Equal-time Spin Tranport
&
Distribution from Detailed Balance

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2020.10.13 @ ECT* online

Equal-time Spin Tranport

Wang, Guo, Shi, Zhuang, Phys.Rev.D 100 (2019) 1, 014015

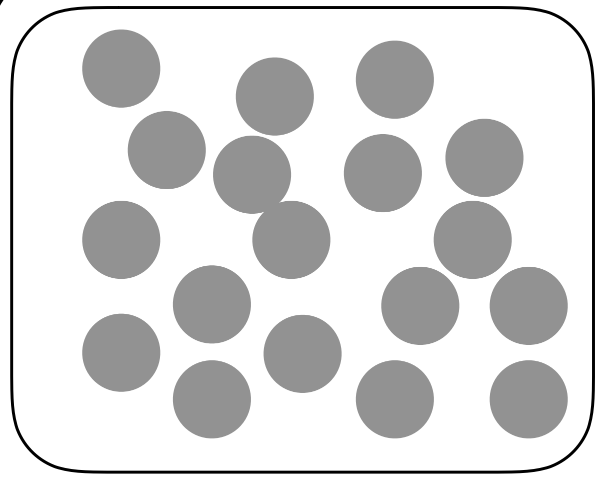
- Wigner function & DHW equation
- Equal-time kinetic theory -- 4 steps
- Spin transport & CKT

Distribution from Detailed Balance

Wang, Guo, 2009.10930 [hep-th]

- Collision term
- Framework
- Detailed balance

Wigner Function



Classical Transport

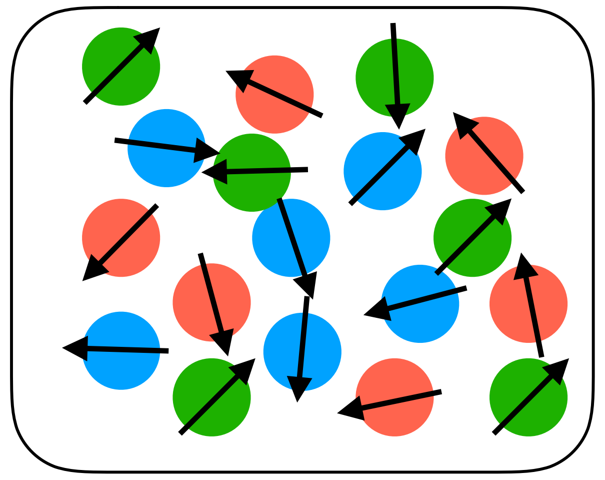
Boltzmann transport equation for particle number f

$$(p^\mu \partial_\mu + F^\mu \partial^p_\mu) f = C[f]$$

Quasi-particle on-shell constraint

$$\delta(p^2 - M^2) f(x, \vec{p}) = 0$$

Quantum Transport



Spin is a quantum effect, and is related to new phenomenon in condensed matter, cold atom, nuclear and particle physics

$$f(x, \vec{p}), \;\; \vec{g}(x, \vec{p})$$

Interaction & medium — generally off-shell

$$\delta(p^2 - m^2) + \hbar \mathcal{A}(p)$$

$$f(x, \vec{p}) \to W(x, \vec{p})$$

Wigner Function

- $\psi(x)\psi^\dagger(x)$ probability distribution in quantum mechanics
  - Wigner operator $\widehat{W}(x,p) = \int d^4y e^{ipy}\hat{\bar{\psi}}(x+y/2)e^{iq\int_{-1/2}^{1/2} ds \hat{A}(x+sy)y}\hat{\psi}(x-y/2)$

    Gauge link to guarantee the gauge invariance
  - Wigner function $W(x,p) = \langle \widehat{W}(x,p) \rangle$

- Kinetic equation for Wigner function

  Dyson-Schwinger equation for quantum field systems

  Dirac & Maxwell equation for quantum mechanics systems

  QED: D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)

  QCD: H.-Th.Elze and U.Heinz, Phys. Rep. 183, 81(1989)

- Problem: Initial $W(x,p)$ is related to the fields $\hat{\psi}(x)$ and $\hat{A}(x)$ at all times.

  Quantum kinetic equations cannot be solved as an initial value problem

Dirac-Heisenberg-Wigner equation

- Equal-time Wigner function $W_0(x,\vec{p}) = \int d^3\vec{y} e^{i\vec{p}\cdot\vec{y}} \left\langle \psi\left(t, \vec{x}+\frac{\vec{y}}{2}\right) e^{ie\int_{-1/2}^{1/2} ds A(x+sy)\cdot y} \psi^\dagger\left(t, \vec{x}-\frac{\vec{y}}{2}\right) \right\rangle$

- In external electromagnetic fields I.Bialynicki-Birula, P.Gornicki and J.Rafelski, PRD44, 1825(1991)

$$(i\gamma^\mu \mathcal{D}_\mu - m)\psi(x) = 0$$

Dirac-Heisenberg-Wigner equation

$$D_t W_0 = -\frac{1}{2}\vec{D}\cdot\left\{\rho_1\vec{\sigma}, W_0\right\} - \frac{i}{\hbar}\left[\rho_1\vec{\sigma}\cdot\vec{\Pi} + \rho_3 m, W_0\right]$$

e.g. Schwinger pair production

C. Best, J.M. Eisenberg Phys.Rev. D47 (1993) 4639-4646

XLSheng, RHFang, QWang, D.Rischke, Phys.Rev. D99 (2019) no.5, 056004

C. Kohlfürst, Phys.Rev. D101 (2020) no.9, 096003

- Equal-time $W_0(x,\vec{p}) = \int dp_0 W(x,p)\gamma^0$ is not equivalent to $W(x,p)$

$W(x,p)$ is equivalent to all energy moments $W_n(x,\vec{p}) = \int dp_0 p_0^n W(x,p)\gamma_0 \qquad (n = 0, 1, 2, \cdots)$

- Only when particles are on the mass shell, $W_n(x,\vec{p}) = E_p^n W_0(x,\vec{p})$

$W_0(x,\vec{p})$ is enough to describe to the system

Spin 1/2 Equal-time Kinetic Theory

4 steps to construct a quantum kinetic theory:

PZhuang and U.Heinz, PRD57, 6525 (1998)

- **Covariant kinetic equations for W(x,p) :** $(\gamma^\mu K_\mu - M)\,W = C$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2} D_\mu$$

$$D_\mu(x,p) \;=\; \partial_\mu - e\int_{-1/2}^{1/2} ds F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu$$

$$\Pi_\mu(x,p) \;=\; p_\mu - ie\hbar\int_{-1/2}^{1/2} dss F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu$$

$$M = M_1 + iM_2$$

$$M_1 = m_0 - \cos\left(\frac{\hbar}{2}\partial_x^\mu\partial_\mu^p\right)(m(x) - m_0)$$

$$M_2 = \sin\left(\frac{\hbar}{2}\partial_x^\mu\partial_\mu^p\right)(m(x) - m_0)$$

Constraint (off-shell) and transport equations

- **Equal-time hierarchy for** $W_n(x,\vec{p})$

$\int dp_0 W(x,p)\gamma^0$ Transport equations for $W_0(x,\vec{p})$; Constraint equations for $W_1(x,\vec{p})$

$\int dp_0 p_0 W(x,p)\gamma^0$ Transport equations for $W_0(x,\vec{p})$; Constraint equations for $W_2(x,\vec{p})$

Spin 1/2 Equal-time Kinetic Theory

- **Truncation of the hierarchy**

  Spin 1/2 particles: W_0 and W_1 form a closed subgroup

  Spin 0 particles: W_0, W_1 and W_2 form a closed subgroup

- **Spin decomposition**

  $W^\dagger = \gamma^0 W \gamma^0 \neq W$ The Wigner function is not a physical quantity

  D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)

$$W_0(x,\vec{p}) = \frac{1}{4}\left[f_0 + \gamma_5 f_1 - i\gamma_0\gamma_5 f_2 + \gamma_0 f_3 + \gamma_5\gamma_0\vec{\gamma}\cdot\vec{g}_0 + \gamma_0\vec{\gamma}\cdot\vec{g}_1 - i\vec{\gamma}\cdot\vec{g}_2 - \gamma_5\vec{\gamma}\cdot\vec{g}_3\right]$$

f_0 : number density $\quad$ f_1 : helicity density $\quad$ f_2 : topologic charge density $\quad$ f_3 : mass density

$\vec{g}_1$: number current $\quad$ $\vec{g}_0$: spin density $\quad$ $\vec{g}_3$: magnetic moment

16 constraint (off-shell) equations + 16 transport equations

Equal-time Equations for spin 1/2 system

W_1

The first moment gives the constraint equations

$$
\begin{aligned}
\int dp_0 p_0 V_0 - \mathbf{\Pi}\cdot\mathbf{g}_1 + \Pi_0 f_0 &= m f_3 \\
\int dp_0 p_0 A_0 + \mathbf{\Pi}\cdot\mathbf{g}_0 - \Pi_0 f_1 &= 0 \\
\int dp_0 p_0 \mathbf{A} + \frac{1}{2}\hbar\mathbf{D}\times\mathbf{g}_1 + \mathbf{\Pi} f_1 - \Pi_0\mathbf{g}_0 &= -m\mathbf{g}_3 \\
\int dp_0 p_0 \mathbf{V} - \frac{1}{2}\hbar\mathbf{D}\times\mathbf{g}_0 + \mathbf{\Pi} f_0 - \Pi_0\mathbf{g}_1 &= 0 \\
\int dp_0 p_0 P + \frac{1}{2}\hbar\mathbf{D}\cdot\mathbf{g}_3 + \Pi_0 f_2 &= 0 \\
\int dp_0 p_0 F - \frac{1}{2}\hbar\mathbf{D}\cdot\mathbf{g}_2 + \Pi_0 f_3 &= m f_0 \\
\int dp_0 p_0 S^{0i}\mathbf{e}_i - \frac{1}{2}\hbar\mathbf{D} f_3 + \mathbf{\Pi}\times\mathbf{g}_3 - \Pi_0\mathbf{g}_2 &= 0 \\
\int dp_0 p_0 S_{jk}\epsilon^{jki}\mathbf{e}_i - \hbar\mathbf{D} f_2 + 2\mathbf{\Pi}\times\mathbf{g}_2 + 2\Pi_0\mathbf{g}_3 &= 2m\mathbf{g}_0
\end{aligned}
$$

W_0

The zeroth moment gives the transport equations

$$
\begin{aligned}
\hbar(D_t f_0 + \mathbf{D}\cdot\mathbf{g}_1) &= 0 \\
\hbar(D_t f_1 + \mathbf{D}\cdot\mathbf{g}_0) &= -2m f_2 \\
\hbar(D_t\mathbf{g}_0 + \mathbf{D} f_1) - 2\mathbf{\Pi}\times\mathbf{g}_1 &= 0 \\
\hbar(D_t\mathbf{g}_1 + \mathbf{D} f_0) - 2\mathbf{\Pi}\times\mathbf{g}_0 &= -2m\mathbf{g}_2 \\
\hbar D_t f_2 - 2\mathbf{\Pi}\cdot\mathbf{g}_3 &= 2m f_1 \\
\hbar D_t f_3 - 2\mathbf{\Pi}\cdot\mathbf{g}_2 &= 0 \\
\hbar(D_t\mathbf{g}_2 - \mathbf{D}\times\mathbf{g}_3) + 2\mathbf{\Pi} f_3 &= 2m\mathbf{g}_1 \\
\hbar(D_t\mathbf{g}_3 + \mathbf{D}\times\mathbf{g}_2) + 2\mathbf{\Pi} f_2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
f_i &= f_i^{(0)} + \hbar f_i^{(1)} + \cdots \\
\mathbf{g}_i &= \mathbf{g}_i^{(0)} + \hbar\mathbf{g}_i^{(1)} + \cdots
\end{aligned}
$$

Semi-classical expansion

Equal-time Constraint Equations

Massless — one independent component

Massive — more degrees of freedom

- two independent components

  f_0 : number density $\mathbf{g}_0$: spin density

- other components depends on $f_0^{(1)}$ & $\mathbf{g}_0^{(1)}$, $f_0^{(0)}$ & $\mathbf{g}_0^{(0)}$

  off-shell effects related to EM field

$$\begin{aligned}
\int dp_0 p_0 V_0 - \mathbf{\Pi}\cdot\mathbf{g}_1 + \Pi_0 f_0 &= m f_3 \\
\int dp_0 p_0 A_0 + \mathbf{\Pi}\cdot\mathbf{g}_0 - \Pi_0 f_1 &= 0 \\
\int dp_0 p_0 \mathbf{A} + \frac{1}{2}\hbar\mathbf{D}\times\mathbf{g}_1 + \mathbf{\Pi} f_1 - \Pi_0 \mathbf{g}_0 &= -m\mathbf{g}_3 \\
\int dp_0 p_0 \mathbf{V} - \frac{1}{2}\hbar\mathbf{D}\times\mathbf{g}_0 + \mathbf{\Pi} f_0 - \Pi_0 \mathbf{g}_1 &= 0 \\
\int dp_0 p_0 P + \frac{1}{2}\hbar\mathbf{D}\cdot\mathbf{g}_3 + \Pi_0 f_2 &= 0 \\
\int dp_0 p_0 F - \frac{1}{2}\hbar\mathbf{D}\cdot\mathbf{g}_2 + \Pi_0 f_3 &= m f_0 \\
\int dp_0 p_0 S^{0i}\mathbf{e}_i - \frac{1}{2}\hbar\mathbf{D} f_3 + \mathbf{\Pi}\times\mathbf{g}_3 - \Pi_0\mathbf{g}_2 &= 0 \\
\int dp_0 p_0 S_{jk}\epsilon^{jki}\mathbf{e}_i - \hbar\mathbf{D} f_2 + 2\mathbf{\Pi}\times\mathbf{g}_2 + 2\Pi_0\mathbf{g}_3 &= 2m\mathbf{g}_0
\end{aligned}$$

$$f_1^{(0)\pm} = \quad f_2^{(0)\pm} = \quad f_3^{(0)\pm} = \quad \mathbf{g}_1^{(0)\pm} = \quad \mathbf{g}_2^{(0)\pm} = \quad \mathbf{g}_3^{(0)\pm} = \quad X\left[f_0^{(0)}, \mathbf{g}_0^{(0)}\right]$$

$$f_1^{(1)\pm} = \quad f_2^{(1)\pm} = \quad f_3^{(1)\pm} = \quad \mathbf{g}_1^{(1)\pm} = \quad \mathbf{g}_2^{(1)\pm} = \quad \mathbf{g}_3^{(1)\pm} = \quad X\left[f_0^{(1)}, \mathbf{g}_0^{(1)}\right] + Y\left[\mathbf{E}, \mathbf{B}, f_0^{(0)}, \mathbf{g}_0^{(0)}\right]$$

Equal-time Transport Equations

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}^{(0)}\right) f_0^{(0)\pm} = 0$$

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}^{(0)}\right) \mathbf{g}_0^{(0)\pm} = I\left[\mathbf{E}, \mathbf{B}, \mathbf{g}_0^{(0)\pm}\right]$$

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}^{(0)}\right) f_0^{(1)\pm} = 0 + J\left[\mathbf{E}, \mathbf{B}, \mathbf{g}_0^{(0)\pm}\right]$$

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}^{(0)}\right) \mathbf{g}_0^{(1)\pm} = I\left[\mathbf{E}, \mathbf{B}, \mathbf{g}_0^{(1)\pm}\right] + K\left[\mathbf{E}, \mathbf{B}, f_0^{(0)\pm}\right]$$

- **Classical: decoupled** The transport equation of f0 has no collision term — no self-interaction

  The transport equation of g0 has effective collision terms from spin interaction with EM field
- **Quantum level: coupled** Effective collision terms from spin interaction with EM field

  Number density modified by spin interaction with EM
- **Valid for all constant quark mass**
- **Can be solved order by order**

Transport Equation of Chiral Component

- **m=0: Axial and Vector still coupled!** Decouple by introducing chiral component!

$$J^\mu_\chi = \frac{1}{2}(V^\mu - \chi A^\mu)$$

$$f_\chi = f_0 + \chi f_1$$
$$\vec{g}_\chi = \vec{g}_1 + \chi\vec{g}_0 = G[f_\chi]$$

- **m: All coupled!** Yet still introduce the same combination.

$$\vec{g}_\chi = \vec{g}_1 + \chi\vec{g}_0 = F[f_\chi, m\vec{g}_3]$$

- **Small m limit:** Taylor expansion, keep to the first order of m

$$\frac{m}{p} << 1$$

$$\partial_t f^\pm_\chi + \dot{\mathbf{x}}\cdot\nabla f^\pm_\chi + \dot{\mathbf{p}}\cdot\nabla_p f^\pm_\chi = \chi m\frac{F_1[\mathbf{E},\mathbf{g}^\pm_3]}{\sqrt{G}} + \hbar m\frac{F_2[\mathbf{E},\mathbf{B},\mathbf{g}^{(0)\pm}_3]}{\sqrt{G}}$$

$$\dot{\vec{x}} = \frac{1}{\sqrt{G}}\left(\vec{v}_p + \hbar(\vec{v}_p\cdot\vec{b})\vec{B} + \hbar\vec{E}\times\vec{b}\right)$$
$$\dot{\vec{p}} = \frac{1}{\sqrt{G}}\left(\vec{v}_p\times\vec{B} + \vec{E} + \hbar(\vec{E}\cdot\vec{B})\vec{b}\right)$$

$$\vec{b} = \chi\frac{\hat{p}}{2p^2}$$

- Same structure
- effective collision (m, E, B, spin)
- E : mass effect @ classical + quantum
- B : mass effect @ quantum
- Can be solved perturbatively

Mass correction to CKT

$$\partial_t f_\chi^\pm + \dot{\vec{x}} \cdot \nabla f_\chi^\pm + \dot{\vec{p}} \cdot \nabla_p f_\chi^\pm = m\hbar \frac{1}{2\sqrt{G} p^4} (\vec{p} \cdot \nabla)(\vec{B} \cdot \vec{g}_3^{(0)\pm})$$

EoM

$$\dot{\vec{x}} = \frac{\hat{p}}{\sqrt{G}} \left(1 + 2Q\hbar \vec{b} \cdot \vec{B}\right)$$

$$\dot{\vec{p}} = \frac{1}{\sqrt{G}} Q\, \hat{p} \times \vec{B}$$

Berry curvature $\vec{b} = \chi \frac{\hat{p}}{2p^2}$

Phase space factor $\sqrt{G} = 1 + Q\hbar \vec{b} \cdot \vec{B}$

- **Small mass (high T):** same structure + effective collisions
- **Analytical solvable:** first solve collision, then transport
- **Mass correction is small — quantum level** $\frac{m\hbar}{\sqrt{G}} (\vec{p} \cdot \nabla) \beta^\pm(\vec{x}, \vec{p}, t)$

1st order, inhomogeneous

Can be solved perturbatively

Equaltime Spin Tranport

Wang, Guo, Shi, Zhuang, Phys.Rev.D 100 (2019) 1, 014015

- Wigner function & DHW equation
- Equal-time kinetic theory -- 4 steps
- Spin transport & CKT

Distribution from Detailed Balance

Wang, Guo, 2009.10930 [hep-th]

- Collision term
- Framework
- Detailed balance

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Collision term

- **Global polarization is robust, local polarization is not. Non equilibrium effect?**
- **Free streaming spin transport well developed, collision terms are under research.**

Mueller, Venugopalan, PRD 99 (2019), 056003
Hattori, Hidaka, Yang, PRD100 (2019), 096011
Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD100 (2019), 056018
Gao, Liang, PRD100 (2019), 056021
Wang, Guo, Shi, Zhuang, PRD100 (2019), 014015

Li, Yee, PRD100 (2019), 056022
Ayala, PLB.801 (2020) 135169
Kapusta, Rrapaj, Rudaz, PRC101 (2020), 024907
Kapusta, Rrapaj, Rudaz, ArXiv: 2004.14807
Zhang, Fang, QWang, XNWang, PRC100 (2019), 064904
Yang, Hattori, Hidaka, JHEP 2020 (2020) 070
Weickgenannt, Speranza, Sheng, Wang, Rischke, arXiv:2005.01506

Chen, Son, Stephanov, PRL115 (2015) 021601
Liu, Sun, Ko, ArXiv:1910.06774

- How does global OAM transfer to spin polarization?
- Does spin d.o.f reach local (global) equilibrium? How? When?
- Spin diffusion

- **Equilibrium spin distribution function.**

Derivation : Density matrix for spin-1/2 particles.
Analyzing free streaming part.
……

Becattini, Chandra, Del Zanna and Grossi, Annals Phys. 338 32–49 (1303.3431)
Fang, Pang, QWang and XNWang Phys. Rev. C94 024904 (1604.04036)
Gao, Pang and Wang Phys. Rev. D100 016008 (1810.02028)

Optimistic : Detailed balance
Maximum entropy

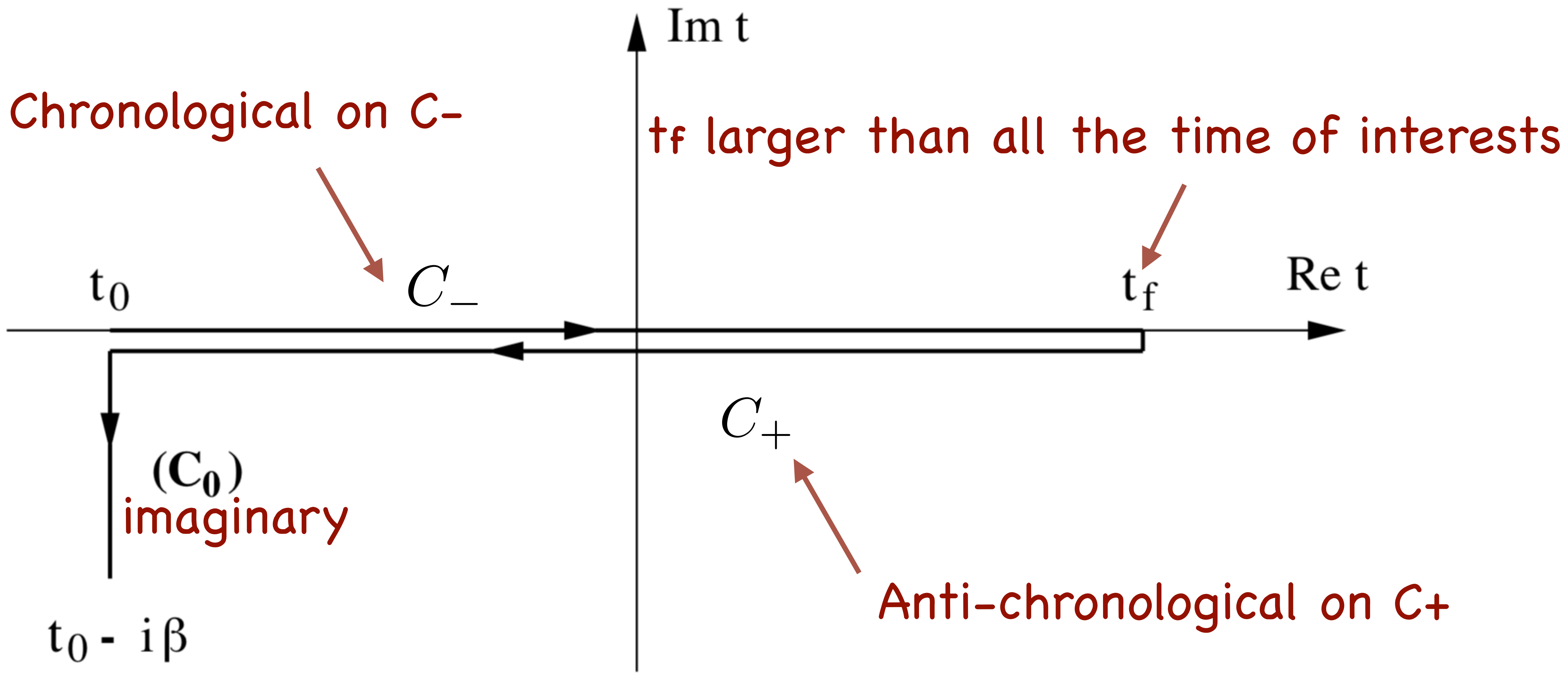
Massless Hidaka, Pu, Yang, Phys.Rev. D97 (2018) no.1, 016004 (1710.00278)

Framework: 1 Contour Green's Function

Complex time path surrounding the real-time axis:

— full power of field theoretical techniques in the calculation of non-equilibrium n-point functions

Schwinger, Keldysh, Kadanoff and Baym Blaizot, Iancu, Phys. Rept 359, 355-528 (2002)



— replace different ordering of operators by placing on different contour.

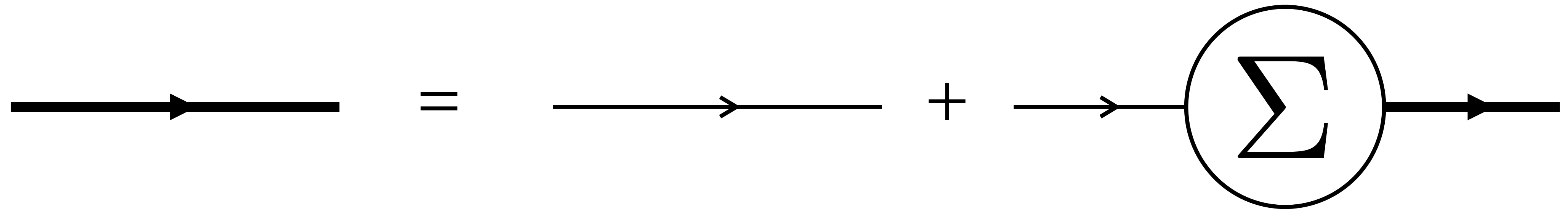
$$i\hbar S^c(x,y) = \langle T\psi(x)\bar{\psi}(y)\rangle$$

$$i\hbar S^a(x,y) = \langle \tilde{T}\psi(x)\bar{\psi}(y)\rangle$$

$$i\hbar S^>(x,y) = \langle \psi(x)\bar{\psi}(y)\rangle$$

$$i\hbar S^<(x,y) = -\langle \bar{\psi}(y)\psi(x)\rangle$$

- **Dyson Equation**



$$S(x,y) = S^0(x,y) + \int d^4z d^4w S^0(x,w)\Sigma(w,z)S(z,y)$$

Framework: 2 Kadanoff-Baym Equation

- **Contour DS Equation**

$$\Big(i\hbar\gamma^\mu\partial^x_\mu - m + \hbar\Sigma^\delta(x)\Big)S^<(x,y) = -\hbar\int_{-\infty}^{\infty} dz\Big(\Sigma_R(x,z)S^<(z,y) + \Sigma^<(x,z)S_A(z,y)\Big),$$
$$S^<(x,y)\Big(i\hbar\gamma^\mu\overleftarrow{\partial}^y_\mu + m - \hbar\Sigma^\delta(y)\Big) = +\hbar\int_{-\infty}^{\infty} dz\Big(S^<(x,z)\Sigma_A(z,y) + S_R(x,z)\Sigma^<(z,y)\Big),$$

- **Kadanoff-Baym Equation (by Wigner transformation)**

$$\Big(\gamma^\mu p_\mu - M(X)\Big)S^< + \frac{i\hbar}{2}\gamma^\mu\nabla_\mu S^< + \frac{i\hbar}{2}(\nabla_\mu M)(\partial^p_\mu S^<) = -\hbar\Sigma^<\hat{\Lambda}\mathrm{Re}S_R + \frac{i\hbar}{2}\Big(\Sigma^<\hat{\Lambda}S^> - \Sigma^>\hat{\Lambda}S^<\Big)$$
$$S^<\Big(\gamma^\mu p_\mu - M(X)\Big) - \frac{i\hbar}{2}S^<\gamma^\mu\overleftarrow{\nabla}_\mu - \frac{i\hbar}{2}(\partial^p_\mu S^<)(\nabla_\mu M) = -\hbar\mathrm{Re}S_R\hat{\Lambda}\Sigma^< - \frac{i\hbar}{2}\Big(S^>\hat{\Lambda}\Sigma^< - S^<\hat{\Lambda}\Sigma^>\Big)$$

Approximation: Chiral restored phase

Quasi-particle approximation

Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612

- **Constraint & Transport**

$$\Big\{(\gamma^\mu p_\mu - m), S^<\Big\} + \frac{i\hbar}{2}\Big[\gamma^\mu, \nabla_\mu S^<\Big] = \frac{i\hbar}{2}\Big([\Sigma^<, S^>]_\star - [\Sigma^>, S^<]_\star\Big),$$
$$\Big[(\gamma^\mu p_\mu - m), S^<\Big] + \frac{i\hbar}{2}\Big\{\gamma^\mu, \nabla_\mu S^<\Big\} = \frac{i\hbar}{2}\Big(\{\Sigma^<, S^>\}_\star - \{\Sigma^>, S^<\}_\star\Big),$$

$$[F,G]_\star \equiv F\star G - G\star F$$
$$\{F,G\}_\star \equiv F\star G + G\star F$$

$$A\star B = AB + \frac{i\hbar}{2}[AB]_{\mathrm{P.B.}} + \mathcal{O}(\hbar^2)$$
$$[AB]_{\mathrm{P.B.}} \equiv (\partial^\mu_q A)(\partial_\mu B) - (\partial_\mu A)(\partial^\mu_q B)$$

Framework: 3 Spin Decomposition

Constraint & Transport

$$\left\{(\gamma^\mu p_\mu - m), S^<\right\} + \frac{i\hbar}{2}\left[\gamma^\mu, \nabla_\mu S^<\right] = \frac{i\hbar}{2}\left(\left[\Sigma^<, S^>\right]_\star - \left[\Sigma^>, S^<\right]_\star\right),$$
$$\left[(\gamma^\mu p_\mu - m), S^<\right] + \frac{i\hbar}{2}\left\{\gamma^\mu, \nabla_\mu S^<\right\} = \frac{i\hbar}{2}\left(\{\Sigma^<, S^>\}_\star - \{\Sigma^>, S^<\}_\star\right),$$

$$[F, G]_\star \equiv F \star G - G \star F$$
$$\{F, G\}_\star \equiv F \star G + G \star F$$

$$A \star B = AB + \frac{i\hbar}{2}[AB]_{\text{P.B.}} + \mathcal{O}(\hbar^2)$$
$$[AB]_{\text{P.B.}} \equiv (\partial_q^\mu A)(\partial_\mu B) - (\partial_\mu A)(\partial_q^\mu B)$$

Spin Decomposition of both sides

Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612

Green's function

$$S^< = \mathcal{S} + i\mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\mathcal{S}_{\mu\nu}\sigma^{\mu\nu},$$
$$S^> = \bar{\mathcal{S}} + i\bar{\mathcal{P}}\gamma^5 + \bar{\mathcal{V}}_\mu\gamma^\mu + \bar{\mathcal{A}}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\bar{\mathcal{S}}_{\mu\nu}\sigma^{\mu\nu},$$

Self-energy

$$\Sigma^< = \Sigma_S + i\Sigma_P\gamma^5 + \Sigma_{V\mu}\gamma^\mu + \Sigma_{A\mu}\gamma^5\gamma^\mu + \frac{1}{2}\Sigma_{T\mu\nu}\sigma^{\mu\nu},$$
$$\Sigma^> = \bar{\Sigma}_S + i\bar{\Sigma}_P\gamma^5 + \bar{\Sigma}_{V\mu}\gamma^\mu + \bar{\Sigma}_{A\mu}\gamma^5\gamma^\mu + \frac{1}{2}\bar{\Sigma}_{T\mu\nu}\sigma^{\mu\nu}.$$

Collision term

$$C = \left[\Sigma^<, S^>\right]_\star - \left[\Sigma^>, S^<\right]_\star = C_S + i\gamma^5 C_P + \gamma^\mu C_{V_\mu} + \gamma^5\gamma^\mu C_{A_\mu} + \frac{1}{2}\sigma^{\mu\nu}C_{T\mu\nu},$$
$$D = \{\Sigma^<, S^>\}_\star - \{\Sigma^>, S^<\}_\star = D_S + i\gamma^5 D_P + \gamma^\mu D_{V_\mu} + \gamma^5\gamma^\mu D_{A_\mu} + \frac{1}{2}\sigma^{\mu\nu}D_{T\mu\nu}.$$

Framework: 4 Component functions

Ten component functions

$$p_\mu \mathcal{V}^\mu - m\mathcal{S} = \frac{i\hbar}{4} C_S,$$

$$2m\mathcal{P} + \hbar\nabla_\mu \mathcal{A}^\mu = -\frac{i\hbar}{2} C_P,$$

$$2p_\mu \mathcal{S} - 2m\mathcal{V}_\mu - \hbar\nabla^\nu \mathcal{S}_{\nu\mu} = \frac{i\hbar}{2} C_{V\mu},$$

$$\hbar\nabla_\mu \mathcal{P} - \epsilon_{\mu\nu\rho\sigma} p^\sigma \mathcal{S}^{\nu\rho} - 2m\mathcal{A}^\mu = \frac{i\hbar}{2} C_{A\mu},$$

$$\hbar\nabla_{[\mu} \mathcal{V}_{\nu]} - 2\epsilon_{\rho\sigma\mu\nu} p^\rho \mathcal{A}^\sigma - 2m\mathcal{S}_{\mu\nu} = \frac{i\hbar}{2} C_{T\mu\nu},$$

$$\nabla_\mu \mathcal{V}^\mu = \frac{1}{2} D_S,$$

$$2p_\mu \mathcal{A}^\mu = \frac{\hbar}{2} D_P,$$

$$2p^\nu \mathcal{S}_{\nu\mu} + \hbar\nabla_\mu \mathcal{S} = \frac{\hbar}{2} D_{V\mu},$$

$$2p_\mu \mathcal{P} + \frac{\hbar}{2}\epsilon_{\mu\nu\rho\sigma} \nabla^\sigma \mathcal{S}^{\nu\rho} = -\frac{\hbar}{2} D_{A\mu},$$

$$2p_{[\mu} \mathcal{V}_{\nu]} + \hbar\epsilon_{\mu\nu\rho\sigma} \nabla^\rho \mathcal{A}^\sigma = -\frac{\hbar}{2} D_{T\mu\nu}.$$

- Take V and A as basis — vector charge & axial charge current
- Semi-classical expansion
  - At each order of $\hbar$, $\mathcal{S}, \mathcal{P}, \mathcal{S}_{\mu\nu}$ can be expressed in terms of V and A (also involving self-energies).
  - V has one independent component

    $$p_{[\mu} \mathcal{V}^{(0)}_{\nu]} = 0 \qquad p_{[\mu} \mathcal{V}^{(1)}_{\nu]} = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma} \nabla^\rho \mathcal{A}^{(0)\sigma} - \frac{1}{4} D^{(0)}_{T\mu\nu}$$

  - A has three independent components

    $$p_\mu \mathcal{A}^{(0)\mu} = 0 \qquad p_\mu \mathcal{A}^{(1)\mu} = \frac{1}{4} D^{(0)}_P$$

- On-shell condition & transport equations of V&A at each order

Framework: 5 Transport of V & A

- **0th order transport**

$$p\cdot\nabla\mathcal{V}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{A}^{(0)\lambda}} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)}_\nu},$$

$$p\cdot\nabla\mathcal{A}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} + \widehat{\Sigma^{(0)}_{A\mu}p^\nu\mathcal{V}^{(0)}_\nu} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(0)\nu}},$$

$\widehat{FG} = \bar{F}G - F\bar{G}$

Local collision term

Dynamical effect ,

e.g. diffusion effect

- **1st order transport**

$$\begin{aligned}p\cdot\nabla\mathcal{V}^{(1)}_\mu =& + m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(1)}_\mu} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(1)}_\nu} - \frac{p_\nu}{m}\epsilon_{\rho\sigma\alpha\mu}p^\rho\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(1)\sigma}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(0)\sigma\nu}_T\mathcal{A}^{(1)\lambda}}\\
&+ m\widehat{\Sigma^{(1)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{V}^{(0)}_\mu} - p_\mu\widehat{\Sigma^{(1)\nu}_A\mathcal{A}^{(0)}_\nu} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(1)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(1)\sigma\nu}_T\mathcal{A}^{(0)\lambda}}\\
&+ \frac{1}{2m}p^\nu[\widehat{\Sigma^{(0)}_{T\mu\nu}(p^\alpha\mathcal{V}^{(0)}_\alpha)}]_{\text{P.B.}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{V}^{(0)\nu}}]_{\text{P.B.}} + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu[\widehat{\Sigma^{(0)\alpha}_A\mathcal{V}^{(0)\beta}}]_{\text{P.B.}}\\
&- \frac{1}{2m}p_\nu\Sigma^{(0)}_{T\alpha\mu}\widehat{\nabla^{[\alpha}\mathcal{V}^{(0)\nu]}} + \frac{1}{2m}p_\nu\Sigma^{\alpha\nu(0)}_T\widehat{\nabla_{[\alpha}\mathcal{V}^{(0)}_{\mu]}} + \frac{1}{2}\epsilon_{\beta\nu\lambda\mu}\Sigma^{(0)\beta}_A\widehat{\nabla^\nu\mathcal{V}^{(0)\lambda}}\\
&+ \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha\Sigma^{(0)\nu}_V)\mathcal{A}^{(0)\beta}} - \frac{1}{2m}p_\mu\widehat{(\nabla^\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\nu} - \frac{1}{2m}\widehat{(p^\nu\nabla_\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\mu} + \frac{p^\nu}{2m}\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha\Sigma^{(0)}_S)\mathcal{A}^{(0)\beta}},\end{aligned}$$

$$\begin{aligned}p\cdot\nabla\mathcal{A}^{(1)}_\mu =& + m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{A\mu}\mathcal{V}^{(1)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(1)\lambda}} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(1)\nu}}\\
&+ m\widehat{\Sigma^{(1)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{A\mu}\mathcal{V}^{(0)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(1)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(1)}_{A\nu}\mathcal{V}^{(0)\nu}}\\
&- \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\widehat{(\nabla^\sigma\Sigma^{(0)\nu}_V)\mathcal{V}^{(0)\rho}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_P\mathcal{V}^{(0)}_\mu}]_{\text{P.B.}} + \frac{1}{2m}p_\mu[\widehat{\Sigma^{(0)}_P(p^\nu\mathcal{V}^{(0)}_\nu)}]_{\text{P.B.}}\\
&+ \frac{1}{2}\epsilon_{\mu\sigma\nu\rho}\nabla^\sigma\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)\rho}} + \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}[\widehat{\Sigma^{(0)\nu}_A(p^\alpha\mathcal{A}^{(0)\beta})}]_{\text{P.B.}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)\nu}}]_{\text{P.B.}} + \frac{p_\mu}{2m}[\widehat{\Sigma^{(0)}_{T\rho\nu}(p^\rho\mathcal{A}^{(0)\nu})}]_{\text{P.B}}\\
&- \frac{1}{2m}p_\sigma\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)\nu}}) + \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)}_\sigma}) + \frac{1}{2m}p_\mu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)\nu}}) - \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)}_\mu}).\end{aligned}$$

- Nonlocal collision term
- Quantum effects
- Related to spatial derivatives
- Correlated transport of V&A
- Polarization can be generated in a initially unpolarized system

the interaction needs to be specified to calculate the off-diagonal self-energy $\Sigma^>$ & $\Sigma^<$

Fermionic 2 by 2 scattering

- ☆ derive the local equilibrium distribution from the detailed balance principle
- ☆ different interaction determines only how fast the system reaches equilibrium state, but not the equilibrium distribution function

NJL-type model with scalar-channel of interaction

$$\mathcal{L} = \bar{\psi}(i\hbar\partial\!\!\!/ - m)\psi + G(\bar{\psi}\psi)^2$$

(fermionic 2 by 2 scattering)

chiral restored phase — consider only the current mass

strong coupling nature — 1/Nc expansion & semiclassical expansion

$\mathcal{O}(1/N_c)$ $$\Sigma^{>}_{\mathrm{LO}}(X,p) = G^2 \int dP\ S^{>}(X,p_1)\mathrm{Tr}\Big[S^{<}(X,p_2)S^{>}(X,p_3)\Big],$$

$\mathcal{O}(1/N_c^2)$ $$\Sigma^{>}_{\mathrm{NL}}(X,p) = -G^2 \int dP\ S^{>}(X,p_1)S^{<}(X,p_2)S^{>}(X,p_3),$$

$$\int dP = \int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^4(2\pi)^4(2\pi)^4}(2\pi)^4\delta(p-p_1+p_2-p_3)$$

Simplification:

- ☆ the detailed balance — gain term and the loss term cancel with each other in arbitrary collision channel
- ☆ consider only self-energy at 1/Nc order

$$\mathrm{Tr}(S^{<}(X,p_2)S^{>}(X,p_3)) = \mathcal{S}^2\bar{\mathcal{S}}^3 - \mathcal{P}^2\bar{\mathcal{P}}^3 + \mathcal{V}^2_\mu\bar{\mathcal{V}}^{3\mu} - \mathcal{A}^2_\mu\bar{\mathcal{A}}^{3\mu} + \frac{1}{2}\mathcal{S}^2_{\mu\nu}\bar{\mathcal{S}}^{3\mu\nu}$$

Detailed Balance at $\mathcal{O}(\hbar^0)$

- **0th order transport**

$$p\cdot\nabla\mathcal{V}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{A}^{(0)\lambda}} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)}_\nu},$$

$$p\cdot\nabla\mathcal{A}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} + \widehat{\Sigma^{(0)}_{A\mu}p^\nu\mathcal{V}^{(0)}_\nu} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(0)\nu}},$$

  - Zeroth order self-energy $\Sigma^{(0)>}_{\text{LO}}(X,p)$
  - Zeroth order solution of V & A

$$\mathcal{V}^{(0)}_\mu(X,p) = \frac{2p_\mu}{(2\pi)^3 2E_{\mathbf{p}}}\Big\{\delta(p^0-E_{\mathbf{p}})f_{Vq}(X,\mathbf{p}) + \delta(p^0+E_{\mathbf{p}})\bar{f}_{V\bar{q}}(X,-\mathbf{p})\Big\},$$

$$\mathcal{A}^{(0)}_\mu(X,p) = \frac{2m}{(2\pi)^3 2E_{\mathbf{p}}}\Big\{\delta(p^0-E_{\mathbf{p}})n^+_\mu(X,\mathbf{p})f_{Aq}(X,\mathbf{p}) - \delta(p^0+E_{\mathbf{p}})n^-_\mu(X,-\mathbf{p})\bar{f}_{A\bar{q}}(X,-\mathbf{p})\Big\}.$$

- The transport equation of $\mathbf{f_V}$ & $\mathbf{f_A}$ contain quark-quark scattering, quark-antiquark scattering

f_{Vq} : incoming quark line
$f_{V\bar{q}}$: incoming antiquark line
$\bar{f}_{Vq}$: outgoing quark lines
$\bar{f}_{V\bar{q}}$: outgoing antiquark line

Off-shell processes involving particle and antiparticle creation and annihilation are not allowed by quasi-particle approximation & energy-momentum conservation

Consider only quark-quark scattering process

- Detailed balance — collision term vanishes — $f_{V\text{eq}}$ & $f_{A\text{eq}}$

Detailed Balance at $\mathcal{O}(\hbar^0)$

- **Detailed balance — collision term vanishes**

$$f^{(0)}_{V\mathrm{eq}}(X,p) = n_F(\beta\cdot p)$$ Comes from $f_V(X,\mathbf{p}_1)\bar{f}_V(X,\mathbf{p})\bar{f}_V(X,\mathbf{p}_2)f_V(X,\mathbf{p}_3) - f_V(X,\mathbf{p})\bar{f}_V(X,\mathbf{p}_1)f_V(X,\mathbf{p}_2)\bar{f}_V(X,\mathbf{p}_3)$

$$f^{(0)}_{A\mathrm{eq}}(X,p) = 0$$ A trivial solution — nontrivial solution may exist (depends on interaction?)

— agrees with the power counting

- **power counting**

Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612
Weickgenannt, Speranza, Sheng, Wang, Rischke, arXiv:2005.01506

  - Spin polarization is generated from coupling between spin and EM field or vorticity, which are quantum effect, $\mathcal{A}^{(0)}_\mu = 0,\ \mathcal{A}^{(1)}_\mu \neq 0$
  - Simplification
    - $\mathcal{A}^{(0)}_\mu = 0$ — power counting
    - $\mathcal{P}^{(0)}_\mu = 0$ — 0th equation
    - $\mathcal{S}^{(0)}_{\mu\nu} = 0$ — relation with A

1st order transport equations required

Detailed Balance at $\mathcal{O}(\hbar^1)$

- Simplified 1st transport, considering $\mathcal{A}_\mu^{(0)\mathrm{eq}} = 0$ (or the power counting)

$$p\cdot\nabla\mathcal{A}_\mu^{(1)} = m\widehat{\Sigma_S^{(0)}\mathcal{A}_\mu^{(1)}} + p_\nu\widehat{\Sigma_V^{(0)\nu}\mathcal{A}_\mu^{(1)}} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}(\nabla^\sigma\widehat{\Sigma_V^{(0)\nu})\mathcal{V}^{(0)\rho}} + p_\nu\widehat{\Sigma_{A\mu}^{(1)}\mathcal{V}^{(0)\nu}} - p_\mu\widehat{\Sigma_{A\nu}^{(1)}\mathcal{V}^{(0)\nu}} + \frac{m}{2}\epsilon_{\rho\nu\lambda\mu}\widehat{\Sigma_T^{(1)\rho\nu}\mathcal{V}^{(0)\lambda}}$$

  - Self-energy $\Sigma^{>(0)}(X,p)$, $\Sigma^{>(1)}(X,p)$
  - Zeroth order solution of V & A $f^{(0)}_{V\mathrm{eq}}(X,p) = n_F(\beta\cdot p)$, $f^{(0)}_{A\mathrm{eq}}(X,p) = 0$

- Detailed balance — vanishing collision term

$$\mathcal{A}_\mu^{\mathrm{LE}}(X,p) = \mathcal{A}_\mu^{\mathrm{LE}(0)} + \hbar\mathcal{A}_\mu^{\mathrm{LE}(1)} = -\frac{\hbar}{(2\pi)^3 2E_p}\epsilon_{\mu\nu\sigma\lambda}p^\nu\nabla^\sigma\beta^\lambda f'_{V,\mathrm{LE}}(X,p).$$

- Spin polarization generated from coupling between vector and axial-vector charge.
- The equilibrium spin polarization is created by a thermal vorticity and is orthogonal to the momentum.
- Gradient expansion of Wigner function — Non local effect — orbital angular momentum
- Agree with previous researches

F. Becattini, V. Chandra, L. Del Zanna and E.Grossi, Annals Phys. 338 32–49 (1303.3431)
RHFang, LGPang, QWang and XNWang Phys. Rev. C94 024904 (1604.04036)
JHGao, JYPang and QWang Phys. Rev. D100 016008 (1810.02028)

Equaltime Spin Tranport

Wang, Guo, Shi, Zhuang, Phys.Rev.D 100 (2019) 1, 014015

- Wigner function & DHW equation
- Equal-time kinetic theory -- 4 steps
- Spin transport & CKT

Distribution from Detailed Balance

Wang, Guo, 2009.10930 [hep-th]

- Collision term
- Framework
- Detailed balance

Thank you !

Q & A

2020.10.13 @ ECT* online