Spin polarization from vorticity through nonlocal collisions

Nora Weickgenannt
NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke,
arXiv:2005.01506 (2020)

Spin and hydrodynamics in relativistic nuclear collisions
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- Large global angular momentum created in noncentral heavy-ion collisions.
- Orbital angular momentum is converted into spin
  $\Longrightarrow$ Spin polarization in hot and dense matter!
  L. Adamczyk et al. (STAR), Nature 548 62-65
- Connect spin polarization and vorticity!
- How to describe this with fluid dynamics? (Talk by Enrico Speranza)
  - Antisymmetric part of energy-momentum tensor describes conversion between spin and orbital angular momentum
  - Different choices of pseudo-gauge imply different physical interpretations
    E. Speranza and NW, arXiv:2007.00138 (2020)
- How to derive spin alignment with vorticity from microscopic theory?
  - Kinetic theory with nonlocal collisions
  - Equilibrium conditions?
  - Calculate nonlocal collision term from quantum field theory.
  - Use Wigner function.

- Nonrelativistic hydrodynamics with spin from kinetic theory studied long time ago.

  S. Hess and L. Waldmann, Zeitschrift für Naturforschung A 26, 1057 (1971)
- Assumes local collision term.
- No orbital angular momentum in collision.
- Spin is conserved separately!
- Hydrodynamic evolution describes diffusion of initial polarization.
- Spin alignment with vorticity cannot be described with local collisions.
- Need nonlocal collision term!
- Spin and orbital angular momentum are converted into one another.
- $\Longrightarrow$ Conversion between vorticity and polarization!
- How to calculate nonlocal collision term?

- Wigner function: Wigner transformation of two-point function:

  H.-Th. Elze, M. Gyulassy, and D. Vasak, Ann. Phys. 173 (1987) 462

$$W(x,p) = \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \langle : \bar{\Psi}(x+\frac{y}{2})\Psi(x-\frac{y}{2}) : \rangle \,.$$

- Dirac equation with general interaction term $\rho = (1/\hbar)\partial\mathcal{L}_{int}/\partial\bar{\psi}$:

$$(i\hbar\gamma\cdot\partial - m)\,\psi = \hbar\rho$$

  $\Longrightarrow$Equation of motion for Wigner function

  S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980)

$$\left[\gamma\cdot\left(p+i\frac{\hbar}{2}\partial\right)-m\right]W = \hbar\mathcal{C}$$

  Collision term

$$\mathcal{C}_{\alpha\beta} \equiv \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \langle : \bar{\psi}_\beta(x_1)\rho_\alpha(x_2) : \rangle \ .$$

- Idea: Include nonlocal collision term $\mathcal{C}$,
  $\Longrightarrow$ Expand Wigner function and collision term up to first order in gradients (formally equivalent to $\hbar$ expansion).

- Convenient decomposition (Clifford Algebra)

$$W = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)$$

also for collision term:

$$\mathrm{Re}\,\mathcal{C} = \frac{1}{4}\left(D_\mathcal{F} + i\gamma^5 D_\mathcal{P} + \gamma^\mu D_{\mathcal{V}\mu} + \gamma^5\gamma^\mu D_{\mathcal{A}\mu} + \frac{1}{2}\sigma^{\mu\nu} D_{\mathcal{S}\mu\nu}\right),$$

$$\mathrm{Im}\,\mathcal{C} = \frac{1}{4}\left(C_\mathcal{F} + i\gamma^5 C_\mathcal{P} + \gamma^\mu C_{\mathcal{V}\mu} + \gamma^5\gamma^\mu C_{\mathcal{A}\mu} + \frac{1}{2}\sigma^{\mu\nu} C_{\mathcal{S}\mu\nu}\right)$$

- Concept for free fields: Decompose equation of motion in Clifford algebra, solve order by order in $\hbar$.
  NW, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, PRD100, 056018 (2019)
  J.-H. Gao and Z.-T. Liang, PRD100, 056021(2019)
  K. Hattori, Y. Hidaka, and D.-L. Yang, PRD100, 096011 (2019)
  Z. Wang, X. Guo, S. Shi, and P. Zhuang, PRD 100 (2019) 014015
  Y.-C. Liu, K. Mameda, and X.-G. Huang (2020),2002.03753
- Now: Additional complication through collision term.

- Equation of motion for Wigner function $\Longrightarrow$

$$
\begin{aligned}
p\cdot\mathcal{V} - m\mathcal{F} &= \hbar D_{\mathcal{F}}, \\
\frac{\hbar}{2}\partial\cdot\mathcal{A} + m\mathcal{P} &= -\hbar D_{\mathcal{P}}, \\
p^{\mu}\mathcal{F} - \frac{\hbar}{2}\partial_{\nu}\mathcal{S}^{\nu\mu} - m\mathcal{V}^{\mu} &= \hbar D^{\mu}_{\mathcal{V}}, \\
-\frac{\hbar}{2}\partial^{\mu}\mathcal{P} + \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}p_{\nu}S_{\alpha\beta} + m\mathcal{A}^{\mu} &= -\hbar D^{\mu}_{\mathcal{A}}, \\
\frac{\hbar}{2}\partial^{[\mu}\mathcal{V}^{\nu]} - \epsilon^{\mu\nu\alpha\beta}p_{\alpha}\mathcal{A}_{\beta} - m\mathcal{S}^{\mu\nu} &= \hbar D^{\mu\nu}_{\mathcal{S}}, \\
\hbar\partial\cdot\mathcal{V} &= 2\hbar C_{\mathcal{F}}, \\
p\cdot\mathcal{A} &= \hbar C_{\mathcal{P}}, \\
\frac{\hbar}{2}\partial^{\mu}\mathcal{F} + p_{\nu}\mathcal{S}^{\nu\mu} &= \hbar C^{\mu}_{\mathcal{V}}, \\
p^{\mu}\mathcal{P} + \frac{\hbar}{4}\epsilon^{\mu\nu\alpha\beta}\partial_{\nu}\mathcal{S}_{\alpha\beta} &= -\hbar C^{\mu}_{\mathcal{A}}, \\
p^{[\mu}\mathcal{V}^{\nu]} + \frac{\hbar}{2}\epsilon^{\mu\nu\alpha\beta}\partial_{\alpha}\mathcal{A}_{\beta} &= -\hbar C^{\mu\nu}_{\mathcal{S}}.
\end{aligned}
$$

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- Want to obtain energy-momentum and spin tensor (see Enrico Speranza's talk)

$$T^{\mu\nu}_{HW} = \frac{1}{m}\int d^4p\, p^\nu \left(p^\mu \mathcal{F} - \hbar D^\mu_{\mathcal{V}}\right) + \mathcal{O}(\hbar^2)$$

$$S^{\lambda,\mu\nu}_{HW} = \frac{1}{2m}\int d^4p\, p^\lambda \mathcal{S}^{\mu\nu}$$

$$= \frac{1}{2m^2}\int d^4p\, p^\lambda \left(\frac{\hbar}{2}\partial^{[\mu}\mathcal{V}^{\nu]} - \epsilon^{\mu\nu\alpha\beta}p_\alpha \mathcal{A}_\beta - \hbar D^{\mu\nu}_{\mathcal{S}}\right)$$

- Problem: $D^\mu_{\mathcal{V}}$ and $D^{\mu\nu}_{\mathcal{S}}$ not immediately given
  $\rightarrow$ expand in most general tensor structure
- Assume: Spin effects at least $\mathcal{O}(\hbar) \implies D^\mu_{\mathcal{V}} \propto p^\mu + \mathcal{O}(\hbar)$

$$p^\mu \mathcal{F} - \hbar D^\mu_{\mathcal{V}} = p^\mu \bar{\mathcal{F}} + \mathcal{O}(\hbar^2)$$

- No antisymmetric rank-two tensor at $\mathcal{O}(1) \implies D^{\mu\nu}_{\mathcal{S}} = \mathcal{O}(\hbar)$
- Can express currents only through $\bar{\mathcal{F}}$ and $\mathcal{A}^\mu$ up to first order.
- Relevant transport equations:

$$p\cdot\partial\bar{\mathcal{F}} = m\, C_F, \qquad p\cdot\partial\mathcal{A}^\mu = m\, C^\mu_A$$

with $C_F = 2C_{\mathcal{F}}$ and $C^\mu_A \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\nu C_{\mathcal{S}\alpha\beta}$.

- In order to account for spin dynamics enlarge phase space
  J. Zamanian, M. Marklund, and G. Brodin, NJP 12, 043019 (2010)
- Introduce new phase-space variable $\mathfrak{s}^\mu$

$$\mathfrak{f}(x,p,\mathfrak{s}) \equiv \frac{1}{2}\left[\bar{\mathcal{F}}(x,p) - \mathfrak{s}\cdot\mathcal{A}(x,p)\right] .$$

- Obtain $\bar{\mathcal{F}}$ and $\mathcal{A}^\mu$ via

$$\bar{\mathcal{F}} = \int dS(p)\, \mathfrak{f}(x,p,\mathfrak{s}) , \quad \mathcal{A}^\mu = \int dS(p)\, \mathfrak{s}^\mu \mathfrak{f}(x,p,\mathfrak{s})$$

  with $dS(p) \equiv \frac{\sqrt{p^2}}{\sqrt{3}\pi} d^4\mathfrak{s}\, \delta(\mathfrak{s}^2+3)\delta(p\cdot\mathfrak{s})$.
- Ensures constraint $p\cdot\mathcal{A}=0$.
- Same formalism as in case of classical spin
  $\Longrightarrow$ suitable for hydrodynamic calculations
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)
  S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar and R. Ryblewski, arXiv: 2002.03937, arXiv:2008.10976 (2020)
- Exact, all quantum information about $\bar{\mathcal{F}}$ and $\mathcal{A}^\mu$ retained.
- Simple, no need to go to matrix-valued distribution function.

- Boltzmann equation

$$p \cdot \partial \mathfrak{f}(x, p, \mathfrak{s}) = m\,\mathfrak{C}[\mathfrak{f}] \,,$$

$$\mathfrak{C}[\mathfrak{f}] \equiv \frac{1}{2}(C_F - \mathfrak{s} \cdot C_A).$$

- Want to obtain collision term up to first order in gradients

$$\mathfrak{C}[\mathfrak{f}] = \mathfrak{C}_l[\mathfrak{f}] + \hbar\,\mathfrak{C}_{nl}[\mathfrak{f}] \;.$$

Local contribution + **Nonlocal** contribution

- Starting point:
S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980)

$$p \cdot \partial W = C$$

with

$$C_{\alpha\beta} = \frac{i}{2}\int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \Big\langle [\bar{\rho}(x_1)(-i\hbar\gamma \cdot \overleftarrow{\partial} + m)]_\beta \psi_\alpha(x_2) - \bar{\psi}_\beta(x_1)[(i\hbar\gamma \cdot \partial + m)\rho(x_2)]_\alpha \Big\rangle$$

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- Modified on-shell condition

$$\left(p^2 - m^2 - \frac{\hbar^2}{4}\partial^2\right) W = \hbar\,\delta M$$

with

$$\delta M_{\alpha\beta} = \frac{1}{2}\int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \left\langle [\bar{\rho}(x_1)(i\hbar\gamma\cdot\overleftarrow{\partial} + m)]_\beta \psi_\alpha(x_2) + \bar{\psi}_\beta(x_1)[(-i\hbar\gamma\cdot\partial + m)\rho(x_2)]_\alpha \right\rangle$$

- By taking traces

$$(p^2 - m^2)\mathfrak{f} = \hbar\,\mathfrak{M} + \mathcal{O}(\hbar^2)$$

with

$$\mathfrak{M} = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{m}{p^2} p\cdot\gamma - \mathfrak{s}\cdot\gamma\gamma^5\right)\delta M\right]$$

- Quasiparticle approximation: assume solution of the form

$$\mathfrak{f} = m\,\delta(p^2 - m^2 - \hbar\delta m^2) f$$

- Expanding δ-function up to first order:

$$\mathfrak{M} = \delta m^2 \delta(p^2 - m^2) m\, f$$

- Next: Calculate C and δM explicitly $\Longrightarrow$ obtain $\mathfrak{C}$ and $\mathfrak{M}$ by taking traces.

Power counting

- "$\hbar$-expansion": gradient expansion
  $\hbar \times$ gradient of Wigner function
  $\ll$ momentum or mass scale $\times$ Wigner function
- We treat all gradients on the same level, i.e.
  - gradients in formal $\hbar$-expansion of Wigner function
  $$W = W^{(0)} + \hbar W^{(1)} + \mathcal{O}(\hbar^2),$$
  - gradients in nonlocal expansion of collision term
  $$\mathfrak{C} = \mathfrak{C}_l + \hbar \mathfrak{C}_{nl} + \mathcal{O}(\hbar^2),$$
  - and gradients in expansion of distribution function around equilibrium
  $$f = f_{eq} + \delta f$$

  considered to be of same order.
- $\Longrightarrow$ $f^{(0)}$ contains only equilibrium contributions.
- $f^{(1)}$ contains equilibrium and off-equilibrium contributions.
- $\mathfrak{C}_{nl}$ is a functional only of $f^{(0)}$,
  $\mathfrak{C}_{nl}[f^{(1)}]$ would enter collision term at second order.

- Expand ensemble average in initial n-particle scattering states.
- Neglect initial correlations (molecular chaos).
- Assume binary scattering ($n = 2$).
- Low-density approximation:
  Identify initial Wigner function in collision term with interacting Wigner function $W_{\text{in}} = W$.

$$C_{\alpha\beta} = \frac{1}{2(4\pi\hbar m^2)^2} \sum_{r_1, r_2, s_1, s_2} \int d^4x_1 \, d^4x_2 \, d^4p_1 \, d^4p_2 \, d^4u_1 \, d^4u_2$$

$$\times \, {}_{\text{in}}\langle p_1 - \frac{1}{2}u_1, p_2 - \frac{1}{2}u_2; r_1, r_2 | \Phi_{\alpha\beta}(p) | p_1 + \frac{1}{2}u_2, p_2 + \frac{1}{2}u_2; s_1, s_2 \rangle_{\text{in}}$$

$$\times \prod_{j=1}^{2} \exp(\frac{i}{\hbar} u_j \cdot x_j) \bar{u}_{s_j}(p_j + \frac{1}{2}u_j) W_{\text{in}}(x + x_j, p_j) u_{r_j}(p_j - \frac{1}{2}u_j),$$

$$\Phi_{\alpha\beta}(p) \equiv \frac{i}{2} \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p \cdot y} \left\{ \left[\bar{\rho}\left(\frac{y}{2}\right) \left(-i\hbar\gamma \cdot \overleftarrow{\partial} + m \right) \right]_\beta \right.$$

$$\left. \times \, \psi_\alpha \left(-\frac{y}{2} \right) - \bar{\psi}_\beta \left(\frac{y}{2} \right) \left[(i\hbar\gamma \cdot \partial + m) \rho \left(-\frac{y}{2} \right) \right]_\alpha \right\}$$

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- Wigner function varies slowly over interaction range
  $\Longrightarrow$ Taylor expansion up to first order

$$W(x + x_j, p_j) = W(x + p_j) + x_j \cdot \partial W(x + p_j)$$

- Integrate over $x_j \to \delta$-functions

$$C_{\alpha\beta} = \frac{(2\pi\hbar)^6}{2(4m^4)} \sum_{r_1, r_2, s_1, s_2} \int d^4 p_1 d^4 p_2 d^4 u_1 d^4 u_2$$

$$\times {}_{\text{in}}\langle p_1 - \frac{1}{2}u_1, p_2 - \frac{1}{2}u_2; r_1, r_2 | \Phi_{\alpha\beta}(p) | p_1 + \frac{1}{2}u_2, p_2 + \frac{1}{2}u_2; s_1, s_2 \rangle_{\text{in}}$$

$$\times \prod_{j=1}^{2} \bar{u}_{s_j}(p_j + \frac{1}{2}u_j) \left[W(x, p_j)\delta^{(4)}(u_j) - i\hbar(\partial^{\mu}_{u_j}\delta^{(4)}(u_j))\partial_{\mu} W(x, p_j) \right] u_{r_j}(p_j - \frac{1}{2}u_j)$$

S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980)

- Consider contribution from local and nonlocal terms at first order in gradients

- Local collision term:

$$\mathfrak{C}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma' \, \mathcal{W} \, [f(x, p_1, \mathfrak{s}_1) f(x, p_2, \mathfrak{s}_2) - f(x, p, \mathfrak{s}) f(x, p', \mathfrak{s}')]$$

$$+ \int d\Gamma_2 \, dS_1(p) \, \mathfrak{W} \, f(x, p, \mathfrak{s}_1) f(x, p_2, \mathfrak{s}_2)$$

$d\Gamma \equiv d^4 p \, \delta(p^2 - m^2) dS(p)$

- Structure: Momentum and spin exchange + Spin exchange only

$$\mathcal{W} \equiv \delta^4(p + p' - p_1 - p_2) \frac{1}{8} \sum_{spins} h_{sr}(p, \mathfrak{s}) h_{s'r'}(p', \mathfrak{s}') \, h_{s_1 r_1}(p_1, \mathfrak{s}_1) \, h_{s_2 r_2}(p_2, \mathfrak{s}_2)$$

$$\times \langle p, p'; r, r' | t | p_1, p_2; s_1, s_2 \rangle \langle p_1, p_2; r_1, r_2 | t^\dagger | p, p'; s, s' \rangle \, ,$$

vacuum scattering amplitude

$$\mathfrak{W} \equiv \hbar \frac{\pi}{4m} \sum_{spins} \epsilon_{\mu\nu\alpha\beta} \mathfrak{s}^\mu \mathfrak{s}_1^\nu p^\alpha n^\beta_{s_1 r} \, h_{s_2 r_2}(p_2, \mathfrak{s}_2) \langle p, p_2; r, r_2 | t + t^\dagger | p, p_2; s_1, s_2 \rangle$$

with

$$h_{sr}(p, \mathfrak{s}) = \left[\delta_{sr} - \frac{1}{2m} \bar{u}_s(p) \, \mathfrak{s} \cdot \gamma \gamma^5 u_r(p) \right]$$

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- Nonlocal collision term:

$$C_{\alpha\beta} = -i\hbar\frac{(2\pi\hbar)^6}{2(4m^4)}\sum_{r_1,r_2,s_1,s_2}\int d^4p_1 d^4p_2 d^4u_1 d^4u_2$$
$$\times\, {}_{\text{in}}\langle p_1-\frac{1}{2}u_1, p_2-\frac{1}{2}u_2; r_1, r_2|\Phi_{\alpha\beta}(p)|p_1+\frac{1}{2}u_2, p_2+\frac{1}{2}u_2; s_1, s_2\rangle_{\text{in}}$$
$$\times\prod_{j=1}^{2}\bar{u}_{s_j}(p_j+\frac{1}{2}u_j)(\partial^{\mu}_{u_j}\delta^{(4)}(u_j))\partial_\mu W(x,p_j)u_{r_j}(p_j-\frac{1}{2}u_j)$$

- Integrate by parts
- Under $\mathfrak{s}$-integration and multiplied by scattering-matrix element:

$$\boldsymbol{\partial}_{u_j}\left[\bar{u}_s(p_j+\frac{1}{2}u)W^{(0)}(x,p_j)u_r(p_j-\frac{1}{2}u_j)\right]_{u_j=0}$$
$$\to \frac{i}{2(p_j^0+m)}(\boldsymbol{p}_j\times\boldsymbol{\mathfrak{s}}_j)\cdot\boldsymbol{\partial}f^{(0)}(x,p_j)$$

- $\Longrightarrow$ Nonlocality of collision term results in position shift of distribution functions $\Delta\cdot\boldsymbol{\partial}f$

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- Integration by parts

$$C_{\alpha\beta} = -i\hbar\frac{(2\pi\hbar)^6}{2(4m^4)}\sum_{r_1,r_2,s_1,s_2}\int d^4p_1 d^4p_2 d^4u_1 d^4u_2$$
$$\times\ {}_{\text{in}}\langle p_1-\frac{1}{2}u_1, p_2-\frac{1}{2}u_2; r_1, r_2|\Phi_{\alpha\beta}(p)|p_1+\frac{1}{2}u_2, p_2+\frac{1}{2}u_2; s_1, s_2\rangle_{\text{in}}$$
$$\times\prod_{j=1}^{2}\bar{u}_{s_j}(p_j+\frac{1}{2}u_j)(\partial^{\mu}_{u_j}\delta^{(4)}(u_j))\partial_\mu W(x,p_j)u_{r_j}(p_j-\frac{1}{2}u_j)$$

three contributions:

- One contribution vanishes after inserting equilibrium distribution function for $f^{(0)}$
- One contribution proportional to momentum derivatives of scattering amplitude

$$\langle p+\frac{1}{2}u_1-\frac{1}{2}u_2, p_2+\frac{1}{2}u_2; r, r_2|t|p+\frac{1}{2}u_1+\frac{1}{2}u_2, p_2+\frac{1}{2}u_2; s_1, s_2\rangle$$

  is neglected: consistent with low-density approximation

  A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics (Courier Corporation, 1975)
- Off-shell contribution $\mathfrak{C}_{\text{off-shell}}$!

- Off-shell contributions both in $\mathfrak{f}$ and in nonlocal collision term

$$p \cdot \partial \mathfrak{f} = m\, \mathfrak{C}_{\text{on-shell}} + \hbar m\, \mathfrak{C}^{(1)}_{\text{off-shell}}$$

- Remember:

$$\mathfrak{f} = m\delta(p^2 - m^2) f - \hbar m\, \delta m^2 \delta'(p^2 - m^2) f$$

with

$$-m\delta'(p^2 - m^2)\delta m^2\, f = \frac{1}{p^2 - m^2} p \cdot \partial \mathfrak{M}$$

- Explicit calculation of $\mathfrak{M}$ and $\mathfrak{C}$

$$\frac{1}{p^2 - m^2} p \cdot \partial \mathfrak{M} = m\, \mathfrak{C}^{(1)}_{\text{off-shell}}$$

$\Longrightarrow$ Same off-shell terms on both sides of Boltzmann equation

- Off-shell contributions cancel!
- On-shell kinetic equation for f

$$\delta(p^2 - m^2) p \cdot \partial f = \mathfrak{C}_{\text{on-shell}}[f]$$

- Collect terms → intuitive result:

$$\mathfrak{C}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma' \, \mathcal{W} \, [f(x+\Delta_1, p_1, \mathfrak{s}_1) \\ \times f(x+\Delta_2, p_2, \mathfrak{s}_2) - f(x+\Delta, p, \mathfrak{s}) f(x+\Delta', p', \mathfrak{s}')] \\ + \int d\Gamma_2 \, dS_1(p) \, \mathfrak{W} \, f(x+\Delta_1, p, \mathfrak{s}_1) f(x+\Delta_2, p_2, \mathfrak{s}_2)$$

- Collision nonlocal, particle positions displaced by

$$\Delta^\mu = -\frac{\hbar}{2m(p\cdot\hat{t}+m)} \epsilon^{\mu\nu\alpha\beta} p_\nu \hat{t}_\alpha \mathfrak{s}_\beta$$

with $\hat{t} = (1, 0)$.

- Interpretation: Particles scatter with finite impact parameter and are shifted before and after the collision.
- Contrast to massless case: Non-covariant distribution functions lead to position shifts after Lorentz transformations even for free fields.
  $\Longrightarrow$ Collision local in one frame → Collision nonlocal in different frame
  J-Y. Chen, D.T. Son, and M. Stephanov, PRL 115 (2015) 021601
- Here: distribution functions covariant, position shifts through nonlocal interactions.

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- Equilibrium condition: Collision term has to vanish.
- Ansatz for distribution function
  F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, AP. 338, 32 (2013)
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$f_{eq}(x, p, \mathfrak{s}) = \frac{1}{(2\pi\hbar)^3} \exp\left[-\beta(x)\cdot p + \frac{\hbar}{4}\Omega_{\mu\nu}(x)\Sigma_{\mathfrak{s}}^{\mu\nu}\right] \delta(p^2 - M^2)$$

  - β^μ - Lagrange multiplier for 4-momentum conservation
  - Spin potential $\Omega^{\mu\nu}$ - Lagrange multiplier for **total** angular momentum conservation
  - M - mass possibly modified by interactions
  - Dipole-moment tensor

$$\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$$

- Insert into $\mathfrak{C}[\mathfrak{f}]$ and expand up to first order in $\hbar$.
  $\Longrightarrow$ Zeroth-order collision term vanishes due to momentum conservation

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- Could Lagrange multiplier of $\Sigma_{\mathfrak{s}}^{\mu\nu}$ also start with zeroth order in $\hbar$?
- For nonlocal collisions spin is no collisional invariant.
- Zeroth-order polarization would lead to large orbital angular momenta in collisions by spin-to-orbital conversion, i.e. gradients of zeroth order.
  $\Longrightarrow$ inconsistent with power-counting scheme
- If collisions are considered to be strictly local, spin is collisional invariant
  $\Longrightarrow$ $\Sigma_{\mathfrak{s}}^{\mu}$ can be multiplied by spin potential with contributions starting at zeroth order.

  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)
- Here: Nonlocal terms are considered, spin is not conserved.
- $\Longrightarrow$ Leading-order polarization would lead to contradictions in power-counting scheme.

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At first order in $\hbar$:

$$\mathfrak{C}[f_{eq}] = -\int d\Gamma' d\Gamma_1 d\Gamma_2\, \widetilde{\mathcal{W}}\, e^{-\beta\cdot(p_1+p_2)}$$

$$\times \left[\partial_\mu\beta_\nu \left(\Delta_1^\mu p_1^\nu + \Delta_2^\mu p_2^\nu - \Delta^\mu p^\nu - \Delta'^\mu p'^\nu\right) - \frac{1}{2}\Omega_{\mu\nu}\frac{\hbar}{2}\left(\Sigma_{\mathfrak{s}_1}^{\mu\nu} + \Sigma_{\mathfrak{s}_2}^{\mu\nu} - \Sigma_{\mathfrak{s}}^{\mu\nu} - \Sigma_{\mathfrak{s}'}^{\mu\nu}\right)\right]$$

$$-\int d\Gamma_2\, dS_1(p) dS'(p_2)\, \mathfrak{W}\, e^{-\beta\cdot(p+p_2)}$$

$$\times \left\{\partial_\mu\beta_\nu \left[(\Delta_1^\mu - \Delta^\mu)p^\nu + (\Delta_2^\mu - \Delta'^\mu)p_2^\nu\right] - \frac{1}{2}\Omega_{\mu\nu}\frac{\hbar}{2}(\Sigma_{\mathfrak{s}_1}^{\mu\nu} + \Sigma_{\mathfrak{s}_2}^{\mu\nu} - \Sigma_{\mathfrak{s}}^{\mu\nu} - \Sigma_{\mathfrak{s}'}^{\mu\nu})\right\}.$$

- Conservation of total angular momentum (orbital+spin) in a collision

$$J^{\mu\nu} = \Delta^\mu p^\nu - \Delta^\nu p^\mu + \frac{\hbar}{2}\Sigma_{\mathfrak{s}}^{\mu\nu}$$

- Conditions for vanishing of collision term at first order:

$$\partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0$$

$$\Omega_{\mu\nu} = \varpi_{\mu\nu} \equiv -\frac{1}{2}\partial_{[\mu}\beta_{\nu]} = \text{const.}$$

$$a_{[\mu}b_{\nu]} \equiv a_\mu b_\nu - a_\nu b_\mu$$

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Discussion

- Collision term vanishes under conditions for global equilibrium!
- But not for (standard) local equilibrium with nonlocal collisions.
- Confirm known result from statistical quantum field theory:
  In global equilibrium spin potential equal to thermal vorticity.
  F. Becattini, PRL 108, 244502 (2012)
- Interpretation: When approaching equilibrium, non-vanishing vorticity converts orbital angular momentum into spin through nonlocal collisions
  $\Longrightarrow$ Initially unpolarized fluid gets polarized!

- Calculate currents in HW pseudo-gauge: Originally derived for free fields by applying Noether's theorem to Klein-Gordon Lagrangian for spinors
  J. Hilgevoord and S. Wouthuysen, Nuclear Physics 40, 1 (1963)
- Here: obtain energy-momentum and spin tensor by pseudo-gauge transformation from canonical tensors.
  See Enrico Speranza's talk
- Choice of pseudo-gauge transformation:
  - Recover HW tensors for zero interactions.
  - Obtain physically meaningful equations of motion (see next slide).
- Result:

$$T^{\mu\nu}_{HW} = \int d\Gamma p^\mu p^\nu f(x, p, \mathfrak{s}) + \mathcal{O}(\hbar^2) ,$$

$$S^{\lambda,\mu\nu}_{HW} = \int d\Gamma p^\lambda \left(\frac{1}{2} \Sigma^{\mu\nu}_{\mathfrak{s}} - \frac{\hbar}{4m^2} p^{[\mu} \partial^{\nu]} \right) f(x, p, \mathfrak{s}) + \mathcal{O}(\hbar^2) .$$

- Using Boltzmann equation

$$\partial_\mu T^{\mu\nu}_{HW} = \int d\Gamma\, p^\nu\, \mathfrak{C}[\mathfrak{f}] = 0 \,,$$

$$\hbar\, \partial_\lambda S^{\lambda,\mu\nu}_{HW} = \int d\Gamma\, \frac{\hbar}{2} \Sigma^{\mu\nu}_{\mathfrak{s}}\, \mathfrak{C}[\mathfrak{f}] = T^{[\nu\mu]}_{HW} \,.$$

- Energy-momentum conserved in a collision
- Spin not conserved in nonlocal collisions $\Leftrightarrow$ $T^{[\nu\mu]}_{\mathrm{HW}} \neq 0$
  $\Rightarrow$ Conversion between spin and orbital angular momentum
- $T^{[\nu\mu]}_{\mathrm{HW}} = 0$
  (i) for local collisions, as spin is collisional invariant
  (ii) in global equilibrium, as collision term vanishes
- With nonlocal collisions out of global equilibrium: dynamics dissipative

- Derivation of nonlocal collisions from quantum field theory
- Start from equations of motion for Wigner function
- Main assumptions: Gradient expansion and low density
- Spin in phase space leads to simple and intuitive treatment of polarization effects
- Collision term contains local and nonlocal contributions
- Off-shell effects cancel on both sides of Boltzmann equation
- Nonlocal contribution to collision term can be expressed as position shifts of distribution functions
- Nonlocal collision term vanishes in global equilibrium
- Then spin potential is equal to thermal vorticity
- Nonlocal collisions are essential to obtain spin alignment with vorticity
- Antisymmetric part of HW energy-momentum tensor describes conversion between spin and orbital angular momentum in presence of nonlocal collisions

Outlook

- Comparison to alternative approach to nonlocal collision term: Kadanoff-Baym equation
  $\Longrightarrow$ See next talk by Xin-li Sheng
  related works:
  D.-L. Yang, K. Hattori, and Y. Hidaka, JHEP 20, 070 (2020)
  Z. Wang, X. Guo, P. Zhuang, arXiv:2009.10930 (2020)
- Derive second-order dissipative hydrodynamics with spin using method of moments.
  G.S. Denicol, H. Niemi, E. Molnar, D.H. Rischke, PRD 85 (2012) 114047
- Dissipative corrections to Pauli-Lubanski vector including effects of nonlocal collisions
- Possible explanation for local polarization of Λ-hyperon?
  J. Adam et al. [STAR Collaboration], PRL 123, 132301 (2019)