

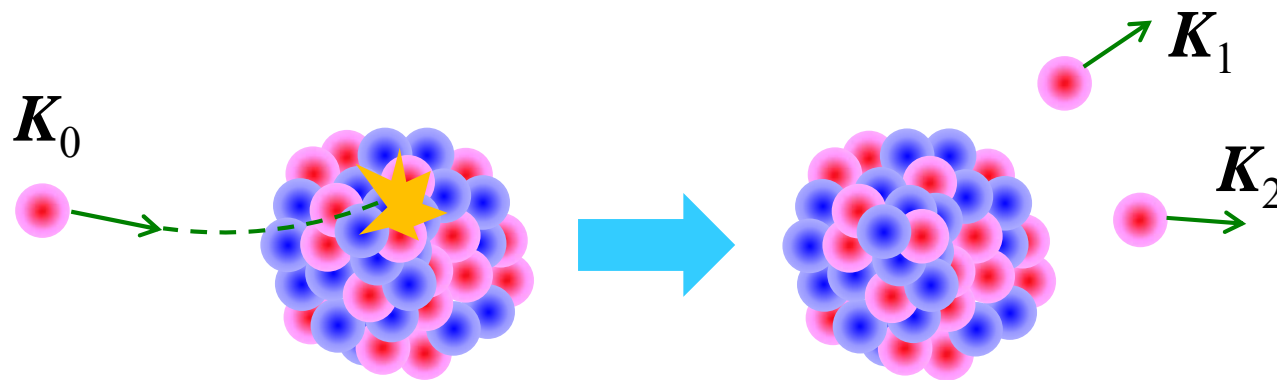
Description of proton-induced inclusive knockout reactions

ECT* workshop on

“Recent advances and challenges in the description of nuclear reactions at the limit of stability”

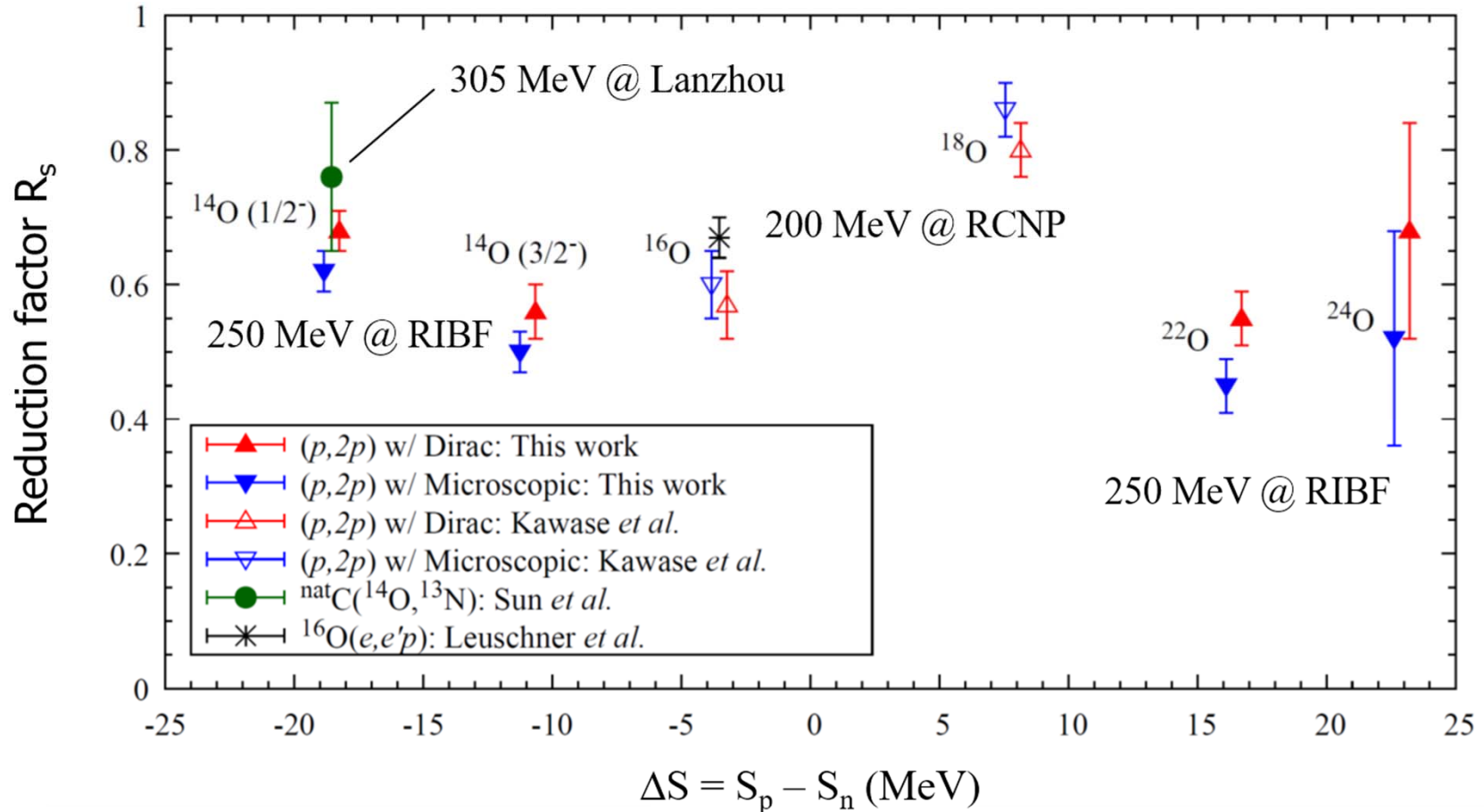
Kazuyuki Ogata

Research Center for Nuclear Physics (RCNP), Osaka University



Recent result from RIBF/RCNP

S. Kawase+, PTEP2018, 021D01 (2018).



- ✓ Collaboration ongoing with R. Tang+ ($^{23,25}\text{F}$), R. Taniuchi+ (^{79}Cu), and L. Olivier+ (^{80}Zn) ...
- ✓ Benchmark study on DWIA with M. Gomez-Ramos, A. M. Moro, K. Yoshida.

Plan of this talk

I. Multistep direct process and quasi-free scattering

II. The semiclassical distorted wave (SCDW) model and its applications

Y. L. Luo and M. Kawai, PRC43, 2367 (1991).

M. Kawai and H. A. Weidenmueller, PRC45, 1856 (1992).

Y. Watanabe et al., PRC59, 2136 (1999).

Sun Weili et al., PRC60, 064605 (1999).

KO et al., PRC60, 054605 (1999).

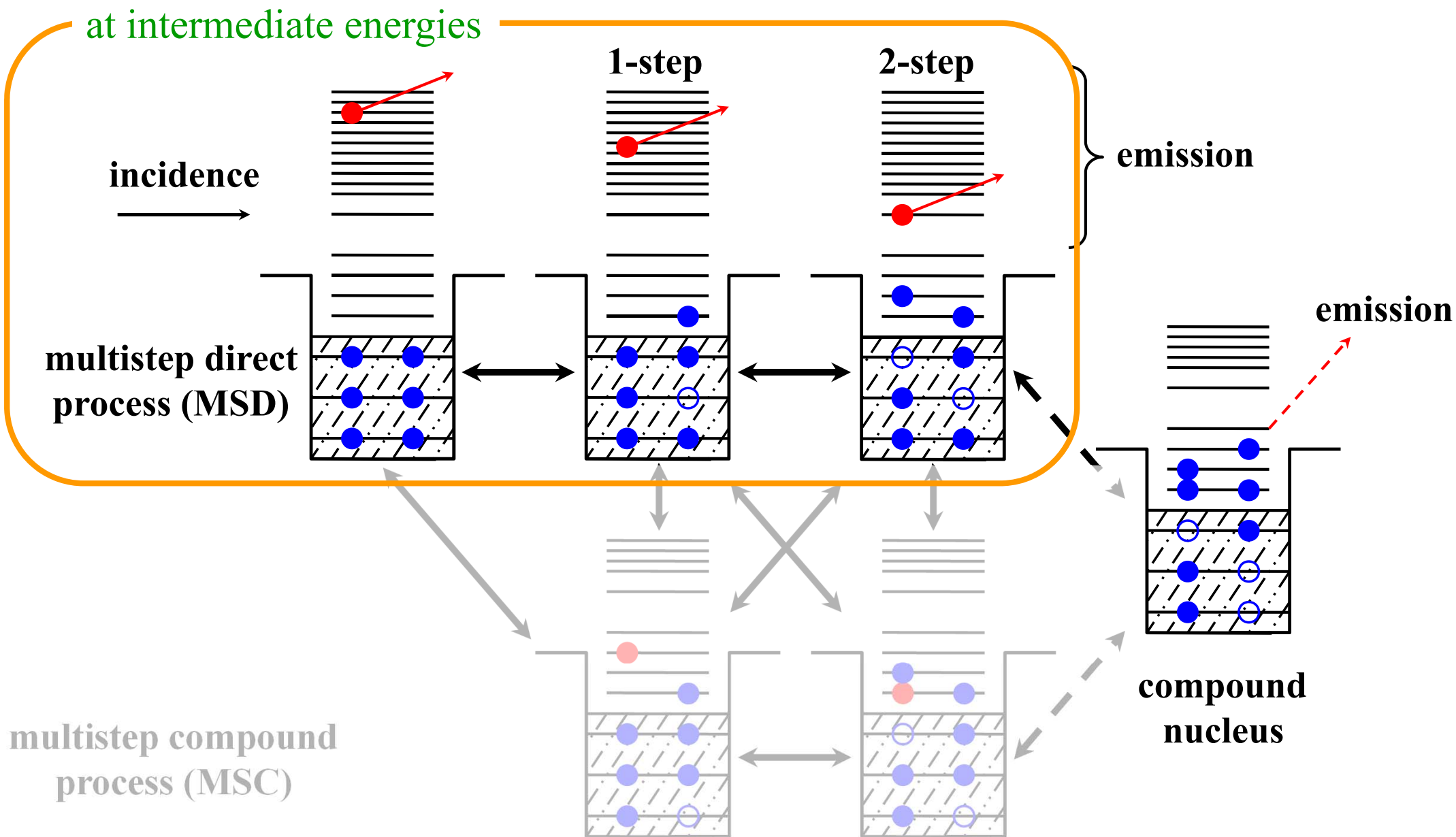
KO, G. C. Hillhouse, and B. I. S. van der Ventel, PRC76, 021602(R) (2007).

M. Kohno et al., PRC74, 064613 (2006).

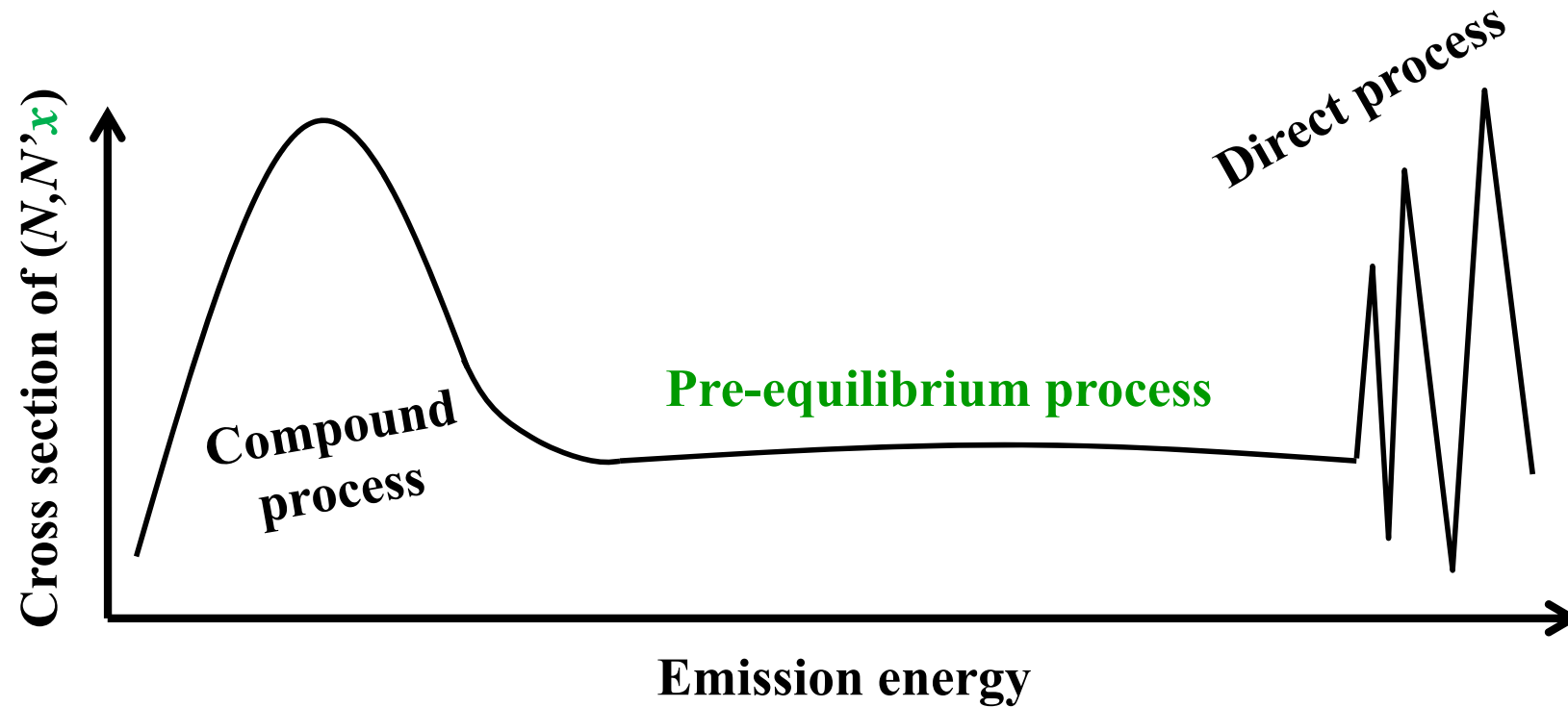
III. Description of inclusive $-1N$ cross section for nuclear transmutation

KO, arXiv:1801.09994.

Pre-equilibrium process



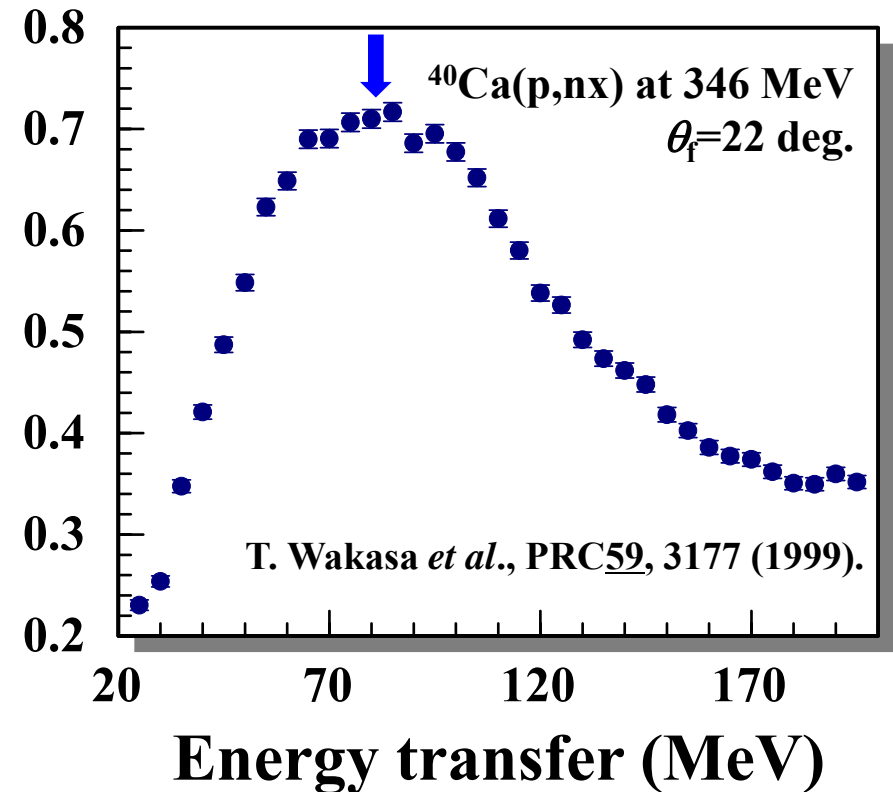
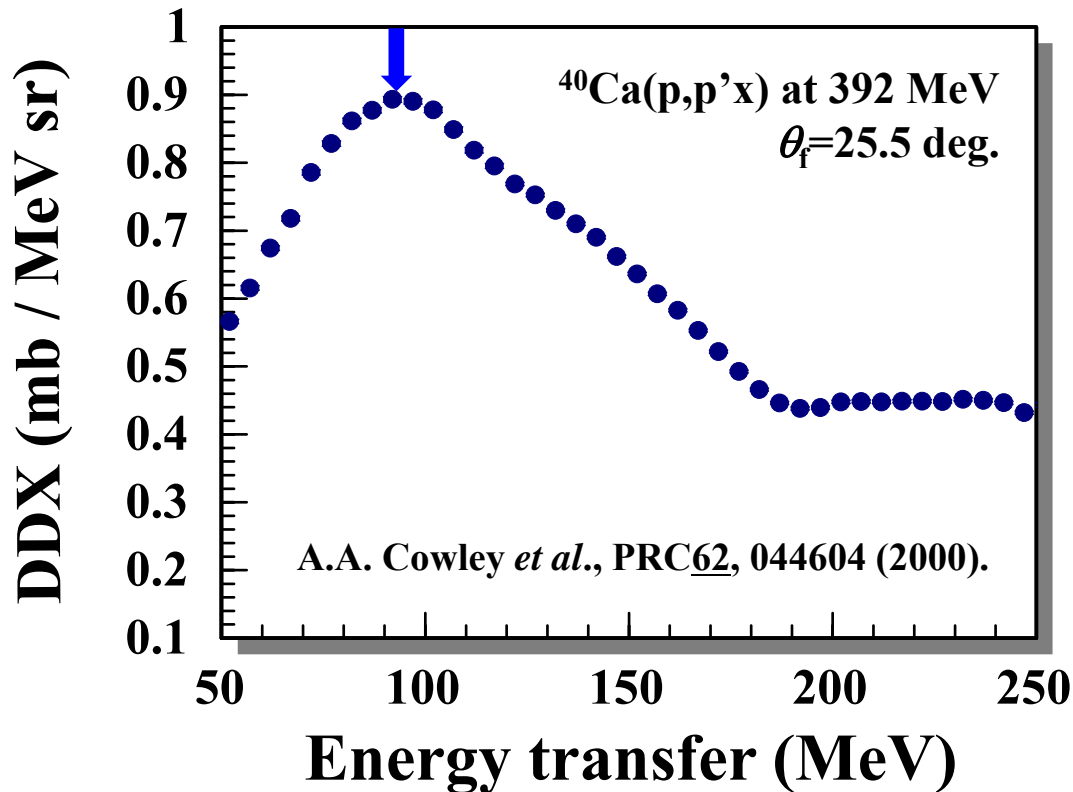
Multistep direct process and QFS



- ❑ MSD is dominant for the pre-equilibrium process for incident energies higher than several dozens of MeV.
- ❑ One-step process is most important.
- ❑ At intermediate energies the impulse picture holds.

➡ **Quasi-Free Scattering**

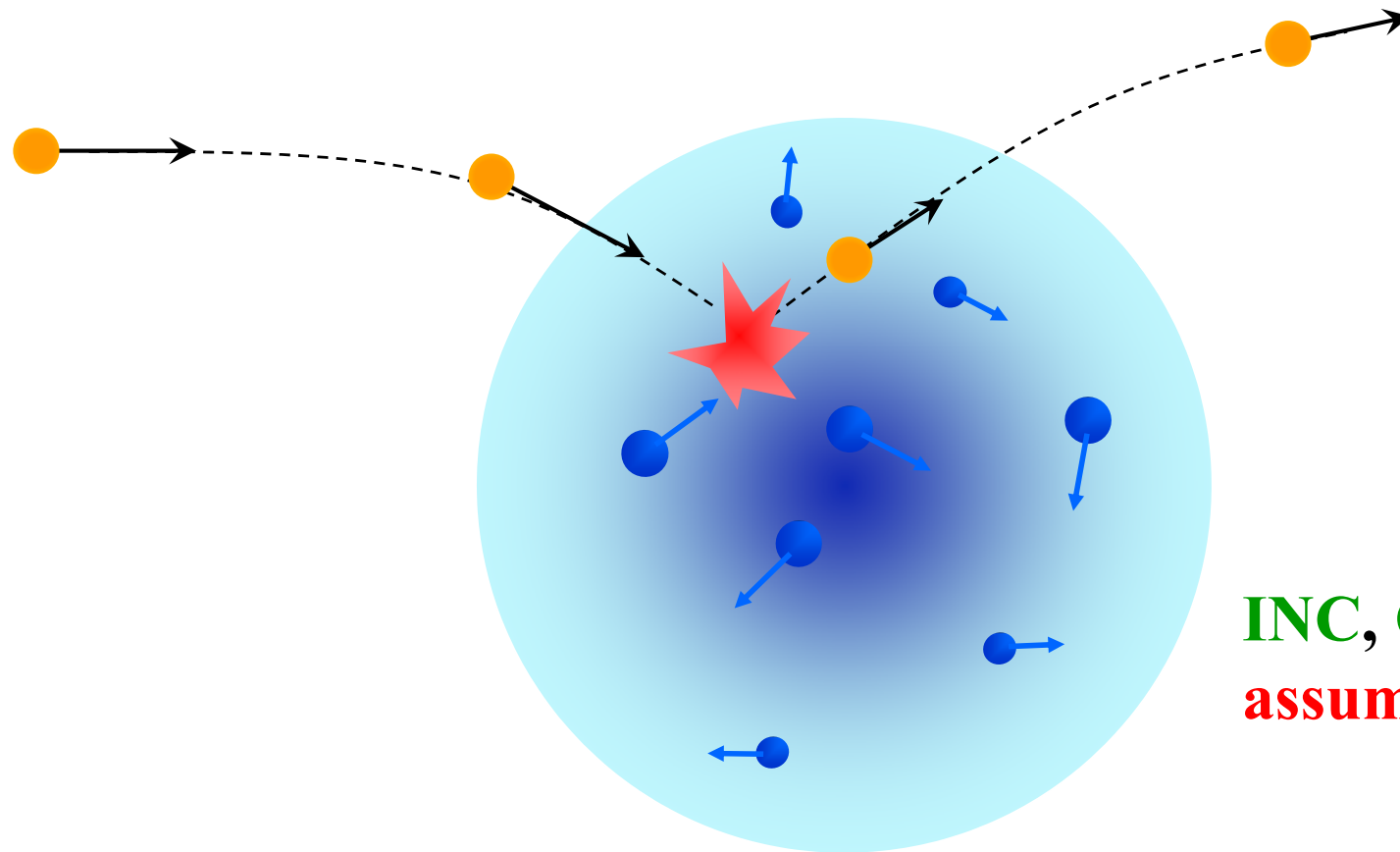
QFS in observables



- We see a peak corresponding to the incident particle with a nucleon inside the nucleus at rest: **quasi-free peak**
- The peak has a **width** due to the **Fermi motion** of the target nucleon.

QFS gives an important contribution to the cross section.

A naïve (intuitive) picture of QFS



INC, QMD, and AMD
assume this picture.

How is this picture **derived** from quantum mechanics?

DWIA formalism for QFS

$$\frac{d^2\sigma}{dE_f d\Omega_f} = C \sum_{\alpha\beta} |T_{f\beta,i\alpha}|^2 \delta(\varepsilon_f - \varepsilon_i),$$

$$T_{f\beta,i\alpha} = \left\langle \chi_f^{(-)}(\mathbf{r}_0) \varphi_\beta(\mathbf{r}) \left| v_{NN}(\mathbf{r} - \mathbf{r}_0) \right| \chi_i^{(+)}(\mathbf{r}_0) \varphi_\alpha(\mathbf{r}) \right\rangle$$

$\begin{matrix} R + s/2 & & R + s/2 \\ \swarrow & & \swarrow \\ \chi_f^{(-)}(\mathbf{r}_0) & \varphi_\beta(\mathbf{r}) & v_{NN}(\mathbf{r} - \mathbf{r}_0) & \chi_i^{(+)}(\mathbf{r}_0) & \varphi_\alpha(\mathbf{r}) \\ \searrow & & \searrow & & \searrow \\ R - s/2 & & s & & R - s/2 \end{matrix}$

□ Local Fermi gas model (LFG) for nuclei (for simple explanation)

$$\sum_{\alpha} \varphi_{\alpha}(\mathbf{r}) \varphi_{\alpha}^{*}(\mathbf{r}') = \int_{k_{\alpha} \leq k_{\text{F}}(\bar{\mathbf{r}})} e^{i\mathbf{k}_{\alpha} \cdot (\mathbf{R} - \mathbf{R}')} e^{i\mathbf{k}_{\alpha} \cdot (\mathbf{s} - \mathbf{s}')/2} d\mathbf{k}_{\alpha}$$

$$\sum_{\beta} \varphi_{\beta}^{*}(\mathbf{r}) \varphi_{\beta}(\mathbf{r}') = \int_{k_{\beta} \geq k_{\text{F}}(\bar{\mathbf{r}})} e^{i\mathbf{k}_{\beta} \cdot (\mathbf{R}' - \mathbf{R})} e^{i\mathbf{k}_{\beta} \cdot (\mathbf{s}' - \mathbf{s})/2} d\mathbf{k}_{\beta}$$

□ Possible issues

- ✓ **Interference** between different “collision points”
- ✓ Two-body kinematics is **not specified**.

Local Semi-Classical Approxⁿ (LSCA)

$$\sum_{\alpha\beta} |T_{f\beta,i\alpha}|^2 = \int_{k_\alpha \leq k_F(\bar{r})} d\mathbf{k}_\alpha \int_{k_\beta \geq k_F(\bar{r})} d\mathbf{k}_\beta \int \int \int \int d\mathbf{R} d\mathbf{R}' ds ds'$$

$$\times \chi_f^{*(-)}(\mathbf{R} - \mathbf{s}/2) e^{-i\mathbf{k}_\beta \cdot (\mathbf{R} + \mathbf{s}/2)} v_{NN}(\mathbf{s}) \chi_i^{(+)}(\mathbf{R} - \mathbf{s}/2) e^{i\mathbf{k}_\alpha \cdot (\mathbf{R} + \mathbf{s}/2)}$$

$$\times \chi_f^{(-)}(\mathbf{R}' - \mathbf{s}'/2) e^{i\mathbf{k}_\beta \cdot (\mathbf{R}' + \mathbf{s}'/2)} v_{NN}(\mathbf{s}') \chi_i^{*(+)}(\mathbf{R}' - \mathbf{s}'/2) e^{i\mathbf{k}_\alpha \cdot (\mathbf{R}' + \mathbf{s}'/2)}.$$

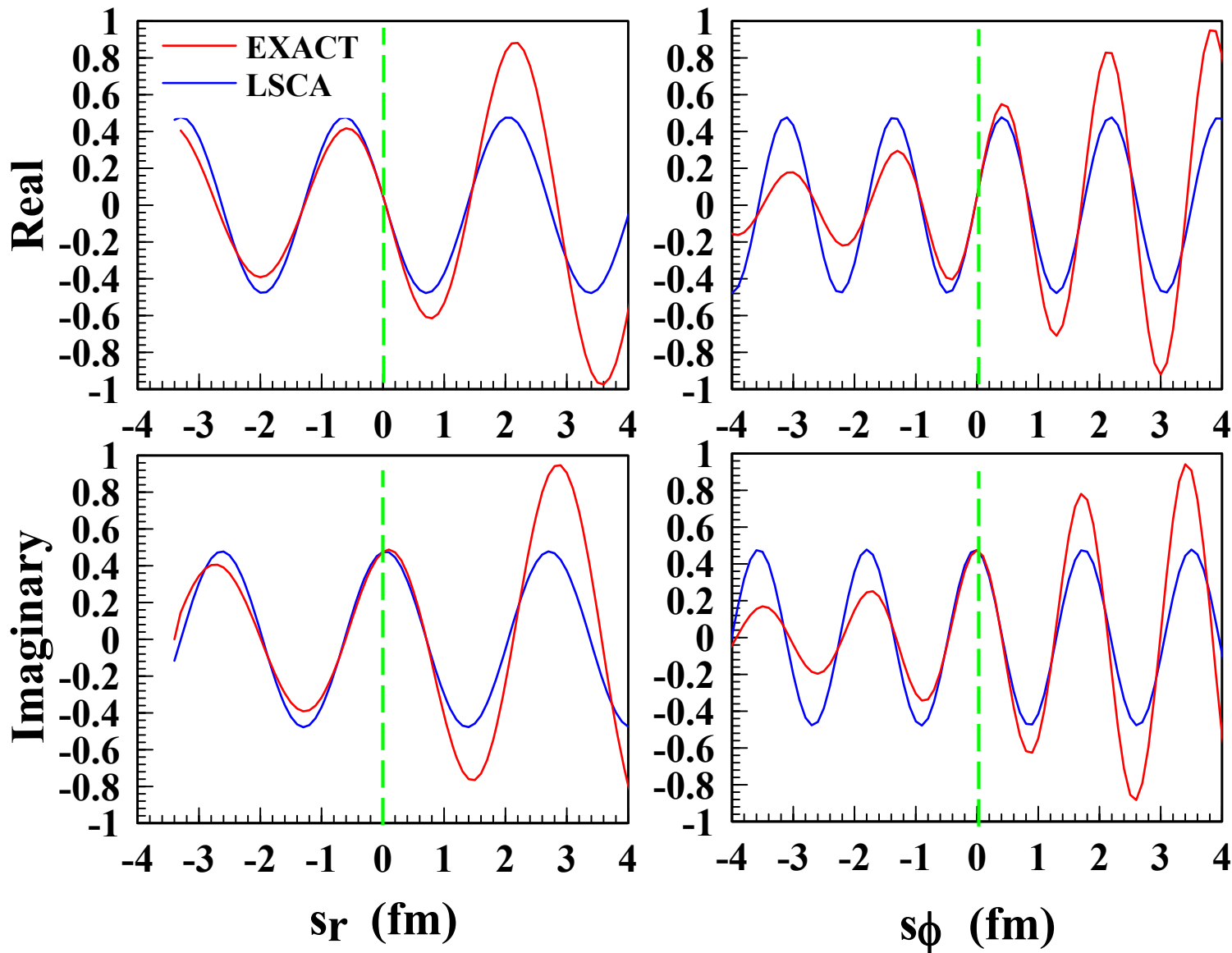
□ LSCA to the distorted waves

Y. L. Luo and M. Kawai, PRC43, 2367 (1991).

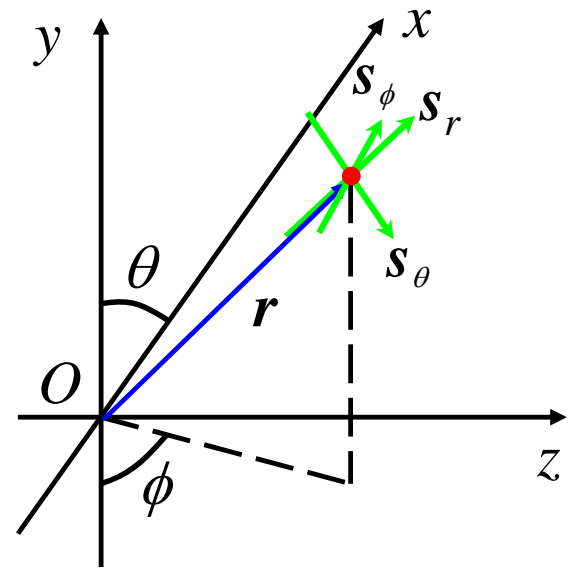
$$\chi_c^{(\pm)}(\mathbf{t}') = \chi_c^{(\pm)}(\mathbf{t} + (\mathbf{t}' - \mathbf{t})) \approx \chi_c^{(\pm)}(\mathbf{t}) e^{i\mathbf{k}_c(\mathbf{t}) \cdot (\mathbf{t}' - \mathbf{t})}, \quad c = i, f$$

$$\mathbf{k}_c(\mathbf{t}) = -i \frac{\nabla \chi_c^{(\pm)}(\mathbf{t})}{\chi_c^{(\pm)}(\mathbf{t})}.$$

Validity of LSCA: p-⁴⁰Ca at 350 MeV



$$\chi(\mathbf{r} + \mathbf{s}) = \chi(\mathbf{r}) e^{i\mathbf{k}(\mathbf{r}) \cdot \mathbf{s}}$$



**LSCA holds within
1–2 fm.**

Determination of the NN kinematics

$$\sum_{\alpha\beta} |T_{f\beta,i\alpha}|^2 = \int_{k_\alpha \leq k_F(\bar{r})} d\mathbf{k}_\alpha \int_{k_\beta \geq k_F(\bar{r})} d\mathbf{k}_\beta \int \int \int \int d\mathbf{R} d\mathbf{R}' ds ds'$$

$$\times \chi_f^{*(-)}(\mathbf{R} - \mathbf{s}/2) e^{-i\mathbf{k}_\beta \cdot (\mathbf{R} + \mathbf{s}/2)} v_{NN}(\mathbf{s}) \chi_i^{(+)}(\mathbf{R} - \mathbf{s}/2) e^{i\mathbf{k}_\alpha \cdot (\mathbf{R} + \mathbf{s}/2)}$$

$$\times \chi_f^{(-)}(\mathbf{R}' - \mathbf{s}'/2) e^{i\mathbf{k}_\beta \cdot (\mathbf{R}' + \mathbf{s}'/2)} v_{NN}(\mathbf{s}') \chi_i^{*(+)}(\mathbf{R}' - \mathbf{s}'/2) e^{i\mathbf{k}_\alpha \cdot (\mathbf{R}' + \mathbf{s}'/2)}.$$



LSCA to $\chi_c^{(\pm)}(\mathbf{R} \pm \mathbf{s}/2)$

$$\sum_{\alpha\beta} |T_{f\beta,i\alpha}|^2 = \int_{k_\alpha \leq k_F(\bar{r})} d\mathbf{k}_\alpha \int_{k_\beta \geq k_F(\bar{r})} d\mathbf{k}_\beta \int \int d\mathbf{R} d\mathbf{R}' e^{i\mathbf{k}_\beta \cdot (\mathbf{R}' - \mathbf{R})} e^{-i\mathbf{k}_\alpha \cdot (\mathbf{R}' - \mathbf{R})}$$

NN kinematics specified

$$\times \chi_f^{*(-)}(\mathbf{R}) \left(\int e^{i\mathbf{k}_f(\mathbf{R}) \cdot \mathbf{s}/2} e^{-i\mathbf{k}_\beta \cdot \mathbf{s}/2} v_{NN}(\mathbf{s}) e^{-i\mathbf{k}_i(\mathbf{R}) \cdot \mathbf{s}/2} e^{i\mathbf{k}_\alpha \cdot \mathbf{s}/2} d\mathbf{s} \right) \chi_i^{(+)}(\mathbf{R})$$

$$\times \chi_f^{(-)}(\mathbf{R}') \int e^{-i\mathbf{k}_f(\mathbf{R}) \cdot \mathbf{s}'/2} e^{i\mathbf{k}_\beta \cdot \mathbf{s}'/2} v_{NN}(\mathbf{s}') e^{i\mathbf{k}_i(\mathbf{R}) \cdot \mathbf{s}'/2} e^{i\mathbf{k}_\alpha \cdot \mathbf{s}'/2} d\mathbf{s}' \chi_i^{*(+)}(\mathbf{R}').$$

$$t_{NN}(\boldsymbol{\kappa}', \boldsymbol{\kappa}) \text{ with } \boldsymbol{\kappa}' = \frac{\mathbf{k}_\beta - \mathbf{k}_f(\mathbf{R})}{2}, \boldsymbol{\kappa} = \frac{\mathbf{k}_\alpha - \mathbf{k}_i(\mathbf{R})}{2}$$

Non-locality of the kernel

$$K(\mathbf{R}, \mathbf{R}') = \sum_{\alpha\beta} \varphi_{\beta}^*(\mathbf{R}) \varphi_{\beta}(\mathbf{R}') \varphi_{\alpha}(\mathbf{R}) \varphi_{\alpha}^*(\mathbf{R}') \delta(\varepsilon_f - \varepsilon_i)$$

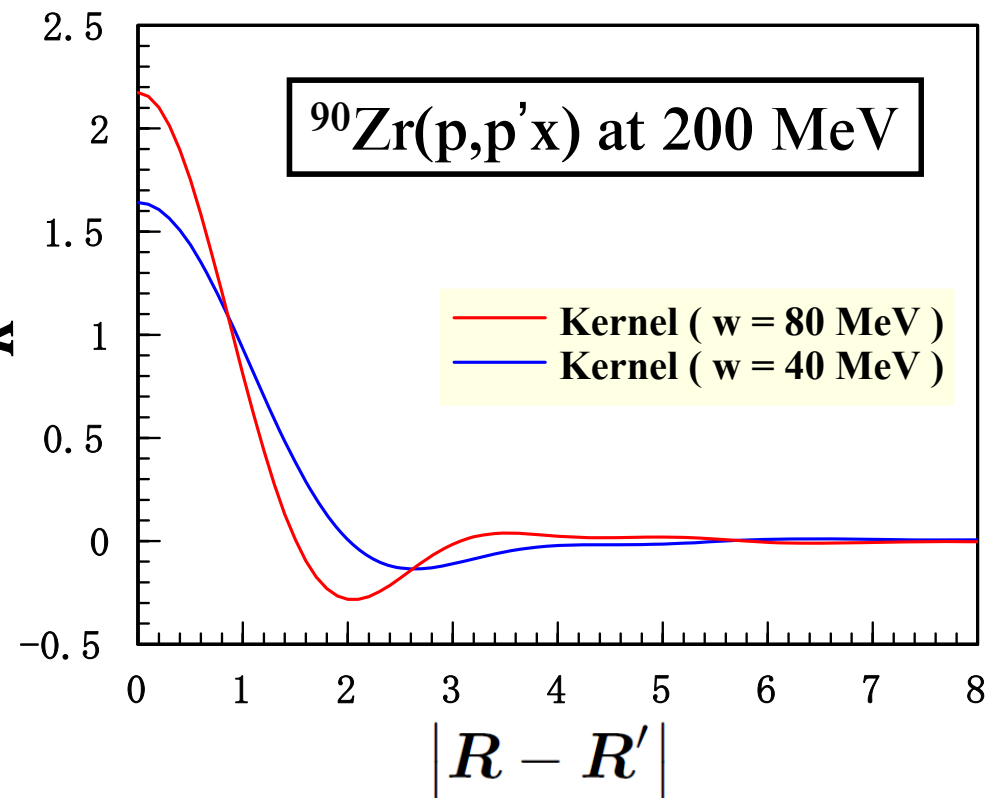
$$\rightarrow \int_{k_{\alpha} \leq k_F(\bar{\mathbf{R}})} d\mathbf{k}_{\alpha} \int_{k_{\beta} \geq k_F(\bar{\mathbf{R}})} d\mathbf{k}_{\beta} \int \int e^{i\mathbf{k}_{\beta} \cdot (\mathbf{R}' - \mathbf{R})} e^{-i\mathbf{k}_{\alpha} \cdot (\mathbf{R}' - \mathbf{R})} \delta(\varepsilon_{\beta} - \varepsilon_{\alpha} - \omega).$$

□ K for inclusive processes

- ✓ If we take the summation over **all the states**, we have a **delta function**.
- ✓ If **many states** contribute, the range \mathbf{K} of non-locality of K should be **short**.

cf. K determines the angular distribution

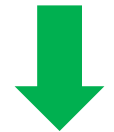
- ~ nuclear radius: **oscillation**
- ~ NN interaction range: **smooth**



Localization of the NN collision

$$\sum_{\alpha\beta} |T_{f\beta,i\alpha}|^2 = \int_{k_\alpha \leq k_F(\bar{\mathbf{R}})} d\mathbf{k}_\alpha \int_{k_\beta \geq k_F(\bar{\mathbf{R}})} d\mathbf{k}_\beta \int \int d\mathbf{R} d\mathbf{R}' e^{i\mathbf{k}_\beta \cdot (\mathbf{R}' - \mathbf{R})} e^{-i\mathbf{k}_\alpha \cdot (\mathbf{R}' - \mathbf{R})}$$

$$\times \chi_f^{*(-)}(\mathbf{R}) t_{NN}(\boldsymbol{\kappa}', \boldsymbol{\kappa}) \chi_i^{(+)}(\mathbf{R}) \chi_f^{(-)}(\mathbf{R}') t_{NN}^*(\boldsymbol{\kappa}', \boldsymbol{\kappa}) \chi_i^{*(+)}(\mathbf{R}').$$


 LSCA to $\chi_c^{(\pm)}(\mathbf{R}')$ with $\mathbf{u} = \mathbf{R}' - \mathbf{R}$.

$$\sum_{\alpha\beta} |T_{f\beta,i\alpha}|^2 = \int_{k_\alpha \leq k_F(\bar{\mathbf{R}})} d\mathbf{k}_\alpha \int_{k_\beta \geq k_F(\bar{\mathbf{R}})} d\mathbf{k}_\beta \int d\mathbf{R}$$

$$\times \int e^{i\mathbf{k}_\beta \cdot \mathbf{u}} e^{-i\mathbf{k}_\alpha \cdot \mathbf{u}} e^{i\mathbf{k}_f(\mathbf{R}) \cdot \mathbf{u}} e^{-i\mathbf{k}_i(\mathbf{R}) \cdot \mathbf{u}} d\mathbf{u}$$

$$\times \chi_f^{*(-)}(\mathbf{R}) \chi_f^{(-)}(\mathbf{R}) |t_{NN}(\boldsymbol{\kappa}', \boldsymbol{\kappa})|^2 \chi_i^{*(+)}(\mathbf{R}) \chi_i^{(+)}(\mathbf{R}).$$

$$\delta(\mathbf{k}_\beta + \mathbf{k}_f(\mathbf{R}) - \mathbf{k}_\alpha - \mathbf{k}_i(\mathbf{R}))$$

Conservation of NN local momentum

Semi-Classical Distorted Wave model

$$\begin{aligned} \frac{d^2\sigma}{dE_f d\Omega_f} &= C \int d\mathbf{R} \left| \chi_f^{(-)}(\mathbf{R}) \right|^2 \left| \chi_i^{(+)}(\mathbf{R}) \right|^2 \\ &\times \int_{k_\alpha \leq k_F(\bar{\mathbf{R}})} d\mathbf{k}_\alpha \int_{k_\beta \geq k_F(\bar{\mathbf{R}})} d\mathbf{k}_\beta |t_{NN}(\boldsymbol{\kappa}', \boldsymbol{\kappa})|^2 \\ &\times \delta(\mathbf{k}_\beta + \mathbf{k}_f(\mathbf{R}) - \mathbf{k}_\alpha - \mathbf{k}_i(\mathbf{R})) \delta(\varepsilon_\beta - \varepsilon_\alpha - \omega). \end{aligned}$$

- The **intuitive picture** of QFS is derived (no free parameter).

Y. L. Luo and M. Kawai, PRC43, 2367 (1991).

- Formulation of multistep processes **up to 3-step** has been completed.

M. Kawai and H. A. Weidenmueller, PRC45, 1856 (1992); Y. Watanabe et al., PRC59, 2136 (1999).

- A Woods-Saxon s.p. wave function can be used (**beyond LFG**).

Sun Weili et al., PRC60, 064605 (1999).

- **Spin observables** can be calculated.

KO et al., PRC60, 054605 (1999); T. Wakasa et al., PRC65, 034615 (2002);

KO, G. C. Hillhouse, and B. I. S. van der Ventel, PRC76, 021602(R) (2007).

- Applicable to **various types** of QFS/knockout reactions, e.g., (π^-, K^+) reaction.

M. Kohno, M. Fujiwara, Y. Watanabe, KO, M. Kawai, PRC74, 064613 (2006).

Extension to the multistep processes

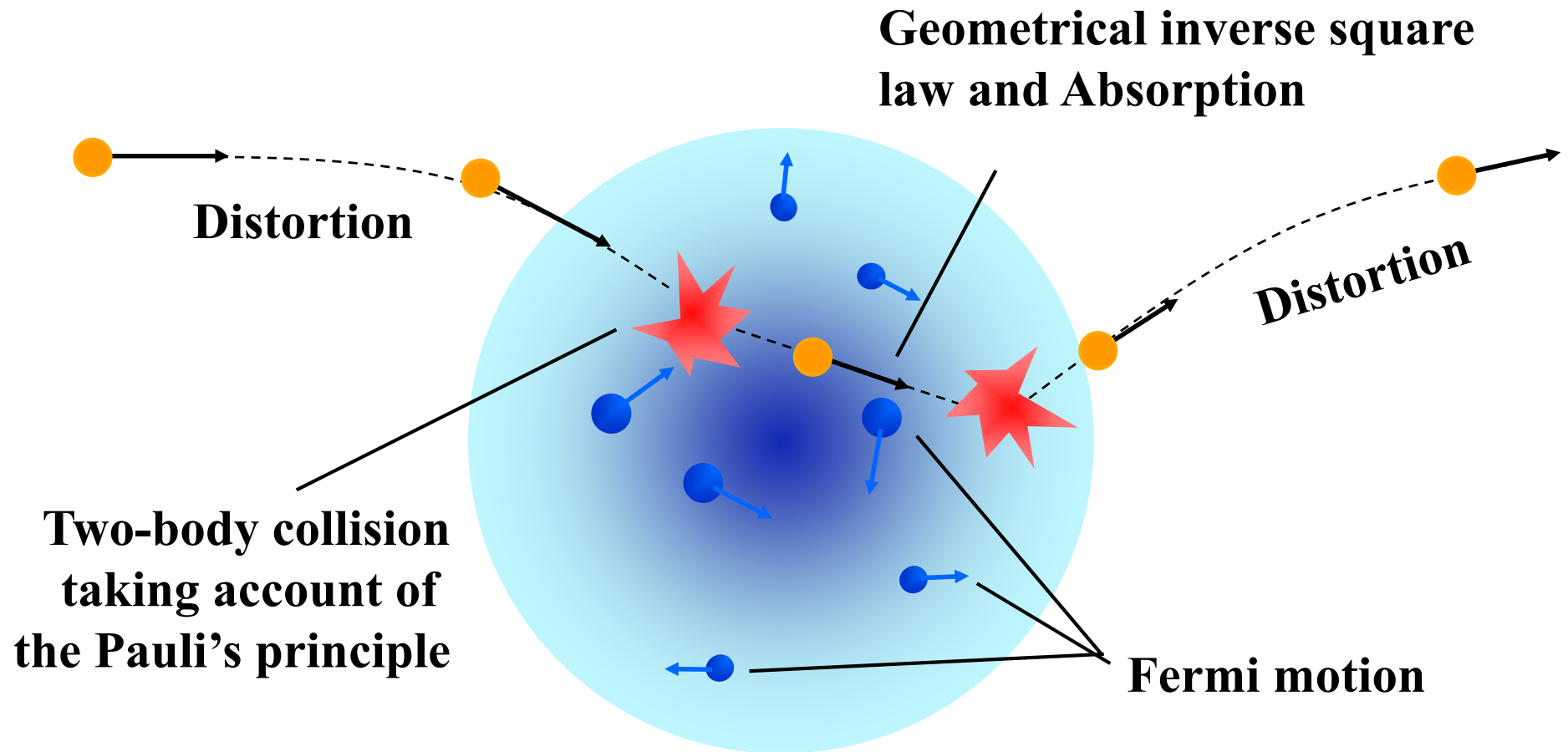
□ Eikonal approximation to the intermediate Green function

$$\left\langle \mathbf{R}_2 \left| (E_m - K_m - U_m + i\eta)^{-1} \right| \mathbf{R}_1 \right\rangle \approx -\frac{m}{2\pi\hbar^2} \frac{e^{i(k_m^R + ik_m^I)|\mathbf{R}_2 - \mathbf{R}_1|}}{|\mathbf{R}_2 - \mathbf{R}_1|}.$$



$$\begin{aligned} \frac{d^2\sigma_{2\text{step}}}{dE_f d\Omega_f} &= C' \int d\mathbf{R}_1 \int dE_m \int d\mathbf{R}_2 \left| \chi_f^{(-)}(\mathbf{R}_2) \right|^2 \left| \chi_i^{(+)}(\mathbf{R}_1) \right|^2 \\ &\times \int_{k_{\alpha 2} \leq k_F(\bar{\mathbf{R}}_2)} d\mathbf{k}_{\alpha 2} \int_{k_{\beta 2} \geq k_F(\bar{\mathbf{R}}_2)} d\mathbf{k}_{\beta 2} |t_{NN}(\boldsymbol{\kappa}'_2, \boldsymbol{\kappa}_2)|^2 \\ &\times \delta\left(\mathbf{k}_{\beta 2} + \mathbf{k}_f(\mathbf{R}_2) - \mathbf{k}_{\alpha 2} - \mathbf{k}_m^R(\mathbf{R}_2)\right) \delta(\varepsilon_{\beta 2} - \varepsilon_{\alpha 2} - \omega_2) \\ &\times \left(\frac{m}{2\pi\hbar^2}\right)^2 \frac{e^{-2k_m^I|\mathbf{R}_2 - \mathbf{R}_1|}}{|\mathbf{R}_2 - \mathbf{R}_1|^2} \\ &\times \int_{k_{\alpha 1} \leq k_F(\bar{\mathbf{R}}_1)} d\mathbf{k}_{\alpha 1} \int_{k_{\beta 1} \geq k_F(\bar{\mathbf{R}}_1)} d\mathbf{k}_{\beta 1} |t_{NN}(\boldsymbol{\kappa}'_1, \boldsymbol{\kappa}_1)|^2 \\ &\times \delta\left(\mathbf{k}_{\beta 1} + \mathbf{k}_m^R(\mathbf{R}_1) - \mathbf{k}_{\alpha 1} - \mathbf{k}_i(\mathbf{R}_1)\right) \delta(\varepsilon_{\beta 1} - \varepsilon_{\alpha 1} - \omega_1) \end{aligned}$$

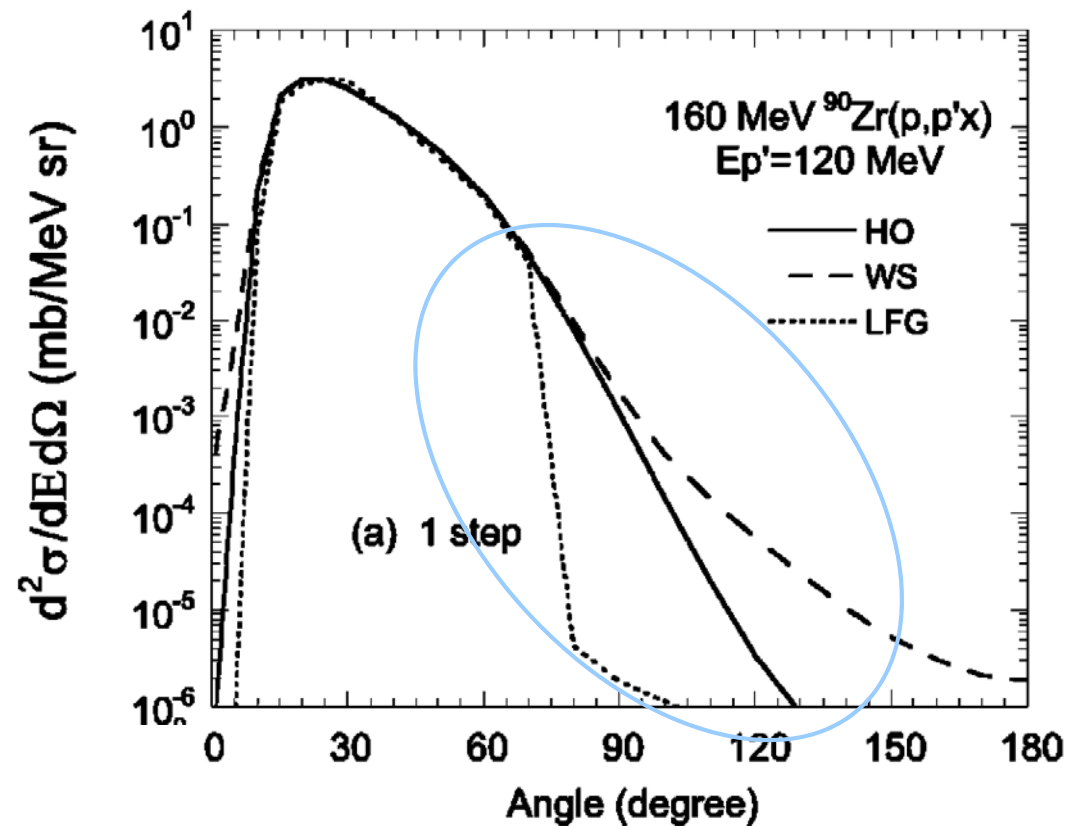
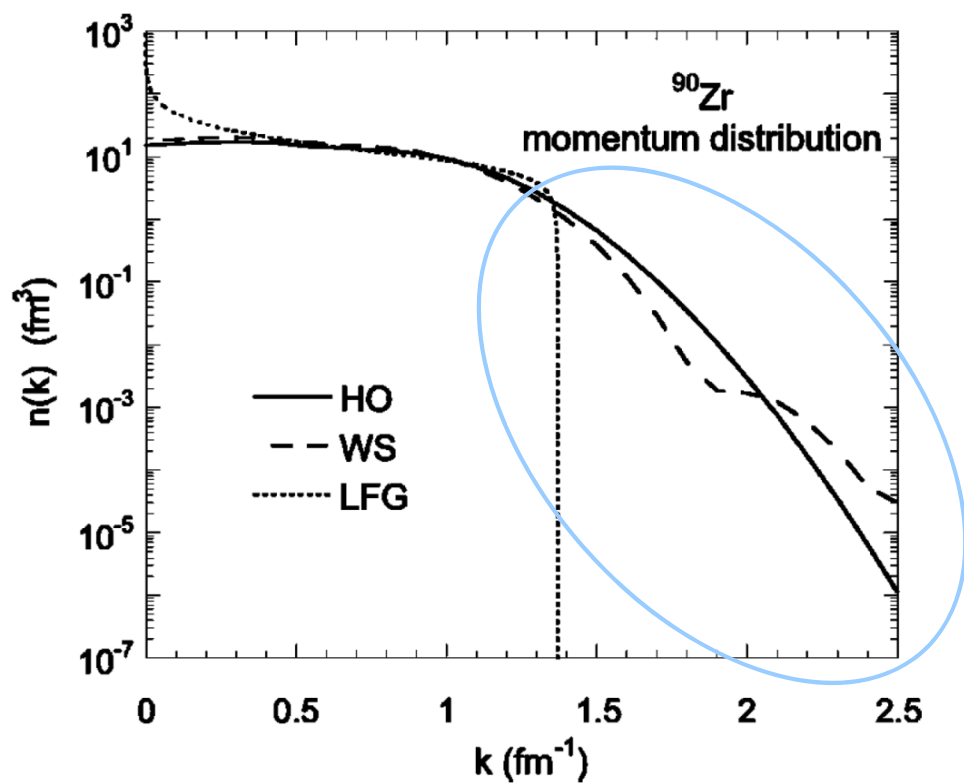
A cascade QFS picture derived by SCDW



**No interference between processes
through different collision points!**

Momentum distribution and observables

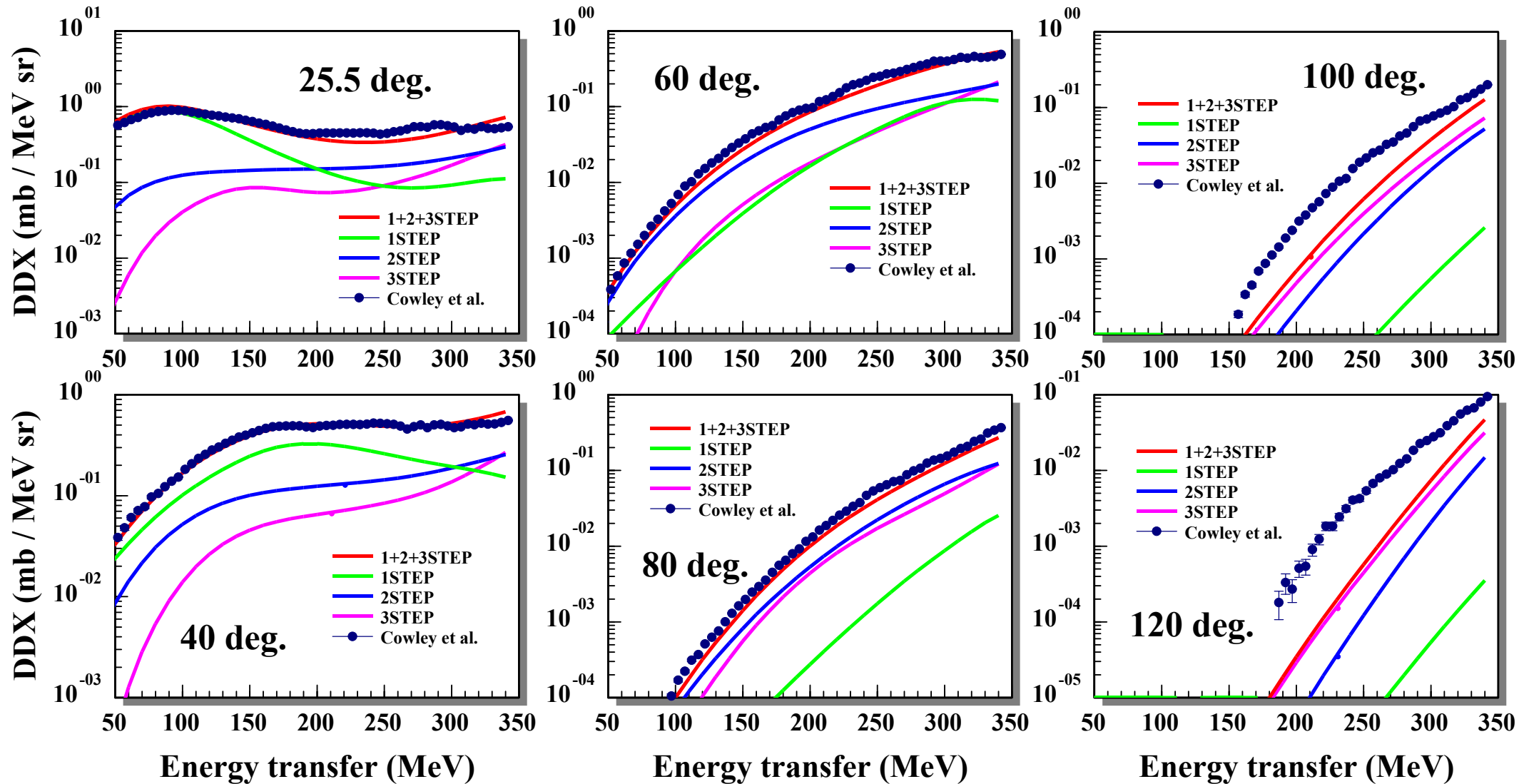
Sun Weili et al., PRC60, 064605 (1999).



$$\frac{d^2\sigma}{dE_f d\Omega_f} = C \int d\mathbf{k}_\beta d\mathbf{k}_\alpha \delta(\mathbf{K}_f + \mathbf{k}_\beta - \mathbf{K}_i - \mathbf{k}_\alpha) \delta(E_f + \varepsilon_\beta - E_i - \varepsilon_\alpha) \\ \times \int d\mathbf{R} \left| \bar{\chi}_{f, \mathbf{K}_f}^{(-)}(\mathbf{R}) \right|^2 \left[2 - f_h^{(\beta)}(\mathbf{k}_\beta, \mathbf{R}) \right] f_h^{(\alpha)}(\mathbf{k}_\alpha, \mathbf{R}) \left| t_{NN}(\boldsymbol{\kappa}', \boldsymbol{\kappa}) \right|^2 \left| \bar{\chi}_{i, \mathbf{K}_i}^{(+)}(\mathbf{R}) \right|^2$$

DDX for $^{40}\text{Ca}(p,p'x)$ at 392 MeV

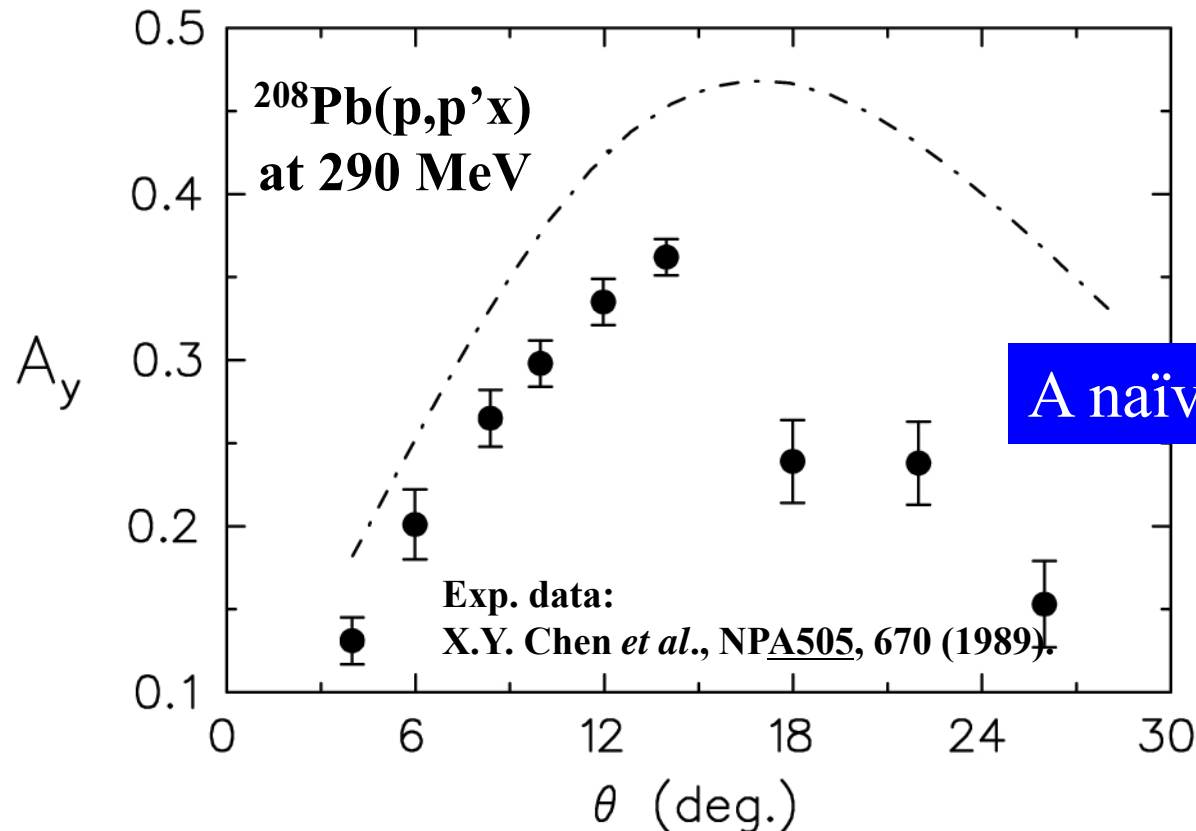
KO, Watanabe, Sun, Kohno, Kawai, Proc. of MEDIUM02, p.231 (2003).



Exp. data: A.A. Cowley et al., PRC62, 044604 (2000).

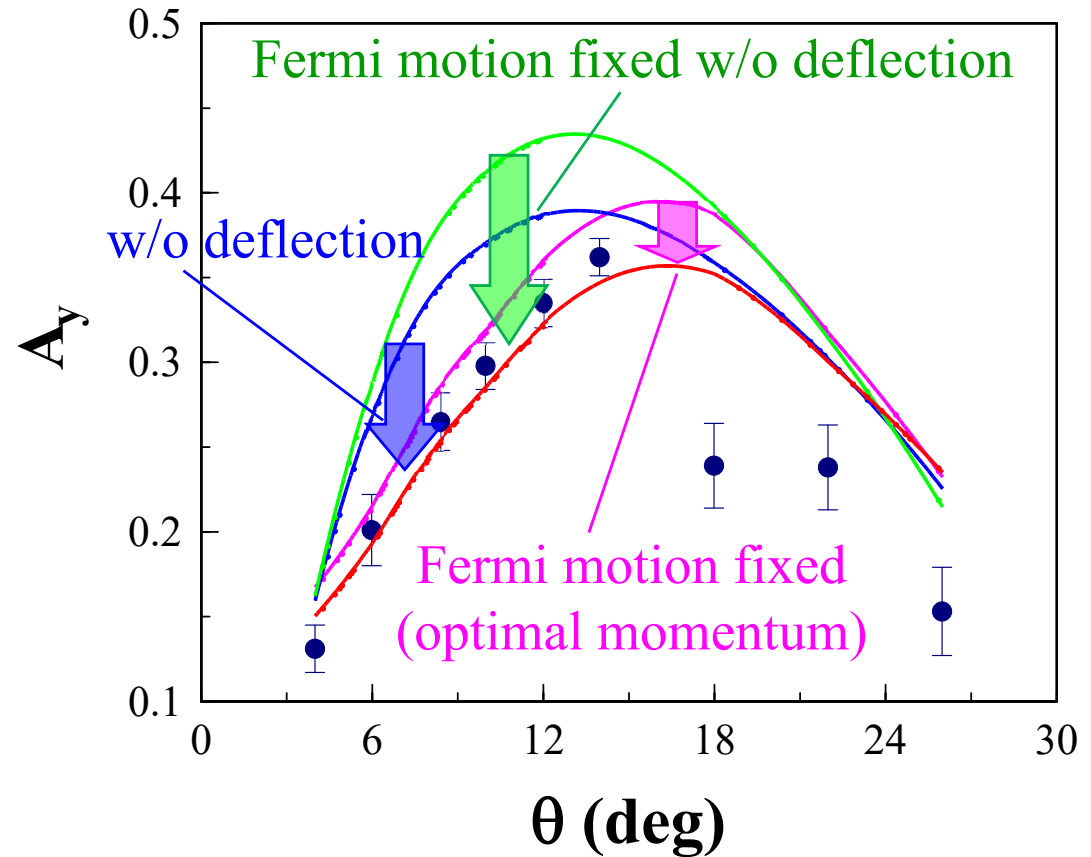
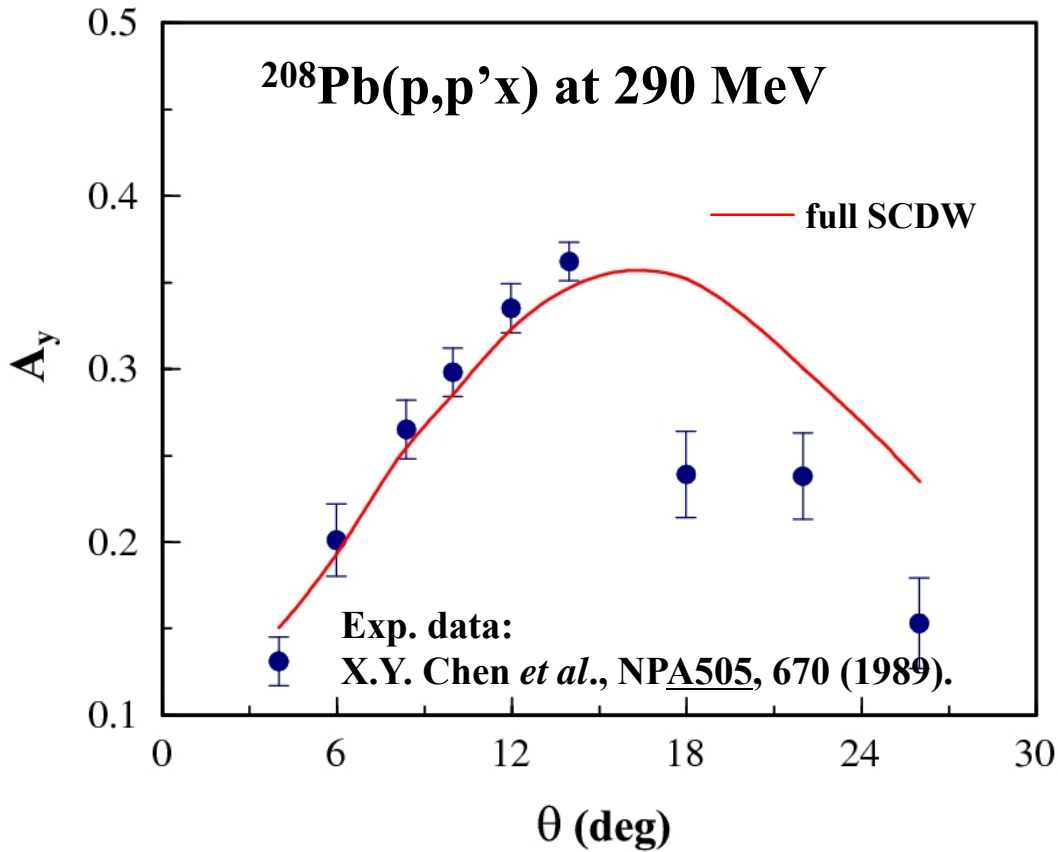
The longstanding quasi-free A_y problem

A common feature of quasielastic scattering is that the experimental (\vec{p}, p') analyzing power A_y at the quasielastic peak is always significantly reduced by about 30–60% relative to the corresponding value for free NN scattering, independent of the target nucleus (for $A > 4$) and incident energy ($E \geq 200$ MeV)



A solution to this problem with SCDW

KO, G. C. Hillhouse, and B. I. S. van der Ventel, PRC76, 021602(R) (2007).



Application of SCDW to (π^-, K^+) reaction

PHYSICAL REVIEW C **74**, 064613 (2006)

Semiclassical distorted-wave model analysis of the $(\pi^-, K^+)\Sigma$ formation inclusive spectrum

M. Kohno,¹ Y. Fujiwara,² Y. Watanabe,³ K. Ogata,⁴ and M. Kawai⁴

¹*Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan*

²*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

³*Department of Advanced Energy Engineering Science, Kyushu University, Kasuga, Fukuoka 816-8580, Japan*

⁴*Department of Physics, Kyushu University, Fukuoka 812-8581, Japan*

(Received 20 January 2006; revised manuscript received 18 September 2006; published 19 December 2006)

(π^-, K^+) hyperon production inclusive spectra with $p_\pi = 1.2$ GeV/c measured at KEK on ^{12}C and ^{28}Si are analyzed by the semiclassical distorted-wave model. Single-particle (s.p.) wave functions of the target nucleus are treated using Wigner transformation. This method is able to account for the energy and angular dependences of the elementary process in nuclear medium without introducing the factorization approximation frequently employed. Calculations of the $(\pi^+, K^+)\Lambda$ formation process, for which there is no free parameter because the Λ s.p. potential is known, demonstrate that the present model is useful to describe inclusive spectra. It is shown that to account for the experimental data of the Σ^- formation spectra a repulsive Σ -nucleus potential is necessary whose magnitude is not so strong as around 100 MeV previously suggested.

Application of SCDW to (π^-, K^+) reaction

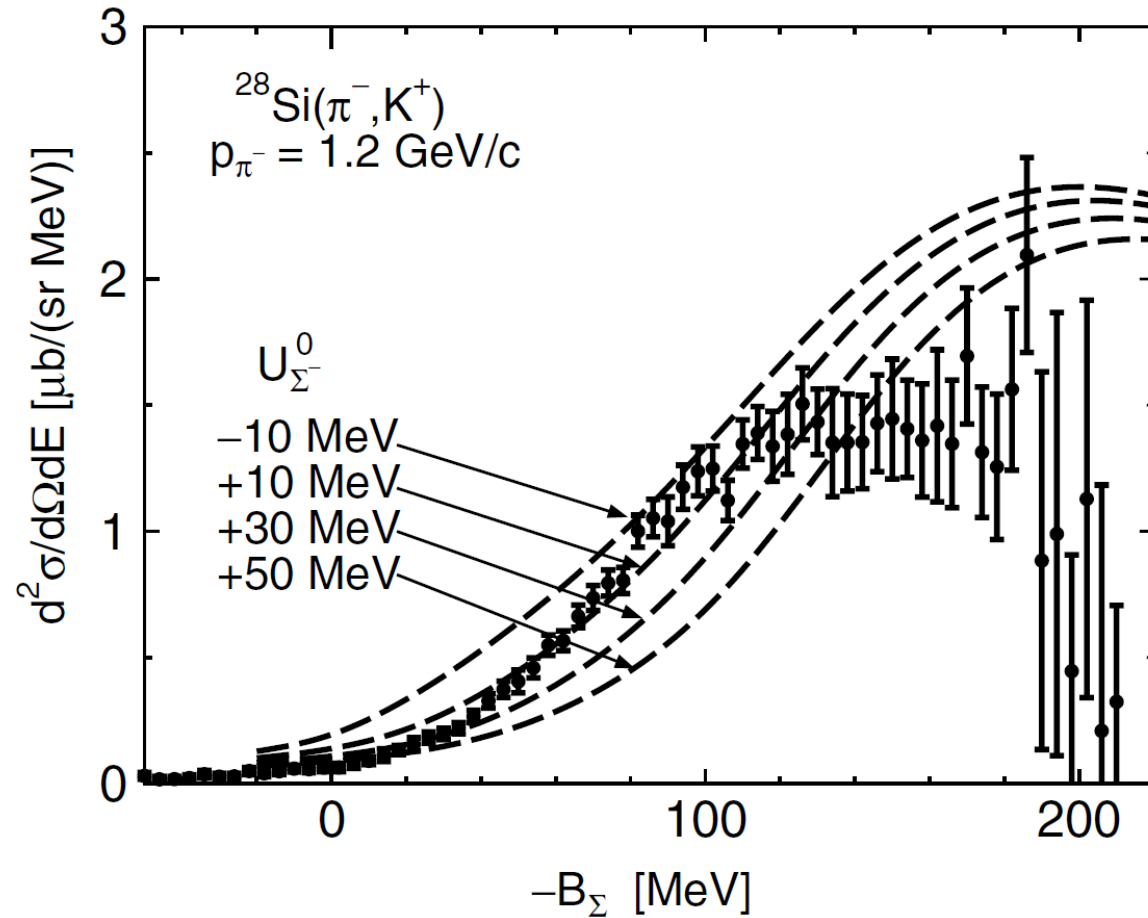


FIG. 9. $(\pi^-, K^+)\Sigma$ formation inclusive spectra with a ^{28}Si target at $\theta_K = 6^\circ \mp 2^\circ$ for pions with $p_\pi = 1.2 \text{ GeV}/c$. These results were obtained with four choices of the strength $U_\Sigma^0 = -10, 10, 30, 50$ in a Woods-Saxon potential form with the geometry parameters of $r_0 = 1.25 \times (A - 1)^{1/3} \text{ fm}$ and $a = 0.65 \text{ fm}$. Experimental data points are taken from Refs. [22,23].

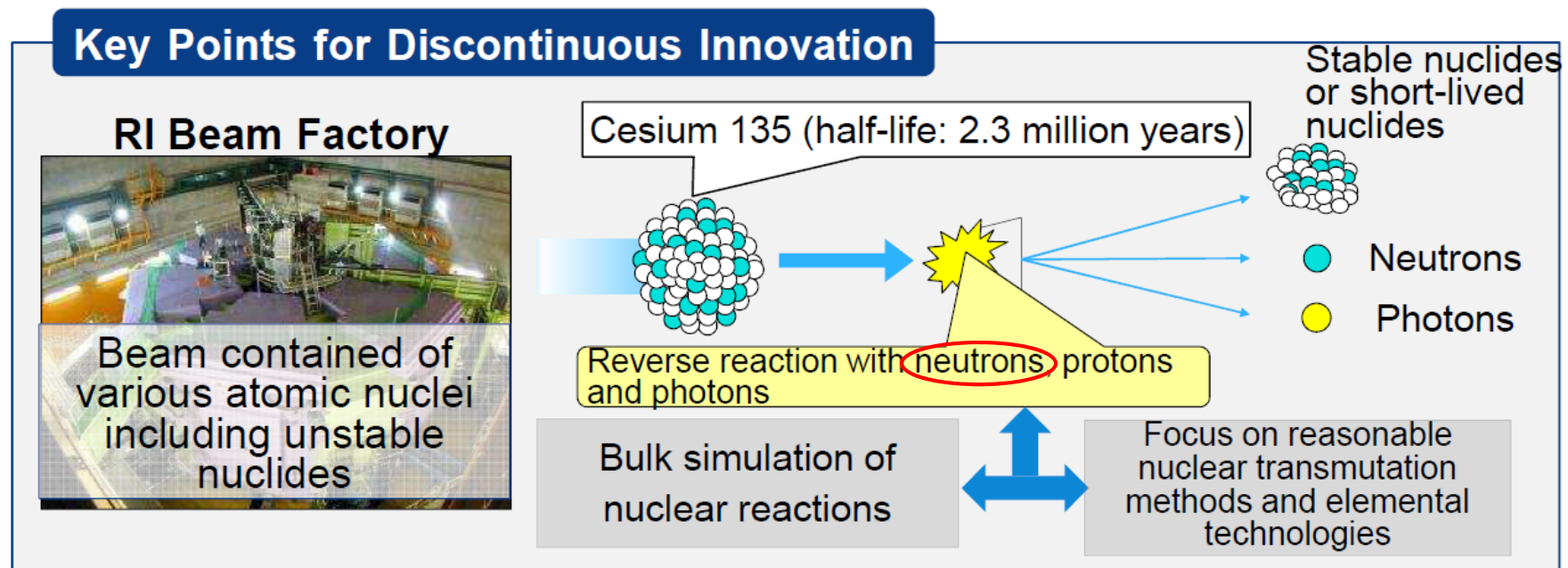
Nuclear Transmutation studies

Impulsing Paradigm Change through Disruptive Technologies Program

- Launched FY2014 and 12 programs approved.
- will end at Dec. 31, 2018.
- Keyword: high risk and high impact



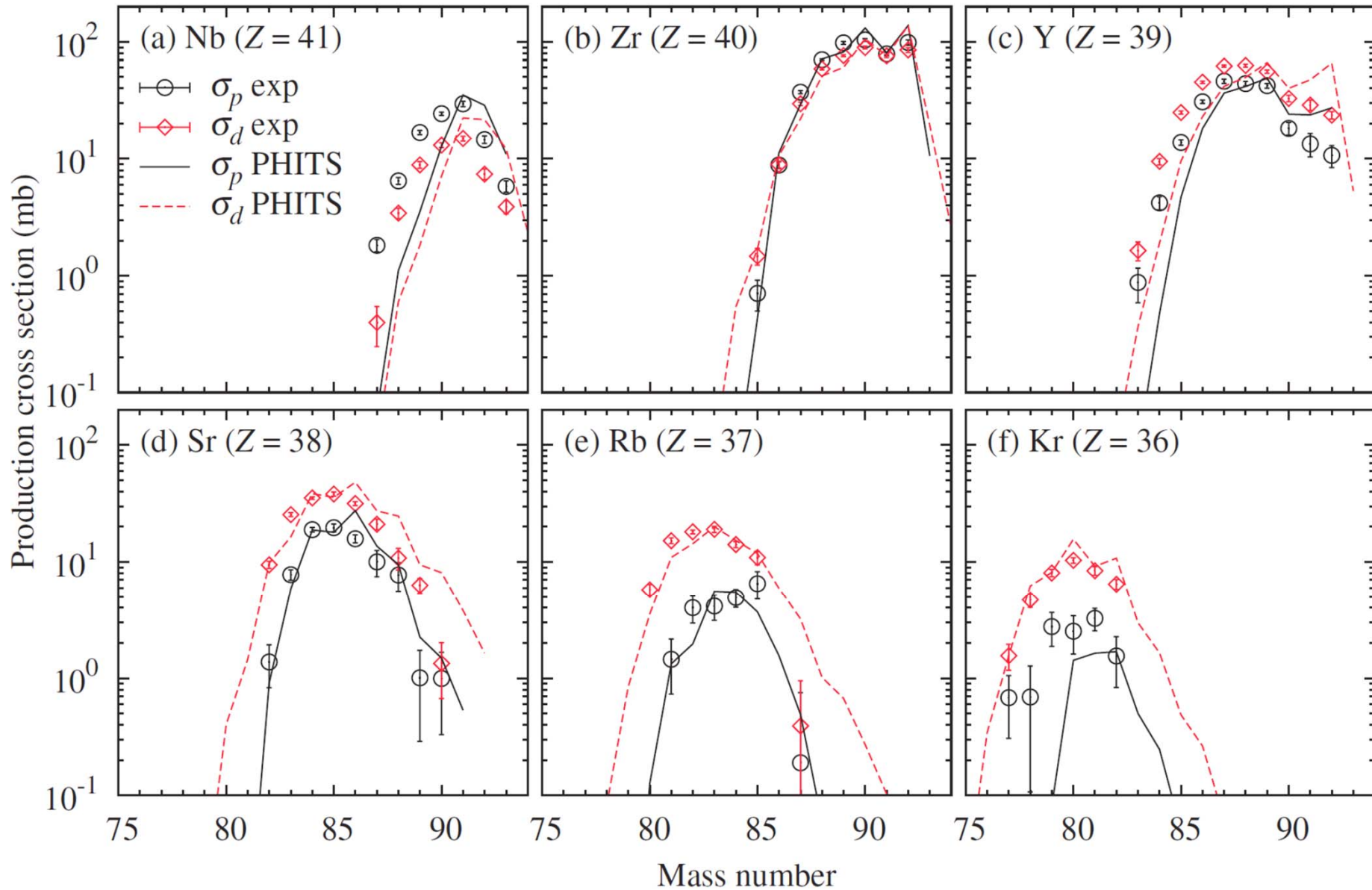
Reduction and Resource Recycle of High Level Radioactive Wastes with Nuclear Transmutation (PM: Reiko Fujita)



Spallation cross section taken at RIBF

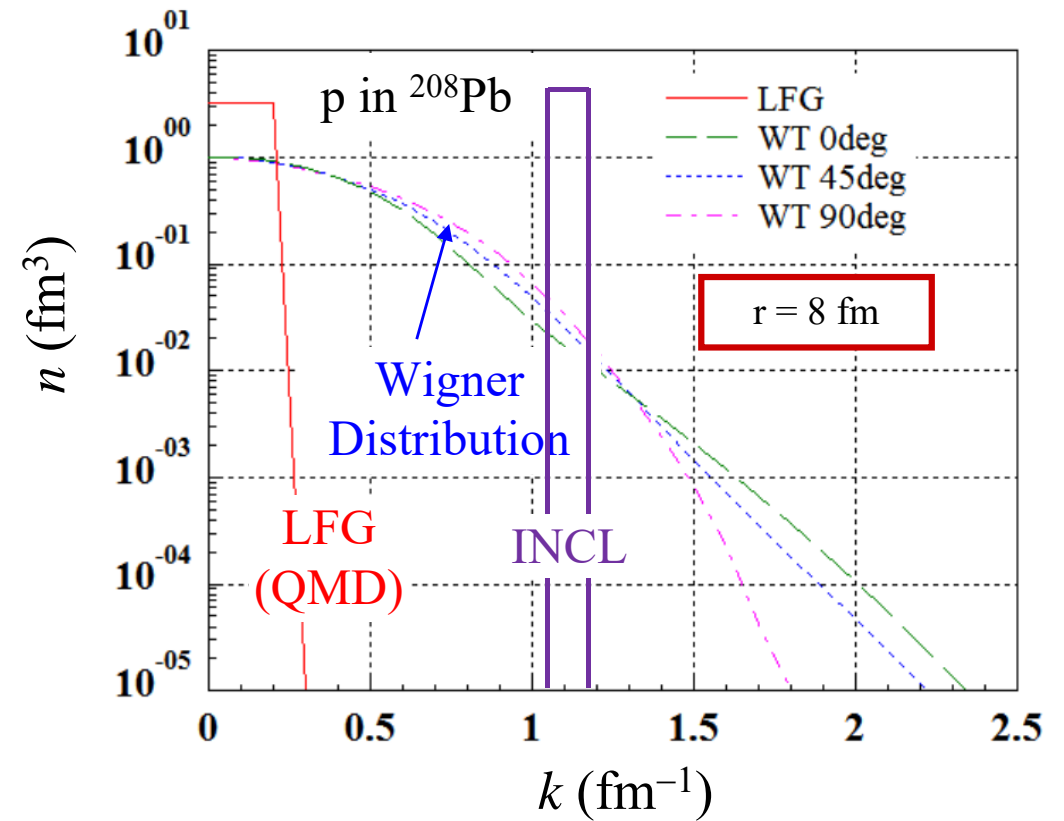
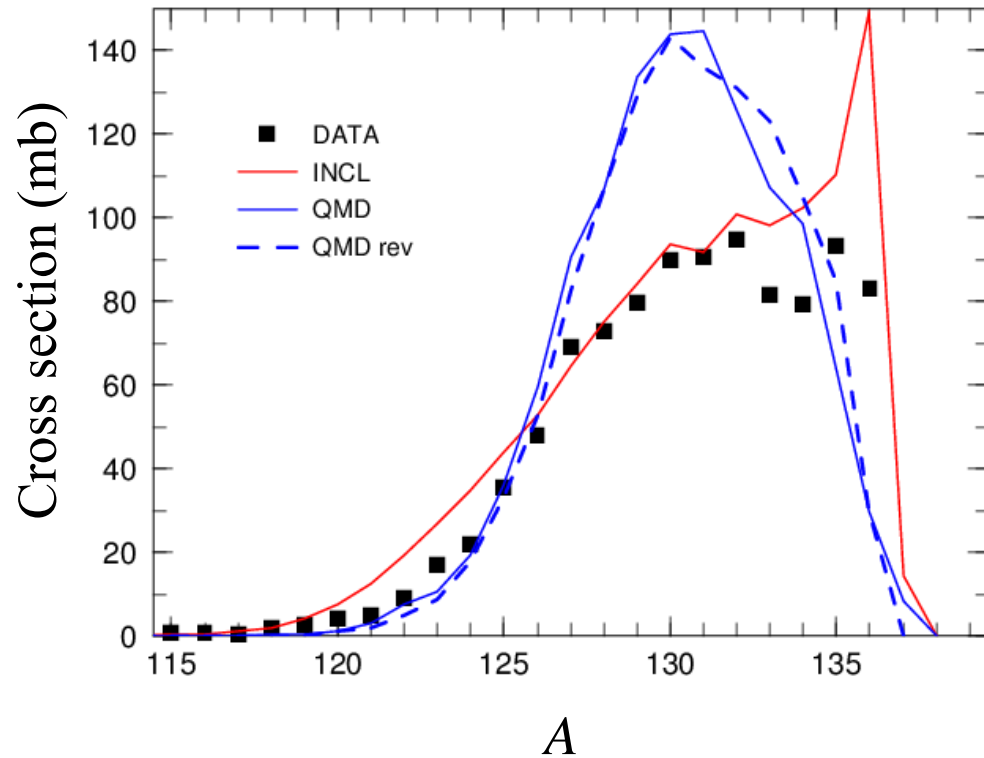
^{93}Zr at 100 MeV/nucleon

S. Kawase et al., PTEP2017, 093D03 (2017).



Problem on $-1N$ process

^{137}Cs 185MeV on proton



cf. D. Mancusi et al., PRC **91**, 034602 (2015).

Outline of the model

KO, arXiv:1801.09994.

$$\frac{d^2\sigma}{dE_f d\Omega_f} = C \int d\mathbf{k}_\beta d\mathbf{k}_\alpha \delta(\mathbf{K}_f + \mathbf{k}_\beta - \mathbf{K}_i - \mathbf{k}_\alpha) \delta(E_f + \varepsilon_\beta - E_i - \varepsilon_\alpha) \\ \times \int d\mathbf{R} \left| \chi_{f, \mathbf{K}_f}^{(-)}(\mathbf{R}) \right|^2 \left[2 - f_h^{(\beta)}(\mathbf{k}_\beta, \mathbf{R}) \right] f_h^{(\alpha)}(\mathbf{k}_\alpha, \mathbf{R}) \left| \tilde{t}(\boldsymbol{\kappa}', \boldsymbol{\kappa}) \right|^2 \left| \chi_{i, \mathbf{K}_i}^{(+)}(\mathbf{R}) \right|^2$$



$$\frac{d^2\sigma}{dk_\alpha dR} = C \int dE_f \int d\Omega_f \frac{k_\alpha m_N}{\hbar^2 q} \int d\phi_{k_\alpha} \int R^2 d\Omega_R \left| \chi_{f, \mathbf{K}_f}^{(-)}(\mathbf{R}) \right|^2 \\ \times \left[2 - f_h^{(\beta)}(\mathbf{k}_\beta, \mathbf{R}) \right] f_h^{(\alpha)}(\mathbf{k}_\alpha, \mathbf{R}) \left| \tilde{t}(\boldsymbol{\kappa}', \boldsymbol{\kappa}) \right|^2 \left| \chi_{i, \mathbf{K}_i}^{(+)}(\mathbf{R}) \right|^2$$

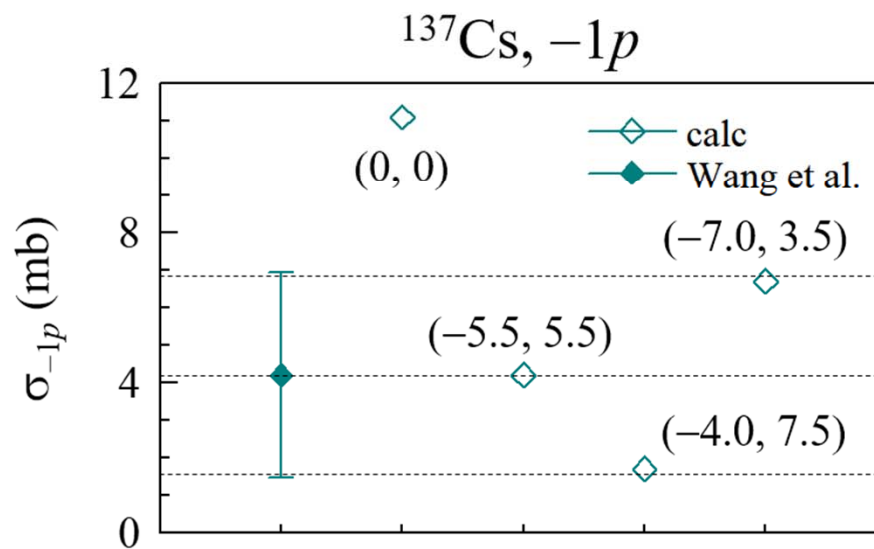
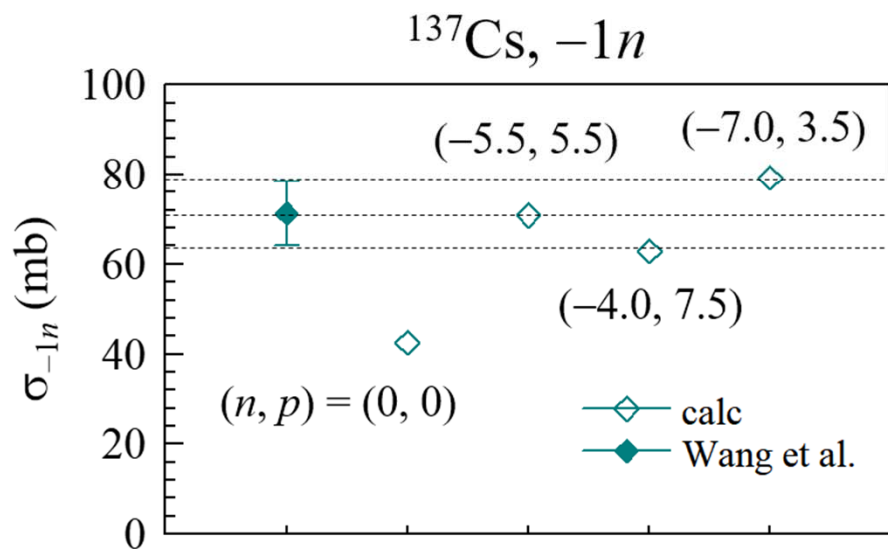
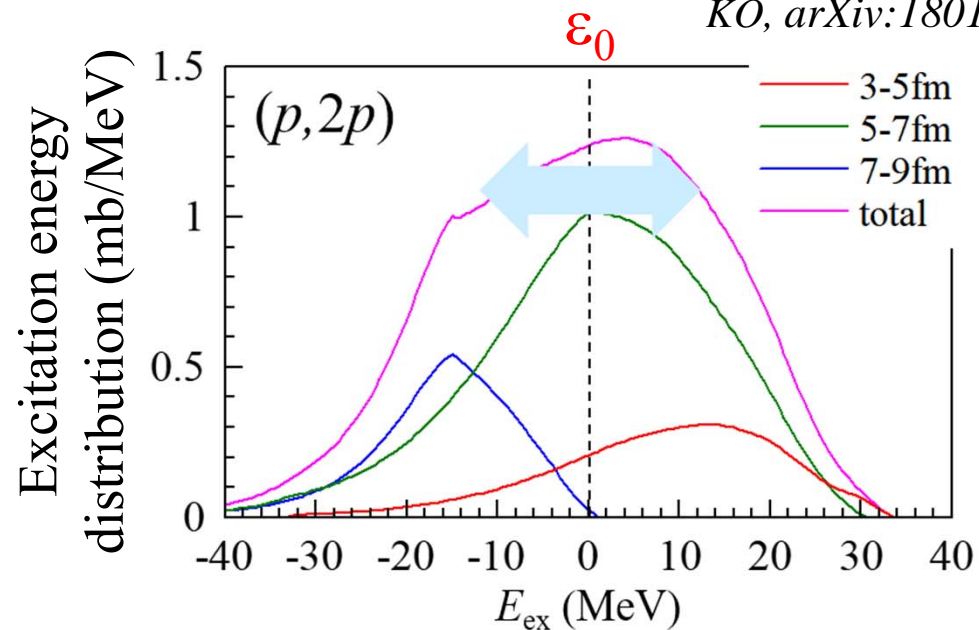
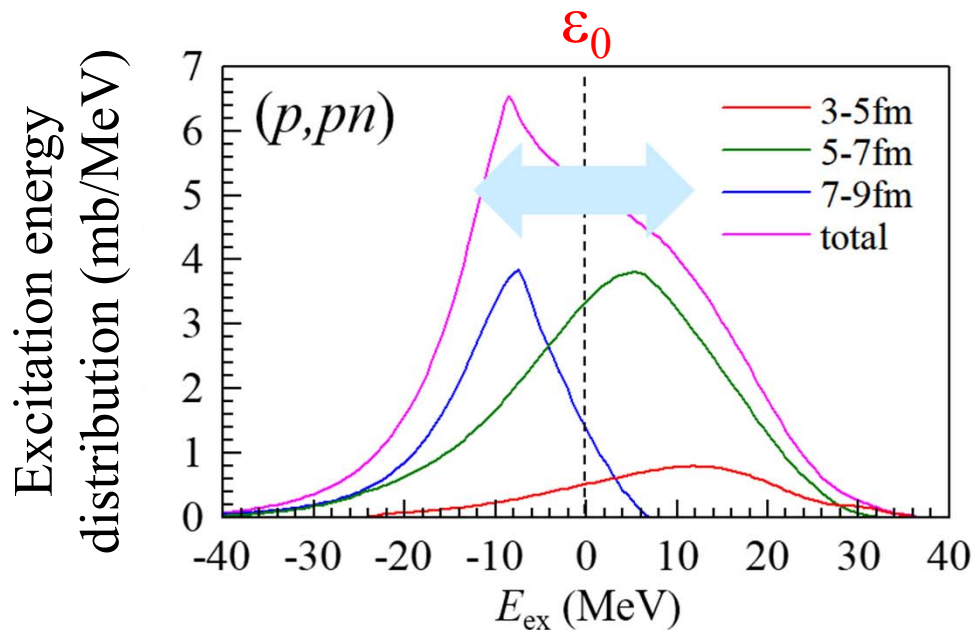


$$\frac{d\sigma}{d\varepsilon_\alpha} \equiv f_{\text{id}} \int dk_\alpha dR \frac{d^2\sigma}{dk_\alpha dR} \delta\left(\frac{\hbar^2}{2m_N} k_\alpha^2 + U_N(R) - \varepsilon_\alpha\right)$$

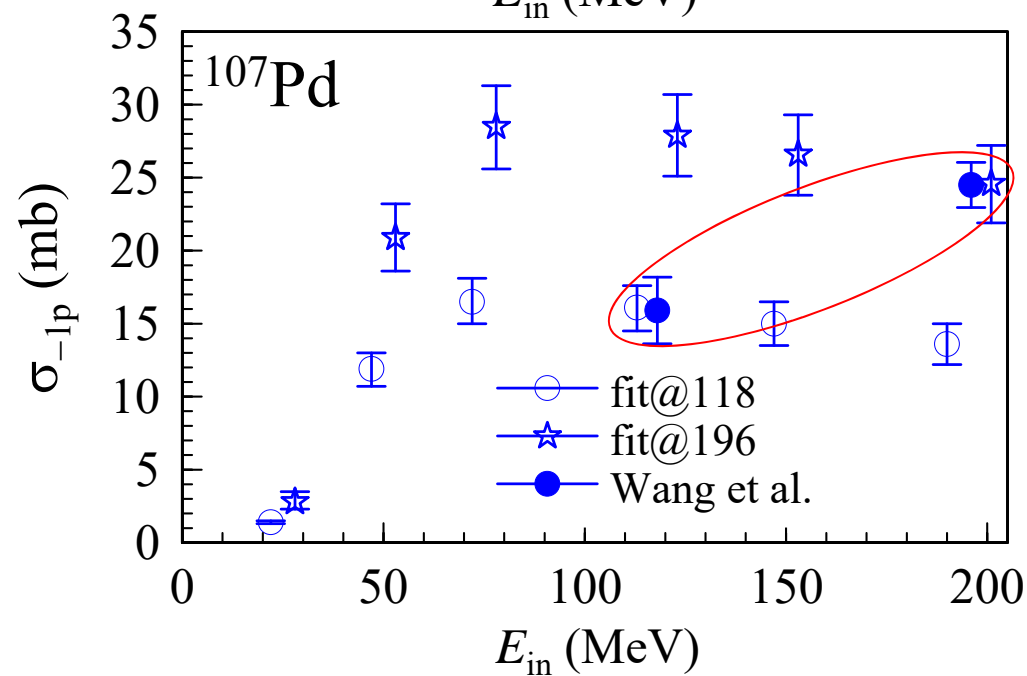
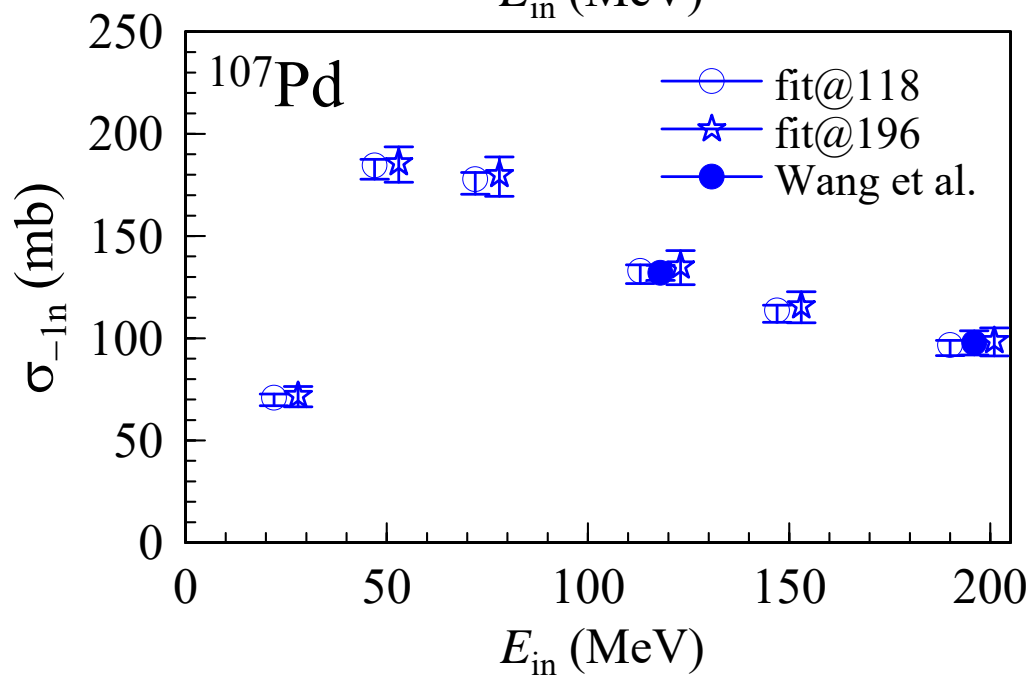
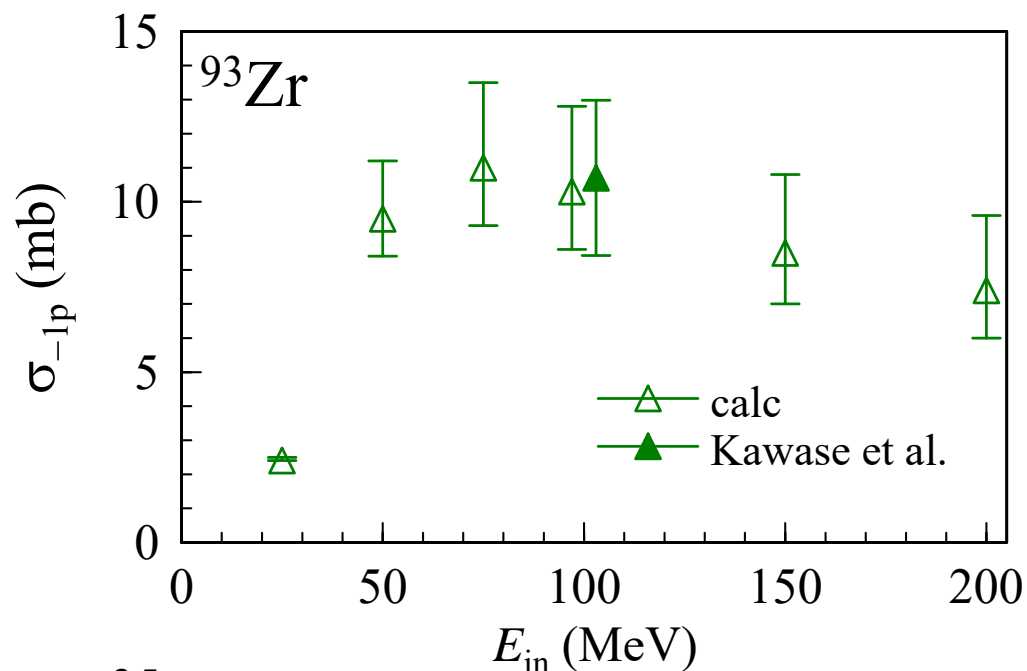
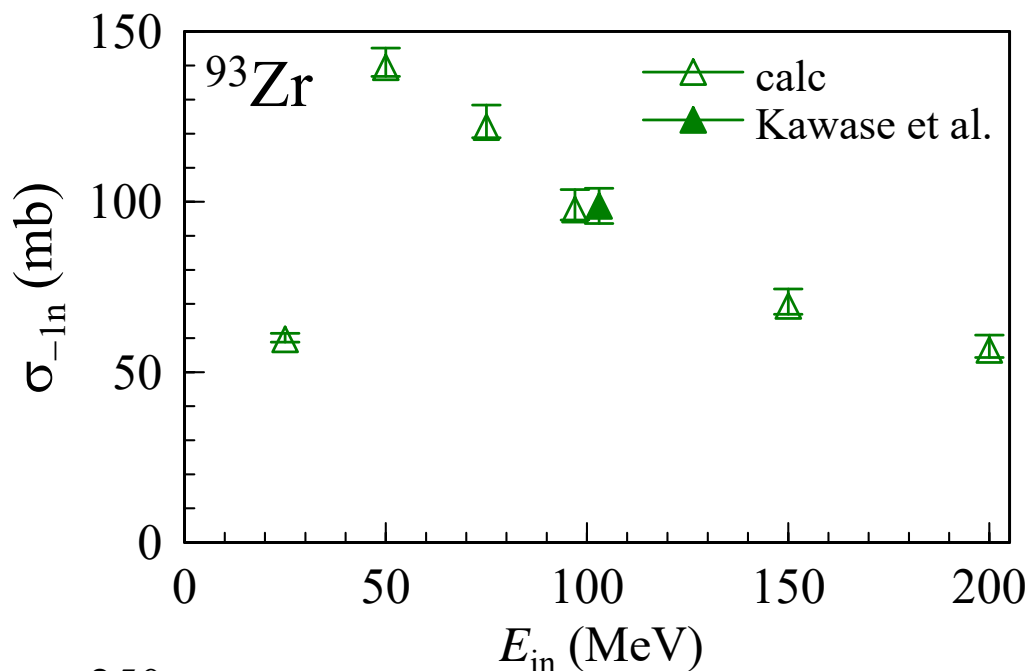
$$\varepsilon_{\text{ex}}^{\text{B}} = -S_N - \varepsilon_\alpha$$

Excitation energy distribution

KO, arXiv:1801.09994.



Incident energy dependence



Summary

- ❑ The Semi-Classical Distorted Wave model (SCDW) for inclusive QFS processes is reviewed.
- ❑ With SCDW,
 - ✓ an intuitive picture of the cascade QFS is derived from DWIA.
 - ✓ multistep direct processes up to 3-step are described.
 - ✓ spin observables also are calculable.
 - ✓ general description of an elementary process inside a nucleus is possible.
- ❑ Future plans:
 - ✓ application of SCDW to other types of inclusive knockout process
 - ✓ better description of the spallation cross sections of LLFP.