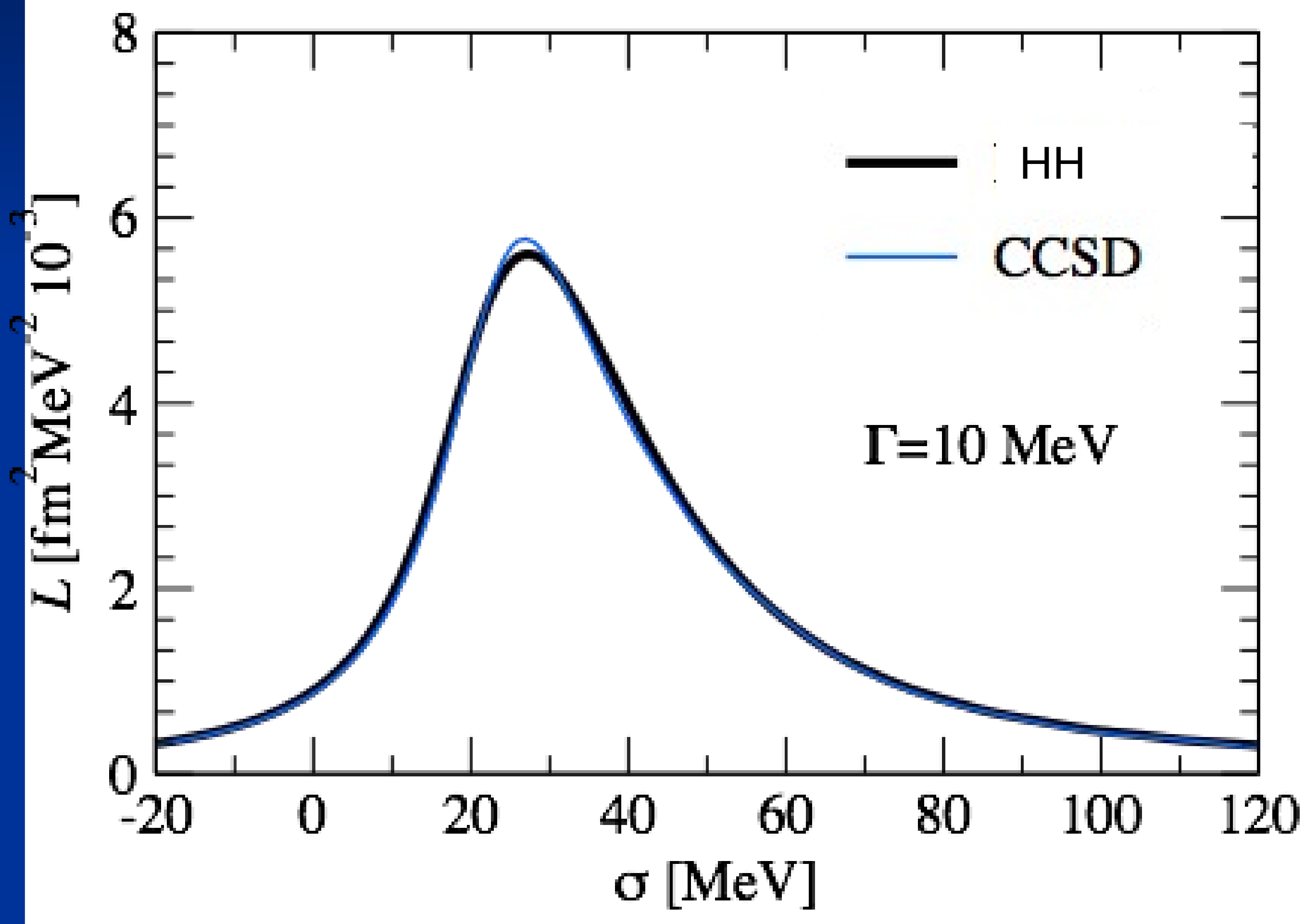
$$T = T_1 + T_2$$

$$R = R_0 + R_1 + R_2$$

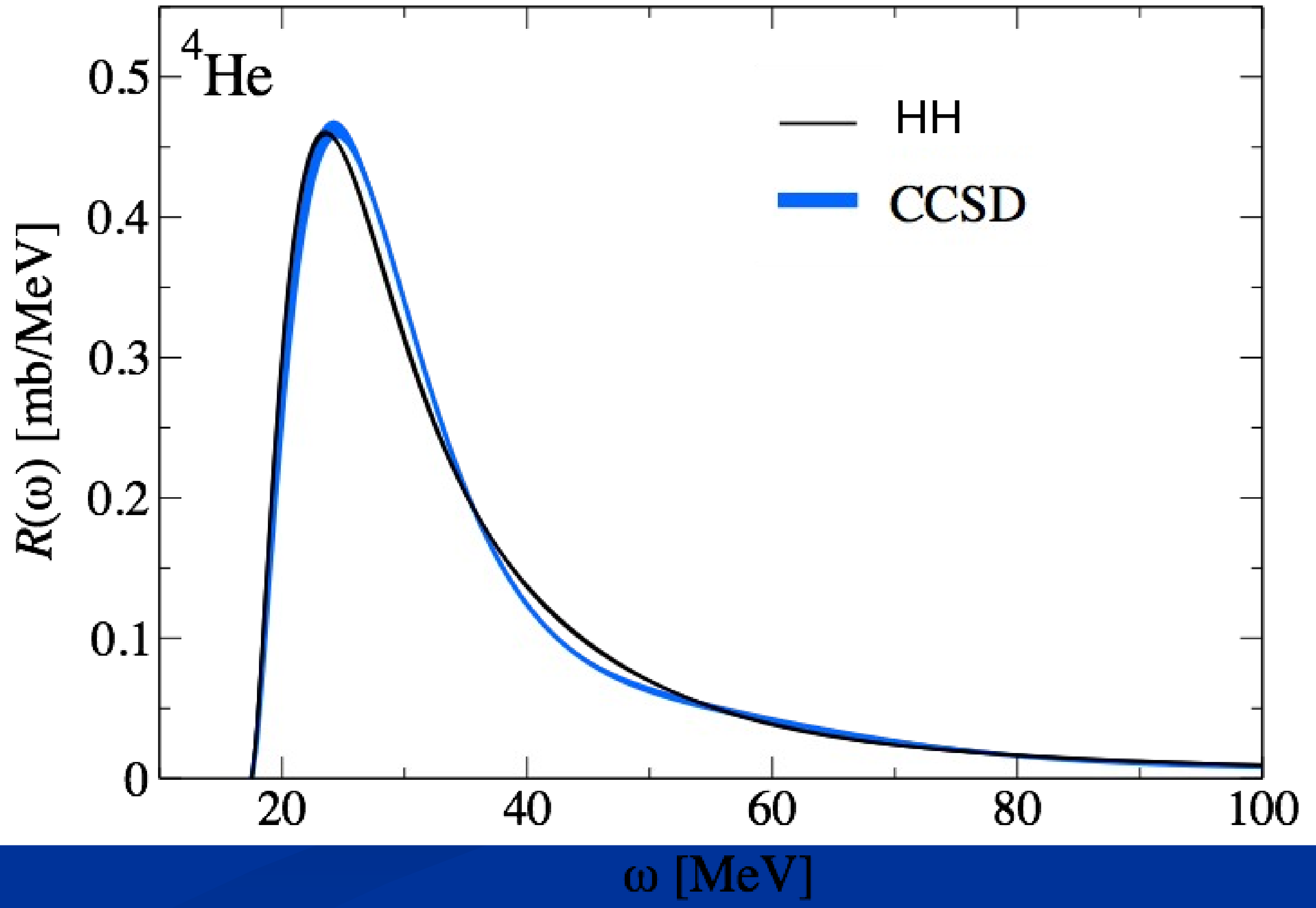
Benchmark

Validation for ^{4}He

Comparison of CCSD with exact hyperspherical harmonics with NN forces at N^{3}LO



The comparison with exact theory is very good

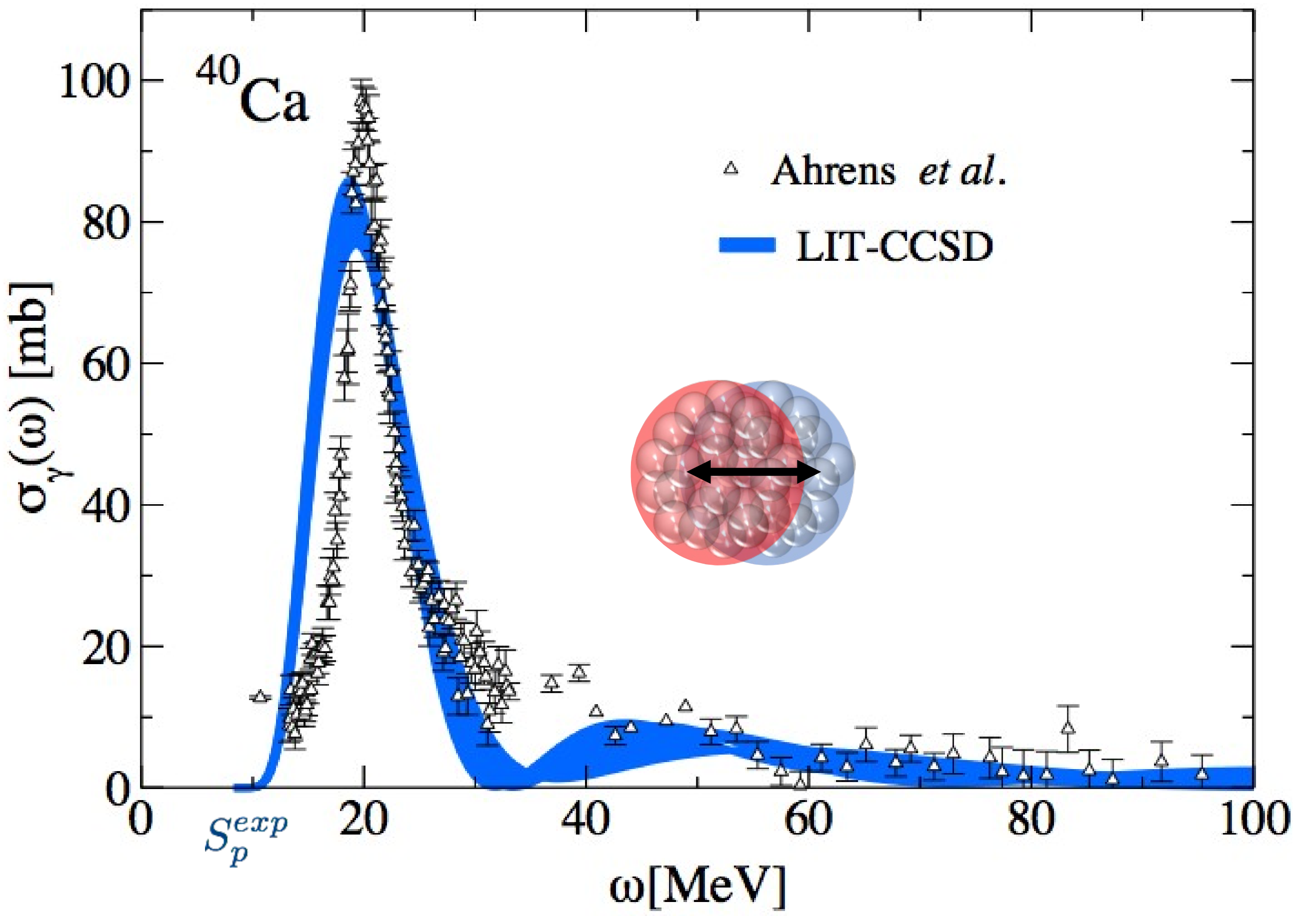


Photonuclear reactions

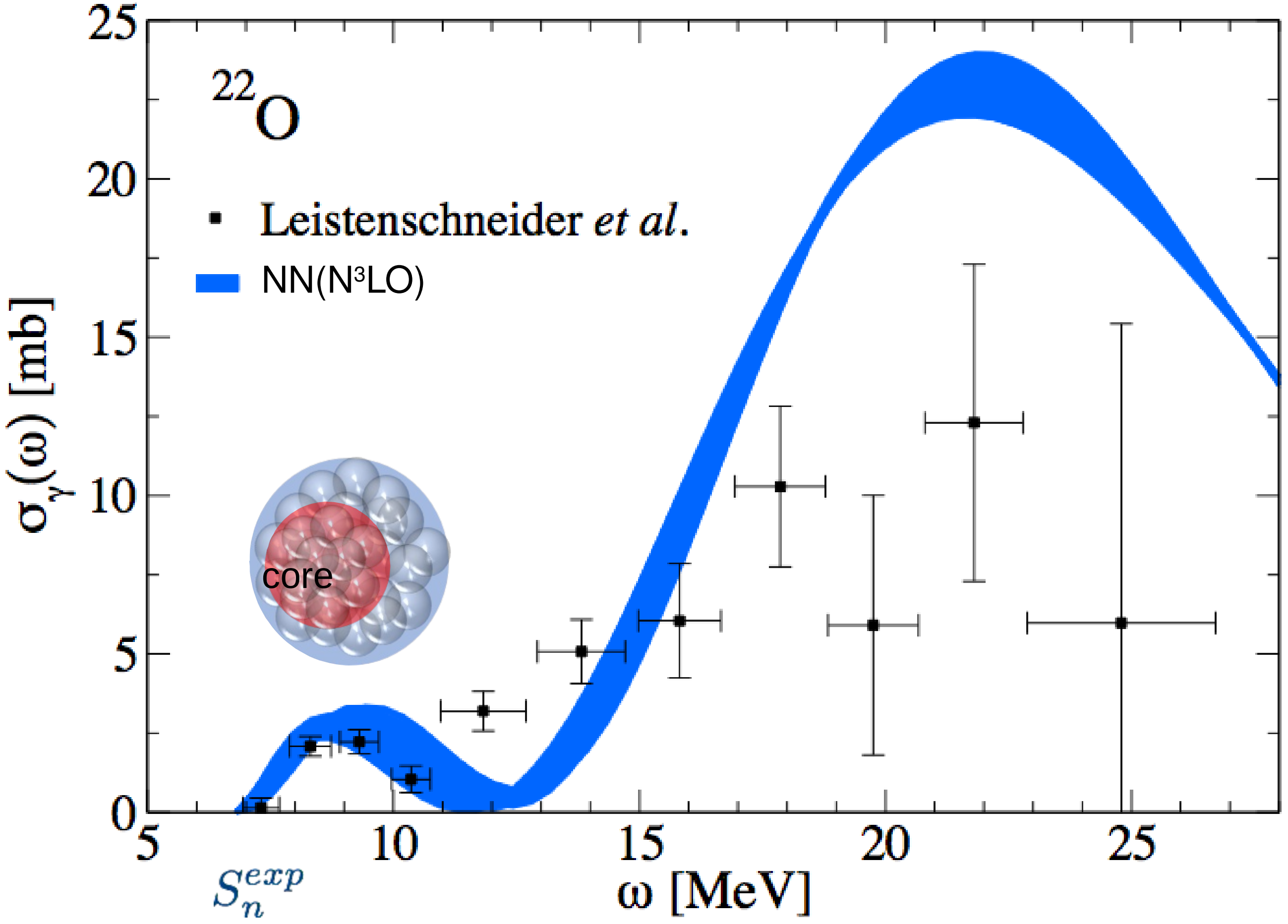
Dipole Response Functions with NN forces from chiral EFT (N^3LO)

PRC **90**, 064619 (2014)

Stable nuclei



Unstable nuclei



Another I.T. with a different Kernel:

The Stieltjes Kernel

$$K(\omega, \sigma) = (\omega + \sigma)^{-1}$$

$\sigma > 0$, real

It may be useful
for a specific purpose:

In fact:

given $S(\omega) = \sum\!\!\!\!\!\!\int_n |\langle n|\Theta|0\rangle|^2 \, \delta(\omega - E_n + E_0)$

$$\lim_{\sigma \to 0} \Phi(\sigma) = \lim_{\sigma \to 0} \int S(\omega)\,(\omega+\sigma)^{-1}\, d\omega = \int \frac{S(\omega)}{\omega}\, d\omega = 2\alpha_\Theta$$

"generalized polarizability"

e.g. electric polarizability, magnetic susceptibility,

compressibility etc... depending on Θ

Main point of the Stieltjes Transform :

Schrödinger-like equation with a source

$$(H - E_0 + \sigma) | \tilde{\Psi} > = \Theta |0>$$

$$\sigma > 0$$

Theorem:

The $\tilde{\Psi}$ solution is unique and has ***bound state*** asymptotic conditions

→ **one can apply *bound state methods***

bound state methods:

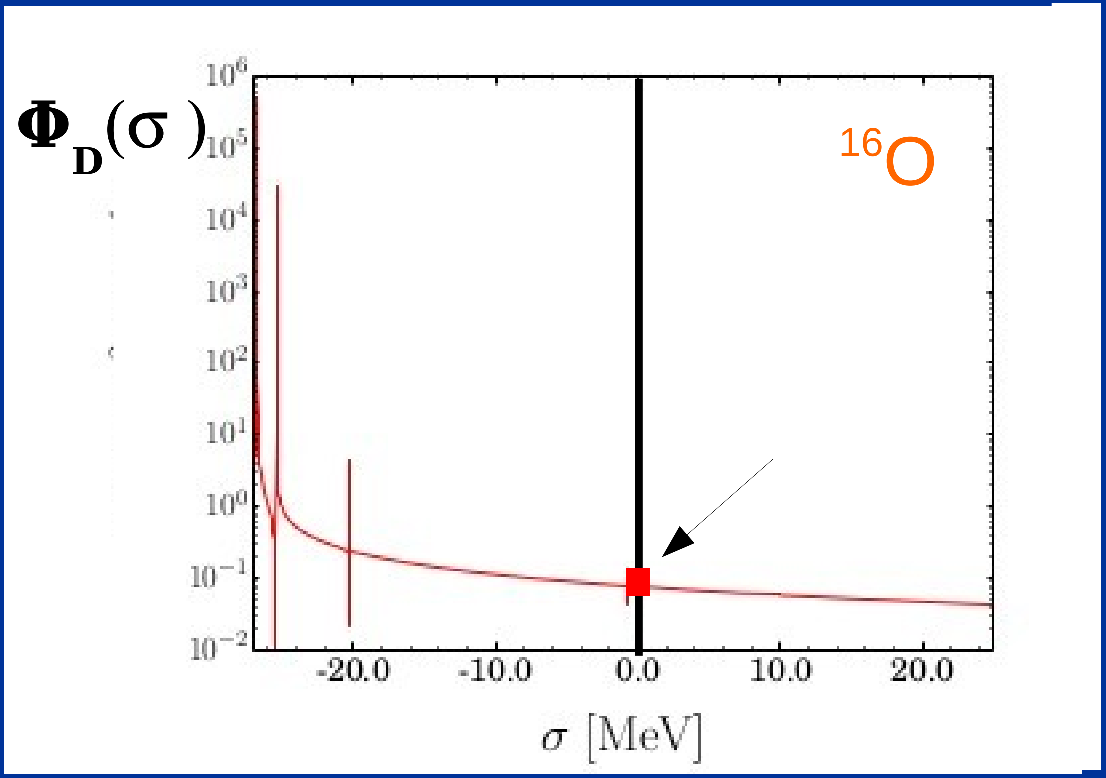
Represent $H, |\tilde{\Psi}>, \Theta|0>$

on a complete b.s. basis

and invert the linear problem

Recent results on α_Θ with $\Theta = D$ (El. Dipole Polarizability)

Electric Dipole Polarizability as limit of the Stieltjes transform for σ ---> 0



b.s. expansion: Coupled Cluster

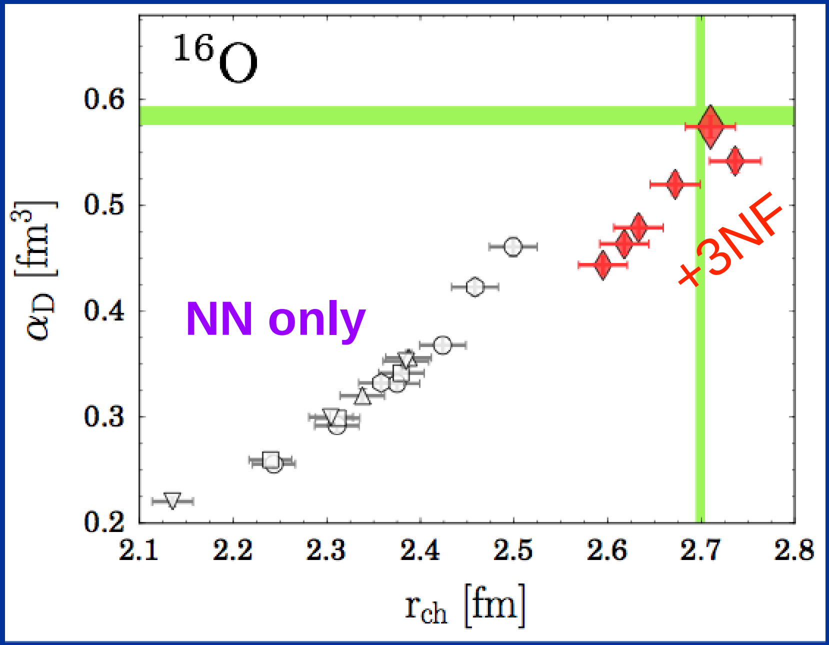
(non hermitian) Lanczos diagonalization

Electric dipole polarizability

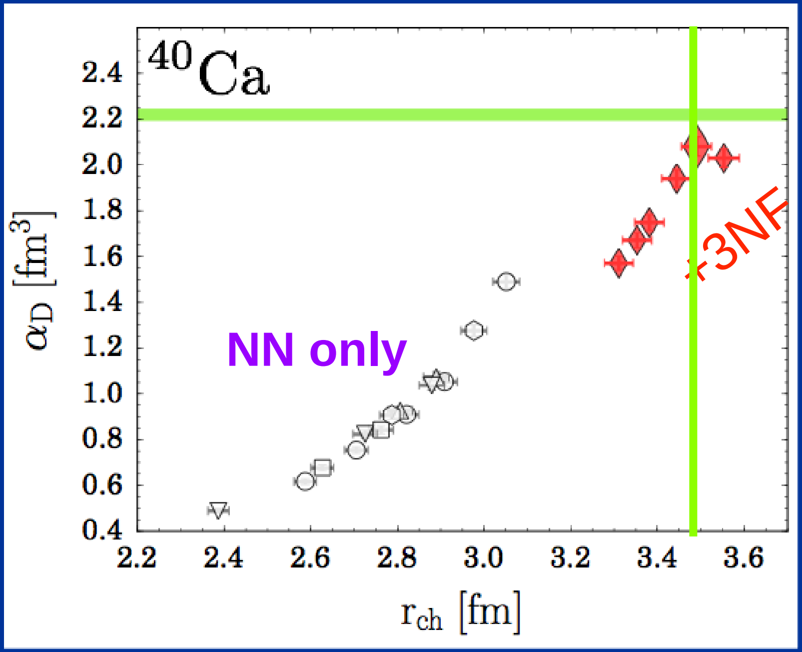
M. Miorelli *et al.*, PRC **94** 034317 (2016)

$$\int \frac{S^D(\omega)}{\omega}\, d\omega = 2\alpha_D$$

Interesting correlation with the proton charge radius



Role of 3b-force



G. Hagen et al. Nature Phys. 2016

A. Ekström *et al*., Phys. Rev. C91, 051301 (2015)

K. Hebeler *et al*., Phys. Rev. C83, 031301 (2011)

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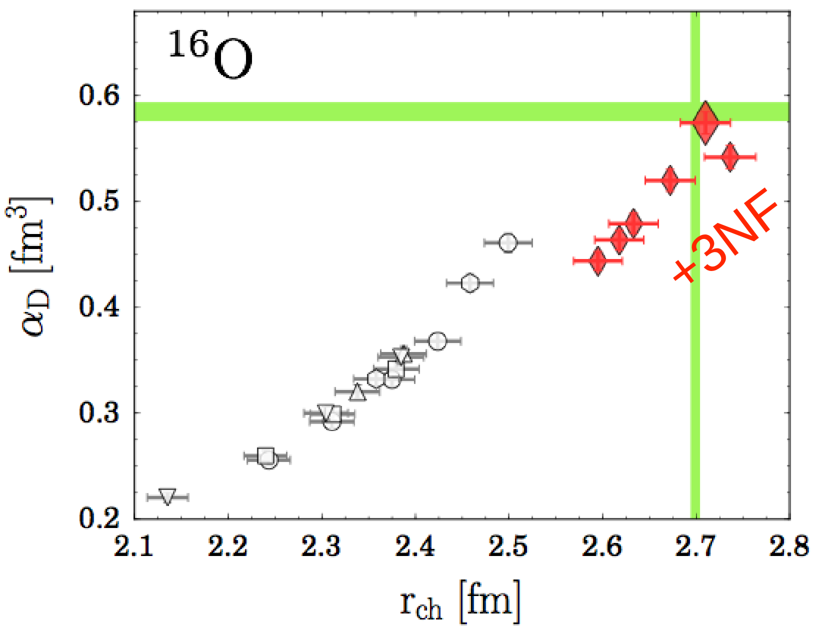

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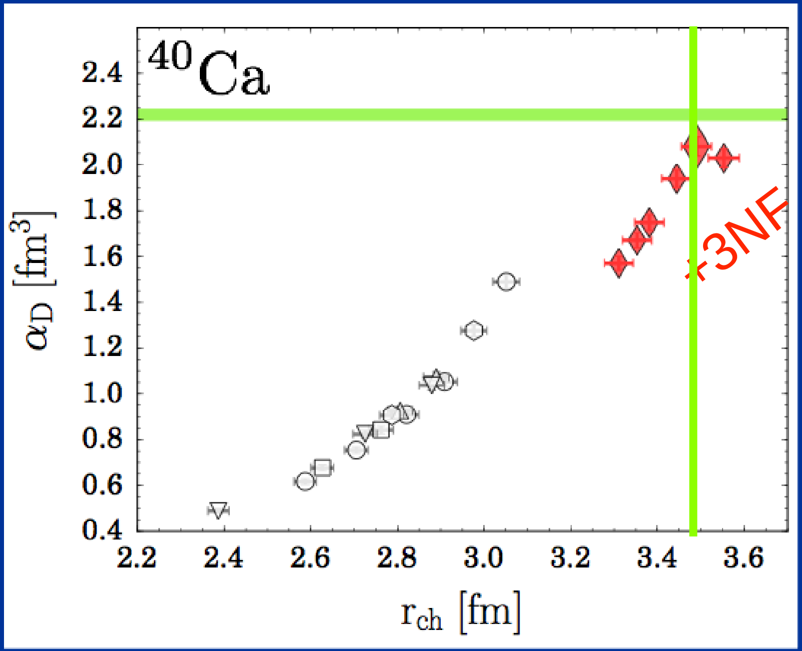


- Much better agreement with experimental data with 3NF
- Variation of Hamiltonian can be used to assess **the theoretical error bar**

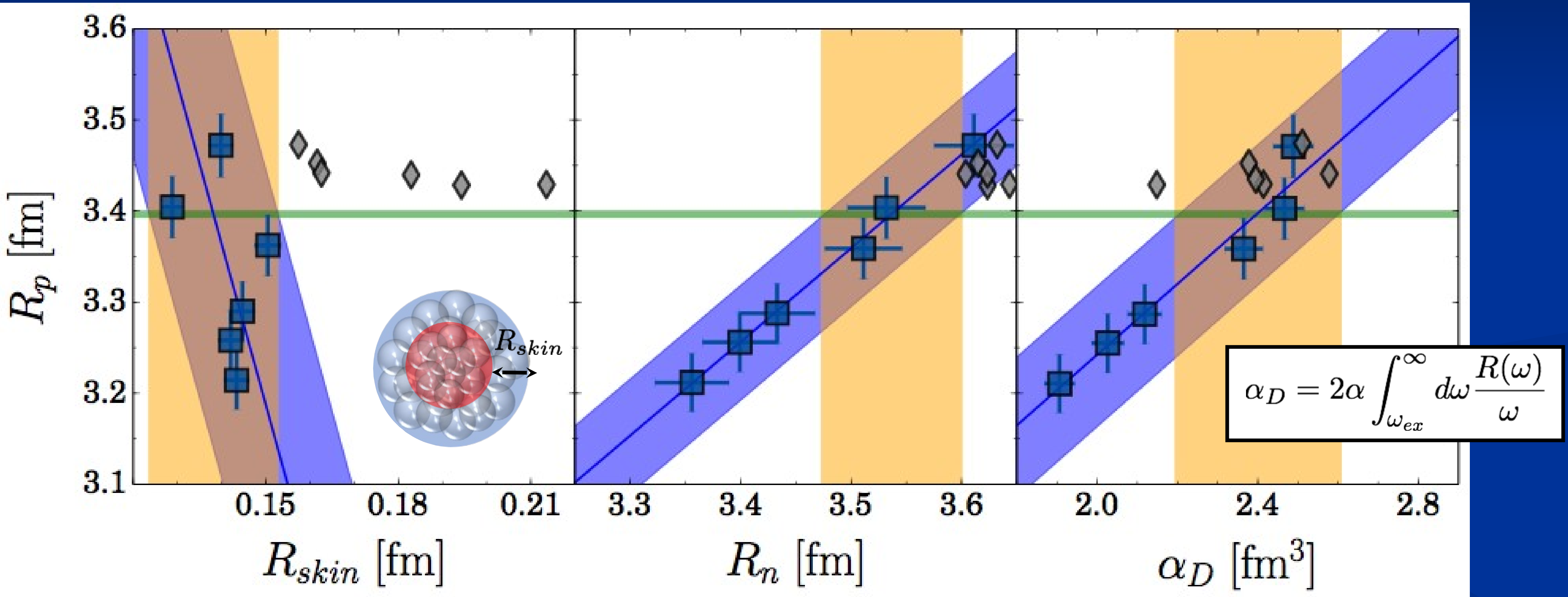
^{48}Ca from first principles

^{48}Ca from first principles

Theory provides predictions for future experiments

International collaboration (USA/Canada/Europe/Israel) using coupled-cluster theory

Hagen *et al.*, Nature Physics 12, 186 (2016)



Ab initio with three nucleon forces from chiral EFT; Density Functional Theory

^{48}Ca from first principles

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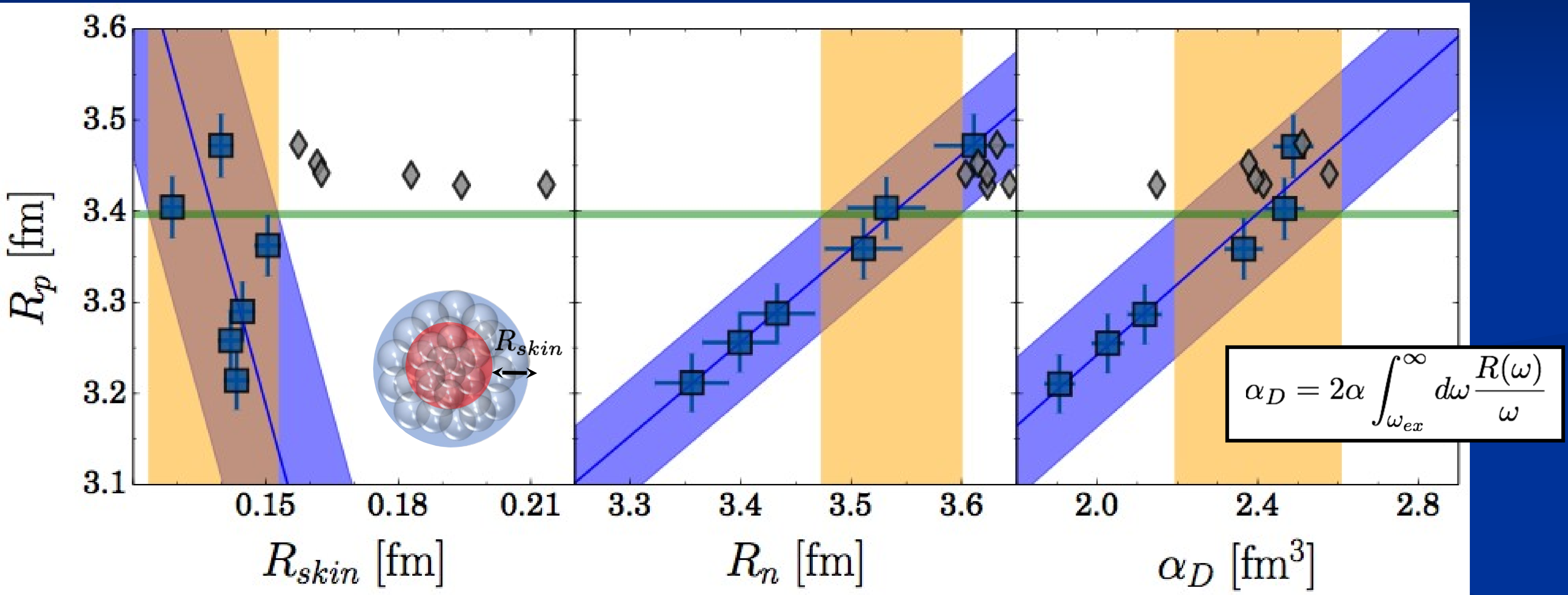

■ Ab initio with three nucleon forces from chiral EFT ◆ Density Functional Theory

Strong correlations with R_p allow to put narrow constraints to R_{skin} and α_D

Ab-initio predictions:

$$0.12 \leq R_{\text{skin}} \leq 0.15 \text{ fm}$$

$$2.19 \leq \alpha_D \leq 2.60 \text{ fm}^3$$

^{48}Ca from first principles

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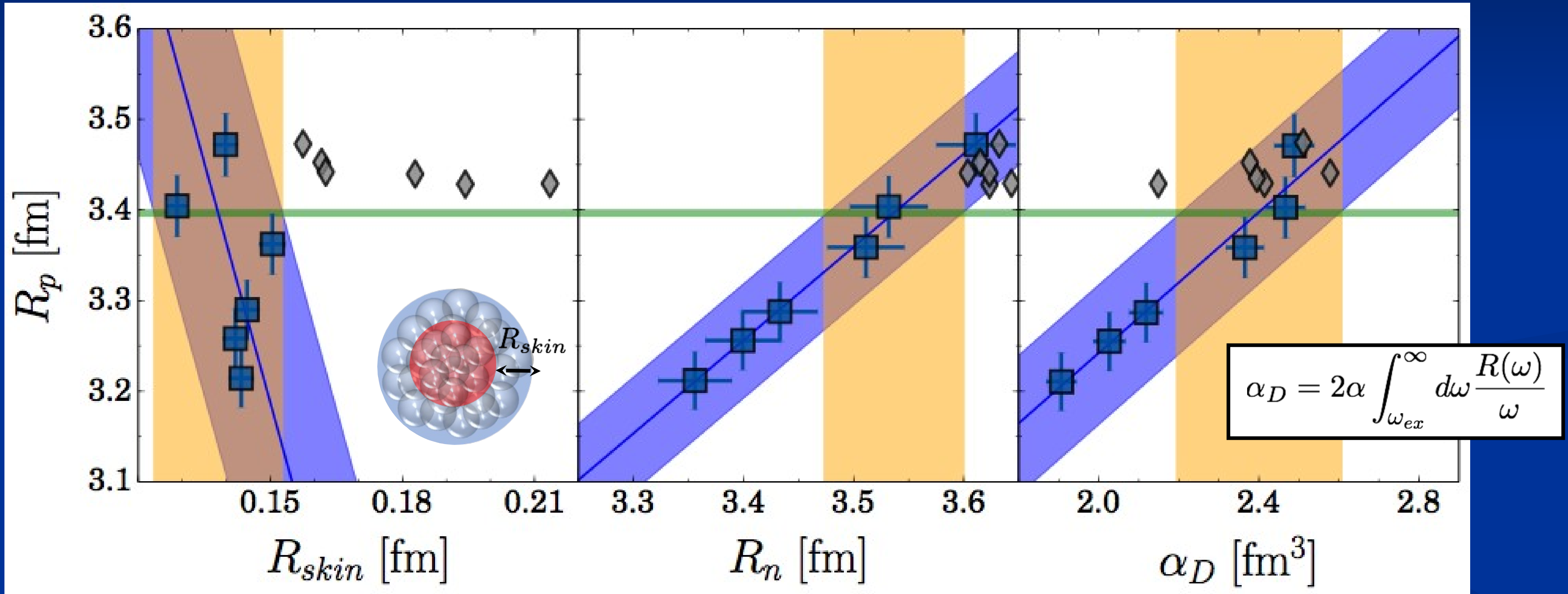

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R_{skin} will be measured at JLab and Mainz with Parity violation electron scattering (CREX and MREX)

Summary:

- The electromagnetic probe is a “clean” mean to investigate nuclear dynamics (pertubation theory is valid)
- **Ab initio** methods help building the bridge between QCD and nuclear phenomena: (**what is the “effective” V ???**)
- They are moving from the traditional few-body (A=2-4) regime to larger systems
- **Integral transform methods** are alternative approaches to overcome the many-body scattering problem