Kadanoff-Baym approach to non-local collision term

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XLS, E. Speranza, Q. Wang,
and D. H. Rischke, in preparation

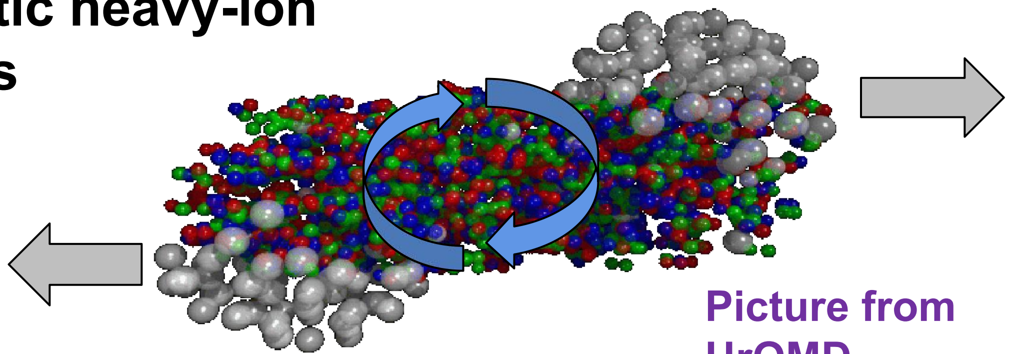
Spin and Hydrodynamics in Relativistic Nuclear Collisions

2020/10/15, ECT* Online workshop

Contents

Introduction

Relativistic heavy-ion collisions



Picture from UrQMD

Large OAM

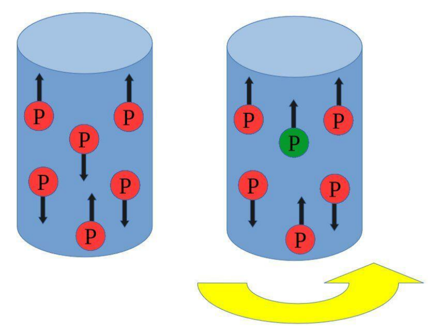
Quarks' spin

Hadrons' polarization, spin alignments of vector mesons

Z.-T. Liang, X.-N. Wang, PLB 629 (2005) 20; PRL 94 (2005) 102301; PRL 96 (2006) 039901.

F. Becattini, V. Chandra, et. al., Annals Phys. 338 (2013) 32.

Barnett effect

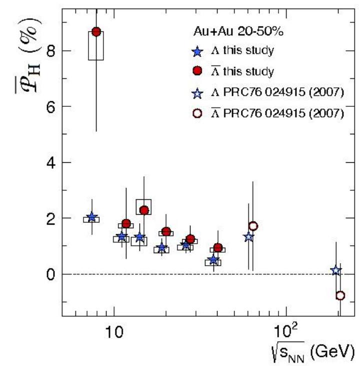


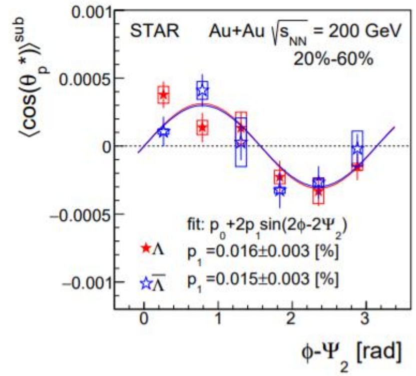
Barnett, Rev. Mod. Phys. 7 (1935) 129

Introduction

Global polarization of Λ and $\bar{\Lambda}$

L. Adamczyk, et al. (STAR), Nature 548 (2017) 62.



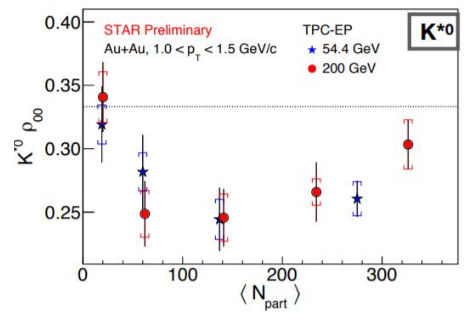


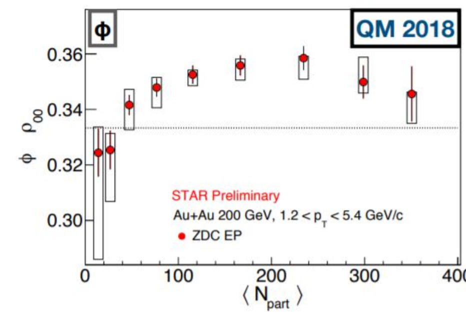
Local polarization of Λ and $\bar{\Lambda}$

J. Adam, et al. (STAR), PRL 123 (2019) 13.

Spin alignments of ϕ, K^{*0}

Talk by Zhou (STAR) at QM2018.
Talk by Singha (STAR) at QM 2019.



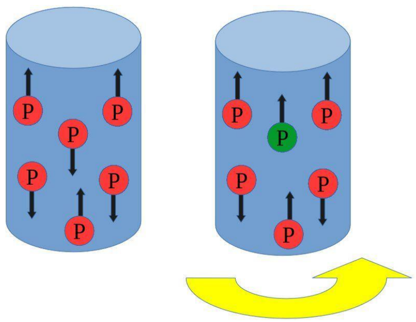


Theoretical developments:

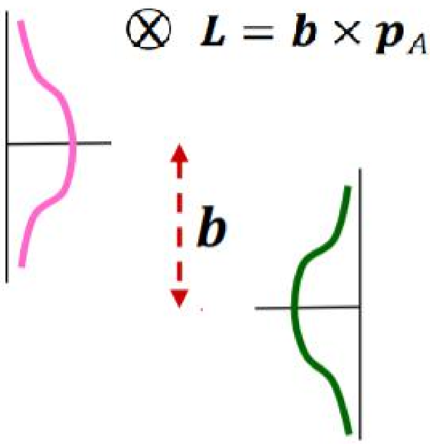
Microscopic models: Liang, Wang (2005); Zhang, Fang, Wang, Wang (2019).
Statistical models: Becattini et al. (2012-2020).
Spin hydrodynamics: Florkowski, Ryblewski, Speranza, et al. (2017-2020).
Kinetic theories: Gao, Liang (2019); Weickgenannt, XLS, Speranza, Wang, Rischke (2019 & 2020); Hattori, Hidaka, Yang (2019 & 2020); Wang, Guo, Shi, Zhuang (2019).

Motivation

Barnett effect



Barnett, Rev. Mod. Phys. 7 (1935) 129



Particles will be polarized along rotation

How this happens microscopically?

We are aiming to derive a Boltzmann equation which includes transfer between spin and OAM.

de Groot's method:

N. Weickgenannt, E. Speranza, XLS, Q. Wang, D. H. Rischke, arXiv: 2005.01506. Talk by Nora Weickgenannt.

Related works:

J.-J. Zhang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 10 (2019) 064904. Talk by Qun Wang.

D.-L. Yang, K. Hattori, Y. Hidaka, JHEP 07 (2020) 070. Talk by Di-Lun Yang.

Z. Wang, X. Guo, P. Zhuang, arXiv: 2009.10930 Talk by Ziyue Wang

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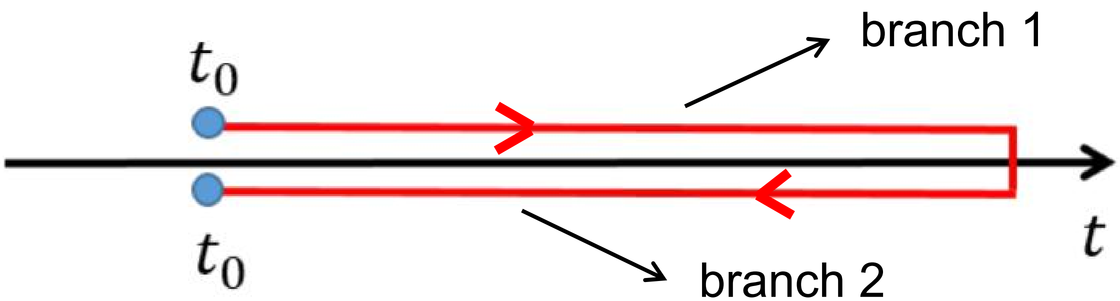


Closed time path (Schwinger-Keldysh) contour

P. Martin, J. S.Schwinger, PR 115 (1959) 1342.

L.V. Keldysh, Zh. Eksp. Teor. Fiz. 47 (1964) 1515.

$$G_{\alpha\beta}(x_1,x_2)=\langle T_C\psi_\alpha(x_1)\bar{\psi}_\beta(x_2)\rangle$$

	t_1	t_2	
$G^F_{\alpha\beta}(x_1,x_2)=\langle T\psi_\alpha(x_1)\overline{\psi}_\beta(x_2)\rangle$,	branch 1	branch 1	time-ordering
$G^{\bar{F}}_{\alpha\beta}(x_1,x_2)=\langle T_A\psi_\alpha(x_1)\overline{\psi}_\beta(x_2)\rangle$,	branch 2	branch 2	anti time-ordering
$G^<_{\alpha\beta}(x_1,x_2)=-\langle\overline{\psi}_\beta(x_2)\psi_\alpha(x_1)\rangle$,	branch 1	branch 2	$t_2>t_1$
$G^>_{\alpha\beta}(x_1,x_2)=\langle\psi_\alpha(x_1)\overline{\psi}_\beta(x_2)\rangle$,	branch 2	branch 1	$t_1>t_2$

$$G^R(x_1,x_2)=\theta(t_1-t_2)\left[G^>(x_1,x_2)-G^<(x_1,x_2)\right] ,$$
$$G^A(x_1,x_2)=-\theta(t_2-t_1)\left[G^>(x_1,x_2)-G^<(x_1,x_2)\right]$$

Retarded / Advanced Green functions

D-S equation

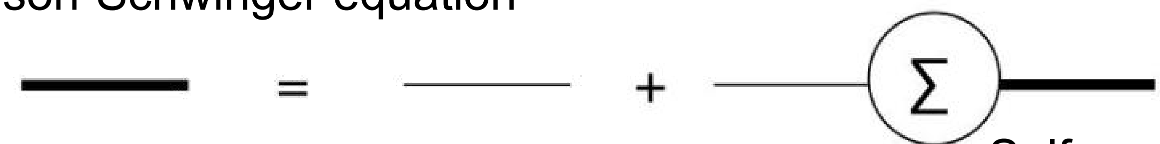
Two point Green function $G^{<}_{\alpha\beta}(x_1, x_2)$

Fourier transform w.r.t. relative position

$$G^{<}_{\alpha\beta}(x,p) \equiv -\frac{1}{2\pi\hbar}\int d^4y\, e^{ip\cdot y/\hbar}\left\langle \bar{\psi}_\beta\left(x-\frac{y}{2}\right)\psi_\alpha\left(x+\frac{y}{2}\right)\right\rangle$$

Wigner function
Heinz(1983);Vasak, Gyulassy, Elze (1987); Zhuang, Heinz (1996); etc...

Dyson-Schwinger equation



Self-energy

$$G = G_0 + G_0 \Sigma G$$

$$G_0^{-1} G = 1 + \Sigma G$$

Explicit expression

Integral over whole CPT

$$\pm i(i\gamma_\mu \partial^\mu_{x_1} - m)G(x_1,x_2) = \delta^{(4)}(x_1 - x_2) + \int_C dx' \Sigma(x_1, x')G(x', x_2)$$

operator G_0^{-1}

$-$ when x_1 is on branch 1

$+$ when x_1 is on branch 2

$$G(x_1,x_2) = \begin{pmatrix} G^F & G^< \\ G^> & G^{\bar{F}} \end{pmatrix}(x_1,x_2)$$

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Kadanoff-Baym equation

$$(\gamma \cdot K - m)\, G^{<}(x,p) = i\hbar \left[\Sigma^{R}(x,p)G^{<}(x,p) + \Sigma^{<}(x,p)G^{A}(x,p)\right]$$
$$+\frac{\hbar^2}{2}\left[\{\Sigma^{R}(x,p), G^{<}(x,p)\}_{\rm PB} + \{\Sigma^{<}(x,p), G^{A}(x,p)\}_{\rm PB}\right]$$

$$K^{\mu} \equiv p^{\mu} + \frac{i\hbar}{2}\partial_x^{\mu}$$

- Corrections from self-energy.
- A gradient expansion is used and higher order contributions are truncated.
- Poisson braket :

$$\{A, B\}_{\rm PB} \equiv (\partial_x A)\cdot(\partial_p B) - (\partial_p A)\cdot(\partial_x B)$$

L. P. Kadanoff, G. Baym, (1962) *Quantum Statistical Mechanics.*

Simplified using on-shell app. $O^{R/A}(x,p) = \pm\frac{1}{2}\left[O^{>}(x,p) - O^{<}(x,p)\right]$

$$(\gamma \cdot K - m)\, G^{<}(x,p) = -\frac{i\hbar}{2}\left[\Sigma^{<}(x,p)G^{>}(x,p) - \Sigma^{>}(x,p)G^{<}(x,p)\right]$$
$$-\frac{\hbar^2}{4}\left[\{\Sigma^{<}(x,p), G^{>}(x,p)\}_{\rm PB} - \{\Sigma^{>}(x,p), G^{<}(x,p)\}_{\rm PB}\right]$$

Decomposition

Generators of Clifford algebra $\Gamma_a \in \{\mathbb{I}_4,\ i\gamma^5,\ \gamma^\mu,\ \gamma^5\gamma^\mu,\ \frac{1}{2}\sigma^{\mu\nu}\}$

Decomposition of Wigner function

$$G^<(x,p) = \frac{1}{4}\left(\mathbb{I}_4\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)$$

Real functions in 8-d phase space $\mathrm{Tr}\left(\Gamma_a G^<\right)$

	Property	Physical meaning (distribution in phase space)
$\mathcal{F}$	Scalar	Mass
$\mathcal{P}$	Pesudoscalar	Pesudoscalar condensate
$\mathcal{V}^\mu$	Vector	Net fermion current
$\mathcal{A}^\mu$	Axial-vector	Polarization (or spin current)
$\mathcal{S}^{\mu\nu}$	Tensor	Electric/magnetic dipole-moment

D. Vasak, M. Gyulassy, H.T. Elze, Annals Phys. 173 (1987) 462.
I. Bialynicki-Birula, P. Gornicki, J. Rafelski, PRD 44 (1991) 1825.

Component equations

KB equation $\left(\gamma\cdot p+\frac{i\hbar}{2}\gamma\cdot\partial_x-m\right)G^<(x,p)=I_{\rm coll}(x,p)$

Decompose in terms of Γ_a

Collisionless: D. Vasak, M. Gyulassy, H.T. Elze, Annals Phys. 173 (1987) 462.

Real parts

$$
\begin{aligned}
p^\mu\mathcal{V}_\mu-m\mathcal{F}&=\mathrm{Re\,Tr}\left(I_{\rm coll}\right)\ ,\\
m\mathcal{P}+\frac{\hbar}{2}\partial_x^\mu\mathcal{A}_\mu&=\mathrm{Re\,Tr}\left(i\gamma^5I_{\rm coll}\right)\ ,\\
p_\mu\mathcal{F}-m\mathcal{V}_\mu+\frac{\hbar}{2}\partial_x^\nu\mathcal{S}_{\mu\nu}&=\mathrm{Re\,Tr}\left(\gamma_\mu I_{\rm coll}\right)\ ,\\
\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu\mathcal{S}^{\alpha\beta}+m\mathcal{A}_\mu-\frac{\hbar}{2}\partial_{x,\mu}\mathcal{P}&=\mathrm{Re\,Tr}\left(\gamma^5\gamma_\mu I_{\rm coll}\right)\ ,\\
\epsilon_{\mu\nu\alpha\beta}p^\alpha\mathcal{A}^\beta+m\mathcal{S}_{\mu\nu}-\frac{\hbar}{2}\partial_{x[\mu}\mathcal{V}_{\nu]}&=-\mathrm{Re\,Tr}\left(\sigma_{\mu\nu}I_{\rm coll}\right)\ ,
\end{aligned}
$$

Imaginary parts

$$
\begin{aligned}
\frac{\hbar}{2}\partial_x^\mu\mathcal{V}_\mu&=\mathrm{Im\,Tr}\left(I_{\rm coll}\right)\ ,\\
p^\mu\mathcal{A}_\mu&=\mathrm{Im\,Tr}\left(-i\gamma^5I_{\rm coll}\right)\ ,\\
p^\nu\mathcal{S}_{\nu\mu}+\frac{\hbar}{2}\partial_{x,\mu}\mathcal{F}&=\mathrm{Im\,Tr}\left(\gamma_\mu I_{\rm coll}\right)\ ,\\
p_\mu\mathcal{P}+\frac{\hbar}{4}\epsilon_{\mu\nu\alpha\beta}\partial_x^\nu\mathcal{S}^{\alpha\beta}&=\mathrm{Im\,Tr}\left(\gamma^5\gamma_\mu I_{\rm coll}\right)\ ,\\
p_{[\mu}\mathcal{V}_{\nu]}+\frac{\hbar}{2}\epsilon_{\mu\nu\alpha\beta}\partial_x^\alpha\mathcal{A}^\beta&=-\mathrm{Im\,Tr}\left(\sigma_{\mu\nu}I_{\rm coll}\right)\ ,
\end{aligned}
$$

Source terms from interactions

Free case has been solved analytically in

J.-H. Gao, Z.-T. Liang, PRD 100 (2019) 5; N. Weickgenannt, XLS, E. Speranza, Q. Wang, D. H. Rischke, PRD 100 (2019) 5; K. Hattori, Y. Hidaka, D.-L. Yang, PRD 100 (2019) 9.

Semi-classical expansion

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Expansion in terms of $\hbar$ $$\mathcal{F}=\sum_{n=0}^{\infty}\hbar^n\mathcal{F}^{(n)}$$

Zeroth order

$$p^\mu\mathcal{V}^{(0)}_\mu - m\mathcal{F}^{(0)} = 0\,, \qquad p^\mu\mathcal{A}^{(0)}_\mu = 0\,,$$
$$\mathcal{P}^{(0)} = 0\,, \qquad p^\nu\mathcal{S}^{(0)}_{\mu\nu} = 0\,,$$
$$p_\mu\mathcal{F}^{(0)} - m\mathcal{V}^{(0)}_\mu = 0\,, \qquad p_{[\mu}\mathcal{V}^{(0)}_{\nu]} = 0\,,$$
$$\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu\mathcal{S}^{(0)\alpha\beta} + m\mathcal{A}^{(0)}_\mu = 0\,,$$
$$\epsilon_{\mu\nu\alpha\beta}p^\alpha\mathcal{A}^{(0)\beta} + m\mathcal{S}^{(0)}_{\mu\nu} = 0\,,$$

Express other components using scalar and axial-vector componenets

$$\begin{aligned}\mathcal{P}^{(0)} &= 0\\ \mathcal{V}^{(0)}_\mu &= \frac{1}{m}p_\mu\mathcal{F}^{(0)}\\ \mathcal{S}^{(0)}_{\mu\nu} &= -\frac{1}{m}\epsilon_{\mu\nu\alpha\beta}p^\alpha\mathcal{A}^{(0)\beta}\end{aligned}$$

Mass-shell conditions

$$\begin{aligned}(p^2-m^2)\mathcal{F}^{(0)} &= 0\\ (p^2-m^2)\mathcal{A}^{(0)}_\mu &= 0\end{aligned}$$

All 0th order terms are on mass-shell

First order equations

First order in $\hbar$

$$p^\mu \mathcal{V}^{(1)}_\mu - m\mathcal{F}^{(1)} = \mathrm{Re\,Tr}\left(I^{(1)}_{\mathrm{coll}}\right),$$

$$\frac{1}{2}\partial_x^\mu \mathcal{A}^{(0)}_\mu + m\mathcal{P}^{(1)} = \mathrm{Re\,Tr}\left(i\gamma^5 I^{(1)}_{\mathrm{coll}}\right),$$

$$\frac{1}{2}\partial_x^\nu \mathcal{S}^{(0)}_{\nu\mu} - p_\mu \mathcal{F}^{(1)} + m\mathcal{V}^{(1)}_\mu = -\mathrm{Re\,Tr}\left(\gamma_\mu I^{(1)}_{\mathrm{coll}}\right),$$

$$\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} p^\nu \mathcal{S}^{(1)\alpha\beta} + m\mathcal{A}^{(1)}_\mu = \mathrm{Re\,Tr}\left(\gamma^5\gamma_\mu I^{(1)}_{\mathrm{coll}}\right),$$

$$\frac{1}{2}\partial_{x[\mu}\mathcal{V}^{(0)}_{\nu]} - \epsilon_{\mu\nu\alpha\beta} p^\alpha \mathcal{A}^{(1)\beta} - m\mathcal{S}^{(1)}_{\mu\nu} = \mathrm{Re\,Tr}\left(\sigma_{\mu\nu} I^{(1)}_{\mathrm{coll}}\right),$$

$$\frac{1}{2}\partial_x^\mu \mathcal{V}^{(0)}_\mu = \mathrm{Im\,Tr}\left(I^{(1)}_{\mathrm{coll}}\right),$$

$$p^\mu \mathcal{A}^{(1)}_\mu = \mathrm{Im\,Tr}\left(-i\gamma^5 I^{(1)}_{\mathrm{coll}}\right),$$

$$\frac{1}{2}\partial_{x,\mu}\mathcal{F}^{(0)} + p^\nu \mathcal{S}^{(1)}_{\nu\mu} = \mathrm{Im\,Tr}\left(\gamma_\mu I^{(1)}_{\mathrm{coll}}\right),$$

$$\frac{1}{4}\epsilon_{\mu\nu\alpha\beta}\partial_x^\nu \mathcal{S}^{(0)\alpha\beta} + p_\mu \mathcal{P}^{(1)} = \mathrm{Im\,Tr}\left(\gamma^5\gamma_\mu I^{(1)}_{\mathrm{coll}}\right),$$

$$\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_x^\alpha \mathcal{A}^{(0)\beta} + p_{[\mu}\mathcal{V}^{(1)}_{\nu]} = -\mathrm{Im\,Tr}\left(\sigma_{\mu\nu} I^{(1)}_{\mathrm{coll}}\right).$$

Boltzmann equation for $\mathcal{F}^{(0)}$, $\mathcal{A}^{(0)}_\mu$

Express $\mathcal{P}^{(1)}$, $\mathcal{V}^{(1)}_\mu$, $\mathcal{S}^{(1)}_{\mu\nu}$ in terms of $\mathcal{F}^{(1)}$, $\mathcal{A}^{(1)}_\mu$ and 0th order terms

$$\mathcal{P}^{(1)} = -\frac{1}{2m}\partial_x^\mu \mathcal{A}^{(0)}_\mu + \frac{1}{m}\mathrm{ReTr}\left(i\gamma^5 I^{(1)}_{\mathrm{coll}}\right)$$

$$\mathcal{V}^{(1)}_\mu = \frac{1}{m}p_\mu \mathcal{F}^{(1)} - \frac{1}{2m}\partial_x^\nu \mathcal{S}^{(0)}_{\nu\mu} - \frac{1}{m}\mathrm{ReTr}\left(\gamma_\mu I^{(1)}_{\mathrm{coll}}\right)$$

$$\mathcal{S}^{(1)}_{\mu\nu} = -\frac{1}{m}\epsilon_{\mu\nu\alpha\beta}p^\alpha \mathcal{A}^{(1)\beta} + \frac{1}{2m}\partial_{x[\mu}\mathcal{V}^{(0)}_{\nu]} - \frac{1}{m}\mathrm{ReTr}\left(\sigma_{\mu\nu} I^{(1)}_{\mathrm{coll}}\right)$$

Mass-shell conditions

$$(p^2 - m^2)\mathcal{F}^{(1)} = 2m\,\mathrm{ReTr}\left(I^{(1)}_{\mathrm{coll}}\right)$$

$$(p^2 - m^2)\mathcal{A}^{(1)}_\mu = -\epsilon_{\mu\nu\alpha\beta}p^\nu \mathrm{ReTr}\left(\sigma^{\alpha\beta} I^{(1)}_{\mathrm{coll}}\right)$$

Mass-shell & Boltzmann Eqs.

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Mass-shell conditions

$$
\begin{aligned}
(p^2-m^2)\mathcal{F}^{(0)} &= 0 \\
(p^2-m^2)\mathcal{F}^{(1)} &= 2m\,\mathrm{ReTr}\left(I^{(1)}_{\mathrm{coll}}\right) \\
(p^2-m^2)\mathcal{A}^{(0)}_\mu &= 0 \\
(p^2-m^2)\mathcal{A}^{(1)}_\mu &= -\epsilon_{\mu\nu\alpha\beta}p^\nu \mathrm{ReTr}\left(\sigma^{\alpha\beta}I^{(1)}_{\mathrm{coll}}\right)
\end{aligned}
$$

Constraint conditions

$$
p^\mu \mathcal{A}^{(0)}_\mu = 0 \quad p^\mu \mathcal{A}^{(1)}_\mu = \mathrm{ImTr}\left(-i\gamma^5 I^{(1)}_{\mathrm{coll}}\right)
$$

Boltzmann equations

$$
\begin{aligned}
p\cdot\partial_x\mathcal{F}^{(0)} &= 2m\,\mathrm{ImTr}\left(I^{(1)}_{\mathrm{coll}}\right) \\
p\cdot\partial_x\mathcal{F}^{(1)} &= 2m\,\mathrm{ImTr}\left(I^{(2)}_{\mathrm{coll}}\right)+\mathrm{ReTr}\left(\gamma\cdot\partial_x I^{(1)}_{\mathrm{coll}}\right)
\end{aligned}
$$

$$
\begin{aligned}
p\cdot\partial_x\mathcal{A}^{(0)}_\mu &= -\epsilon_{\mu\nu\alpha\beta}p^\nu \mathrm{ImTr}\left(\sigma^{\alpha\beta}I^{(1)}_{\mathrm{coll}}\right) \\
p\cdot\partial_x\mathcal{A}^{(1)}_\mu &= -2p^\mu \mathrm{ImTr}\left(\gamma^5 I^{(2)}_{\mathrm{coll}}\right)-2\mathrm{ImTr}\left(\gamma\cdot p\gamma^5\gamma^\mu I^{(2)}_{\mathrm{coll}}\right)-\mathrm{ReTr}\left(\gamma^5\partial^\mu_x I^{(1)}_{\mathrm{coll}}\right)
\end{aligned}
$$

$\mathcal{P},\ \mathcal{V}_\mu,\ \mathcal{S}_{\mu\nu}$ can be expressed in terms of $\mathcal{F},\ \mathcal{A}_\mu$

Interactions induce mass corrections at first order

Are discussed in

Z. Wang, X. Guo, P. Zhuang, arXiv: 2009.10930.

➡ Ziyue Wang's talk.

Alternative method

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KB equation $\left(\gamma\cdot p+\frac{i\hbar}{2}\gamma\cdot\partial_x-m\right)G^<(x,p)=I_{\text{coll}}(x,p)$

Multiplying $\gamma\cdot p+\frac{i\hbar}{2}\gamma\cdot\partial_x+m$ from left-hand-side

$$\left[\left(p^2-m^2-\frac{\hbar^2}{4}\partial_x^2\right)+i\hbar p\cdot\partial_x\right]G^<(x,p)=\left(\gamma\cdot p+\frac{i\hbar}{2}\gamma\cdot\partial_x+m\right)I_{\text{coll}}(x,p)$$

Decompose in terms of Γ_a

$$\left(p^2-m^2-\frac{\hbar^2}{4}\partial^2\right)\text{Tr}(\Gamma_a G^<) = \text{Re}\,\text{Tr}\left[\Gamma_a\left(\gamma\cdot K+m\right)I_{\text{coll}}\right]$$

Mass-shell conditions

$$\hbar p\cdot\partial\text{Tr}(\Gamma_a G^<) = \text{Im}\,\text{Tr}\left[\Gamma_a\left(\gamma\cdot K+m\right)I_{\text{coll}}\right]$$

Boltzmann equation

★ Mass-shell conditions and Boltzmann equations may have various forms

$$p\cdot\partial_x\mathcal{F}^{(1)} = \text{Im Tr}\left[\left(\gamma\cdot p+m\right)I^{(2)}_{\text{coll}}\right]+\frac{1}{2}\text{Re Tr}\left[\gamma\cdot\partial_x I^{(1)}_{\text{coll}}\right]$$

$$p\cdot\partial_x\mathcal{F}^{(1)}=2m\,\text{Im Tr}\left(I^{(2)}_{\text{coll}}\right)+\text{Re Tr}\left(\gamma\cdot\partial_x I^{(1)}_{\text{coll}}\right)$$

Equivalent at order $\mathcal{O}(\Sigma)$

Constraint

Adjoint Dirac equation

$$G(x_1, x_2)(i\hbar\gamma \cdot \overleftarrow{\partial}_{x_2} + m) = \mathcal{O}(\Sigma)$$

Multiplying with self-energy
and integrating over intermediate position

$$\int dx' \Sigma(x_1, x')G(x', x_2)(i\hbar\gamma \cdot \overleftarrow{\partial}_{x_2} + m) = \mathcal{O}(\Sigma^2)$$

Wigner transform $I_{\text{coll}}(x, p)$

Constraint for collision term in KB equation

$$I_{\text{coll}}(x, p)\left(\gamma \cdot p - \frac{i\hbar}{2}\gamma \cdot \overleftarrow{\partial}_x - m\right) = \mathcal{O}(\Sigma^2)$$

$\mathcal{O}(\Sigma)$

Contents

WF for free particles

Free Dirac field

$$\psi(x) = \sqrt{\Omega}\sum_s \int \frac{d^3\mathbf{k}}{(2\pi\hbar)^3}\frac{1}{\sqrt{2E_k}}\left[a(s,\mathbf{k})u(s,\mathbf{k})e^{-ik\cdot x/\hbar} + b^\dagger(s,\mathbf{k})v(s,\mathbf{k})e^{ik\cdot x/\hbar}\right]$$

Inserting into definition of $G^<_{\alpha\beta}(x,p)$

$$G^<_{\alpha\beta}(x,p) = -\theta(p_0)\delta(p^2-m^2)\sum_{r,s}\Big\{\bar{u}_\beta(s,\mathbf{p})u_\alpha(r,\mathbf{p})f^{(+)}_{sr}(x,\mathbf{p})$$
$$-i\hbar\lim_{\mathbf{q}\to\mathbf{0}}\nabla_\mathbf{q}\left[\bar{u}_\beta\left(s,\mathbf{p}-\frac{\mathbf{q}}{2}\right)u_\alpha\left(r,\mathbf{p}+\frac{\mathbf{q}}{2}\right)\right]\cdot\nabla_\mathbf{x}f^{(+)}_{sr}(x,\mathbf{p})\Big\}$$

- Momentum is on-shell because interactions are not included yet.
- Quantum nonlocal correlations for wave-packets; Berry connection
- Local contribution from point-like particles

F. Becattini, V. Chandra, et. al., Annals Phys. 338 (2013) 32.

Matrix-valued distribution

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Matrix-valued distribution

$$f_{sr}^{(\pm)}(x,\pm\mathbf{p}) \equiv \Omega \int \frac{d^3\mathbf{q}}{(2\pi\hbar)^3} e^{i\mathbf{q}\cdot\mathbf{x}/\hbar} e^{-i(E_{\pm\mathbf{p}+\mathbf{q}/2}-E_{\pm\mathbf{p}-\mathbf{q}/2})x_0/\hbar} n_{sr}^{(\pm)}\left(\pm\mathbf{p}-\frac{\mathbf{q}}{2},\pm\mathbf{p}+\frac{\mathbf{q}}{2}\right)$$

2-d Hermitian matrix

$$\left\langle a^\dagger\left(s,\mathbf{p}-\frac{\mathbf{q}}{2}\right) a\left(r,\mathbf{p}+\frac{\mathbf{q}}{2}\right)\right\rangle \equiv n_{sr}^{(+)}\left(\mathbf{p}-\frac{\mathbf{q}}{2},\mathbf{p}+\frac{\mathbf{q}}{2}\right),$$

$$\left\langle b^\dagger\left(s,-\mathbf{p}-\frac{\mathbf{q}}{2}\right) b\left(r,-\mathbf{p}+\frac{\mathbf{q}}{2}\right)\right\rangle \equiv n_{sr}^{(-)}\left(-\mathbf{p}-\frac{\mathbf{q}}{2},-\mathbf{p}+\frac{\mathbf{q}}{2}\right)$$

$\mathrm{tr}\left[f^{(+)}\right]$ intrinsic probability density in phase space

$\mathrm{tr}\left[\tau_j^T f^{(+)}\right]$ intrinsic spin density in rest frame

A simple example: Gaussian-type wave-packet

$$|\mathbf{p}_0,s_0,+\rangle_{\mathrm{wp}} = \frac{1}{N}\int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\exp\left[-\frac{(\mathbf{p}'-\mathbf{p}_0)^2}{4\sigma_p^2}+\frac{i}{\hbar}p'\cdot x_0\right] a^\dagger_{\mathbf{p}',s_0}|0\rangle$$

$$f_{rs}^{(+)}(x,\mathbf{p}) = 8\exp\left\{-\frac{(\mathbf{p}-\mathbf{p}_0)^2}{2\sigma_p^2}-\frac{2\sigma_p^2}{\hbar^2}\left[(\mathbf{x}-\mathbf{x}_0)-\frac{\mathbf{p}}{E_\mathbf{p}}(t-t_0)\right]^2\right\}\delta_{rs}\delta_{ss_0} \quad \begin{pmatrix}1&0\\0&0\end{pmatrix} \text{ or } \begin{pmatrix}0&0\\0&1\end{pmatrix}$$

Gaussian type distribution — spin-up/-down

Important for connecting massive to massless case:
XLS, Q. Wang, X.-G. Huang, PRD 102 (2020) 2.

Modified distribution

Another distribution defined by sandwiched $G^{<}(x,p)$ between wave functions

$$g_{sr}^{(+)}(x,\mathbf{p}) \equiv -\frac{E_p}{2m^2}\int dp^0\,\theta(p^0)\,\overline{u}(r,\mathbf{p})G^{<}(x,p)u(s,\mathbf{p})$$

Comparing with intrinsic distribution $f_{sr}^{(\pm)}(x,\pm\mathbf{p})$

$$g_{sr}^{(\pm)}(x,\pm\mathbf{p}) = f_{sr}^{(\pm)}(x,\pm\mathbf{p}) + \hbar\, j_{sr}^{(\pm)}(x,\pm\mathbf{p}) \ ,$$

$$j_{sr}^{(+)}(x,\mathbf{p}) = -\frac{1}{4m(E_p+m)}\mathbf{n}_j\cdot(\mathbf{p}\times\nabla_{\mathbf{x}})\left[\tau_j^T f^{(+)}(x,\mathbf{p}) + f^{(+)}(x,\mathbf{p})\,\tau_j^T\right]_{sr}$$

Generator of SU(2) group in 2×2 spin representation

spin polarization vector in rest frame

Considering a Gaussian-type wave packet, we find $(\mathbf{p}\times\nabla_{\mathbf{x}})\,f^{(+)}(x,p) \sim (\mathbf{p}\times\mathbf{x})\,f^{(+)}(x,p)$

Orbital angular momentum, spin-orbit coupling

Wigner function components can be expressed in terms of $g_{sr}^{(\pm)}(x,\pm\mathbf{p})$

$$\mathcal{F}(x,p) = -\frac{m}{E_p}\left\{\delta(p_0-E_p)\mathrm{tr}\left[g^{(+)}(x,\mathbf{p})\right] + \delta(p_0+E_p)\mathrm{tr}\left[g^{(-)}(x,\mathbf{p})-1\right]\right\}$$

Continuous spin

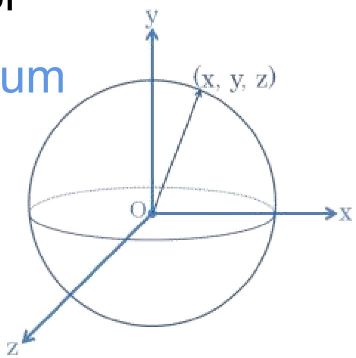
Extend the phase space by introducing a continuous spin vector

$$[d\mathfrak{s}] \equiv \frac{1}{C} d^4\mathfrak{s}\, \delta(\mathfrak{s}\cdot p)\, \delta(\mathfrak{s}^2 + 3/c_0)$$

Perpendicular to momentum

Normalized to $-3/c_0$

$$\int [d\mathfrak{s}] = 1, \quad \int [d\mathfrak{s}]\mathfrak{s}^\mu = 0, \quad \int [d\mathfrak{s}]\mathfrak{s}^\mu \mathfrak{s}^\nu = -\frac{1}{c_0}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$$



$\mathfrak{s}^\mu$ is located on a spherical shell in rest frame

Spin dependent distribution function

$$g_+(x,\mathbf{p},\mathfrak{s}) = \frac{1}{2}\mathrm{tr}\left\{(1 - \mathfrak{s}\cdot n_j^{(+)}\tau_j^T)g^{(+)}(x,\mathbf{p})\right\}$$

Projects matrix-valued distribution onto direction of $\mathfrak{s}^\mu$

$$n_j^{(+)\mu} = \left(\frac{\mathbf{n}_j\cdot\mathbf{p}}{m}, \mathbf{n}_j + \frac{\mathbf{n}_j\cdot\mathbf{p}}{m(E_p+m)}\mathbf{p}\right)^T$$

spin vectors related to τ_j^T

Wigner function expressed as

$$G_+^<(x,p) = -2\pi\delta(p^2-m^2)\theta(p^0)\int [d\mathfrak{s}]\,(1+\gamma^5\gamma\cdot\mathfrak{s})\,(\gamma\cdot p + m)\, g_+(x,\mathbf{p},\mathfrak{s})$$

A covariant definition is used in talks by Nora Weickgenannt; Radoslaw Ryblewsk.

Space shift

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Since matrix $g^{(+)}(x,\mathbf{p})$ contains intrinsic probability and spin densities as well as contributions of spatial gradient, $g_+(x,\mathbf{p},\mathfrak{s})$ is cast into the following form

$$
\begin{aligned}
g_+(x,\mathbf{p},\mathfrak{s}) &= g_+^{\mathrm{local}}(x,\mathbf{p},\mathfrak{s}) + \Delta x^\mu \partial_{x\mu} g_+^{\mathrm{local}}(x,\mathbf{p},\mathfrak{s}) \\
&= g_+^{\mathrm{local}}(x+\Delta x,\mathbf{p},\mathfrak{s})
\end{aligned}
$$

Nonlocal part takes this simple form only when spin is normalized with $c_0 = 1$

The spatial shift is first order in $\hbar$

$$
\Delta x^\mu = \left(0,\ \frac{\hbar}{2m(E_p+m)}\mathbf{p}\times\mathfrak{s}\right)^T
$$

The local distribution in general contains both zeroth and first order parts

$$
g_+^{\mathrm{local}}(x,\mathbf{p},\mathfrak{s}) = g_+^{(0)}(x,\mathbf{p},\mathfrak{s}) + \hbar g_+^{(1)}(x,\mathbf{p},\mathfrak{s})
$$

Anti-particles are similar, see
XLS, E. Speranza, Q. Wang, and D. H. Rischke, in preparation.

Contents

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- Introduction
- Kadanoff-Baym equation
  and component equations
- Spin-dependent distributions
- **Boltzmann equation
  with local and non-local collisions**
- Summary

In absence of interactions, relation between Wigner function and the spin-dependent distribution is

$$g_+(x,\mathbf{p},\mathfrak{s}) = -\frac{E_p}{4\pi m}\int dp_0\theta(p_0)\mathrm{Tr}\left[\Pi(\mathfrak{s})G^<\right]$$

We assume that this relation holds even when we have interactions between particles. (dilute gas approximation)

spin projection operator

$$\Pi(\mathfrak{s}) = \frac{1}{2}\left(1+\gamma^5\mathfrak{s}\cdot\gamma\right)$$

Boltzmann equation

$$\hbar p\cdot\partial_x g_+(x,\mathbf{p},\mathfrak{s}) = -\frac{E_p}{2\pi}\int dp_0\theta(p_0)\mathrm{Im}\mathrm{Tr}\left[\Pi(\mathfrak{s})I_{\mathrm{coll}}\right]$$

3-momentum because of energy integral

$$I_{\mathrm{coll}} \equiv -\frac{i\hbar}{2}\left[\Sigma^<(x,p)G^>(x,p) - \Sigma^>(x,p)G^<(x,p)\right]$$

Self-energy

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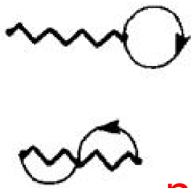
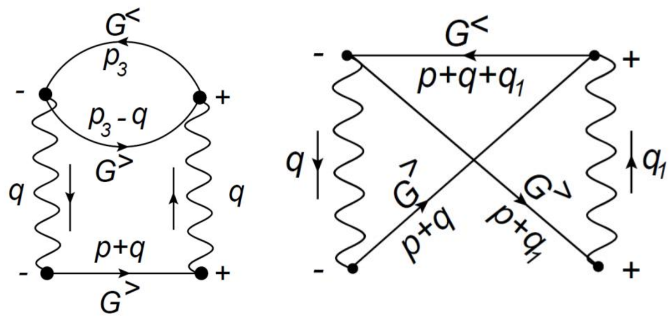
Leading order:
mean-field contributions



neglected

Next-to-leading order: Born diagrams,
scattering contributions



NJL model as an example: $\mathcal{L}_{\rm NJL} = \bar{\psi}(i\hbar\gamma\cdot\partial_x - m)\psi + \sum_a G_a\left(\bar{\psi}\Gamma_a\psi\right)^2$

$$\Sigma^>(x,p) = 4G_aG_b\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}(2\pi)^4\delta^{(4)}(p+p_3-p_1-p_2)$$
$$\times\mathrm{Tr}\left[\Gamma_aG^<(p_3)\Gamma_bG^>(p_1)\right]\Gamma_bG^>(p_2)\Gamma_a$$
$$-4G_aG_b\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}(2\pi)^4\delta^{(4)}(p+p_3-p_1-p_2)$$
$$\times\Gamma_bG^>(p_1)\Gamma_aG^<(p_3)\Gamma_bG^>(p_2)\Gamma_a,$$

coupling constants ← G_aG_b

$\Gamma_b\ldots\Gamma_a$ → Product of Dirac, flavor, and color matrices

Collision term

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$$\hbar p\cdot\partial_x g_+(x,\mathbf{p},\mathfrak{s}) = -\frac{E_p}{2\pi}\int dp_0\theta(p_0)\mathrm{Im}\mathrm{Tr}\left[\Pi(\mathfrak{s})I_{\mathrm{coll}}\right]$$

$$I_{\mathrm{coll}} \equiv -\frac{i\hbar}{2}\left[\Sigma^<(x,p)G^>(x,p)-\Sigma^>(x,p)G^<(x,p)\right]$$

Expressed in terms of $G^{</>}$

$$G_+^<(x,p) = -2\pi\delta(p^2-m^2)\theta(p^0)\int[d\mathfrak{s}]\left(1+\gamma^5\gamma\cdot\mathfrak{s}\right)(\gamma\cdot p+m)g_+(x,\mathbf{p},\mathfrak{s})$$

$$G_+^>(x,p) = 2\pi\delta(p^2-m^2)\theta(p^0)\int[d\mathfrak{s}]\left(1+\gamma^5\gamma\cdot\mathfrak{s}\right)(\gamma\cdot p+m)\left[1-g_+(x,\mathbf{p},\mathfrak{s})\right]$$

Collision term

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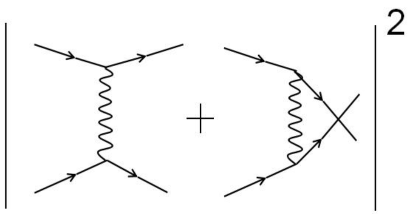
$$\hbar p\cdot\partial_x g_+(x,\mathbf{p},\mathfrak{s}) = -\frac{E_p}{2\pi}\int dp_0\theta(p_0)\mathrm{Im}\mathrm{Tr}\left[\Pi(\mathfrak{s})I_{\mathrm{coll}}\right]$$

$\hbar\mathcal{C}\left[g_+\right]$

$$\mathcal{C}\left[g_+\right] = \frac{1}{2E_p}\int Dp_1Dp_2Dp_3\int[d\mathfrak{s}_1][d\mathfrak{s}_2][d\mathfrak{s}_3][d\mathfrak{s}_p]$$
$$\times(2\pi)^4\delta^{(4)}(p+p_3-p_1-p_2)$$
$$\times\left[g_1g_2(1-g_3)(1-g_p)-g_3g_p(1-g_1)(1-g_2)\right]$$
$$\times\mathrm{Re}\left[M_{(s,a)}+M_{(s,b)}\right]$$

momentum conservation

vanishes in local equilibrium



$$M_{(s,a)} = 64G_aG_b\mathrm{Tr}\left[\Gamma_a\Pi(c_0\mathfrak{s}_3)(p_3\cdot\gamma+m)\Gamma_b\Pi(c_0\mathfrak{s}_1)(p_1\cdot\gamma+m)\right]$$
$$\times\mathrm{Tr}\left[\Pi(\mathfrak{s})\Gamma_b\Pi(c_0\mathfrak{s}_2)(p_2\cdot\gamma+m)\Gamma_a\Pi(c_0\mathfrak{s}_p)(p\cdot\gamma+m)\right],$$
$$M_{(s,b)} = -64G_aG_b\mathrm{Tr}\left[\Pi(\mathfrak{s})\Gamma_b\Pi(c_0\mathfrak{s}_1)(p_1\cdot\gamma+m)\Gamma_a\Pi(c_0\mathfrak{s}_3)(p_3\cdot\gamma+m)\right.$$
$$\left.\times\,\Gamma_b\Pi(c_0\mathfrak{s}_2)(p_2\cdot\gamma+m)\Gamma_a\Pi(c_0\mathfrak{s}_p)(p\cdot\gamma+m)\right],$$

Detailed balance

Detailed balance condition gives the local equilibrium distribution

$$\begin{aligned} & g_+(x,\mathbf{p}_1,\mathfrak{s}_1)g_+(x,\mathbf{p}_2,\mathfrak{s}_2)\left[1-g_+(x,\mathbf{p}_3,\mathfrak{s}_3)\right]\left[1-g_+(x,\mathbf{p},\mathfrak{s}_p)\right] \\ = \; & g_+(x,\mathbf{p}_3,\mathfrak{s}_3)g_+(x,\mathbf{p},\mathfrak{s}_p)\left[1-g_+(x,\mathbf{p}_1,\mathfrak{s}_1)\right]\left[1-g_+(x,\mathbf{p}_2,\mathfrak{s}_2)\right] \end{aligned}$$

Semi-classical expansion:

$\hbar$ order

$$g_+(x,\mathbf{p},\mathfrak{s}) = g_+^{(0)}(x,\mathbf{p},\mathfrak{s}) + \underbrace{\hbar g_+^{(1)}(x,\mathbf{p},\mathfrak{s}) + \Delta x^\mu \partial_{x\mu} g_+^{(0)}(x,\mathbf{p},\mathfrak{s})}_{\hbar \text{ order}}$$

From nonlocal correlation inside Wigner function

We assume that zeroth order distributions do not depend on spins

$$g_+^{(0)}(x,\mathbf{p},\mathfrak{s}) = g_+^{(0)}(x,\mathbf{p})$$

$$g_+^{(0)}(x,\mathbf{p}) = \frac{1}{1+\exp(\beta\cdot p)} \simeq \exp(-\beta\cdot p)$$

First order

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Semi-classical expansion:

$\hbar$ order

$$g_+(x,\mathbf{p},\mathfrak{s}) = g_+^{(0)}(x,\mathbf{p},\mathfrak{s}) + \underbrace{\hbar g_+^{(1)}(x,\mathbf{p},\mathfrak{s}) + \Delta x^\mu \partial_{x\mu} g_+^{(0)}(x,\mathbf{p},\mathfrak{s})}_{\hbar \text{ order}}$$

In the presence of vortical field, the equilibrium distribution should contain a spin-vorticity coupling, which is a first-order contribution in our $\hbar$ - counting.

$$\begin{aligned} g_+^{(1)}(x,\mathbf{p},\mathfrak{s}) &= g_+^{(1)}(x,\mathbf{p}) - \mathfrak{s}_\alpha g_{+,s}^{(1)\alpha}(x,\mathbf{p}) \\ &= g_+^{(1)}(x,\mathbf{p}) - \frac{1}{2}\mathfrak{s}_\alpha \mu_s^\alpha g_+^{(0)}(x,\mathbf{p}) \\ &= \boxed{g_+^{(1)}(x,\mathbf{p})} + \frac{1}{4}\Omega_{\alpha\beta}\Sigma^{\alpha\beta}(\mathbf{p},\mathfrak{s}) g_+^{(0)}(x,\mathbf{p}) \end{aligned}$$

Expand in terms of $\mathfrak{s}_\alpha$

spin-independent part can be merged with zeroth order

Spin potential

$$\mu_s^\alpha \equiv 2\frac{g_{+,s}^{(1)\alpha}(x,\mathbf{p})}{g_+^{(0)}(x,\mathbf{p})} = -\frac{1}{2m}\epsilon^{\alpha\beta\mu\nu}p_\beta\Omega_{\mu\nu}$$

Spin dipole moment

$$\Sigma^{\alpha\beta}(\mathbf{p},\mathfrak{s}) \equiv -\frac{1}{m}\epsilon_{\mu\nu\alpha\beta}p^\alpha \mathfrak{s}^\beta$$

Semi-classical expansion:

$\hbar$ order

$$g_+(x,\mathbf{p},\mathfrak{s}) = g_+^{(0)}(x,\mathbf{p},\mathfrak{s}) + \hbar g_+^{(1)}(x,\mathbf{p},\mathfrak{s}) + \Delta x^\mu \partial_{x\mu} g_+^{(0)}(x,\mathbf{p},\mathfrak{s})$$

Nonlocal part: $\Delta x^\mu \partial_{x\mu} g_+^{(0)}(x,\mathbf{p}) = -(\omega_{\mu\nu} + \delta^K_{\mu\nu})\Delta x^\mu p^\nu g_+^{(0)}(x,\mathbf{p})$

Boltzmann distribution

Detailed balance condition at first order:

$$\begin{aligned} 0 = \; & \frac{\hbar}{2}\left(\Omega_{1,\mu\nu}\Sigma_1^{\mu\nu} + \Omega_{2,\mu\nu}\Sigma_2^{\mu\nu} - \Omega_{3,\mu\nu}\Sigma_3^{\mu\nu} - \Omega_{p,\mu\nu}\Sigma_p^{\mu\nu}\right) \\ & -(\omega_{\mu\nu} + \delta^K_{\mu\nu})\left[(\Delta x_1^\mu p_1^\nu - \Delta x_1^\nu p_1^\mu) + (\Delta x_2^\mu p_2^\nu - \Delta x_2^\nu p_2^\mu)\right. \\ & \left. - (\Delta x_3^\mu p_3^\nu - \Delta x_3^\nu p_3^\mu) - (\Delta x_p^\mu p^\nu - \Delta x_p^\nu p^\mu)\right] \end{aligned}$$

$$\omega_{\mu\nu} \equiv \frac{1}{2}(\partial_{x\mu}\beta_\nu - \partial_{x\nu}\beta_\mu)$$

$$\delta^K_{\mu\nu} \equiv \frac{1}{2}(\partial_{x\mu}\beta_\nu + \partial_{x\nu}\beta_\mu)$$

Also been obtained using de Groot's mothod:

N. Weickgenannt, E. Speranza, XLS, Q. Wang, D. H. Rischke, arXiv: 2005.01506.

→ Talk by Nora Weickgenannt.

Global equilibrium

Detailed balance condition at first order:

$$
\begin{aligned}
0 \; = \; & \frac{\hbar}{2}\left(\Omega_{1,\mu\nu}\Sigma_1^{\mu\nu} + \Omega_{2,\mu\nu}\Sigma_2^{\mu\nu} - \Omega_{3,\mu\nu}\Sigma_3^{\mu\nu} - \Omega_{p,\mu\nu}\Sigma_p^{\mu\nu}\right) \\
& -(\omega_{\mu\nu} + \delta^K_{\mu\nu})\left[(\Delta x_1^\mu p_1^\nu - \Delta x_1^\nu p_1^\mu) + (\Delta x_2^\mu p_2^\nu - \Delta x_2^\nu p_2^\mu)\right. \\
& \left. - (\Delta x_3^\mu p_3^\nu - \Delta x_3^\nu p_3^\mu) - \left(\Delta x_p^\mu p^\nu - \Delta x_p^\nu p^\mu\right)\right]
\end{aligned}
$$

Assuming that spin potential equals thermal vorticity and Killing condition is satisfied

$$\Omega_{a,\mu\nu} = -\omega_{\mu\nu}, \;\; (a = 1, 2, 3, p) \quad \delta^K_{\mu\nu} = 0$$

Global equilibrium

detailed balance condition is shown to be equivalent with angular momentum conservation

$$
\begin{aligned}
& \left(\frac{\hbar}{2}\Sigma_1^{\mu\nu} + \Delta x_1^\mu p_1^\nu - \Delta x_1^\nu p_1^\mu\right) + \left(\frac{\hbar}{2}\Sigma_2^{\mu\nu} + \Delta x_2^\mu p_2^\nu - \Delta x_2^\nu p_2^\mu\right) \\
= & \left(\frac{\hbar}{2}\Sigma_3^{\mu\nu} + \Delta x_3^\mu p_3^\nu - \Delta x_3^\nu p_3^\mu\right) + \left(\frac{\hbar}{2}\Sigma_p^{\mu\nu} + \Delta x_p^\mu p^\nu - \Delta x_p^\nu p^\mu\right)
\end{aligned}
$$

Summary

- From Kadanoff-Baym equation, obtained mass-shell conditions and Boltzmann equations for components of $G^<$ in presence of interactions.
- Expressed $G^<$ in terms of a matrix-valued distribution and then in terms of a spin-dependent distribution $g_+(x, \mathbf{p}, \mathfrak{s})$ which includes nonlocal correlations.
- Calculated self-energy for Born diagrams (direct and exchange).
- Derived Boltzmann equation for $g_+(x, \mathbf{p}, \mathfrak{s})$ and detailed balance conditions, where nonlocal correlation enters as a contribution from orbital angular momentum.

  Similar results obtained from de Groot's method:
  N. Weickgenannt, E. Speranza, XLS, Q. Wang, D. H. Rischke, arXiv: 2005.01506.

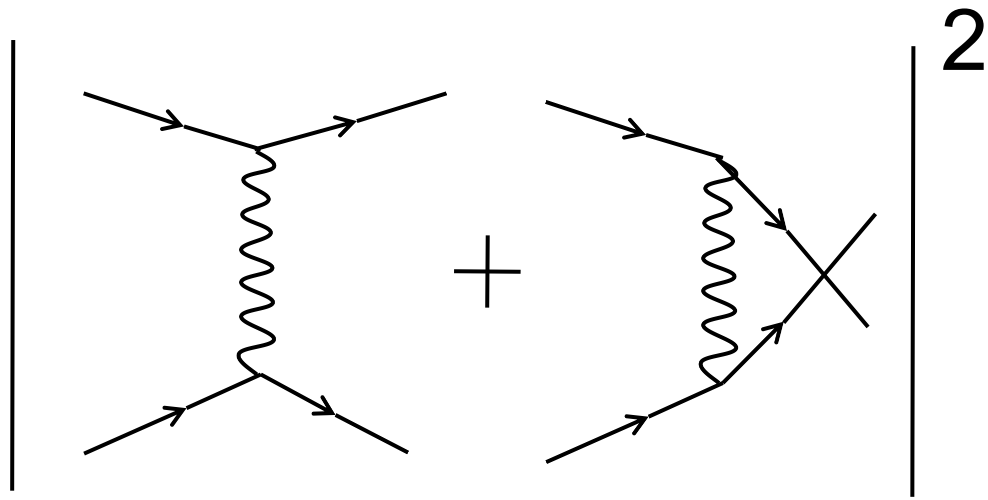
  ↓ Talk by Nora Weickgenannt.

- Anti-particles are included in our upcoming paper.

  XLS, E. Speranza, Q. Wang, and D. H. Rischke, in preparation.

Thanks for your attention !

Kinetic equation



Introduction

Point-particle picture (classical)

No interaction if impact parameter is non-zero (we only consider short-range interactions)

wave-packet picture

Finite spatial uncertainty, interaction exsit even for nonvanishing impact parameter

Distribution function

Some figures to show the physics
cite Jun-Jie's paper

Poisson bracket term

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$$I_{\rm coll}(x,p) = -\frac{i\hbar}{2(2\pi\hbar)^8}\int d^4y'\int d^4x'\int d^4p_1 d^4p_2 \exp\left[-\frac{i}{\hbar}(p_1-p)\cdot y' - \frac{i}{\hbar}(p_2-p)\cdot x'\right]$$
$$\times\left[\Sigma^<\left(x+\frac{x'}{2},p_1\right)G^>\left(x+\frac{y'}{2},p_2\right) - \Sigma^>\left(x+\frac{x'}{2},p_1\right)G^<\left(x+\frac{y'}{2},p_2\right)\right]$$

Gradient expansion w.r.t. $x'^\mu,\ y'^\mu$

$$I_{\rm coll} \equiv -\frac{i\hbar}{2}\left[\Sigma^<(x,p)G^>(x,p) - \Sigma^>(x,p)G^<(x,p)\right]$$
$$-\frac{\hbar^2}{4}\left[\{\Sigma^<(x,p),G^>(x,p)\}_{\rm PB} - \{\Sigma^>(x,p),G^<(x,p)\}_{\rm PB}\right]$$

The first term is dominated by short-range interactions, while Poisson bracket term by long-range interactions.
P. Danielewicz, Ann. of Phys. 152 (1984) 239.

In heavy-ion collisions, strong interaction is short ranged, thus Poisson bracket term is neglected in this work.