MHD from QED

Masaru Hongo, KH, arXiv:2005.10239
Cf. KH, Yuji Hirono, Ho-Ung Yee, Yi Yin, arXiv:711.08450

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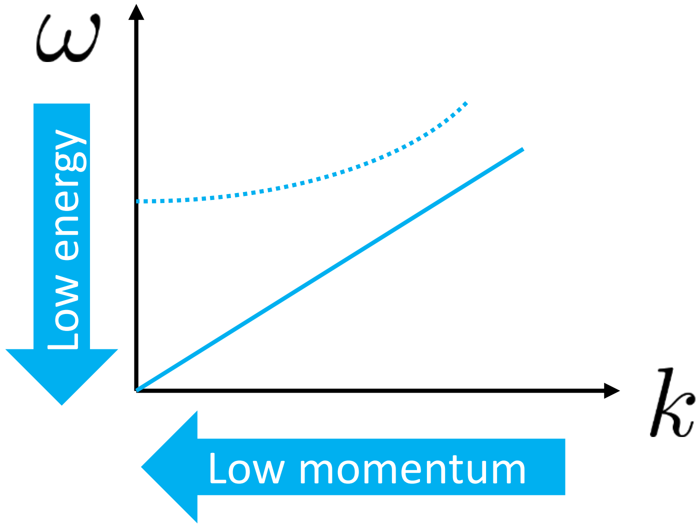
What is hydrodynamics in the theoretical perspective?

Consider constructing a low-energy effective theory of a given system.

Need to identify the relevant dof. = Gapless modes.
In the coordinate space, they are conserved charges
surviving in an (infinitely) long spacetime scale.

Hydrodynamics can be constructed based on symmetries of the system.
(Equations of motion are composed of a set of conservation laws.)



Therefore, hydrodynamics can be regarded as a universal low-energy EFT on the basis of the symmetries of the system.

Issues in the conventional formulation of the magnetohydrodynamics (MHD)

Coupled dynamics of fluid and electromagnetism

$$\partial_\mu T^{\mu\nu}_{\mathrm{matt}} = F^{\nu\mu} j_\mu$$ Joule heat/Lorentz force

$$\partial_\mu F^{\mu\nu} = j^\nu$$ Current

--- Based on the non-conservation laws

--- Contains a gapped mode (or dissipative mode)

Does NOT match the concept of hydrodynamics.

Dynamics of conserved charges

Hydrodynamic variables

{Energy density, fluid flow velocity}

$$\{\epsilon,\ u^\mu\}$$

$\Longleftrightarrow$

Conservation laws

Translational symmetries

$$\partial_\mu T^{\mu\nu} = 0$$

Electromagnetism

$$\boldsymbol{E},\ \boldsymbol{B}?$$

$\Longleftrightarrow$

$$\partial_\mu F^{\mu\nu} = j^\nu ?$$
$$\partial_\mu \tilde{F}^{\mu\nu} = 0?$$

E-field is screened by a static charge distribution, i.e. $E \to 0$
Debye screening effect.

A static charge distribution does not screen the B-field.
(No "magnetic Coulomb field" without a magnetic monopole).

$$B \not\to 0$$

Dynamics of conserved quantities with electromagnetism

MHD variables

$$\{\epsilon,\ u^\mu,\ B^\mu\}$$

No E-field in the global equilibrium
$(E^\mu \sim \mathcal{O}(\partial^1))$

Conservation laws

$$\{\partial_\mu T^{\mu\nu} = 0,\ \partial_\mu \tilde{F}^{\mu\nu} = 0\}$$

No need for $j^\nu = \partial_\mu F^{\mu\nu}$
(Time derivative of $\boldsymbol{E}$).

E^μ is induced by dynamics of the hydro-variables:
$\rightarrow$ Constitutive eq. for E^μ

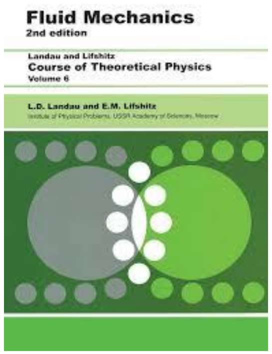
$$E^\mu(u^\mu, B^\mu)$$
$$j^\mu(u^\mu, B^\mu)$$

$\partial_\mu \tilde{F}^{\mu\nu} = 0$ is the conservation law of the magnetic lines.

(Re-)formulation of relativistic magnetohydrodynamics as a universal low-energy effective theory

1. *Macroscopic formulation based on the laws of thermodynamics*

   "Entropy-current analysis"

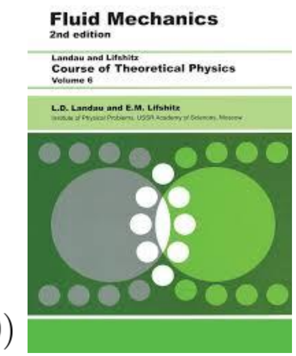


KH, Hirono, Yee, Yin, 1711.08450

2. Microscopic formulation based on non-equilibrium statistical method

Masaru Hongo, KH, arXiv:2005.10239

“Phenomenological derivation” *a la* Landau and Lifshitz
--- *How it works for non-derivative terms*



KH, Hirono, Yee, Yin

Writing down all possible tensors that meet symmetries of the system.

$$
\begin{aligned}
T^{\mu\nu}_{(0)} &= \epsilon u^\mu u^\nu - X\Delta^{\mu\nu} - Y B^\mu B^\nu \\
\tilde{F}^{\mu\nu}_{(0)} &= Z(B^\mu u^\nu - B^\nu u^\mu)
\end{aligned}
$$

$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \;\; (u_\mu \Delta^{\mu\nu} = 0)$

E-field is first order.

$B^{(\mu}u^{\nu)}$ is absent in $T^{\mu\nu}$ when $n_V = 0$.

In general, they, however, do not satisfy the second law of thermodynamics.
→ Additional constraints on the constitutive equations (Landau & Lifshitz).

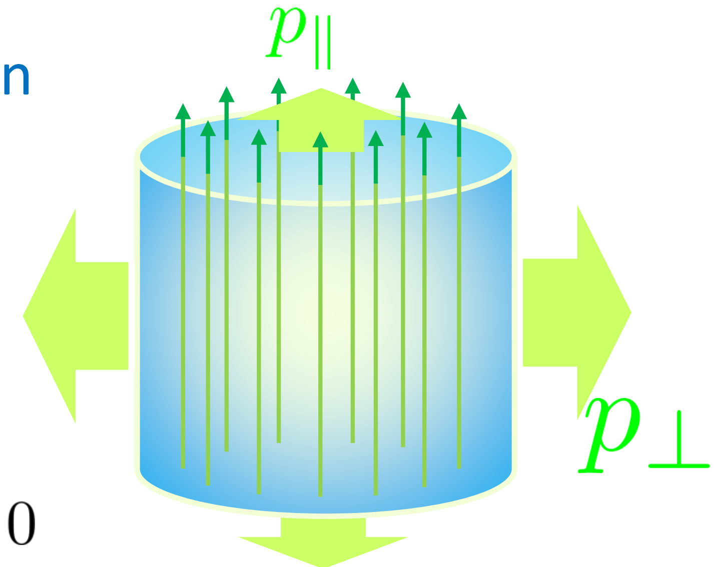
$$
T\partial_\mu s^\mu_{(0)} = \theta(p - X) - (YB^\nu - ZH^\nu)B^\mu \partial_\mu u_\nu + (Z-1)B^\nu D H_\nu = 0
$$

From EoM + thermodynamic relation $\quad ds = \frac{1}{T}(d\epsilon - H_\mu dB^\mu)$

Therefore, $\quad T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} - H^\mu B^\nu \,, \quad Z = 1$

ϵ and p are the total (fluid+magnetic) energy and pressure.

Relation to the conventional formulation



If we assume

$\epsilon \sim \epsilon_{\rm matt} + \boldsymbol{B}^2/2 - \boldsymbol{M}\cdot\boldsymbol{B}$

$p \sim p_{\rm matt} + \boldsymbol{B}^2/2 - \boldsymbol{M}\cdot\boldsymbol{B}$

$$\partial_\mu (T^{\mu\nu}_{\rm matt} + \boxed{T^{\mu\nu}_{\rm Maxwell}}) = 0 \qquad \text{at } E = 0$$

$$T^{\mu\nu}_{\rm Maxwell} = -(F^{\mu\alpha}F^{\nu}{}_{\alpha} - g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}/4)$$

$\Updownarrow$

$$\boxed{\text{NB)}\ F^{\nu\mu}j_\mu = \partial_\mu T^{\mu\nu}_{\rm Maxwell}}$$

Conventional formulation:

$$\partial_\mu T^{\mu\nu}_{\rm matt} = \boxed{F^{\nu\mu}j_\mu}$$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

Separation btw the matter and EM parts is not necessary
and even may not be possible for strongly-coupled systems.

The translation symmetry tells the conservation of the *total* energy.

First-order constitutive eqs

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)}$$

$$\tilde{F}^{\mu\nu} = \tilde{F}^{\mu\nu}_{(0)} - \epsilon^{\mu\nu\alpha\beta} u_\alpha E_{(1)\beta}$$

$$T^{\mu\nu}_{(1)},\ E^{\mu}_{(1)},\ \sim \mathcal{O}(\partial^1)$$

The second law of the thermodynamics $\partial_\mu (s u^\mu) \geq 0$ constrains the tensor structures of the first order corrections.

Computing the entropy current,

$$\partial_\mu \left(s u^\mu + \mathcal{O}(\partial^1) \right)$$

$$= T^{\mu\nu}_{(1)} \partial_\mu (\beta u_\nu) - E^{\mu}_{(1)} \{ \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha (\beta H^\beta) \}$$

Each term should take a semi-positive value.

$$= E^{\mu}_{(1)} X_{\mu\nu} E^{\nu}_{(1)},$$ for example.

Constitutive eqs. of the electric field and current

Positivity is ensured by a bilinear form: $E^{\mu}_{(1)} X_{\mu\nu} E^{\nu}_{(1)} \geq 0$

$$X_{\mu\nu} = \sigma_{\parallel} b_\mu b_\nu - \sigma_{\perp}(g_{\mu\nu} - u_\mu u_\nu + b_\mu b_\nu) - \sigma_{\mathrm{Hall}}\, \epsilon_{\mu\nu\alpha\beta} u^\alpha b^\beta$$

$b^\mu = -B^\mu / B^2$ breaks a spatial rotational symmetry.

$\sigma_{\parallel,\perp} \geq 0$, but $\sigma_{\mathrm{Hall}} \propto \mu_V$.

Therefore, we get a "constitutive eq." of the E-field:

$$E^{\mu}_{(1)} = -X^{-1\mu\rho} \epsilon_{\rho\nu\alpha\beta} u^\nu \partial^\alpha (\beta H^\beta)$$

The relation to the current is given by the Maxwell eq:

$$\begin{aligned} J^\mu_V &= \partial_\nu F^{\nu\mu} = \partial_\nu (\epsilon^{\nu\mu\alpha\beta} u_\alpha H_\beta) + \mathcal{O}(\partial^2) \\ &= \left[\sigma_{\parallel} E^\mu_{\parallel} + \sigma_{\perp} E^\mu_{\perp} + \sigma_{\mathrm{Hall}} \epsilon^{\mu\nu\alpha\beta} u_\nu b_\alpha E_\beta\right] + \cdots \end{aligned}$$

Chiral MHD: Chiral magnetic effect is required in MHD if one includes an axial charge and redo the entropy-current analysis.

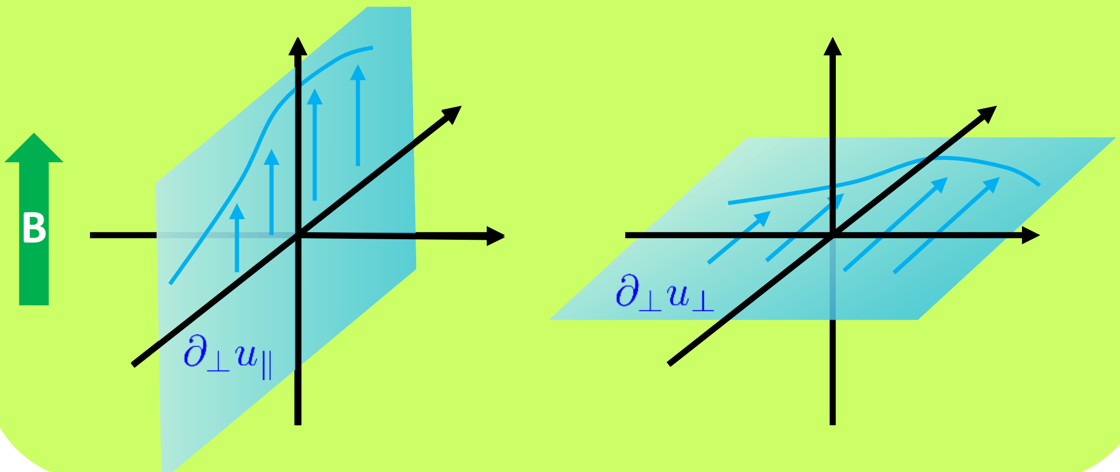
KH, Hirono, Yee, Yin

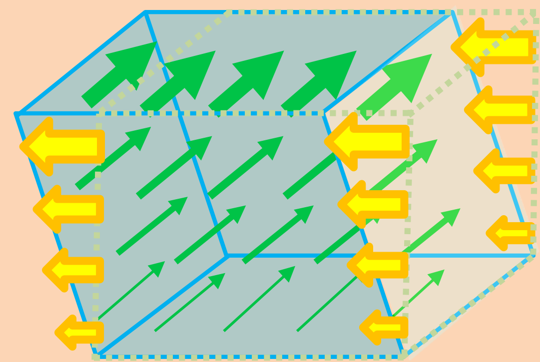
Seven viscous coefficients

Shear and bulk viscosities

$$\eta \to \eta_{\parallel,\perp}$$
$$\zeta \to \zeta_{\parallel,\perp}$$

Without the full spatial rotational symmetry, the viscosities split into the parallel and transverse components.





Hall viscosities with $\epsilon^{\mu\nu\rho\sigma}$: ($\parallel$ and $\perp$ components)

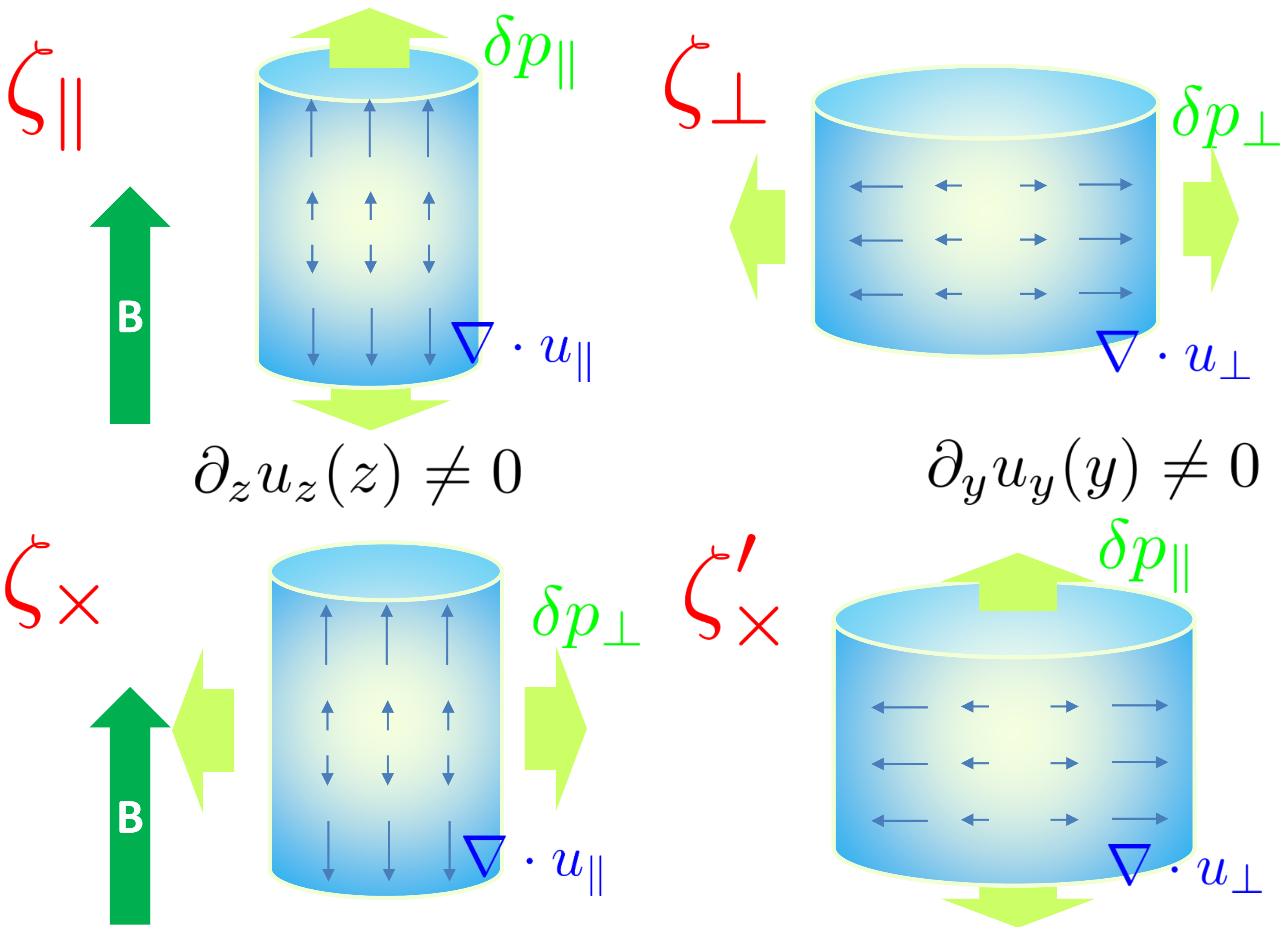
where $\bar{T}^{\mu\nu}_{(1)}\partial_\mu(\beta u_\nu) = 0$

Deformation of fluid cells induced by an inhomogeneous Lorentz force

What would be the last one?

Grozdanov et al.,
KH, Itakura & Ozaki

Bulk viscosities:



Onsager's relation: $\zeta_\times = \zeta'_\times$ (7th coefficient)

Lessons from the macroscopic derivation

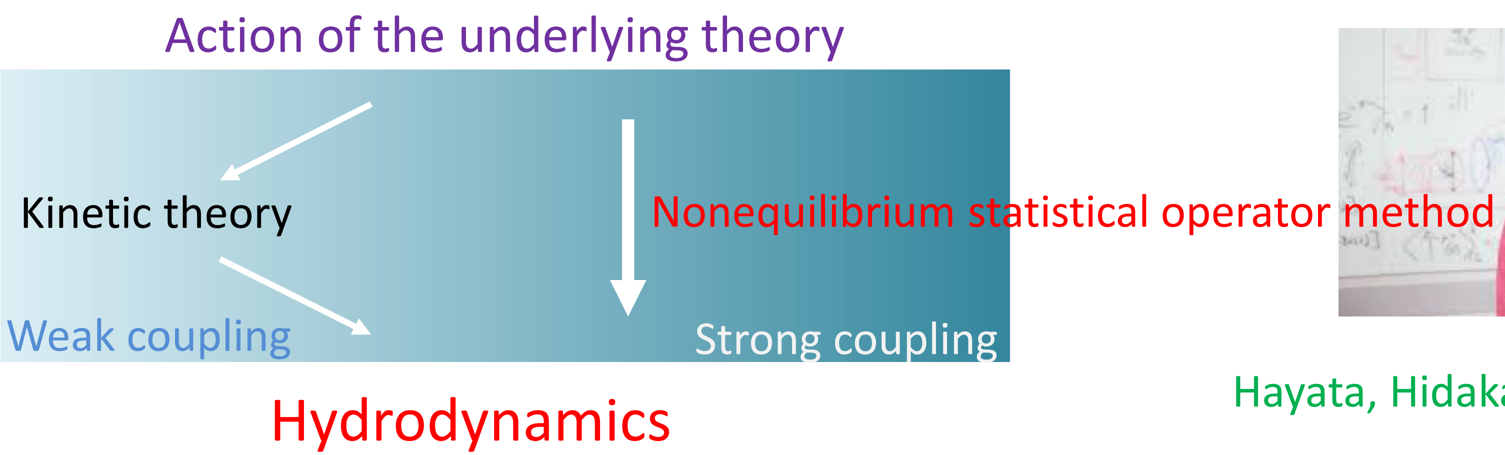
- *Relativistic magnetohydrodynamics can be (re)formulated as dynamics of correctly identified gapless modes.*
- Many transport coefficients from the entropy-current analysis when a spatial rotational symmetry is broken by B-field.

Relativistic MHD from
the nonequilibrium statistical method

Derivation from the microscopic theory

The “phenomenological” strategy of Landau & Lifshitz assumes the laws of thermodynamics.

How can we establish microscopic foundation of MHD from QED with statistical mechanical methods?



Hayata, Hidaka, Noumi, Hongo

Application to MHD formulation, Masaru Hongo, KH, arXiv:2005.10239 [hep-th].

Setup of the problem

Starting with a given Lagrangian, one can identify the symmetries of a theory and get the conservation laws at the operator level.

$$\partial_\mu \hat{Q}^\mu = 0$$

$$\hat{Q}^0 = \hat{T}^{0\nu},\ \hat{j}^0\ \ldots$$

To get a closed set of eqs., we need to express $\langle \hat{Q}^i(t) \rangle$ by $\langle \hat{Q}^0(t) \rangle$.

How can we evaluate the expectation values to get a "consitutive equation", a functional relation: $\langle \hat{Q}^i(t) \rangle = F^i[\langle \hat{Q}^0(t) \rangle]$?

“Local” Gibbs ensemble

$$\hat{\rho}_{\mathrm{LG}}(t,\boldsymbol{x}) = \frac{\exp\left[\int_{\partial\Sigma_t}[\mu(t,\boldsymbol{x})\hat{Q}^0(t,\boldsymbol{x})]\right]}{\mathrm{tr}[\exp\left[\int_{\partial\Sigma_t}[\mu(t,\boldsymbol{x})\hat{Q}^0(t,\boldsymbol{x})]\right]}$$

$\partial\Sigma_t$: Equal-time hypersurface

Assumption: The system is in the local equilibrium at the initial time t_i.

$$\hat{\rho}(t_i) = \hat{\rho}_{\mathrm{LG}}(t_i)$$

From this assumption, one can get the constitutive eq. at $t = t_i$.

$$\langle \hat{Q}^i(t_i) \rangle^{\mathrm{LG}}_{t_i} = \mathrm{tr}[\hat{\rho}(t_i)\hat{Q}^i(t_i)] = F^i[\mu(t_i)]$$

But, how can we evaluate the expectation value at later time t?

$$\langle \hat{Q}^\mu(t) \rangle = \mathrm{tr}[\hat{\rho}(t_i)\hat{Q}^\mu(t)] = \,?$$

--- The density operator does NOT evolve in the Heisenberg picture.

Updates of the "unperturbed" basis

t= t_i

$$\hat{\rho}(t_i) = \hat{\rho}_{\rm LG}(t_i)$$

$$\langle \hat{Q}^i(t_i)\rangle^{\rm LG}_{t_i} = {\rm tr}[\hat{\rho}(t_i)\hat{Q}^i(t_i)] = F^i[\langle \hat{Q}^0\rangle^{\rm LG}_{t_i};\mu(t)]$$

We are ready to solve the eqs. to get a charge density in the next time step.

t_i + dt

$$\langle \hat{Q}^0(t_i + dt)\rangle \quad = \quad c_0\mu(t_i) + c_1\partial\mu(t_i) + \cdots$$

t_i + 2dt

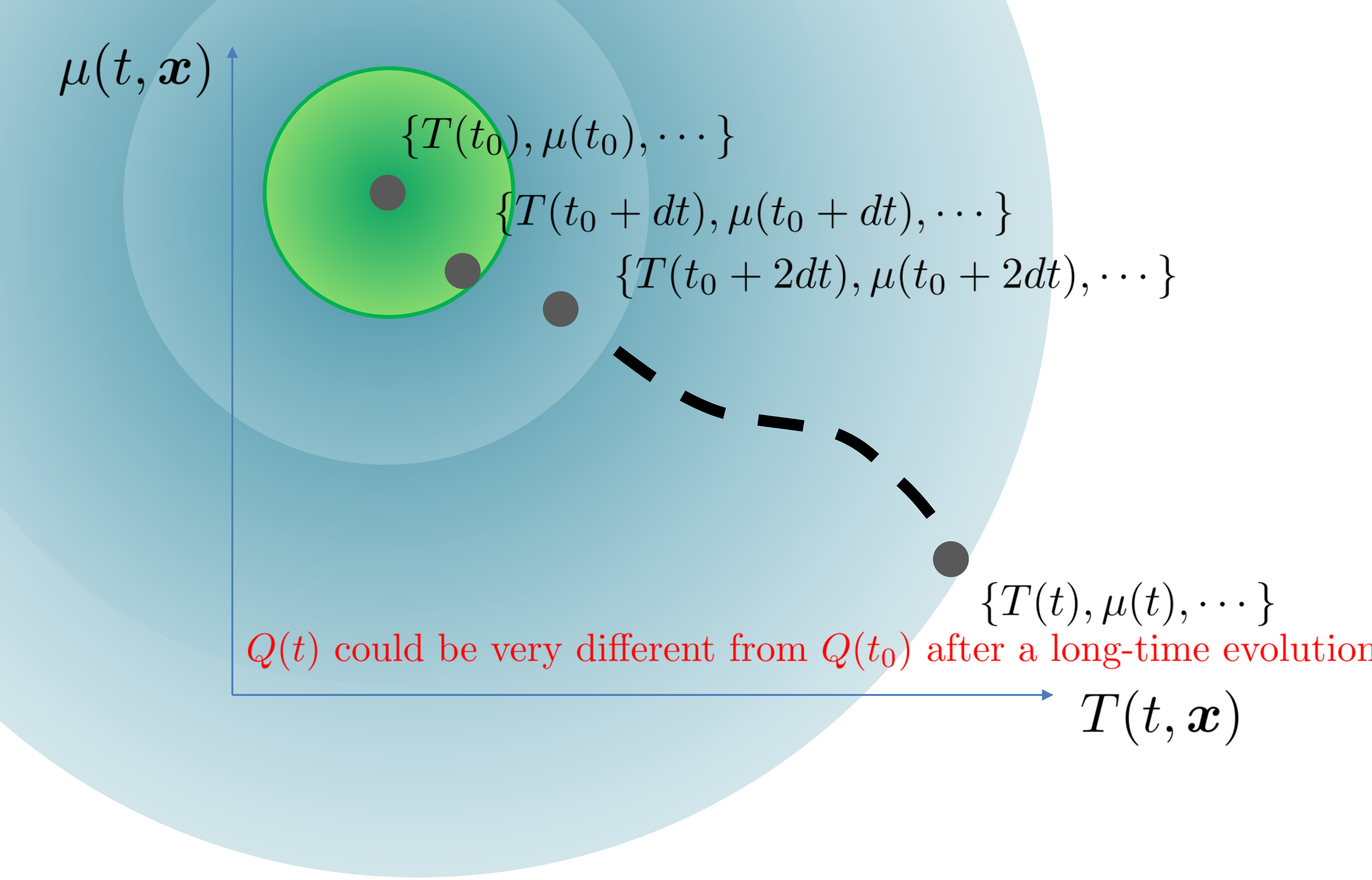
But, what local equilibrium should we refer once the system starts evolving?
E.g., what is the relation between $\mu(t_i)$ and $\mu(t_i + dt)$?

$$\langle \hat{Q}^0(t_i + 2dt)\rangle \quad = \quad ?$$

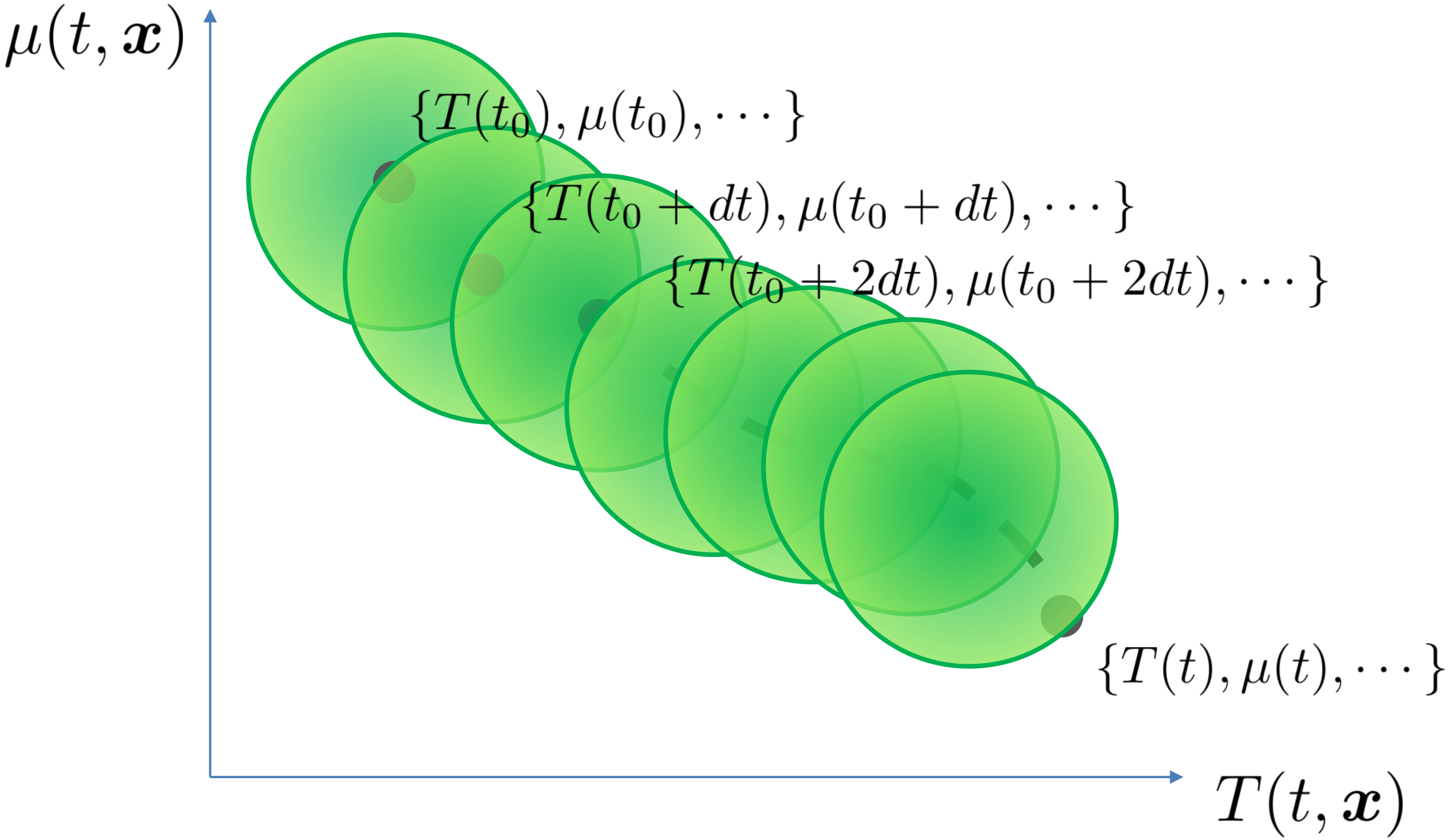
✘ Dead end?

Time

Time evolution and update of the thermodynamic parameters



Time evolution and update of the thermodynamic parameters



Update condition

$$\mathrm{tr}[\hat{\rho}_{\mathrm{LG}}(t_i + dt)\hat{Q}^0(t_i + dt)] = \langle \hat{Q}^0(t_i + dt)\rangle$$

"Thermodynamics parameters" in the dissipative hydrodynamics are not unique (Cf. the frame choice).

How the derivative expansion and the update condition work

$$\hat{\rho} = \hat{\rho}_{\mathrm{LG}}(t_i)$$

Is our $\hat{\rho}$ rewritten by $\hat{\rho}_{\mathrm{LG}}(t)$ at an arbitrary time t?

$$\hat{\rho} = \hat{\rho}_{\mathrm{LG}}(t) \times \hat{U}$$

How the derivative expansion and the update condition work

$$\delta\hat{S}_\tau(t) = e^{\tau\hat{S}(t)}\delta\hat{S}(t)e^{-\tau\hat{S}(t)}$$

"Time" ordering

$$\hat{\rho} = \hat{\rho}_{\mathrm{LG}}(t_i)$$

$$\begin{aligned}\hat{\rho}_{\mathrm{LG}}(t_i) &= e^{-\hat{S}(t_i)} \\ &= e^{-\hat{S}(t)+\delta\hat{S}(t)} = \hat{\rho}_{\mathrm{LG}}(t) \times \mathrm{T}_\tau e^{\int_0^1 d\tau\delta\hat{S}_\tau}\end{aligned}$$

Cf. Transition from the Heisenberg to interaction pictures

$$\delta\hat{S}(t) = \hat{S}(t) - \hat{S}(t_i) \sim \int_{\partial\Sigma_t}[\delta\hat{Q}^\mu\nabla_\mu\mu]$$

Derivative expansion

$$\langle\hat{\mathcal{O}}(t)\rangle = \langle\hat{\mathcal{O}}(t)\rangle_t^{\mathrm{LG}} + \sum_{n=1}^{\infty}\langle T_\tau\left[\int_0^1 d\tau\delta\hat{S}_\tau\right]^n\hat{\mathcal{O}}(t)\rangle_t^{\mathrm{LG}}$$

$$\hat{\rho} = \hat{\rho}_{\mathrm{LG}}(t) \times \hat{U}$$

$$\delta\hat{\mathcal{O}} = \hat{\mathcal{O}} - \langle\hat{\mathcal{O}}\rangle_{\mathrm{LG}}$$

Constitutive equation and Kubo formula

- Constitutive eq. from the systematic derivative expansion

$$\langle \hat{Q}^\mu(t) \rangle = \langle \hat{Q}^\mu(t) \rangle_t^{\rm LG} + \sum_{n=1}^{\infty} \langle\langle [\delta \hat{S}_\tau]^n, \delta \hat{Q}^\mu(t) \rangle\rangle$$

Ideal hydro

Derivative corrections

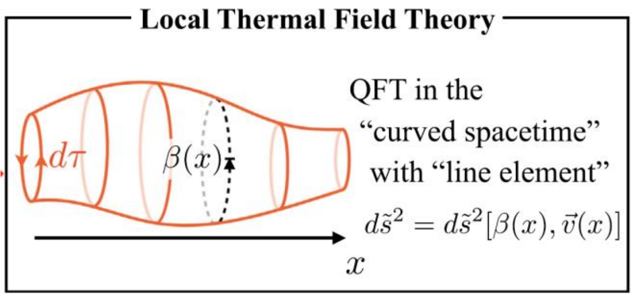
1. Non-dissipative parts: Partition function with the path-integral formalism

$$\Psi = \ln \mathrm{tr} \exp \left[\int_{\partial \Sigma_t} [\mu(t) \hat{Q}^0(t)] \right]$$

Path integral for the trace

$$\hat{\rho}_{\rm LG}(t) = \frac{\exp\left[\int_{\partial\Sigma_t} [\mu(t)\hat{Q}^0(t)]\right]}{\mathrm{tr}[\exp\left[\int_{\partial\Sigma_t} [\mu(t)\hat{Q}^0(t)]\right]}$$

Variation of Ψ generates the current $\langle \hat{Q}^\mu(t) \rangle_t^{\rm LG}$.



M.Hongo, 1611.07074 [hep-th]

2. Dissipative parts: Kubo formulas for the first-order transport coefficients

$$\langle \hat{Q}^\mu_{(1)} \rangle = \lambda \nabla^\mu_\perp \mu$$

Kubo formula

$$\lambda = \langle\langle \tilde{\delta} \hat{Q}_\mu, \tilde{\delta} \hat{Q}^\mu \rangle\rangle$$

MHD from the global symmetries of QED

Masaru Hongo, KH
arXiv:2005.10239 [hep-th]

We argued that E is screened in the equilibrium, but B is not.

→ MHD = fluid dynamics + magnetic flux

Q1: Can we understand MHD from any symmetry
of the underlying microscopic theory? Grozdanov et al.

Q2: How does the statistical method support
the phenomenological derivation of MHD?

Symmetries of the Maxwell part

$$\mathcal{L}_{\text{Maxwell}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$F^{\mu\nu}$ is invariant under a **global** shift by a 1-form parameter θ_μ: $A_\mu \to A_\mu + \theta_\mu$

So is $\tilde{F}^{\mu\nu}$ under $\tilde{A}_\mu \to \tilde{A}_\mu + \tilde{\theta}_\mu$.

NB) E-M duality: $F \leftrightarrow \tilde{F}$
$\tilde{F}^{\mu\nu} = \partial \tilde{A}_\mu - \partial \tilde{A}_\mu$

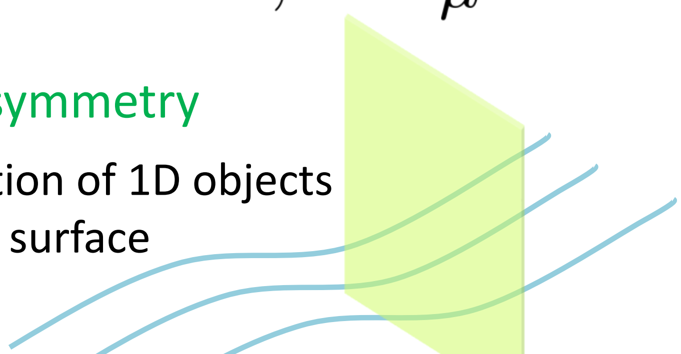
Electric and magnetic fluxes as conserved currents

$$\partial_\mu F^{\mu\nu} = 0\,, \quad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

Gaiotto et al.

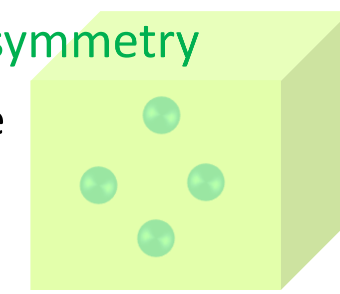
1-form symmetry

Conservation of 1D objects through a surface



Conventional (0-form) symmetry

Conservation of point-like charges in a box



Conservation laws from the global symmetries of QED

$$\mathcal{L}_{\rm QED} = -\bar{\psi}(\partial\!\!\!/ + iq A\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Symmetries of QED

Translational symmetry
Magnetic one-form symmetry

Electric ~~one-form~~ symmetry

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

Explicitly broken by the current

Ans. 1: MHD can be still understood as the dynamics of the conserved charges from the generalized global symmetry in addition to the Poincare symmetry.

Grozdanov et al.

Highlights from the results

Nondissipative part at the zeroth order (Ideal order)

Ans. 2-1. Same tensor structure as the phenomenological derivation and clarifies the relation to the partition function (EoS).

$$\langle \hat{T}^{\mu\nu} \rangle = p g^{\mu\nu} - \beta \frac{\partial p}{\partial \beta} u^\mu u^\nu - \tilde{H} \frac{\partial p}{\partial \tilde{H}} b^\mu b^\nu$$

$$\langle \tilde{F}^{\mu\nu} \rangle = \beta \frac{\partial p}{\partial \tilde{H}} (u^\mu b^\nu - u^\nu b^\mu)$$

$\beta,\ \tilde{H}$ are related to the Lagrange multipliers for $T^{0\mu},\ \tilde{F}^{0i}$.

$$e = -\partial(\beta p)/\partial\beta,\ \ B = \partial(\beta p)/\partial\tilde{H}$$

The partition functional establishes all the relations to QED:

$$\Psi = \int d^3x \beta\, p(\beta, \tilde{H})$$

No first-order corrections for a charge-neutral plasma.

Dissipative part at the first order

Ans. 2-2. Same constitutive eqs. as the phenomenological derivation, and "verification" of the thermodynamic inequalities from the statistical method.

$$\langle\langle \tilde{\delta}\hat{\mathcal{O}}, \tilde{\delta}\hat{\mathcal{O}} \rangle\rangle \geq 0 \qquad s = -\mathrm{tr}[\hat{\rho}_{\mathrm{LG}} \ln \hat{\rho}_{\mathrm{LG}}]$$

Inequalities for the seven viscous coefficients:

$$\zeta_{\parallel} \geq 0 \, , \quad \zeta_{\perp} \geq 0 \, , \quad \eta_{\parallel} \geq 0 \, , \quad \eta_{\perp} \geq 0 \, .$$

$$\zeta_{\parallel}\zeta_{\perp} - \zeta_{\times}^2 \geq 0$$

The positivity requires 5 inequalitites.
(But, either $\zeta_{\parallel,\perp}$ is redundant when $\zeta_{\parallel}\zeta_{\perp} - \zeta_{\times}^2 \geq 0$.
$\rightarrow$ 4 inequalities as the minimal set)

"Proof of the semi-positivity" within the first order

As a consequence of the inequalities from the statistical mechanics , the entropy production rate should take a semi-positive value:

$$\nabla_\mu s^\mu_{(1)} \geq 0$$

Comparison to other phenomenological derivations

Annals of Physics 326 (2011) 3075–3094

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Kubo formulas for relativistic fluids in strong magnetic fields

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Requiring all positive coefficients.
This is a too strong requirement.

Relativistic magnetohydrodynamics

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There is a redundancy
in the set of inequalities.

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Generalized global symmetries and dissipative magnetohydrodynamics

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Supported by our results
from statistical mechanics.

Cf. Appendix in Masaru Hongo, KH
arXiv:2005.10239 [hep-th]

Transport coefficients from perturbative QED × QCD

Only $\sigma_{||}$, $\zeta_{||}$ are finite in the strong-field limit.

Carriers' motion is restricted to the parallel motion along the magnetic field as the cyclotron radius shrinks.

[1] KH, S.Li, D.Satow, H.-U. Yee, 1610.06839 [hep-ph]
[2] KH, D.Satow, 1610.06818 [hep-ph]
[3] KH, X.-G.Huang, D.Satow, D.Rischke, 1708.00515 [hep-ph]

	$\sigma_{\parallel}$	$\sigma_{\perp}$	$\sigma_{\rm Hall}$	$\eta_{\parallel}$	$\eta_{\perp}$	$\zeta_{\parallel}$	$\zeta_{\perp}$	$\zeta_{\times}$	$\xi^{\parallel}_{\rm Hall}$	$\xi^{\perp}_{\rm Hall}$
Strong B	**[1,2]**	---	---	---	---	**[3]**	---	---	---	---
	[4]	[4]	[4]	**? ? ? ? ? ? ? ?**						
Weak B				[5]	**To be explored!**					

[4] Fukushima and Hidaka, 1711.01472 [hep-ph]; 1906.02683 [hep-ph]
[5] Li and Yee, 1707.00795 [hep-ph]

Summary

1-1. Relativistic MHD from the entropy-current analysis

1-2. Relativistic MHD from the nonequilibrium statistical method

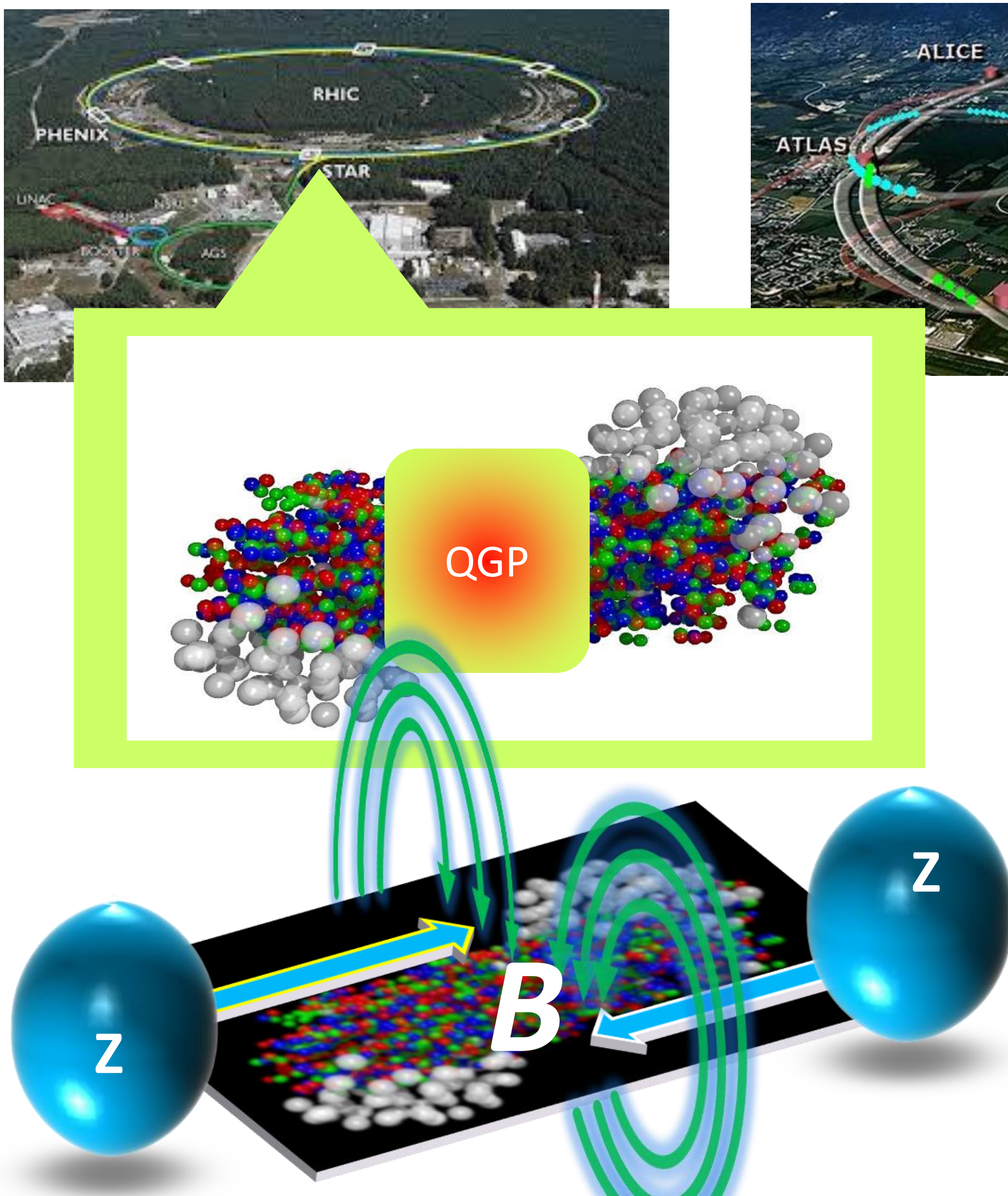
--- MHD is dynamics of conserved magnetic-field lines
(associated with the magnetic one-form symmetry of QED).

--- Semi-positive entropy production is verified within the first-order in derivative expansion thanks to the statistical method.

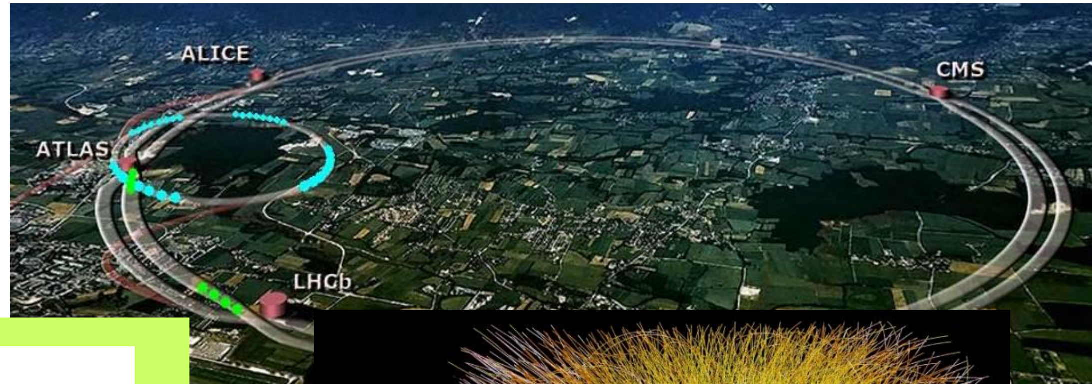
Back-up slides

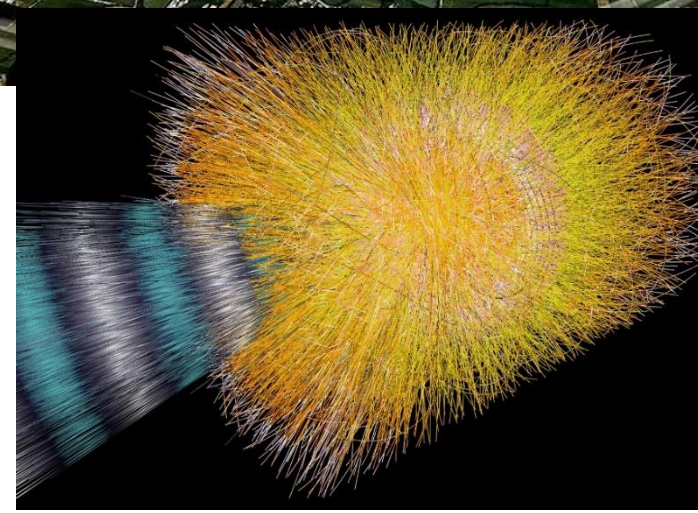
Relativistic heavy-ion colliders

Relativistic Heavy Ion Collider (RHIC)



Large Hadron Collider (LHC)





Z ~ 80, v > 0.99999 c,

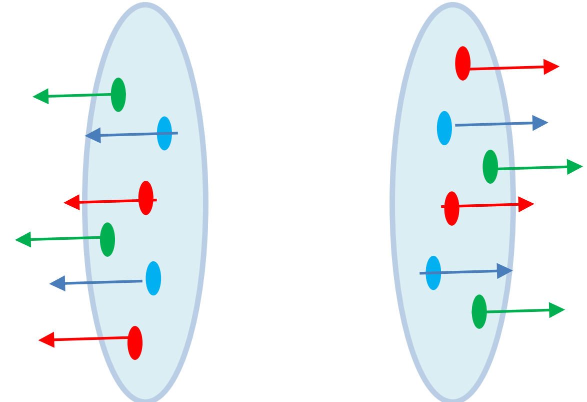
Length scale ~ $1/\Lambda_{QCD}$

$$eB \sim \Lambda_{\mathrm{QCD}}^2$$

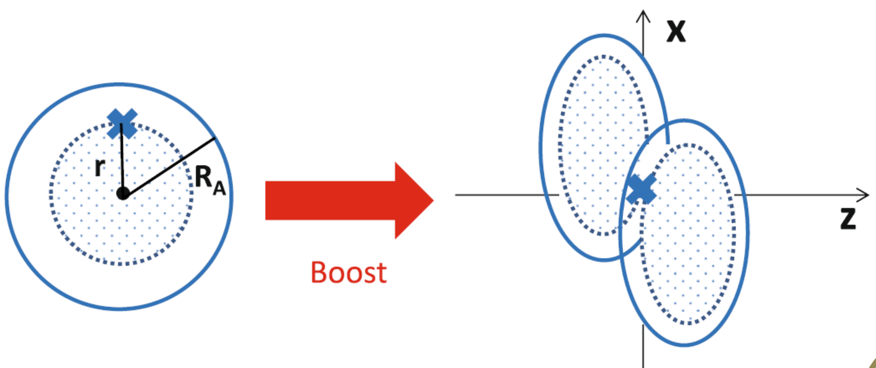
$$\mathrm{QED} \otimes \mathrm{QCD}$$

Analytic and numerical estimates of the strong B
-- Impact parameter dependences

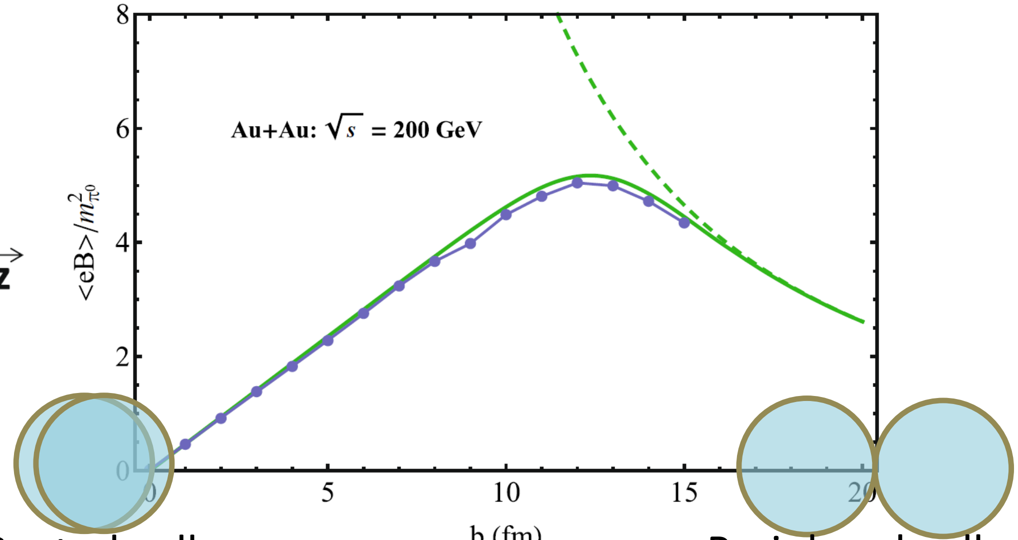
0. Free streaming of protons
1. Nuclear stopping power
2. Medium response



(Gluons do not carry electric charges.)



Static Coulomb field
→ Lienard-Wiechert potential



Central coll.

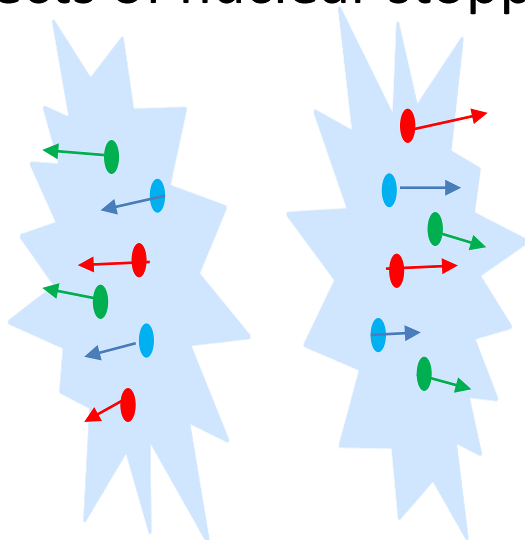
Peripheral coll.

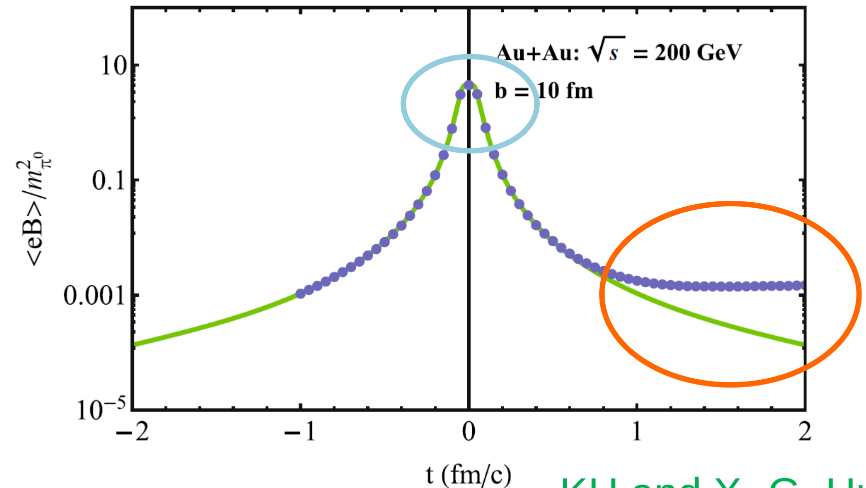
t = 0 (at the collision)

Similar for Pb+Pb $\sqrt{s} = 2.67$ TeV (LHC)

W.-T. Deng & X.-G. Huang,   KH & X.-G. Huang

1. Effects of nuclear stopping power in time dependences

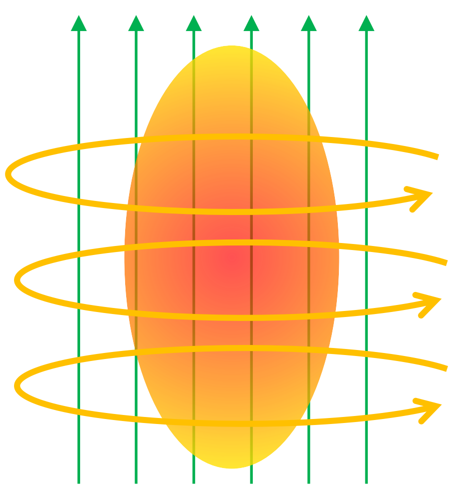




KH and X.-G. Huang

2. Possible medium effects in time dependences

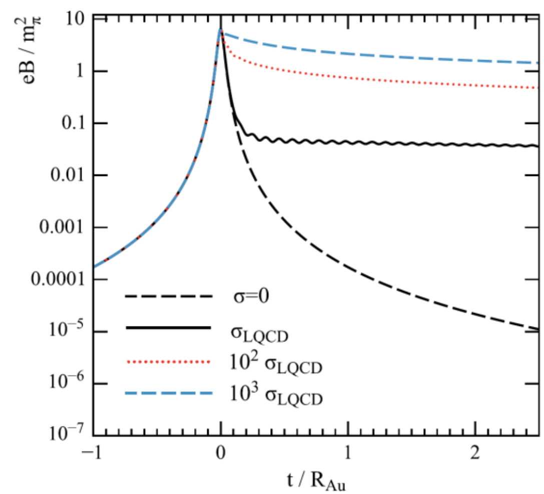
B(t)



A longer lifetime due to the Lenz's law? Tuchin

$$\partial_t B(t) < 0$$

1. Time dependent B induces E.
2. E induces J if QGP is conducting.
3. Induced J = σ E sustains B.



McLerran & Skokov

Important to know the conductivity σ of QGP in magnetic fields.

KH & Satow; KH, Li, Satow, Yee; Fukushima & Hidaka

Matching between # of variables and eqs.

# of variables

$$\epsilon\ (1),\ u^{\mu}\ \ (3),\ B^{\mu}\ \ (3)$$

$$(u^2 = 1,\ \ B^{\mu} u_{\mu} = 0)$$

# of eqs.

$$\partial_{\mu} T^{\mu\nu} = 0\ (4),\ \ \partial_{\mu} \tilde{F}^{\mu\nu} = 0\ (3)$$

$$(\partial_{\mu} \partial_{\nu} \tilde{F}^{\mu\nu} = 0)$$

$\partial_{\mu} \tilde{F}^{\mu 0} = 0$ is a non-dynamic constraint ($\nabla \cdot \boldsymbol{B} = 0$).

6 dynamic equations for 6 variables (+ EoSs).

“Phenomenological derivation” *a la* Landau and Lifshitz

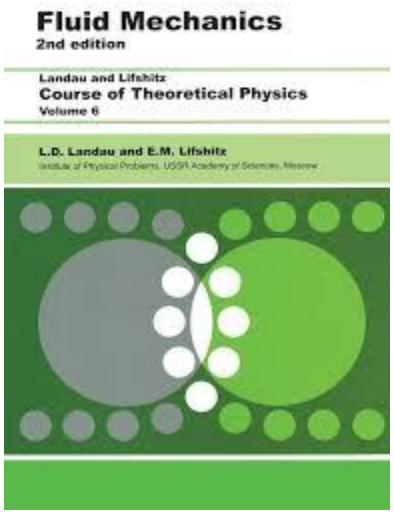
0. Perform the derivative expansion for the long time and wave-length limit.

1. Write down all possible tensor structures which are constructed from hydro-variables and derivatives and meet the symmetries of the system.

$$\text{e.g., } u^\mu u^\nu,\ \partial^{(\mu} u^{\nu)} \text{ for } T^{\mu\nu}.$$

--- However, those terms do not necessarily satisfy the second law of thermodynamics:

$$\partial_\mu s^\mu \geq 0$$



2. Constrain the tensor structures so that they never violate the second law.

Approaching an equilibrium state in E and B

Consider the time evolution of charge fluctuation δn(t, x) in a neutral plasma.

$$\frac{\partial \delta n(t,\boldsymbol{x})}{\partial t} = -\nabla \cdot \boldsymbol{j}(t,\boldsymbol{x}) = -\frac{\sigma}{\epsilon}\delta n(t,\boldsymbol{x})$$

- $\boldsymbol{j} = \sigma \boldsymbol{E}$
- $\nabla \cdot \boldsymbol{E} = \delta n/\epsilon$

$$\delta n(t,\boldsymbol{x}) = \delta n(0,\boldsymbol{x}) \exp(-\frac{\sigma}{\epsilon}t)$$

Inhomogeneity decays within $t \sim \epsilon/\sigma$.

E-field is screened by a static charge redistribution.

$$E \to 0$$

A static charge distribution does not screen the B-field.
(No "magnetic Coulomb field" without a magnetic monopole).

$$B \not\to 0$$

Auxiliary background fields: $g_{\mu\nu}(e_\mu^{\ a})$ and $b_{\mu\nu}$

$$\begin{aligned}\mathcal{L}_{\mathrm{QED}}[e_a^{\ \mu}, b_{\mu\nu}] &= -\frac{1}{2}\bar{\psi}(\gamma^a e_a^{\ \mu} D_\mu - \gamma^a e_a^{\ \mu} D_\mu - m)\psi \\ &\quad -\frac{1}{4} g^{\mu\nu} g^{\alpha\beta} F_{\mu\nu} F_{\alpha\beta} + \frac{1}{2} b_{\mu\nu} \tilde{F}^{\mu\nu})\end{aligned}$$

Fermions in a curved spacetime
(D contains a spin connection)

Magnetic flux coupled to
an external 2-form field

$$\nabla_\mu \langle T^{\mu\nu} \rangle = \frac{2}{e} \frac{\delta S_{\mathrm{QED}}}{\delta g_{\mu\nu}} = \frac{1}{2} \langle \tilde{F}^{\alpha\beta} \rangle H^\nu{}_{\alpha\beta}$$

$$\nabla_\mu \langle \tilde{F}^{\mu\nu} \rangle = \frac{2}{e} \frac{\delta S_{\mathrm{QED}}}{b_{\mu\nu}} = 0$$

Dissipative terms in the Landau frame: Entropy production $T_{(1)}^{\mu\nu} w_{\mu\nu} \geq 0$

Decompositions $w^{\mu\nu} \equiv \nabla^{(\mu} u^{\nu)}/2$

$$T_{(1)\mathrm{dis}}^{\mu\nu} = \delta p_{\|}(-b^{\mu}b^{\nu}) + \delta p_{\perp}\Xi^{\mu\nu} + f^{(\mu}b^{\nu)} + \tau^{\mu\nu}$$

$$w^{\mu\nu} = -\theta_{\|} b^{\mu}b^{\nu} + \{\theta_{\perp}\Xi^{\mu\nu} + (\Xi^{\alpha(\mu}\Xi^{\nu)\beta} - \Xi^{\mu\nu}\Xi^{\alpha\beta})w_{\alpha\beta}\}/2 - b^{\alpha}b^{(\mu}\Xi^{\nu)\beta}w_{\alpha\beta}$$

Each tensor has good orthogonal properties when both two indices are contracted with three tensors out of four.

$\theta_{\|} \equiv (-b^{\mu}b^{\nu})w_{\mu\nu}$ and $\theta_{\perp} \equiv \Xi^{\mu\nu}w_{\mu\nu}$

$\{\delta P_{\|}, \delta P_{\perp}, f^{\mu}, \tau^{\mu\nu}\} \sim \mathcal{O}(\partial^1)$

$f, \tau \perp b, u, \quad \tau^{\mu}_{\mu} = 0$

$\Downarrow$

$$T_{(1)\mathrm{dis}}^{\mu\nu} w_{\mu\nu} = \delta p_{\|}\theta_{\|} + \delta p_{\perp}\theta_{\perp} + f^{\mu}(b^{\alpha}\Xi^{\beta}_{\mu}w_{\alpha\beta}) + \frac{1}{2}\tau^{\mu\nu}(\Xi^{\alpha}_{(\mu}\Xi^{\beta}_{\nu)} - \Xi_{\mu\nu}\Xi^{\alpha\beta})w_{\alpha\beta}$$

Positivity and inequalities

$$T^{\mu\nu}_{(1)\mathrm{dis}}\nabla_{(\mu}u_{\nu)} = \begin{pmatrix}\theta_{\|} & \theta_{\perp}\end{pmatrix}\begin{pmatrix}\zeta_{\|} & \zeta_{\times}\\ \zeta_{\times} & \zeta_{\perp}\end{pmatrix}\begin{pmatrix}\theta_{\|}\\ \theta_{\perp}\end{pmatrix} + \frac{(-f^{\mu}f_{\mu})}{\eta_{\|}} + \frac{\tau^{\mu\nu}\tau_{\mu\nu}}{\eta_{\perp}}$$

Should have $\zeta_{\|,\perp} \geq 0$ when $\theta_{\perp,\|} = 0$ as necessary conditions.

Positivity of the general form of matrix

$$(\zeta_{\|} + \zeta_{\perp}) \pm \sqrt{(\zeta_{\|} - \zeta_{\perp})^2 + 4\zeta_{\times}^2} \geq 0 \ \&\ \zeta_{\|,\perp} \geq 0$$

$$\Rightarrow \quad \zeta_{\|}\zeta_{\perp} - \zeta_{\times}^2 \geq 0$$

$$\eta_{\|} \geq 0\,, \quad \eta_{\perp} \geq 0\,, \quad \zeta_{\|} \geq 0\,, \quad \zeta_{\perp} \geq 0\,, \quad \zeta_{\|}\zeta_{\perp} - \zeta_{\times}^2 \geq 0\,.$$

$\zeta_{\perp} \geq 0$ if $\zeta_{\|} \geq 0$ and $\zeta_{\|}\zeta_{\perp} - \zeta_{\times}^2 \geq 0$,
and so is $\zeta_{\|}$ when $\zeta_{\perp}$.

Let us Start from Huang, Sedrakian, & Rischke,
relativistic extension of Landau & Lifshitz (Physical kinetics)

$$
\begin{aligned}
T^{\mu\nu}_{(1)} \;=\;& -3\zeta_{\parallel}^{\rm HSR} b^\mu b^\nu \theta_\parallel + \frac{3}{2}\zeta_\perp^{\rm HSR}\Xi^{\mu\nu}\theta_\perp + 2\eta_0^{\rm HSR}\left(w^{\mu\nu} - \frac{1}{3}\Delta^{\mu\nu}\theta\right) - \eta_1^{\rm HSR}\left(b^\mu b^\nu + \frac{1}{2}\Xi^{\mu\nu}\right)\left(\theta_\parallel - \frac{1}{2}\theta_\perp\right) \\
& + 2\Big\{ -\eta_2^{\rm HSR} b^\alpha b^{(\mu}\Xi^{\nu)\beta} - \eta_3^{\rm HSR}\Xi^{\alpha(\mu}\bar{b}^{\nu)\beta} + \eta_4^{\rm HSR} b^\alpha b^{(\mu}\bar{b}^{\nu)\beta}\Big\} w_{\alpha\beta}
\end{aligned}
$$

$$
\boxed{w^{\mu\nu} = -\theta_\parallel b^\mu b^\nu + \{\theta_\perp \Xi^{\mu\nu} + (\Xi^{\alpha(\mu}\Xi^{\nu)\beta} - \Xi^{\mu\nu}\Xi^{\alpha\beta}) w_{\alpha\beta}\}/2 - b^\alpha b^{(\mu}\Xi^{\nu)\beta} w_{\alpha\beta}}
$$

$$
\begin{aligned}
T^{\mu\nu}_{(1)} \;=\;& (\zeta_\parallel \theta_\parallel - \zeta_\times \theta_\perp)(-b^\mu b^\nu) + (\zeta_\times \theta_\parallel - \zeta_\perp \theta_\perp)\Xi^{\mu\nu} \\
& + \Big\{ -\eta_\parallel b^\alpha b^{(\mu}\Xi^{\nu)\beta} + \frac{\eta_\perp}{2}(\Xi^{\alpha(\mu}\Xi^{\nu)\beta} - \Xi^{\mu\nu}\Xi^{\alpha\beta}) + \xi^{\parallel}_{\rm Hall} b^\alpha b^{(\mu}\bar{b}^{\nu)\beta} + \xi^{\perp}_{\rm Hall}\Xi^{\alpha(\mu}\bar{b}^{\nu)\beta}\Big\} w_{\alpha\beta}\,.
\end{aligned}
$$

where

$$
\begin{aligned}
&\zeta_\parallel = 3\zeta_\parallel^{\rm HSR} + \left(\frac{4}{3}\eta_0^{\rm HSR} + \eta_1^{\rm HSR}\right), \quad \zeta_\perp = \frac{3}{2}\zeta_\perp^{\rm HSR} + \frac{1}{4}\left(\frac{4}{3}\eta_0^{\rm HSR} + \eta_1^{\rm HSR}\right), \quad \zeta_\times = -\frac{1}{2}\left(\frac{4}{3}\eta_0^{\rm HSR} + \eta_1^{\rm HSR}\right), \\
&\eta_\perp = 2\eta_0^{\rm HSR}, \quad \eta_\parallel = 2(\eta_0^{\rm HSR} + \eta_2^{\rm HSR}), \quad \xi^{\parallel}_{\rm Hall} = 2\eta_4^{\rm HSR}, \quad \xi^{\perp}_{\rm Hall} = -2\eta_3^{\rm HSR}. \qquad (598)
\end{aligned}
$$

Inequalities found by Hernandez & Kovtun

① $\eta_{0,\mathrm{HSR}} \geqslant 0\,,$ ② $\eta_{0,\mathrm{HSR}} + \eta_{2,\mathrm{HSR}} \geqslant 0\,,$ ③ $\frac{1}{3}\eta_{0,\mathrm{HSR}} + \frac{1}{4}\eta_{1,\mathrm{HSR}} + \frac{3}{2}\zeta_{\perp,\mathrm{HSR}} \geqslant 0\,,$

④ $3\eta_{0,\mathrm{HSR}} + \frac{9}{4}\eta_{1,\mathrm{HSR}} + \frac{3}{2}\zeta_{\perp,\mathrm{HSR}} + 3\zeta_{\parallel,\mathrm{HSR}} \geqslant 0\,,$

⑤ $18\zeta_{\parallel,\mathrm{HSR}}\zeta_{\perp,\mathrm{HSR}} + 4\zeta_{\parallel,\mathrm{HSR}}\eta_{0,\mathrm{HSR}} + 3\zeta_{\parallel,\mathrm{HSR}}\eta_{1,\mathrm{HSR}} + 8\zeta_{\perp,\mathrm{HSR}}\eta_{0,\mathrm{HSR}} + 6\zeta_{\perp,\mathrm{HSR}}\eta_{1,\mathrm{HSR}} \geqslant 0$

They look complicated, but are so simplified in our basis.

① $\eta_\perp \geq 0\,,$ ② $\eta_\parallel \geq 0\,,$ ③ $\zeta_\perp \geq 0\,,$ ④ $\cancel{\zeta_\parallel + \zeta_\perp + 2\zeta_\times \geq 0}\,,$ ⑤ $\zeta_\parallel\zeta_\perp - \zeta_\times^2 \geq 0$

Redundant (Automatically satisfied when 3 and 5 are satisfied.)

① $\eta_\perp \geq 0\,,$ ② $\eta_\| \geq 0\,,$ ③ $\zeta_\perp \geq 0\,,$ ④ $\zeta_\| + \zeta_\perp + 2\zeta_\times \geq 0\,,$ ⑤ $\zeta_\|\zeta_\perp - \zeta_\times^2 \geq 0$

$$3 + 5 \quad \Rightarrow \quad \text{③'}\ \zeta_\| \geq 0$$

Inequality ④ is redundant.

$$(\zeta_\| + \zeta_\perp)^2 - (2\zeta_\times)^2 = (\zeta_\| - \zeta_\perp)^2 + 4(\zeta_\|\zeta_\perp - \zeta_\times^2) \geq 0$$

③ ③'

$$\&\ \zeta_{\|,\perp} \geq 0 \quad \Rightarrow \quad \zeta_\| + \zeta_\perp + 2\zeta_\times \geq 0$$

$$\boxed{\eta_\perp \geq 0\,, \quad \eta_\| \geq 0\,, \quad \zeta_\perp \geq 0\,, \quad \zeta_\|\zeta_\perp - \zeta_\times^2 \geq 0\,.}$$

Grozdanov et al., KH