Ideal hydrodynamics limit extensions explored

Fluctuations, Polarization and gauge theory

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Based on 1810.12468,1807.02796,1701.08263,1604.05291,1502.05421,1112.4086 (PRD,PRC,EPJA) with D.Montenegro,L.Tinti. Speaking in a personal capacity (my collaborators do not necessarily agree with my interpretation)

First of all.... thanks to the organizers for putting this together, and all participants for a high level stimulating profound discussion which illustrates how fascinating this subject is! Seriously, only thing missing is...

Where all is

understood!

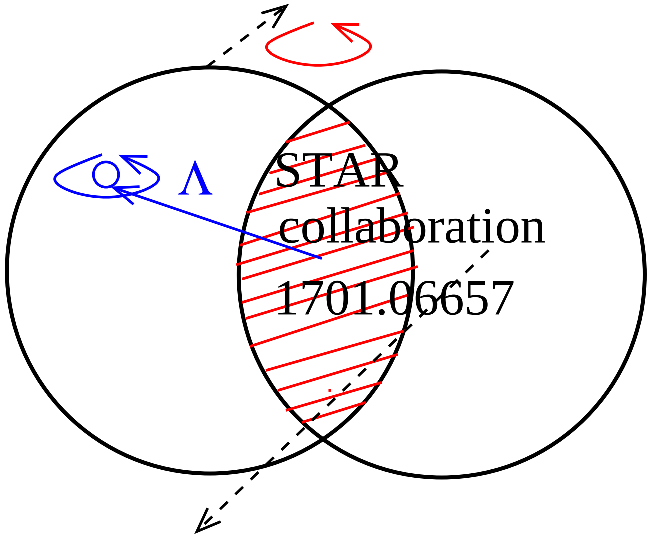
What is this talk about

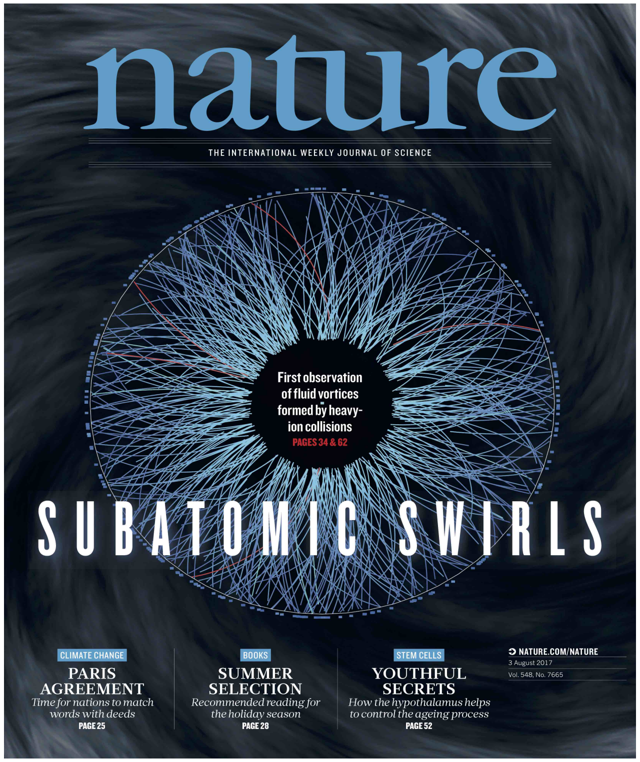
The necessity of a field theory perspective
Hydrodynamics is neither transport nor string theory!

Introduction to the field theory of hydrodynamics
Our knowledge of hydrodynamics rewritten as symmetries and free energy minimization

Disadvantage Unlike other approaches, EoS,viscosity,polarizability etc. black boxes. Limited insight on microscopic physics (beyond "thermalizes fast, has these symmetries" and causality consequences)

Advantage Some insights easy (viscosity,polarization,E-M tensor, role of gauge symmetries...)
Considering the ambiguities apparent in last few weeks, very useful!

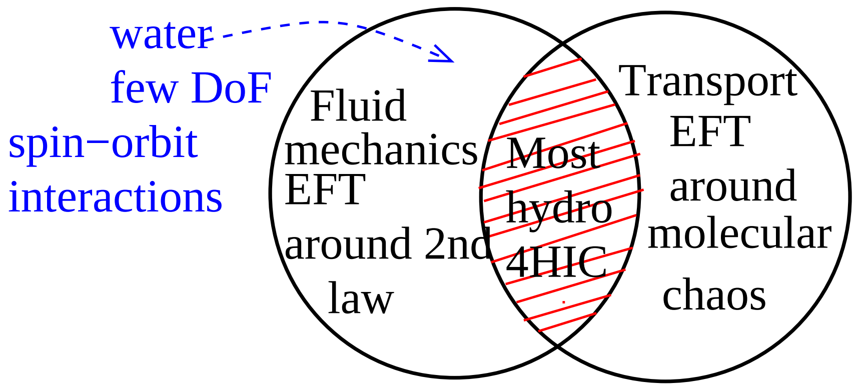




A spectacular experimental result. A common question: but what have we learned? My take is...

Hydro is not (just) transport! Nor string theory! Hydro is hydro!

Its constitutents are usually neither billiard balls not black holes!



$\langle f_{micro} f_{micro} \rangle \sim \langle f \rangle \langle f \rangle$

Hydro very good but "micro" and "macro" DoFs talk to each other!

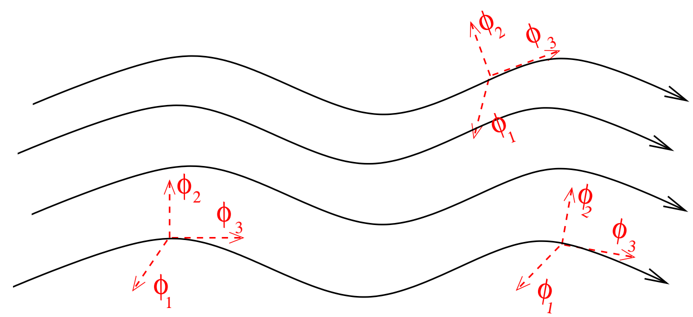
Convergence of both Boltzmann (reliant on molecular chaos, corrections expanded by Occupancy number) and AdS/CFT (N_c) suspect

But fluid appears "perfect", viscosity low to vanishing!

Lagrangian hydrodynamics , assuming "instant thermalization"

Action $\leftrightarrow$ Free energy, Fluctuations/correlations $\leftrightarrow$ Functional integrals

Lets set-up EFT around local equilibrium (Nicolis et al,1011.6396 (JHEP))
Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t = 0, x^i), I = 1...3$. (Lagrangian hydro . NB: no conserved charges)



Local equilibrium$\Rightarrow$ Maximize s $\Rightarrow$ Minimize F $\rightarrow S \underbrace{\Leftrightarrow}_{Legendre} F$

The system is a Fluid if it's Lagrangian obeys some symmetries subject to local minima of free energy. Solutions generally break these, Excitations (Sound waves, vortices etc) can be thought of as "Goldstone bosons".
Causality can be checked by linearizing and finding dispersion relation

Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \mathrm{det} B^{IJ}, \mathrm{diag} B^{IJ}$
The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B (actually $b = \sqrt{B}$)
In all fluids a cell can be infinitesimally deformed
(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

A few exercises for the bored public Check that L = -F(B) leads to

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B) \;, \qquad p = F(B) - 2F'(B)B \;, \qquad u^\mu = \frac{1}{6\sqrt{B}}\epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\,\partial_\alpha\phi^I\partial_\beta\phi^J\partial_\gamma\phi^K$$

(A useful formula is $\frac{db}{d\partial_\mu\phi_I}\partial_\nu\phi_I = u^\mu u^\nu - g^{\mu\nu}$)

Equation of state chosen by specifying $F(b)$. "Ideal": $\Leftrightarrow F(B) \propto b^{2/3}$

b is identified with the entropy and $b\frac{dF(B)}{dB}$ with the microscopic temperature.

u^μ fixed by $u^\mu\partial_\mu\phi^{\forall I} = 0$. Vortices become Noether currents of diffeomorphisms!

This is all really smart, but why?

Hydrodynamics is based on <u>three</u> scales

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

l_{micro} stochastic, l_{mfp} dissipative. If $l_{micro} \sim l_{mfp}$ soundwaves

Of amplitude so that <u>momentum</u> $P_{sound} \sim (area) \lambda \, (\delta\rho) \, c_s \gg T$

And wavenumber $k_{sound} \sim P_{sound}$

Survive (ie their amplitude does not decay to $E_{sound} \sim T$) $\tau_{sound} \gg 1/T$

Transport: Beyond <u>Molecular chaos</u> AdS/CFT: Beyond <u>large N_c</u>
It turns out Polarization, gauge symmetries mess this l_{micro} hyerarchy!

Ideal hydrodynamics and the microscopic scale

The most general Lagrangian is

$$L = T_0^4 F\left(\frac{B}{T_0^4}\right) \quad , \quad B = T_0^4 \det B^{IJ} \quad , \quad B^{IJ} = \left|\partial_\mu \phi^I \partial^\mu \phi^J\right|$$

Where $\phi^{I=1,2,3}$ is the comoving coordinate of a volume element of fluid.

NB: $T_0 \sim \Lambda g$ microscopic scale, includes thermal wavelength and $g \sim N_c^2$ (or μ/Λ for dense systems). $T_0 \to \infty \Rightarrow$ classical limit
It is therefore natural to identify T_0 with the microscopic scale!

Kn behaves as a gradient, T_0 as a Planck constant!!!

At $T_0 < \infty$ quantum and thermal fluctuations can produce sound waves and vortices, "weighted" by the usual path integral prescription!

$$L \to \ln \mathcal{Z} \quad \mathcal{Z} = \int \mathcal{D}\phi_i \exp\left[-T_0^4 \int F(B) d^4x\right], \langle \mathcal{O} \rangle \sim \frac{\partial \ln \mathcal{Z}}{\partial \ldots}$$

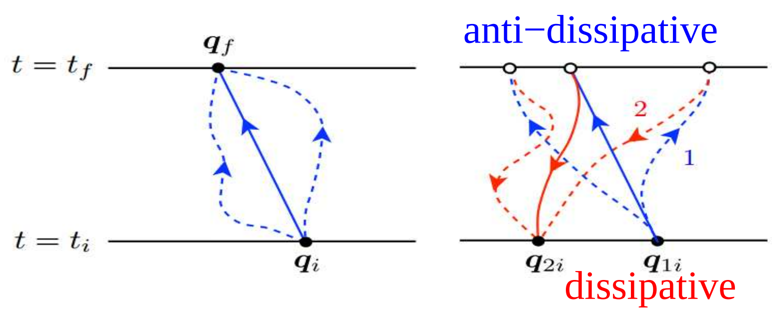
$$\left(eg. \quad \left\langle T_{\mu\nu}^{x} T_{\mu\nu}^{x'} \right\rangle = \frac{\partial^2 \ln \mathcal{Z}}{\partial g_{\mu\nu}(x) \partial g_{\mu\nu}(x')}\right)$$

$T_0 \sim n^{-1/3}$, unlike Knudsen number, behaves as a "Planck constant".EFT expansion and lattice techniques should give all allowed terms and correlators. Coarse-graining will be handled here!

The big problem with Lagrangians... usually only non-dissipative terms

But there are a few ways to fix it. We focus on coordinate doubling (Galley, but before Morse+Feschbach)

Dissipative extension of Hamiltons principle

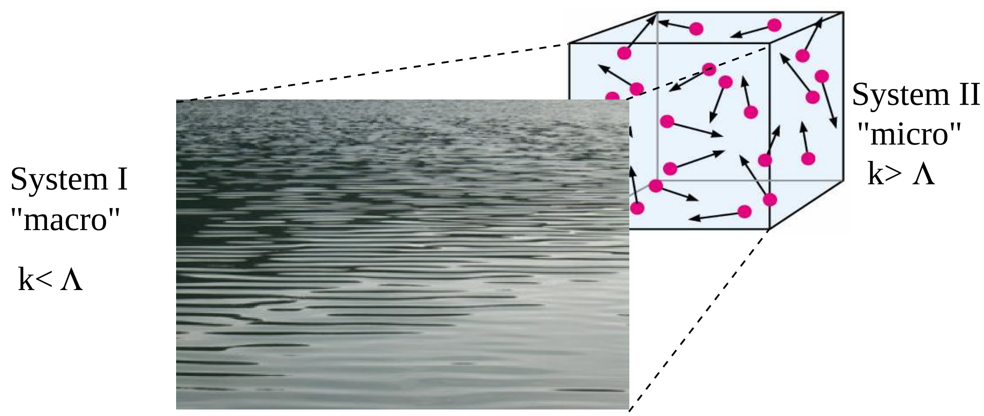


$$L = \frac{1}{2}\Big(\underbrace{m\dot{x}^2 - w x^2}_{SHO}\Big) \rightarrow \underbrace{\left(m\dot{x}_+^{\,2} - w x_+^2\right)}_{\mathcal{L}_1} - \underbrace{\left(m\dot{x}_-^{\,2} - w x_-^2\right)}_{\mathcal{L}_2} + \underbrace{\alpha\left(\dot{x}_+ x_- - \dot{x}_- x_+\right)}_{\mathcal{K}}$$

two sets of equations, one with a damped harmonic oscillator, the other "anti-damped". Navier-Stokes and Israel-Stewart (GT, D.Montenegro, PRD, (2016)) Functional integrals/Lattice also possible!

For analytical calculations fluid can be perturbed around a hydrostatic ($\phi_I = \vec{x}$) background

$$\phi_I = \vec{x} + \underbrace{(\vec{\pi}_L)}_{sound} + \underbrace{(\vec{\pi}_T)}_{vortex}$$



Kolmogorov cascade, Viscosity from turbulence when frequency$\simeq$ energy?

And we discover a fundamental problem: Vortices carry arbitray small energies but stay put! No S-matrix in hydrostatic solution!

$$L_{linear} = \underbrace{\dot{\vec{\pi}}_L^{\,2} - c_s^2(\nabla.\vec{\pi}_L)^2}_{sound\quad wave} + \underbrace{\dot{\pi_T}^2}_{vortex} + Interactions(\mathcal{O}\left(\pi^3, \partial\pi^3, ...\right))$$

Unlike sound waves , Vortices can not give you "free particles", since they do not propagate: They carry energy and momentum but stay in the same place! Can not expand such a quantum theory in terms of free particles.

Physically: "quantum vortices" can live for an arbitrary long time, and dominate any vacuum solution with their interactions. This does not mean the theory is ill-defined, just that its strongly non-perturbative!
Lattice: Tommy Burch,GT, 1502.05421 In ideal limit, Indications of a 1st order transition between turbulent and hydrostatic phases! Need viscous corrections, fluctuation/dissipation on lattice (BIG project!) But also Polarization might help here!

And chemical potential?

Within Lagrangian field theory a <u>scalar</u> chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding <u>chemical shift symmetry</u>

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

What is ideal hydro? A conceptual difficulty!

Entropy conserved always at maximum at each point in spacetime

Local isotropy in the comoving frame
But polarization non-zero at equilibrium if particles have spin!

Circulation is conserved (Kelvins theorem/Nother current for deformations)
But polarization "absorbs and emits" angular momentum!

Continuum limit when you break up cells, intensive results stay the same
But each particle carries discrete spin unit!

With polarization, only the first has a chance of being realized even in the ideal limit

Back to those length scales... $\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$

It is clear first 'stochastic' scale controls polarization:

- Vorticity is a "collective" excitation, while polarization is given by microstate counting, $\leftrightarrow$ fluctuations
- Polarization a 2-particle correlation, reducing entropy $f(s_1|s_2)$
- In planar limit fermion polarization typically N_c-suppressed
  Gauge boson polarization not gauge invariant!

Understanding role of polarization is "similar" to understanding role of fluctuations. Not a conserved quantity so lagrangians help!

Combining polarization with the ideal hydrodynamic limit, defined as

(i) The dynamics within each cell is faster than macroscopic dynamics, and it is expressible only in term of local variables and with no explicit reference to four-velocity u^μ (gradients of flow are however permissible, in fact required to describe local vorticity).

(ii) Dynamics is dictated by local entropy maximization, within each cell, subject to constraints of that cell alone. Macroscopic quantities are assumed to be in local equilibrium inside each macroscopic cell

(iii) Only excitations around a hydrostatic medium are sound waves,vortices

(i-iii) ,with symmetries and EFT define the theory

So how do we implement polarization?

In comoving frame, polarization described by a representation of a "little group" of the volume element.

Need local $\sim SO(3)$ charges <u>and</u> unambiguus definition of u^μ $(s^\mu \propto J^\mu)$

$$\Psi_{\mu\nu}|_{comoving} = -\Psi_{\nu\mu}|_{comoving} = \exp\left[-\sum_{i=1,2,3} \alpha_i(\phi_I) \hat{T}_i^{\mu\nu} \right]$$

For particle spinor, vector, tensor... repreentations possible.

For "many incoherent particles" RPA means only vector representation remains

Similar to Xin-Li Sheng's "continuus spin"

Chemical shift symmetry, $SO(3)_{\alpha_{1,2,3}} \rightarrow SO(3)_{\alpha_{1,2,3}(\phi^I)}$

$$\alpha_i \rightarrow \alpha_i + \Delta\alpha_i\left(\phi_I\right) \Rightarrow L(b, y_{\alpha\beta} = u_\mu \partial^\mu \Psi_{\alpha\beta})$$

$y_{\mu\nu} \equiv \mu_i$ for polarization vector components in comoving frame

This way we ensured spin current flows with u^μ.

Note that it is not a proper chemical potential (it it would be there would be 3 phases attached to each ϕ_I) as $y_{\mu\nu}$ not invariant under symmetries of ϕ_I. $y_{\mu\nu}$ "auxiliary" polarization field

How to combine polarization with local equilibrium?

Since polarization decreases the entropy by an amount proportional to the DoFs and independent of polarization direction

$$b \to b\left(1 - c y_{\mu\nu} y^{\mu\nu} + \mathcal{O}\left(y^4\right)\right) \quad , \quad F(b) \to F(b,y) = F\left(b\left((1 - c y^2\right)\right)$$

$c > 0$ ferrovortetic $c < 0$ antiferrovortetic (like ferromagnetic but for vorticity!)

Other terms break requirement (i)

First law of thermodynamics,

$$dE = TdS - pdV - Jd\Omega \to dF(b) = db\frac{dF}{db} + dy\frac{dF}{d(yb)}$$

Energy-momentum tensor

Not uniquely defined

Canonical defined as the Noether charge for translations, could be negative because of $\sim \frac{\partial L}{\partial(\partial\psi_i)}\partial\psi_j$

Belinfante-Rosenfeld $\sim \frac{\delta S}{\delta g_{\mu\nu}}$ symmetric independent of spin, no non-relativistic limit

Which is the source for $\partial_\mu T^{\mu\nu} = 0$? Not clear as...

The problem: Too many degrees of freedom

8 degrees of freedom,5 equations $(e, p, u_{x,y,z}, y^{\mu\nu})$. One can include the antisymmetric part of $T_{\mu\nu}$ and match equations but...

No entropy maximization If spin waves and sound waves separated, in comoving volume their ratio is arbitrary... but it should be decided by entropy maximization!

I suspect EFTs based on $T_{\mu\nu}$ (Hong Liu,Florkowski and collaborators) will have this problem

Solution clear: make polarization always proportional to vorticity,

$$y^{\mu\nu} \sim \chi(T)(e+p)\left(\partial^{\mu}u^{\nu} - \partial^{\nu}u^{\mu}\right)$$

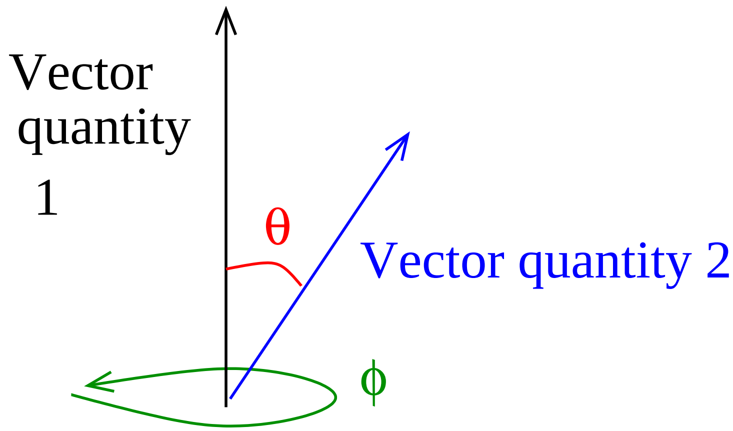
extension of Gibbs-Duhem to angular momentum uniquely fixes χ via entropy maximization. For a free energy $\mathcal{F}$ to be minimized

$$d\mathcal{F} = \frac{\partial \mathcal{F}}{\partial V} dV + \frac{\partial \mathcal{F}}{\partial e} de + \frac{\partial \mathcal{F}}{\partial \left[\Omega_{\mu\nu}\right]} d\left[\Omega_{\mu\nu}\right] = 0$$

where $[\Omega_{\mu\nu}]$ is the vorticity in the comoving frame.
THis fixes χ . It also constrains the Lagrangian to be a Legendre transform of the free energy just as in the chemical potential case, in a straightforward generalization of Nicolis,Dubovsky et al. Free energy always at (local) minimum! (requirement (ii))

A qualitative explanation

Instant thermalization means vorticity instantly adjusts to angular momentum, and is parallel to angular momentum. Corrections to this will be of the relaxation type a-la Israel-Stewart



Microscopic physics allows an arbitrary angle between vorticity and polarization. but such systems would have no hydrodynamic limit due to requirement (iii) and the necessity for stability of relaxation dynamics

Radoslaw had a Killing-vector argument, here's a qualitative expanation!

These techniques lead to a well-defined Euler-Lagrange equation of motion

$$\partial_\mu J_I^\mu = 0 \quad , \quad J_I^\mu = 4\,c\,\partial_\nu \left\{ F' \left[\chi \left(\chi + 2\,\partial_{\Omega^2}\chi \right) \omega_{\alpha\beta}\, g^{\alpha\{\mu} P_I^{\nu\}\beta} \right] \right\} -$$

$$-F' \left[u_\rho P_I^{\rho\mu} \left(1 - c y^2 - 2cb\chi\omega^2\,\partial_b\chi \right) \right] - 2c \left(\chi + 2\,\omega^2\,\partial_{\Omega^2}\chi \right) F' \times$$

$$\times \left\{ \left[\chi\,\omega^2 - \frac{1}{b} y_{\rho\sigma} \left(u_\alpha \partial^\alpha K^\rho - u_\alpha \nabla^\rho K^\alpha \right) \right] P_I^{\sigma\mu} - \frac{1}{6b} y_{\rho\sigma} \varepsilon^{\mu\rho\alpha\beta} \epsilon_{IJK} \nabla^\sigma \partial_\alpha \phi^J \partial_\beta \phi^K \right\}.$$

$$P_K^{\mu\nu} = \partial K^\mu / \partial(\partial^\nu \phi^K) \quad , \quad \nabla^\alpha = \Delta^{\alpha\beta}\partial_\beta$$

NB depends on accelleration, so $\Delta S = 0 \Rightarrow \partial_\mu \partial_\nu \frac{\partial F}{\partial(\partial_\mu \partial_\nu \phi^I)} = \partial_\mu \frac{\partial F}{\partial(\partial_\mu \phi^I)}$

J_I^μ : Co-moving total angular momentum components!

Which can be linearized, $\phi_I = X_I + \pi_I$
The "free" (sound wave and vortex kinetic terms) part of the equation will be

$$\mathcal{L} = (-F'(1))\left\{\frac{1}{2}(\dot{\pi})^2 - c_s^2[\partial\pi]^2\right\} +$$

$$+ f\zeta\left\{\ddot{\pi}^i\partial_i\dot{\pi}_j + \ddot{\pi}_i\ddot{\pi}_j + \partial_j\dot{\pi}^i\partial_i\dot{\pi}_j + \partial_j\dot{\pi}_i\ddot{\pi}_j + \right.$$

$$\left. + (2\ddot{\pi}^i\partial_j\dot{\pi}_i - 2\ddot{\pi}_j\partial^i\dot{\pi}_j) + (\ddot{\pi}_i^2 - \ddot{\pi}_j^2) + (\partial_j\dot{\pi}_i^2 - \partial_i\dot{\pi}_j^2)\right\}$$

- Accelleration terms survive linearization
- Vortices and sound wave modes mix at "leading" order. Change in temperature due to sound wave changes polarizability, and that changes vorticity

We decompose perturbation into sound and vortex $\phi_I = \nabla\phi + \nabla \times \vec{\Omega}$

$$\left(\begin{array}{c} \varphi \\ \vec{\Omega} \end{array} \right) = \int dwd^3k \left(\begin{array}{c} \varphi_0 \\ \vec{\Omega}_0 \end{array} \right) \exp\left[i \left(\vec{k_{\phi,\Omega}}.\vec{x} - w_{\phi,\Omega} t \right) \right]$$

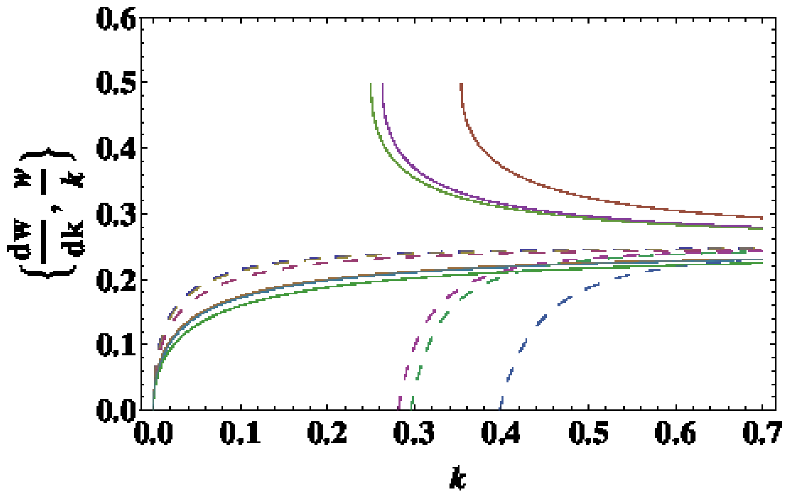
The part parallel to k ("sound-wave") will have a dispersion relation

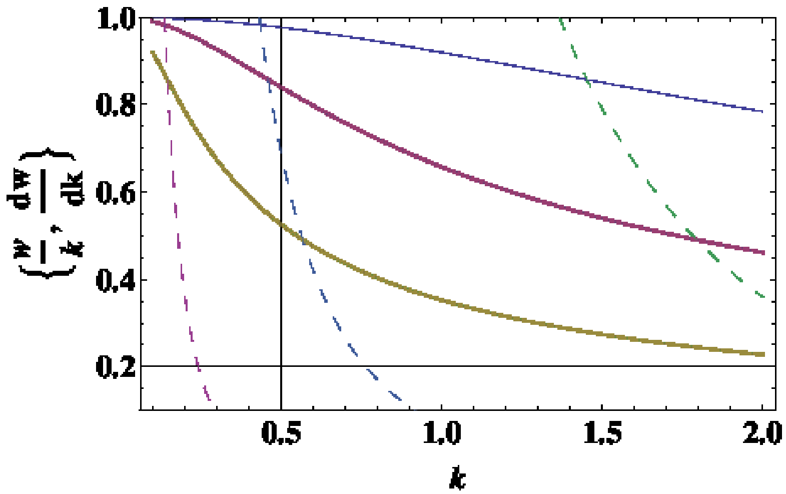
$$w_\phi^2 - c_s^2 k_\phi^2 + 2\beta k_\phi w_\phi^3 = 0$$

The vector part will be

$$(3k_\Omega^2 - 2k_\Omega w_\Omega)_j (\vec{k}_\Omega \times \vec{\Omega}_0)_i w_\Omega^2 + w^4 \Omega = 0$$

Dispersion relations show violation of causality!





Both phase and group velocity will generally go above unity

What I think is going on I: A lower limit of viscosity for polarized hydro

the Free energy $\mathcal{F}$, and hence the local dynamics, is sensitive to an accelleration. As is well-known (Ostrogradski's theorem, Dirac runaway solutions) such Lagrangians are unstable and lead to causality violation. Note that one needs Lagrangians to see this!

To fix this issue, one would need to update the proportionality of y on Ω to an Israel-Stewart type equation

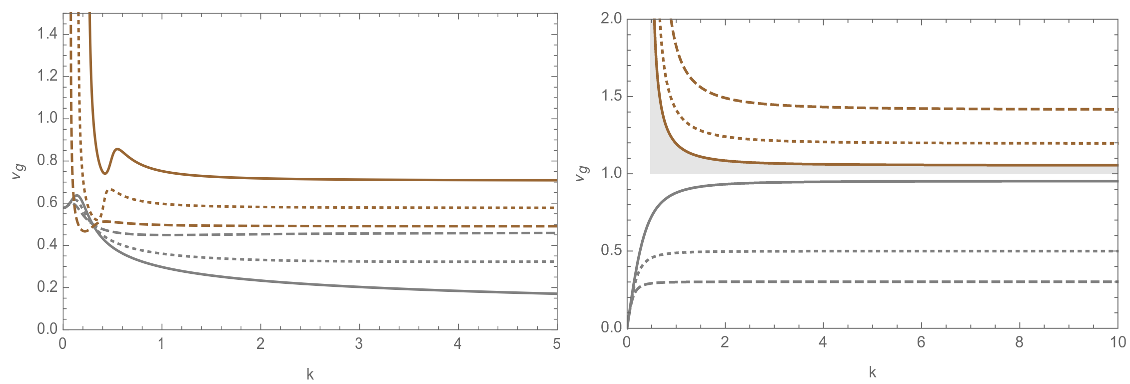
$$\tau_\Omega u_\alpha \partial^\alpha y_{\mu\nu} + y_{\mu\nu} = \chi(T, y)\Omega_{\mu\nu}$$

with an appropriate relaxation time τ_Ω would resolve this issue. Just like with Israel-Stewart, this requires the introduction of new DoFs with relaxation-type dynamics, but, unlike non-polarized hydro,such terms are required from the idea limit

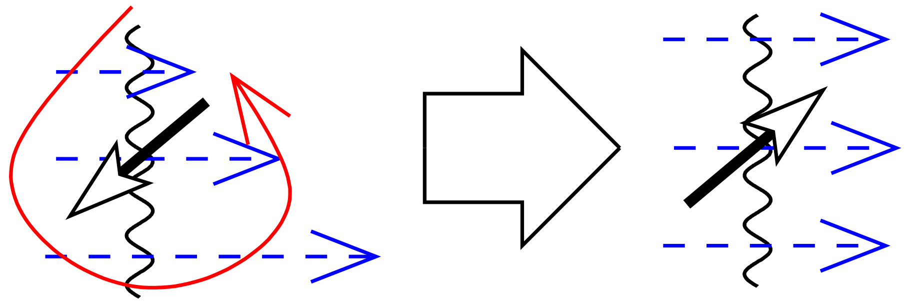
G Torrieri,D Montenegro, 1807.02796 : Polarization are independent DoFs which relax to vorticity

Anti-"Ferrovortetic" fluid non-causal mode ($|dw/dk| \geq 1$) in UV unless

$$\tau_Y^2 \geq \frac{8c\chi^2(b_o,0)}{(1-c_s^2)b_o F'(b_o)} \quad , \quad \frac{\eta}{s} \geq T\tau_Y$$



τ_Y regulates quenching of vortices into polarization!

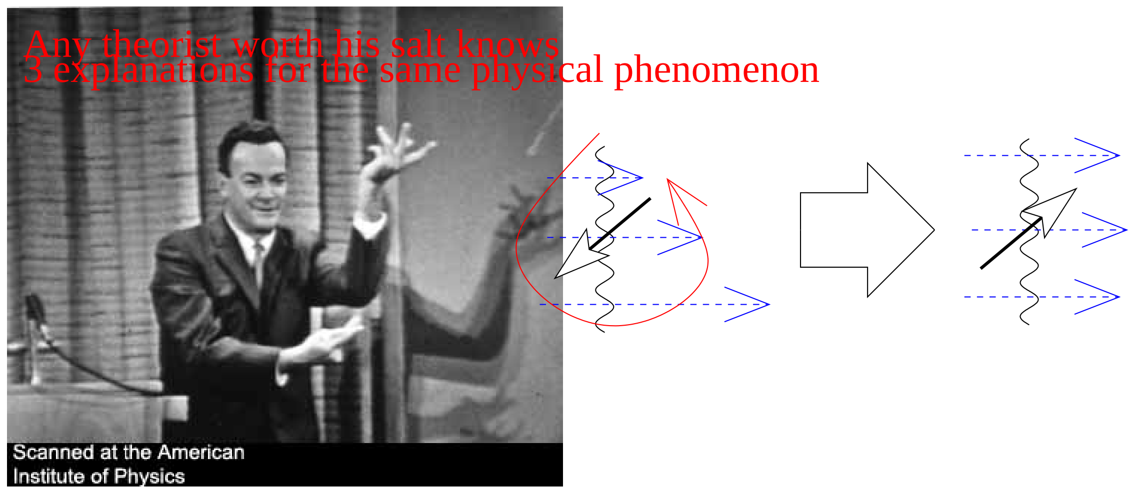


A bottom-up limit on viscosity from polarization!

$$\tau_Y^2 \geq \frac{8c\chi^2(b_o,0)}{(1-c_s^2)b_o F'(b_o)} \quad , \quad \frac{\eta}{s} \geq T\tau_Y$$

Heuristically: At strong coupling vorticity quenches gradients on a timescale $1/T$

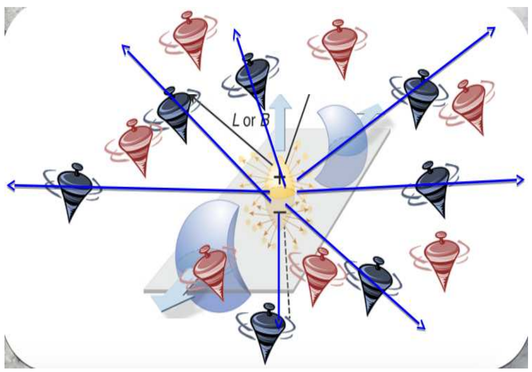
Dispersion relations show violation of causality!

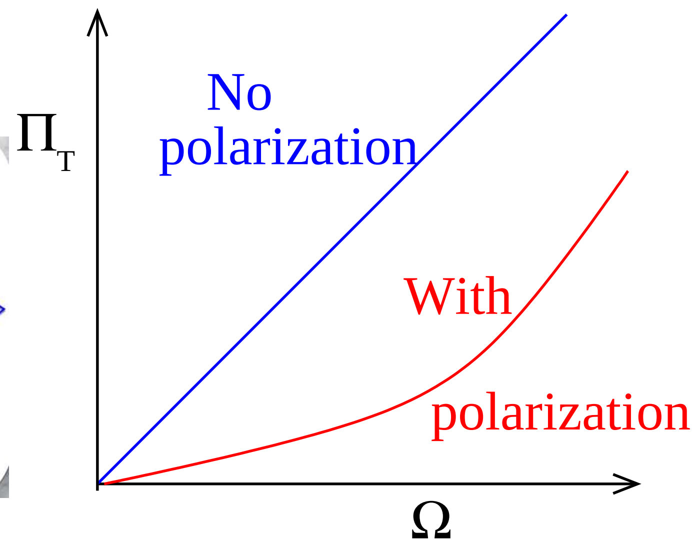


Mathematically : vorticity is an accelleration, instantaneus equilibration to it non-causal. Causality from Relaxation , of spin-flip dynamics “non-local collision term N. Weickgenannt et al , gradient expansion of entropy M.Hongo,S.Shi ,...yield same conclusion! Top-down dissipation constraint from polarizeability!

What I <u>think</u> is going on II

Fluctuation-dissipation: $\tau_\Omega \sim \lim_{\omega\to 0}\omega^{-1}\int dt\,\langle y_{\mu\nu}\Omega_{\mu\nu}\rangle \exp(i\omega t)$





Polarization makes vorticity aquire a "soft gap" wrt angular momentum. At small amplitudes, creating polarization is more advantageus than creating vorticity. This means small amplitude vortices get quenched.

A more rigorous derivation

GT,Montenegro, 2004.10195 A tour de force calculation

τ_Ω, χ related by Kramers-Konig relation

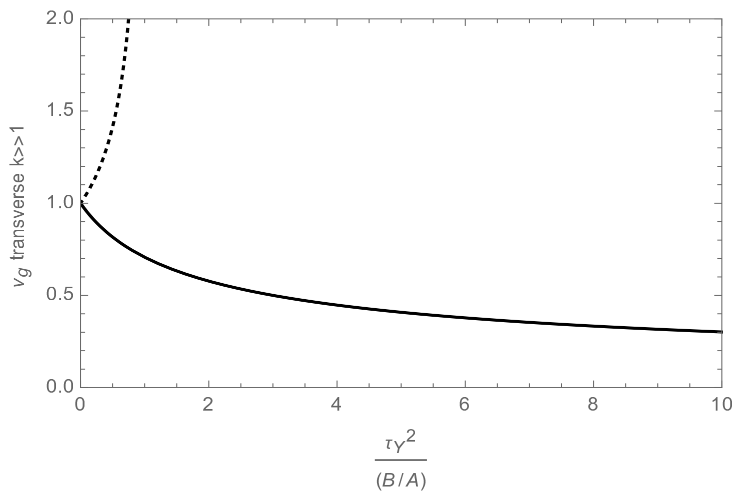
$$\chi = Re[F] \quad , \quad \tau_\Omega = Im[F] \quad , \quad F = Lim_{w\rightarrow\infty} \int e^{iwt} \langle [\Omega_{\mu\nu}(t), y_{\mu\nu}(0)$$

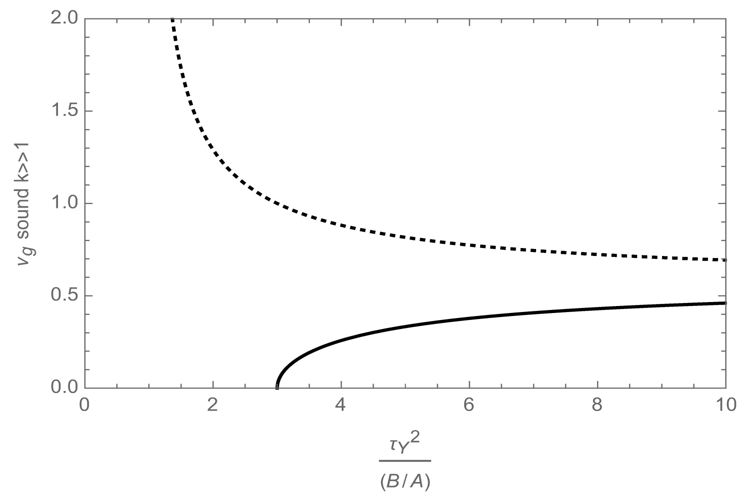
Higgs-like mechanism spontaneus breaking rotation symmetries and giving "mass" to vortices

Tails of corelation functions have same behavior as fluctuating hydrodynamics

Kubo formulae and correlators of both $T_{\mu\nu}, J_I^\mu$

Ferrovortetic fluid   Causal mode in IR remains





But remember that ferromagnetic vaccum is unstable and

$$\langle y_{\mu\nu} \rangle \equiv \lim_{k \to 0} \rho(\omega_{T,L}(k))$$

equivalent to Banks-Casher mode for unstable vacua!

EFT expansion unreliable in this case. Must construct gap equation, fluid with ferro-magnetic like (ferrovortetic) phase transition!

Phenomenology

Dilepton and photon polarization sensitive to spin density in the early phase, if measurable (can experiment double as a SG detector for dileptons? Speranza et al, 1802.02479,Baym et al, 1702.05906

Vector mesons could help in both vector meson and longitudinal spin discrepancies with hydro!

- Longitudinal polarization timescale different from transverse (longitudinal spin might not be in equilibrium)
- Cooper-Frye formula cannot work if spin and vorticity already present. Coalescence Wigner functions? Coherence?

Vector mesons might allow us to test this quantitatively since there is a more information in their decay distribution...

$$W(\theta,\phi) \sim \cos^2\theta\rho_{00} + \sin^2\theta\left(\frac{\rho_{11}+\rho_{-1-1}}{2}\right) - \sin 2\theta\left(\frac{\cos\phi Re\rho_{10} - \sin\phi Im\rho_{10}}{\sqrt{2}}\right)$$

$$+\sin 2\theta\left(\frac{\cos\phi Re\rho_{-10} + \sin\phi Im\rho_{-10}}{\sqrt{2}}\right)$$

$$\rho = U^{-1}_{\theta,\pi}\begin{pmatrix} 1+n_8 & \sqrt{3}(n_1 - in_2) & 0 \\ \sqrt{3}(n_1+in_2) & 1+n_8 & 0 \\ 0 & 0 & 1-2n_8 \end{pmatrix} U_{\theta,\phi}$$

To verify would neet both both θ and ϕ of vector mesons and photons.

$$\Psi^M_S = \sum_{S_1 S_2} (C)^L_{S_1 S_2}\Psi^{q_1}_{S_1}\Psi^{2_2}_{S_2} \quad , \quad L \equiv Vorticity$$

Coherence: $\rho^2 = \rho$ Vorticity: $S_{1,2}$ vs L . Work with Kayman Jhosef

Theory: Global/local equilibrium: 2007.09224

https://www.youtube.com/watch?v=oLYouz0YMHM (2 days ago)

Need to include fluctuations and pseudo-gauge dependence. Only way I can see it possible is Zubarev hydrodynamics

expanded around equilibrium $L \sim \ln Z \quad , \quad Z = Z_{T_0} \times Z_{\Pi}$

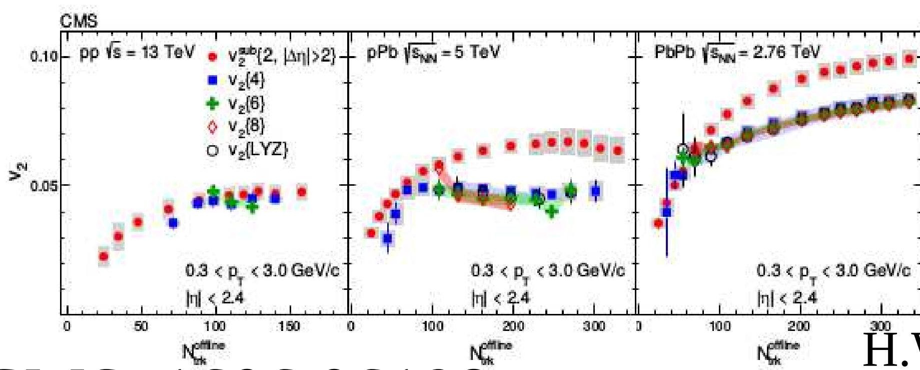
$$Z_{T_0} \Rightarrow \hat{\rho} = \exp\left[-\beta_\mu d\Sigma_\nu \left(\hat{T}^{\mu\nu} - \hat{n}^\nu - \omega^{\mu\nu\alpha} \hat{J}_\alpha\right)\right]$$

Each part of the free energy is pseudo-Gauge dependent, but free energy invariant! $\hat{\Pi}_{\mu\nu}$ also an operator determined by Crooks theorem

$$Z_\Pi \Rightarrow \frac{P(W)}{P(-W)} \sim \exp\left[\Delta S(\langle \Pi \rangle)\right]$$

Any local equilibrium state will tend to global equilbrium via fluctuations obeying detailed balance. Speaking of pseudo-gauge symmetries...

Another spectacular experimental result



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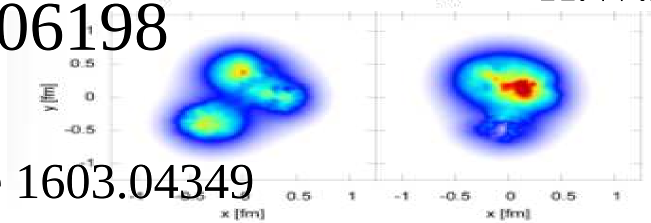

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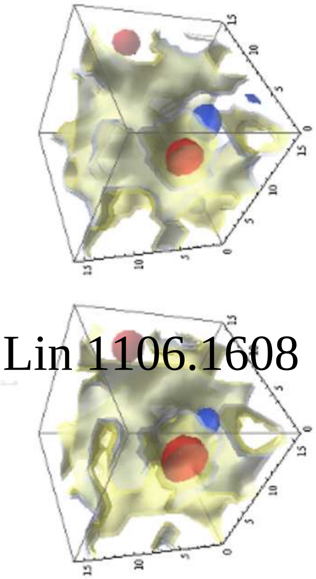


H.W.Lin 1106.1608

1606.06198 (CMS) : When you consider geometry differences, hydro with $\mathcal{O}(20)$ particles "just as collective" as for 1000. So mean free path is really small. What about thermal fluctuations? Nothing here is infinite, not even N_c Also hydro applicability scale below color domain scale. colored hydro?

The formalism we introduced earlier is ok for quark polarization but not gluon polarization: Gauge symmetry means one can exchange locally angular momentum states for spin states. So vorticity vs polarization is ambiguus. Separation in optics, parton spin structure requires a preferred static frame,different from comoving frame

Using the energy-momentum tensor for dynamics is even more problematic for spin $T_{\mu\nu}$ aquires a "pseudo-gauge" transformation

$$T_{\mu\nu} \to T_{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda} \right)$$

Φ fully antisymmetric. T.Brauner, 1910.12224:
$\phi \to \phi + \zeta(x), x_\mu \to x_\nu + \omega_\mu(x), S \to S$ But in a gauge theory, pseudo-Gauge transformations are gauge transformations ($\hat{\Pi}_\mu \to \hat{\Pi}_\mu + \hat{A}_\mu$)!
Large gauge configurations change $T^{\mu\nu}$

From global to gauge conserved currents

A reminder: Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

Generalization from $U(1)$ to generic group easy

$$\alpha \to \{\alpha_i\} \quad , \quad \exp(i\alpha) \to \exp\left(i \sum_i \alpha_i \hat{T}_i \right)$$

One subtlety: Currents stay parallel to u_μ but chemical potentials become adjoint, since rotations in current space still conserved

$$y = J^\mu \partial_\mu \alpha_i \to y_{ab} = J_a^\mu \partial_\mu \alpha_b$$

Lagrangian still a function of $dF(b, \{\mu\})/dy_{ab}$, "flavor chemical potentials"

If color was just a global symmetry same thing happens see CFL literature!
But need to covariantize w.r.t. local gauge symmetries

From global to gauge invariance! Lagrangian invariant under

$$\{y_{ab}\} \to y'_{ab} = U^{-1}_{ac}(x) y_{cd} U_{db}(x) \quad , \quad U_{ab}(x) = \exp\left(i \sum_i \alpha_i(x) \hat{T}_i \right)$$

However, gradients of x obviously change y .

$$y_{ab} \to U^{-1}_{ac}(x) y_{cd} U_{bd}(x) = U^{-1}(x)_{ac} J^{\mu}_{f} U_{cf} U^{-1}_{fg} \partial_\mu \alpha_g U_{bg} =$$

$$= U^{-1}(x)_{ac} J^{\mu}_{f} U_{cf} \partial_\mu \left(U^{-1}_{fg} \alpha_d U_{bd}(x) \right) - J^{\mu}_{a} \left(U \partial_\mu U \right)_{fb} \alpha_f$$

Only way to make lagrangian gauge invariant is

$$F\left(b, J^{\mu}_{j} \partial_\mu \alpha_i\right) \to F\left(b, J^{\mu}_{j} \left(\partial_\mu - U(x) \partial_\mu U(x)\right) \alpha_i\right)$$

Which is totally unexpected, profound and crazy

The swimming ghost!

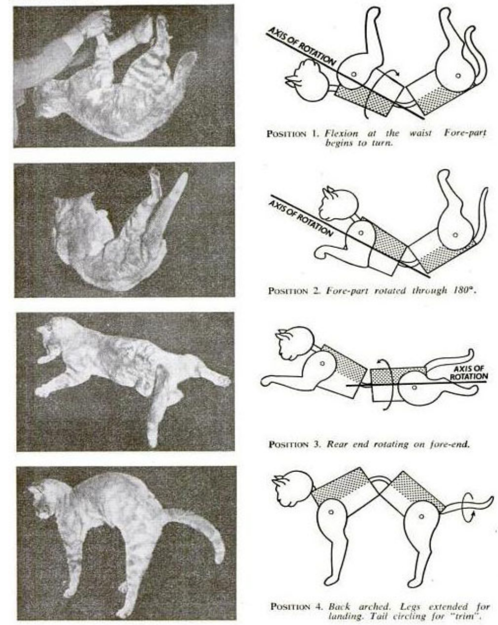
$$F\left(b, J_j^\mu \partial_\mu \alpha_i\right) \to F\left(b, J_j^\mu \left(\partial_\mu - U(x)\partial_\mu U(x)\right)\alpha_i\right)$$

Means the ideal fluid lagrangian depends on velocity!. no real ideal fluid limit possibl the system "knows it is flowing" at local equilibrium! NB: For U(1)

$$\hat{T}_i \to 1 \quad , \quad y_{ab} \to \mu_Q \quad , \quad u_\mu \partial^\mu \alpha_i \to A_\tau$$

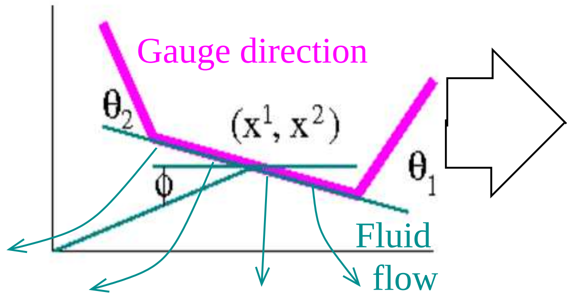
So second term can be gauged to a redefinition of the chemical potential (the electrodynamic potentials effect on the chemical potential).

Cannot do it for Non-Abelian gauge theory, "twisting direction" in color space It turns out this has an old analogue...



S. Montgomery (2003): How does a cat always fall on its feet without anything to push themsevles against? The shape of spaces a cat can deform themselves into defines a “set of gauges” a cat can choose without change of angular momentum.

Purcell,Shapere+Wilczek,Avron+Raz : A similar process enables swimmers to move through viscous liquids with no applied force



Now imagine each fluid cell filled with a “swimmer”, with arms and legs outstretched in “gauge” directions...

In ideal limit all currents proportional to u_μ . But gauge symmetry requires "ghost" excitations, proportional to gradients of currents, to not be physical. So free energy HAS to depend on flow.
Classic on this, B. Bistrovic, R. Jackiw, H. Li, V. P. Nair and S. Y. Pi, Phys. Rev. D **67**, 025013 (2003) ,"NonAbelian fluid dynamics in Lagrangian formulation," missed this subtlety as no local equilibrium defined!

Whats going on? A more statistical mechanics perspective

We perturb the hydrostatic limit, where $\phi_I = X_I$, and isolate a transverse mode (vortex) and a longitudinal mode (sound wave)

$$\phi_I = X_I + \vec{\pi}_I^{sound} + \vec{\pi}_I^{vortex} \quad , \quad \nabla . \vec{\pi}_I^{vortex} = \nabla \times \vec{\pi}_I^{sound} = 0$$

Since the derivative of the free energy w.r.t. b is positive, sound waves and vortices do "work". Let us now assume the system has a "color chemical potential" in some direction Let us change the color chemical potential in space according to

$$\Delta\mu(x) = \sum_i \left(\mu_i(x)^{swim} + \mu_i(x)^{swirl} \right) \hat{T}_i \quad , \quad \nabla_i . \mu_i^{swim} = \nabla_i \times \mu_i^{swirl} = 0$$

Because of gauge redundancy, the derivatives of the free energy with respect to color ("color susceptibility") will typically be negative. So the two can balance!!!!

But this breaks the "hyerarchy" of statistical mechanics

It mixes micro and macro perturbations!

In statistical mechanics, what normally distinguishes “work” from “heat” is coarse-graining, the separation between micro and macro states. Quantitatively, probability of thermal fluctuations is normalized by $1/(c_V T)$ and microscopic correlations due to viscosity are $\sim \eta/(Ts)$. Since for a usual fluid, there is a hyerarchy between microscopic scale, Knudsen number and gradient

$$\frac{1}{c_V T} \ll \frac{\eta}{(Ts)} \ll \partial u_\mu$$

Gauge symmetry breaks it, since it equalizes perturbations at both ends of this!

Is there a Gauge-independent way of seeing this? Perhaps!

One can write the effective Lagrangian in a Gauge-invariant way using Wilson-Loops . But the effective Lagrangian written this way will have an infinite number of terms, in a series weighted by the characteristic Wilson loop size. For a locally equilibrated system, this series does not commute with the gradient. Just like with Polymers, the system should have multiple anisotropic non-local minima which mess up any Knuden number expansion. Some materials are inhomogeneus and anisotropic at equilibrium, YM could be like this!

Lattice would not see it , as there are no gradients there. There is an entropy maximum, and it is the one the lattice sees. The problems arise if you "coarse-grain" this maximum into each microscopic cell and try to do a gradient expansion around this equilibrium, unless you have color neutrality.

Future project Rewrite all of this using

Zubarev hydrodynamics incorporating both microscopic and macroscopic fluctuations

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[-\int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

Crooks fluctuation theorem giving a dynamical off-equilibrium definition of entropy

$$P(-W)/P(W) = \exp[\Delta S] \quad , \quad S \simeq \Pi_{\mu\nu} \partial^\mu \beta^\nu$$

Conclusions

Hydrodynamics is not a limit of transport, AdS/CFT or any other microscopic theory

Hydrodynamics is an EFT built around symmetries and entropy maximization and should be treated as such

Once you realize this , generalizing it to theories with extra DoFs, symmetries etc. becomes straight-forward.

Lots of things to do Gauge symmetry looks particularly interesting!